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Comparative Study to Estimate the Effect of Parameter and Reliability of the Exponential Distribution on the Failure time of the ATM using Bayes Method (Case study: A.T.M in Sudan)

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ABSTRACT

The consequent of non-application the reliability system on the ATM will lead to big problems which are non-increase of productivity and the lack of optimal utilization of machines and reflect on economic development. The aim of this paper is to apply lifetime models on the failure time of the automatic teller machine (ATM) in Sudan and estimate the parameter and reliability of the machines, in order to compare between machines. failure's data has been taken from Central Bank of Sudan, which is, type of machine, type of failure, Downtime, Uptime and outage duration in Hrs. during the period of time (1/1/2017-30/6/2017).The comparison between five machines selected randomly out of 28 machines have done. Through the lifetime models estimation (failure distribution, reliability, hazard rate, and mean time to failure (MTTF)), using Bayes method in estimate the parameters and reliability of exponential distribution. The comparison is executed through reliability values; from analysis results it is clear that, the failure time of all machines follow exponential distribution with one-parameter, according to the Kolmogorov-Smirnov test and Chi-Squared test for goodness of fit result. The machines no (B5 and B35) have high reliability compered by other machines. When we predict the reliability according to the time we found that the reliability decrease and hazard rate increase and there is relationship between MTTF and reliability. Finally to the extending of study span the authors recommended to include all types of ATM faults (out of cash and out of serves). Furthermore, the authors recommended that when expanding or adding a new machine, it is preferable to buy the machine with high reliability.

المستخلص:

عدم تطبيق نظام الموثوقية على ماكينات الصراف الالي (ATM) سيؤدي إلى مشاكل كبيرة مثل عدم الاستخدام الأمثل للماكينات ، عدم زيادة الإنتاجية والتأثير علي التنمية الاقتصادية . والهدف من هذه الدراسة هو تطبيق نماذج الموثوقة على أوقات الفشل لماكينات الصراف الآلي في السودان وتقديرمعالم و موثوقية الماكينات، للمقارنة بين الماكينات المختلفة . تم أخذ بيانات الفشل(بيانات الدراسة) من بنك السودان المركزي، والمتمثله في: نوع الجهاز، نوع الفشل، وقت التوقف، مدة التشغيل وانقطاع الخدمة بالساعات . خلال الفترة الزمنية (1/1/2017 -30/6/2017) ، وقد أجريت المقارنة بين خمسة آلات مختارة عشوائيا من أصل 28 آلة . من خلال تقديرنماذج الفشل توزيع الفشل

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،الموثوقية ،معدل الخطر ومتوسط الوقت للفشل (MTTF)، وذلك باستخدام طريقة بييز في تقدير معلمات وموثوقية التوزيع الأسي . تم تنفيذ المقارنة من خلال قيم دالة الموثوقية، ومن خلال نتائج التحليل إتضح أن وقت الفشل لجميع الآلات يتبع التوزيع الأسي ذو معلمة واحدة، وفقا لاختباري كولموجوروف سميرنوف و كاي تربيع لجودة التوفيق ايضا اتضح أن الماكينات (B5 و B35) لديهما موثوقية عالية مقارنة بالماكينات الاخرى كذلك عندما نتوقع الموثوقية وفقا للوقت نجد أن هنالك انخفاض في الموثوقية وزيادة في معدل الخطر ايضاهناك علاقة طردية بين متوسط زمن الانتظار حتي حدوث الفشل للماكينة ودرجة الموثوقية واخيرا أوصت الدراسة بتوسيع الدراسة لتشمل كل انواع الأعطال في ماكينات الصراف الالي،كذلك عند شراء ماكينات جديدة يفضل شراء ماكينات ذات الموثوقية العالية.

KEY WORD: Reliability, failure rate, mean time to failure, exponential distribution, Bays method, goodness of fit.

INTRODUCTION

Reliability is defined as the ability of the individual device or system or component to perform its required functions under stated conditions for a specified period of time [1, 2]. Reliability can be somewhat abstract in that it involves much statistics; yet it is engineering in its most practical form [3]. Will the design perform its intended mission? Product reliability is seen as a testament to the robustness of the design as will the safety of the quality and manufacturing commitments of an organization.

The study of reliability appeared in the first decade of the twentieth century. The concentration on this type of study has been crystallized during the world war (II), via studying the military devices reliability. This type has expected recently to include the study of commercial products as a result of extraordinary developments on the one hand; and the use of electronic devices and the complex systems on the other. This sort of development has imposed an increasing concern on studying the reasons of breakdowns that may lead to the stoppage of devices and sets in their various kinds [10]Reliability has gained increasing importance in the last few years in manufacturing organizations, the government and civilian communities. With recent concern about government spending, agencies are trying to buy systems with higher reliability and lower maintenance costs [6]. As consumers ،we are mainly concerned with buying products that last longer and are cheaper to maintain, or have higher reliability. There are many reason for wanting high product or component or system reliability such as higher customer satisfaction، increased sales، improved safety، decreased warranty costs, decreased maintenance costs.

Moreover the importance of reliability has risen recently as it is appropriate to most of applications represented in the repairable systems and equipment in case of its sudden stop and becoming out of service and then reiterates them. So the importance of this study rises due to the importance of the subject of reliability of the systems and machines on which the wheel of production depends so that we may be able to satisfy the basic needs of commodities and services in a perfect technique.

The research problem is that, there is no lifetime (reliability) models application to predict the sudden failures that occur in ATM and the consequent of non-application the reliability system on the

ATM will lead to big problems which are non-increase of productivity and the lack of optimal utilization of machines and reflect on economic development.

The objective of this paper is to apply lifetime models on the failure time of the automatic teller machine (ATM) in Sudan and estimate the parameter and reliability of the machines, in order to compare between machines.

Reliability function:

The most frequently used function in life data analysis and reliability engineering is the reliability function. This function gives the probability of an item operating for a certain amount of time without failure. As such, the reliability function is a function of time, in that every reliability value has an associated time value. The reliability function of an item is defined by [1,7,11]:

$$R(t) = 1 - F(t) = \Pr(T > t) \quad \text{for } t > 0 \text{ .... (1)}$$

Hence $R(t)$ is the probability that the item dos not fail in the time interval$(0, t]$, or, in other words, the probability that the time survives the time interval (0, t] and is still functioning at time t.

Time to failure:
By the time to failure of an item we mean the time elapsing from when the item is put into operation until it fails for the first time. We set $t = 0$ as the starting point. At least to some extent the time to failure is subject to chance variations it is therefore natural to interpret the time to failure as a random variable (T) [12].

Failure rate function:
The probability that an item will fail in the time interval $(t, t + \Delta t)$ when we know that the item functioning at time t is [1]:

$$\Pr(t < t + \Delta t / T > t) = \frac{\Pr(t < T \leq t + \Delta t)}{\Pr(T > t)} = \frac{F(t + \Delta t) - F(t)}{R(t)}$$

By dividing this probability by the length of the time interval, Δt, and letting, $\Delta t \to 0$,we get the failure rate function h(t) of the item

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr(t<t+\Delta t/T>t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{F(t+\Delta t)-F(t)}{\Delta t} = \frac{1}{R(t)} = \frac{f(t)}{R(t)} \text{.................... (2)}$$

The exponential distribution:
Exponential distribution plays an essential role in reliability engineering because it has constant failure rate [5]. This distribution has been used to model the life time of electronic and electrical components and systems. This distribution is appropriate when a used component that has not failed is as good as a new component – a rather restrictive assumption. Therefore it must be used diplomatically since numerous exist where the restriction of the memory less property may not apply.

Consider an item that is put into operation at time t=0. The time to failure T of the item has probability density function is [1,8]

$$f(t) = \begin{cases} \frac{1}{\theta} e^{-\frac{1}{\theta} t} & \text{for } t > 0, \theta > 0 \\ 0 & \text{otherwise} \end{cases} \text{ ....(3)}$$

The cumulative distribution function,cdf, is a function $F(t)$ of a random variable T, and is defined for a number t by:

$$F(t) = P(T \leq t) = \int_0^t f(u)du = \int_0^t \frac{1}{\theta} e^{-\frac{1}{\theta}u} du = 1 - e^{-\frac{1}{\theta}t} \text{.............................. (4)}$$

The reliability function of the item ($R(t)$) is:

$$R(t) = \Pr(T > t) = \int_t^{\infty} f(u)du = e^{-\frac{1}{\theta}t}, \quad \text{for } t > 0 \ldots\ldots(5)$$

The mean time to failure (MTTF) is:

$$MTTF = \int_0^{\infty} R(t)dt = \int_0^{\infty} e^{-\frac{1}{\theta}t}\, dt = \theta \ldots\ldots(6)$$

And the variance of T is

$$var(t) = \theta^2 \ldots\ldots(7)$$

The failure rate function ($h(t)$) is:

$$h(t) = \frac{f(t)}{R(t)} = \frac{\frac{1}{\theta}e^{-\frac{1}{\theta}t}}{e^{-\frac{1}{\theta}t}} = \frac{1}{\theta} \ldots\ldots(8)$$

Accordingly, the failure rate function of an item with exponential life distribution is constant (i.e., independent of time).

Bayes Estimation:

Suppose that, $t_1, t_2, \ldots\ldots t_n$, be the life time of a random sample size n with distribution function $F(t, \theta)$ and probability function $f(t; \theta)$.

In the exponential distribution, we assume that the probability density function of the life time is given by [(4)]:

$$f(t; \theta) = \frac{1}{\theta} e^{-\frac{1}{\theta}t} \qquad \text{for } \theta > 0, t > 0$$

We find Jeffery prior by taking $g(\theta) \propto \sqrt{I(\theta)}$

Where

$$I(\theta) = nE\left[\frac{\partial^2 \ln f(t;\theta)}{\partial\theta^2}\right] = \frac{n}{\theta^2} \ldots\ldots(9)$$

Taking: $g(\theta) \propto \sqrt{I(\theta)}, g(\theta) = k\sqrt{I(\theta)}$, with k a constant.

The joint probability density function $f(t_1, t_2, \ldots\ldots t_n, \theta)$ is given by;

$$H(t_1, t_2, \ldots\ldots t_n, \theta) = \prod_{i=1}^{n} f(t_i; \theta)\, g(\theta) = L(t_1, t_2, \ldots. t_n|\theta)g(\theta)$$

Where

$$L(t_1, t_2, \ldots. t_n|\theta) = \prod_{i=1}^{n}(t_i|\theta)\, g(\theta) = \frac{1}{\theta^n} e^{-\frac{1}{\theta}\sum t_i}$$

$$H(t_1, t_2, \ldots\ldots t_n, \theta) = \frac{1}{\theta^n} e^{-\frac{1}{\theta}\sum t_i}. k\frac{\sqrt{n}}{\theta} = k\frac{\sqrt{n}}{\theta^{n+1}}.e^{-\frac{1}{\theta}\sum t_i}$$

Then the marginal probability density function of θ given the data $(t_1, t_2, \ldots\ldots t_n)$ is:

$$p(t_1, t_2, \ldots \ldots . t_n) = \int_0^\infty H(t_1, t_2, \ldots ., t_n, \theta)\, d\theta$$

$$\int_0^\infty k \frac{\sqrt{n}}{\theta^{n+1}} . e^{-\frac{1}{\theta}\Sigma t_i}\, d\theta = \frac{k\sqrt{n}\Gamma(n)}{(\sum_{i=1}^{n} t_i)^n}$$

Where

$$\int_0^\infty \frac{1}{\theta^{n+1}} . e^{-\frac{1}{\theta}\Sigma t_i}\, d\theta = \frac{\Gamma(n)}{(\sum_{i=1}^{n} t_i)^n}$$

The conditional probability density functions of θ given data $(t_1, t_2, \ldots \ldots . t_n, \theta)$ is given by:

$$H(\theta | t_1, t_2, \ldots . . t_n) = \frac{H(t_1, t_2, \ldots ., t_n, \theta)}{Pr(t_1, t_2, \ldots . . t_n)}$$

$$= \frac{k \frac{\sqrt{n}}{\theta^{n+1}} . e^{-\frac{1}{\theta}\Sigma t_i}}{\frac{k\sqrt{n}\Gamma(n)}{(\sum_{i=1}^{n} t_i)^n}} = \frac{e^{-\frac{1}{\theta}\Sigma t_i}}{\theta^{n+1}} . \frac{(\sum_{i=1}^{n} t_i)^n}{\Gamma(n)}$$

By using squared error loss function

$$\ell(\theta, \hat{\theta}) = C(\hat{\theta} - \theta)^2$$

We can obtained the Risk function, such that $R(\hat{\theta} - \theta)$

$$R(\hat{\theta} - \theta) = E[C(\hat{\theta} - \theta)] = \int_0^\infty \ell(\hat{\theta} - \theta) \prod (\theta | t_1, t_2, \ldots . . t_n) d\theta$$

$$c\hat{\theta}^2 - 2c\hat{\theta} \int_0^\infty \frac{\sum_{i=1}^{n} t_i}{\Gamma(n)} \theta^{-n}\, e^{-\frac{1}{\theta}\Sigma t_i}\, d\theta + \zeta(\theta)$$

Where

$$\zeta(\theta) = \frac{c(\sum_{i=1}^{n} t_i)^2}{(n-1)(n-2)}$$

Let $\frac{\partial R(\hat{\theta}, \theta)}{\partial \hat{\theta}} = 0$ then the Bayes estimation is

$$\hat{\theta}_{Bayes} = \frac{\sum_{i=1}^{n} t_i}{\Gamma(n)} \int_0^\infty \int_0^\infty \frac{\sum_{i=1}^{n} t_i}{\Gamma(n)} \theta^{-n}\, e^{-\frac{1}{\theta}\Sigma t_i}\, d\theta \quad = \frac{\Sigma t_i}{n-1} \ldots\ldots\ldots\ldots (10)$$

This unbiased estimation with variance

$$Var(\hat{\theta}_{Bayes}) = \frac{n\theta^2}{(n-1)^2} \ldots\ldots\ldots\ldots (11)$$

And mean square error is

$$MSE(\hat{\theta}_{Bayes}) = \frac{n\theta^2}{(n-1)^2} + \frac{\theta^2}{(n-1)^2} = \frac{(n+1)\theta^2}{(n-1)^2} \dots\dots\dots\dots\dots\dots (12)$$

Then the Bayes reliability function is steamed by:

$$\hat{R}(t) = \int_0^\infty e^{-\frac{t}{\theta}} h(\theta|t)d\theta = \left(\frac{\sum t_i}{\sum t_i + t}\right)^n \dots\dots\dots\dots\dots\dots\dots (13)$$

MATERIALS and METHODS

The technical fault data collocated from the Sudan central Bank and it was, (type of machine or thermal name, type of failure, Downtime, Uptime and outage duration in Hrs.), during the period between (1/1/2017-30/6/2017).

The data have been collocated from the five deferent banks in Sudan selected randomly out of 28 public banks. Because of difficulty of collection the data of study we decided to select randomly five public banks out of 28 public banks by using the simple random sample in the selection of the machines.

There are three types of failures in ATM, out of journals, out of serves and out of cash; in this study we applied the data of out of journal.

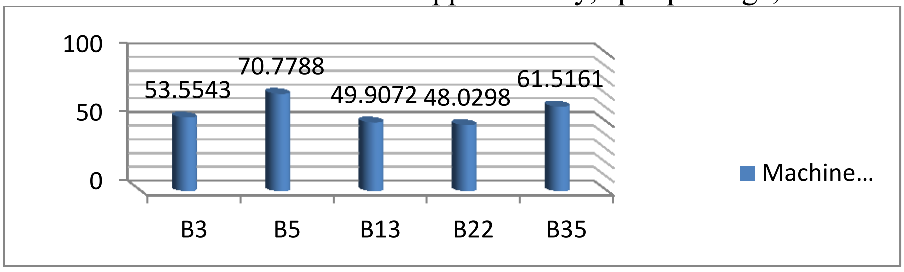
This study depends on the statistical descriptive approach and analytical approach through describing the nature of machines work in the automatic teller service and estimating parameter of exponential distributions of the failure and the reliability used EasyFit, SPSS, Excel and other suitable statistical software.

RESULTS and DISCATION

Table (1) Descriptive Statistics of out of journal time for each machines

Machine code	N	Mean		Std. Deviation
	Statistic	Statistic	Std. Error	Statistic
B3	51	53.5543	6.50364	46.44527
B5	25	70.7788	12.79589	63.97943
B13	123	49.9072	4.43891	49.22987
B22	51	48.0298	6.65307	47.51243
B35	44	61.5161	8.97677	59.54515

Source: the researcher from applied study, spss package, 2017



Source: the researcher from applied study, Excel package, 2017

Figure (1) Descriptive Statistics of out of journal time for each machine

From table(1) and figure(1), it is shows that according to the mean values for the five machines, the machine (B5) have the highest mean failure time (out of journals) with the mean time of failure value (70.7788) hours and it has Std. Error of mean(12.79589) hours and Std. Deviation (63.97943) hours, followed by machine (B35) with the mean failure time (out of journals) (61.5161) hours and it has Std. Error of mean(8.97677) hours and Std. Deviation (59.54515) hours, followed by machine (B3) with the mean failure time (out of journals) (53.5543) hours and it has Std. Error of mean(6.50364) hours and Std. Deviation (46.44527) hours, followed by machine (B13) with the mean failure time (out of journals) (49.9072) hours and it has Std. Error of mean(4.43891) hours and Std. Deviation (49.22987) hours, and the machine (B22) have the less mean failure time (out of journals) (48.0298) hours and it has Std. Error of mean(6.65307) hours and Std. Deviation (47.51243) hours.

Goodness of Fit – Details for the machines:

The goodness of fit (GOF) tests measures the compatibility of a random sample with a theoretical probability distribution function. In other words, these tests show how well the distribution you selected fit's to your data [(5)].

The general procedure consists of defining a test statistic which is some function of the data measuring the distance between the hypothesis and the data, and then calculating the probability of obtaining data which have a still larger value of this test statistic than the value observed, assuming the hypothesis is true. This probability is called the confidence level.

To test the failure time data follow exponential distribution we used Kolmogorov-Smirnov test and Chi-Squared test as the following:

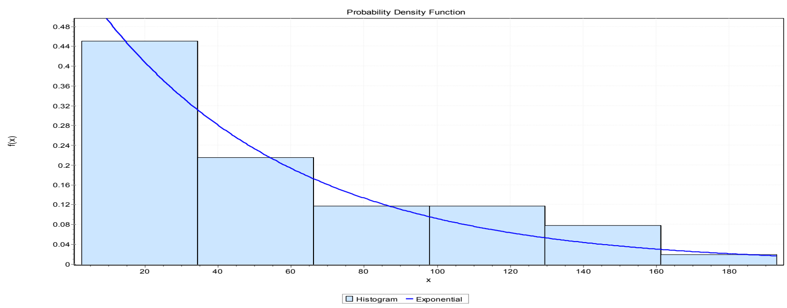
H_0: The failure time data follow exponential distribution

H_1: The failure time data not follow exponential distribution

Table (2) Kolmogorov-Smirnov test and Chi-Squared test for the machines

Machine code	Kolmogorov-Smirnov				Chi-Squared				Reject H_0 at $\alpha = 0.05$
	N	Statistic	P-Value	Critical Value	df	Statistic	P-Value	Critical Value	
B3	51	0.06238	0.98161	0.18659	5	0.53138	0.99093	11.07	No
B5	25	0.09653	0.95646	0.26404	2	0.48266	0.78558	5.9915	No
B13	123	0.06647	0.62448	0.12245	6	1.1613	0.97875	12.592	No
B22	51	0.07683	0.9015	0.18659	5	1.6685	0.89285	11.07	No
B35	44	0.0725	0.96189	0.20056	4	3.6612	0.45379	9.4877	No

Source: the researcher from applied study, EasyFit package, 2017



Source: the researcher from applied study, EasyFit package, 2017

Figure (2) density function of Exponential Vs. time for machine (B3)

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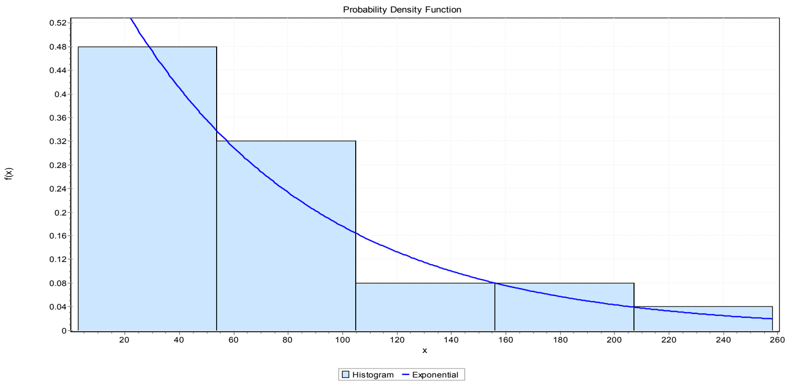
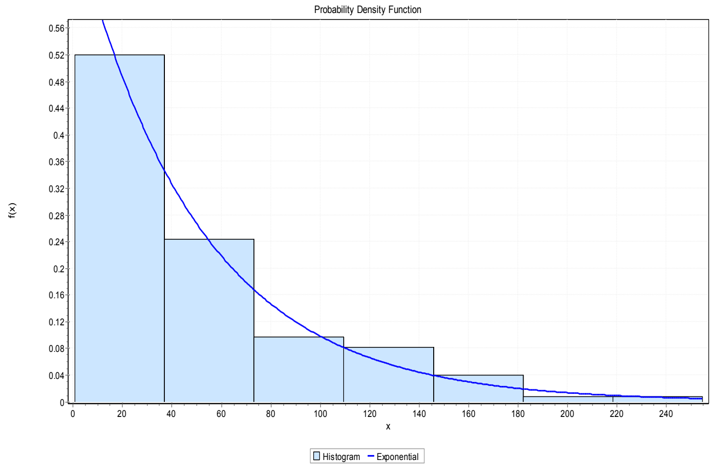
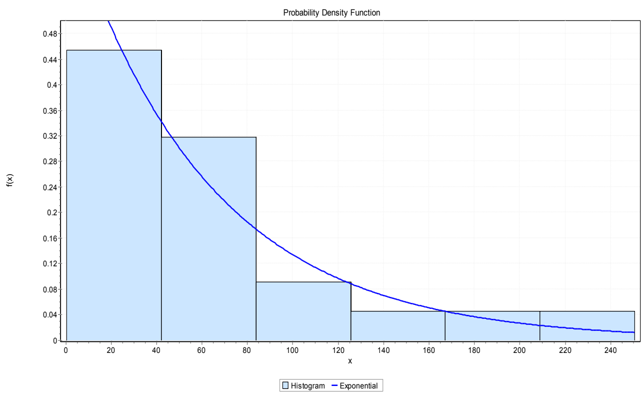


Source: the researcher from applied study, EasyFit package, 2017
Figure (3) density function of Exponential Vs. time for machine (B5)



Source: the researcher from applied study, EasyFit package, 2017
Figure (4) density function of Exponential Vs. time for machine (B13)



Source: the researcher from applied study, EasyFit package, 2017
Figure (5) density function of Exponential Vs. time for machine (B22)

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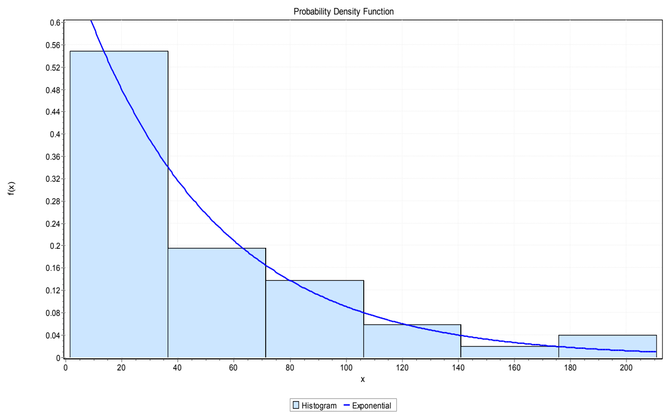


Source: the researcher from applied study, EasyFit package, 2017

Figure (6) density function of Exponential Vs. time for machine (B35)

From table (2), it is clear that the failure time data of the machines follow exponential distribution with one-parameter according to Kolmogorov-Smirnov test and Chi-Squared test result (statistic, p-value and critical value.

For machine B3, the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.98161) and (0.99093) are graters than significant level (0.05) and their critical values are greater than calculated values that mean we are not reject the H_0 or the failure time data of machine (B3) follow exponential distribution with parameter ($\lambda = 0.018307$) and the probability density function is illustrated in figure (2).

For machine B5, the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.95646) and (0.78558) are graters than significant levels 0.05 and their critical values are greater than calculated values that mean we are not reject the H_0 or the failure time data of machine (B5) follow exponential distribution with parameter ($\lambda = 0.013563$) and the probability density function is illustrated in figure (3).

For machine B13, the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.62448) and (0.97875) are graters than significant level (0.05) and their critical values are greater than calculated values that mean we are not reject the H_0 or the failure time data of machine (B13) follow exponential distribution with parameter (λ =0.01987), and the probability density function is illustrated in figure (4).

For machine B22, the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.9015) and (0.89285) are graters than significant level (0.05) and their critical values are greater than calculated values that mean we are not reject the H_0 or the failure time data of machine (B22) follow exponential distribution with parameter (λ= 0.020412), and the probability density function is illustrated in figure (5).

For machine B35, the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.96189) and (0.45379) are graters than significant level (0.05) and their critical values are greater than calculated values that mean we are not reject the H_0 or the failure time data of machine (B35) follow exponential distribution with parameter (λ=0.015886), and the probability density function is illustrated in figure (6).

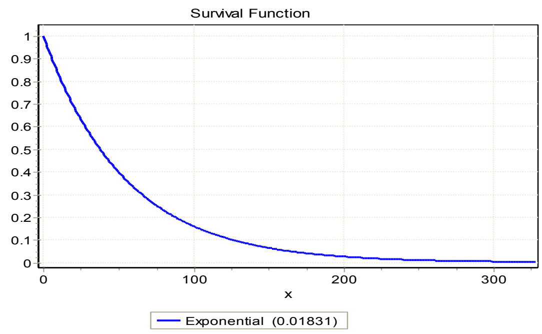
Reliability models of the machines:

The Reliability models of machines have been conducted from all machines for a period of time (t= 0, 24, 48, 72, 96, 120 and144 hours) using method and Bayes method in estimate the following measures:

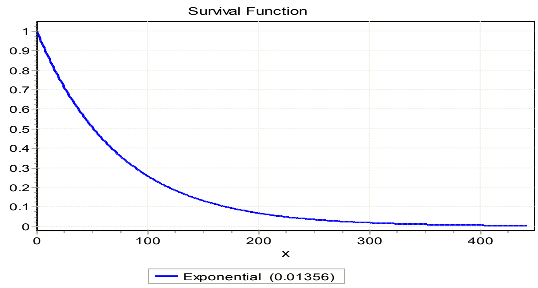
Table (8) Result of life time's test of the machines

M. Code	Masseur	Time/hour						
		0	24	48	72	96	120	144
B3	R(t)	1	0.6400	0.41127	0.26527	0.17174	0.11159	0.07277
	h(t)	0.018307	0.01867	0.01867	0.01867	0.01867	0.01867	0.01867
	Cum.h(t)	0.00	0.43937	0.87888	1.3183	1.7578	2.1972	2.6366
B5	R(t)	1.00	0.7140	0.51215	0.36895	0.266915	0.193901	0.141429
	h(t)	0.013563	0.013563	0.013563	0.013563	0.013563	0.013563	0.013563
	Cum.h(t)	0.00	0.32544	0.65102	0.97632	1.3018	1.6272	1.9531
B13	R(t)	1.00	0.6188	0.38364	0.238286	0.148275	0.092433	0.057726
	h(t)	0.019874	0.019874	0.019874	0.019874	0.019874	0.019874	0.02004
	Cum.h(t)	0.00	0.47698	0.95376	1.4306	1.9075	2.386344	2.8613
B22	R(t)	1.00	0.6082	0.37168	0.228216	0.140775	0.087232	0.054295
	h(t)	0.020412	0.020412	0.020412	0.020412	0.020412	0.020412	0.020412
	Cum.h(t)	0.00	0.48989	0.97968	1.4695	1.9594	2.4492	2.939
B35	R(t)	1.00	0.6781	0.46142	0.315017	0.215772	0.148272	0.102212
	h(t)	0.015886	0.015886	0.015886	0.015886	0.015886	0.015886	0.015886
	Cum.h(t)	0.00	0.38136	0.76272	1.1441	1.5254	1.9068	2.2876

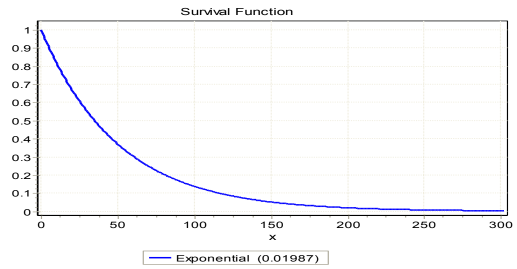
Source: the researcher from applied study, EasyFit package, 2017



Source: the researcher from applied study, EasyFit package, 2017
Figure (8): Reliability function of Exponential Vs. time for machine (B3)

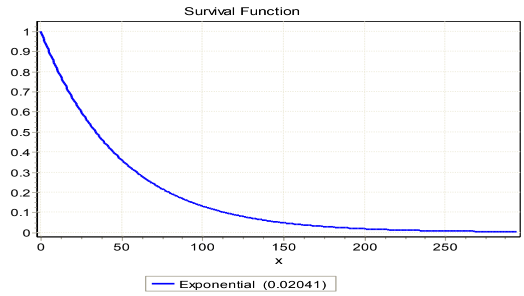


Source: the researcher from applied study, EasyFit package, 2017
Figure (9): Reliability function of Exponential Vs. time for machine (B5)



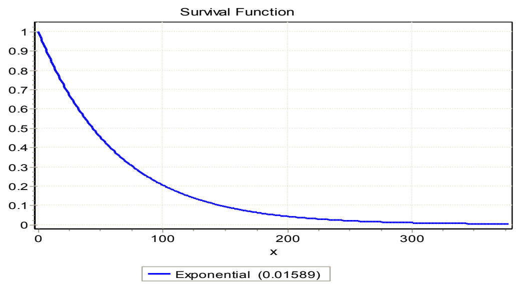
Source: the researcher from applied study, EasyFit package, 2017

Figure (10): Reliability function of Exponential Vs. time for machine (B13)



Source: the researcher from applied study, EasyFit package, 2017

Figure (11): Reliability function of Exponential Vs. time for machine (B22)



Source: the researcher from applied study, EasyFit package, 2017

Figure (12): Reliability function of Exponential Vs. time for machine (B35)

From table (8), it is shows that the reliability of machines decreases whenever the working time of the machines increases.

Fore machine (B3), at time (t=0) hour the reliability is(100%), at time (t=24) hours the reliability is about(64%), at time (t=48) hours the reliability is about(41%), at time (t=72) hours the reliability is about(27%), at time (t=96) hours the reliability is about(17%), at time (t=120) hours the reliability is about(11%)and at time (t=144) hours the reliability is about(07%), that mean the reliability of machine (B3) decreases whenever the working time of the machine increases. The reliability function is illustrated in figure (8).

Fore machine (B5), at time (t=0) hour the reliability is(100%), at time (t=24) hours the reliability is about(0.71%), at time (t=48) hours the reliability is about(0.51%), at time (t=72) hours the reliability is about(37%), at time (t=96) hours the reliability is about(27%), at time (t=120) hours the reliability is about(19%) and at time (t=144) hours the reliability is about(14%), that mean the reliability of machine (B5) decreases whenever the working time of the machine increases. The reliability function is illustrated in figure (9).

Fore machine (B13), at time (t=0) hour the reliability is(100%), at time (t=24) hours the reliability is about(62%), at time (t=48) hours the reliability is about(38%), at time (t=72) hours the reliability is about(24%), at time (t=96) hours the reliability is about(15%), at time (t=120) hours the reliability is about(09%) and at time (t=144) hours the reliability is about(06%), that mean the reliability of machine (B13) decreases whenever the working time of the machine increases. The reliability function is illustrated in figure (10).

Fore machine (B22), at time (t=0) hour the reliability is(100%), at time (t=24) hours the reliability is about(61%), at time (t=48) hours the reliability is about(0.37%), at time (t=72) hours the reliability is about(23%), at time (t=96) hours the reliability is about(14%), at time (t=120) hours the reliability is about(09%) and at time (t=144) hours the reliability is about(05%), that mean the reliability of machine (B22) decreases whenever the working time of the machine increases. The reliability function is illustrated in figure (11).

Fore machine (B35), at time (t=0) hour the reliability is(100%), at time (t=24) hours the reliability is about (68%), at time (t=48) hours the reliability is about(46%), at time (t=72) hours the reliability is about(32%), at time (t=96) hours the reliability is about(22%), at time (t=120) hours the reliability is about(15%) and at time (t=144) hours the reliability is about(10%), that mean the reliability of machine (B35) decreases whenever the working time of the machine increases. The reliability function is illustrated in figure (12).

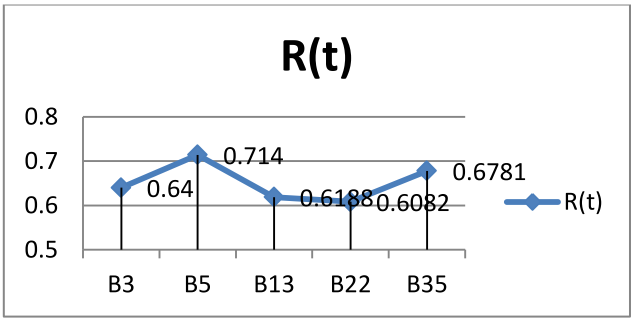
Comparison between machines:

We compare the machines according to the reliability values of machines at time t=24 hours by using Maximum likelihood and Bayes method in estimate the parameter and the reliability values of the machines.

Table (3) Result of Reliability model test of the machines at time (t=24) hours

Machine code	Measure		
	Hazard rate h(t)	Reliability R(t)	failure distribution f(t)
B3	0.018307	0.6400	0.0118
B5	0.013563	0.7140	0.00979
B13	0.019874	0.6188	0.01233
B22	0.020412	0.6082	0.01251
B35	0.015886	0.6781	0.01251

Source: the researcher from applied study, Easy Fit package, 2017



Source: the researcher from applied study, Excel package, 2017

Figure (7) Result of Reliability model test of machines at time (t=24) hours

From table(3) and figure(7), it is clear that according to the reliability values for the five machines ,the machine (B5) have the highest reliability R(t=24=0.7140) with probability fault (f(t=24=0.01007) and hazard rate h(t=24= 0.013563), this mean that the probability of work(24)hours without fault for machine (B5) is (0.7140) followed by machine (B35) with R(t=24=0.67681) and probability fault (f(t=24=0.01251) and hazard rate h(t=24= 0.015886), this mean that the probability of work(24)hours without fault for machine (B35) is (0.6781), followed by machine (B3) with R(t=24=0.6400) and probability fault f(t=24=0.0118) and hazard rate h(t=24= 0.018307), this mean that the probability of work(24)hours without fault for machine (B3) is (0.6400).

CONCLUIONS:

The main findings of this study are:

1. Time of failures follows exponential distribution with one parameter for all machines.

2. The reliability of machines decreases whenever the working time of the machines increases.

3. The hazard rate of machines is constant according to the time.

4. The machines with high reliability have a low faults probability and hazard rate.

5. The machines with low reliability have a high fault probability and hazard rate.

6. Whenever mean time between failures for machines increase that indicate the machine has high reliability.

7. The machines no (B5 and B35) have high reliability compered by other machines.

RECONDATIONS

1- To the extending of study span the authors recommended to include all types of ATM faults (out of cash and out of serves).

2-The authors recommended that when expanding or adding a new machine, it is preferable to buy the machine with high reliability.

3- The authors recommended that it is better to follow the remedial policy of maintenance for the machines which mentions that maintenance is made when a defect occurs. It is not recommended to follow precautionary maintenance followed by the ATM because the bulk of defect takes place shortly after the precautionary maintenance.

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APPENDIX:

The technical fault data for five machines collocated from the Sudan central Bank, during the period of time (1/1/2017-30/6/2017).

B3	B5	B13	B22	B35
11.10	54.93	10.96	5.23	65.97
86.68	63.44	0.59	83.27	128.05
60.46	24.24	34.30	28.94	45.94
16.03	116.59	11.17	150.27	14.27
24.87	12.17	60.33	15.88	250.71
51.79	37.11	30.75	117.50	168.86
33.74	57.24	18.80	6.11	17.26
2.58	178.40	2.15	92.54	0.24
29.38	53.18	12.27	68.75	29.75
67.27	28.00	68.69	45.57	10.98
63.31	79.69	83.87	25.19	36.91
2.72	203.47	8.24	50.05	2.55
40.23	10.39	97.99	4.01	51.77
100.32	15.58	9.08	10.99	177.25
43.51	92.97	113.94	1.35	2.70
120.07	2.70	106.12	28.04	62.52
14.48	26.80	17.80	5.59	40.67
78.81	88.00	20.14	32.18	2.64
28.96	91.95	31.71	70.99	99.05
5.54	21.60	12.63	106.75	10.34
131.65	20.51	49.83	27.21	136.38
35.51	258.14	41.49	12.07	72.39
53.28	64.11	11.79	15.22	4.66
78.00	48.35	75.20	46.78	10.85
69.44	119.91	55.82	23.06	90.80
99.53		19.50	2.19	49.87
140.60		60.77	210.93	38.77
21.89		137.75	186.48	19.02
40.48		103.54	83.57	76.26
102.55		2.70	13.53	11.39
12.02		5.44	25.00	98.15
34.98		168.42	23.34	49.09
8.34		2.34	26.01	244.67
118.52		9.57	18.14	32.84
98.44		159.16	46.88	81.24
9.28		9.81	9.62	67.96
193.00		45.67	93.49	9.28
4.90		14.01	19.10	69.66
148.36		23.77	77.83	54.17

17.61		51.17	86.91	52.27
17.73		30.88	11.92	25.51
23.46		131.10	37.46	93.48
5.08		10.67	12.39	67.03
66.36		40.86	24.90	32.54
57.37		16.26	19.62	
13.46		4.89	39.07	
14.27		112.39	128.43	
10.96		139.58	46.35	
50.62		18.36	45.34	
24.43		21.50	81.82	
147.30		29.24	5.66	
		7.23		
		34.14		
		61.84		
		133.52		
		86.69		
		90.27		
		41.00		
		3.82		
		13.66		
		57.46		
		73.06		
		4.45		
		5.15		
		10.32		
		6.12		
		12.88		
		149.61		
		10.05		
		17.37		
		40.40		
		5.59		
		4.30		
		27.28		
		169.07		
		42.57		
		72.43		
		17.48		
		48.80		
		20.74		
		174.88		
		50.93		

		62.90		
		5.69		
		107.31		
		45.74		
		39.51		
		49.10		
		110.38		
		17.78		
		14.01		
		72.86		
		84.73		
		86.65		
		20.36		
		20.67		
		1.82		
		39.03		
		24.10		
		112.34		
		31.30		
		11.25		
		37.80		
		114.68		
		2.81		
		47.47		
		52.77		
		254.70		
		32.92		
		62.71		
		1.89		
		36.89		
		116.33		
		6.40		
		28.36		
		14.62		
		63.04		
		76.41		
		19.49		
		2.18		
		71.26		
		205.42		
		84.78		

Source: Sudan central Bank, 2017