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Investigation of Density Perturbation Growth within Generalized Field Equations

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Abstract

We extend a previous work where a nonlinear perturbation equation based on Generalized Field Equation was deduced assuming a flat universe. In this paper a more general nonlinear perturbation equation which can be utilized in both flat and non-flat universes was derived. It consists of nonlinear terms in density perturbation and its derivatives. The deduced fourth order nonlinear equation consists of terms that are supposed to be responsible of strong and weak gravitational fields. Thus the equation can be utilized to study density perturbations in the presence of either strong or weak fields. The equation was used to investigate the possibility of perturbation growth using different scales of expansion of the universe assuming the sole presence of matter energy density. The different scales of expansion were obtained from the solution of the contracted version of the Generalized Field Equation. Constant nonlinear density perturbations were obtained in a curved universe, while constant or decaying ones were obtained in a flat expanding universe, the obtained results were comparable with previous works.

المستخلص

هذا البحث يعتبر امتداداً لبحث سابق حيث استنتجت معادلة اضطراب لاخطية مبنية على معادلة الحقل لتثاقلي المعمم في فضاء لا منحنى. في هذه الورقة استنتجت معادلة اضطراب لاخطية أعم من سابقتها ويمكن استخدامها في كلا الفضاءين المنحنى وغير المنحنى.تحتوى هذه المعادلة على حدود لاخطية بالنسبة لاضطراب الكثافة ومشتقاتها.هذه المعادلة هى معادلة من الدرجة الرابعة ; تحتوي على معاملات يمكن اعتبارها ممثلة للمجال التثاقلي الشديد وللمجال التثاقلي الضعيف . لذلك يعتقد أنه من الممكن استخدام المعادلة لدراسة الاضطراب في الكثافة بوجود أي من المجالين الشديد أو الضعيف. استخدمت المعادلة لدراسة إمكانية تنامي الاضطرابات بتطبيق معدا ت تمدد مختلفة للكون بافتراض وجود كثافة طاقة المادة فقط. معدلات تمدد الكون المختلفة تم الحصول عليها من حل معادلة الحقل التثاقلى المعمم. توصل الباحثان إلى إمكانية وجود اضطرابات ثابتة لاخطية في حالة الكون المنحنى بينما حصلا على اضطرابات ثابتة أو مضمحلة في حـ لة الكون المسطح المتمدد هذه النتائج تبدو متفقة مع ما توصل إليه سابقاً.

Keywords: nonlinear density perturbation; cosmology ;perturbation growth; rate of expansion.

Introduction

The structures we observe in the universe nowadays, formed from the gravitational instability of initial perturbations in the matter density. This idea refers to the 1940s [1]. However, it was only in the early 1980s that a physical mechanism capable of producing small perturbations was identified. This is the mechanism of inflation, an idea due to Guth [2].

For any small density perturbation, there will be competition between its self-gravity which is attempting to increase the density, and the general expansion of the universe which acts to decrease the growth rate of density fluctuations. This growth rate depends on the content of the universe and the values of the cosmological parameters. In a static space exponential growth of perturbations is observed, this gets reduced to only a power law behavior in time in an expanding space–time[3].

Over dense fluctuations expand linearly, at a retarded rate relative to the Universe as a whole, until eventually they reach a maximum size and collapse non-linearly to form an equilibrium object[4].
The density contrast is $\delta >> 1$ today and as the equation of standard cosmology based on Einstein's General Relativity (GR) can account only for $\delta << 1$, the derivation of a nonlinear equation that can account for large perturbations is necessary. Several attempts were performed to solve the density perturbation problem[5].

Einstein's General Relativity is widely accepted as a fundamental theory to describe the geometric properties of spacetime. However, some problems in general relativity, e.g., the non renormalizability of general relativity and the singularity problems in black hole physics and in the early universe, imply that general relativity may not be the final gravitational theory [6].General Relativity should be considered a special theory in the more general class of theories that one could consider[7].
As a result several new theories of gravity were introduced. Different approaches and models were proposed to solve these problems [8,9]. Some of these models suggest to promote the Einstein tensor to a more general form able to account for the tested predictions of Einstein model within the weak gravity of solar system scales and further to justify both inflationary and current acceleration of the Universe.
One of these approaches is the Generalized Field Equations (GFE) [10] one of the recently known f(R) theories and the model based on it[11].In contrast to previous works for example in Ref. [12] , the gravitational collapse of a uniform dust cloud in f(R) gravity was analyzed; the scale factor and the collapsing time were computed. In Ref. [13] , the junction conditions through the hypersurface separating the exterior and the interior of the global gravitational field in f(R) theory were derived. In Ref. [14], a charged black hole from gravitational collapse in f(R) gravity was obtained. Here we calculate the scale factor and investigate the possibility of perturbation growth numerically.
In the next section of this paper, the contracted equation of GFE was solved assuming a power law of the scale factor $a \; \alpha \; ct^n$ [15] in an isotropic universe . In section 3 a nonlinear equation based on GFE for solving the perturbation problem in a non-flat universe was deduced. In section 4 the resulting solution of the contracted equation of GFE was applied to the deduced nonlinear perturbation equation to investigate the possibility of perturbation growth.

The Model

In this section the contracted version of GFE is utilized to get the possible allowed cases of the scale expansion dependence on time, assuming an isotropic and a homogeneous universe.

The GFE equation[16]

$$L'''\left(R_{;\mu}R_{;\nu} - g_{\mu\nu}g^{\rho\sigma}R_{;\rho}R_{;\sigma}\right) + L''\left(R_{;\mu;\nu} - g_{\mu\nu}\ {}^{2}R\right) + L'R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}L = 0 \qquad (1)$$

where L is the most general form of the Lagrangian L', L'', L''' are its partial derivatives of the scalar curvature R.

Equation(1) is contracted by $g^{\mu\nu}$ to yield

$$^{2}\square = \frac{L'R - 2L}{3L''} - \frac{L'''}{L''}R_{;\rho}R^{;\rho} \qquad (2)$$

where $^{2}\square$ is the invariant d'Alembertian which is defined by

$$^{2}R = g^{\rho\sigma}\frac{\partial R_{;\rho}}{\partial x^{\sigma}} - \Gamma^{\lambda}_{\rho\sigma}R_{;\lambda} \qquad (3)$$

In (1) we have the only non vanishing components of the derivatives R in the spherical coordinates(r, ϑ, φ) and time read as

$$R_{;t} = \dot{R} \quad , \quad R_{;t;t} = \ddot{R} \quad , \quad R_{;r;r} = -\frac{\dot{a}}{a}\dot{R}g_{rr}$$

$$R_{;\theta;\theta} = -\frac{\dot{a}}{a}\dot{R}g_{\theta\theta} \quad , \quad R_{;\phi;\phi} = -\frac{\dot{a}}{a}\dot{R}g_{\phi\phi} \qquad (4)$$

where a is the cosmic scale factor and overdot stands for the derivative with time.

Setting $L = -\alpha R^2 + \beta R + \gamma$ as one choice of the non linear Lagrangian , α , β and γ are constants. α and β are assumed somehow to be responsible for strong and weak gravitational fields respectively and γ is supposed to stand for the total universe content of energy density.

The following Robertson-Walker metric in a spatially homogeneous and isotropic universe was used: $ds^2 = c^2dt^2 - a^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$

Using the coordinate condition $\Gamma^{\lambda}_{\rho\sigma} = 0$ [17] in (2) yields

$$-\ddot{R} = \frac{\beta R + 2\gamma}{6\alpha} \qquad (5)$$

Now equation(5) will be employed to describe the unperturbed portion of the background universe reading

$$\ddot{R}_0 = \frac{-\beta R_0}{6\alpha} - \frac{\rho_0}{3\alpha} \qquad (6)$$

Also ρ was identified for γ.

Writing R in terms of a differentiating R twice with respect to time, substituting and assuming $k=0,1,-1$ [18] one gets

$$-6\left(\frac{-8\ddot{a}\dot{a}^2}{a^3}+\frac{6\dot{a}^4}{a^4}+\frac{\ddot{\ddot{a}}}{a}+\frac{\ddot{a}^2}{a^2}\right)+12k\left(\frac{\ddot{a}}{a^3}-\frac{3\dot{a}^2}{a^4}\right)=\frac{\beta}{\alpha}\left(\frac{\ddot{a}a+\dot{a}^2+k}{a^2}\right)-\frac{\rho}{3\alpha} \qquad (7)$$

The equation may reduce to $\rho=\frac{3\beta k}{a^2}$ in a static universe giving the same result as standard cosmology if $\beta=\frac{1}{8\pi G}$ [19] otherwise if α is set to zero it reduces to Friedman equations added together if the pressure is neglected[20].

To solve equation (7) it was assumed that the energy density is made up of matter alone i.e. $\rho=\rho_m$ with no radiation or vacuum energy densities and this matter energy density scales as $\rho_m\ \alpha\ a^{-3}$ [21]. Also assuming that $a=ct^n$ where c is a constant, inserting this relationship and its derivatives in (7), one gets

$$\left[48n^3(n-1)-36n^4-6n(n-1)(n-2)(n-3)-6n^2(n-1)^2\right]t^{-4}$$
$$+12\frac{k}{c^2t^{2n+2}}\left[n(n-1)-3n^2\right]=\frac{\beta}{\alpha}\left[n(n-1)+n^2\right]t^{-2}+\frac{\beta k}{\alpha c^2t^{2n}}-\frac{C_m}{3\alpha c^3t^{3n}} \qquad (8)$$

The above equation is satisfied for the following cases:

Case1: $n=0$ resulting in (i) a static flat universe with no matter: $C_m=k=0$, or (ii) a non-flat static universe in which $C_m=\pm 3\beta c$ for $k=\pm 1$.

Case2: $n=2/3$ resulting in α=k=0, $C_m=\frac{2}{3}\beta c^3$ which is approximately the same result as that obtained within the frame work of general relativity.

Case3: $n=1$ the equation yields (i) $c=\pm 1$ for $k=-1$, or (ii) $c\pm i$ for $k=1$, and $C_m=0$, i.e. the model describes an open universe in which the expansion rate scales with time or a closed universe in which the expansion rate is imaginary but with no matter in both cases.

Case4: $n=1/2$ the equation yields $C_m=k=0$ both fields are present.

Case5: $n=4/3$ resulting in : β=k=0, $C_m=-15800\alpha c^3$.

Deduction of the Nonlinear Perturbation Equation

The perturbation will be calculated using method [22]. The equation

$$\ddot{R}_0=\frac{-\beta R_0}{6\alpha}-\frac{\rho_0}{3\alpha} \qquad (9)$$

describes the unperturbed model, which we shall take to be the background universe. Consider now a spherical region of the universe, which at time t_0 has a radius r_0 where $r_0=la(t)$, and l is the commoving label of the unperturbed test particle, here l is assumed to be a constant.

Suppose now we make a small positive perturbation, so that the same amount of particles originally in the radius r_0 is now in $r=r_0-\Delta r<<1$, r is the perturbed radius of the sphere given by $r=la(t)\left[1-\delta(t)\right]$, where δ is the commoving perturbation. Hence the perturbed model will be given by

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$$\ddot{R} = \frac{-\beta R}{6\alpha} - \frac{\rho}{3\alpha} \quad (10)$$

Using the scalar curvature R and its derivatives in terms of the scale factor a, for the perturbed and unperturbed models respectively and subtracting one gets the following nonlinear perturbation equation:

$$-6\left\{-4\left[\frac{\dddot{\delta}\dot{a}+2\ddot{\delta}\ddot{a}+\dot{\delta}\dddot{a}}{a(1-\delta)}\right]-\left[\frac{8\dot{\delta}^2\ddot{a}+12\dot{\delta}\ddot{\delta}\dot{a}}{a(1-\delta)^2}\right]+\left[\frac{20\ddot{\delta}\dot{a}^2+36\dot{\delta}\dot{a}\ddot{a}}{a^2(1-\delta)}\right]+\frac{8\dot{a}^2\dot{\delta}^2}{a^2(1-\delta)^2}-\frac{8\dot{a}^3\dot{\delta}}{a^3(1-\delta)}\right.$$

$$\left.\frac{-8\dot{a}\dot{\delta}^3}{a(1-\delta)^3}+\frac{6\dot{\delta}^4}{(1-\delta)^4}+\frac{8\dot{\delta}^2\ddot{\delta}}{(1-\delta)^3}+\frac{\ddot{\delta}^2}{(1-\delta)^2}-\frac{\ddddot{\delta}}{1-\delta}\right\}+$$

$$+\frac{12k}{a^2l^2(1-\delta)^2}\left[\frac{8\dot{\delta}\dot{a}}{a(1-\delta)}-\frac{\ddot{\delta}}{1-\delta}-\frac{3\dot{\delta}^2}{(1-\delta)^2}+\left(\frac{-\ddot{a}}{a}+\frac{3\dot{a}^2}{a^2}\right)\left(-2\delta+\delta^2\right)\right]$$

$$=\frac{\beta}{\alpha}\left[\frac{-4\dot{\delta}\dot{a}}{a(1-\delta)}+\frac{\dot{\delta}^2}{(1-\delta)^2}-\frac{\ddot{\delta}}{1-\delta}+\frac{k\left(2\delta-\delta^2\right)}{l^2a^2(1-\delta)^2}\right]-\frac{1}{3\alpha}\left\{\frac{3\delta-3\delta^2+\delta^3}{l^3(1-\delta)^3}\right\}\rho_{m_0} \quad (11)$$

This is a fourth order nonlinear differential equation which can be treated as a generalization to the second order linear differential equation based on standard big bang model .It consists of a term α which is assumed to be attributed to strong gravitational fields, and it also consists of a term β that is assumed to be attributed to weak gravity. The equation is supposed to describe density perturbations in a flat or a curved universe as it consists of the term k. The equation contains nonlinear terms in δ and its derivatives, thus δ does not have to be small. Only the linear evolution of the density perturbations can be studied within standard cosmology. Therefore we hopefully think that the above equation will be also good for studying nonlinear perturbations.

Here we assumed the unperturbed matter energy density is $\rho_{m0} = \frac{C_m}{a_0^3}$ and the perturbation in the matter energy density was calculated to be

$$\Delta\rho_m = \frac{C_m\delta}{l^3a^3}\left\{\frac{3-3\delta+\delta^2}{(1-\delta)^3}\right\} \quad (12)$$

Multiplying equation(11) by $\alpha(1-\delta)^4$, confining to terms without α that accounts for a weak gravitational field only, linearizing and assuming $\beta = \frac{1}{8\pi G}$ in a flat universe only, one obtains

$$\ddot{\delta}+\frac{4\dot{\delta}\dot{a}}{a}+\frac{8\pi G\rho_m\delta}{l^3}=0 \quad (13)$$

The above equation can be compared with the linear perturbation equation in standard cosmology [23] which can be used in a weak gravitational field only. Otherwise if we set *k=0* in equation (11) it reduces to the nonlinear perturbation equation in a flat universe obtained in [24].

Solution of the Nonlinear Perturbation Equation

In this section the possibility of perturbation growth will be investigated using the different scales of expansion obtained from equation (8).This will be achieved by applying the results of equation (8) to (11). Hence:

Case1i

For a static flat Universe and $C_m = 0$, equation (11) yields

$$-6\left\{\frac{6\dot{\delta}^4}{(1-\delta)^4}+\frac{8\dot{\delta}^2\ddot{\delta}}{(1-\delta)^3}+\frac{\ddot{\delta}^2}{(1-\delta)^2}-\frac{\ddddot{\delta}}{1-\delta}\right\}=\frac{\beta}{\alpha}\left[\frac{\dot{\delta}^2}{(1-\delta)^2}-\frac{\ddot{\delta}}{1-\delta}\right] \quad (14)$$

This equation is satisfied for a solution of the form $\delta = 1 - bt^n$,where b is a constant, yielding $n = 0$ or $n = \frac{1}{2}$ .Hence the resulting perturbation is either equal to $\delta = 1 - b$ which is a constant or $\delta = 1 - b_1 t^{\frac{1}{2}}$ which is growing with time much slower than the exponential rate of growth obtained in the standard model in a static Universe .

Case1ii

The derivatives of the density perturbation were written in terms of the scale factor , hence the first derivative reads $\dot{\delta} = \frac{d\delta}{da}\dot{a}$. Inserting this and the higher derivatives in equation (11), yields

$$-6\left\{-4\left[\frac{2\dddot{a}\dot{a}\frac{d\delta}{da}+4\dot{a}^2\ddot{a}\frac{d^2\delta}{da^2}+\dot{a}^4\frac{d^3\delta}{da^3}+2\ddot{a}^2\frac{d\delta}{da}}{a(1-\delta)}\right]-\left[\frac{20\dot{a}^2\ddot{a}\left(\frac{d\delta}{da}\right)^2+12\dot{a}^4\frac{d^2\delta}{da^2}\frac{d\delta}{da}}{a(1-\delta)^2}\right]\right.$$

$$+\left[\frac{56\ddot{a}\dot{a}^2\frac{d\delta}{da}+20\dot{a}^4\frac{d^2\delta}{da^2}}{a^2(1-\delta)}\right]+\frac{8\dot{a}^4\left(\frac{d\delta}{da}\right)^2}{a^2(1-\delta)^2}-\frac{8\dot{a}^4\frac{d\delta}{da}}{a^3(1-\delta)}$$

$$\frac{-8\dot{a}^4\left(\frac{d\delta}{da}\right)^3}{a(1-\delta)^3}+\frac{6\dot{a}^4\left(\frac{d\delta}{da}\right)^4}{(1-\delta)^4}+\frac{8\left(\dot{a}^2\ddot{a}\left(\frac{d\delta}{da}\right)^3+\dot{a}^4\frac{d^2\delta}{da^2}\left(\frac{d\delta}{da}\right)^2\right)}{(1-\delta)^3}+\frac{\ddot{a}^2\left(\frac{d\delta}{da}\right)^2+\dot{a}^4\left(\frac{d^2\delta}{da^2}\right)^2+\dot{a}^2\ddot{a}\frac{d\delta}{da}\frac{d^2\delta}{da^2}}{(1-\delta)^2}$$

$$\left.-\frac{1}{1-\delta}\left(\ddddot{a}\frac{d\delta}{da}+4\dot{a}\dddot{a}\frac{d^2\delta}{da^2}+6\dot{a}^2\ddot{a}\frac{d^3\delta}{da^3}+3\ddot{a}^2\frac{d^2\delta}{da^2}+\dot{a}^4\frac{d^4\delta}{da^4}\right)\right\}$$

$$+\frac{12k}{a^2l^2(1-\delta)^2}\left[\frac{8\dot{a}^2\frac{d\delta}{da}}{a(1-\delta)}-\frac{\left(\ddot{a}\frac{d\delta}{da}+\dot{a}^2\frac{d^2\delta}{da^2}\right)}{1-\delta}-\frac{3\dot{a}^2\left(\frac{d\delta}{da}\right)^2}{(1-\delta)^2}+\left(\frac{-\ddot{a}}{a}+\frac{3\dot{a}^2}{a^2}\right)\left(-2\delta+\delta^2\right)\right]$$

$$=\frac{\beta}{\alpha}\left[\frac{-4\dot{a}^2\frac{d\delta}{da}}{a(1-\delta)}+\frac{\dot{a}^2\left(\frac{d\delta}{da}\right)^2}{(1-\delta)^2}-\frac{\ddot{a}\frac{d\delta}{da}+\dot{a}^2\frac{d^2\delta}{da^2}}{1-\delta}+\frac{k\left(2\delta-\delta^2\right)}{l^2a^2(1-\delta)^2}\right]-\frac{1}{3\alpha}\left\{\frac{3\delta-3\delta^2+\delta^3}{l^3(1-\delta)^3}\right\}\rho_{m_0} \quad (15)$$

For a static non flat Universe $C_m=\pm3\beta c$, $k=\pm1$ from(8), the above equation was solved to give

$$\delta=\frac{3}{2}\pm\frac{1}{2}\sqrt{9-4\frac{(2l-3)}{l+1}} \quad (16)$$

One can see that in this case the perturbation is a nonlinear constant $\delta >> 1$ for $l\geq 0$. The density perturbation in (16) was calculated for *l=0* to be either $\delta=3.79$ for the positive case or $\delta=-0.79$ for the negative case. Non-linearity of equation(11) is assumed to allow the growth of a perturbation and makes its growth faster than the linear case.

Also as the universe is still static, the change from flat to curved geometry may be considered to slow the growth of density perturbations as the strength of the gravitational force is reduced [25]. As a result the growing mode in case 1i above did not show up.

Case2: This case reduces to a version comparable with that obtained within standard cosmology based on general relativity.

Case3: Inserting $a=ct$ and its derivatives in (11) also $\beta=0$ and $C_m=0$ the same equation for both cases (i) $c=\pm1$ for $k=-1$,or (ii) $c\pm i$ for $k=1$ was obtained, hence

$$\frac{-24\dot{x}}{xt}+\frac{72\dot{x}\dot{x}}{tx^2}+\frac{120}{xt^2}\ddot{x}-48\left(\frac{\dot{x}^2}{x^2t^2}+\frac{\dot{x}}{xt^3}-\frac{\dot{x}^3}{tx^3}\right)-\frac{36\dot{x}^4}{x^4}+\frac{48\dot{x}^2\ddot{x}}{x^3}-6\frac{\ddot{x}^2}{x^2}-6\frac{\ddddot{x}}{x}$$

$$+\frac{12k}{c^2L^2t^2}\left[\frac{-8n}{t^2x^2}+\frac{n(n-1)}{t^2x^2}-\frac{3n^2}{t^2x^2}+\left(-\frac{3}{t^2}\right)\left(1-x^2\right)\right]=0 \quad (17)$$

where $x=1-\delta$ and its derivatives was used for simplification. We note that the same differential equation (17) governs both the *k* = +1 and the *k* = −1 cases .Assuming again a power law solution $x=bt^n$ the equation is found to be satisfied for *n=0*, and *b=-1*, therefore $\delta=2$ for both non flat cases . Hence the perturbation is a constant nonlinear perturbation.

One notes here again that non-linearity enhances the growth of a perturbation and makes the growth faster when compared with linear one. On the other hand the expansion of the Universe slows the growth rate of perturbations. As a result of these two effects one supposes that the power-law in case 1i changed into a nonlinear constant. This case may be compared with the case in the standard model for an Open Universe, without a cosmological constant, where the scale factor evolves as $a \propto t$ at late times when curvature dominates the dynamics. Because there is no matter density, perturbations have stopped growing altogether; hence here the perturbation is a constant $\delta=2$.

It is also identified that the critical linear density $\delta_c=1.686$ [26], above this value the object collapses. Full non-linear collapse will occur when the linear over-density reaches 1.7. The above two results may suggest that this fraction of space may have separated from the rest of the expanding part of the Universe, and formed a galaxy.

Case4: Using $a = ct^{\frac{1}{2}}$ and its derivatives in (11), $C_m = 0$, $k = 0$ one obtains

$$-6\left\{-4\left[\frac{\dddot{\delta}}{2t}-\frac{\ddot{\delta}}{2t^2}+\frac{3\dot{\delta}}{8t^3}\right]\left[\frac{1}{(1-\delta)}\right]-\left[\frac{-2\dot{\delta}^2}{t^2}+\frac{6\dot{\delta}\ddot{\delta}}{t}\right]\left[\frac{1}{(1-\delta)^2}\right]\right.$$
$$+\left[\frac{5\ddot{\delta}}{t^2}-\frac{36\dot{\delta}}{8t^3}\right]\left[\frac{1}{(1-\delta)}\right]+\frac{2\dot{\delta}^2}{t^2(1-\delta)^2}-\frac{\dot{\delta}}{t^3(1-\delta)}$$
$$\left.\frac{-4\dot{\delta}^3}{t(1-\delta)^3}+\frac{6\dot{\delta}^4}{(1-\delta)^4}+\frac{8\dot{\delta}^2\ddot{\delta}}{(1-\delta)^3}+\frac{\ddot{\delta}^2}{(1-\delta)^2}-\frac{\ddddot{\delta}}{1-\delta}\right\}=\frac{\beta}{\alpha}\left[\frac{-2\dot{\delta}}{t(1-\delta)}+\frac{\dot{\delta}^2}{(1-\delta)^2}-\frac{\ddot{\delta}}{1-\delta}\right] \quad (18)$$

In this case both α and β are present. Solving one gets $\delta = 1 - b_0$ or $\delta = 1 - b_1 t^{\frac{-1}{2}}$ for $\alpha = 0$ i.e. when we switch off momentarily the strong gravitational field a constant or a decaying mode is produced in a weak gravitational field .The behavior is qualitatively the same as in the radiation-dominated era: the decaying mode becomes negligible, and the amplitude of the non-decaying mode remains constant. Otherwise if the strong gravitational is huge such that we can assume that $\beta \approx 0$ one obtains $\delta = 1 - b$ or, $\delta = 1 - b_2 t^{-1.22}$ which are again a constant ,and a decaying mode but with a different rate of growth.

Comparing with case 1i it seems that due to the expansion of the Universe the rate of growth of perturbations is lowered, so constant perturbations changed to decaying ones whereas growing modes changed to constant perturbations due to the Hubble drag. Also as there is no matter energy density no perturbation growth is predicted, that is why perturbations are either constant or decaying with time which is quite reasonable. The only difference between the presences of a weak or a strong field is associated with the rate of decay.

Case5: Using $a = ct^{\frac{4}{3}}$ and its derivatives in (11) also β=k=0 , $C_m = -15800\alpha c^3$ the equation yields $\delta = \frac{1}{2}\left(3 \pm \sqrt{3}i\right)$. This case is rather interesting for two reasons: first, it is the only case in which the scale of expansion is accelerating in agreement with recent observations and second the resulting perturbation is a constant made up of a real part *>>1* and an imaginary part which may suggest the presence of effects that may be unperceivable [27].

Discussion and Conclusion

Using the contracted GFE, different rates of expansions were obtained. In case1 the universe is static hence there is no expansion. In case3 the rate of expansion is constant ,which means there is no acceleration or deceleration, in cases 2 and 4 the universe is decelerating whereas in case 5 it is accelerating. As recent observations suggest that cosmic scale factor is accelerating[28], $a = ct^{\frac{4}{3}}$ appears to be the most interesting result as it implies a positive second derivative of the scale of expansion *a* and further investigation of this case is needed.

Also it can be noticed that the matter energy density dependence on time differs from one case to another. It changed from $\rho_m \ \alpha \ t^{-2}$ when $a = ct^{\frac{2}{3}}$ to $\rho_m \ \alpha \ t^{-4}$ when $a = ct^{\frac{4}{3}}$. As the universe expands the scale factor increases, hence the density of matter decreases, whereas in a static universe, matter energy density was a constant which is quite reasonable[29].

The deduced perturbation equation obtained from GFE is a fourth order nonlinear equation. As it is a non linear equation in δ and its derivatives it is expected to account for perturbations >>*1* .In case3 ,for example, the calculated perturbation equals *2* . The equation seems able of describing structures in either strong or weak gravitational fields or both as it consists of the terms α and β that allow that possibility. Also it can describe density perturbations in a flat or a curved universe as it consists of the parameter *k*. The equation was found to reduce to that based on GR when the strong gravitational field was set to zero and linearization was applied.

Different possibilities of perturbation growth were obtained for different rates of expansion. In some cases constant, decaying or growing perturbations were obtained. In a static empty universe a perturbation that is growing with time is obtained. Its growth is much slower than the exponential rate of growth obtained in the standard model in a static universe . Of special interest are cases 1ii and case3 which resulted directly in a nonlinear constant perturbation in a curved static Universe in which $C_m = \pm 3\beta c$ or a curved expanding empty Universe. Also case5 which yielded a complex constant nonlinear perturbation in a flat universe, if the gravitational field is strong. We believe that this case needs further investigation. Case4 resulted in decaying perturbations with different rates of decay in weak or strong gravitational fields in a flat empty expanding universe which can be accepted as expansion lowers the rate of growth.

The obtained results in flat and curved universes were as follows. In a flat empty universe the perturbations where either constant or growing in a static universe, this rate of growth was lowered to either decaying or constant perturbations in a decelerating empty Universe whereas these perturbations changed to complex nonlinear ones in an accelerating non empty universe. In a curved universe only constant nonlinear perturbations were allowed whether the universe is non-empty and static or empty and expanding universe.

Hence our conclusion is that within GFE nonlinear density perturbations are allowed in a curved non empty , static universe or an empty decelerating curved universe or in an accelerating non empty flat universe if matter energy density is present alone. No perturbation growth is allowed unless the universe is static and the obtained rate of growth is much slower than the exponential growth allowed within the standard model.

Density perturbation growth if other kinds of energy densities are present together with matter energy density is open for investigation.

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