CHAPTER THREE

METHDOLOGY

3.1 power system planning

Power system planning is done to ensure adequate and reliable power supply to meet the estimated load demand in both near and distant future. this must be done at minimum possible cost keeping the quality of supply satisfactory.

Power system planning is needed to develop and build modern electric power systems. In general, planning time horizons lie in one of the following ranges:

1. short term (up to 1 year).
2. medium term (up to 2-3 years).
3. long term (between 20-30 years).

The power system planning process starts by forecasting the anticipated future loads.

3.1.1 The goals of planning

1. Cover the service territory, reaching all customers.

2. Have sufficient capacity to meet the peak demands of its customers.

3. Provide highly reliable delivery to its customers.

4. Provide stable voltage quality to its customer.

3.1.2 The System Approach

The economics and electrical behaviour of the substation, the feeder system, and the transmission levels need to be added together to determine cost and performance as a whole.

It is more important that the parts fit together well that any of them be individually ideal.

Action when the substation does not meet loading criteria:

- Transfer load to another.
- Reinforce an existing one.
- Build a new substation.

3.1.3 Route selection

The route considerations are:

- OH or UG.
- Right of Way.
- Lowest Present Worth costs.

3.1.4 Site Consideration

Two of the most critical factors in the design of a substation are its location and siting. Failure to carefully consider these factors can result in excessive investment in the number of substations and associated transmission and distribution facilities.

It is becoming increasingly important to perform initial site investigations prior to the procurement of property. Previous uses of a property might render it very costly to use as a substation site. Such previous uses might include its use as a dumping ground where buried materials or toxic waste has to be removed prior to any grading or installation of foundations.

The following factors should be evaluated when selecting a substation site:

1- Location of present and future load center

2-Location of existing and future sources of power

3- Availability of suitable right-of-way and access to site by overhead or underground transmission and distribution circuits

4- Alternative land use considerations

5- Location of existing distribution lines

6- Nearness to all-weather highway and railroad siding, accessibility to heavy equipment under all weather conditions, and access roads into the site

7- Possible objections regarding appearance, noise, or electrical effects

8- Site maintenance requirements including equipment repair, watering, mowing, land scaping, storage, and painting

9- Possible objections regarding present and future impact on other private or public facilities

3.1.5 Recent load calculation

The data of recent load was taken from reading meters of distribution substations at area of center of Khartoum

3.1.6 Feeders calculation

Number of feeders calculating from equation below

Number of feeders $= \frac{MVA*DF}{FC}$ (3.1)

Where:

MVA: increasing power.

DF: diversity factor.

FC: average capacity of 185sq.mm. standard feeder (MVA).

FC = $\frac{\sqrt{3}*V*I}{1000}$ MVA (3.2)

Where

I: maximum current can flow in feeder it equal 520 A taken from IEC standard

V: rated voltage of busbar

3.1.7 Forecasting Load calculation

The load forecasting was calculated from equation below

Load = $recent\ load(1 + GR)^n$ MVA (3.3)

Where:

Gr: growth factor.

n: number of years.

3.1 Introduction to Load Flow:

The power system is assumed to be operating under balanced condition and can be represented by single line diagram. The power system network contains hundreds of buses and branches with impedances specified in per-unit on a common MVA base. Power flow studies commonly referred to as long flow, are essential of power system analysis and design. Load flow studies are necessary for planning, economic operation, scheduling and exchange of power between utilities. Load flow study is also required for many other analyses such as transient stability, dynamic stability, contingency and state estimation.

Network equations can be formulated in variety of forms. However, node voltage method is commonly used for power system analysis. The network equations which are in the nodal admittance form results in complex linear simultaneous algebraic equations in terms of node currents. The load flow result gives the bus magnitude and phase angle and hence the power flow through the transmission lines, line losses and power injection at all the busses.

3.2.1 Bus Classifications:

Four quantities are associated with each bus. These are voltage magnitude $|V|$, phase angle δ, real power P and reactive Q. in a load flow study, two out of four quantities are specified and the remaining two quantities are to be classified into three categories.

Slack bus: Also known as swing bus and taken as a reference where the magnitude and phase angle of the voltage are specified. This bus provides the additional real and reactive power to supply the transmission losses, since these are unknown until the final solution is obtained.

Load bus: Also known as PQ bus. At these buses the real and reactive powers are specified. The magnitude and phase angle of the bus voltage are unknown until the final solution is obtained.

Voltage controlled buses: Also known as generator buses or regulated buses or PV buses. At these buses, the real power and voltage magnitude are specified. The phase angles of the voltages and the reactive power are unknown until the final solution is obtained. The limits on the value of reactive power are also specified.

Table 3.1: Bus Classification

Bus type	Specified quantities	Unknown quantities
Slack bus	$\lvert V \rvert, \delta$	P,Q
Load bus	P,Q	$\lvert V \rvert, \delta$
Voltage bus	P, $\lvert V \rvert$	Q, δ

3.1.2 Bus Admittance Matrix:

In order to obtain the bus-voltage equations, consider the simple 4-bus power system as shown in figure below

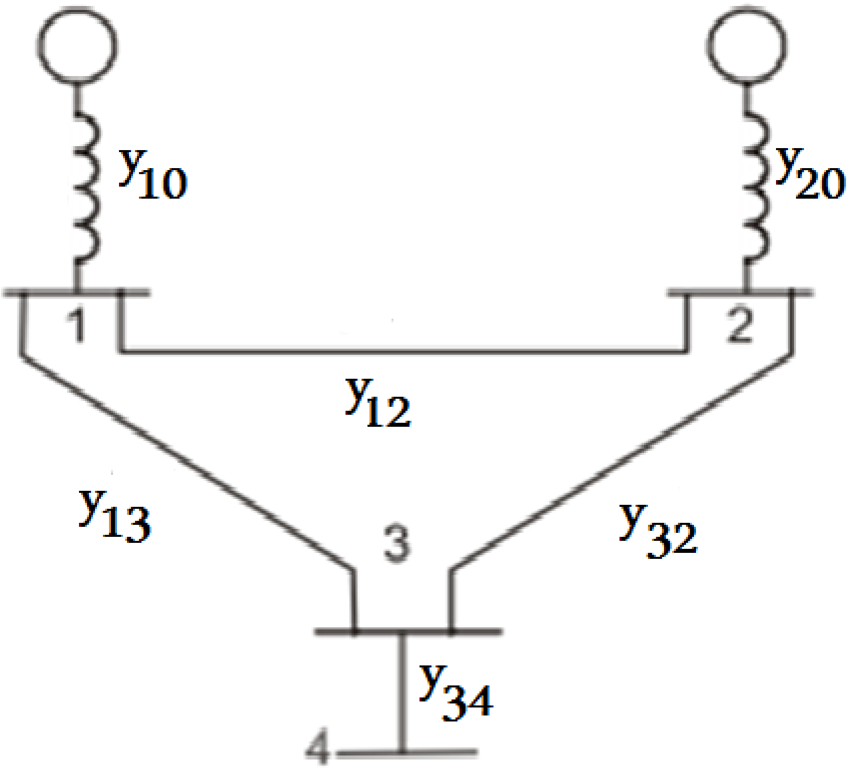

Figure 3.1: The impedance diagram of sample 4-bus power system

for simplicity resistance of the line are neglected and the impedances shown in above figure are expressed in per-unit on common MVA base.

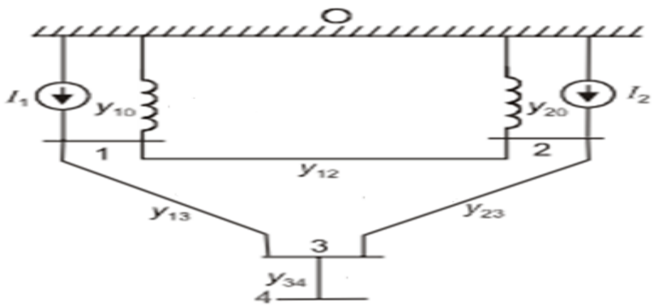

Figure 3.2: The admittance diagram of Figure 3.1

Applying KCL to the independent nodes 1, 2, 3, 4 we have,

$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$

$I_2 = y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3)$

$0 = y_{23}(V_3 - V_2) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4)$

$0 = y_{34}(V_4 - V_3)$

Rearranging the above equations, we get

$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$

$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3$

$0 = -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4$

$0 = -y_{34}V_3 + y_{34}V_4$

Let,

$Y_{11} = (y_{10} + y_{12} + y_{13})$

$Y_{22} = (y_{20} + y_{21} + y_{23})$

$Y_{33} = (y_{31} + y_{32} + y_{34})$

$Y_{44} = y_{43}$

$Y_{12} = Y_{21} = -y_{12}$

$Y_{13} = Y_{31} = -y_{13}$

$Y_{23} = Y_{32} = -y_{23}$

$Y_{34} = Y_{43} = -y_{43}$

The node equations reduce to

$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$

$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$

$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$

$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$

Above equations can be written in matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \tag{3.4}$$

Or in general

$$I_{bus} = Y_{bus}V_{bus} \tag{3.5}$$

$V_{bus} \equiv \textit{vector of bus voltages}$

I_{bus} ≡vector of the injected currents

Y_{bus} ≡admittance matrix

Diagonal element of Y matrix:

$$Y_{ii} = \sum_{k=0}^{n} y_{ik} \ , j \neq i \tag{3.6}$$

Off-diagonal element of Y matrix:

$$Y_{ik} = Y_{ki} = -y_{ik} \tag{3.7}$$

$$\mathrm{V_{bus}} = \mathrm{Y_{bus}^{-1}}\ \mathrm{I_{bus}} \tag{3.8}$$

3.2.3 BUS LOADING EQUATIONS

Consider ith bus of a power system as shown infigure(3.3)Transsion lines arerepresented by their equivalent p models π models. yi0 is the total charging admitance at bus i.

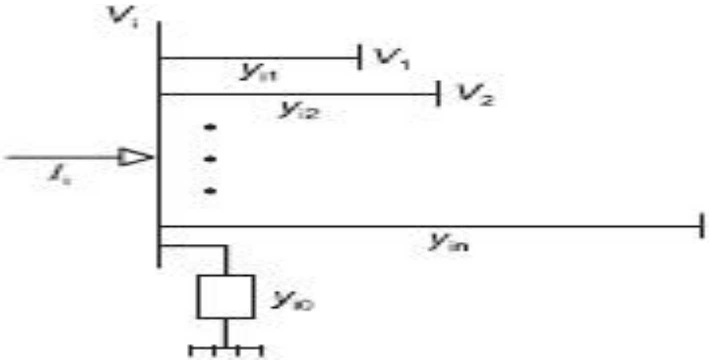

Figure 3.3: i-th bus of a power system

Net injected current $\mathbf{I}_i$ into the bus i can be written as:

$$I_i = y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n)$$

$$I_i = (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i \ - y_{i1}V_1 - y_{i1}V_2 - \cdots - y_{i1}V_n \tag{3.9}$$

Let us define

$$Y_{ii} = y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in}$$

$$Y_{i1} = -y_{i1}$$

$$Y_{i2} = -y_{i2}$$

$$\vdots$$

$$Y_{in} = -y_{in}$$

$$I_i = Y_{ii}V_i + Y_{i1}V_1 + Y_{i2}V_2 + \cdots + Y_{in}V_n \tag{3.10}$$

Or

$$I_i = Y_{ii}V_i + \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k \tag{3.11}$$

The real and reactive power injected at the bus i is

$$Pi - jQi = V_i * I_i$$

$$I_i = \frac{P_i - jQ_i}{v_i^*} \tag{3.12}$$

From eqns. (3.9) and (3.10) we get

$$\frac{P_i - jQ_i}{v_i^*} = Y_{ii}V_i + \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k \tag{3.13}$$

$$Y_{ii}V_i = \frac{P_i - jQ_i}{v_i^*} - \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k$$

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{v_i^*} - \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k\right] \tag{3.14}$$

3.2.4 Calculation of Net Injected Power:

From eqn. (3.11), we get

$$\frac{P_i - jQ_i}{v_i^*} = Y_{ii}V_i + \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k$$

$$P_i - jQ_i = v_i^*\left[Y_{ii}V_i + \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k\right] \tag{3.15}$$

$$\therefore P_i - jQ_i = |V_i|^2|Y_{ii}|\cos\theta_{ii} + j|V_i|^2|Y_{ii}|\sin\theta_{\mathrm{ii}}$$

$$+\sum_{\substack{k=1\\k\neq i}}^{n}|Y_{ik}||V_i||V_k|\cos(\theta_{ik}+\delta_k-\delta_i)+\mathrm{j}\sum_{\substack{k=1\\k\neq i}}^{n}|Y_{ik}||V_i||V_k|\sin(\theta_{ik}+\delta_k-\delta_i)$$

(3.16)

separating real and imaginary part of equation (3.16)

$$P_i=\sum_{k=1}^{n}|Y_{ik}||V_i||V_k|\cos(\theta_{ik}+\delta_k-\delta_i) \quad (3.17)$$

And

$$Q_i=-\sum_{k=1}^{n}|Y_{ik}||V_i||V_k|\sin(\theta_{ik}+\delta_k-\delta_i) \quad (3.18)$$

3.2.6 Load Flow Using Newton-Raphson Method:

Newton-Raphson (NR) method is more suitable and practical for large power systems. The advantage of this method is that the number of iterations required to obtain a solution is independent of the size of the problem and computationally it is very fast.

Rewriting equations (3.17) and (3.18):

$$P_i=\sum_{k=1}^{n}|Y_{ik}||V_i||V_k|\cos(\theta_{ik}+\delta_k-\delta_i) \quad (3.19)$$

$$Q_i=-\sum_{k=1}^{n}|Y_{ik}||V_i||V_k|\sin(\theta_{ik}+\delta_k-\delta_i) \quad (3.20)$$

By expanding eqns. (3.19) and (3.20) in Taylor-series and neglecting H.O.T

$$\therefore \begin{bmatrix} \Delta P_2^{(p)} \\ \vdots \\ \vdots \\ \vdots \\ \Delta P_n^{(p)} \\ \hline \Delta Q_2^{(p)} \\ \vdots \\ \vdots \\ \vdots \\ \Delta Q_n^{(p)} \end{bmatrix}$$

$$= \left[\begin{array}{ccc|ccc} \left[\frac{\partial P_2}{\delta_2}\right]^{(p)} & \cdots & \left[\frac{\partial P_2}{\delta_n}\right]^{(p)} & \left[\frac{\partial P_2}{\partial|V_2|}\right]^{(p)} & \cdots & \left[\frac{\partial P_2}{\partial|V_n|}\right]^{(p)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left[\frac{\partial P_n}{\delta_2}\right]^{(p)} & \cdots & \left[\frac{\partial P_n}{\delta_n}\right]^{(p)} & \left[\frac{\partial P_n}{\partial|V_2|}\right]^{(p)} & \cdots & \left[\frac{\partial P_n}{\partial|V_n|}\right]^{(p)} \\ \hline \left[\frac{\partial Q_2}{\delta_2}\right]^{(p)} & \cdots & \left[\frac{\partial Q_2}{\delta_n}\right]^{(p)} & \left[\frac{\partial Q_2}{\partial|V_2|}\right]^{(p)} & \cdots & \left[\frac{\partial Q_2}{\partial|V_n|}\right]^{(p)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left[\frac{\partial Q_n}{\delta_2}\right]^{(p)} & \cdots & \left[\frac{\partial Q_n}{\delta_n}\right]^{(p)} & \left[\frac{\partial Q_n}{\partial|V_2|}\right]^{(p)} & \cdots & \left[\frac{\partial Q_n}{\partial|V_n|}\right]^{(p)} \end{array}\right] \begin{bmatrix} \Delta\delta_2^{(p)} \\ \vdots \\ \vdots \\ \vdots \\ \Delta\delta_n^{(p)} \\ \hline \Delta|V_2|^{(p)} \\ \vdots \\ \vdots \\ \vdots \\ \Delta|V_n|^{(p)} \end{bmatrix} \tag{3.21}$$

Bus 1 in above eqn. assumed to be slack bus, and this eqn. can be written as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{3.26}$$

3.3 Short-Circuit Study

Short-circuits occur in well-designed power systems and cause large transient currents of magnitudes, much above the system load currents. These result in disruptive electrodynamics and thermal stresses that are potentially damaging.

Fire risks and explosions are inherent. When a fault occurs in a system, switching devices are used to isolate the fault only the faulty section is isolated so that the fault is not escalated. The faster the operation of the sensing and switching devices, the smaller is the fault damage and the better is the chance of the system holding together without loss of synchronism [1].

3.3.1 Short-Circuit Studies

Short circuits can be studied from the following angles:

1. Calculation of short-circuit currents.

2. Determination of the ratings of switching devices.

3. Effects of short-circuit currents.

4. Limitation of short-circuit currents.

5. Transient stability of interconnected systems after isolation of the faulty section of the power system.

3.3.2 Objectives of Short Circuit Calculations

Objectives of short circuit calculation can be summarized as follows:

1. Determination of short-circuit duties on switching devices.

2. Calculation of short-circuit currents required for protective relaying and coordination of protective devices.

3. Evaluations of the adequacy of short-circuit withstand ratings of static equipment.

3.3.3 Benefits of Short Circuit Analysis:

1. Reduces the risk a facility could face and helps avoid catastrophic losses.

2. Increases the safety and reliability of the power system equipment and the maintenance personnel.

3. Evaluates the application of protective devices and equipment.

4. Identifies problem areas in the system.

5. Identifies recommended solutions to existing problems.

3.3.4 Types of Short Circuits and Faults

3.3.4.1 Symmetrical Faults:

1. Three phase ungrounded fault

2. Three phase grounded fault

3.3.4.2 Unsymmetrical Shunt Type Faults:

1. Single phase-to-ground fault

2. Double line-to-ground fault

3. phase-to-phase fault

3.3.4.3 Unsymmetrical Series Type Faults:

1. One conductor open

2. Two conductors open

Figure 3.4 shows type of faults. The broken conductors may be grounded on one side or on both sides of the break. An open conductor fault can occur due to operation of a fuse in one of the phases.

Unsymmetrical faults are more common. The most common type is a line-to ground fault. Approximately 70% of the faults in power systems are single phase-to ground faults.

While applying symmetrical component method to fault analysis the load currents are ignored and this makes the positive sequence voltages of all the generators in the system identical and equal to the pre fault voltage.

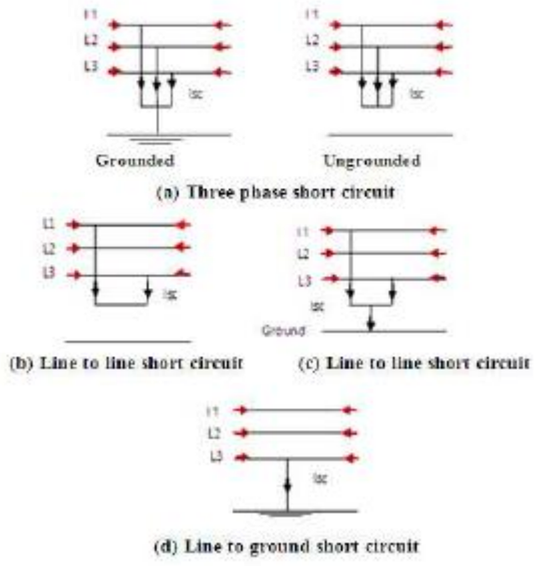

Figure 3.4: Types of Short Circuits

3.3.5 Short Circuit Current Wave Shapes

There are two distinct components of a short-circuit current:

(1) Non decaying ac component or the steady-state component.

(2) Decaying dc component at an exponential rate.

The initial magnitude of which is a maximum of the ac component and it depends on the time on the voltage wave at which the fault occurs.

Figures 3.5, 3.6 shows non decaying ac component and Decaying dc component

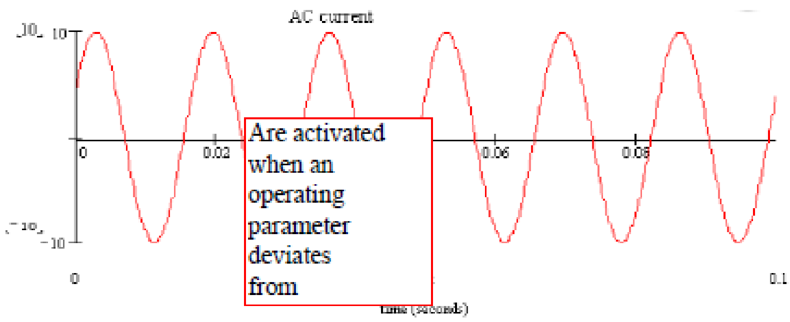

Figure 3.5: AC Current (symmetrical) with no AC decay

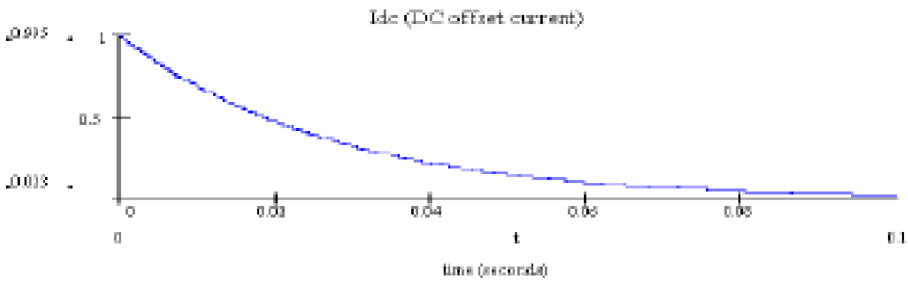

Figure 3.6: DC current

Figure 2.7 and figure 2.8 shows the fault current including DC offset or ac decay current

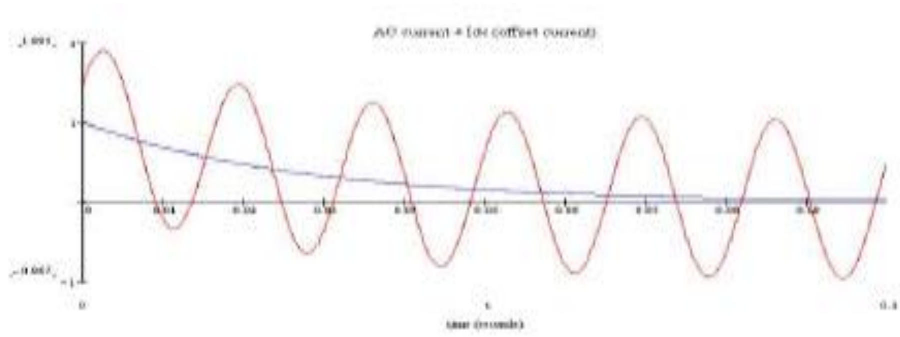

Figure 3.7: AC fault current including the DC offset (no AC decay)

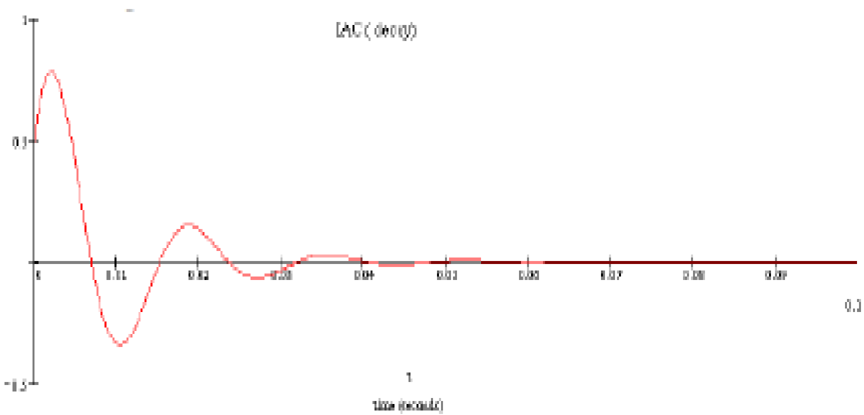

Figure 3.8: AC decay current

Figure 2.9 shows the fault current including AC and DC decay

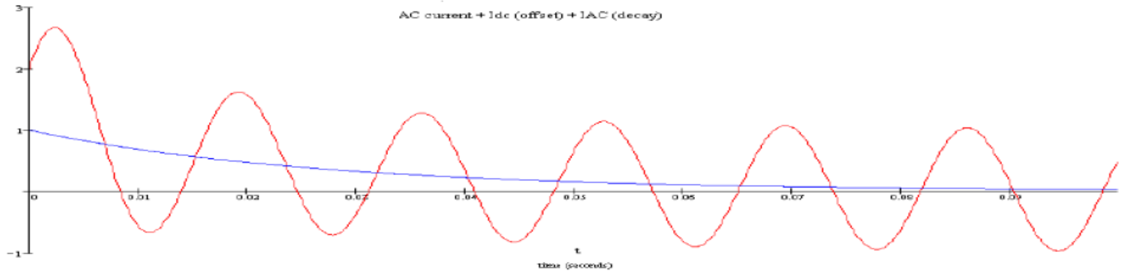

Figure 3.9: Fault current including AC and DC decay

3.3.6 Short-Circuit Calculation Standards

There are three categories of international standards for short circuit calculation:

1. IEC 60909.

2. European Standard EN 60909.

i. German National Standard DIN VDE 0102.

ii. further National Standards.

iii. Engineering Recommendation G74 (UK).

2. ANSI IEEE Std. C37.5 (US).

Only IEC standards shall be elaborate.

3.3.6.1 Short-Circuit Calculations Standard IEC 60909

This comprises four parts:

Part 0: Calculation of currents

Part 1: Factors for the calculation of short-circuit currents

Part 2: Electrical equipment; data for short-circuit current calculations

Part 3: Currents during two separate simultaneous line-to-earth short circuits and Partial short-circuit currents flowing through earth

Part 4: Examples for the calculation of short-circuit currents.

3.3.6.2 Scope of IEC 60909

1. Three-phase A.C. systems
2. low voltage and high voltage systems up to 500 kV
3. Nominal frequency of 50 Hz and 60 Hz
4. Balanced and unbalanced short circuits:
5. Maximum short circuit currents
6. Minimum short circuit currents

3.3.7 Type of Short Circuits

IEC 60909 and the associated standards classify short circuit currents according to their magnitudes (maximum and minimum) and fault distances from the generator (far and near). Maximum short circuit currents determine equipment ratings, while minimum currents dictate protective device settings. Near-to generator and far-from-generator classifications determine whether or not to model the AC component decay in the calculation, respectively.

3.3.7.1 Near-to-Generator Short Circuit

This is a short circuit condition to which at least one synchronous machine contributes a prospective initial short circuit current which is more than twice the generator s rated current ,or a short circuit condition to which synchronous and asynchronous motors contribute more than 5% of the initial symmetrical short circuit current I_k'' .

Figure 2.10 shows near-to-generator short circuits current

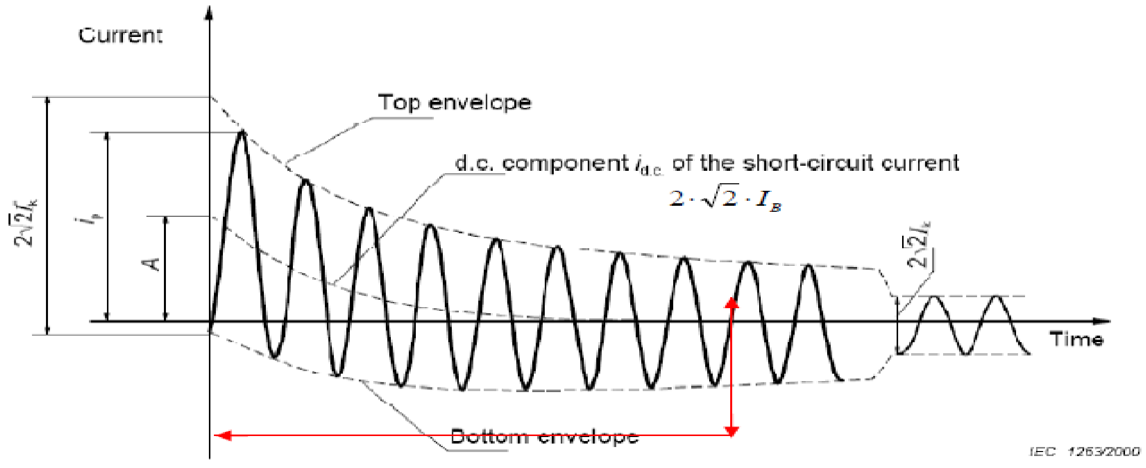

Figure 3.10: Near –to- generator short circuits

3.2.7.2 Far-From-Generator Short Circuit

This is the short circuit condition during which the magnitude of symmetrical ac component of available short circuit current remains essentially constant. Figure3.11 shows far-from-generator short circuits

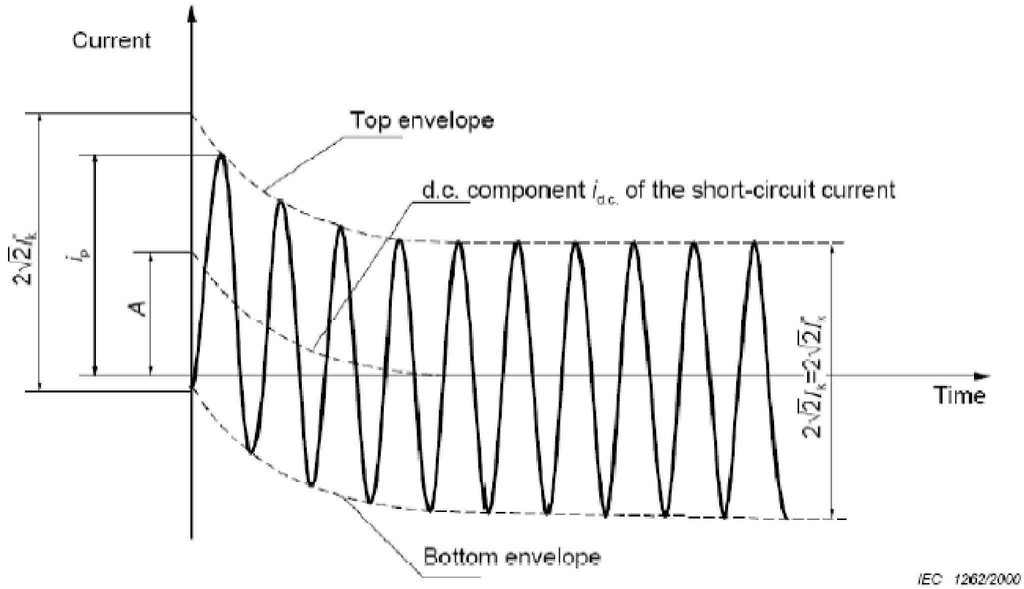

Figure 3.11: Far-from-generator short circuits

3.3.8 Short Circuit Definitions According to IEC 60909

1. Initial symmetrical short-circuits currentI_k'':

R.M.S value of the A.C symmetrical component of a prospective (available) short-circuit current, applicable at the instant of short circuit if the impedance remains at zero-time value

2. Steady-state short-circuit current I_k

R.M.S value of the short-circuit current which remains after the decay of the transient phenomena

3. Symmetrical short-circuit breaking current I_b

R.M.S value of an integral cycle of the symmetrical a.c. component of the prospective short-circuit current at the instant of contact separation of the first pole to open of a switching device

4. Decaying (a periodic) component $i_{d \cdot c}$. Of short-circuit current:

Mean value between the top and bottom envelope of a short circuit current decaying from an initial value to zero.

5. Peak short-circuit current Ip :

Maximum possible instantaneous value of the prospective (available) short circuit current

NOTE: The magnitude of the peak short-circuits current varies in accordance with the moment at which the short circuit occurs.

3.3.9 Faults calculation

1-Three-phase fault

The three phases are short-circuited through equal fault impedances Zf, Figuer3.12

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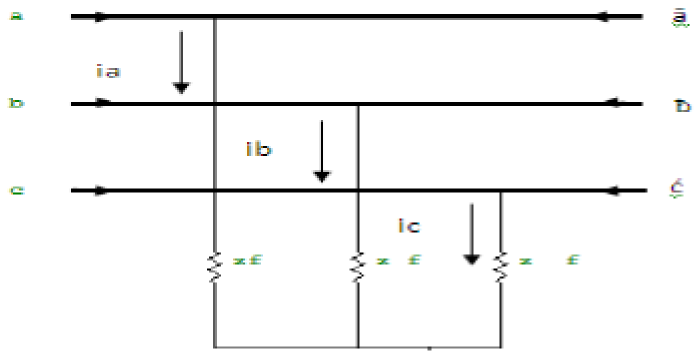

Figure 3.12: Three-phase symmetrical fault

Three phase short circuit current expressed in equation (3.27)

$$I_a = I_1 = \frac{V}{Z_f + Z_1} \quad (3.27)$$

2- Single Line-to-Ground Fault

This fault occurs on one phase through impedance Zf. As shown in Figure3.13

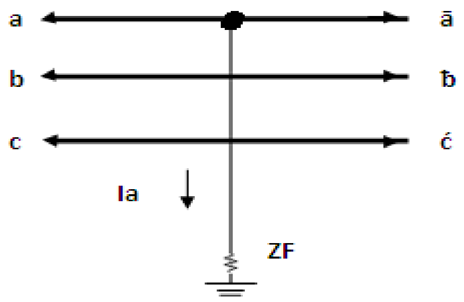

Figure 3.13: Line-to-ground fault in a three-phase system

The fault current is:

$$I_f = \frac{3Ea}{Z^{1+}Z^{2+}Z^{0}+3Zf} \qquad (3.28)$$

3- Line-to-Line Fault

Figure 3.14 shows a line-to-line fault. A short-circuit occurs between phases b and c, through a fault impedance. The fault current circulates between phases b and c, flowing back to source through phase b and returning through phase c

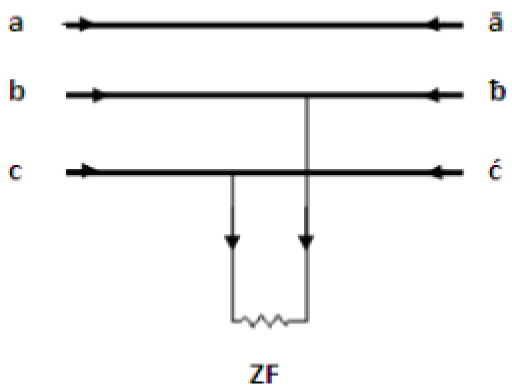

Figure3.14: Line-to-line fault in a three-phase system

The faults current is

$$I_f = \frac{\mathrm{Ea}}{\mathrm{Z}^1 + \mathrm{Z}^2 + \mathrm{Zf}} \qquad (3.29)$$

4- Double Line-to-Ground Fault

A double line-to-ground fault is shown in Figure 3.15 Phases b and c go to ground through a fault impedance. The current in the ungrounded phase is zero

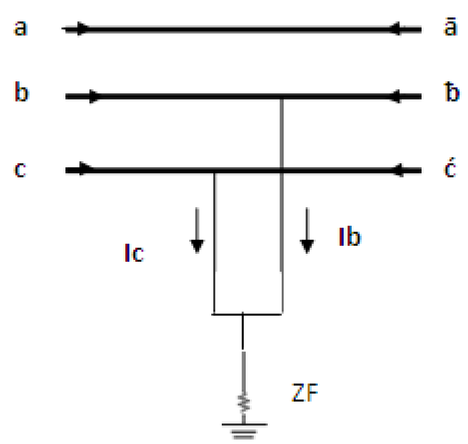

Figure3.15: Double line-to-ground fault in a three-phase system

The faults current is

$$I_f = \frac{\mathrm{Ea}}{\mathrm{Z}^1 + \frac{\mathrm{Z}^2(\mathrm{Z}^0 + 3\mathrm{Zf})}{\mathrm{Z}^2 + \mathrm{Z}^0 + 3\mathrm{Zf}}} \qquad (3.30)$$

3.4 Models of three phases three windings transformers

There is two model of transformers for the two studies discussed below

3.4.1 Models of three phases three windings transformers at short circuit

Three identical single phase three winding transformers can be connected to form a three phase bank. Figure 3.15 and Figure 3.16 show the general per unit sequence networks of a three phase three winding transformer. Instead of labelling the windings 1, 2, and 3 as was done for the single phase transformer, the letters H, M, and X are used to denote the high, medium, and low voltage windings, respectively. By convention, a common Sbase is selected for the H, M, and X terminals, and voltage bases VbaseH, VbaseM, and VbaseX are selected in proportion to the rated line-to-line voltages of the transformer.

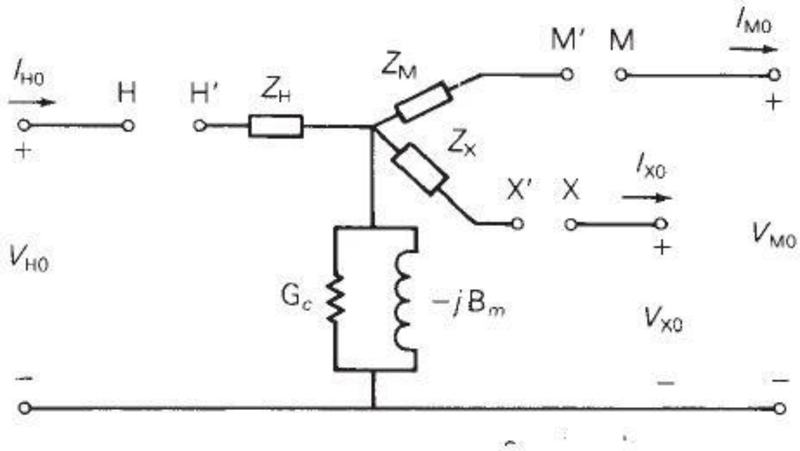

Figure 3.15: Per unit zero sequence network

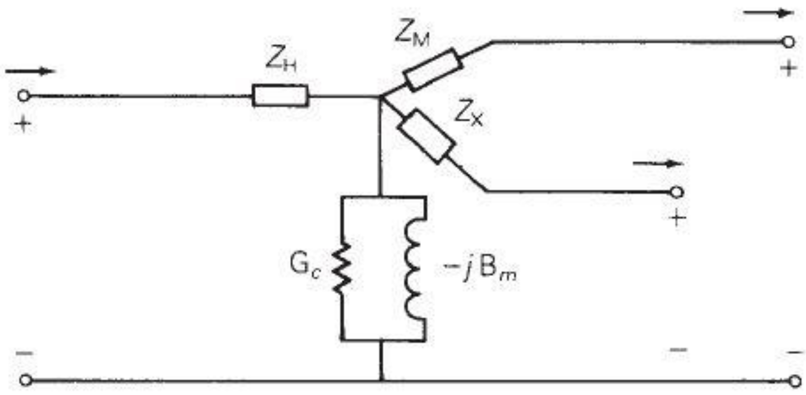

Figure 3.16: Per unit positive or negative network

3.4.2 Models of three phases three windings transformers at short circuit

The flow of real power along a transmission line is determined by the angle difference of the terminal voltage, and the flow of reactive power is determined by the magnitude difference of the terminal voltage. Real and reactive powers can be controlled by use of tap changing transformers and regulating transformers.

In a tap changing transformer, when the ratio is at the nominal value, the transformer is represented by series admittance y_t in per unit. With off nominal ratio, the per unit admittance is different from both sides of the transformer, and the admittance must be modified to include the effect of the off nominal ratio.

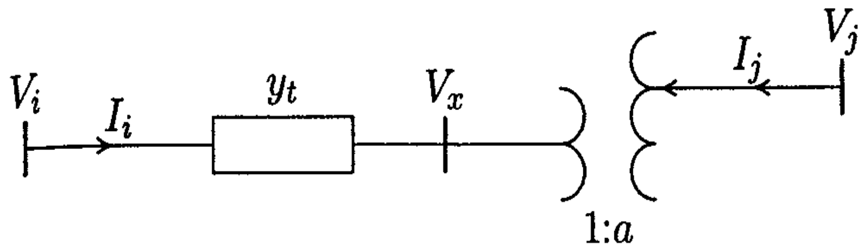

Figure 3.17: Transformer model with tap setting ratio a:1

$$V_x = \frac{1}{a} V_j \quad (3.31)$$

$$I_i = -a^* I_j$$

$$I_i = y_t(V_i - V_x)$$

$$I_i = y_t V_i - \frac{y_t}{a} V_j \quad (3.32)$$

$$I_j = -\frac{1}{a^*} I_i$$

$$I_j = -\frac{y_t}{a^*} V_i + \frac{y_t}{|a|^2} V_j \tag{3.33}$$

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_t & -\frac{y_t}{a} \\ -\frac{y_t}{a^*} & \frac{y_t}{|a|^2} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{3.34}$$

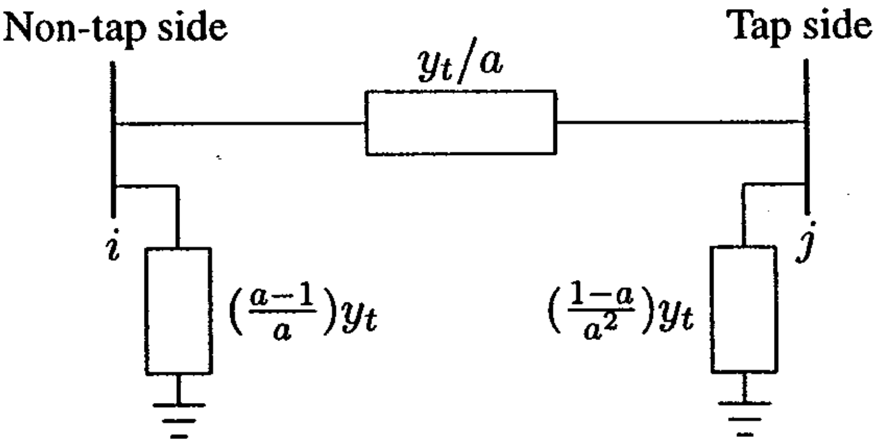

Figure 3.12: Equivalent circuit for a tap changing transformer