

# **Dedication**

*I dedicate this research to my parents who have given me love and support .*

# Acknowledgments

*First I thank my teacher*

*Prof mohammed Ali Bashir*

*Who helped me to complete this research .I also  
thank my wife who supported me.*

## *Abstract*

Adomian analytical decomposition method is one of the important method in solving integral and differential equations.

In this study we handled the analysis of the method and then the solution of Volterra integral equation .Also we treated the solution of nonlinear integral equations.

The integro–differential equations has been studied with applications to the wave and heat equations in three dimensions, using Adomian method.

## الخلاصة

تعتبر طريقة تفكيك ادوميان أحد الطرق التحليلية الهامة في حلول المعادلات التكاملية والتفاضلية لذلك تناول هذا البحث طريقة ادوميان. تمت دراسة تحليل هذه الطريقة و حل معادلة فولتيرا التكاملية بطريقة ادوميان وكذلك علاج حل المعادلات التكاملية غير الخطية. أيضاً درست المعادلات التفاضلية التكاملية مع التطبيق على حلول معادلة الموجة والحرارة في ثلاثة ابعاد بطريقة ادوميان.

## Introduction

The Adomian decomposition method (ADM) is a semi analytical method for solving ordinary and partial non linear differential equation the method was developed from 1970 to the 1990 by George Adomian the chair of the center for applied mathematics at the university of Georgia in (USA) [21] it's further extensible to (stochastic systems) by using the integral [21] The aim of the method is to wards a unified theory for the solutions of partial differential equations (PDE) and aim which has been superseded by the more general theory of the homology analysis method[21] the crucial aspect of the method employment of the Adomian polynomial which allow for solution convergence the nonlinear partial of equation. Without simply linearizing the system. These polynomials mathematically generalize to a MacLoren series about on arbitrary external parameter, which gives the solution method more flexibility than direct Taylor series expansion[21].

The Adomian Decomposition Method has been receiving much attention in recent years in applied mathematical general, and in the area of series solutions in particular. The method proved to be powerfull. Effective and can easily handle a wide class of linear or nonlinear ordinary or partial differential equation, and linear or non linear integral equations the decomposition method demonstrates fast convergence of the solution and therefore provides several significant advantages.

the show that the method will successfully used to handle most types of partial differential equation that appear in several physical models and scientific applications the method attacks the problem in a direct way and straight forword fashion without using the linearization, perturbation or any

other restrictive assumption that may change the physical behavior of the model under discussion[21].

Adomian decomposition method was introduced and developed in USA by George Adomian and is well addressed in the literature. Considerable amount of research work has been invested recently in applying this method to ordinary partial differential equations, and partial differential equation and integral equation as well and the non linear partial differential equation can be found in wide variety scientific and engineering application. Many important mathematical models can be expressed in terms of the non linear partial differential equations, the most general form of non linear partial equation is given by:

$$F(u, u_t, u_x, u_y, x, y, t) = 0$$

With initial and boundary conditions

$$u(x, y, 0) = \phi(x, y), \quad \forall x, y \in \partial\Omega, \quad \Omega \in \mathbb{R}^2$$

$$u(x, y, t) = f(x, y, t), \quad \forall x, y \in \partial\Omega, \quad (\text{IC})$$

where  $(\Omega)$  the solution region and  $\partial\Omega$  is the boundary of  $\Omega$ .

In the recent years, much research has been focused on the numerical solution of non linear partial equation by using numerical method and developing these methods some persons some of them (Alsaif , 2007; levegue 2006; Rosser & Husner 1997, Wescot & Rizwan- Uddin, 2001)[21].

In the numerical methods, which are commonly used for solving these kinds of equations large size or difficult of computations and appeared and

usually the round of error causes the loss of accuracy. The Adomain Decomposition Method which needs less computation was employed to solve in any problems therefore, we applied the Adomian decomposition method to solve some models of non linear partial equation, this study and search reveals that the steps of the Adomian Decomposition for solving and how this method of very efficient for the non linear equation. And most of result gives and shows using given evidence that high accuracy and flexibility for the non linear can be achieved[21].

In this research our scheme is as follows:

In chapter one Adomian technique method , and chapter two solving Volterra integral equation .

Also we treated the solution of nonlinear integral equations in chapter three , also we studied the integro – differential equations with applications to the wave and heat equations in three dimensional , using Adomian method in chapter four and five .

## Table of Contents

Dedication	I
Acknowledgments	Ii
Abstract	iii
Abstract(Arabic)	iv
Introduction	V
<b>Table of Contents</b>	<b>Viii</b>
<b><i>Chapter One</i></b>	
<b>Analysis of Adomian Decomposition Method</b>	
1-1 Adomian Decomposition Formula	1
1-2 Formalism Of the Adomian Decomposition	2
<b><i>Chapter Two</i></b>	
<b>Volterra Integral Equations</b>	
2.1 Volterra Integral Equations	9
2.2 Volterra Integral Equations of the Second Kind:	10
2.3 The Adomian Decomposition Method	11
2.4 Classification of Integral Equations:	13
2.5 Classification of Integro-Differential Equations:	18
2.6 Examples	20
2.7 Systems of Volterra Integral Equations	28
<b><i>Chapter Three</i></b>	
<b>Non linear integral Equation</b>	
3.1 Introduction	38
3.2 The method of successive approximation	39
3.3 Picard's method of successive approximation	40



3.4 The Adomian decomposition method.	45
<b><i>Chapter Four</i></b> <b>Integro-Differential Equations</b>	
4.1 Introduction.	59
4.2 Volterra integro – differential equation.	60
4.3 fredholm integro- differential equations.	75
<b><i>Chapter Five</i></b> <b>Three Dimensional Wave Equation and Heat flow</b>	
5-1 Introduction	93
5-2 Homogeneous and inHomogenous PDEs	93
5-3 Adomian Decomposition method	94
References	117