Sudan University of Science and Technology

College of Graduate Studies

Tri-linear Soft Couplings Running in Five Dimensional Minimal Super-symmetry Standard Models

نشأة ثابت التفاعل الثلاثي الخطي في نموذج التماثل الفائق في خمسة أبعاد

A dissertation submitted to the college of graduate studies, in partial fulfillment of the requirements for the degree of M. Sc. in Physics.

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الآية

قَالَ تَعَالَى:

﴿ قُلْ سِيرُوا۟ فِى ٱلْأَرْضِ فَٱنظُرُوا۟ كَيْفَ بَدَأَ ٱلْخَلْقَ ثُمَّ ٱللَّهُ يُنشِئُ ٱلنَّشْأَةَ ٱلْءَاخِرَةَ إِنَّ ٱللَّهَ عَلَىٰ كُلِّ شَىْءٍ قَدِيرٌ ﴾

صدق الله العظيم
سورة العنكبوت الآية (20)

I

Dedication

To

Dearest people in my life

I dedicate this work

Acknowledgments

Abstract

In this dissertation, we derived the Renormalization Group Equations (RGEs) for five dimensional Minimal Super-symmetric Standard Model (MSSM) plus additional fields $F^{\pm}$. We found a successful unification of gauge coupling constants in this model very close to: $10^{5.6}$ GeV. Yukawa couplings seemed to be tending to unify at some high scale and approximately vanish beyond the unification scale of the gauge couplings, i.e. $10^{5.6}$ GeV. We showed that a five dimensional (5D) MSSM+$F^{\pm}$ with compactification scale of 15 TeV through powers law running generate a large tri-linear soft coupling A_t at low scales ($\sim 10^{5.6}\ GeV$). This value of A_t is governed and driven by the size of the gluino mass M_3, which is necessary to be above collider bounds, however its large value dose not depends on how super-symmetry is broken. We have assumed that A_t vanish at the unification energy scale and it is entirely generated through renormalization group equation.

ملخص البحث

في هذه الاطروحة, تم إشتقاق معادلات المجموعة المعايرة في نموذج التماثل الفائق زائدا جسيمات اضافية في خمسة ابعاد. وجد ان ثوابت توحيد القوى توحدت في هذا النموذج عند طاقة $10^{5.6}$ قيقا الكترون فولت. أيضا ثوابت يوكاوا يمكن توحيدهم عند مقياس التوحيد تقريبا وتتلاشي هذه الثوابت عند مقياس طاقة التوحيد $10^{5.6}$ قيقا الكترون فولت. أثبت ان في نظرية الخمس ابعاد لنموذج التماثل الفائق زائدا جسيمات اضافية عند طاقة 15 تيرا الكترون فولت خلال قانون القوة الاسي يولد ثابت تقاعل فائق ثلاثي A_t كبير عند طاقة منخفضة. وهذه القيمة الكبيرة للثابت A_t يتحكم ويقودها كتلة القولينو M_3 ، وبالضرورة يجب ان تكون هذه القيمة أعلي من حد المصادم وهذه القيمة الكبيرة لاتعتمد علي كيفية كسر التماثل الفائق. وقد فرض أن هذا الثابت A_t يتلاشي عند مقياس طاقة التوحيد وتتولد قيمته عن طريق معادلة المجموعة المعايرة.

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Introduction

1.1 Introduction

According to present knowledge at low energy the standard model (SM) of particle physics has no inconsistencies and agrees with the experiments, however the standard model have no explanation to several issues, such as the origin of fermion masses and their associated mixing angles, gauge couplings unification, hierarchy problem that is why the weak scale is much smaller than Planck scale, and other incomplete list of unsolvable problems that will be discussed later in chapter two. These issues imply the existence of some new physics beyond the standard model, among other model super-symmetry (SUSY) is considered to be the best candidate as a theory beyond the standard model. Super-symmetry has a minimal version called Minimal Super-symmetric Standard Model (MSSM) dealt with some problems with the SM; it solved the gauge couplings unification and solves the hierarchy problem order by order in the perturbation theory. SUSY contain a good candidate for dark matter (Ammar Ibrahim Abdalgabar, May 21,2014).

An important feature of extra-dimensional models is the impact of the large number of KK modes on the renormalization group (RG) running of physical parameters. The RG running in explain extra-dimensional models has been investigated and studied. It has been shown that the RG evolution changes from the typical logarithmic running in four-dimensional standard models to an effective power-law running at high energies in extra dimension model. This means that sizable running could take place at relatively low energy scales. As such Extra space-time dimensions naturally lead to unification to gauge coupling constant at intermediate mass scales, and moreover it provides a natural mechanism for explaining the Yukawa fermion couplings hierarchy this has potential to address

the fermion masses hierarchy problem. In extra dimension Yukawa couplings too evolve with a power-law dependence on the mass scale (N.~Maru, 2010).

1.2   Aim of the Research:

The main objectives is to derive the renormalization group equations for the Yukawa coupling constants at one loop level in extra dimension model, and to study the evolution of Yukawa couplings as function of energy scale in universal extra dimension model.

1.3   Problem of the Research:

Many unified theories have been proposed, but we need data to choose which, if any, of these theories describes nature. In many grand unified theory the gauge couplings constants (which define the electromagnetic, weak and strong interactions or forces, are combined into one single force) are predicted to meet at some high-energy unification scale. In the standard model the gauge; couplings do not meet at single point. However, the unification works very well in Super-symmetry theory but at high scale approximately10^{16}GeV through the lograthm, such a high-energy scale is beyond the reach of any present or future experiments. Extra dimensions offer power law running, that brings down the unification scale to low energy range (N.~Maru, 2010). For MSSM plus additional fields $F^{\pm}$ we found a successful unification of gauge coupling constants in this model very close to: $10^{5.6}$ GeV, through power law .Standard model have no explanation to several issues, such as the origin of fermion masses and their associated mixing angles, gauge couplings unification, hierarchy problem that is why the weak scale is much smaller than Planck scale.

1.4 Scope of Methodology of the Research:

We solve the set of RGEs equations by using a dedicated computer soft were program mathematic version 9.

1.5 Outlines of the Dissertation:

The outline of the dissertation is as follow:

Chapter one dealt with a general introduction to the topic. In chapter two we will discuss the theory of the standard model, super-symmetry and extra dimension models. Chapter three shall concern with the technique of renormalization group equations. Chapter four will present our numerical results, discussions and conclusions.

Chapter Two

Introduction to the Standard Model and Beyond

2.1 Introduction

Before trying to go beyond the standard model, let us examine it more closely, paying attention to those features that might be relevant to our attempts to go beyond. This chapter will present a general review of the standard model of particle physics. Its mathematical foundation and why do we need new physics (Beyond the standard model).

2.2 What is The Standard model?

The Standard Model (SM) of particle physics is a remarkably successful theory even through it has some limits describing the interactions among elementary particles. Its predictions have been tested experimentally to a high level of accuracy, such as structure of neutral and charged current which agree with experiment. The standard model asserts that material in universe is made up of elementary fermions interaction through field; the particle associated with the interaction field is called boson. This elementary particle in the standard model is arranged by the following:

$$Quarks = \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \tag{2·1}$$

$$Leptons = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{2·2}$$

$$Gauge\ bosons \left\{ \begin{pmatrix} photon\ \gamma \\ Weak\ gauge \\ gluon\ g \end{pmatrix} W^{\pm}, Z^{0}, Higges\ boson\ H \right\}$$

Quarks and leptons are fundamental building blocks of matter; all of them are fermions and have spin ($\frac{1}{2}$), they are classified as left- handed isospin doublet and right- handed isospin singlet's and will be described by the Dirac equation.

Quarks interact through the electromagnetic and weak interaction and also through the strong interaction. Leptons interact only through the electromagnetic interaction (if they are charged) and the weak interaction. Gauge bosons having spin (1) are mediators of interactions between quarks or leptons; there is massless boson, the photon, and three massive ones, the W^{+}, W^{-} and Z_{0} boson. Electromagnetic, weak, strong interactions are mediated by photon (γ), weak bosons $W^{\pm}$, Z_{0} and gluons respectively. Interaction strength depends on which gauge bosons propagate between quarks or leptons. The Higgs boson with spin (0) is introduced for the Higgs mechanism to work, which is operative in the theories with spontaneous symmetry breaking of local gauge symmetry (J.donoghue, 1994).

Table 2·1 The SM fields with their representation under $SU(3)_c$ and $SU(2)_L$ and their charges under $U(1)_Y$ and $U(1)_{EM}$ Q is electric charge and S is the spin of the field.

Field	Notation	$SU_{(3)c}$	$SU_{(2)L}$	$U_{(1)Y}$	$U_{(1)EM}$
Quarks $S = \frac{1}{2}$	$Q_L = \begin{pmatrix} u_l \\ d_l \end{pmatrix} \begin{pmatrix} c_l \\ b_l \end{pmatrix} \begin{pmatrix} t_l \\ s_l \end{pmatrix}$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
	u_R, c_R, t_R	3	1	$-2/3$	$2/3$
	$b_R,\ s_R, d_R$	3	1	$1/3$	$-1/3$

Leptons $S = \frac{1}{2}$	$L_l = \begin{pmatrix} \nu_e \\ e_l \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_l \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	1	2	$-2/3$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	e_R, μ_R, τ_R	1	1	1	-1
Gauge $S = 1$	g	8	1	0	0
	$W^3, W^\pm$	1	3	0	$0, \pm 1$
	B	1	1	0	0
Higgs $S = 0$	$\Phi = \begin{pmatrix} \Phi_+ \\ \Phi_0 = \frac{1}{\sqrt{2}}(v + h + i\Phi_0) \end{pmatrix}$	1	2	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

2.3 Symmetries and particle content in standard model:

We have made all the preparations to write down a gauge invariant lagragian. We now only have to pick the gauge group and matter content of the theory. It should be noticed that there are no theoretical reasons to pick a certain group or certain matter contact. To match experimental observations we pick the gauge group for the standard model to be

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \qquad (2 \cdot 3)$$

Where

1- The $SU(3)_C$ gauge group or color group is strong interaction, this group acts on the quarks which are the elementary constituents of matter and the interaction force is mediated by gluons which are the gauge boson of the group. The quarks and the gluons are colored fields (L.F.Li, 1991). The

corresponding coupling is denoted by (α), the $SU(3)_C$ color symmetry is exact and consequently the gluons are massless.

2- The theory of the strong interaction based on colors of $SU(3)$ is called Quantum Chromo Dynamics (QCD).

3- The $SU(2)_L \otimes U(1)_{EM}$ is the gauge group of the unified weak and electromagnetic interactions where $SU(2)_L$ is the weak isospin group, acting on left-handed fermions, and $U(1)_Y$ is the hypercharge group.

4- The $SU(2)_L$ group has three gauge bosons are denoted by $W_\mu^1, W_\mu^2, W_\mu^3$. None of these gauge bosons (and neither B_μ) are physical particles, linear combinations of these gauge bosons will make up the photon as well as the $W^\pm$ and Z boson – as matter content for the first family, we have

$$q_l \equiv \begin{pmatrix} u_l \\ d_l \end{pmatrix}; u_R; d_R \text{ And } l_l \equiv \begin{pmatrix} \nu_e \\ e_l \end{pmatrix}; e_R; \nu_R \tag{2 · 4}$$

Under $SU(3)$ the lepton fields are l_l, e_R, ν_R, singlets, i.e. they do not couple to gluons. Quarks on the other hand form triplets under $SU(3)$. The strong interaction dose not distinguishes between left and right handed particles. However, since we ultimately want massive weak gauge bosons, we will have to break $U(1)_Y \otimes SU(2)_L$ gauge group spontaneously, by introducing some type of Higgs scalar (P.~W.~Higgs, 1964).

2.4 The Higgs Mechanism:

As will be presented in the next section, a Dirac mass term will violate the local gauge symmetries. As such we need a mechanism that give mass to the SM particles and keeps the lagragian invariant under gauge symmetries this can be done through the mechanism of spontaneous gauge symmetry breaking also known as the Higgs mechanism. This mechanism adds a new complex scalar field Φ

which is doublet under the $SU(2)_L$ group, a singlet with respect to $SU(3)_c$ and has hypercharge $Y_\Phi = 1$ (P.~W.~Higgs1964).

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \begin{pmatrix} \Phi_1 & + & i\Phi_2 \\ \Phi_3 & + & \Phi_4 \end{pmatrix} \qquad (2\cdot6)$$

Where $\Phi_1, \Phi_2, \Phi_3 \ and \ \Phi_4$ are real scalar. This new scalar Φ adds extra terms to the SM lagragian:

$$\mathcal{L}_{Higgs} = \left(D_\mu\Phi\right)^\dagger\left(D_\mu\Phi\right) - V(\Phi) \qquad (2\cdot7)$$

Where the covariant derivative D_μ is defined as

$$D_\mu = \partial_\mu - \frac{\overline{ig}}{2}B_\mu - \frac{ig\sigma^a}{2}W_\mu^a \qquad (2\cdot8)$$

The general gauge invariant renormalizable potential involving ϕ is given by

$$V(\Phi) = \frac{1}{2}\mu^2\Phi^\dagger\,\Phi + \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right) \qquad (2\cdot9)$$

Equation (2.9) describes the potential, which involves two new real parameters μ and λ. We demanded $\lambda>0$ for the potential to be bounded; otherwise the potential is unbounded from below and there will be no stable vacuum state. μ takes the following two values.

- $\mu^2 >0$ then the vacuum corresponding to $\Phi =0$, the potential has a minimum at origin.
- $\mu^2 <0$ then the potential develops a non-zero vacuum Expectation value (VEV) and the minimum is a long a circle of radius $\frac{v}{\sqrt{2}} = \frac{246}{\sqrt{2}}$

Minimizing the potential we get

$$\Phi_1^2 + \Phi_2^2 + \Phi_3^2 + \Phi_4^2 = \frac{\mu^2}{\lambda} = v^2 \qquad (2\cdot 10)$$

As such, we need to choose one of these minima as the ground state($\Phi_3 = v$ and $\Phi1=0,\Phi2=0,\Phi4=0$. Thus the vacuum doses not have origin have the original symmetry of the lagragian, and therefore spontaneously break the symmetry.

In other words, the lagragian is still invariant under the neutral direction as photon is neutral, so Φ becomes.
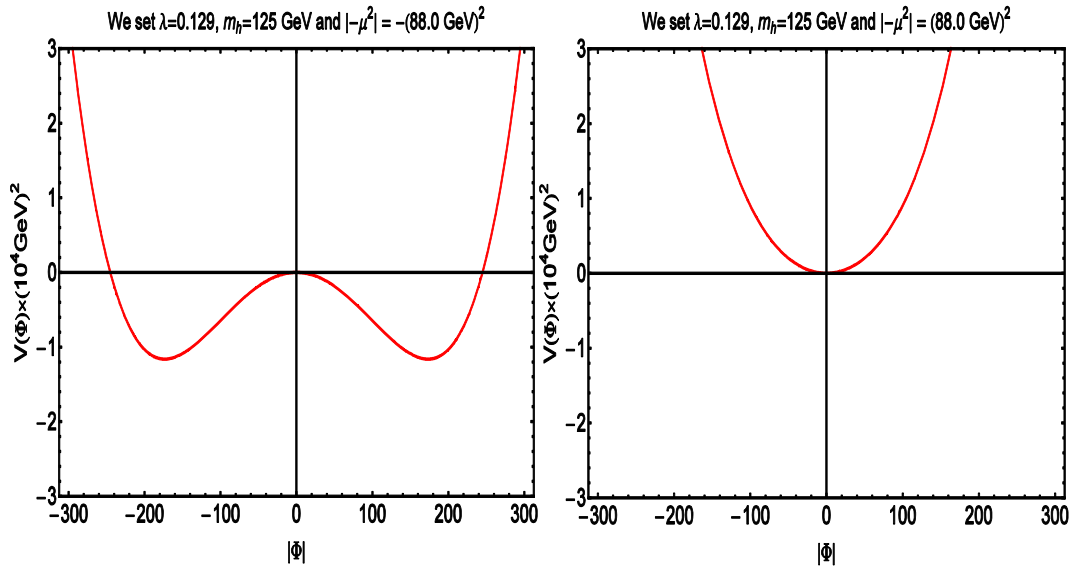
Fig 2.1 shows that The Higgs potential $V(\Phi)$ with: in the left panel, the case $\mu^2 < 0$ and the right panel for the case$\mu^2 > 0$ as a function of $|\Phi| = \sqrt{\Phi^\dagger \Phi}$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad (2\cdot 11)$$

With this particular choice of the ground state, the electroweak gauge group

$U(1)_Y \otimes SU(2)_L$ Is broken to electromagnetism, $U(1)_{em,}$

$$U(1)_Y \otimes SU(2)_L \xrightarrow{\langle \Phi \rangle} U(1)_{em} \qquad (2 \cdot 12)$$

2.5 The Lagragian of the SM:

The SM lagragian can be written as

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge\ sector} + \mathcal{L}_{fermion\ sector} + \mathcal{L}_{Higgs\ sector} + \mathcal{L}_{Yukawa\ sector}$$
$$+ \mathcal{L}_{Ghost\ sector} \qquad (2 \cdot 13)$$

2.5.1 Gauge Sector

The gauge sector is composed of 12 gauge fields A_μ^a which responsible for the interaction among the fermion fields; the photon (γ) mediate the electromagnetic interaction, the weak interactions mediated by the three weak gauge bosons $(W^\pm \text{and } Z)$ and eight gluons $\left(g_{\alpha,} \alpha = 1,2,3,4,5,6,7,8\right)$ mediate the strong interactions (A.J.G.hey, 1993). The gauge field lagragian is given in terms of field strength tensors as:

$$\mathcal{L}_{gauge\ bosons} = \frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \qquad (2 \cdot 14)$$

Where repeated means summation over that index, and μ, v takes 0, 1, 2, 3. Where the field strength tensors for non-Abelian theories are given by:

$$G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G^{A\mu} - ig_s F^{ABC} G_\mu^B G_\nu^C \qquad (2 \cdot 15)$$

Represents the $SU(3)_c$ field strength, g_s is the coupling strength of the strong interaction, A, B, C run from 1 to 8 and F^{ABC} are the (anti-symmetric) structure constants of $SU(3)$, and obey the lie algebra for the group generator t^A

$$[t^A, t^B] = iF^{ABC}t^C \tag{2 · 16}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - ig\epsilon^{abc}W_\mu^b W_\nu^c \tag{2.17}$$

Represents the $SU(2)_L$ field strength, a, b, c, run from 1 to 3 and $\in^{abc}$ is totally antisymmetric there index tensor with $\in^{123} = 1$, g is the coupling strength of the weak interaction.

The field strength of the $U(1)_Y$ gauge boson which has the same form as electromagnetism is given by:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2 · 18}$$

2.5.2 Fermion Sector

The SM contains three copies of chiral fermions (generations) with different gauge transformations. The fermionic lagragian written in the usual covariant Dirac equation

$$\mathcal{L}_{fermonic} = \sum_f if\gamma_\mu D_\mu F \tag{2 · 19}$$

The covariant derivative is given:

$$D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^{-a} - ig\frac{\sigma^a}{2}W_\mu^a - i\bar{g}\frac{1}{6}B_\mu \right) \begin{pmatrix} u \\ d \end{pmatrix}_L \tag{2 · 20}$$

$$D_\mu u_R = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - i\bar{g}\frac{2}{3}B_\mu \right) u_R \tag{2 · 21}$$

And

$$D_\mu \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a - i\bar{g} \frac{1}{2} B_\mu \right) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad (2 \cdot 22)$$

$$D_\mu e_R = \left(\partial_\mu + ig' B_\mu \right) e_R \qquad (2 \cdot 23)$$

Here γ_μ are the usual Dirac matrices, g' is coupling strength of the hypercharge, σ^a are the generators of $SU(2)_L$(simply the Pauli matrices), and λ^a are generators of $SU(3)_c$(the Gell man matrices).

Note that gauge symmetry forbids a mass term for fermions (quarks and leptons) and gauge bosons. A mass term would break the gauge invariance$SU_L(2) \otimes U(1)_Y$.

The mass term of gauge bosons W and Z and the fermions experimentally, so we need to give mass to these particles.

The masses in The SM are generated through a different mechanism, the Higgs mechanism, which will be discussed at length in pervious sections.

2.5.3 Higgs Sector:

The Higgs –gauge boson interactions generated by the covariant derivative

$$\mathcal{L}_H = \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a + ig' B_\mu \right) e_R \qquad (2 \cdot 23)$$

Where

$$D_\mu = \partial_\mu - ig_\mu W^{\mu,a} t^a - ig_\mu Y_\mu B^\mu \qquad (2 \cdot 24)$$

2.5.4 Yukawa Sector:

From symmetry considerations, we are free to add gauge-invariant interaction between the scalar fields and fermions; these are called the Yukawa terms in the lagragian and they are responsible of generating fermions masses and mixing between different families:

$$\mathcal{L}_y = Y_d^{ij} q_L^i \Phi d_R^j + Y_u^{ij} q_L^i \ \tilde{\Phi} u_R^j + Y_e^{ij} L_L^i \Phi e_R^j + h \cdot c \tag{2 \cdot 25}$$

Where the Y are 3×3 complex matrices, the so-called Yukawa coupling constants, h.c. means the Hermitian conjugate and $\tilde{\Phi}$ is defined by:

$$\tilde{\Phi} = \begin{pmatrix} \overline{\Phi_2^*} \\ \Phi_1^* \end{pmatrix} \tag{2.26}$$

2.5.5 Gauge Fixing and Ghosts:

Gauge fixing is necessary when the gauge fields are quantized. Quantization means to develop a path integral formalism for the gauge theory. The path integral is diverging as one integrate over an infinite set of gauge-equivalent configuration, here the gauge fixing is used to pick up on arbitrary representative, therefore, giving meaning to the path integral. On other hands, the gauge invariance we look for in gauge theory, a naive path integral approach would spoil it. The solution is given by what is called the fadder-popov procedure, where they introduced an identity expression consisting of a functional integral over a gauge fixing conductions. Time a functional determinant over anti commuting fields in the path integral. The latter gives the rise to what is known as ghost fields, which keep the gauge freedom within the theory, but are not physical particles (because ghost violate the spin-statistics relation). As such, we need to add terms in the lagragian like

$$\mathcal{L}_{Gauge-fixing} = -\frac{\xi}{2}\left(\partial_\mu A^\mu\right)^2 \tag{$2\cdot 27$}$$

And

$$\mathcal{L}_{Ghost} = C'_b \partial^\mu D_\mu C_a^{ab} \tag{$2\cdot 28$}$$

2.6 Problems with the Standard Model:

The SM is a mathematically-consistent renormalizable gauge field theory which is consistent with all experimental facts (Majee, March, 2008). However the SM dose not has answer of the following list:

2.6.1 Unification of Coupling:

At present the most direct phenomenological evidence in favour of super-symmetry is obtained from the unification of coupling in Grand Unification Theory (GUTs).

Precise LEP (Large Electron Positron) data on $\alpha_s(M_z)$ and $sin\,\theta_w^2$ ·Confirm what was already known with less accuracy: Standard one-scale GUTs. Are in agreement with the present, very precise and experimental result. According to a recent analysis; if one starts from the known values of $sin\,\theta_w^2$ and $\alpha(M_z)$the results:

$$\alpha_s = 0\cdot 073 \pm 0\cdot 002 \; (Standard\; GUTs) \tag{$2\cdot 29$}$$

$$\alpha_s = 0\cdot 129 \pm 0\cdot 010 \quad (SUSY\; GUTs) \tag{$2\cdot 30$}$$

2.6.2 Dark Matter:

The SM is treated as an effective theory at a natural cut-off scale. The ultimate goal in particle physics is to unify all the fundamental forces in nature. Moreover, the SM does not have any dark matter candidates, as opposed to observation or observed cosmology (Collaboration], 2011) .

2.6.3 Neutrino Masses:

The SM does not account for the neutrino oscillations and their non-zero masses. Neutrinos are massless particles. However, neutrino oscillation experiments have shown that neutrinos do have masses (A.D.Martin, 1984).

2.6.4 Gauge Hierarchy Problem:

The hierarchy problem is the equation of why there is such a huge difference between the electro weak scale $M_{Ew} = O(100)$ GeV and the Planck scale $M_{pl} = O(10^{18})$ GeV. This is also known as the naturalness problem (Weinberg S. , 1996).

2.6.5 Gravity is not included:

Though the unification of the electromagnetic and weak interactions was achieved in the SM and the strong interaction appears to be part of the unification, the SM does not include the effects of gravity. Note that the effects of gravity become important at energies of the order of the Planck scale, $M_{PL} = 10^{18}$ GeV (J.donoghue, 1994).

2.7 Beyond the Standard Model:

Most of the Beyond the Standard Model (BSM) physics have been constructed to solve the gauge hierarchy problem. The models that have been discussed in the

literature maybe categorized and (BSM) refers to the theoretical developments needed to explain some unanswered issues in SM raised in previous section, such as super-symmetry, extra dimensions or a combination between them 5D super-symmetric (H.~-U.~Yee, 2003). Here in this dissertation we will try to study all of them to search for unification.

Follows:

2.7.1 Super-symmetric (SUSY):

Super-symmetry is space-time symmetry, which relates the bosonic degree of freedom to the fermionic degree of freedom (Ammar Ibrahim Abdalgabar, May 21,2014).

- The beautiful idea of (SUSY) helps to solve the gauge hierarchy problem the one loop radiative correction for Higgs mass due to scalar particles in the loop (J. Louis, 1998).
- Super-symmetry leads to unification of gauge coupling when the gauge couplings are extrapolated to high scale from their measured values at the weak scale.
- Super-symmetry triggers Electroweak Spontaneous Breaking (EWSB):

The drive spontaneous symmetry breaking in SM, one requires to set the scalar mass in the lagragian, to negative value by hand.

- In super-symmetry transformation a bosons changes to a fermion and vise versa. Thus Q is generator of this transformation then:
- $Q|fermion> \equiv |boson> \ and \ Q|boson> \equiv |fermion>$
- Super-symmetry is Idea that there is a super partner for each elementary particle. Selection smuon, stau, photino and higgsino, etc.

Table 2·2 Super-symmetry partners with the standard model (Ray, Beyond The Standard Model:Some Aspects of Supersymmetry and Extra Dimension, 2010)**.**

Name		$Spin0$	$Spin\left(\dfrac{1}{2}\right)$	$SU(3)_c, SU(2)_L, U(1)_Y$
Squarks Quarks	$\begin{cases} Q \\ \bar{u} \\ \bar{d} \end{cases}$	$(\bar{u}_l, \bar{d}_l)$ $u_R^{\widetilde{~}*}$ $d_R^{\widetilde{~}*}$	(u_l, d_l) $u_R^{\dagger}$ $d_R^{\dagger}$	$\left(3,2,\dfrac{1}{6}\right)$ $\left(3,1,\dfrac{-2}{3}\right)$ $\left(3,1,\dfrac{1}{3}\right)$
Sleptons Leptons	L E	$(\tilde{v}, \tilde{e}_l)$ $e_R^{\widetilde{~}*}$	(v, e_l) $e_R^{\dagger}$	$\left(1,2,\dfrac{-1}{2}\right)$ $(1,1,1)$
Higgs, Higgsions	H_1 H_2	(H_1^+, H_1^0) $(H_2^0, \overline{H_2})$	$(H_1^{+\sim}, H_1^{0\sim})$ $(H_2^{0\sim}, H_2^{-\sim})$	$\left(1,2,\dfrac{+1}{2}\right)$ $\left(1,2,\dfrac{-1}{2}\right)$
		$Spin\left(\dfrac{1}{2}\right)$	$Spin(1)$	
Gluino,		$\tilde{g}$	G	$(8,1,0)$
Gluon		$W^{\sim\pm}, W^{\sim 0}$	$W^{\sim\pm}, W^{0\sim}$	$(1,3,0)$
Winoso w-boson Bino, B-boson		$\widetilde{B_0}$	B_0	$(1,1,0)$

2.7.2 Extra Dimensions:

In the SM we have seen that the hierarchy problem is arising due to the huge ratio of the Planck scale (mpl) or the Guts scale to electroweak scale. As discussed in the previous section, super-symmetry provides a beautiful way to solve this hierarchy problem. In that case, the super-symmetry particles are situated around the TeV scale. Actually to solve the hierarchy problem if we incorporate any new physics it should appear around that scale to address the huge ratio. More recently, a new kind of physics, Extra Dimension (ED), was introduced in particle physics. If we can distinguish a fermion from a bosonic particle by measuring the spin of the particle at Large Hadron Collider (LHC) or International Linear Collider (ILC), then we can have distinct signature of the physics of extra dimension from that of super-symmetry.

2.7.2.1 Scalar Particle in ED:

In addition to the four space-time co-ordinates $X_{(x,t)}$, let us denote the extra space-type co-ordinate by y, compactified on circle or radius R. Thus, the lagragian of a free complex scalar $\Phi_{(x,y)}$ with mass m will be a function of both x and y co-ordinates with a condition that at $Y=2\pi R$, will match with that at $Y=0$, i.e. it has a periodicity of $2\pi R$ along the Y direction. So one can expend it in a Fourier series as

$$\Phi(x,t) = \frac{1}{\sqrt{2\pi R}} \Phi_{0(x)} + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left[\Phi_{n(x)}^{+} \cos\left(\frac{ny}{R}\right) + \overline{\Phi}_{n(x)} \sin\left(\frac{ny}{R}\right) \right] \qquad (2 \cdot 31)$$

The five dimensional actions is given by

$$S_{\Phi}^{5} = \frac{1}{2} \int d_{x}^{4} \, d_{y} \{ (\partial_{\Phi}^{A})^{\dagger} (\partial_{\Phi}^{A}) - m^{2} \Phi^{\dagger} \Phi \ \} \qquad (2 \cdot 32)$$

With $A = 0, 1, 2, 3, 5$

With the use of equation $(2 \cdot 30)$ if we replace the scalar field ϕ and integrate out the extra dimension Y, then the action will correspond to a large number kaluza–klein (KK) modes as

$$S_{[\Phi]}^4 = \frac{1}{2}\int d_x^4 \left\{ (\partial^\mu \Phi^0)^\dagger (\partial_\mu \phi) - m^2 \Phi^{0\,\dagger}\Phi \right\}$$

$$+ \int d_x^4 \left\{ (\partial^\mu \Phi_n^-)^\dagger (\partial_\mu \Phi_n^-) - m^2 \Phi_n^{-\dagger}\Phi_n^- \right\} \qquad (2 \cdot 33)$$

Where the n-th KK mode mass is given as

$$M_n^2 = \frac{m^2 + n^2}{R^2} \qquad (2 \cdot 34)$$

In four dimensional effective theory, thus, in addition to the zero mode field, we are getting two different sets –one is even and another odd under the transformation $Y \to -y$– of field when the extra space dimension is compactified on the circle S^1 (H. $\sim - $ U. $\sim$Yee, 2003)$^.$

2.7.2.2 Fermion Particle in ED

In some models only the scalar bosons are allowed to access the extra dimensions while fermions are kept in a fixed point of the extra dimension, called "brane". In such cases the above compactification is quite natural but what happens if we intend to allow the fermions as well to access the extra dimension? Do we have the same set of Kaluza-Klein modes for the fermionic fields or something else?

Let us consider a fermion in the five-dimensional field, where the extra space dimension is compactified the way we discussed. The five-dimensional spinor can be written as a two component four-dimensional spinor

$$\Psi = \begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix} \tag{2.35}$$

Note that in the five-dimensional field theory, one can construct the five Γ^A matrices with

A=0, 1,2,3,5, from the usual four-dimensional ones as follows:

$$\Gamma_\mu = Y_\mu \quad and \quad \Gamma^5 = iY^5 \tag{2·36}$$

In 5d the fifth component of the Γ_A is constructed from the γ^5 matrix, which is used, in four dimensions, to define the chiral operator $P_{R/l} = (1 \pm \gamma^5)$. So, in five-dimensions, and it is true for any odd number of dimensions, there is no chiral operator. To be clear, in eqn. (2·30) the Subscripts L and R are just two component notations only.

Let us consider the action for a massless fermion as:

$$S = \int d^4 x dy\, i\Psi\Gamma^A\partial_A\Psi = \int d_x^4 d_y \left(i\Psi Y_\mu \partial_\mu \Psi + \Psi Y^5 \partial y \Psi\right) \tag{2·37}$$

Due to the symmetry of the fermion field at the point $y = 0$ and $y = 2\pi R$, we can have the:

Fourier expansion of the field as:

$$\Psi_{L/R(x,y)} = \frac{1}{\sqrt{2\pi R}} \Psi_{L/R(x)}^0$$

$$+ \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left\{ \Psi_{L/R(x)}^{+n} cos\left(\frac{ny}{R}\right) + \Psi_{L/R(x)}^{-n} sin\left(\frac{ny}{R}\right) \right\} \tag{2·38}$$

Once we put these fermions – eqn. (2.37) in the above action eqn. (2·38) we end up with a few phenomenological problems.

For example, let us use the zero mode term in eqn. (2.37), then we have

$$S_0 = \int d^4x \, \gamma \left\{ \Psi_L^{0-} i \Upsilon_\mu \partial_\mu \Psi_L + \Psi_R^{0-} i \Upsilon_\mu \partial_\mu \Psi_R \right\} \tag{2·39}$$

Thus, for each massless field in five-dimension we are having two massless zero modes in the four-dimensional effective theory. The four-dimensional fermion is thus vector like in nature.

It is well-known that fermions in the SM are chiral in nature, the left chiral part transforms as a doublet under $SU(2)$ gauge transformations and the right chiral part transforms trivially. If the dimensional reduction doubles the state can we regain our chiral nature of the fermion in its zero modes?

To regain the chiral nature we have to compactify on an $\frac{S_1}{Z_2}$ orbifold instead of a circle. The expansions of different kind of field for the $\frac{S_1}{Z_2}$ orbifold will be discussed. In that case, although the higher KK modes of the chiral fermion behave as vector but the zero modes remains a chiral one.

2.7.2.3. Gauge Fields and Gauge fixing

The lagrangian for an Abelian gauge field and gauge fixing given by:

$$\mathcal{L}_{Gauge+GF} = \int d_{x_5} \left(-\frac{1}{4} F^{MN} F_{MN} - \frac{1}{2\xi} (\partial_\mu A^\mu - \xi (\partial_5 A_5)^2 \right. \tag{2·40}$$

Where ξ is the gauge fixing parameter and $F_{MN} = \partial_M A_N - \partial_N A_M$. The gauge fixing term eliminates the mixing between A_μ and the extra polarization A_5.

In the Feynman-'t Hooft gauge $\xi = 1$, the equations of motion for A_5 can be obtained:

$$(\partial_5^2 - \partial_\mu^2) A_5 = 0 \tag{2·41}$$

Where

$$A_\mu^a(x, y) = (G_\mu^a(x, y), W_\mu^a(x, y), B_\mu(x, y)) \tag{2.42}$$

$$A_\mu^a(x, y) = \frac{1}{\sqrt{2\pi R}} A_\mu^{(0)a}(x) + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left[A_\mu^{(n)a}(x) \cos(\frac{ny}{R}) \right] \tag{2.43}$$

$$\begin{aligned}
\mathcal{L}_{4D} \quad &= \quad -\frac{1}{4} \left(G_{\mu\nu}^{(0)a} G^{(0)a\mu\nu} + G_{\mu\nu}^{(n)a} G^{(n)a\mu\nu} + 2G_{\mu5}^{(n)a} G^{(n)a\mu5} \right) \\
&\quad -\frac{1}{4} \left(W_{\mu\nu}^{(0)a} W^{(0)a\mu\nu} + W_{\mu\nu}^{(n)a} W^{(n)a\mu\nu} + 2W_{\mu5}^{(n)a} W^{(n)a\mu5} \right) \\
&\quad -\frac{1}{4} \left(B_{\mu\nu}^{(0)} B^{(0)\mu\nu} + B_{\mu\nu}^{(n)} B^{(n)\mu\nu} + 2B_{\mu5}^{(n)} B^{(n)\mu5} \right) \tag{2.44}
\end{aligned}$$

2.8 The 5D MSSM Models:

In the 5D MSSM, the Higgs super fields and gauge super fields always propagate in the fifth dimension. However, different possibilities of localization for the matter super fields can be studied. We shall consider the two limiting cases of super fields with SM matter fields all in the bulk or all super fields containing SM matter fields restricted to the brane. When all fields propagate in the bulk, the action for the matter fields Φ_i is (Ammar Ibrahim Abdalgabar, May 21,2014)

$$\begin{aligned}
S_{matter} = \int d_z^8 \, d_{x5} \big[&\bar{\Phi}_i \Phi_i + \Phi_i^c \overline{\Phi_i^c} + \Phi_i^c \partial_\mu \delta(\bar{\theta}) - \bar{\Phi}_i \partial_5 \bar{\Phi}_i \delta(\theta) + \tilde{g}(2\Phi_i^c) V\Phi_i \\
&- 2\Phi_i^c V\overline{\Phi_i^c} + \Phi_i^c X\Phi_i \delta(\bar{\theta}) + \bar{\Phi}_i X^- \overline{\Phi_i^c} \delta(\theta) \big] \tag{2.45}
\end{aligned}$$

Similarly, when

All super fields containing SM fermions are restricted to the brane; the part of the action involving only gauge and Higgs field is not modified, whereas the action for the super fields containing the two becomes:

$$S_{matter} = \int d_z^8\, d_{x5}\delta_y \tag{2 \cdot 46}$$

For N = 1 4D SUSY, the super field formalism is well established: super fields describe quantum fields and their super partners, as well as auxiliary fields as a single object. This simplifies the notation and the calculation considerably. A similar formulation for a 5D vector super field and the super field formulation for matter super-multiples have been developed in Ref. (J. Louis, 1998)Note that in our model the Yukawa couplings in the bulk are forbidden by the 5D N = 1 SUSY.

However, they can be introduced on the branes, which are 4D subspaces with reduced SUSY. We will write the following interaction terms, called brane interactions, containing Yukawa-type couplings:

$$S_{brane} = \int d_z^8 dx_5\delta(x^5)\left(\frac{1}{6}\lambda_{\widetilde{ijk}}\Phi_i\Phi_j\Phi_k\right)\delta(\bar{\theta}) + h\cdot c \tag{2 \cdot 47}$$

Chapter Three

Renormalization Group Equations

3.1 Introduction

This chapter shall discuss the Renormalization Group Equations (RGEs) method and calculation of beta function for gauge couplings and Yukawa couplings in Standard model and Extra dimension model.

3.2 Renormalization Group:

The renormalization group in quantum field theory (QFT) tells us how different couplings behave with energy (L.~-X.~Liu A. a., 2011).

3.2.1 What is Renormalization?

In Quantum Field Theory (QFT), Green function is the most important thing to be calculated. In perturbative QFT these quantities are divergent. The systematic way to remove these divergences is known as **renormalization** (Collins, 1984).

The renormalization theory is implemented to remove all the divergences in loop integrals from the physical measurable quantities. These loop diagrams are supposed to give finite results to the physical quantities but they give infinities instead. This tells us that our theory has missed some information. One might ask where these infinities come from**.** These infinities arise from the integration over all momentum. In other words, the infinities occur because we let our theory go to arbitrary high energy (UV) (Majee, March, 2008).

There are different ways to cancel these infinities. In order to renormalize the theory we need a reference point which is also arbitrary, Different choices of this reference point lead to different sets of parameters for the theory, but physics

should not depend on the arbitrary choice of the reference point and be invariant. This invariance leads to the **renormalization group.**

In quantum field theory it is a useful method to examine the behavior of physics at a different scale knowing the same at some other scale. Thus, measuring the observables in a low energy experiment one can compare with the values predicted from a theory at a higher scale, *e.g.* at the GUT scale and certify about the correctness of the theory. In the standard model, variations of the gauge coupling constants with energy are given by the following renormalization group equations (**RGEs**) (A.~Abdalgabar, 2013).

$$16\pi^2 \frac{dg_i}{dt} = \beta_{SM}(g_i) + \beta_{ED}(g_i) \tag{3.1}$$

$$\beta(g_i) = b_i g_i^3 \tag{3.2}$$

Where i stands for $U(1)_Y, SU(2)_L \text{ and } SU(3)_C$ and the right-hand-side is known as the β function of the corresponding coupling.

In the above equations the coefficient b_i can be calculated for any $SU(N)$ group as

$$b_i = -\frac{11}{3} C_2(G) + \frac{2}{3} n_f C(R) + \frac{1}{3} n_s C_2(R) \tag{3.3}$$

Where:

$C_2(G), C(R) \text{ and } C_2(R)$ refer to the gauge boson, fermionic and Higgs scalar contribution respectively. n_f is number of fermion flavor and n_s is the number of scalar field.

3.3 The Numerical Result for Gauge Couplings Evolution In 5D MSSM:

In 5D MSSM, the one-loop corrections to gauge couplings are given by:

$$16\pi^2 \frac{dg_i}{dt} = bi + \left(S(t) - 1)\tilde{b}_i\right) gi^3 \tag{3.4}$$

$\tilde{b}_i = \left(\frac{33}{5}, 1, -3\right)$, for matter field in the bulk and $\tilde{b}_i = \left(\frac{6}{5}, -2, -6\right)$ for matter field in brane, $t = \ln\left(\frac{\mu}{M_z}\right)$ in the Z boson mass (Z.~-z.~Xing H. a., 2008).

The generic structure of the one-loop RGEs for the gauge couplings is given by:

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 + \pi(S(t) - 1)b_i^{5D} g_i^3 \tag{3.5}$$

Where

$$t = \ln\left(\frac{E}{M_z}\right) \quad and \quad S(t) = e^t M_z R \quad for \quad M_z < E < \Lambda \tag{3.6}$$

Where Λ is the cut-off energy scale, $s(t)$ is the sum of KK states and $b_i^{SM} = \left[-\frac{41}{10}, \frac{19}{6}, 7\right]$, in more details in Appendix.

3.4. Soft Masses:

We expect the gaugino soft masses to run in following:

$$\beta_{M_i}^{(1)} = 2b_{MSSM}^i M_i[t]g_i^2[t] + 2b_{(5D)}^i M_i[t]g_i^2[t](S[t] - 1) \tag{3.7}$$

3.5 Yukawa and Tri-linear Soft Breaking Parameters

The beta function of Yukawa couplings maybe related to the matrices of anomalous dimensions (Ammar Ibrahim Abdalgabar, May 21,2014) .

$$\beta_Y^{ijk} = \gamma_n^i Y^{nij} + \gamma_n^i Y^{ink} + \gamma_n^K Y^{ijn} \tag{3.8}$$

3.5.1 The 4D MSSM Soft Breaking Parameter:

The beta functions of Yukawa couplings are given by

$$\beta_u^{(1)} = 3Y_u Y_u^\dagger Y_u + Y_u Y_d^\dagger Y_d$$

$$- \frac{1}{15} Y_u \left(13g_1^2 + 45g_2^2 + 80g_3^2 - 45\text{Tr}(Y_u Y_u^\dagger) \right) \tag{3.9}$$

$$\beta_{Y_d}^{(1)} = 3Y_d Y_d^\dagger Y_d + Y_d Y_u^\dagger Y_u$$

$$+ Y_d \left(-3g_2^2 - \frac{16}{3} g_3^2 - \frac{7}{15} g_1^2 + \text{Tr}(Y_e Y_e^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger) \right) \tag{3.10}$$

$$\beta_{Y_e}^{(1)} = 3Y_e Y_e^\dagger Y_e + Y_e \left(-3g_2^2 - \frac{9}{5} g_1^2 + \text{Tr}(Y_e Y_e^\dagger) + 3\text{Tr}(Y_d Y_d^\dagger) \right) \tag{3.11}$$

3.5.2 The Four Dimensional Contributions:

The beta functions of tri-linear soft breaking couplings are given by

$$\beta_{A_u}^{(1)} = +2Y_u Y_d^\dagger A_d + A_u Y_d^\dagger Y_d + 3A_u Y_u Y_u^\dagger + 6Y_u Y_u^\dagger A_u + 6\text{Tr}(Y_u^\dagger A_u)$$

$$+ 3A_u \text{Tr}(Y_u^\dagger Y_u) - \frac{13}{15} g_1^2 A_u - 3g_2^2 A_u - \frac{16}{3} g_3^2 A_u$$

$$+ Y_u \left(\frac{26}{15} g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3} g_3^2 M_3 \right) \tag{3.12}$$

$$\beta_{A_d}^{(1)} = +2Y_d Y_u^\dagger A_u + A_d Y_u^\dagger Y_u + 3A_d Y_d^\dagger Y_d + 6Y_d Y_d^\dagger A_d + 6\text{Tr}(Y_u^\dagger A_u)$$

$$+ 3A_u \text{Tr}(Y_u^\dagger Y_u) - 3g_2^2 A_d - \frac{16}{3} g_3^2 A_d - \frac{7}{15} g_1^2 A_d$$

$$+ Y_d \left(\frac{14}{15} g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3} g_3^2 M_3 \right) \tag{3.13}$$

$$\beta_{A_e}^{(1)} = +3A_e Y_e^\dagger Y_e + 6Y_e Y_e^\dagger A_e - 3g_2^2 A_e - \frac{9}{5}g_1^2 A_e + 2Y_e \mathrm{Tr}\left(Y_e^\dagger A_e\right) + 6\mathrm{Tr}\left(Y_d^\dagger A_d\right)$$

$$+ Y_e \left(6g_2^2 M_2 + \frac{18}{5}g_1^2 M_1\right) \tag{3.14}$$

3.5.3 The 5D MSSM Soft Breaking Parameter:

The beta functions of Yukawa couplings in 5D are given by:

$$\beta_{(5D)Y_u}^{(1)}[t] = Y_u\left[\left(6Y_u^\dagger Y_u + 2Y_d^\dagger Y_d\right) - \left(\frac{34}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right] \tag{3.15}$$

$$\beta_{(5D)Y_d}^{(1)}[t] = Y_d\left[\left(6Y_d^\dagger Y_d + 2Y_u^\dagger Y_u\right) - \left(\frac{19}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right)\right] \tag{3.16}$$

$$\beta_{(5D)Y_e}^{(1)}[t] = Y_e\left[6Y_e^\dagger Y_e - \left(\frac{33}{10}g_1^2 + \frac{9}{2}g_2^2\right)\right] \tag{3.17}$$

3.5.4 The Five Dimensional Contributions:

The beta functions of tri-linear soft breaking couplings are given by:

$$\beta_{(5D)A_u}^{(1)}[t] = 6A_u Y_u^\dagger Y_u + 12Y_u Y_u^\dagger A_u + 2A_u Y_d^\dagger Y_d + 4Y_u Y_d^\dagger A_d - \frac{34}{15}g_1^2 A_u - \frac{9}{2}g_2^2 A_u$$

$$- \frac{32}{3}g_3^2 A_u + Y_u\left(\frac{34}{15}g_1^2 M_1 + 9g_2^2 M_2 + \frac{64}{3}g_3^2 M_3\right) \tag{3.18}$$

$$\beta_{(5D)A_d}^{(1)}[t] = 6A_d Y_d^\dagger Y_d + 12Y_d Y_d^\dagger A_d + 2A_d Y_u^\dagger Y_u + 4Y_d Y_u^\dagger A_u - \frac{19}{30}g_1^2 A_d - \frac{9}{2}g_2^2 A_d$$

$$- \frac{32}{3}g_3^2 A_d + Y_d \tag{3.19}$$

$$\beta_{(5D)A_e}^{(1)}[t] = 6A_e Y_e^\dagger Y_e + 12Y_e Y_e^\dagger A_e - \frac{33}{10}g_1^2 A_e - \frac{9}{2}g_2^2 A_e - \frac{32}{3}g_2^2 A_e$$

$$+ Y_e\left(\frac{33}{5}g_1^2 M_1 + 9g_2^2 M_2\right) \tag{3.20}$$

Chapter Four

Numerical Results, Discussions and Conclusions

4.1 Numerical Results:

Chapter four devoted to our numerical result and discussion for the gauge coupling constants and Yukawa couplings behavior in 5D MSSM+F. We set the compactification energy scale to be $R^{-1} = 15$ TeV. Only some selected plots will be. We quantitatively analyzed and explored these quantities in five dimensional MSSM+F. The initial value we shall adopt at the M_{SUSY} scale is given in table 4.1.

Table 4.1 shows the initial values at $M_{SUSY} = 1$ TeVscale and unification scale that used in our numerical calculations. *Data is taken from Ref* (Z.~-z.~Xing H. a., 2008).

Parameter	Value (90% CL)
$g_1(M_{SUSY})$	$0.360945804 \times \sqrt{3/5}$
$g_2(M_{SUSY})$	0.633371083
$g_3(M_{SUSY})$	1.02739852
$Y_t(M_{SUSY})$	0.849348847
$Y_b(M_{SUSY})$	0.128188819
$Y_\tau(M_{SUSY})$	0.099965377

4.2 Discussions of the Results:

As depicted in figure 4.1 the running of the three gauge couplings constants g_1, g_2 and g_3 in five dimensional $MSSM + F^\pm$ changes from logarithmic running (4D MSSM) to power law running this is due to the effect of Kaluza Klein particles at energy greater than $Q = \frac{1}{R}$. This is one should expected from extra dimension scenarios. We noted that the unification scale is lowered around $10^{5.6}$ GeV compare to the usual 4D MSSM unification scale 10^{16} GeV.
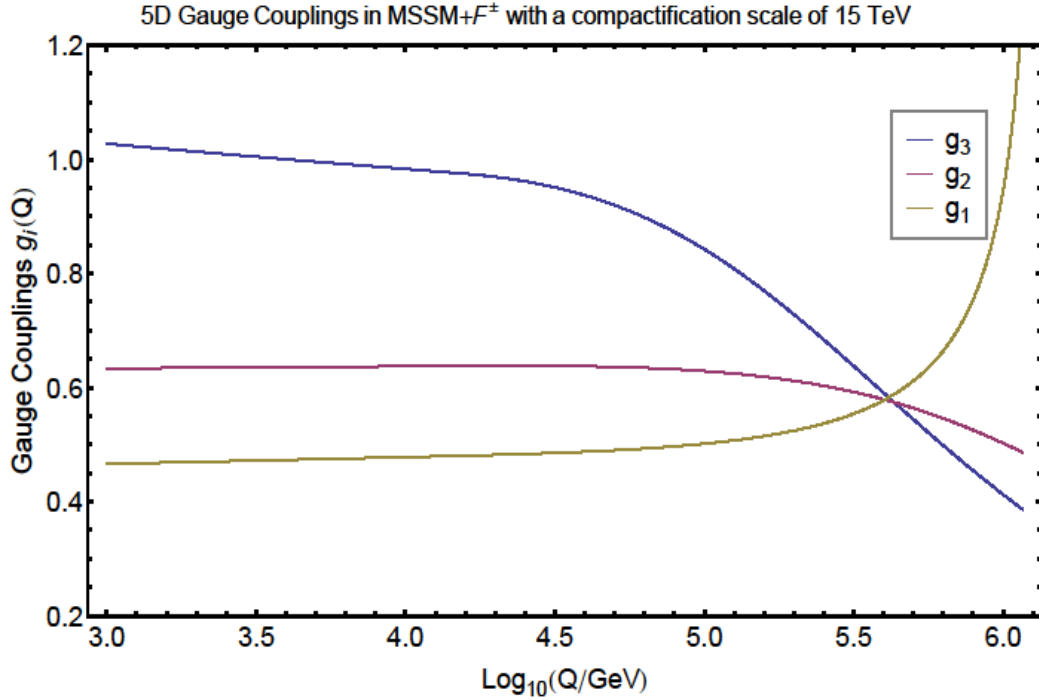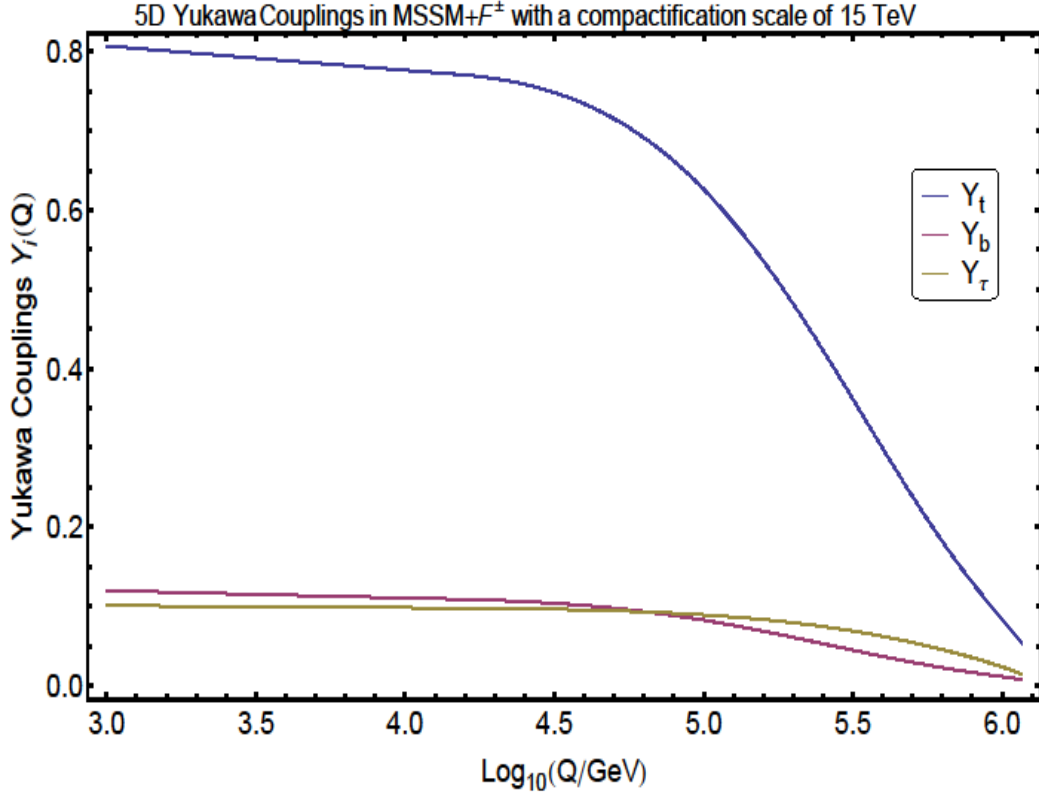

Figure 4.1 present the evolution of the 5D Yukawa couplings in MSSM as function of energy scale.

The evolution of Yukawa couplings in 5D $MSSM + F^\pm$ is shown in figure 4.2.As can be seen from this figure that the three Yukawa couplings try to unify at some

high energy scale compactification scale at $15\,TeV$. We may make Yukawa couplings to vanish at high scale.

Figure 4.2: evolution of the Yukawa couplings constants as function of energy scale in 5D MSSM+F$^{\pm}$.

We specified the Yukawa coupling RGEs boundary conditions at M_{SUSY}, which interestingly appear to vanish when evolved to the unification scale as highlighted in figure 4.2. Let us now pay attention on the evolution of the tri-linear soft breaking terms A_t . We also, specified a low scale value of the gluino mass M_3 and set a high scale boundary condition that the A_t vanish, and then we solved the set of equations. The results are shown in figure 4.3 for compactification radius of 15 TeV.Figure 4.3 shows that after a reasonable period of renormalization evolution the A_t mimics the magnitude of the final value of the gluino mass, at $\frac{1}{R} \sim 15$TeV,

such that at low scales $|A_t| \sim M_3$. Thus, for this compacitification radius an O (2) TeV gluino can achieve a reasonably large size A_t at low scales, but with an initially low unification scale.
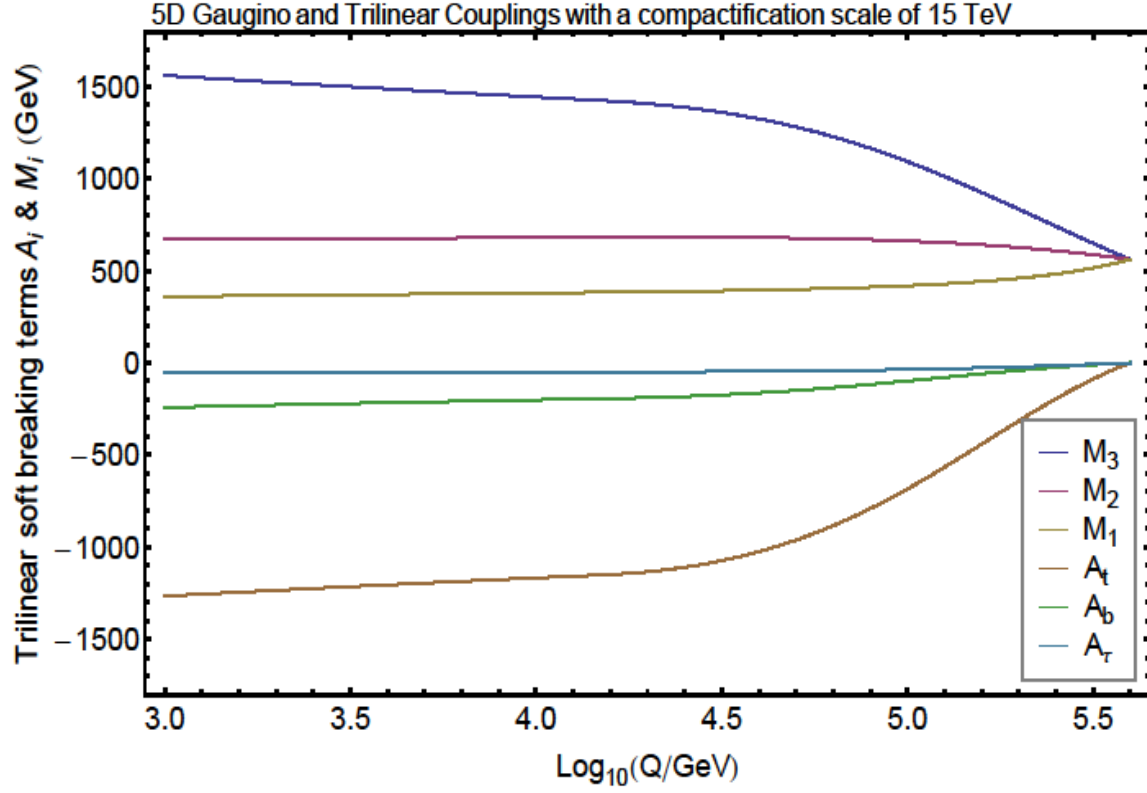
Figure 4.3: Evolution of the 4D gaugino masses and tri-linear soft braking couplings in 5D MSSM+F$^{\pm}$ as function of energy scale.

4.3 Conclusions:

We derived the renormalization group equation at one loop for gauge, Yukawa, tri-linear soft breaking couplings in 5D MSSM plus additional fields. We discussed the evolution of these parameters in five dimensional super-symmetric contexts. We found that due to the KK state in five dimensions the evolution of these parameters run as power law instead of logarithmic running. We found successful unification of gauge couplings in this model around $10^{5.6}$ GeV. Furthermore, Yukawa couplings may be made to unify at some high scale and approximately

vanish at the unification scale of the gauge couplings $10^{5.6}\ GeV$. We may generate a large tri-linear A_t coupling by exchanging a high initial super-symmetry breaking scale such as in the four dimensional MSSM, for a larger compactification scale and a lower initial super-symmetry breaking scale.

4.4 Recommendation

To extend this work, it would be interesting to explore if warped (Randall Sundrum model) or holographic picture may also generate a largeA_t, as one expects logarithmic running rather than power law running in these models.

Appendix:

In this appendix we give a detail calculation of the one loop diagram for the gauge coupling constants and how to calculate the coefficients of beta function in the case of the standard model.
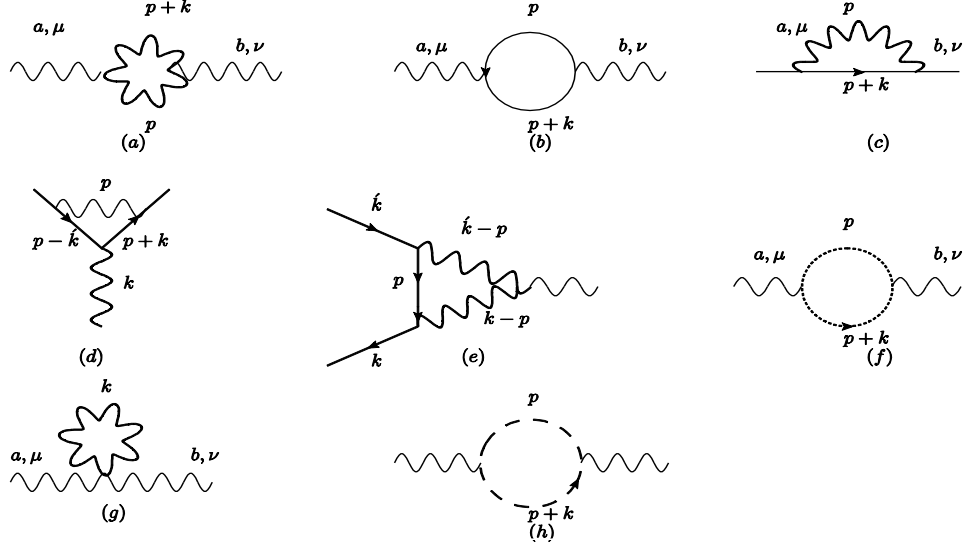
Figure (1): show all diagrams contributing to self-gauge boson

We will calculate the contribution from figure (1) in details: Using Feynman rules (Guigg, 1983) and converts it to an integral we get

$$F_a = \int_{-\infty}^{+\infty} \frac{d^4p}{(2\pi)^2} Tr[-ig\gamma^\mu \lambda^a \times \frac{ig^{\mu\nu}}{(p+k)^2 - m^2} \times -ig\gamma^\nu \lambda^b \times \frac{ig^{\mu\nu}}{p^2 - m^2} \qquad (1)$$

$$F_a = g^2 Tr(\lambda^a \lambda^b) \int_{-\infty}^{+\infty} \frac{d^4p}{(2\pi)^2} Tr\left[\frac{\gamma^\mu g^{\mu\nu}}{(p+k)^2 - m^2} \times \frac{\gamma^\nu g^{\mu\nu}}{p^2 - m^2}\right] \qquad (2)$$

Put:

$$Tr(\lambda^a \lambda^b) = C_2(R), and\ Tr(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu} \qquad (3)$$

Then equation (2)becomes:

$$F_a = \frac{4g^2(g^{\mu\nu})^3 C_2(R)^2}{(2\pi)^4} \int\limits_{-\infty}^{+\infty} d^4p \ Tr\left[\frac{1}{((p+k)^2 - m^2)(p^2 - m^2)}\right] \tag{4}$$

To calculate the above integral we using Feynman integral parameterization:

$$\frac{1}{ab} = \int\limits_{0}^{1} dz \frac{1}{[b + (a-b)z]^2} \tag{5}$$

Now let us call:

$$a = (p+k)^2 - m^2 = p^2 - 2pk + k^2 - m^2, b = p^2 - m^2 \tag{6}$$

From equations (4)and equation (5)equation (2) becomes:

$$F_a = \frac{4g^2(g^{\mu\nu})^3 C_2(R)^2}{(2\pi)^4} \int\limits_{-\infty}^{+\infty} d^4p \ \left[\frac{1}{[(p^2 - m^2) + (2pk + k^2)z]^2}\right] \tag{7}$$

Now let:

$$D = [(p^2 - m^2) + (2pk + k^2)z]^2 \tag{8}$$

Introduce new variable q:

$$q = p + kz \rightarrow p = q - kz \tag{9}$$

Then equation (8)can be written as:

$$D = [(q - kz)^2 - m^2 + (2(qk - k^2z) + k^2)z]^2 \tag{10}$$

After a little algebra, the above equation simplified to:

$$D = [q^2 + k^2z(1-z) - m^2]^2 = [q^2 + a]^2 \tag{11}$$

Where:

$$a = k^2z(1-z) - m^2 \tag{12}$$

Insert equation(11) in equation (9)we obtain:

$$F_a = \frac{4g^2(g^{\mu\nu})^3 C_2(R)^2}{(2\pi)^4} \int\limits_{-\infty}^{+\infty} d^4p \ \left[\frac{1}{q^2 + a}\right] \tag{13}$$

Using the standard integral of beta function:

$$\int_{-\infty}^{+\infty} d^4p \left[\frac{1}{q^2+a}\right]^n = \frac{i(\pi)^{\frac{d}{2}}\Gamma\left(n-\frac{d}{2}\right)}{a^{2-\frac{d}{2}}} \tag{14}$$

but we have $n=2, d=4-\varepsilon$ \hfill (15)

Substituting equation (15) in equation(14), we get

$$\int_{-\infty}^{+\infty} d^4p \left[\frac{1}{q^2+a}\right]^n = \frac{i\pi^2\pi^{-\frac{\varepsilon}{2}}\Gamma\frac{\varepsilon}{2}}{a^{\frac{\varepsilon}{2}}} = i\pi^2\pi^{-\frac{\varepsilon}{2}}\Gamma\frac{\varepsilon}{2}\int_0^1 a^{-\frac{\varepsilon}{2}}\,dz \tag{16}$$

We have also

$$\Gamma(\frac{\varepsilon}{2}) = \frac{2}{\varepsilon} - \gamma, a^{-\frac{\varepsilon}{2}} = 1 - \frac{\varepsilon}{2}\log a \tag{17}$$

Then equation (16) becomes

$$\int_{-\infty}^{+\infty} d^4p \left[\frac{1}{q^2+a}\right] = i\pi^2\pi^{-\frac{\varepsilon}{2}}(\frac{2}{\varepsilon}-\gamma)\int_0^1(1-\frac{\varepsilon}{2}\log a)dz = i\pi^2\pi^{-\frac{\varepsilon}{2}}\left(\frac{2}{\varepsilon}-\gamma\right) \tag{18}$$

Therefore, our figure (1)gives us

$$F_a = \frac{4g^2(g^{\mu\nu})^3 C_2(R)}{16\pi^4}\left[i\pi^2\pi^{-\frac{\varepsilon}{2}}\left(\frac{2}{\varepsilon}-\gamma\right)\right] = \frac{4ig^2(g^{\mu\nu})^3 C_2(R)\left(\frac{2}{\varepsilon}-\gamma\right)}{16\pi^4\pi^{\frac{\varepsilon}{2}}} \tag{19}$$

Other figures give us

$$F_b = i(k^2 g^{\mu\nu} - k^\mu k^\nu)\delta^{ab}\left[\frac{-g^2}{(4\pi)^2}\frac{4}{3}n_f C(r)\right]\left(\Gamma\left(2-\frac{d}{2}\right)\right) \tag{20}$$

$$F_c = i\frac{g^2}{(4\pi)^2}\left[C_2(R)\not{k}\right]\Gamma\left(2-\frac{d}{2}\right) \tag{21}$$

$$F_d = \frac{ig^3}{(4\pi)^2}t^a\gamma^\mu\left[C_2(r)-\frac{1}{2}C_2(G)\right]\Gamma\left(2-\frac{d}{2}\right) \tag{22}$$

$$F_e = \frac{3}{2}\frac{ig^3}{(4\pi)^2}[C_2(G)t^a\gamma^\mu]\Gamma\left(2-\frac{d}{2}\right) \tag{23}$$

$$F_f = \frac{ig^2 C_2(G)\delta^{ab}}{6(4\pi)^2}\left[\frac{g^{\mu\nu}k^2}{2}+k^\mu k^\nu\right]\Gamma\left(2-\frac{d}{2}\right) \tag{24}$$

$$F_g = 0 \tag{25}$$

$$F_h = \frac{-ig^2 C_2(R)\delta^{ab}}{6(4\pi)^2} [g^{\mu\nu}k^2 - k^\mu k^\nu]\Gamma\left(2 - \frac{d}{2}\right)$$ (26)

Therefore summing all these contributions yield:

$$b_G^i = \left[\frac{11}{3} C_2(G) - \frac{4}{3} n_g\, C(R) - \frac{1}{3} n_h C_2(R)\right]$$ (27)

For the representation R; $C_2(G)$, $C(R)$ and $C_2(R)$ refer to the gauge bosons, fermions and Higgs scalar contributions, respectively. Normally gauge bosons are in the ad joint representation of the group(for SU (N), $C_2(G) = N$). The fermions and the Higgs fields are in the fundamental representation; we choose for the fundamental representation in which $C(R) = C_2(R) = \frac{1}{2}$.

- **Coefficients b_G^i in the SM ($SU(3) \times SU(2) \times U(1)$):**

Consider first the strong interaction $SU(3)$, here which $C_2(G) = 3$, $C(R) = \frac{1}{2}, n_g = 3$ and $C_2(R) = 0$ because the Higgs fields does not carry color. Substitute these group factors in to equation (3.29), we get

$$b_3 = \left[\frac{11}{3} \times 3 - \frac{4}{3} \times 3 \times 1\right] = 11 - 4 = 7$$ (28)

Now for the weak interaction $SU(2)$; we have which $C_2(G) = 2$, $C(R) = C_2(R) = \frac{1}{2}$ and $n_g = 3, n_h = 1$, we obtain:

$$b_2 = \left[\frac{11}{3} \times 2 - \frac{4}{3} \times 3 \times 1 - \frac{1}{3} \times 1 \times \frac{1}{2}\right] = \frac{44 - 24 - 1}{6} = \frac{19}{6}$$ (29)

Finally we consider the $U(1)$ hypercharge, here we set $C_2(G) = 0$, $C(R) = C_2(R) = (\frac{Y}{2})^2$ and $n_g = 3, n_h = 1$ yields

$$b_1 = -\frac{4}{3} \times 3 \left[2 \times \left(\frac{1}{6}\right)^2 \times 3 + \left(\frac{2}{3}\right)^2 \times 3 + \left(-\frac{1}{3}\right)^2 \times 3 + 2 \times \left(-\frac{1}{2}\right)^2 + (-1)\right] \times \frac{1}{2}$$

$$-\frac{1}{3} \times \left(\frac{1}{2}\right)^2 \times 2 = -\frac{41}{6}$$ (30)

b_1 should be normailized according to SU(5) unified model by the factor $\left(\frac{3}{5}\right)$.

Therefore

$$b_1 = -\frac{41}{6} \times \frac{3}{5} = -\frac{41}{10} \tag{31}$$

Bibliography

A.~Abdalgabar, A. A. (2013). Evolution of Yukawa couplings and quark flavour mixings in 2UED models. *Phys.\ Rev.\ D, 88*, 056006.

A.~S.~Cornell, L.-X. a. (2010). Scaling of Yukawa Couplings and Quark Flavor Mixings in the UED Model. *PoS KRUGER, 2010*, 045.

A.D.Martin, F. a. (1984). *Quarks and leptons:An introductory course in modern particle physices.* new york : John Wiley.

A.J.G.hey, I. a. (1993). *Gague theories in particlphysics* (second edition ed.). Bristol and philadelphia.

al, G. A. (2012). ATLAS collaboration . *Phys. Lett, 1*, B716.

al, S. C. (2012). CMS collaboration . *Phys.Lett, 30*, B716.

Ammar Abdalgabar, A. C. (2016). Natual Suppersymmetry and unification in five diemsion. *JHEP, 083*, 01.

Ammar Ibrahim Abdalgabar, A. S. (May 21,2014). *Large At without the desert.* F-69622 Lyon,France: National Institute for physics, School of Physics, University of the Witwatersrand, Wits 2050, South Africa, Universite de lyon.

Collaboration], E. {. (2011). Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. *Astrophys.\ J.\ Suppl., 192*, 18.

Collins, J. C. (1984). Renormalization. An Introduction to Renormalization, the renormalization Group and the operator product expansion. *Cambridge, UK: Univ. Pr.*

E.~Poppitz, B. a. (2001). Number of fermion generations derived from anomaly cancellation. *Phys.\ Rev.\ Lett., 87*, 031801.

Falcone, D. (2002). Fermion masses and mixings in gauge theories. *Int. J. Mod. Phys., A17*, 3981.

Guigg, c. (1983). *Gauge theories of the strong,weak and electromagnetic interaction.* Benjiamin/cummings.

H.~-U.~Yee, T. a. (2003). Universal extra dimensions and the Higgs boson mass. *Phys.\ Rev.\ D, 67*, 055002.

J. Louis, I. B. (1998). The Supersymmetric Standard model. *Arxiv:9811341*.

J.donoghue, E. a. (1994). *Dynamics of the standard model,Cambridge Monographs on particle ,Nuclear physics and Cosmology.*

K.~S.~Babu. (1987). Renormalization Group Analysis Of The Kobayashi-maskawa Matrix. *Z.\ Phys. \ C, 35*, 69.

L.~-X.~Liu. (2010). Renormalization Invariants and Quark Flavor Mixings. *Int.\ J.\ Mod.\ Phys., A25*, 4975.

L.~-X.~Liu, A. a. (2011). Evolution of the CKM Matrix in the Universal Extra Dimension Model. *Phys.\ Rev.\ D, 83*, 033005.

L.F.Li, T. a. (1991). *Gauge theory of elementary particle physics.* oxford univ.

Majee, S. K. (March, 2008). Some Explorations of New Physics beynod the SM. 38,39,40.

McMahon, D. (2008). *Quantum Field theory Demystified.* McGraw-Hill companies.

N.~Maru, T. J. (2010). The Universal Extra Dimensional Model with S**2/Z(2) extra-space. *Nucl.\ Phys.\ B, 830*, 414.

P.~W.~Higgs. (1964). Broken symmetries, massless particles and gauge fields. *Phys.\ Lett., 12*, 132.

perkins, D. (2009). *oxford master series in particles physics,astrophysics and cosmology.*

Quigg, C. (2007). spontaneous symmetry Breaking as a Basic of particle mass. arXIV:0704.2232.

Ray, T. S. (2010). *Beyond The Standard Model:Some Aspects of Supersymmetry and Extra Dimension.* Kolkata 700 064, India: Saha Institute of Nuclear Physics, India.

S.L.Glashow. (1961). *Nucl. Phys, 22*, 579.

Salam, A. (1968). Elementary Partcile Physics (Nobel Symp. N.8). 367.

T.~Appelquist, H.-C. a. (2001). Bounds on universal extra dimensions. *Phys.\ Rev.\ D, 64*, 035002.

T.~Kaluza. (1921). On the Problem of Unity in Physics. *Sitzungsber.\ Preuss.\ Akad.\ Wiss.\ Berlin (Math.\ Phys.\ ), 1921*, 966.

Veltman, M. (2003). Facts and mysteries in elementary particls physics. Co.Pte.Ltd.

Weinberg, S. (1967). *Phys. ReV. Lett, 19*, 1264.

Weinberg, S. (1996). *the Quantum theory of field.* Cambridge University.

Z.~-z.~Xing, H. a. (2000). Mass and flavor mixing schemes of quarks and leptons. *Prog.\ Part.\ Nucl.\ Phys., 45*, 1.

Z.~-z.~Xing, H. a. (2008). Updated Values of Running Quark and Lepton Masses. *Phys.\ Rev.\ D, 77*, 113016.