Dedication

I would like to dedicate my dissertation work to:

My Wife: She is my rock, for standing beside me throughout my life and writing this dissertation. She has been my inspiration and motivation for continuing to improve my knowledge and move my career forward.

MyChildren: You have made me stronger, better and more fulfilled than I could have ever imagined. I hope that one day they can read this research and understand why I spent so much time in front of my computer. I love you to the moon and back.

MyFamily: My family, including my in-laws, have always supported me throughout my career and I really appreciate it.

Acknowledgments

Firstly, I would like to express my sincere gratitude to my advisor **Prof. Mohamed Ali Bashir** for the continuous support of my Ph.D study and related research, for his patience, motivation and immense knowledge. His guidance helped me all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D study.

Abstract

Geometrical control theory has been handled in this research. In fact we consider this topic in light of symplectic geometry where a symplectic form has been studied with the property that it is invariant under the dynamics. In our study we consider geometrical interpretations and solutions of nonoholonomic dynamical systems. Controllability is investigated utilizing symplectic geometry in particular nonlinear behavior. Several examples are given illustrating controllability and optimal control.

الخـــلاصــة

في هذه الاطروحة تم التعامل مع نظرية التحكم الهندسي. في الواقع، تناولنا هذا الموضوع في ضوء الهندسة المتناظرة حيث تمت دراسة شكل التناظر مع خاصية الثبات في ظل الديناميكية. في دراستنا تناولنا بعض التفسيرات والحلول الهندسية للانظمة الديناميكية غير التحليلية. قابلية التحكم تم التحقق منها باستخدام الهندسة المتناظرة في سلوكيات غير خطية معينة. العديد من الامثلة تم طرحها توضح قابلية التحكم والتحكم الامثل.

Preface

Our goal in this thesis is to explore some of the connections between control theory and geometric mechanics; that is, we link control theory with a geometric view of classical mechanics in both its Lagrangian and Hamiltonian formulations and in particular with the theory of mechanical systems subject to motion constraints.

We have achieved our study via five chapters. These chapters as follows:

Chapter 1 consists of a little preliminary mechanics but mainly of examples that are used later in the thesis. Key examples include the vertical and falling rolling disks and various versions of a skate on ice as well as the rolling ball. More complicated examples include the roller racer and rattle back top. There are also mechanical examples that are holonomic but that are used later to illustrate basic Lagrangian and Hamiltonian mechanics and control.

Chapter 2 is devoted to various analytical mechanics including Newtonian mechanics, Lagrangian mechanics and the equation of Hamilton.

These ideas are also illustrated with very simple examples such as the geometrical relationship and Hamiltonian formalisms.

Chapter 3 gives general background in geometric mechanics by introducing the foundation of Symplectic manifolds and its basics structure. Various examples of symplectic manifolds are introduced in addition to some properties including symmetries and integrability.

Chapter 4 gives general background in nonlinear control theory including basic definitions of controllability and accessibility. We extended the discussion to various aspects of control and stabilization of nonholonomic systems both for kinematic and dynamic systems. Control of nonholonomic systems on Riemannian manifolds are discussed in details.

Chapter 5 is devoted to optimal control. It begins by discussing the relationship of variational nonholonomic control systems and the classical Lagrange problem to optimal control. We then discuss sub-Riemannian (kinematic) optimal control problems.

The Contents