Sudan University of Science and Technology
College of Graduate Studies

Faculty of Science

Neutrino Mass in Grand Unified Models

كتلة النيوترينو في نماذج التوحيد الكبير

A Dissertation Submitted to College of Graduate Studies for the Partial Fulfillment Requirement for the Degree of M. Sc. in Physics

By: Taj addeen Abdallah Mohamed Alhag

Supervisor: Dr. Ammar Ibrahim Abdalgabar

Khartoum, December 2018

الآيـة

(يا بُنـيَّ إنَّـهَا إِنْ تَكُ مِثْقَالَ حَبَّةٍ مِنْ خَرْدَلٍ فَتَكُنْ فِي صَخْرَةٍ أَوْ فِي السَّمَاوَاتِ أَوْ فِي الأَرْضِ يَأتِ بِهَا اللهُ إِنَّ اللهَ لَطِيْفٌ خَبِيْر).

[لقمان:16]

Acknowledgements

I would like to express my great appreciation to **Dr. Ammar Ibrahim Abdalgabar** for his encouragement, useful advice, valuable and interesting suggestions during the development of this dissertation.

I would like to thanks all academic staffs at department of physics, college of science for their efforts and kindness.

And I also wish to thank my beloved families for their support and patience.

I would also like to extend my thanks to my friends for their assistance.

I thank Allah who made all these possible.

Abstract

In spite of all the successes of the Standard Model, it is unlikely to be the final theory. It leaves many unanswered questions. The minimal SM predicted vanishing of the neutrino mass, but the recent observations of neutrino oscillations confirm that neutrinos have finite masses providing a likely window to new physics beyond the standard model. There we will discuss possible theories of neutrino mass, from changes of the minimal SM all the way to the $SU(5)$ and $SO(10)$ grand unified theory. We shall discuss at length the see-saw mechanism which leads to neutrino Majorana mass, its realization in left-right symmetric theory and in $\mathrm{SO}(10)$.

ملخص البحث

بالرغم من النجاحات التي حققتها نظرية النموذج العياري ولكن ليس هناك إحتمال أن تكون النظرية الآخيرة حيث تركت هذه النظرية العديد من الأسئلة من دون إجابات .

النموذج العياري المصغر توقع تلاشي كتلة النيوترينو ولكن المشاهدات الآخيرة لتذبذب النيوترينو أكدت أن للنيوترونات كتل صغيرة فتحت باب لفيزياء جديدة مابعد النموذج العياري .

في هذا البحث سوف نناقش النظريات المحتملة لكتلة النيوترينو من التغيرات في النموذج العياري وهي SU(5) ونظرية التوحيدالكبير وكذلك سنناقش بعض الآلية المتأرجحة see-saw mechanism التي تؤدي إلى كتلة نيوترينو مايورانا وهذا مايتحقق في نظرية التماثل وفي ال SO(10)

Table of contents

LIST OF THE FIGURES

Chapter One

INTRODUCTION

(1.1) Introduction

The theory of Standard Model (SM) was established for massless neutrinos and thus without including a mechanism for generating neutrino masses. In the SM neutrinos came purely with left-handed neutrinos as they have been introduced to solve the problem of a continuous spectrum in the beta decay in weak decay. As such the right-handed neutrinos are strictly speaking not part of the SM particle contents, no renormalisable neutrino mass term is possible. However, they are often considered to be a trivial extension, there was experimental evidence that at least two of those have a small but non-vanishing mass. Thus, a mechanism for generating neutrino masses has to be implemented in the SM and theories beyond. See-saw mechanism introduced three right-haded neutrinos into the standard model with very large Majorana masses and predicts that observed neutrinos are their own anti-particles. The Majorana masses of these right handed are expected to be either at TeV scale or at a higher scale. Together with the Dirac mass, this Majorana mass introduces a see-saw mechanism which is described as in follow. In addition to the standard see-saw mechanism (see-saw Type I), we consider the double see-saw and a see-saw with single right-handed neutrino dominance in order to generate a phenomenologically valid flavour structure for the SM

neutrinos. The see-saw mechanism provides an explanation for light masses of neutral fields. It was first introduced in the late 70's (Senjanovi´c, 1981) and is mainly discussed in the context of neutrinos. There are three commonly used realizations for SM neutrinos (Type I to III) leading to similar results. Here, we explain the Type I and only comment on Type II and III.
In the other hand, the Grand Unified Theories (GUTs) are one of the most appealing extensions of the SM where one can understand the origin of SM interactions. Here we will discuss the implementation of the different mechanisms for neutrino masses in the context of renormalizable SU(5) and SO (10) theories.
SU(5) and Neutrino Masses : The original model proposed by Georgi and Glashow (Georgi & Glashow, 1974) in 1974 has been considered as the simple grand unified theory. This model is based on SU(5), the SM matter fields live in the 5 and 10 representations, and the minimal Higgs sector is composed of H_5 and H_{24}.

(1.2)The Importance of the Study

Neutrino oscillation experiments have confirmed that neutrino changes flavor after propagating a finite distance. Thus neutrino has finite mass, but this mass is tiny ≈ 1 eV The smallness of neutrino mass provides a window to the physics beyond standard model (SM).

(1.3) Aim and main Objectives of the Study

In this dissertation, we will discuss possible theories of neutrino mass, from the minimal changes of the SM all the way to grand unified theory; specifically we will limit ourselves with Pati-Salam group. We shall implement at length the see-saw mechanism which leads to neutrino Majorana mass, its realization in left-right symmetric theory and GUTs.

(1.4)The Outline of the Dissertation

This Dissertation is orgnazied as follows: In chapter one we gave brief introduction to the subject, and we discussed the standard model of particle physics in details in chapter two , in Chapter three dealt with the implmentation of the three type of see saw mechanisms, SU(5) and SO(10) GUTs, finally we presented in chapter, four discussions and conclusions.

Chapter Two

INTRODUCTION TO THE STANDARD MODEL

(2.1) Introduction

This chapter deals with the structure of the standard model and its mathematical foundation, morever we will introduce the mechanism of spontaneous symmetry breaking (known also the higgs mechanism) to see how elementary particles obtian their masses.

(2.2) What is the SM?

The standard model is developed to describe the weak, strong and electromagnetic interactions in terms of "gauge theories". It was dated back to the latter half of the 20th century, as a collaborative effort of scientists around the world.(Georgi, H. & Glashow, S., 1974) .The current formulation was finalized in the mid-1970s upon experimental confirmation of the existence of quarks. Since then, discoveries of the top quark (1995), the tau neutrino (2000), and more recently the Higgs boson (2013) (al, 2012), have given further credence to the Standard Model. Because of its success in explaining many experimental results, the Standard Model is sometimes regarded as a "theory of almost everything". Mathematically, the standard model is a quantized Yang–Mills theory. In 1950's Yang and Mills considered (as purely mathematical exercise) extending gauge invariance to include non-abelian (i.e. non-commuting) transformations such as SU(2). In this case one needs a set of massless vector fields (three in the case of SU (2)), which were formally called "Yang-Mills" fields, but are now known as "gauge fields" (L.F.Li, 1991).

(2.3)The Standard Model Lagrangian

Quantum field theory provides the mathematical framework for the standard model in which a lagrangian controls the dynamics and kinematics of the theory. Each kind of particle is discribed in terms of dynamical field that pervades space-time. The construction of the standard model based on the modren method of construction of field theories by first postulating a set of symmetries of the system and then by writing down the most general renormlizabel lagrangian from it's particle (L.F.Li, 1991).

The standard model is a gauge theory representing the fundamental interactions as changes in a Lagrangian function of quantum fields. It contain spinless, spin-(1/2) and spin -1 fields interacting with one another in a way governed by the Lagrangian which is invariant by Lorentz transformations (Weinberg, 1996).

The Lagrangian of the standard model contains kinetic terms, coupling and interaction terms related to the gauge symmetries of the force carriers , mass terms and the Higgs mechanism term (Quigg, 2007).

(2.3.1)The Fermion Sector

The fermionic sector consist of quarks and leptons come in three families with identical properties except for mass. The particle content in each family is:

$$1^{\mathrm{st}}\ \text{family: lepton; } \mathrm{l} = \begin{pmatrix} \nu_{\mathrm{e}} \\ e \end{pmatrix}_{\mathrm{L}}, e_R^- \tag{2.1}$$

$$\text{Quark; } q = \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R \tag{2.2}$$

$$2^{nd}\text{family: lepton; l} = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \mu_R^- \quad (2.3)$$

$$\text{Quark;} q = \begin{pmatrix} s \\ c \end{pmatrix}_L , c_R , s_R \quad (2.4)$$

$$3^{rd}\text{ family: lepton; l} = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L , \tau_R^- \quad (2.5)$$

$$\text{Quark; } q = \begin{pmatrix} t \\ b \end{pmatrix}_L , t_R , b_R \quad (2.6)$$

(2-3-2) Gauge Boson Sector

The gauge boson and the scalar lagrangians give rise to the free lagrangian for the photon, W, Z. and the Higgs boson .The standard model gauge boson lagrangian (gauge fields) is given by

$$\mathcal{L}_G = -\frac{1}{4}\mathrm{B}_{\mu\nu}B^{\mu\nu} - \frac{1}{4}tr\left(W_{\mu\nu}W^{\mu\nu}\right) - \frac{1}{4}tr\left(G_{\mu\nu}G^{\mu\nu}\right) \quad (2.7)$$

$\mathbf{G_{\mu\nu}}$ is the gauge field strength of the strong SU(3) gauge field.

$\mathbf{W_{\mu\nu}}$is the gauge field strength of the weak isospin SU(2) gauge field.

$\boldsymbol{B_{\mu\nu}}$is the gauge field strength of the weak hypercharge U(1) gauge field.

These fields respectively are defined as

$$G_{\mu\upsilon}^a = \partial_\mu G_\upsilon^a - \partial_\upsilon G_\mu^a + g_s f_{abc} G_\mu^b G_\upsilon^c \quad (2.8)$$

$$W_{\mu\upsilon}^i = \partial_\mu W_\upsilon^i - \partial_\upsilon W_\mu^i + g\epsilon_{ijk} W_\mu^i W_\nu^i \quad (2.9)$$

$$B_{\mu\upsilon} = \partial_\mu B_\upsilon - \partial_\upsilon B_\mu \quad (2.10)$$

(2.4) Higgs Mechanism

The masses of elementary particle can not be included in the lagrangian because they will break the gauge symmetry. Therefor we need a mechanism that give masses to these particle. An extra field called the Higgs field has to be added by hand to give the particles masses (Quigg, 2007). The Higgs field has a spin-0 particle called Higgs boson. The higgs boson is electrically neutral. The extra field if it exists, is believed to fill all of empty space throughout the entire universe. Elementary particles acquire their mass through their interaction with the Higgs field. Mathematically we introduce mass into a theory by adding interaction terms into the Lagrangian that couple the field of the particle to the Higgs field. Basically, the lowest energy state of a field would have an expectation value of zero. By symmetry breaking we introduce a nonzero lowest energy state of the field. This procedure leads to the acquisition of mass by particles in the theory (Quigg, 2007).

We can imagine the movement of elementary particles being resisted by the Higgs field, with each particle interact with the Higgs field at a different stregth. If the coupling between the Higgs field and the particle is strong then the mass of the particle is large. if it is weak then the particle has a smaller mass. A particle like the photon with zero rest mass doesn't interact with the Higgs field at all only through a loop. If the Higgs field didn't exist at all then all particles would be massless. This scalar particle has been discovered by the ATLAS (al, 2012) and CMS (al, 2012) experiments, which is compatible with the SM Higgs expectations with a mass 126 GeV.

The addition of this new particle will add new terms into the lagrangian:

$$\mathcal{L}_{Higgs} = \frac{1}{2}\left(D_{\mu}\phi\right)\left(D_{\mu}\phi\right) - V(\phi) \tag{2.11}$$

Where the potential is given by

$$V(\phi) = \frac{\mu^2}{2}\phi^*\phi - \frac{\lambda}{4}\phi^4 \qquad (2.12)$$

Therefore equation (2.11) becomes:

$$\mathcal{L}_{Higgs} = \frac{1}{2}\left(D_\mu\phi\right)\left(D_\mu\phi\right) - \left(\frac{\mu^2}{2}\phi^*\phi + \frac{\lambda}{4}\phi^4\ \right) \qquad (2.13)$$

Where $\lambda \equiv$ Higgs self coupling.

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} \qquad (2.14)$$

Minimizing V(ϕ) that is by taking the first derivative with respect to ϕ:

$$\frac{\partial V}{\partial \phi} = 0 \qquad (2.15)$$

We get

$$\frac{\partial V}{\partial \phi} = -(\mu^2\phi + \lambda\phi^3\ ) \qquad (2.16)$$

This equation has two solutions

$$\phi(-\mu^2 + \lambda\phi^2) = 0 \qquad (2.17)$$

$$\phi = 0\ (trivial\ solution) \quad or \quad (-\mu^2 + \lambda\phi^2) = 0 \qquad (2.18)$$

Therefore

$$\langle\phi^2\rangle = \frac{\mu^2}{\lambda} \qquad (2.19)$$

$$\langle\phi\rangle = \sqrt{\frac{\mu^2}{\lambda}} = \nu \qquad (2.20)$$

Where ν is known as the vacuum expectation value (VEV), $\nu = 246\ GeV$.

$$\left(\phi_1{}^2 + \phi_2{}^2 + \phi_3{}^2 + \phi_4{}^2\right) = \nu^2 \qquad (2.21)$$

Equation (2.12) represents the Higgs potential, which involves two new real parameters μ and λ. We require that $\lambda > 0$ for the potential to be bounded; otherwise the potential is unbounded from below and there will be no stable vacuum state. μ takes the following two values:

1- $\mu^2 > 0$ in this case the vacuum corresponds to $\phi = 0$, the potential has a minimum at the origin (see Figure 2.1).

2- $\mu^2 < 0$ in this case the potential develops a non-zero Vacuum Expectation Value (VEV) and the minimum is along a circle of radius of 246 (see Figure 2.2).

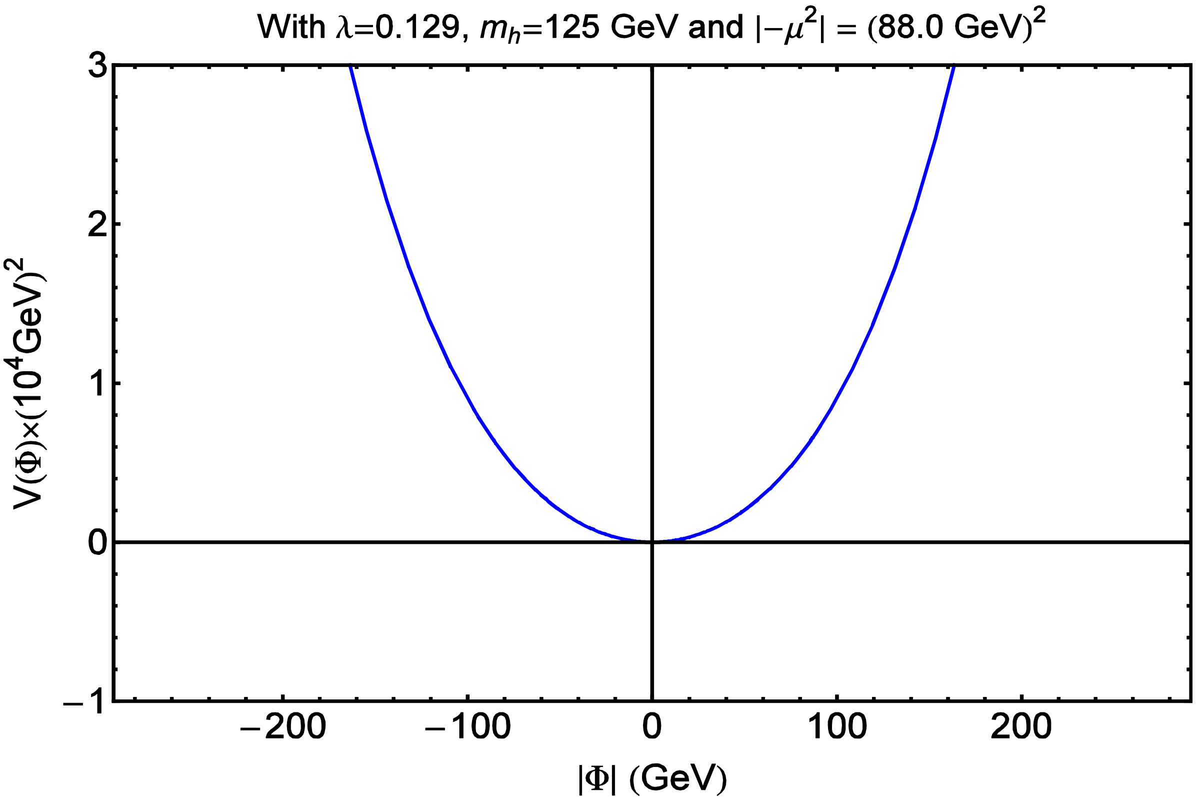

Figure2.1. The Higgs potential with: the case $\mu^2 > 0$; as function of $|\Phi|$.

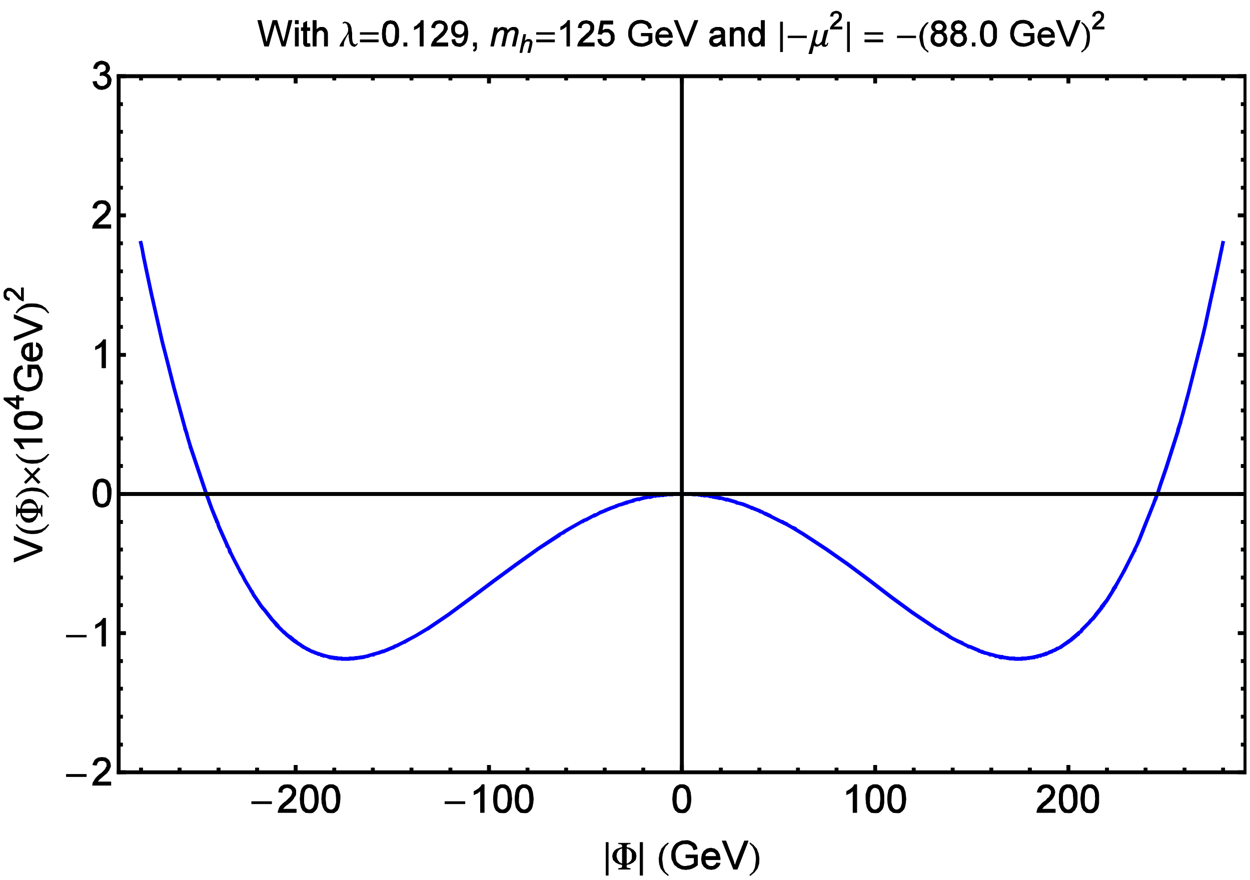

Figure2.2. The Higgs potential with: the case $\mu^2 < 0$; as function of $|\Phi|$.

$$\phi = \frac{1}{\sqrt{2}}\binom{0}{\nu} \tag{2.22}$$

Next we will see how to use this technique to give bosons and fermions a mass.

(2-4-1) Gauge Bosons Mass

To obtain the masses for the gauge bosons we will only need to study the scalar part of the lagrangian

$$\mathcal{L} = \frac{1}{2}\left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) - V(\phi) \tag{2.23}$$

Where $D_\mu$is the covariant derivative.

$$D_\mu = \left(\partial_\mu + ig\tau^a W_\mu^a + i\acute{g}\frac{Y_\phi}{2}B_\mu\right) \quad (2.24)$$

$$D_\mu = \left[\partial_\mu + ig\begin{pmatrix} W_{\mu 3} & W_\mu{}^- \\ W_\mu{}^+ & -W_{\mu 3}\end{pmatrix} + i\acute{g}\frac{Y_\phi}{2}B_\mu\right] \quad (2.25)$$

Then

$$D_\mu\phi = \left[\partial_\mu\phi + ig\begin{pmatrix} W_{\mu 3} & W_\mu{}^- \\ W_\mu{}^+ & -W_{\mu 3}\end{pmatrix}\phi + i\acute{g}\frac{Y_\phi}{2}B_\mu\phi\right] \quad (2.26)$$

But we have

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad (2.27)$$

Therefore, after carrying out some algebra we get

$$D_\mu\phi = \frac{ig}{\sqrt{2}}\begin{pmatrix} W_{\mu 3} & W_\mu{}^- \\ W_\mu{}^+ & -W_{\mu 3}\end{pmatrix}\begin{pmatrix} 0 \\ \nu \end{pmatrix} + \frac{i\acute{g}Y_\phi B_\mu}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad (2.28)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} igW_{\mu 3} & igW_\mu{}^- \\ igW_\mu{}^+ & -igW_{\mu 3}\end{pmatrix}\begin{pmatrix} 0 \\ \nu \end{pmatrix} + \frac{i\acute{g}}{\sqrt{2}}\begin{pmatrix} 0 \\ B_\mu\nu \end{pmatrix} \quad (2.29)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} igvW_\mu{}^- \\ -igvW_{\mu 3}\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i\acute{g}\mathrm{B}_\mu\nu \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} igvW_\mu{}^- \\ -igvW_{\mu 3} + i\acute{g}vB_\mu \end{pmatrix} \quad (2.30)$$

$$\Rightarrow D_\mu\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} igvW_\mu{}^- \\ -igvW_{\mu 3} + i\acute{g}vB_\mu \end{pmatrix} \quad (2.31)$$

Since $\left(D_\mu\phi\right)^\dagger$ is the complex conjugate of $D_\mu\phi$ then

$$\left(D_\mu\phi\right)^\dagger = \frac{1}{\sqrt{2}}\left(-igvW_\mu{}^+ \quad igvW_{\mu 3} - i\acute{g}v\mathrm{B}_\mu\right) \quad (2.32)$$

Therefore

$$\left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right)=\frac{1}{\sqrt{2}}\left(-igvW_\mu{}^{+}\quad igvW_{\mu3}-i\acute{g}v\mathrm{B}_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} igvW_\mu{}^{-}\\ -igvW_{\mu3}+i\acute{g}vB_\mu\end{pmatrix}$$

$$=\frac{1}{2}\left[g^2\nu^2W_\mu{}^{+}W_\mu{}^{-}+\nu^2(gW_{\mu3}-\acute{g}B_\mu)^2\right] \tag{2.33}$$

So

$$\frac{1}{2}\left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right)=\frac{1}{4}g^2\nu^2W_\mu{}^{+}W_\mu{}^{-}+\frac{1}{4}\nu^2\left(gW_{\mu3}-\acute{g}B_\mu\right)^2 \tag{2.34}$$

From the above equation we obtain

$$m_w^2=\frac{1}{4}g^2\nu^2$$

$$m_w=\frac{1}{2}\nu g$$

For Z boson, we use the orthogonal combination as

$$z_\mu=\frac{gW_{\mu3}-\acute{g}B_\mu}{\sqrt{g^2+\acute{g}^2}}=\left(\cos\theta_w\,W_{\mu3}-\sin\theta_w\,B_\mu\right) \tag{2.35}$$

And the photon:

$$A_\mu=\frac{1}{\sqrt{g^2+\acute{g}^2}}\left(\acute{g}W_{\mu3}+gB_\mu\right) \tag{2.36}$$

By using a rotation transformation

$$\begin{pmatrix} z_\mu\\ A_\mu\end{pmatrix}=\begin{pmatrix}\cos\theta_w & -\sin\theta_w\\ \sin\theta_w & \cos\theta_w\end{pmatrix}\begin{pmatrix} W_{\mu3}\\ B_\mu\end{pmatrix} \tag{2.37}$$

$$\cos\theta_w=\frac{g}{\sqrt{g^2+\acute{g}^2}}\quad and\quad \sin\theta_w=\frac{\acute{g}}{\sqrt{g^2+\acute{g}^2}} \tag{2.38}$$

Multiply the second part of equation (2.34) by $\frac{\sqrt{g^2+\acute{g}^2}}{\sqrt{g^2+\acute{g}^2}}$ we obtain

$$\frac{1}{4}\nu^2\left(gW_{\mu 3}-\acute{g}B_\mu\right)^2\cdot\frac{\sqrt{g^2+\acute{g}^2}}{\sqrt{g^2+\acute{g}^2}}=\frac{1}{4}\nu^2(\sqrt{g^2+\acute{g}^2})^2 z_\mu z^\mu \qquad (2.39)$$

Thus

$$m_{z=}^2\frac{1}{4}\nu^2(g^2+\acute{g}^2\,)^2$$

$$m_z=\frac{1}{2}\nu\sqrt{g^2+\acute{g}^2}$$

Although since g and $\acute{g}$ are free parameters. The SM makes no absolute predictions for $M_w$and M_z, it has been possible to set a lower limit before the W- and Z-boson were discovered. Their measured values are $M_w = 80.4\ GeV$ and $M_z = 91.2\ GeV$ (Weinberg ،1967).

(2.4.2) Yukawa Interaction and Fermions Mass:

In particle physics, Yukawa's interaction, named after Hideki Yukawa is an interaction between a scalar field and a Dirac field, the Yukawa interaction can be used to describe; the nuclear force between nucleons (which are fermions), mediated by Pions (which are pseudo scalar mesons) (J.donoghue, 1994).

The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, as result these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field (J.donoghue, 1994).

The Yukawa interaction is uniquely fixed by the dynamic of the system. It is given by

$$\mathcal{L}_{yukawa} = Y_d \bar{q}_L \phi d_R + Y_U \bar{q}_L \phi^* u_R \tag{2.40}$$

Using equation (2.22) we get

$$\mathcal{L}_{yukawa} = Y_d (\bar{u}_L \quad \bar{d}_L) \frac{1}{\sqrt{2}} \binom{0}{\nu} d_R + Y_u (\bar{u}_L \quad \bar{d}_L) \frac{1}{\sqrt{2}} \binom{\nu}{0} u_R$$
$$+ Y_e (\bar{\nu}_L \quad \bar{e}_l) \frac{1}{\sqrt{2}} \binom{0}{\nu} e_R \tag{2.41}$$

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}} (\bar{u}_L \quad \bar{d}_L) \binom{0}{\nu} d_R + \frac{Y_u}{\sqrt{2}} (\bar{u}_L \quad \bar{d}_L) \binom{\nu}{0} u_R$$
$$+ \frac{Y_e}{\sqrt{2}} (\bar{\nu}_L \quad \bar{e}_l) \binom{0}{\nu} e_R \tag{2.42}$$

Then

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}} \nu \bar{d}_L d_R + \frac{Y_u}{\sqrt{2}} \nu \bar{U}_L U_R + \frac{Y_e}{\sqrt{2}} \nu \bar{e}_l e_R \tag{2.43}$$

From the last equation and analog to previous section we find that

$$m_d = \frac{Y_d}{\sqrt{2}} \nu$$

$$m_u = \frac{Y_u}{\sqrt{2}} \nu$$

$$m_e = \frac{Y_e}{\sqrt{2}} \nu$$

Where **Y** is Yukawa coupling. You can see that Neutrino remain massless because it's right partner does not exist in the SM.

(2.5) Full SM Lagrangian:

To summarize the full standard model we can write it as:

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{L}\gamma^{\mu}\left(i\partial_{\mu} - g\frac{1}{4}\tau W_{\mu} - \acute{g}\frac{Y}{2}B_{\mu}\right)L$$
$$+ \bar{R}\gamma^{\mu}\left(i\partial_{\mu} - \acute{g}\frac{Y}{2}B_{\mu}\right)R + \left|\left(i\partial_{\mu} - g\frac{1}{4}\tau W_{\mu} - \acute{g}\frac{Y}{2}B_{\mu}\right)\phi\right|^{2}$$
$$- V(\phi) + (Y_d\bar{q}_L\phi d_R + Y_U\bar{q}_L\phi^{*}U_R + Y_e\bar{l}_L\phi e_R + \text{h.c.} \qquad (2.44)$$

L denotes a left-handed fermion (lepton or quark) doublet, and R a right-handed fermion singlet (J.donoghue, 1994).

Chapter Three

Generating Neutrino mass through the See Saw Mechanisms

(3.1) Introduction

In this chapter we will implement the technique that give mass to neutrino, that can be seen by studying the three types of the see-saw mechanism which leads to Majoran neutrino at length (B. Bajc, 2010).

(3.1.2) The See-Saw Mechanism

The see-saw mechanism is a generic model used to understand the relative sizes of observed neutrino masses, of the order of few eV, compared to those of quarks and charged leptons, which are millions times heavier (Pal, 1998).

The see-saw mechanism is the most likely way to explain how neutrinos got their masses, and why they are so small (W. Marciano, 1982).

(3.2) Types of the See-Saw Mechanism

There are several types of the see-saw mechanism each extending the standard model.

(3.2.1) Type I See-Saw Mechanism

Since the right hand neutrino doesn't present in the SM, simply we introduce the right-handed neutrino ν_R, which allows inserting additional term into Yukawa interaction.

$$\mathcal{L}_Y^{\nu} = Y_D \bar{\ell}_L \sigma_2 \Phi^* \nu_R + \frac{M_R}{2} \nu_R^T C \nu_R + h.c. \tag{3.1}$$

The Yukawa interaction defines the quantum numbers of the right-handed neutrino, it carries the lepton number. Majorana mass term is allowed for the right-handed neutrinos, consistent with the gauge symmetries of the theory (Majorana, 1937).

The Yukawa interaction, which couples left-handed and right-handed neutrinos yields after spontaneous symmetry breaking as usual the Dirac neutrino mass matrix $m_D = Y_D \upsilon$, so that the complete mass terms are given by

$$\mathcal{L}_M^{\nu} = m_D \bar{\nu}_L \nu_R + \frac{1}{2} M_R \nu_R^T C \nu_R + h.c. \tag{3.2}$$

We can rewrite equation (3.2) in terms of two components spinners.

$$\nu \equiv \nu_L + C \bar{\nu}_L^T \tag{3.3}$$

$$N \equiv \nu_R + C \bar{\nu}_R^T \tag{3.4}$$

Using the properties of charge-conjugation matrix (which can be found in any quantum field books (McMahon, 2008)).

$$C^T \gamma^{\mu} C = -\gamma_{\mu}^T \,, \qquad C^T = -C \tag{3.5}$$

We obtain the following relation

$$\bar{\nu}N = \bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L = \bar{N}\nu \tag{3.6}$$

$$\bar{N}N = \nu_R^T C\nu_R + h.c. \tag{3.7}$$

$$\nu\bar{\nu} = \nu_L^T C\nu_L + h.c. \tag{3.8}$$

So that the full lagrangian with the kinetic terms will be

$$\mathcal{L} = \frac{1}{2}\,[i\bar{\nu}\gamma^\mu\partial_\mu\nu + i\bar{N}\gamma^\mu\partial_\mu N - M_L\bar{\nu}\nu - M_R\bar{N}N - m_D\bar{\nu}N - m_D\bar{N}\nu] \tag{3.9}$$

We now summarize the masses in the above equation with the following form of mass matrix

$$\mathcal{L}_m^{(\nu)} = \frac{1}{2}(\bar{\nu},\bar{N})\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu \\ N \end{pmatrix} + h.c. \tag{3.10}$$

The mass matrix for the neutrino fields ν and N is

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \tag{3.11}$$

Where $m_D = Y_D v$. One can diagonalize this matrix by a similarity transformation using the orthogonal matrix

$$\begin{pmatrix} 1 & \epsilon \\ -\epsilon^T & 1 \end{pmatrix} \tag{3.12}$$

Where $\epsilon = \frac{m_D}{M_R}$. This diagonalization is correct up to terms smaller than of order ϵ^2, one obtains the mass matrix for the light neutrino to be

$$m_\nu^{light} = -m_D\frac{1}{M_R}m_D^T \tag{3.13}$$

This is the original see-saw formula called type I.

The diagrammatic representation of the see-saw in Figure 3.1.of the type I. show that the heavy neutrino propagator gives the see-saw result.

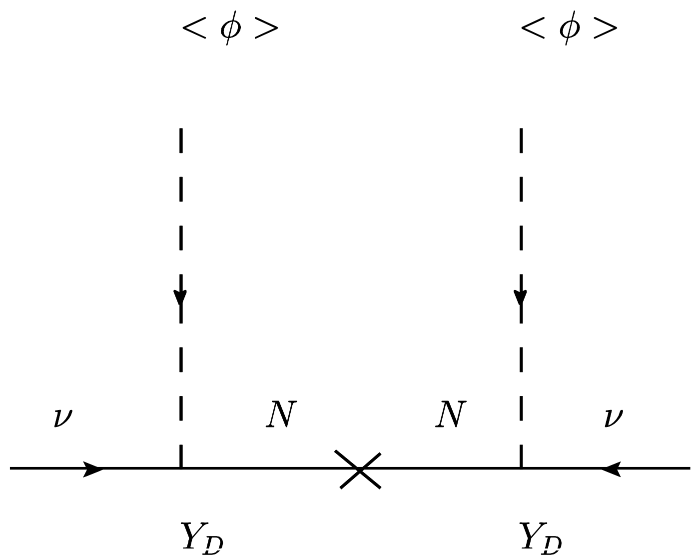

Figure 3.1. Diagrammatic representation of the Type I see-saw mechanism

(3.2.2) Type II See-Saw Mechanism

Instead of adding right-handed neutrino ν_R into the SM. We could choose Y=2, triplet scalar Δ_L. This will lead to new term in Yukawa interaction as

$$\mathcal{L}_\Delta = Y_\Delta^{-1}\ell^T C\sigma_2\Delta_L\ell + h.c \tag{3.14}$$

See figure 3.2

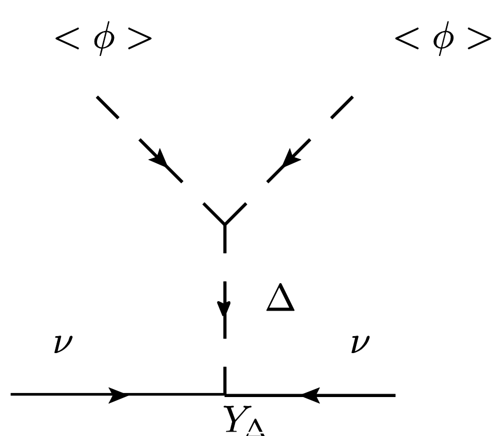

Figure 3.2. Diagrammatic representation of the Type II see-saw

Neutrino gets mass when Δ_L gets a vacuum expectation value (VEV) as

$$M_\nu = Y_\Delta \langle \Delta \rangle \tag{3.15}$$

The VEV $\langle \Delta \rangle$ results from the cubic scalar interaction

$$\Delta V = \mu \Phi^t \sigma_2 \Delta_L^* \Phi + M_\Delta^2 Tr \Delta_L^* \Delta_L + \cdots \tag{3.16}$$

With

$$\langle \Delta \rangle \equiv \frac{\mu \upsilon^2}{M_\Delta^2} \tag{3.17}$$

The mass of the neutrino is

$$m_\nu = Y_\Delta \frac{\mu \upsilon^2}{M_\Delta^2} \tag{3.18}$$

Where one would expect μ of order M_Δ. If the scale $M_\Delta >> \upsilon$, then neutrinos are naturally light.

(3.2.3) Type III See-Saw Mechanism

In this type we introduce triplet fermions $\vec{T}_F$ in Majorana notation, (where for simplicity the generation index is suppressed and also an index counting the number of triplet).

$$\Delta \mathcal{L}(T_F) = Y_T \ell^T C \sigma_2 \vec{\sigma}.\vec{T}_F \Phi + M_T \vec{T}_F^T C \vec{T}_F \tag{3.19}$$

Exactly the same manner as before in type I, one gets a type III see-saw mechanism for $M_T >> \upsilon$

$$M_\nu = -Y_T^T \frac{1}{M_T} Y_T \, v^2 \qquad (3.20)$$

(3-3) Neutrino Mass in Left- Right Symmetric Model

This section is devoted to the left right symmetric SM and the issue of the origin of the breaking of parity. This model played an important historic role in leading automatically to nonzero neutrino mass and the see-saw mechanism; there are two different possible left- right symmetries: party and charge conjugation. The latter is the finite gauge transformation in $SO(10)$ and is thus rather suggestive still parity is normally identified with L-R symmetry so we will discuss next parity (Senjanovi´c, 1981).

(3.3.1) Parity as L-R symmetry:

Parity is the fundamental symmetry between left and right and its breaking we believe should be understood. In the SM parity is broken explicitly. So in order to break parity spontaneously we must enlarge the gauge group. The minimal model based on the gauge group is

$$G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

With quarks and lepton completely symmetric under L-R transformation

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \overset{R}{\leftrightarrow} Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \qquad (3.21)$$

$$L_L = \begin{pmatrix} N \\ e \end{pmatrix}_L \overset{P}{\leftrightarrow} \ell_R = \begin{pmatrix} v \\ e \end{pmatrix}_R \qquad (3.22)$$

Notice: that the requirement of left- right symmetry leads to the existence of the right handed neutrino which means that the neutrino mass becomes a dynamical issue related to the pattern of symmetry breaking.

In the SM the right hand neutrino is absent, M_R: here instead we have to explain why neutrinos are so much lighter than the corresponding charged leptons. In left right model the formula for the electromagnetic charge becomes.

$$Q_{em} = I_{T_L} + I_{T_R} + \frac{B-L}{2} \tag{3.23}$$

This is in contrast with standard model where the hyper charge is completely devoid of any physical meaning. Therefore L-R symmetry is deeply connected with L-R symmetry, the existence of right handed neutrinos impelled by L-R symmetry which is necessary to cancel anomalies when gauging B-L normally the B-L symmetry is global anomaly face symmetry of the SM but without v_R the gauged version would have $(B-L)^3$ anormaly.

So our task is to break L-R symmetry to account for the fact that $M_{W_R} >> M_{W_L}$, W_R and W_L denoting right handed and left handed gauge boson respectively. In order to do so we need asset of left- handed and right handed higgs scalars whose quantum number will be specified later, imagine for the moment two scalars $\phi_L$and ϕ_R with

$$\phi_L \overset{P}{\rightarrow} \phi_R \tag{3.24}$$

Assuming now no terms liners in the fields (since $\phi_L$and $\phi_R$should carry quantum numbers under $SU(2)_R$) we can write down the left – right symmetric potential as

$$V = -\frac{\mu^2}{2}(\phi_L^2 + \phi_R^2) + \frac{\lambda}{4}(\phi_L^4 + \phi_R^4) + \frac{\bar{\lambda}}{2}(\phi_L^2 - \phi_R^2)^2 \quad (3.25)$$

Where we require $\lambda > 0$ for the potential to be bounded from below and we choose $\mu^2 > 0$ in order to achieve symmetry breaking in the usual manner.

Let us rewrite the potential as:

$$V = -\frac{\mu^2}{2}(\phi_L^2 + \phi_R^2) + \frac{\lambda}{4}(\phi_L^2 + \phi_R^2)^2 + \frac{\bar{\lambda} - \lambda}{2}(\phi_L^2 - \phi_R^2) \quad (3.26)$$

Which tell us that the pattern of symmetry breaking depends crucially on the sign of $\bar{\lambda} - \lambda$ since the first two term don't depends on the direction of symmetry breaking (of course $\mu^2 > 0$ quarantines that $< \phi_L > = < \phi_R > \geq 0$ is a maximum not a minimum of the potential).

(3.3.2) Left – Right symmetry and massive neutrinos:

What should we choose for the role of ϕ_L and ϕ_R scalars? From the neutrino mass point of view the ideal candidates should be triplets.

$$\Delta_L(\bar{3}_L, I_R, 2)\ ;\ \ \Delta_R(1_L, 3_R, 2) \quad (3.27)$$

Where the quantum numbers denote $SU(2)_L$ and SU $(2)_R$, Δ_L and Δ_R are $SU(2)_L$ and $SU(2)_R$ triplets respectively with $B - L$ numbers equal to two.

Writing $\Delta_{L,R} = \Delta_L^l\, T_i\, /2(T_i$ being the Pauli matrices) as usual for adjoint representations we found Yukawa couplings

$$L_\Delta = h_\Delta(\ell_L^T\, CiT_2\, \Delta_L\, \ell_L + L \to R) + h.c. \quad (3.28)$$

To check the invariance of (3.28) under the Lorentz group and the group symmetry

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Recall That $\Psi_L^T C\Psi_L$ is a Lorentz invariant quantity for a chiral Weyl spinor Ψ_L (and similarly for Ψ_R).

Under the gauge symmetry $SU(2)_L$:

$$\ell_L \to U_L \ell_L , \Delta_L \to U_L \Delta_L U_L^4 \, U_L^T \, (iT_2) = (iT_2) U_L^4 \tag{3.29}$$

The same apply to $SU(2)_R$. This proves the invariance of (3.28) under all the relevant symmetry now from their definition; the fields $\Delta_{L,R}$ have the following decomposition under the charge eigenstates:

$$\Delta_{L,R} = \begin{bmatrix} \frac{\Delta_+}{\sqrt{2}} & \Delta_{++} \\ \Delta^o & -\frac{\Delta_+}{\sqrt{2}} \end{bmatrix}_{L,R} \tag{3.30}$$

Where we have used the fact that $T_r \Delta_{L,R} = D$ and the charge is computed from the following formula

$$Q = I_{3L} + I_{3R} + (B-L)/2$$

Note that an important consequence of doubly charged physical Higgs scalars of the spontaneous $L - R$ symmetry breaking we know that for arrange of parameters of the potential the minimum of the theory can be chosen as:

$$< \Delta_L >= 0 \text{ and } < \Delta_R >= \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \tag{3.31}$$

From the equation (3.28) we obtain the mass for the right handed neutrino υ_R

$$\ell_{mass} = h_\Delta \upsilon_R \, (\nu_R^T \; C\nu_R + \nu_R^+ \; C^+ \nu_R^*) \tag{3.32}$$

Therefore the right handed neutrino gets a large mass $M_R = \; h_\Delta \, \upsilon_R$, which identify with the scale of parity breaking. At the same time the original gauge symmetry is broken down to the standard model one

$$SU(2)_L \; \times \; SU(2)_R \; \times \; U(1)_{B-L} \xrightarrow{<\Delta_R>} SU(2)_L \; \times \; U(1)_Y \tag{3.33}$$

This can be seen by computing the gauge boson mass matrix and define the right-handed charged gauge boson as

$$W_R^{\pm} = \frac{A_R^1 \pm iA_R^2}{\sqrt{2}} \tag{3.34}$$

We get the masses of gauge bosons

$$M_{W_R}^2 = g_R^2 \; \upsilon_R^2 \tag{3.35}$$

$$M_{Z_R}^2 = 2(g^2 + g_{B-L}^2)\upsilon_R^2 \tag{3.36}$$

Where

$$Z_R = \frac{g_{\,B-L} \, A_R^3 + g_R A_{B-L}}{\sqrt{g^2 + g_{B-L}^2}} \tag{3.37}$$

Here the new massive neutral gauge filled, the g_R and g_{B-L} gauge couplines correspond to $SU\;(2)_R$ and $(B-L)/2$ respectively.

To complete the theory one needs a Higgs bi-doublet $\phi\epsilon(2_L, 2_{R,0})$ which contains the SM Higgs, give masses to quarks and leptons. At the next stage of symmetry breaking the neutral componets of ϕ develop a vev and break the SM symmetry down to $U(1)_{em}$ as usual.

$$< \phi > = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \tag{3.38}$$

Which yield the mass of the $M_W^2 = g^2 v^2 = g^2(v_1^2 + v_2^2)$.

In the process we get the Dirac neutrino mass between ν_L and ν_R we end up with the type I see –saw mechanism for light neutrino masses. The Type 1 see-saw from Dirac Yukawa can be written as

$$L = h_Q\, \bar{\ell}_L\, \phi L_R + h.c. \tag{3.39}$$

After the symmetry breaking the neutrino Dirac mass term is $m_D = h_Q < \phi >$ the neutrino mass terms become.

$$m_\nu = m_D \bar{\ell}_L \ell_R + M_R\, \ell_R^T\, c\, \ell_R + h.c. \tag{3.40}$$

So the neutrino mass matrix takes clearly the see-saw form as discussed earlier. The important point here is that the mass of ν_R is determined by the scale of parity breaking and the smallness of the neutrino mass is reflection of the predominant $V - A$ structure of the weak interaction and provides a probe of parity restoration at high energies $E > M_{WR}$.

Type II see-saw:

The gauge symmetry of the left- right model allows also for the following term in the potential that we have ignored before for simplicity.

$$\Delta_{\nu} = \propto \Delta_L^{+} \, \phi \Delta_R \phi^{+} \tag{3.41}$$

Which implies that $< \Delta_L >$can not vanish in equation (3.31)

$$< \Delta_L > = \propto \frac{M_W^2 < \Delta_R >}{\mu \Delta_L} \approx \propto \frac{M_W^2}{M_R} \tag{3.42}$$

This leads to type II see-saw.

This also tell us that the prediction for neutrino mass depends crucially or M_{W_R}, however the left right symmetric model cannot give us its value. This will be cured in SO(10) grand unified theory, where we will see that this scales tends to be very large, above the Tera energy scale Large Hadron Collider (LHC).

(3.4) Neutrino Mass in $\mathbf{SU(5)}$ GUT Model

The minimal group that can unify the standard model SM in single coupling is $\mathbf{SU(5)}$, a group of rank four. In the $\mathbf{SU(5)}$ model (Georgi & Glashow, 1974), the fermions of each generation are assigned to $\overline{\mathbf{5}}$ and $\mathbf{10}$ representations, denoted by $\mathbf{\Psi_5}$and $\mathbf{\Psi_{10}}$ (Georgi & Glashow, 1974):

$$\mathbf{\Psi_5} = \begin{pmatrix} d^r \\ d^g \\ d^b \\ e^+ \\ -\nu^c \end{pmatrix} \text{ and } \mathbf{\Psi_{10}} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} \begin{pmatrix} 0 & u^c{}_b & -u^c{}_g & -u^r & -d^r \\ -u^c{}_b & 0 & u^c{}_r & -u^g & -d^g \\ u^c{}_g & -u^c{}_r & 0 & -u^b & -d^b \\ u^r & u^g & u^b & 0 & e^+ \\ d^r & d^g & d^b & -e^+ & 0 \end{pmatrix} \tag{3.44}$$

The breaking dynamics down of the gauge symmetry $\mathbf{SU(5)}$ to $\mathbf{SU(3)}_{\boldsymbol{c}} \times \mathbf{U(1)}_{\boldsymbol{Q}}$ is achieved by choosing only two Higgs multiplets H_5 and Φ_{24}. As such the vacuum expectation value of Φ_{24} chosen such that it breaks $\mathbf{SU(5)}$ to $\mathbf{SU(3)}_{\boldsymbol{c}} \times \mathbf{SU(2)}_{\boldsymbol{L}} \times \mathbf{U(1)}_{\boldsymbol{Y}}$ as follows

$$< \Phi_{24} > = v\, diag\left(1, 1,1, -\frac{3}{2}, -\frac{3}{2}\right) \tag{3.45}$$

Let the gauge bosons of SU(5) model to be written as

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\sum_{a=1}^{8} \lambda^a G^a + \sqrt{\frac{2}{15}}\mathrm{B}_{24} & \begin{matrix} \mathrm{X_1}^{4/3} & \mathrm{Y_1}^{1/3} \\ \mathrm{X_2}^{4/3} & \mathrm{Y_2}^{1/3} \\ \mathrm{X_3}^{4/3} & \mathrm{Y_3}^{1/3} \end{matrix} \\ \begin{matrix} \mathrm{X_1}^{-4/3} & \mathrm{X_2}^{-4/3} & \mathrm{X_3}^{-4/3} \\ \mathrm{Y_1}^{-1/3} & \mathrm{Y_2}^{-1/3} & \mathrm{Y_3}^{-1/3} \end{matrix} & \frac{1}{\sqrt{2}}\sum_{a=1}^{3} \tau^a W^a - \sqrt{\frac{10}{3}}\mathrm{B}_{24} \end{pmatrix}$$

Then the masses of heavy boson s X and Y will be

$$M_X{}^2 = M_Y{}^2 = \frac{25}{8} g^2 v^2$$

The final stage of symmetry breaking down to electromagnetic U(1) occurs via the non-zero VEV of the **5** dimensional Higgs field H (Georgi & Glashow, 1974):

$$H_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\mathrm{v}}{\sqrt{2}} \end{pmatrix} \tag{3.46}$$

The masses of gauge boson will be

$$M_W{}^2 = \frac{1}{4} g^2 v^2 \; and \; M_Z{}^2 = \frac{g^2 v^2}{4 \cos \theta_W{}^2}.$$

The most general gauge invariant Yukawa couplings in the SU(5) group is given by

$$\mathcal{L}_Y^{SU(5)} = Y_1 T_{ij}^T C^{-1} \Psi^i H^j + Y_2 \varepsilon^{ijklm} T_{ij}^T T_{kl} H_m + h.c. \tag{3.47}$$

Here we displayed indices of SU(5), and suppressed generation indices for the sake of simplicity. Also, complex conjugation is implied by lowering and raising of the indices. Thus, for example, Ψ^i denotes the complex conjugate of Ψ_i. After the spontaneous symmetry breaking occur equation (3.47) yield the mass foe fermions

$M_d = M_l = Y_1 v/\sqrt{2}$ and $M_u = Y_2 v/\sqrt{2}$

As can be seen from above equation the neutrinos remain massless. Indeed, in the **5 + 10** representation there is no room for the right-handed neutrino and so a mass term would be necessarily a Majorana mass term with a violation of B−L of two units. As such the SU(5) model is a fail tailor of the SM. We can also have a mass term for the neutrino by implementing the see-saw in any of its three forms but we do not do it here as the neutrino mass cured in this model by the same way as we did in the Standard model. It is therefore, phenomenologically not viable.

(3.5) Neutrino Mass in $SO(10)$ GUT Model

$SO(10)$ Family Unified:

The minimal gauge group that unifies the gauge interactions of the standard model was seen in the previous subsection is $\mathbf{SU(5)}$. The masses of neutrino turned out to be masses just like the SM, in the minimal version of the theory neutrinos get neither Dirac nor Majorana mass terms. We found that the simple extension with the adjoint fermion representation provides a minimal and remarkably predictive theory with light fermionic triplet expected at LHC and whose decay rates probe the Dirac Yukawa couplings of neutrinos. We have a theory that works and furthermore gives serious hope for an old dream of verifying seesaw mechanism at colliders. So why should one ever wish to go beyond $\mathbf{SU(5)}$? We can think of at least two reasons. First, if one is to worry about the Higgs mass naturalness and it is not an interesting theory of fermion masses and mixings. This is where $\mathbf{SO(10)}$ fits ideally, for it also unifies matter besides the interactions. It works nicely without supersymmetry too; it provides a natural unification of gauge couplings through the intermediate scale of left right symmetry breaking (Senjanovi´c, 1981).

One of the most important representations of $\mathbf{SO(10)}$ is a 16-dimensional spinor representation, which can be decomposed under $\mathbf{SU(5)}$ as $\mathbf{16 = 10 + 5 + 1}$. it unifies a family of fermions with an addition of a right handed neutrino per family. This minimal grand unified theory unifies matter on top of interactions suggests naturally small neutrino masses through the seesaw mechanism. Furthermore, it relates neutrino masses and mixings to the ones of charged fermions, and is predictive in its minimal version (K. R. Balaji, 2000). In this Section we discuss some important features in this theory while focusing on its minimal realizations. The crucial representation is a self-dual five index anti-symmetric one responsible for right-handed neutrino masses. A number of different minimal realizations of $\mathbf{SO(10)}$ depend on this construction, and what follows summarizes a few of them (B. Bajc, 2010).

There are a number of features that make $\mathbf{SO(10)}$ special:

1. A family of fermions is unified in a 16-dimensional spinorial representation; naturally right-handed neutrinos exist.
2. Left Right symmetry is a finite gauge transformation in the form of charge conjugation. This is a consequence of both left-handed fermions f_L and its charged conjugated counterparts residing in the same 16_F representation.

3. Its other maximal subgroup, besides $SU(5) \times U(1)$, and $SO(4) \times SO(6) = SU(2)_L \times SU(2)_R \times SU(4)_c$ symmetry of Pati and Salam.
4. The unification of gauge couplings can be achieved with or without supersymmetry.

In order to understand some of these results, and in order to address the issue of construction of the theory, we turn now to the Yukawa sector (B. Bajc, 2010).

(3.5.1) Yukawa Sector:

The decomposition of 16_F spinor representation is

$$16 \otimes 16 = 10 \oplus 120 \oplus 126$$

Therefore the most general Yukawa sector in general will contain 10_H, 120_H and $\overline{126}_H$, respectively the fundamental vector representation, the three-index antisymmetric representation and the five-index antisymmetric and anti-self-dual representation. Thus Yukawa couplings is

$$\mathcal{L}_y = y_{10}\psi^T B\Gamma_{\mathrm{i}}\psi\Phi_i + y_{120}\psi^T B\Gamma_{\mathrm{i}}\Gamma_{\mathrm{k}}\psi\Phi_{[\mathrm{ijk}]} + \mathrm{y}_{120}\psi^{\mathrm{T}}\mathrm{B}\Gamma_{\mathrm{i}}\Gamma_{\mathrm{j}}\Gamma_{\mathrm{k}}\Gamma_{\mathrm{l}}\Gamma_{\mathrm{m}}\psi\bar{\Phi}_{[\mathrm{ijklm}]}$$

Note that the $\overline{126}_H$ is necessarily complex, 10_H and $\overline{120}_H$ Yukawa matrices are symmetric in generation space, while the 120_H one is antisymmetric.

To understand the fermion masses, it is easier to work in the Pati-Salam language of one of the two maximal subgroups of $\mathrm{SO}(10)$, $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$(the other being $\mathrm{SO}(5) \times \mathrm{U}(1)$. Let us decompose the relevant representations under Pati Salam group G_{PS} (Pati & Salam, 1974).

$16 = (4, 2, 1) + (4, 1, 2)$

$10 = (1,2,2) + (6,1,1)$

$120 = (1,2,2) + (6,3,1) + (6,1,3) + (15,2,2) + (10,1,1) + (10,1,1)$

$\overline{126} = (\overline{10},3,1) + (10,1,3) + (15,2,2) + (6,1,1)$

Clearly, the see-saw mechanism, whether type I or II, requires $\overline{126}$: it contains both (10,1,3) whose VEV gives a mass to υ_R (type I), and ($\overline{10}$, 3,1), which contains a color singlet, $B - L = 2$ field Δ_L, that can give directly a small mass to ν_L (type II). A reader familiar with the SU(5) language sees this immediately from the decomposition under this group.

$$\overline{126} = 1 + 5 + 15 + \overline{45} + 50$$

The singlet of SU(5) belongs to the (10,1,3) of G_{PS} and gives a mass for ν_R, while 15 corresponds to the ($\overline{10}$, 3,1) and gives the direct mass to ν_L.

Normally the light Higgs is chosen to be the smallest one, 10_H. Since $\langle 10_H \rangle = \langle (1,2,2) \rangle_{PS}$ is a $SU(4)c$ singlet, the relation $m_d = m_e$ follows immediately, independently of the number of 10_H you wish to have. Thus we must add either 120_H or $\overline{126_H}$ or both in order to correct the bad mass relations. Both of these fields contain (15,2,2) in PS, and its VEV gives the relation $m_e = -3m_d$.

As $\overline{126}_H$ is needed anyway for the see-saw, it is natural to take this first. The crucial point here is that in general (1,2,2) and (15,2,2) mix through ((10,1,3) and thus the light Higgs is a mixture of the two. In other words, ((15,2,2)) in $\overline{126}_H$ is in general non-vanishing. It is rather appealing that 10_H and $\overline{126}_H$ may be sufficient for all the fermion masses, with only two sets of symmetric Yukawa coupling matrices.

An instructive failure:

Before proceeding, let us emphasize the crucial point of the necessity of 120_H or $\overline{126}_H$ in the charged fermion sector on an instructive failure: a simple and beautiful model by cite. here model is non-supersymmetric and the SUSY lovers may place the blame for the failure here. It uses $\langle 16_H \rangle$ in order to break $B - L$, and the "light" Higgs is 10_H. Witten noticed an ingenious and simple way of generating an effective mass for the right-handed neutrino, through a two-loop effect which gives.

$$M_{vR} \approx y_{up} \left(\frac{\alpha}{\pi}\right)^2 M_{GUT}$$

Where one takes all the large mass scales, together with $\langle 16_H \rangle$, of the order M_{GUT}. Since $\langle 10_H \rangle = \langle (1,2,2)_{PS} \rangle$ preserves quark-lepton symmetry, it is easy to see that

$$M_v \propto M_u$$

$$M_e = M_d$$

$$M_u \propto M_d$$

So that $V_{lopton} = V_{quark} = 1$. The model fails badly.

The original motivation here was a desire to know the scale of M_{vR} and increase M_v, at that time neutrino mases were expected to be larger. But the real achievement of this simple, elegant, minimal SO(10) theory is the predictivity of the structure of M_{vR} and thus M_v.

Chapter Four

DISCUSSIONS AND CONCLUSIONS

(4.1) Discussions

When considering the SM family plus an additional SM gauge singlet, the right-handed neutrino, such theory allows type I and type II see-saw mechanisms for generating light neutrino masses.

In type I see-saw mechanism, we can infer from the formula presented in equation (3.13) the following:
If the scale $M_R << m_D$, then neutrino would be predominantly Dirac particles. But if the scale $M_R \approx M_D$, we will have a messy combination of Majorana and Dirac particles, whereas for $m_D << M_R$ we would have a Majorana case. The Majorana neutrino mass is rather suggestive from the theoretical point of view. As such, it provides a window to new physics at scale M_R. The crucial prediction of this picture is the $L = 2$ lepton number violation in processes such as neutrino less double beta decay $\beta\beta 0\nu$. However, $\beta\beta 0\nu$ depends in general on the new physics at scale M_R, and it is desirable to have a direct probe of lepton number violation.

What happens if the neutrino has a pure Dirac mass nature? In this case and the smallness of Dirac mass essentially requires the smallness of Yukawa. The smallness of Dirac mass remains a puzzle as controlled by small Yukawa, as much as the smallness of electron mass is controlled by a small electron Yukawa coupling.

In left right symmetric model, the neutrino mass matrix takes clearly the see-saw form as discussed earlier. The important point in this model is that the mass of neutrino ν_R is determined by the scale of parity breaking and the smallness of the neutrino mass is reflection of the predominant $V - A$ structure of the weak interaction and provides a probe of parity restoration at high energies $E > M_{WR}$.

The $SU(5)$ model is a fail tailor of the SM. We can also have a mass term for the neutrino only (renormalizable theory) by implementing the see-saw in any of its three forms. As such the neutrino mass will be cured in this model by the same way as the Standard model. It is therefore, phenomenologically not viable.

In SO(10) the see-saw mechanism, whether type I or II, requires $\overline{126}$: it contains both (10,1,3) whose VEV gives a mass to υ_R (type I), and $(\overline{10}, 3,1)$, which contains a color singlet, $B - L = 2$ field Δ_L, that can give directly a small mass to ν_L (type II).

(4.2) Conclusions

In conclusion, we have studied the theory of standard model in some details and its extension to accommodate the neutrino mass by implementing the three type of sees-saw mechanisms and the explanation of the smallness of neutrino masses.

From a strict minimal standard model point of view masses could vanish if no right-handed neutrinos existed (so no Dirac mass) and lepton number will be conserved (so no Majorana mass).

We also discussed the see-saw mechanism that explains the smallness of neutrino masses in term of the large scale where B-L violated. Thus neutrino masses are important to a probe into the physics at GUTs scale.

$\mathrm{SU}(5)$ model has no right handed neutrino, therefore there will be no mass term for the neutrino, so it's a fail tailor of the SM. But we can have mass term for the neutrino only (at renormalizable level) by implementing the three types of see-saw. However in $\mathrm{SO}(10)$ we have seen that naturally neutrino have mass due to the fact that the model is left right symmetric model. (M. Gell-Mann, 1979)

Recommendation: This work may be extended by including the supersymmetric version of grand unified models.

Bibliography

A.D.Martin, F. a., 1984. *Quarks and leptons:An introductory course in modern particle physices.* new york : John Wiley.

A.J.G.hey, I. a., 1993. *Gague theories in particlphysics.* second edition ed. s.l.:Bristol and philadelphia.

al, G. A. e., 2012. ATLAS collaboration. *Phys. Lett,* Volume 1, p. B716.

al, S. C. e., 2012. CMS collaboration. *Phys.Lett,* Volume 30, p. B716.

Anon., 1980. *Phys.Rev.Lett.,* Volume 44, p. 912.

B. Bajc, G. S. a. F. V., 2010. *arXiv: hepph,* p. 0110310.

Georgi, H. & Glashow, S., 1974. "Unity of All Elementary-Particle Forces". *Physical Review Letters,* Volume 32, p. 438.

Guigg, c., 1983. *Gauge theories of the strong,weak and electromagnetic interaction.* s.l.: Benjiamin/cummings.

J.donoghue, E. a. B., 1994. *Dynamics of the standard model,Cambridge Monographs on particle ,Nuclear physics and Cosmology.* s.l.:s.n.

K. R. Balaji, A. S. D. R. N. M. a. M. K., 2000. *Phys. Rev. Lett. ,* Volume 5034, p. 84.

K. S. Babu, J. C. P. a. F. W., 2000. *Nucl. Phys. B ,* Volume 566, p. 33.

L.F.Li, T. a., 1991. *Gauge theory of elementary particle physics.* s.l.:oxford univ.

M. Gell-Mann, P. R. a. R. S., 1979. *proceedings of the Supergravity Stony Brook Workshop New York.*

Majorana, E., 1937. *Nuovo Cim.,* Volume 14, p. 171.

McMahon, D., 2008. *Quantum Field theory Demystified.* s.l.:McGraw-Hill companies.

Pal, R. N. M. a. P. B., 1998. *World Sci. Lect. Notes Phys. 60,* Volume 1, p. 60.

Pati, J. C. & Salam, A., 1974. "Lepton number as the fourth "color". *Physical Review D,* Volume 10, p. 275–289.

Quigg, C., 2007. *spontaneous symmetry Breaking as a Basic of particle mass,* s.l.: arXIV:0704.2232.

S.L.Glashow, 1961. *Nucl. Phys,* Volume 22, p. 579.

Salam, A., 1968. Elementary Partcile Physics (Nobel Symp. N.8). p. 367.

Senjanovi´c, R. N. M. a. G., 1981. *Phys. Rev. D,* Volume 23, p. 165.

Veltman, M., 2003. *Facts and mysteries in elementary particls physics,* s.l.: Co.Pte.Ltd.

W. Marciano, G. S., 1982. *Phys.Rev.D,* Volume 25, p. 3092.

Weinberg, S., 1967. *Phys. ReV. Lett,* Volume 19, p. 1264.

Weinberg, S., 1996. *the Quantum theory of field.* s.l.:Cambridge University.