Sudan University of Science and Technology

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كلية الدراسات العليا

Comparative study to estimate the parameters of Failure probability Distributions and the Reliability using Maximum Likelihood and Bayes Methods (Case study: ATM in Sudan)

دراسة مقارنة لتقدير معالم التوزيعات الإحتمالية للفشل والموثوقية بإستخدام طريقتي الإمكان الأعظم وبيز (دراسة حالة:ماكينات الصراف الآلي في السودان)

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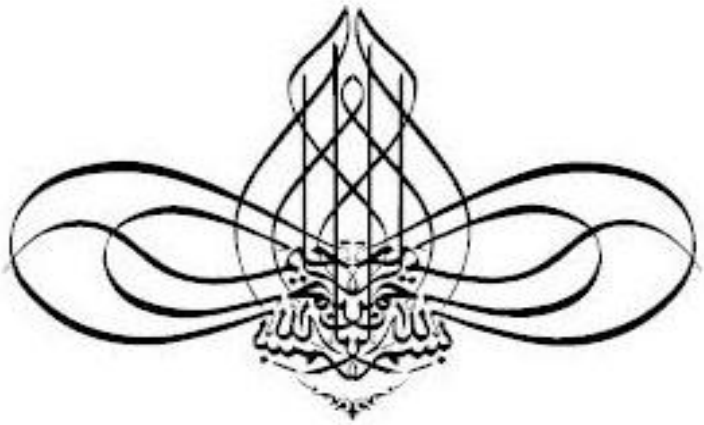

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I am here by declaring that all information in this thesis which has been obtained and presented in accordance to academic rules. I also declare that, as required by these rules, I have fully cited and referenced all materials and results that are not original to this thesis.

The researcher

Dedication:

This thesis is especially dedicated to my:

Mother

Father

Brothers

Sisters

Wife

Son

Friends

And

For all those who have guided and inspired me throughout my journey of education.

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My gratitude goes to my supervisor ***D: Ahmed Mohamed Abdallah Hamdi and to the CO-Supervisor D: Afra Hashim Abdelateef,*** *those who are responsible for the successful completion of my dissertation. Their untiring effort, commitment, encouragement, guidance and support helped me greatly in the understanding and writing of the dissertation. I would have been lost without them.*

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ABSTRACT

The aim of this study is to apply lifetime models on the failure time of automatic teller machines (ATM) in Sudan and to estimate the reliability of machines, in order to compare between machines and methods of estimation, through use the failure models and the suitable statistical methods for failure data analysis. This research depends on Some hypothesis are that: Time to failure in ATM follows exponential distribution with one parameter, and decrease the reliability of the machines coincides with the increase of the machine's life and the increase of operating hours, the methods of statistical and mathematical approaches help in obtaining correct estimates of reliability if the data used in the estimations is correct and there is no significant differences between the mean of Maximum likelihood and Bayes parameter and the reliability estimators. Also the researcher presented the theoretical principles for Basic concepts and important measures for the reliability and the Lifetime-distributions and some methods of estimate the exponential distribution's lifetime models and Goodness of fit techniques. These it will be explained in chapter two and, three. The failure data has been taken from the Central Bank of Sudan, during the period of time **(1/1/2017-31/5/2017).** The statistical packages have been used for data analysis and models constricting are: **Easy Fit, SPSS** and **Microsoft Excel**. The comparison between 15 machines selected randomly (used simple random sample in selection) out of 28 machines from public banks in Sudan has done, through the lifetime models estimation used the Maximum likelihood and Bayes methods in estimate the parameters and reliability of exponential distribution. The comparison between methods of estimation has executed through mean square error values (MSE).And the most important results found out by researcher are: The failure-time of all machines follows exponential distribution with one-parameter and the hazard rate of machines is constant or independence of time. The mean square error values (MSE) of exponential parameter which is estimated by Maximum likelihood method is less than MSE of Bayes' estimator or the Maximum likelihood method is better than Bayes method in estimate the parameter and the reliability function for exponential distribution of automatic teller machines (ATM) in Sudan, The machines with high reliability have a low faults probability and hazard rate, while the machines with low reliability have a high fault probability and hazard

rate, Whenever the mean time to failure for machine increases it indicates that the machine has high reliability and the reliability of the machines decreases when the working time of machines increases, the machines no (B28, B5 and B35) have high reliability compared to the other machines and the machines (B23and B33) have low reliability, and lastly there is no significant differences between the mean of Maximum likelihood and Bayes parameter and the reliability estimators at $\alpha = 0.05$ and 95% Confidence Interval of the Difference. Ultimately, as the researcher suggests that to emphasize the importance of the subject of reliability in the studies and evaluations the machines or the differentiation between one system and another. And it recommends that possibly, to depend on Maximum likelihood method to estimate the parameter of the Exponential distribution and the reliability of the ATM machines in Sudan. Furthermore, the researcher suggests that to extend the study span to include all types of ATM faults (out of cash and out of serves).Also the researcher recommends that when expanding or adding a new machine, it is preferable to buy the machine with high reliability. In addition to that, the researcher recommends that it is better to follow the remedial policy of maintenance for the machines which says that maintenance is made when a defect occurs and the precautionary policy should be replaced because the great percentage of failure will occur shortly after the operation of precautionary maintenance and finally the researcher suggests that to accurate recording of failures which occur in all machines that leads to determine the time, interval, type of failure as well as the cause of failure.

مستخلص الدراسة

. الهدف من هذه الدراسة هو تطبيق نماذج الموثوقية على أوقات الفشل لماكينات الصراف الآلي في السودان ، من أجل المقارنة بين الماكينات من حيث الموثوقية وطرق تقدير المعالم، وذلك من خلال تطبيق نماذج الفشل و الطرق الإحصائية المناسبة لتحليل بيانات الأعطال وتحديد نسبة الإستخدام الأمثل للماكينات المختلفة .ومن أهم الفرضيات التي إستند إليها الباحث هي: أن زمن الفشل لماكينات الصراف الآلي (ATM) يتبع التوزيع الأسي ذو معلمة واحدة ، تتزامن إنخفاض موثوقية الماكينات مع زيادة عمر الماكينة وزيادة ساعات التشغيل ،إيضا الطرق والأساليب الإحصائية والرياضية الحديثة تساعد في الحصول على تقديرات صحيحة للموثوقية إذا كانت البيانات المستخدمة في التقديرات صحيحة ودقيقة ولا توجد فروق معنوية بين متوسط قيم الموثوقية و المعالم المقدرة بإستخدام طريقتي الإمكان الأعظم وبيز . كما عرض الباحث المبادئ النظرية للمفاهيم الأساسية والمقاييس الهامة للموثوقية وبعض توزيعات الفشل و طرق تقديرمعالم نماذج الفشل للتوزيع الاسي وتقنيات إختبار جودة التوفيق وذلك في الفصل الثاني والثالث . كما تم أخذ بيانات الفشل من بنك السودان المركزي خلال الفترة الزمنية **(1/1/2017-31/5/2017)**. ولتطبيق نماذج الموثوقية على بيانات الفشل الخاصة بماكينات الصراف الآلي. تم إستخدام الحزم الإحصائية الجاهزة (Easy Fit و SPSS و Excel) في تحليل البيانات وبناء النماذج. و تم المقارنة بين 15 ماكينة تمت إختيارها عشوائيا بإستخدام المعاينة العشوائية البسيطة من بين 28 ماكينة من البنوك العامة في السودان ،ومن خلال تقييم نماذج الفشل بإستخدام طريقة الإمكان الأعظم وطريقة بيز في تقديرمعالم التوزيع الأسي وموثوقية الماكينات تم تنفيذ المقارنة بين طرق التقدير من خلال قيم متوسط مربع الخطأ (MSE)للمعالم المقدرة. وتوصلت الدراسة إلى مجموعة من النتائج ، أبرزها: زمن الفشل لجميع الآلات يتبع التوزيع الأسي بمعلمة واحدة وأن معدل الفشل للماكينات ثابت اي مستقل عن الزمن ، وأن قيم متوسط مربع الخطأ (MSE) لمعلمة التوزيع الاسي التي تم تقديرها عن طريق الأمكان الأعظم(MLE) أقل من قيم متوسط مربعات الخطأ للمعالم المقدرة عن طريق أسلوب بيز (Bayes) ، إي طريقة الأمكان الأعظم أفضل من طريقة بيز في تقدير معالم التوزيع الأسي وموثوقية ماكينات الصراف الآلي (ATM) في السودان ، كذلك الماكينات ذات الموثوقية العالية لديها إحتمالات ومعدلات فشل منخفضة والماكينات ذات الموثوقية المنخفضة لديها إحتمالات ومعدلات فشل عالية ،ايضا كلما زاد متوسط الوقت بين الفشل في الماكينات تشير إلى أن الماكينة تتمتع بموثوقية عالية و تنخفض موثوقية الماكينات كلما زاد زمن عملها ، كذلك

وجد ان الماكينات(B28و B5 و B35) تتمتع بموثوقية عالية مقارنة بالماكينات الأخرى والماكينات (B23 و B33) لها موثوقية منخفضة، ولاتوجد فروق معنوية بين متوسط قيم الموثوقية والمعالم المقدرة بإستخدام طريقتي الامكان الأعظم وبيز.وأخيرًا عرضت الدراسة بعض التوصيات المهمة منها : التأكيد على أهمية موضوع الموثوقية في دراسة وتقييم الماكينات أو التفاضل بين نظام وآخر و إمكانية الإعتماد على طريقة الإمكان الأعظم في تقدير معالم التوزيع الأسي و موثوقية ماكينات الصراف الآلي في السودان .كما يوصي الباحث بتوسيع نطاق الدراسة لتشمل جميع أنواع الأعطال في ماكينات الصراف الآلي ايضا يوصى الباحث بأنه عند توسيع أو إضافة ماكينات جديدة ، يفضل شراء ماكينات ذات الموثوقية العالية .بالإضافة إلى ذلك ، يوصي الباحث بأنه من الأفضل إتباع سياسة الصيانة العلاجية،والتي تنص علي أن الصيانة تتم عند حدوث العطل،وتعديل سياسة الصيانة الوقائية وذلك لأن نسبة كبيرة من الأعطال سوف تحدث بعد فترة قصيرة من إنجاز عملية الصيانة الوقائية.إيضا يقترح الباحث التسجيل الدقيق للأعطال التي تحدث في جميع الماكينات ، مما يؤدي إلى تحديد زمن الفشل ،فترة الفشل ، نوع الفشل وسبب حدوث الفشل بصورة جيدة.

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