

## Chapter Five

### 5. Conclusion and recommendation

#### 5.1 Conclusion

In this thesis it is clearly appears that the more number of antennas in the base station the more energy efficiency and area throughput, and the transmit power per base station decrease as the number of antenna increase. Contrary to common belief, the transmit power should increase with  $M$ . This indicates that massive MIMO can be built using low-power consumer grade transceiver equipment at the BSs instead of conventional industry-grade high-power equipment. The EE (in bit/Joule) is a quasi-concave function of  $M$  and  $K$ , thus it has a finite global optimum. Our numerical results show that deploying 100–200 antennas to serve a relatively large number of UEs is the EE-optimal solution using today's circuit technology, we conclude that massive MIMO is the EE-optimal architecture.

#### 5.2 Recommendation

As massive MIMO is still a relatively new research area in wireless communications, there are many topics for future work, including theoretical investigations, propagation measurements and implementation issues.

Regarding new channel measurements, outdoor-to-indoor scenarios are particularly interesting, as indoor coverage from outdoor base stations is a challenge for the operators. Massive MIMO is far from fully-exploited. With respect to propagation channels in real-life environments, there are many interesting aspects to study, and there are many new possibilities that have not been exploited yet.

investigate the performance of linear pre-coding techniques by taking into account the imperfect channel state information (CSI) and to Study the nonlinear pre-coding techniques for massive MIMO system. Compare the performance the linear and non

linear pre-coding techniques for massive MIMO system. Study the linear pre-coding techniques by taking into account pilot contamination phenomena.

On “massive” uses of the spatial domain, personally, I foresee a promising future of this technology.

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# Appendix

```
%Initialization
close all;
clear all;

%%Simulation parameters

rng('shuffle'); %Initiate the random number generators with a random seed
%%If rng('shuffle'); is not supported by your Matlab version, you can use
%%the following commands instead:
%randn('state',sum(100*clock));

%Ranges of optimization parameters
Mmax = 220; %Consider all number of BS antennas from 1 to 220 in simulation
Kmax = 150; %Consider all number of active UEs from 1 to 150 in simulation

%Should Monte Carlo simulations be used for MRT/MRC processing? (true or false)
runMonteCarloSimulationsMRT = true;

%Should Monte Carlo simulations be used for MMSE processing? (true or false)
%Beware: This part of the simulation is extremely slow, due to the iterative
%power allocation algorithm! It takes weeks to finish.
runMonteCarloSimulationsMMSE = false;

%Should the sequential/alternating optimization algorithm from Section V.E
%be used? (true or false). This option should only be turned on if the
%global optimum is within the range 1,...,Mmax and 1,...,Kmax, otherwise
%there will be an error.
runSequentialAlgorithm = true;

%Geometric scenarios (From Table 2)
d_max = 250; %Cell radius for a circular cell in the single-cell scenario. It is also the distance
from BS to a side of the square cells in the multi-cell scenario
d_min = 35; %Minimum distance between UE and BS
areaSinglecell = pi*(d_max/1000).^2; %Coverage area in km^2
areaMulticell = 4*(d_max/1000).^2; %Coverage area in km^2

%Large-scale fading parameters (From Table 2)
dbar = 10^(-3.53); %Regulates channel attenuation at minimum distance (see Example 1)
kappa = 3.76; %Path-loss exponent (see Example 1)

%Spectral resources and properties (From Table 2)
B = 20e6; %Transmission bandwidth (Hz)
Bc = 180e3; %Channel coherence bandwidth (Hz)
Tc = 10e-3; %Channel coherence time (s)
U = Bc * Tc; %Coherence block (number of channel uses)
sigma2B = 10^(-9.6-3); %Total noise power (B*sigma2 in W)

%Traffic assumptions (From Table 2)
zetaDL = 0.6; %Fraction of downlink transmission
zetaUL = 0.4; %Fraction of uplink transmission

%Relative lengths of pilot sequences (From Table 2)
tauDL = 1; %Relative pilot length in the downlink
tauUL = 1; %Relative pilot length in the uplink

%Hardware characterization (From Table 2)
etaDL = 0.39; %PA efficiency at the BSs
etaUL = 0.3; %PA efficiency at the UEs
L_BS = 12.8e9; %Computational efficiency at BSs (flops/W)
L_UE = 5e9; %Computational efficiency at UEs (flops/W)
P_FIX = 18; %Fixed power consumption (control signals, backhaul, etc.) (W)
P_SYN = 2; %Power consumed by local oscillator at a BS (W)
P_BS = 1; %Power required to run the circuit components at a BS (W)
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P_UE = 0.1; %Power required to run the circuit components at a UE (W)
P_COD = 0.1e-9; %Power required for channel coding (W/(bit/s))
P_DEC = 0.8e-9; %Power required for channel decoding (W/(bit/s))
P_BT = 0.25e-9; %Power required for backhaul traffic (W/(bit/s))

%System parameters computed from the parameters defined above
eta = 1/(zetaDL/etaDL + zetaUL/etaUL); %Effective PA efficiency, averaged over uplink and downlink.
Defined in Eq. (19)
S_x = (d_max^(kappa+2)-d_min^(kappa+2))/dbar/(1+kappa/2)/(d_max^2-d_min^2); %Average inverse channel
attenuation in the single-cell scenario (see Eq. (3))

Bsigma2SxetaSinglecell = sigma2B*S_x/eta; %Precomputation of B*sigma^2*S_x/eta for the single-cell
scenario. This term appears in Eq. (19) and other places that defines the total RF power
Bsigma2SxetaMulticell = 1.602212311888643; %Value for B*sigma^2*S_x/eta in the multi-cell scenario,
computed numerically. This term appears in Eq. (19) and other places that defines the total RF power

%%The rest of the script takes care of plotting the results.

%Density of the lines that are used in the 3d plots to make it easier to
%see the shape
gridDensity = 25;

%Plot Figure 3: Energy efficiency (in Mbit/Joule) with ZF processing in
%the single-cell scenario with perfect CSI.
figure(3); hold on; box on;
title('Figure 3: ZF processing, Single-cell, Perfect CSI')

surface(1:Kmax,1:Mmax,EEoptZF/1e6, 'EdgeColor', 'none'); %Plot the 3d surface
colormap(autumn);

%Compute and plot the optimal point
[EEvalues,indM] = max(EEoptZF,[],2);
[EEoptimal,indK] = max(EEvalues);
plot3(indM(indK),indK,EEoptimal/1e6, 'k*', 'MarkerSize', 10);

if runSequentialAlgorithm == true

plot3(K_sequential(1:itrSequential),M_sequential(1:itrSequential),EE_sequential(1:itrSequential)/1e6, '
ko-');
end

%Plot lines on top of the 3d surface, to make it easier to see the shape
for m = [1 gridDensity:gridDensity:Mmax]
plot3(1:Kmax,m*ones(1,Kmax),EEoptZF(m,:)/1e6, 'k-');
end

for k = [1 gridDensity:gridDensity:Kmax]
plot3(k*ones(1,Mmax),1:Mmax,EEoptZF(:,k)/1e6, 'k-');
end

plot3(1:Kmax,1:Kmax,zeros(Kmax,1), 'k-');

view([-46 24]);
axis([0 Kmax 0 Mmax 0 35]);

ylabel('Number of Antennas (M)');
xlabel('Number of Users (K)');
zlabel('Energy Efficiency [Mbit/Joule]');

if runMonteCarloSimulationsMMSE == true

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%Plot Figure 4: Energy efficiency (in Mbit/Joule) with MMSE processing in
%the single-cell scenario with perfect CSI.
figure(4); hold on; box on;
title('Figure 4: MMSE processing, Single-cell, Perfect CSI');

surface(1:Kmax,1:Mmax,EEoptMMSE/1e6,'EdgeColor','none'); %Plot the 3d surface
colormap(autumn);

%Compute and plot the optimal point
[EEvalues,ind] = max(EEoptMMSE,[],2);
[EEoptimal, ind2] = max(EEvalues);
plot3(ind(ind2),ind2,EEoptimal/1e6,'k*','MarkerSize',10);

%Plot lines on top of the 3d surface, to make it easier to see the shape
for m = [1 gridDensity:gridDensity:Mmax]
    plot3(1:Kmax,m*ones(1,Kmax),EEoptMMSE(m,:)/1e6,'k-');
end

for k = [1 gridDensity:gridDensity:Kmax]
    plot3(k*ones(1,Mmax),1:Mmax,EEoptMMSE(:,k)/1e6,'k-');
end

view([-46 24]);
axis([0 Kmax 0 Mmax 0 35]);

ylabel('Number of Antennas (M)');
xlabel('Number of Users (K)');
zlabel('Energy Efficiency [Mbit/Joule]');

end

if runMonteCarloSimulationsMRT == true

%Plot Figure 5: Energy efficiency (in Mbit/Joule) with MRT/MRC processing in
%the single-cell scenario with perfect CSI.
figure(5); hold on; box on;
title('Figure 5: MRT/MRC processing, Single-cell, Perfect CSI');

surface(1:Kmax,1:Mmax,EEoptMRT/1e6,'EdgeColor','none'); %Plot the 3d surface
colormap(autumn);

%Compute and plot the optimal point
[EEvalues,ind] = max(EEoptMRT,[],2);
[EEoptimal, ind2] = max(EEvalues);
plot3(ind(ind2),ind2,EEoptimal/1e6,'k*','MarkerSize',10);

%Plot lines on top of the 3d surface, to make it easier to see the shape
for m = [1 gridDensity:gridDensity:Mmax]
    plot3(1:Kmax,m*ones(1,Kmax),EEoptMRT(m,:)/1e6,'k-');
end

for k = [1 gridDensity:gridDensity:Kmax]
    plot3(k*ones(1,Mmax),1:Mmax,EEoptMRT(:,k)/1e6,'k-');
end

view([-46 24]);
axis([0 Kmax 0 Mmax 0 12]);

ylabel('Number of Antennas (M)');
xlabel('Number of Users (K)');
zlabel('Energy Efficiency [Mbit/Joule]');

end

```

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%Plot Figure 6: Energy efficiency (in Mbit/Joule) with ZF processing in
%the single-cell scenario with imperfect CSI.
figure(6); hold on; box on;
title('Figure 6: ZF processing, Single-cell, Imperfect CSI')

surface(1:Kmax,1:Mmax,EEoptZFimperfect/1e6,'EdgeColor','none'); %Plot the 3d surface
colormap(autumn);

%Compute and plot the optimal point
[EEvalues,indM] = max(EEoptZFimperfect,[],2);
[EEoptimal,indK] = max(EEvalues);
plot3(indM(indK),indK,EEoptimal/1e6,'k*','MarkerSize',10);

%Plot lines on top of the 3d surface, to make it easier to see the shape
for m = [1 gridDensity:gridDensity:Mmax]
    plot3(1:Kmax,m*ones(1,Kmax),EEoptZFimperfect(m,:)/1e6,'k-');
end

for k = [1 gridDensity:gridDensity:Kmax]
    plot3(k*ones(1,Mmax),1:Mmax,EEoptZFimperfect(:,k)/1e6,'k-');
end

plot3(1:Kmax,1:Kmax,zeros(Kmax,1),'k-');

view([-46 24])
axis([0 Kmax 0 Mmax 0 30])

ylabel('Number of Antennas (M)');
xlabel('Number of Users (K)');
zlabel('Energy Efficiency [Mbit/Joule]');

%Placeholders for storing the EE-optimal sum rates for different M and for
%each processing scheme.
optEEsumratesZF = zeros(Mmax,1);
optEEsumratesMRT = zeros(Mmax,1);
optEEsumratesMMSE = zeros(Mmax,1);
optEEsumratesZFimperfect = zeros(Mmax,1);

%Go through all different M
for M = 1:Mmax

%Store the maximal EE for different number of antennas M (normalized to Mbit/Joule)
    optEEsZF(M) = EEoptZF(M,optKzf(M))/1e6;
    optEEsMRT(M) = EEoptMRT(M,optKmr(M))/1e6;
    optEEsMMSE(M) = EEoptMMSE(M,optKmmse(M))/1e6;
    optEEsZFimperfect(M) = EEoptZFimperfect(M,optKzfimperfect(M))/1e6;

%Store the corresponding EE-optimal RF power
    optRFpowersZF(M) = RFpowersZF(M,optKzf(M));
    optRFpowersMRT(M) = RFpowersMRT(M,optKmr(M));
    optRFpowersMMSE(M) = RFpowersMMSE(M,optKmmse(M));
    optRFpowersZFimperfect(M) = RFpowersZFimperfect(M,optKzfimperfect(M));

%Store the corresponding EE-optimal sum rates (normalized to Gbit/s)
    optEEsumratesZF(M) = sumRatesZF(M,optKzf(M))/1e9;
    optEEsumratesMRT(M) = sumRatesMRT(M,optKmr(M))/1e9;
    optEEsumratesMMSE(M) = sumRatesMMSE(M,optKmmse(M))/1e9;
    optEEsumratesZFimperfect(M) = sumRatesZFimperfect(M,optKzfimperfect(M))/1e9;

end

%Compute and store the value of M that maximizes the EE globally. It is
%used to draw circles at the optimal values in Figures 11-13.
[~,MoptimalReuse1] = max(optEEsMulticellReuse1);
[~,MoptimalReuse2] = max(optEEsMulticellReuse2);
[~,MoptimalReuse4] = max(optEEsMulticellReuse4);

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%Plot Figure 11: Maximal EE in the multi-cell scenario for different number
%of antennas.
figure(11); hold on; box on;
title('Figure 11: Multi-cell, Comparison of EE values');

plot(Mrange,optEESMulticellReuse4,'b--','LineWidth',1);
plot(Mrange,optEESMulticellReuse2,'k','LineWidth',1);
plot(Mrange,optEESMulticellReuse1,'r-.','LineWidth',1);

plot(MoptimalReuse4,optEESMulticellReuse4(MoptimalReuse4),'bo','LineWidth',1);
plot(MoptimalReuse2,optEESMulticellReuse2(MoptimalReuse2),'ko','LineWidth',1);
plot(MoptimalReuse1,optEESMulticellReuse1(MoptimalReuse1),'ro','LineWidth',1);

axis([0 Mmax 0 8]);

legend('ZF (Imperfect CSI): Reuse 4','ZF (Imperfect CSI): Reuse 2','ZF (Imperfect CSI): Reuse
1','Location','SouthEast');

xlabel('Number of Antennas (M)');
ylabel('Energy Efficiency [Mbit/Joule]');

%Plot Figure 12: Total RF power at the EE-maximizing solution in the
%multi-cell scenario for different number of antennas. The radiated power per
%BS antennas is also shown.
figure(12); hold on; box on;
title('Figure 12: Multi-cell, Comparison of RF power and radiated power/antenna')

plot(Mrange,optRFpowersMulticellReuse4,'b--','LineWidth',1);
plot(Mrange,optRFpowersMulticellReuse2,'k','LineWidth',1);
plot(Mrange,optRFpowersMulticellReuse1,'r-.','LineWidth',1);

plot(MoptimalReuse4,optRFpowersMulticellReuse4(MoptimalReuse4),'bo','LineWidth',1);
plot(MoptimalReuse2,optRFpowersMulticellReuse2(MoptimalReuse2),'ko','LineWidth',1);
plot(MoptimalReuse1,optRFpowersMulticellReuse1(MoptimalReuse1),'ro','LineWidth',1);

plot(Mrange,zetaDL*eta*optRFpowersMulticellReuse1./Mrange,'r-.','LineWidth',1);
plot(Mrange,zetaDL*eta*optRFpowersMulticellReuse2./Mrange,'k','LineWidth',1);
plot(Mrange,zetaDL*eta*optRFpowersMulticellReuse4./Mrange,'b--','LineWidth',1);

plot(MoptimalReuse4,zetaDL*eta*optRFpowersMulticellReuse4(MoptimalReuse4)./MoptimalReuse4,'bo','LineWi
dth',1);
plot(MoptimalReuse2,zetaDL*eta*optRFpowersMulticellReuse2(MoptimalReuse2)./MoptimalReuse2,'ko','LineWi
dth',1);
plot(MoptimalReuse1,zetaDL*eta*optRFpowersMulticellReuse1(MoptimalReuse1)./MoptimalReuse1,'ro','LineWi
dth',1);

set(gca,'YScale','Log');
axis([0 Mmax 1e-2 1e2]);

text(20,5.5,'Total RF power');
text(20,0.15,'Radiated power per BS antenna');
legend('ZF (Imperfect CSI): Reuse 4','ZF (Imperfect CSI): Reuse 2','ZF (Imperfect CSI): Reuse
1','Location','Best');

xlabel('Number of Antennas (M)');
ylabel('Average Power [W]');

%Plot Figure 13: Area throughput at the EE-maximizing solution in the
%multi-cell scenario, for different number of BS antennas.
figure(13); hold on; box on;
title('Figure 13: Multi-cell, Comparison of area throughput')

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```

plot(Mrange,optEESumratesMulticellReuse4/areaMulticell,'b--','LineWidth',1);
plot(Mrange,optEESumratesMulticellReuse2/areaMulticell,'k','LineWidth',1);
plot(Mrange,optEESumratesMulticellReuse1/areaMulticell,'r-','LineWidth',1);

plot(MoptimalReuse4,optEESumratesMulticellReuse4(MoptimalReuse4)/areaMulticell,'bo','LineWidth',1);
plot(MoptimalReuse2,optEESumratesMulticellReuse2(MoptimalReuse2)/areaMulticell,'ko','LineWidth',1);
plot(MoptimalReuse1,optEESumratesMulticellReuse1(MoptimalReuse1)/areaMulticell,'ro','LineWidth',1);

axis([0 Mmax 0 9]);

legend('ZF (Imperfect CSI): Reuse 4','ZF (Imperfect CSI): Reuse 2','ZF (Imperfect CSI): Reuse
1','Location','NorthWest')

xlabel('Number of Antennas (M) ');
ylabel('Area Throughput [Gbit/s/km^2]');

%Plot Figure 14: Energy efficiency (in Mbit/Joule) with ZF processing
%in multi-cell scenario with pilot reuse 4.
figure(14); grid on; hold on;
title('Figure 14: ZF processing, Multi-cell, Pilot reuse 4')

surface(1:Kmax,1:Mmax,EEoptZFMulticellReuse4(1:Mmax,:)/1e6,'EdgeColor','none'); %Plot the 3d surface
colormap(autumn);

%Compute and plot the optimal point
[EEvalues,indM] = max(EEoptZFMulticellReuse4,[],2);
[EEoptimal,indK] = max(EEvalues);
plot3(indM(indK),indK,EEoptimal/1e6,'k*','MarkerSize',10);

%Plot lines on top of the 3d surface, to make it easier to see the shape
for m = [1 gridDensity:gridDensity:Mmax]
    plot3(1:Kmax,m*ones(1,Kmax),EEoptZFMulticellReuse4(m,:)/1e6,'k-');
end

for k = [1 gridDensity:gridDensity:Kmax]
    plot3(k*ones(1,Mmax),1:Mmax,EEoptZFMulticellReuse4(:,k)/1e6,'k-');
end

plot3(1:Kmax,1:Kmax,zeros(Kmax,1),'k-');

view([-46 24])
axis([0 Kmax 0 Mmax 0 8])

ylabel('Number of Antennas (M)');
xlabel('Number of Users (K)');
zlabel('Energy Efficiency [Mbit/Joule]');

```