



**Sudan University of Science and Technology**  
**College of Graduate Studies**



**Light Super Partner Top Quark Mass and Higgs Mass in Five  
Dimensions Minimal Super Symmetric Standard Model**

**صغر كتلة نظير الكوارك العلوي الفائق وكتلة الهيجز في نموذج التماثل الفائق**

**في خمسة أبعاد**

**A submitted in partial fulfillment of the requirements of M.Sc. Degree**

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# الآية

قال تعالى: ((وَمَا يَعْزُبُ عَنْ رَبِّكَ مِنْ مِثْقَالِ ذَرَّةٍ فِي الْأَرْضِ وَلَا فِي السَّمَاءِ وَلَا أَصْغَرَ مِنْ ذَلِكَ وَلَا أَكْبَرَ إِلَّا فِي كِتَابٍ مُبِينٍ (61)).

صدق الله العظيم

سورة يونس الآية (61)

# Dedication

*I dedicate this work to Mom and dad*

# Acknowledgments

I would thank after Allah, my supervisor Dr. Ammar Ibrahim Abdalgabar who guide me and help me get through this research. I thank him for taking me as his student and supporting me without hesitation through this work, I will forever remain indebted to him.

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# Abstract

In this dissertation, the renormalization group equations in five dimensional minimal super symmetric standard model plus additional fields were derived. the gauge couplings constants was studied, the evolution was founded does not follow a logarithmic running, but follow power law running, this is because of the presence of KK states which at contributes at energy greater than the compactification scale  $Q = \frac{1}{R}$ . The coupling constant were noticed unified at energy  $Q = 10^{5.6} GeV$ , which is lowered in comparison with the unification scale in the 4D MSSM. the tri-linear soft breaking terms,  $A_{t,b,\tau}$  was vanish at the unification scale and generating large  $A_t$  at electroweak scale through the renormalization group equation. However this large value of  $A_t$  is driven by the gluino mass. We have shown that, in five dimensional models that one can accommodate the correct Higgs mass and producing light stop quark mass below 2 TeV.

## ملخص البحث

في هذه الأطروحة اشتقت معادلات المجموعة المعاييرة في نموذج التماثل الفائق زائدا جسيمات اضاافية في خمسة ابعاد. درست ثوابت الاقتران ووجد ان التوحيد لا يتبع اللوغريتم بل يتبع قانون الاسي وذلك بسبب وجود حالات KK, لوحظ ان ثابت الاقتران موحد عند الطاقة  $Q = 10^{5.6} \text{Gev}$  والتي تم تخفيضها مقارنة بمقياس التوحيد في 4D MSSM وضع  $A_{t,b,\tau}$  لتتلاشى عند مقياس التوحيد وتوليد مقدار كبير ل  $A_t$  على مقياس electroweak من خلال معادلة مجموعة اعادة التشكيل ومع ذلك فان هذه القيمة الكبيرة ل  $A_t$  تنتج بواسطة كتلة Gluino ووضح ان النماذج ذات الابعاد الاضاافية يمكن ان تستوعب كتلة Higgs الصحيحة وتنتج كتلة الكوارك التي تكون اقل من 2Tev.

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# Chapter One

## Introduction

### 1.1 Introduction:

In the context of super-symmetric and through the prism of the naturalness aesthetic, the discovery of a Standard Model-like scalar particle of mass  $m_h = 125 \text{ GeV}$  (Ammar Ibrahim Abdalgabar, May 21,2014), and no direct evidence so far of super particles has motivated renewed interest in non-minimal extensions of the Super-symmetric Standard Model (SSM) that can help to compellingly explain such results. Within the Minimal-SSM (MSSM), for the lightest CP even charge neutral scalar to be the discovered scalar then requires either multi-TeV stops, which is disfavored from naturalness, an enhancement to the tree-level Higgs mass such as for example (Ammar Ibrahim Abdalgabar ‘May 21,2014), or a near maximal mixing scenario whereby  $|T_u(m_Z)| \gtrsim 1\text{TeV}$ . There are few models that compellingly achieve a large enough  $T_u$  if one first assumes  $T_u$  to vanish at some initial super-symmetric breaking scale. Even if one obtains such a large  $T_u$  one must still explain why stops are lighter than their first and second-generation counterpart s-quarks, consistent with collider bounds (Ammar Ibrahim Abdalgabar ‘May 21,2014). One such framework that can address both problems is a five dimensional-SSM.

In five dimensional (5D) SSMs, power law running for a sufficiently low compactification radius  $R$ , generates at low energies a large enough  $T_u$  to explain the observed Higgs mass. Furthermore, through spatially localizing different generations along the extra dimension(s), one can explain geometrically why the third generation can be consistently lighter than its first and second-generation counterparts. This framework is sufficiently compelling that it should

understandably endure further scrutiny. In particular, five dimensional theories are effective field theories with a cutoff and are (often over-dramatically) defined as non-renormalizable, as many parameters such as gauge couplings can be sensitive to this UV scale (J. Louis ‘1998).

## **1.2 Problem of Research:**

In the context of the Minimal Super-symmetric Standard Model (MSSM), this motivates considering models of super-symmetry breaking in which the stop super-partner is heavy (beyond the reach of the LHC) or a model in which a large tri-linear  $T_u$  soft supersymmetry breaking parameter can be generated at low energies. The first option is disfavored by fine-tuning arguments, while the second one allows for lighter stops  $\sim 1$  TeV, and it is thus preferred by naturalness arguments.

## **1.3 Objectives:**

The main objective is to derive the renormalization group equations for the gauge, Yukawa coupling constants, Tri-linear soft breaking and soft mass terms at one loop level in extra dimension model, how to obtain the correct 125 GeV Higgs mass, with stops lighter than 2 TeV.

## **1.4 Outlines of the Dissertation:**

This dissertation content four chapter:

Chapter one include a general introduction to topics. Chapter two will discuss the theory of the standard model, super-symmetry and extra dimension model. Chapter three will concern with the technique of renormalization group equations for various parameters. Chapter four devoted to our numerical results, discussions and conclusion.

# Chapter Two

## The Standard Model and Beyond

### 2.1 Introduction:

In this chapter, we are going to present a general introduction to the Standard Model of particle physics and will discuss some problems with the standard model, and present alternative solution beyond the standard model, super- symmetry and extra dimension models.

### 2.2 Standard Model:

The standard model(SM) of particle physics is a mathematical theory that deals with the week electromagnetic and strong interaction between leptons and quarks, the basic particle of the standard model. (wood, 2007).

The standard model, as we stated in the previous section, is a gauge field theory based on the group  $SU(2)_L \times U(1)_Y \times SU(3)_C$  the particle content of the SM is enlisted in Table 2.1 with their corresponding gauge group representations. The left-handed particles are doublet under the  $SU(2)_L$  gauge transformation while the right- handed ones are singlet. As  $SU(3)_C$  group does not distinguish left or right so both type of quarks are triplets while leptons are singlet. The hypercharge quantum number Y is normalized to (Collins, 1984):

$$Q = I_3 + y \tag{2.1}$$

**Table2.1: The fundamental matter and mediator members of particle physics**  
(Majee, March, 2008)

Nature	Particles	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Quarks (left-handed) : $Q_L$	$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$	(3,2,1/6)
Quarks(right hand): $Q_R$	$U_R, C_R, T_R$ $d_R, s_R, b_R$	$\begin{pmatrix} 3 \\ 1 \\ -1/3 \end{pmatrix}$
Lepton(left-handed): $L_L$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	(1,2,-1/2)
Lepton(right-handed): $L_R$	$e_R, \mu_R, \tau_R$	(1,1,-1)
$SU(3)_C$ gauge boson	$G_\mu^a$	(8,1,0)
$SU(2)_L$ gauge boson	$W_\mu^+, W_\mu^-, Z_\mu$	(1,3,0)
$U(1)_Y$ gauge boson	$B_\mu$	(1,1,0)

Quarks and leptons are fundamental building of matter all of them are fermions and have spin (1/2).they are classified as left-hand Isospin doublets and right-hand isospin singlet and will be described by the Dirac equation (Collins, 1984)

### 2.3 SM Lagrangian:

The lagrangian of standard model can be split in to six parts the matter sector or the fermions sector  $L_f$ , the gauge sector  $L_g$ , the Higgs sector  $L_H$ , the Yukawa sector  $L_y$ , the gauge fixing sector  $L_{gauge\ fixing}$  and the ghost sector  $L_{ghost}$  (A.J.G.HEY,1993).

$$L_{SM} = L_G + L_F + L_Y + L_H + L_{Gauge.fix} + L_{Ghosts} \quad (2.2)$$

### 2.3.1 The Matter Sector:

The matter field of the standard model are the leptons and quarks, carrying spin  $\pm 1/2$  they are classified as left-handed isospin doublets and right hand isospin singlet; moreover, quarks are color triplets.

Depending on the fermions, species the covariant derivative take the form:

$$D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L = \left( \partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig \frac{\sigma^a}{2} W_\mu^a - i \frac{1}{6} g' B_\mu \right) \begin{pmatrix} u \\ d \end{pmatrix}_L \quad (2.3)$$

$$D_\mu u_R = \left( \partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a + ig' \frac{2}{3} B_\mu \right) u_R \quad (2.4)$$

$$D_\mu d_R = \left( \partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a + ig' \frac{1}{3} B_\mu \right) d_R \quad (2.5)$$

$$D_\mu \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \left( \partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a + i \frac{1}{2} g' B_\mu \right) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad (2.6)$$

And

$$D_\mu e_R = \left( \partial_\mu + ig' B_\mu \right) e_R \quad (2.7)$$

Where  $g'$  is the coupling strength of the hypercharge interaction,  $\sigma^a$  are the generators of  $SU(2)_L$  (simply the three Pauli matrices) and  $\lambda^a$  are generator of  $SU(3)_C$  (The eight Gell Mann matrices).

### 2.3.2 The Gauge Sector:

The symmetries associated with isospin, hypercharge and color are realized as local gauge symmetries. The corresponding spin -1 gauge fields are the following vector fields:

$SU(2)_L$  Isospin  $w_i^\mu$  Isotriplets  $I=1, 2, 3$

$U(1)_Y$  Hypercharge  $\beta_\mu$

$SU(3)_C$  Color  $G_\mu^a$  gluon color octet  $a=1, 2, 3, 4, 5, 6, 7, 8$

$$\mathcal{L}_G = -\frac{1}{4} W_{\mu\nu}^i W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a} \quad (2.8)$$

Where,  $B_\mu$ ,  $G_\mu^a$  and  $W_\mu^i$  is the field strength of associated gauge field given by:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (2.9)$$

$$W_{\mu\nu}^i = \partial_\nu W_\mu^i - \partial_\mu W_\nu^i - g_w \varepsilon^{ijk} W_\mu^j W_\nu^k \quad (2.10)$$

$$G_{\mu\nu}^a = \partial_\nu G_\mu^a - \partial_\mu G_\nu^a - g_s f^{abc} G_\mu^a G_\nu^b \quad (2.11)$$

### 2.3.3 The Higgs Sector:

To combine left-hand doublets and right-hand singlet in the fermions –Higgs Yukawa interaction, the Higgs field must be an is doublets field  $\Phi = [\Phi^0, \Phi^-]$ .

The value of the field in the ground state is determined by the minimum of the self-interaction potential  $V(\Phi)$ . A field component  $H$  that describes small oscillations about the ground state defines the physical Higgs field. Thus, the scalar isodoublet field may be parameterized as:



$$\Phi = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \nu \end{bmatrix} \quad (2.12)$$

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad (2.13)$$

### 2.3.4 Yukawa Sector:

From symmetry considerations, we are free to add gauge-invariant interactions between the Scalar fields and the fermions, these are called the Yukawa terms in the Lagrangian and they are responsible of generating fermions masses and the mixing between different families (Falcone, 2002)

$$\mathcal{L}_Y = Y_{ij}^d q_L^i \Phi d_R^j + Y_{ij}^u q_L^i \Phi \tilde{u}_R^j + Y_{ij}^e L_L^i \Phi e_R^j + h.c \quad (2.14)$$

Where the  $Y$   $3 \times 3$  complex matrices, the so-called Yukawa coupling constants, h.c means the Hermitian conjugate and  $\tilde{\Phi}$  is define by:

$$\tilde{\Phi} = \begin{pmatrix} -\Phi_2^* \\ \Phi_1^* \end{pmatrix}$$

### 2.3.5 Gauge Fixing and Ghosts:

Gauge fixing is necessary when the gauge fields are quantized. Quantization means to develop a path integral formalism for the gauge theory. The path integral us diverging as one integrate over an infinite set of gauge-equivalent configuration, here the gauge fixing is used to pick up one arbitrary representative therefore, giving meaning to the path integral. On the other hand, the gauge invariance we look for in gauge theory, a native path integral approach would spoil it. The solution is given by what is called the Faddeev-Popv procedure, where they introduce an identity expression consisting of a functional integral over a gauge fixing condition time a functional determinant over anti commuting field in the

path integral. The latter gives rise to what is known as ghost fields, which keep the gauge freedom within the theory. However, are not physical particles (because ghosts violate the spin-statistics relation). (Collins, 1984).

As such, we need to add terms in the lagrangian like

$$\mathcal{L}_{Gaugefixing} = -\frac{\zeta}{2}(\partial_\mu A^\mu)^2 \quad (2.15)$$

$$\mathcal{L}_{Ghost} = \bar{c}_b \partial^\mu D_\mu^{ab} \quad (2.16)$$

## 2.4 Symmetries and Particle Content:

We have made all the preparation to write down a gauge invariant lagrangian. We now only have to pick the gauge group and the matter content of the theory. It should be noticed that there are no theoretical reasons to pick a certain group or certain matter content.

To match experimental observation we pick the gauge group for the standard model to be  $U(1)_Y \times SU(2)_L \times SU(3)_C$ .

To indicated that a blain  $U(1)_Y$ group of QED but of hypercharge a subscript Y has been added. The corresponding coupling and gauge boson is denoted by  $I_g$  and  $B_\mu$ respectively.

The  $SU(2)_L$ group has three generations  $T^a = \frac{\sigma^a}{2}$ , the coupling is denoted by g and the three gauges are denoted by  $W_{\mu 1}$ ,  $W_{\mu 2}$ ,  $W_{\mu 3}$ .

None of these gauges boson and neither  $B_\mu$ are physical particle. As we see, linear combination of boson well makes up the photon as well as thee  $W^\pm$ eand Z bosons.

$SU(3)_C$  Is the group of the strong interaction? The corresponding eight gauge Bosons are the gluons (Dr.Teubner, 2009).

## 2.5 Higgs Mechanism:

As was presented in the previous section, a Dirac mass term will violate the gauge symmetry. As such, we need a mechanism that gives mass to the SM particles and keeps the Lagrangian invariant under gauge symmetries. This can be done through the mechanism of spontaneous gauge symmetry breaking also known as the Higgs mechanism. This mechanism adds a new complex scalar field  $\Phi$  which is a doublet under the  $SU(2)_L$  group, a singlet with respect to and has  $SU(3)_C$  hypercharge  $Y_\Phi = 1$

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_4 \end{pmatrix} \quad (2.17)$$

Where  $\Phi_1, \Phi_2, \Phi_3$  and  $\Phi_4$  are real scalars. This new scalar  $\Phi$  adds extra terms to the lagrangian

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad (2.18)$$

Where the covariant derivative  $D_\mu$  is, define as:

$$D_\mu = \partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\sigma^a}{2}W_\mu^a - ig_s\lambda^a w_\mu^a \quad (2.19)$$

The general gauge invariant renormalization potential involving  $\Phi$  is given by

$$V(\Phi) = -\frac{1}{2}\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 \quad (2.20)$$

### 2.5.1 Gauge Boson Masses:

The gauge boson masses can be obtained from the kinetic term of the Higgs field.

$$L_H = \frac{1}{\sqrt{2}} (0 \ v) \left( g T^a W_\mu^a + \frac{1}{\sqrt{2}} g^- \beta_\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} W_\mu^b + \frac{1}{\sqrt{2}} g^- \beta_\mu \quad (2.21)$$

From the definition of:

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \pm W_\mu^2) \quad (2.22)$$

$$Z_\mu = W_\mu^3 \cos \theta - \beta_\mu \sin \theta \quad (2.23)$$

Moreover:

$$A_\mu = W_\mu^3 \sin \theta - \beta_\mu \cos \theta \quad (2.24)$$

We found:

$$M_W^2 = \frac{g^2 v^2}{4} \quad (2.25)$$

$$M_Z^2 = (g^2 + g^{-2}) \frac{v^2}{4} \quad (2.26)$$

### 2.5.2 Fermions Masses:

Fermions masses originate from Yukawa interaction, which are the couplings between the fermions doublets and scalar field  $\Phi$ . these Yukawa coupling are uniquely fixed by gauge invariance and the lagrangian, as given by

$$L_y = Y_d q_L^- \Phi d_R + Y_u q_L \sigma^a \Phi \tilde{u}_R + Y_e L^- \Phi e_R \quad (2.27)$$

When the Higgs doublet acquires an on vanishing the mass term for fermions as follow:

$$L_y = m_u u^-_L u_R + m_d d^-_L d_R + m_e e^-_L e_R \quad (2.28)$$

### 2.5.3 Higgs Boson:

The spontaneous symmetry breaking predicted anew particle, the Higgs boson that must be scalar and neutral, the lagrangian for the new scalar come from kinetic term.

$$L_{Higgs.boson} = \frac{1}{2}(\partial_\mu h)(\partial_\mu h) - \frac{1}{2}m_h^2 h^2 + interaction \quad (2.29)$$

Where  $m_h^2 = \sqrt{\lambda}v$  is the Higgs boson mass. The interaction term contain both Higgs-self interaction and interaction with gauge bosons and fermions.

Note that as a consequence of the Higgs mechanism all the Higgs couplings are completely determined in term of the coupling constants and mass. The Higgs boson, which was the last missing piece of the SM, has been confirmed by the ATLAS and CMS experiments, and this is compatible with the SM Higgs expectations with a mass of about 126Gev.

## 2.6 Problem of Standard Model:

Despite being the most successful theory of particle physics, the SM is not perfect it has some problem:

### **2.6.1 Dark Matter:**

Dark matter cannot be explained in the (SM), possibly will be directly produced at the LHC, dark matter has been observed only indirectly, through its gravitational effects.

### **2.6.2 Gravity:**

The SM does not explain gravity. The approach of simply adding a graviton to the standard model does not recreate what is observed experimentally without other modification, yet undiscovered, to the SM. Moreover, instead the standard model is widely considered incompatible with the most successful theory of gravity to date, general relativity.

### **2.6.3 Neutrino Masses:**

According to the standard model; neutrino is a massless particle. However, neutrino oscillation experiments have shown that neutrinos do have mass.

### **2.6.4 Strong CP Problem:**

Theoretically it can be argued that the SM should contain a term that breaks CP symmetry relating matter to antimatter – in the strong interaction sector, experimentally, however, no such violation has been found implying that the coefficient of this term is very close to zero. This fine tuning is also considered unnatural.

### **2.6.5 Matter-Antimatter Asymmetry:**

The universe is made of mostly matter. However, the standard model predicts that matter and antimatter should have been created in (almost)

Equal amount if the initial conditions of the universe did not involve disproportionate matter relative to antimatter. Yet, no mechanism sufficient to explain this asymmetry exists in the standard model.

### **2.6.6 Hierarchy Problem:**

The SM introduces particle masses through a process known spontaneous symmetry breaking caused by the Higgs field. within the SM , the mass of the Higgs gets some very large quantum correction due to the presence of virtual particles .these correction are much larger than the actual mass of the Higgs.

This means that the bare mass parameter of the Higgs in the SM must be find tuned in such a way that almost completely cancels the quantum correction.

## **2.7 Beyond of the Standard Model:**

Is theoretical developments needed to explain the deficiencies of the standard model:

### **2.7.1 Super-symmetry (SUSY):**

Super-symmetry is the idea that there a super-partner for each elementary particle: s electron, s quark, etc. many super partner could be discovered at the LHC. SUSY is generalization of space-time transformation that fermions and bosons. In SUSY,

fermions transform into bosons and vice versa. It admits super-multiplets with fermions and bosons.

SUSY is the most popular extension of the (SM) because it provides a very aesthetic way to address the gauge hierarchy problem and ameliorate various other shortcomings of the standard model.

Some of the attractive features of the SUSY model are:

- SUSY solves the gauge hierarchy problem, the quantum correction to the Higgs mass from a boson loop and fermions loop have opposite sign. So if the couplings are identical and bosons are mass degenerate with the fermions, the net contribution would cancel, SUSY fits this bill very well, as for every particle, SUSY provides a mass degenerate partner differing by spin  $1/2$  and having identical coupling.
- SUSY leads to unification of gauge coupling, in the SM when the gauge couplings are extrapolated to high scale from their measured value at the weak scale, they come close to each other but do not meet at a single point.
- SUSY provides a cold dark matter candidate
- SUSY provides a framework to turn on gravity. (Sankararray, n.d.)

#### **2.7.1.1 Minimal Super-symmetry (MSSM):**

The MSSM is an extension of the SUSY to the standard model. The left-handed and right-handed SM fermions will be associated with scalar degrees of freedom (s fermions), the model also contains the fermion partners of the SM gauge bosons and Higgs bosons see table 2.2.



**Table 2.2: Super symmetric with the standard model**

Nature	spin 0	spin $\frac{1}{2}$	spin $\frac{1}{2}$	$SU(3)_C, SU(2)_L, U(1)_Y$
S quarks , quarks	$\begin{cases} Q \\ \bar{u} \\ d \end{cases}$	$\tilde{u}_l, d_l^{\sim}$	$u_l, d_l$	$(3, 2, \frac{1}{3})$
		$\tilde{u}_R^*$	$u_R^+$	$(3, 1, \frac{4}{3})$
		$d_R^{\sim*}$	$d_R^+$	$(\bar{3}, 1, -\frac{2}{3})$
S leptons, leptons	L	$\tilde{\nu}, \tilde{e}_l$	$\nu, e_l$	$(1, 2, -1)$
	$\bar{E}$	$\tilde{e}_R^*$	$e_R^+$	$(1, 1, -2)$
Higgs, Higgs boson	$H_1$	$H_1^+, H_1^0$	$H_1^{\sim+}, H_1^{\sim 0}$	$(1, 2, +1)$ $(1, 2, -1)$
	$H_2$	$H_2^0, H_2^-$	$H_2^{\sim 0}, H_2^{\sim -}$	
		spin $\frac{1}{2}$	spin 1	
Gluon, gluino		$g^{\sim}$	$g$	$(8, 1, 0)$
winos, W- bosons		$W^{\sim\pm}, W^{\sim 0}$	$W^{\pm}, W^0$	$(1, 3, 0)$
bino, B- bosons		$B^{\sim 0}$	$B^0$	$(1, 1, 0)$

### 2.7.2 Extra dimensions:

In the SM the hierarchy problem is arising due to the huge ratio of the Planck scale,  $M_{pl}$ , or the GUT scale,  $M_G$ , to the electroweak scale. As discussed in the previous section, SUSY provides a natural way to solve this hierarchy problem (Majee, March, 2008). In that case, the super symmetric particles are situated around the TEV scale. Actually to solve the hierarchy problem if we incorporate any new physics it should appear around that scale to address the huge ratio. More recently, a new kind of physics, Extra Dimension (ED), was introduced in particle physics. One might ask a question how do we distinguish super symmetry particles from extra dimension particles Practically we can distinguish a fermions from a boson particle by measuring the spin of the particle at the Large Hadrons Collider (LHC) or the International Linear Collider (ILC), and then we can have a distinct signature of the physics of extra dimension from that of super symmetry (Majee, March, 2008).

Historically, Extra dimension was first introduced by (KK) in 1920, to unify the electromagnetic interaction with the gravitational one by generating the photon from the extra components of the five-dimensional metric (T.~Kaluza, 1921). Nowadays in a more popular and fundamental theory, namely, string theory, it is common to use more than one space dimension, as the theory is consistent only in the extra-dimensional scenario. There are many open questions about the extra dimension. Example what would be nature of the extra dimension, what is the size of it and many more, A huge number of phenomenological studies have been pursued in this subject in this decade (H.~U.~Yee, 2003). Let us have a closer look on some of these.

### 2.7.3 Universal Extra Dimensions:

A model in which all the standard model particles are allowed to access the extra dimensions is known as the Universal Extra Dimension (UED) model. Construction-wise it is very similar to the ADD model, but as in this case, in addition to gravity, scalars, fermions and vector gauge bosons are also accessing the extra dimension so, as discussed earlier, it should be Compact on an orbifold  $S^1/Z_2$  instead of a circle. This orbifold is nothing but equivalent to the compact on a circle of radius  $R$  with a  $Z_2$  symmetry- identifying  $y \rightarrow -y$ , where  $y$  denotes the fifth compact coordinate. The orbifold is crucial in generating chiral zero modes for fermions. The motivations of universal extra dimensions are quite speculative. Besides providing viable dark matter candidates, the six-dimensional theory can explain from anomaly cancellation why we have only three generations. Only three generation of fermions can remove the  $SU(2)$  global gauge anomaly. Another good feature about universal extra dimensions is to provide a natural way to explain the long lifetime for the proton. They could lead to a new mechanism of supersymmetry breaking, address the fermions mass hierarchy in an alternative way provide a cosmologically viable dark matter candidate, stimulate power law renormalization group running, admit substantial evolution of neutrino mixing angles defined through an effective Major neutrino mass operator, etc. The interesting point is that in this case the discrete symmetry which removes operators providing dangerous contributions to the proton decay is not imposed externally but is an essential ingredient for the theory (Majee, March, 2008).

# Chapter Three

## Renormalization Group Equations

### 3.1 Introduction:

In this chapter will discuss Renormalization Group Equations (RGES) and Gauge coupling constant in standard model and Extra Dimension.

### 3.2 Renormalization and Renormalization Group Equation:

In Quantum Field Theory (QFT), Green function is a most important thing to be calculated. In perturbative (QFT), these quantities are divergent. The systematic way to remove these divergences is known as renormalization. The renormalization theory was implemented to remove all the divergences in loop integrals from the physical measurable quantities. These loop diagrams are supposed to give finite results to the physical quantities but they give infinities instead. This tells us that our theory has missed some information. One might ask a question where these infinities are come from. These infinities arise from the integration over all momentum. In other words, the infinities occur because we let our theory go to arbitrary high energy (UV). There are different ways to cancel these infinities. In order to renormalize the theory we need a reference point which is also arbitrary, Different choices of this reference point lead to different sets of parameters for the theory, but physics should not depend on the arbitrary choice of the reference point and be invariant. This invariance leads to the renormalization group equation. In quantum field theory, it is a useful method to examine the behavior of physics at a different scale knowing the same at some other scale. Thus, measuring the observables in a low energy experiment one can compare with the values predicted from a theory at a higher scale, *e.g.* at the GUT scale and

certify about the correctness of the theory. In the standard model, variations of the gauge coupling constants with energy are given by the following renormalization group equations (RGEs) (Collins, 1984).

$$16\pi^2 E \frac{dg_i}{dE} = b_i g_i^3 = \beta_{SM}(g_i) \quad (3.1)$$

Where  $i$  stands for  $U(1)_Y$ ,  $SU(2)_L$  and  $SU(3)_C$  and the right-hand-side is known as the  $\beta$  function of the corresponding coupling constants. In the above equation, the co-efficient  $b_i$  can be calculated for any  $SU(N)$  group as:

$$b_i = \left[ \frac{11}{3} C_2(G) - \frac{4}{3} n_f C(R) - \frac{1}{3} n_s C_2(R) \right] \quad (3.2)$$

Where:

In the above equation  $n_f$  is the number of fermions and  $n_s$  is the number of Higgs scalar and  $i = 1, 2, 3$ . For the representation the  $C_2(G)$ ,  $C(R)$ ,  $C_2(R)$  refer to the gauge boson, fermions, and Higgs scalar contribution respectively (A.~Abdalgabar, 2013).

### 3.3 Calculation of Beta functions for the Gauge Coupling Constants:

#### 3.3.1 Minimum super symmetry case:

$$B_G^{MSSM} = \frac{11}{3} C_2(G) - \frac{2}{3} C_2(G) - \frac{4}{3} n_g C(R) - \frac{2}{3} n_g C(R) - \frac{1}{3} n_h C_2(R) - \frac{2}{3} n_h C_2(R) \quad (3.3)$$

For the strong interaction ( $b_3$ ):  $SU(3)_c$

$$C_2(G) = 3, n_g = 3, C(R) = 1$$

$$b_3 = 3 \quad (3.4)$$

For the weak interaction ( $b_2$ ):

$$C_2(G) = 2, n_g = 3, C(R) = 1, C_2(R) = 1, n_h = 1.$$

$$b_2 = -1 \tag{3.5}$$

For the electromagnetic interaction( $b_1$ ):

To calculate  $b_1$  we need to calculate the hypercharge from equation (2.1) and put:

$$C(R) = \left(\frac{Y}{2}\right)^2$$

$$b_1 = -\frac{33}{5} \tag{3.6}$$

$$\therefore b_i = \left[ \frac{-33}{5}, -1, 3 \right] \tag{3.7}$$

### 3.3.2 Extra Dimension super-symmetry:

The beta function of five dimensions minimum super-symmetry is given by:

$$b_G^{5DMSSM} = -\frac{11}{3}C_2(G) + \frac{2}{3}C_2(G) + C_2(G) + 4C(R)\eta + 2n_h C_2(R) \tag{3.8}$$

We have

$$C_2(G) = G, C(R) = 1, C_2(R) = \frac{1}{2}$$

After a little algebra we get

$$b_3 = -6 + 4\eta \tag{3.9}$$

$$b_2 = -2 + 4\eta \tag{3.10}$$

$$b_1 = \frac{6}{5} + 4\eta \tag{3.11}$$

$\eta$  is the number of fermions in bluk

### 3.4 Yukawa Couplings in MSSM and 5D MSSM:

The one loop RGES for Yukawa coupling in the 4DMSSM are given by:

$$B_{y_u} = 3y_u y_u^\dagger y_u + y_u y_d^\dagger y_d + y_u \left( \frac{-13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 + Tr(y_e y_e^\dagger + 3y_d y_d^\dagger) \right) \quad (3.12)$$

$$B_{y_d} = 3y_d y_d^\dagger y_d + y_d y_u^\dagger y_u + y_d \left( \frac{-7}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 + Tr(y_e y_e^\dagger + 3y_d y_d^\dagger) \right) \quad (3.13)$$

$$B_{y_e} = 3y_e y_e^\dagger y_e + y_e \left( \frac{-9}{5} g_1^2 - 3g_2^2 + Tr(y_e y_e^\dagger + 3y_d y_d^\dagger) \right) \quad (3.14)$$

The five dimensional contributions are given by:

$$B_{(5D)Y_u} = Y_u \left[ (6Y_u^\dagger Y_u + 2Y_d^\dagger Y_d) - \left( \frac{34}{30} g_1^2 + \frac{9}{2} g_2^2 + \frac{32}{3} g_3^2 \right) \right] \quad (3.15)$$

$$B_{(5D)Y_d} = Y_d \left[ (6Y_d^\dagger Y_d + 2Y_u^\dagger Y_u) - \left( \frac{19}{30} g_1^2 + \frac{9}{2} g_2^2 + \frac{32}{3} g_3^2 \right) \right] \quad (3.16)$$

$$B_{(5D)Y_e} = Y_e \left[ (6Y_e^\dagger Y_e) - \left( \frac{33}{30} g_1^2 + \frac{9}{2} g_2^2 \right) \right] \quad (3.17)$$

### 3.5 Tri-linear Soft Breaking Parameters in 4D and 5D MSSM:

The 4DMSSM soft breaking parameters at one loop are given by:

$$B_{A_u} = 3A_u y_u^\dagger y_u + 6y_u y_u^\dagger A_u + A_u y_d^\dagger y_d + 2y_u y_u^\dagger A_d + A_u \left( -\frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right) + y_u \left( \frac{26}{15} g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3} g_3^2 M_3 \right) + 3A_u Tr(y_u^\dagger y_u) + 6y_u Tr(y_u^\dagger A_u) \quad (3.18)$$

$$\begin{aligned}
B_{A_d} = & 3A_d y^+_d y_d + 6y_d y^+_d A_d + A_d y^+_u y_u + 2y_d y^+_u A_u + A_d \left( -\frac{7}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right) \\
& + y_d \left( \frac{14}{15} g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3} g_3^2 M_3 \right) + 3A_d \text{Tr}(y^+_d y_d) + \\
& 6y_d \text{Tr}(y^+_d y_d) + A_d \text{Tr}(y^+_e y_e) + 2y_d \text{Tr}(y^+_e A_e)
\end{aligned} \tag{3.19}$$

$$\begin{aligned}
B_{A_e} = & 3A_e y^+_e y_e + 6y_e y^+_e A_e + A_e \left( -\frac{9}{5} g_1^2 - 3g_2^2 \right) + y_e \left( \frac{18}{5} g_1^2 M_1 + 6g_2^2 M_2 \right) \\
& + 3A_e \text{Tr}(y^+_d y_d) y_e + 6\text{Tr}(y^+_d A_d) + A_e \text{Tr}(y^+_e y_e) \\
& + 2y_e \text{Tr}(y^+_e A_e)
\end{aligned} \tag{3.20}$$

For five dimension case

$$\begin{aligned}
B^{5D}_{A_u} = & 6A_u y^+_u y_u + 12y_u y^+_u A_u + 2y_u y^+_d y_d + 4y_u y^+_d A_d \\
& + A_u \left( -\frac{34}{30} g_1^2 - \frac{9}{2} g_2^2 - \frac{32}{3} g_3^2 \right) \\
& + y_u \left( \frac{34}{15} g_1^2 M_1 + 9g_2^2 M_2 + \frac{64}{3} g_3^2 M_3 \right) + 3A_u \text{Tr}(y^+_u y_u) \\
& + 6y_u \text{Tr}(y^+_u A_u)
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
B^{5D}_{A_d} = & 6A_d y^+_d y_d + 12y_d y^+_d A_d + 2y_d y^+_u y_u + 4y_d y^+_u A_u \\
& + A_d \left( -\frac{19}{30} g_1^2 - \frac{9}{2} g_2^2 - \frac{32}{3} g_3^2 \right) \\
& + y_d \left( \frac{19}{15} g_1^2 M_1 + 9g_2^2 M_2 + \frac{64}{3} g_3^2 M_3 \right) + 3A_e \text{Tr}(y^+_d y_d) \\
& + 6y_e \text{Tr}(y^+_d A_d) + A_e \text{Tr}(y^+_e y_e) \\
& + 2y_e \text{Tr}(y^+_e A_e)
\end{aligned} \tag{3.22}$$

$$\begin{aligned}
B^{5D}_{A_e} = & 6A_e y^+_e y_e + 12y_e y^+_e A_e + A_e \left( -\frac{33}{10} g_1^2 - \frac{9}{2} g_2^2 \right) + y_e \left( \frac{33}{5} g_1^2 M_1 + 9g_2^2 M_2 \right) \\
& + 3A_e \text{Tr}(y^+_d y_d) + 6y_e \text{Tr}(y^+_d A_d) + A_e \text{Tr}(y^+_e y_e) \\
& + 2y_e \text{Tr}(y^+_e A_e)
\end{aligned} \tag{3.23}$$



### 3.6 Soft Masses in 4D MSSM and 5D MSSM:

The four dimensional MSSM contribution is given by:

$$\begin{aligned}\beta_{m_l^2} = & -\frac{6}{5}g_1^2|M_1^2| - 6g_2^2|M_2^2| + 2m_{H_d}^2 y_e y_e^+ + 2A_e^+ A_e + m_L^2 y_e^+ y_e + 2y_e^+ m_e^2 y_e \\ & + y_e^+ y_e m_L^2 - \sqrt{\frac{3}{5}}g_1\sigma_{1,1}\end{aligned}\quad (3.24)$$

$$\begin{aligned}\beta_{m_e^2} = & -\frac{24}{5}g_1^2|M_1^2| + 2(2m_{H_d}^2 y_e y_e^+ + 2A_e A_e^+ + m_e^2 y_e y_e^+ + 2y_e m_l^2 y_e^+ + y_e^+ y_e m_e^2 \\ & + 2\sqrt{\frac{3}{5}}g_1\sigma_{1,1}\end{aligned}\quad (3.25)$$

$$\begin{aligned}\beta_{m_q^2} = & -\frac{2}{15}g_1^2|M_1^2| - 6g_2^2|M_2^2| - \frac{32}{3}g_3^2|M_3^2| + 2m_{H_d}^2 y_d^+ y_d + 2m_{H_d}^2 y_u^+ y_u \\ & + 2A_d^+ A_d + 2A_u^+ A_u + m_q^2 y_d^+ y_d + m_q^2 y_u^+ y_u + 2y_d^+ m_d^2 y_d + y_d^+ y_d m_q^2 \\ & + 2y_u^+ m_u^2 y_u + y_u^+ y_u m_q^2 + \frac{1}{\sqrt{15}}g_1\sigma_{1,1}\end{aligned}\quad (3.26)$$

$$\begin{aligned}\beta_{m_u^2} = & -\frac{32}{15}g_1^2|M_1^2| - \frac{32}{3}g_3^2|M_3^2| + 4m_{H_u}^2 y_u y_u^+ + 4A_u A_u^+ + 2m_u^2 y_u y_u^+ \\ & + 4y_u m_q^2 y_u^+ + 2y_u y_u^+ m_u^2 - 4\frac{1}{\sqrt{15}}g_1\sigma_{1,1}\end{aligned}\quad (3.27)$$

$$\begin{aligned}\beta_{m_d^2} = & -\frac{8}{15}g_1^2|M_1^2| - \frac{32}{3}g_3^2|M_3^2| + 4m_{H_d}^2 y_d y_d^+ + 4A_d A_d^+ + 2m_d^2 y_d y_d^+ \\ & + 4y_d m_q^2 y_d^+ + 2y_d y_d^+ m_d^2 + 2\frac{1}{\sqrt{15}}g_1\sigma_{1,1}\end{aligned}\quad (3.28)$$

Where

$$\begin{aligned}\sigma_{1,1} = & \sqrt{\frac{3}{5}}g_1 \left( -2\text{Tr}(m_u^2) - \text{Tr}(m_l^2) - m_{H_d}^2 + m_{H_u}^2 + \text{Tr}(m_d^2) + \text{Tr}(m_e^2) \right. \\ & \left. + \text{Tr}(m_q^2) \right)\end{aligned}\quad (3.29)$$

In the five dimension

$$\beta_{(5D)m_q^2} = - \left[ \frac{4}{15} g_1^2 |M_1^2| - 9g_2^2 |M_2^2| - \frac{64}{3} g_3^2 |M_3^2| + \frac{\sqrt{2}}{\sqrt{15}} g_1 \sigma_{1,1} \right] \quad (3.30)$$

$$\beta_{(5D)m_l^2} = - \left[ \frac{12}{5} g_1^2 |M_1^2| - 9g_2^2 |M_2^2| - \sqrt{\frac{6}{5}} g_1 \sigma_{1,1} \right] \quad (3.31)$$

$$\beta_{(5D)m_u^2} = - \left[ \frac{64}{15} g_1^2 |M_1^2| - \frac{64}{3} g_3^2 |M_3^2| - 4 \frac{\sqrt{2}}{\sqrt{15}} g_1 \sigma_{1,1} \right] \quad (3.32)$$

$$\beta_{(5D)m_d^2} = - \left[ \frac{16}{15} g_1^2 |M_1^2| - \frac{64}{3} g_3^2 |M_3^2| + 2 \frac{\sqrt{2}}{\sqrt{15}} g_1 \sigma_{1,1} \right] \quad (3.33)$$

$$\beta_{(5D)m_e^2} = \frac{48}{5} g_1^2 |M_1^2| + 2 \sqrt{\frac{6}{5}} g_1 \sigma_{1,1} \quad (3.34)$$

The one loop RGEs for the two Higgs doublet soft masses in the 4DMSSM are given by:

$$\begin{aligned} \beta_{m_{H_d}^2} = & \frac{6}{5} g_1^2 |M_1^2| - 6g_2^2 |M_3^2| - \sqrt{\frac{3}{5}} g_1 \sigma_{1,1} + 6m_{H_d}^2 \text{Tr}(y_d y_d^+) + 2m_{H_d}^2 \text{Tr}(y_e y_e^+) \\ & + 6\text{Tr}(A_d^* A_d^T) + 2\text{Tr}(A_e^* A_e^T) + 6\text{Tr}(m_d^2 y_d y_d^+) + 2\text{Tr}(m_e^2 y_e y_e^+) \\ & + 2\text{Tr}(m_l^2 y_e y_e^+) + 6\text{Tr} m_q^2 y_d^+ y_d \end{aligned} \quad (3.35)$$

$$\begin{aligned} \beta_{m_{H_u}^2} = & \frac{6}{5} g_1^2 |M_1^2| - 6g_2^2 |M_3^2| - \sqrt{\frac{3}{5}} g_1 \sigma_{1,1} + 6m_{H_u}^2 \text{Tr}(y_u y_u^+) + m_q^2 y_u y_u^+ \\ & + 6\text{Tr}(A_u^* A_u^T) + 6\text{Tr}(m_u^2 y_u^+ y_u) \end{aligned} \quad (3.36)$$

In the 5D MSSM, the two Higgs doublet soft masses obey the RGEs

$$\beta_{(5D)m_{H_d}^2} = - \left[ \frac{12}{5} g_1^2 |M_1^2| - 9g_2^2 |M_2^2| - 2 \sqrt{\frac{3}{5}} g_1 \sigma_{1,1} \right] \quad (3.37)$$

$$\beta_{(5D)m_{H_u}^2} = - \left[ \frac{12}{5} g_1^2 |M_1^2| - 9g_2^2 |M_2^2| + 2 \sqrt{\frac{3}{5}} g_1 \sigma_{1,1} \right] \quad (3.38)$$

### 3.7 The One-loop Higgs Mass:

The leading one-loop self-energy contributions to the lightest CP even Higgs mass are given by:

$$m_H^2 \approx m_Z^2 \cos 2\beta^2 + \frac{3m_t^4}{4\pi^2 v_{ew}^2} \left( \ln \frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right) \quad (3.39)$$

Where  $v_{ew}$  is the electroweak vacuum expectation value,  $X_t = A_t - \mu \cot \beta$ ,  $M_S$  is the stop mass,  $m_t$  is the top quark mass.

# Chapter Four

## Numerical Results, Discussions and Conclusion

### 4.1 Numerical Results:

Chapter four present the numerical result and discussion for the gauge coupling constants and Soft breaking terms and Higgs mass in 5D MSSM+F. We set the compactification energy scale to be  $R^{-1} = 15 \text{ TeV}$ . The initial value we shall adopt at the  $M_{SUSY}$  scale is given in table 4.1.

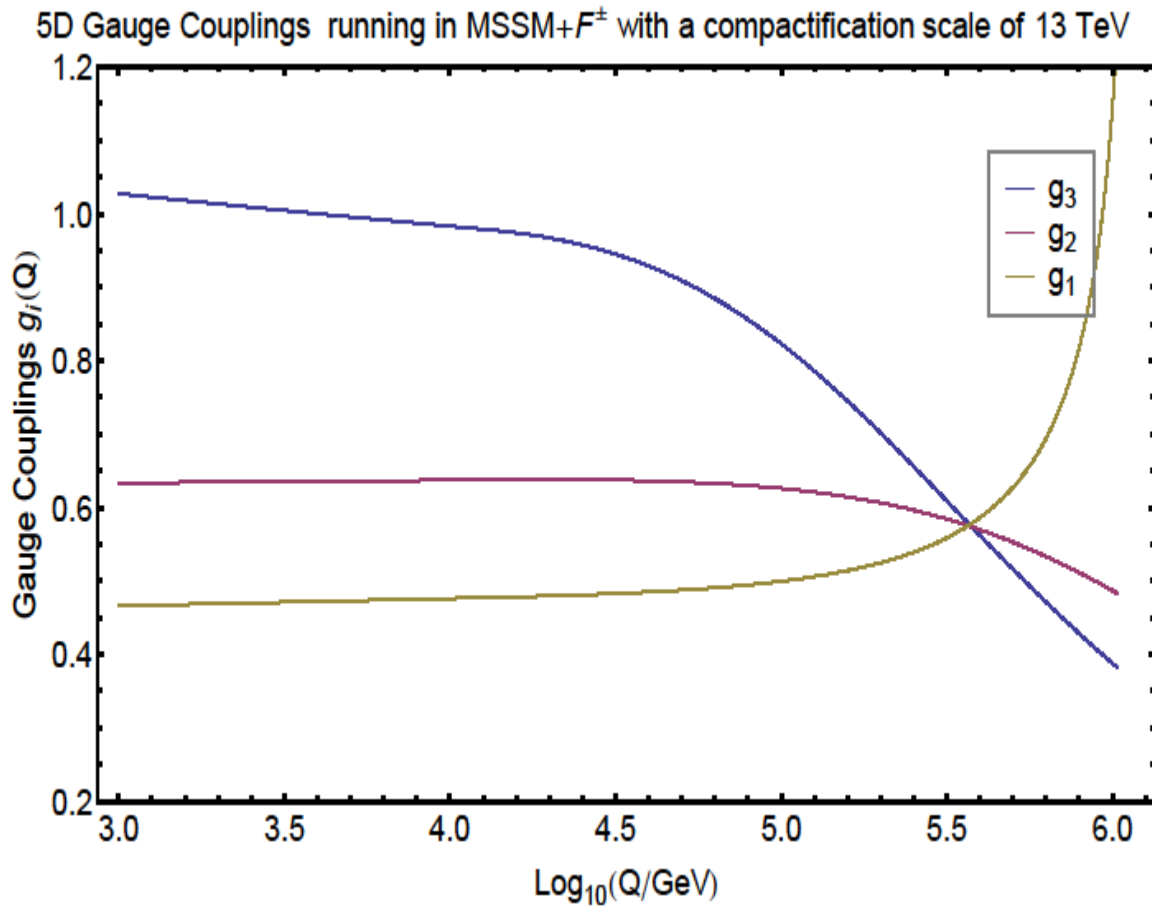
*Table 4.1 shows the initial values at  $M_{SUSY} = 1 \text{ TeV}$  scale used in our numerical calculations. Data is taken from Ref (Z.-z.-Xing, 2008).*

Parameter	Value (90% CL)
$g_1(M_{SUSY})$	$0.360945804 \times \sqrt[3]{3/5}$
$g_2(M_{SUSY})$	0.633371083
$g_3(M_{SUSY})$	1.02739852
$Y_t(M_{SUSY})$	0.849348847
$Y_b(M_{SUSY})$	0.128188819
$Y_\tau(M_{SUSY})$	0.099965377

### 4.2 Discussions:

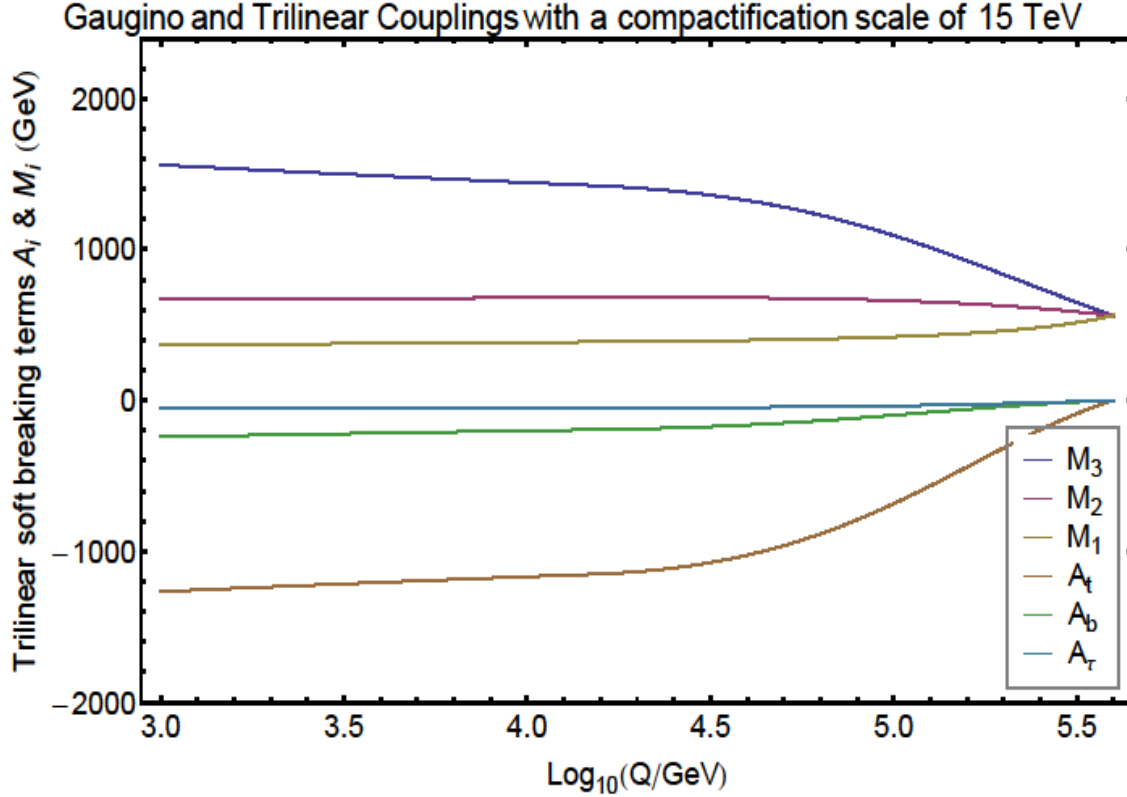
Figure 4.1 show the running of the three gauge couplings  $g_1$ ,  $g_2$  and  $g_3$  in 5D MSSM plus additional fields as can be seen from the figure that the evolution of gauge coupling constants is not a logarithmic running, follow power law running,

this is because of the presence of KK states at contributes at energy greater than  $Q = \frac{1}{R}$  . We noticed that the coupling constant unified at energy  $Q = 10^{5.6} GeV$ , which is lowered in comparison with the unification scale in the 4D MSSM, which occurred at  $Q = 10^{16} GeV$ .



**Figure 4.1:** shows the evolution of the down quark Yukawa coupling as function in energy scale in 5D MSSM+F $^{\pm}$  for three different values of the compactification scales R.

Figure 4.1: shows the evolution of the 5D gauge coupling constants in MSSM as function of energy scale in gauge coupling constants As depicted in figure 4.1 the running of the three gauge couplings constant  $g_1$ ,  $g_2$  and  $g_3$  in gauge couplings in 5D MSSM changes from logarithmic running to power law running this is due to the contribution of KK states at energy greater than  $E = \frac{1}{R}$ . As expected extra dimension, lower the unification scale. We find that the evolution of  $g_3$  decrease faster,  $g_1$  increase faster and  $g_2$  approximately remain constant before  $10^4$  GeV and suddenly increase faster after  $10^4$  GeV and the unification scale occur exactly at  $E = 10^{5.6}$  GeV which is lowered compare to the unification scale in the Yukawa couplings at  $E = 10^{14}$  GeV .

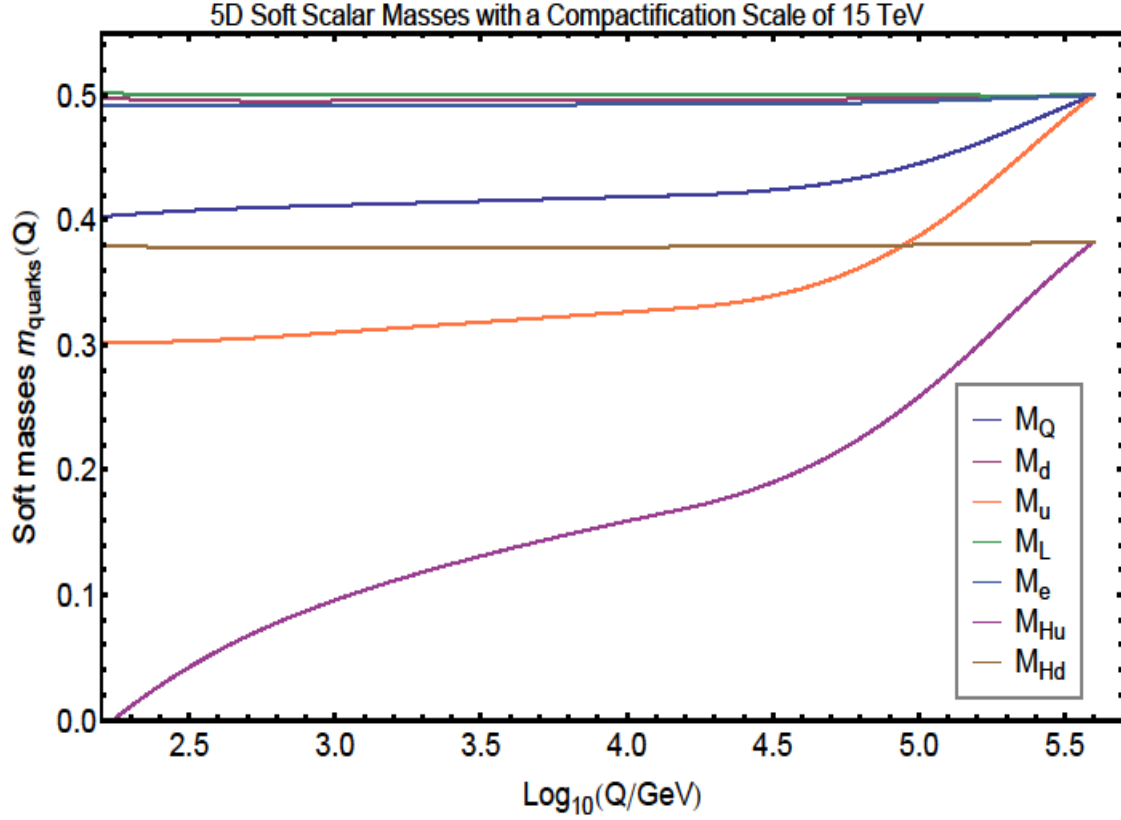


**Figure 4.2.** Show the evolution of gaugino masses and Tri-linear soft breaking term in 5D MSSM+F as function of energy.

We have assumed super-symmetry breaking occurred at the unification scale, which is can be found by obtaining the scale at which  $g_1 = g_2 = g_3$ . This is pictured in figure 4.1.

In figure 4.2 we have specified the value of the gluino mass,  $M_3(Q)$  at  $Q = 10^3 \text{ GeV}$ . We then find the bino and wino soft masses  $M_1$  and  $M_2$  such that all masses of gauging  $M_1 = M_2 = M_3$

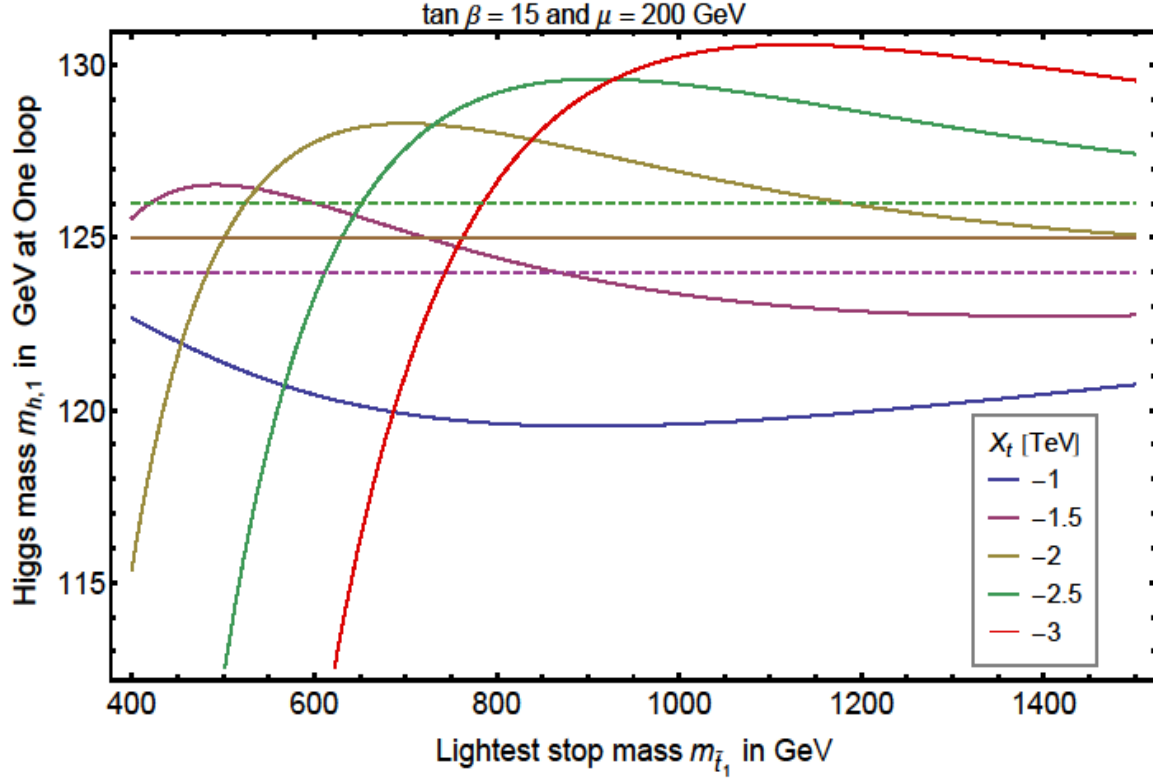
at the unification scale. We set the tri-linear soft breaking terms,  $A_{t,b,\tau}$ , to vanish at the unification scale.



**Figure 4.3:** show the running of scalars soft mass terms as function of energy in 5D MSSM+F.

As depicted in figure 4.3, the running of scalar soft mass terms is plotted against the energy scale. Where we used  $m_{h_u}^2 = m_{h_d}^2$  not from the condition  $(\mu^2 + m_0^2)$  to get electroweak symmetry breaking and correct Higgs mass, where  $m_0^2$  set the scalar soft mass boundary condition.





**Figure 4.4.** The one loop Higgs mass at function of lightest stop mass; for various mixing  $X_t$ .

We show, in figure 4.4, the one loop Higgs mass formula appeared in equation (3.38) for representative values of  $A_t$ . As can be seen that one can accommodate the correct Higgs mass and having light stop s-quark masses which can be below 2 TeV.

### 4.3 Conclusions:

In conclusion, the five dimensional MSSM+F with compactification radius near the TeV scale lead to interesting phenomenology for collider physics. We found that the evolution of the gauge coupling constants has a rapid variation in the presence of the KK states and this leads to a much lower unification scale compare

to the 4D MSM. We showed that the scale dependence is not logarithmic anymore; but shows a power law behavior. Therefore, the five dimensional models has substantial effects on the various and promises exciting phenomenology for upcoming collider physics results, especially with the Large Hadron Collider now being operational and already started explorations of the TeV scale. We used the one loop Higgs mass formula to generate a light stop mass at TeV scale.

#### **4.4 Recommendation:**

This work can be extended in a number of ways; we should include the effect of two-loop Higgs mass formula to get the precise values. Or else we may consider a warped geometry (Randall Sundrum model).

## Bibliography

- A.~Abdalgalbar, A. A. a. A., 2013. Evolution of Yukawa couplings and quark flavour mixings in 2UED models. *Phys. \ Rev. \ D*, Volume 88, p. 056006.
- A.~S.~Cornell, L.-X. a., 2010. Scaling of Yukawa Couplings and Quark Flavor Mixings in the UED Model. *PoS KRUGER*, Volume 2010, p. 045.
- A.D.Martin, F. a., 1984. *Quarks and leptons:An introductory course in modern particle physics*. new york : John Wiley.
- A.J.G.hey, I. a., 1993. *Gauge theories in particlphysics*. second edition ed. s.l.:Bristol and philadelphia.
- al, G. A. e., 2012. ATLAS collaboration. *Phys. Lett*, Volume 1, p. B716.
- al, S. C. e., 2012. CMS collaboration. *Phys.Lett*, Volume 30, p. B716.
- Ammar Ibrahim Abdalgalbar, A. S. A. D. a. M. M., May 21,2014. *Large At without the desert*. F-69622 Lyon,France: National Institute for physics, School of Physics, University of the Witwatersrand, Wits 2050, South Africa, Universite de lyon.
- Collaboration], E. { e. a. [, 2011. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. *Astrophys. \ J. \ Suppl.*, Volume 192, p. 18.
- Collins, J. C., 1984. Renormalization. An Introduction to Renormalization, the renormalization Group and the operator product expansion. *Cambridge, UK: Univ. Pr.*.
- Dr.Teubner, 2009. *The standard model*. s.l., s.n.
- E.~Poppitz, B. a., 2001. Number of fermion generations derived from anomaly cancellation. *Phys. \ Rev. \ Lett.*, Volume 87, p. 031801.
- Falcone, D., 2002. Fermion masses and mixings in gauge theories. *Int. J. Mod. Phys.*, Volume A17, p. 3981.
- Guigg, c., 1983. *Gauge theories of the strong,weak and electromagnetic interaction*. s.l.: Benjamin/cummings.
- H.~U.~Yee, T. a., 2003. Universal extra dimensions and the Higgs boson mass. *Phys. \ Rev. \ D*, Volume 67, p. 055002.
- J. Louis, I. B. a. S. J. H., 1998. The Supersymmetric Standard model. *Arxiv:9811341*.
- J.donoghue, E. a. B., 1994. *Dynamics of the standard model,Cambridge Monographs on particle ,Nuclear physics and Cosmology*. s.l.:s.n.
- K.~S.~Babu, 1987. Renormalization Group Analysis Of The Kobayashi-maskawa Matrix. *Z. \ Phys. \ C*, Volume 35, p. 69.

L.-X. Liu, 2010. Renormalization Invariants and Quark Flavor Mixings. *Int. J. Mod. Phys.*, Volume A25, p. 4975.

L.-X. Liu, A. a., 2011. Evolution of the CKM Matrix in the Universal Extra Dimension Model. *Phys. Rev. D*, Volume 83, p. 033005.

L.F. Li, T. a., 1991. *Gauge theory of elementary particle physics*. s.l.:oxford univ.

Majee, S. K., March, 2008. Some Explorations of New Physics beyond the SM. pp. 38,39,40.

McMahon, D., 2008. *Quantum Field theory Demystified*. s.l.:McGraw-Hill companies.

N. Maru, T. J. a. M., 2010. The Universal Extra Dimensional Model with  $S^2/Z(2)$  extra-space. *Nucl. Phys. B*, Volume 830, p. 414.

P. W. Higgs, 1964. Broken symmetries, massless particles and gauge fields. *Phys. Lett.*, Volume 12, p. 132.

Quigg, C., 2007. *spontaneous symmetry Breaking as a Basic of particle mass*, s.l.: arXIV:0704.2232.

S.L.Glashow, 1961. *Nucl. Phys*, Volume 22, p. 579.

Salam, A., 1968. Elementary Particle Physics (Nobel Symp. N.8). p. 367.

sankarray, T., n.d. *Beyond the standard model*. s.l., s.n.

T. Appelquist, H.-C. a. B., 2001. Bounds on universal extra dimensions. *Phys. Rev. D*, Volume 64, p. 035002.

T. Kaluza, 1921. On the Problem of Unity in Physics. *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys. )*, Volume 1921, p. 966.

Veltman, M., 2003. *Facts and mysteries in elementary particles physics*, s.l.: Co.Pte.Ltd.

Weinberg, S., 1967. *Phys. Rev. Lett*, Volume 19, p. 1264.

Weinberg, S., 1996. *the Quantum theory of field*. s.l.:Cambridge University.

wood, W. c. H. a. D. G., 2007. *An introduction to the standard model of particle physics*. s.l., s.n.

Z.-z. Xing, H. a., 2000. Mass and flavor mixing schemes of quarks and leptons. *Prog. Part. Nucl. Phys.*, Volume 45, p. 1.

Z.-z. Xing, H. a. S., 2008. Updated Values of Running Quark and Lepton Masses. *Phys. Rev. D*, Volume 77, p. 113016.