Sudan University of Science and Technology
College of Graduate Studies
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Higgs Decay to Gluons in the Standard Model

تحلل جسيم هيغز للقلونات في النموذج القياسي

A Dissertation Submitted to College of Graduate Studies for the Degree of Master of Science (M. Sc.) in Physics

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الآية

قال تعالي :

{اللَّهُ نُورُ السَّمَاوَاتِ وَالْأَرْضِ ۚ مَثَلُ نُورِهِ كَمِشْكَاةٍ فِيهَا مِصْبَاحٌ ۖ الْمِصْبَاحُ فِي زُجَاجَةٍ ۖ الزُّجَاجَةُ كَأَنَّهَا كَوْكَبٌ دُرِّيٌّ يُوقَدُ مِنْ شَجَرَةٍ مُبَارَكَةٍ زَيْتُونَةٍ لَا شَرْقِيَّةٍ وَلَا غَرْبِيَّةٍ يَكَادُ زَيْتُهَا يُضِيءُ وَلَوْ لَمْ تَمْسَسْهُ نَارٌ ۚ نُورٌ عَلَىٰ نُورٍ ۗ يَهْدِي اللَّهُ لِنُورِهِ مَنْ يَشَاءُ ۚ وَيَضْرِبُ اللَّهُ الْأَمْثَالَ لِلنَّاسِ ۗ وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٌ}

سورة النور الآية 35

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Abstract:

In this thesis we calculated the total Tree level two body width of the Higgs as function of its mass from $m_h = 100\ GeV$ all the way to $m_h = 400\ GeV$. In particular we calculate the Higgs decay rates into W and Z gauge bosons and f fermions in the Standard Model of Particle Physics. We find that in order for the Higgs to decay to any particle in the Standard Model its mass must be $m_h \geq 2m_{W,Z,f}\ GeV$. Also we found that, the total Higgs decay rate increase with Higgs mass increases. Moreover, We calculated the Higgs decay to gluons up to one-loop level, and found that its decay rate significantly large and must be added to the total Higgs decay rate.

ملخص البحث :

في هذا البحث تم حساب معدل للتحلل الكلي لجسيم الهيغز في النموذج القياسي للجسيمات الاوليه وذلك حسب كتله الهيغز حيث استخدمنا $m_h = 100\ GeV$ الي $m_h = 400\ GeV$. ايضا حسب معدل التحلل لجسيم الهيغز الي بوزونات W و Z و الي فيرميونات f. ووجد ان جسيم الهيغز يتحلل لهذه الجسيمات فقط في حاله. $m_h \geq 2m_{W,Z,f}\ GeV$ و وجد ايضا ان معدل التحلل يزيد بزياده كتله الهيغز. تم حساب تحلل جسيم الهيغز لقولونات للرتبه الاولي في النموذج القياسي. ووجد ان معدل تحلله لقولونات كبير لايمكن تجاهله ويجب اضافته لمعدل التحلل الكلي لجسيم الهيغز.

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Chapter I

(1-1) Introduction

In this chapter, we shall discuss and explore the importance of studying the Higgs boson decays in the Standard Model. The Higgs is not just a new particle in the particle physics, but really forms one of the foundations of the electroweak sector of the standard model; it is responsible of giving masses to both fermions and gauge bosons in a local gauge invariance theory. There are several reasons to believe that the standard model is just the low energy limit of more fundamental theory. The standard model has been successfully tested to high level of accuracy and provides at present our best fundamental understanding phenomenology of particle physics [1].

(1-2)The importance of higgs decay rates:

The decay rate is the way to probe the properties of the higgs. All higgs decay rates are modified by electroweak (EW) and Quantum chromodynamics (QCD) corrections. QCD corrections are important for higgs decay into$H \rightarrow \bar{\psi}\,\psi$. The next process is a one-loop process. We could naively think that it's decay rate must be very small compared to the tree-level ones, but that is not the case. Because top quark mass is heavy, this diagram produces a high enough decay rate that necessarily must be taken into account. We shall see at the end of our computation that for a massless quark this diagram does not contribute.

(1-3)The main objectives of the research:

The main object ective of this thesis is to explain what is higgs boson ,and what is standard model. It is also show how the higgs field interacts with particels and gives them mass and calculate the decay rates of the Higgs in the Standard Model up to one-loop level.

(1-4)The outline of the research:

This reserach project is structured as follow: In chapter one we give brief introduction and chapter two introduce the standard model, in chapter three we calculate the Higgs decay rates into gluons, we present in chapter four our numerical results, discussion and conclussion our results.

Chapter II

(2-1)The Standard Model

The Standard Model (SM) of particle physics is a theoretical framework that describes fundamental particles and their interactions. The SM is currently accepted theory and its prediction has been confirmed experimentally. A single Lagrangian equation is the common representation of the Standard Model. The fundamental particles are divided according to their spin into fermions (spin$\frac{1}{2}$), the matter forming particles that have half integer spin, and bosons (spin 1), the force mediators that have integer spins. Fermions are further divided into quarks, which experience the strong force, and leptons that do not. Both quarks and leptons come with three generations. The family of quarks consists of the up (u), down (d), charm (c), strange (s), top (t) and bottom (b). The properties of these six quarks are summarized in Table 2.1. The family of leptons consists of the electron (e), muon (μ), tau (τ) and a corresponding neutrino (ν) for each. The properties of these six leptons are summarized in Table 2.2. Each of the fermions has an anti-particle partner with the same properties apart from having equal and opposite charge and internal quantum numbers. The first generation of quarks and leptons are the matter that makes up the majority of our universe [1,2].

Table 2.1: Properties of the six quarks in the Standard Model. Charge is expressed as a fraction of the electron charge e.

Quark	Generation	Mass	EM Charge
Up	1	$2.3^{+0.7}_{-0.5}$ MeV	$+2/3$
Down	1	$4.8^{+0.5}_{-0.3}$ MeV	$-1/3$
Charm	2	1.275 ± 0.025 GeV	$+2/3$
Strange	2	95 ± 5 MeV	$-1/3$
Top	3	173.1 ± 1.3 GeV	$+2/3$
Bottom	3	4.18 ± 0.03 GeV	$-1/3$

Table 2.2: Properties of the six leptons in the Standard Model. Charge is expressed as a fraction of the electron charge e.

Lepton	Generation	Mass	EM Charge
e	1	0.511 MeV	-1
ν_e	1	<2 eV	0
μ	2	105.66 MeV	-1
ν_μ	2	<2 eV	0
τ	3	1.78 GeV	-1
ν_τ	3	<2 eV	0

The bosons act as mediators for the fundamental forces of nature, allowing the interactions between quarks, leptons and other bosons to occur. The photon carries the electromagnetic force, the 8-fold family of gluons carries the strong force and the $W^{\pm}$ and Z carry the weak force. All charged particles can experience electromagnetic interactions, fermions experience the weak force and particles carrying color charge (quarks and gluons) experience the strong force. The properties of the bosons are summarized in Table 2.3.

Table 2.3: Properties of the Bosons in the Standard Model. Charge is expressed as a fraction of the electron charge e.

Force	Boson	Mass	EM Charge
Electromagnetic	γ	0	0
Weak	Z	91.2 GeV	0
Weak	W	80.4 GeV	±1
Strong	g	0	0

(2-2) Standard Model described by a Lagrangian

The Standard Model is described by a Lagrangian that is the sum of the gauge, matter, Higgs, and Yukawa interactions:

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_{higgs} + \mathcal{L}_{fermion} + \mathcal{L}_{Yukawa} \qquad (2-1)$$

This Lagrangian is not written initially in terms of the (very) low energy degrees of freedom we observe in our ground state, but in terms of

- massless states
- fundamental symmetries

$$L_{\text{Gauge}} = \frac{1}{2g_s^2}\, tr\left[G_{\mu\nu}G^{\mu\nu}\right] + \frac{1}{2g^2}\, tr\left[A_{\mu\nu}A^{\mu\nu}\right] + \frac{1}{2\acute{g}^2}\, tr\left[B_{\mu\nu}B^{\mu\nu}\right] \qquad (2-2)$$

$$\mathcal{L}_{\text{Yukawa}} = -\Gamma_u^{ij}\overline{Q}_L^i\epsilon\phi^* u_R^j - \Gamma_d^{ij}\overline{Q}_L^i\phi d_R^j - \Gamma_e^{ij}\overline{L}_L^i\phi e_R^j + H.c. \qquad (2-3)$$

Γ_u , Γ_d , Γe are 3×3 complex matrices

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \qquad (2-4)$$

$$\text{where } D_\mu = (\partial_\mu + i\tfrac{g}{2}\sigma^i A_\mu^i + i\tfrac{g'}{2}B_\mu) \qquad (2-5)$$

$$\mathcal{L}_{\text{Mat}} = i\overline{Q}_L^i\, \not{D}Q_L^i + i\overline{(u^c)}_L^i\, \not{D}(u^c)_L^i + i\overline{(d^c)}_L^i\, \not{D}(d^c)_L^i + i\overline{L}_L^i\, \not{D}L_L^i + i\overline{(e^c)}_L^i\, \not{D}(e^c)_L^i \qquad (2-6)$$

(2-3)The Higgs Boson

On the 4th of July 2012, the two LHC experiments ATLAS and CMS reported the discovery of a new particle in searches for the SM Higgs boson. Until now the measurements of its couplings and its properties have strengthened the assumption that the observed particle with a mass around 125 GeV is indeed the SM Higgs boson. However, in order to verify the SM hypothesis, all possible production and decay rates need to be measured and compared to the SM prediction [3]

(2-3-1) The Higgs mechanism

We can now consider the spontaneous symmetry breaking of a “local gauge symmetry”. The simplest case is the U(1) gauge symmetry

$$\emptyset(x) \rightarrow e^{i\alpha(x)}\,\emptyset(x) \quad \text{with } \emptyset = \frac{1}{\sqrt{2}}(\emptyset_1 \pm i\emptyset_2) \qquad (2-7)$$

We introduce in the lagrangian

$$L = \left(\partial_\mu \emptyset\right)^* (\partial^\mu \emptyset) - \mu^2 \emptyset^* \emptyset - \lambda(\emptyset^* \emptyset)^2 \qquad (2\text{-}8)$$

the covariant derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - i\, e\, A_\mu \qquad (2\text{-}9)$$

with the gauge field A_μ transforming according to

$$A^\mu \rightarrow A^\mu + \frac{1}{e}\, \partial^\mu\, \alpha(x) \qquad (2\text{-}10)$$

The Lagrangian takes then the form

$$L = (\partial_\mu + i\, e\, A_\mu)\emptyset^* (\partial^\mu - i\, e\, A^\mu)\, \emptyset - \mu^2\, \emptyset^* \emptyset - \lambda(\emptyset^* \emptyset)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (2\text{-}12)$$

If $\mu^2 > 0$, then this is just the QED Lagrangian for a charged scalar particle of mass m, with the addition of a $\emptyset^4$ self-interaction term.

But we take $\mu^2 < 0$, since we want to generate mass terms through the spontaneous symmetry breaking mechanism. In this case we have to translate the field $\emptyset$ (x) to the ground state. With the same substitution as before

$$\emptyset\, (x) = \frac{1}{\sqrt{2}} \left[v + \eta(x) + i\, \xi(x)\right] \qquad (2\text{-}13)$$

the Lagrangian becomes

mass term mass term strange off-diagonal term

$$L' = \frac{1}{2}\left[\partial_\mu\, \xi\right]^2 + \frac{1}{2}[\partial^\mu \eta]^2 - v^2\, \lambda\, \eta^2 + \frac{1}{2} e^2 v^2 A_\mu A^\mu - e\, v A_\mu\, \partial^\mu \xi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \text{ interaction}$$

(2-14)

The particle spectrum in L' contains

- a massless Goldstone boson ξ (x) $\quad m_\xi = 0$
- a massive scalar field η(x) $\quad m_\eta = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$
- a massive vector field $A\mu$ $\quad m_A = ev$

Let us consider the following simple Lagrangian describing the self interaction of a scalar particle associated to the field $\emptyset$ (x)

$$L = T - V \qquad (2\text{-}15)$$

The Higgs potential is given by

$$V(x) = -\frac{1}{2}\, \mu^2 \emptyset^* \emptyset - \frac{\lambda}{4}\, (\emptyset^* \emptyset)^2 \qquad (2\text{-}16)$$

Which involve two new real parameters μ and λ we demand $\lambda > 0$ for potential to be bounded; otherwise the potential is unbounded from below and there will be no stable vacuum state.

μ is takes the following two value:

- $\mu^2 > 0$ then the vacuum corresponding to $\emptyset = 0$, the potential has a minimum at the origin (see figure 2.2).
- $\mu^2 < 0$ then the potential develops a non-zero vacuum expectation value and the minimum is along a circle of radius $\frac{v}{\sqrt{2}} = \frac{246}{\sqrt{2}}$ (see figure 2.1).

We set λ=0.129, m_h=125GeV and $|-\mu^2| = (88.0GeV)^2$

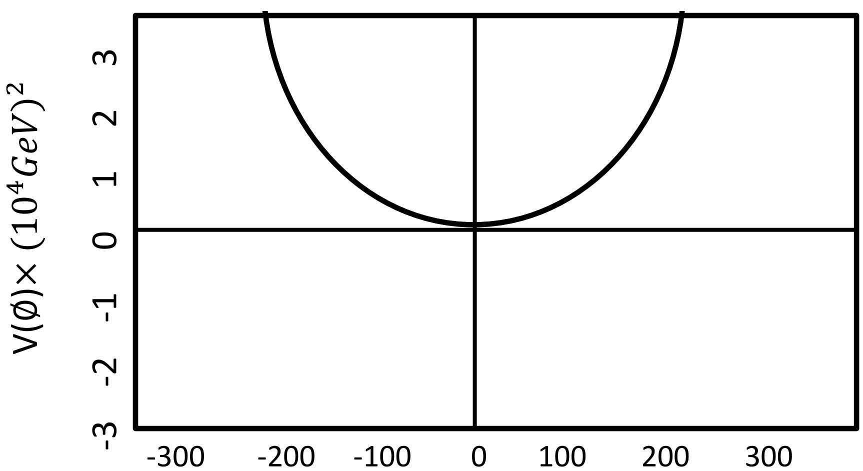

Figure (2.1): the Higgs potential V(∅) with the case $\mu^2 > 0$; as a function of $|\emptyset| = \sqrt{\emptyset^* \emptyset}$

We set λ=0.129, m_h=125GeV and $|-\mu^2| = (88.0GeV)^2$

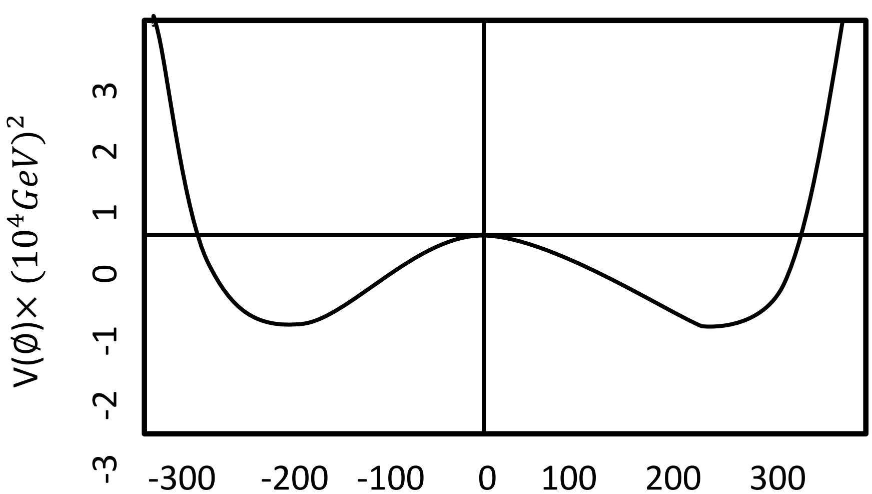

Figure (2.1): the Higgs potential V(∅) with the case $\mu^2 < 0$; as a function of $|\emptyset| = \sqrt{\emptyset^* \emptyset}$

(2-4) The problem of standard model:

Despite the success of standard model. Below we list some of unsolved problem in standard model.

1. Cosmologic consideration: the observed matter density of galaxies falls short of the measured matter as measured by the rotation curves. It is theorized that the baryon matter density is ~ 4% . the rest of the universe is made up ~24% dark matter and ~ 72% dark energy. In the last decade , the direct observation of gravitational lensing and observations in galactic collision event have provided hard evidence for the existence of Dark Matter . the WMAP probe has measured the dark matter density to be between (0.087$< DMh2 <$0.138) at 3σ range . SM neither provides any explanation for dark energy nor dose it have a suitable dark matter condition.
2. Gauge Hierarchy problem: the Gauge Hierarchy problem is the question of why there is such a huge difference between the electroweak scale M_{EW} = O(100)GeV and plank scale M_{PL} = O(10^{18}) GeV. This is also known as the naturalness problem.
3. Gravity is not included: Gravity is not put on the same footing as other interaction in the SM.
4. Fermion mass: in particle physics one of the major issues is to explain the fermion mass hierarchy and their mixings. The practical feature of fermion mass spectrum gives us $M_u \ll M_c \ll M_t$, $M_d \ll M_s \ll M_b$, $M_e \ll M_\mu \ll M_\tau$

Chapter III

This section is devoted to the main calculation of our result. We are now in position to calculate the rates of some simple decay processes. The decay width, Z, is a measure of the probability of a specific decay process under some given set of initial and final conditions, such as momenta and spin polarization. The calculation involves the following steps:

3-1-1 Calculation of the amplitude M

Firstly, We calculate the so called the matrix element, and denoted by M_{fi}, to indicate that in a matrix representation of the transformation process, with the initial and final states as bases, this is the element that connects a particular final state f to a given initial state i. A process can be a combination of sub processes, in which case, the total amplitude is the sum of the sub process amplitudes. Each simple (sub) process is represented by a unique Feynman diagram. Its amplitude is a point function in the phase space of all the particles involved, including any intermediate propagator, and depends on the nature of the coupling at each vertex (of the diagram). For a given diagram, the amplitude can be obtained by using the Feynman rules for combining the elements a factor for each external line (representing a free particle in the initial or final state), one for each internal line (representing a virtual propagator particle),
and one for each vertex point where the lines do meet (A.D.Martin, 1984).

3-1-2 Integrating the amplitude

Secondly, we should integrate the amplitude over the allowed phase space to get Γ_Z. The integral can be constructed, easily in principle, by utilizing Fermis golden rule. This section will describe the above rules and use them to calculate the decay rates (L.F.Li, 1991).

3-1-3 Physical meaning of decay width

One of the most important characteristics of a particle is the lifetime. It depends, of course, on the available decay modes or channels, which are subject to conservation laws for appropriate quantum numbers, coupling strength of the decay process, and kinematic constraints. The decay rate is the probability per unit time that a given particle will decay. The probability that a single unstable entity will cease to exist as such after an interval is proportional to that interval.

The time after which the ensemble is expected to reduce to $\frac{1}{e}$, of its original size is called the lifetime:

$$\tau_Z = \frac{1}{\Gamma_Z},$$

If multiple decay modes are available, as is often the case, then one can associate a decay rate for each mode, and the total rate, will be the sum of the rates of the individual modes (Weinberg, 1996).

$$\Gamma_{total} = \sum_{i=1}^{N} \Gamma_i,$$

The particles lifetime is given by

$$\tau_Z = \frac{1}{\Gamma_{total}},$$

In such cases, we are often interested in the branching fractions, i.e. the probabilities of the decay by individual modes. The branching fraction of mode i is:

$$Br_i = \frac{\Gamma_i}{\Gamma_{total}},$$

Since the dimension of $\Gamma$is the inverse of time, in our system of natural units, it has the same dimension as mass (or energy). When the mass of an elementary particle is measured, the total rate shows up as the irreducible width of the shape of the distribution. Hence the name decay width

3-2 Calculation of decay widths

The matrix element between the initial state $|i>$and the final state $|f>$ is called the S matrix:

$$S_{fi} = (2\pi)^4 \delta^4 (p_f - p_i) M_{fi}$$

where $p_i$is the total initial momentum, $p_f$the total final momentum, andδ the 4-dimensional function expresses the conservation of 4-momentum$(E, \vec{p})$. The quantity M_{fi}, called the (reduced) matrix element or amplitude of the process, contains the non-trivial physics of the problem, including spins and couplings. It is usually calculated by perturbative approximation (L.F.Li, 1991).

3-3 Feynman rules for calculating the amplitude

In the previous sections, the formula for decay rates and scattering cross sections are given in terms of the amplitude M_{fi}. Here we give the recipe to calculate $iM_{fi}$for a given Feynman diagram for tree-level processes:

3-1-4 External lines

(a) For an incoming fermion, anti-fermion, or gauge boson, associate a factor u,$\bar{v}$ or ε_τ, respectively.

(b) For an outgoing fermion, anti-fermion, or gauge boson, associate a factor ū,v, or ε_τ^* respectively.

3-1-5 Vertices

For each vertex, include a factor of $ig\gamma^\mu$for an fermion or $ig\gamma^\mu$or a anti-fermion.

3.4 Internal lines

(a) For an gauge boson connecting two vertices, include a term

$$\frac{ig_{\mu\nu} - q_\mu q_\nu / m_Z^2}{q^2 - m_Z^2}$$

(b) Integrate over all undetermined internal momenta.

In this chapter we shall calculate the Higgs decay rates to Fermions and Bosons in the Standard Model of particle physics as well as the branching ratio of the Higgs decay in the SM.

First the Higgs decay into two gauge bosons

(3-5) Higgs decay to gauge boson

$1 - H \rightarrow W^{+}W^{-}$

The decay rate is given by the following equation

$$\Gamma = \frac{1}{2m_h}\int (2\pi)^2\,\delta^4(p-k-k')\,\frac{d^3k}{(2\pi)^3}\,\frac{d^3k`}{(2\pi)^3\,2EE`}\sum\nolimits_{spin}|M|^2 \qquad (3\text{-}1\text{-}1)$$

Where M is the matrix element and in this case is equal to

$M = -igM_w\varepsilon_1\varepsilon_2$

$$\sum_{spin}|M|^2 = g^2M_w^2(\varepsilon_1\varepsilon_2)^2$$

We shall consider two cases for $\varepsilon_1\varepsilon_2$

Firstly longitudinal polarization case:

$$\varepsilon_L(k) = (\frac{|k|}{m_w}, 0,0, \frac{E}{M_w})$$

$$\varepsilon'(k) = \left(\frac{k'}{M_w}, 0, \frac{E'}{M_w}, 0\right)$$

Therefore

$$\varepsilon_L^1(k)\varepsilon_L^2(\acute{k}) = \left(\frac{k-\acute{k}}{M_w^2}\right)$$

In the center of mass frame, we have

$$m_h^2 = \left(\frac{k+k'}{m_h^2}\right) = k + k'^2 + 2kk' = 2M_w^2 + 2kk' = 2M_w^2 + 2k\acute{k}$$

$$k.\acute{k} = {}^1\!/_2\,(m_h^2 - 2m_w^2)$$

$$\varepsilon_1\varepsilon_2 = \frac{{}^1\!/_2\ (m_h^2 - 2m_w^2)}{M_w^2}$$

Thus

$$\sum_{\text{Spin}}|\mathrm{M}|^2 = g^2 M_w^2 \left(\frac{{}^1\!/_2\,(m_h^2-2m_w^2)}{M_w^2}\right)^2$$

$$= \frac{g^2}{4m_w^2}(m_h^2 - 2m_w^2)^2$$

$$= \frac{g^2}{4m_w^2}(m_h^4 - 4m_h^2 m_w^2 + 4m_w^2)$$

$$= \frac{g^2}{4m_w^2} m_h^4 \left(1 - 4\frac{m_w^2}{m_h^2} + 4\frac{m_w^2}{m_h^2}\right)$$

$$= \sum |M|^2 = 2 \cdot \frac{G_f}{\sqrt{2}} m_h^4 \left(1 - 4\frac{m_w^2}{m_h^2} + 4\frac{m_w^2}{m_h^2}\right)$$

Now let's evaluate phase space ρ:

$$\rho = \int (2\pi)^4 \delta^4\big(p - k - \acute{k}\big)\frac{d^3k}{(2\pi)^3 2E}\cdot\frac{d^3\acute{k}}{(2\pi)^3 2\acute{E}}$$

$$= \frac{1}{(2\pi)^2}\int \delta^3\big(\rho - k - \acute{k}\big)d^3\acute{k}\cdot\delta^\circ\big(m_h - E - \acute{E}\big)d^3k$$

$$= \frac{1}{(2\pi)^2}\int \delta^\circ\big(m_h - E - \acute{E}\big)\frac{d^3k}{2E\cdot 2\acute{E}}$$

But we know that $\mathrm{d}^3\mathrm{k} = \mathrm{k}^2\mathrm{dk}(4\pi) = kE \cdot dE\ 4\pi$

Then

$$\rho = \frac{1}{4\pi^2}\times 4\pi \int \delta\,\big(m_h - E - \acute{E}\big)K\frac{E\ dE}{4E\ \acute{E}} =$$

$$\rho = \frac{1}{4\pi} \times \mathrm{K} \int \delta\left(m_h - (E + \acute{E})\right) \frac{dE}{\acute{E}} = \frac{\mathrm{K}}{4\pi} \times \frac{1}{2\mathrm{m_h}}$$

$$E = \frac{1}{2m_h} = (K^2 + M_W^2)^{\frac{1}{2}}$$

$$\frac{1}{4} m_h^2 = K^2 + M_W^2$$

So

$$K = \sqrt{\frac{1}{4}} (m_h^2 - 4m_w^2)$$

$$\mathrm{K} = \frac{1}{2} \mathrm{m_h} \left(1 - 4\frac{\mathrm{m_w^2}}{\mathrm{m_h^2}}\right)^{\frac{1}{2}}$$

Therefore

$$\rho = \frac{1}{8\pi} \mathrm{m_h} \left(1 - 4\frac{\mathrm{m_w^2}}{\mathrm{m_h^2}}\right)^{\frac{1}{2}}$$

Substitute the above equation into the equation of the decay rate we get

$$\Gamma = \frac{1}{2m_h} \ \rho \sum |M|^2$$

$$\Gamma = \frac{1}{2m_h} \frac{m_h}{8\pi} \left(1 - 4\frac{m_w^2}{m_h^2}\right)^{\frac{1}{2}} \left(1 - 4\frac{m_w^2}{m_h^2} + 4\frac{m_w^2}{m_h^2}\right)$$

Secondly Transverse case:

In this case we have

$$\varepsilon_1 = \frac{1}{\sqrt{2}} (0,1,i,0)$$

$$\varepsilon_2 = \frac{1}{\sqrt{2}}(0,1,-i,0)$$

$$\varepsilon_1\varepsilon_2 = \frac{1}{2}(1+1) = 1$$

Hence

$$\sum|\mathrm{M}|^2 = \mathrm{g}^2\mathrm{m}_\mathrm{w}^2(\varepsilon_1\varepsilon_2)^2 = \mathrm{g}^2\mathrm{m}_{\mathrm{w}=}^2\frac{8\mathrm{G_F}\mathrm{m}_\mathrm{w}^4}{\sqrt{2}}$$

Therefore the decay rate in this case is

$$\Gamma = \frac{1}{2\mathrm{m_h}}\ \rho\sum|\mathrm{M}|^2$$

$$= \frac{1}{2\mathrm{m_h}} \times \frac{\mathrm{m_h}}{8\pi}\left(1-4\left(\frac{\mathrm{m_w}}{\mathrm{m_h}}\right)^2\right)^{1/2}\left(\frac{4\mathrm{G_f}\mathrm{m}_\mathrm{w}^4}{\sqrt{2}}\right)$$

$$\Gamma_\mathrm{T} = \frac{1}{2\mathrm{m_h}} \times \frac{\mathrm{m_h}}{8\pi}\left(1-4\left(\frac{\mathrm{m_w}}{\mathrm{m_h}}\right)^2\right)^{1/2}\left(\frac{4\mathrm{G_f}\mathrm{m}_\mathrm{w}^4}{\sqrt{2}}\right)$$

Add the longitudinal and transverse cases to yield

$$\Gamma_{\mathrm{tot}}(\boldsymbol{H} \to \boldsymbol{W}^+\boldsymbol{W}^-) = \Gamma_\mathrm{L} + 2\Gamma_\mathrm{T}$$

$$\Gamma_{\mathrm{tot}}(\boldsymbol{H} \to \boldsymbol{W}^+\boldsymbol{W}^-) = \frac{\mathrm{G_f}}{\sqrt{2}}\frac{1}{8\pi}\left(1-4\left(\frac{\mathrm{m_w}}{\mathrm{m_h}}\right)^2\right)^{1/2}\left(1-4\frac{m_w^2}{m_h^2}+12\frac{m_w^2}{m_h^2}\right)$$

2- $H \to ZZ$

The Higgs decay into two ZZ is half of the Higgs decay into two WW because Z Z are neutral and identical particle cannot be distinguished from each other unlike charged WW gauge boson

$$\Gamma(H \to ZZ) = \frac{1}{2}\Gamma(H \to WW)$$

$$= \frac{1}{2}\frac{G_F}{\sqrt{2}}\frac{m_h^2}{8\pi}\left(1-4\left(\frac{m_w}{m_h}\right)^2\right)^{\frac{1}{2}}\left(1-4\left(\frac{m_w}{m_h}\right)^2+12\left(\frac{m_w}{m_h}\right)^4\right)$$

(3-6) **Higgs decay to fermion**

Higgs decay to fermions:

Higgs can decay to fermion and anti-fermion if the it's mass is greater the mass of the fermion

$$H \rightarrow \acute{f}f$$

The mass matrix can be written as

$$M = \frac{-ig}{2}\frac{m_f}{m_w}v(K)U(K)$$

Therefore the sum over all possible spin is

$$\sum_{spin}|M|^2 = \frac{g^2}{4}\frac{m_f^2}{m_w^2}|Tr[(k \acute{-} m_f)(k - m_f)]^2$$

$$= \frac{g^2}{4}\frac{m_f^2}{m_w^2}\left[Tr\left(k.k + m_f k \acute{-} m_f \acute{k} - m_f^2\right)\right]$$

$$= \frac{g^2}{4}\frac{m_f^2}{m_w^2}\left[4(k.k \acute{-} m_f^2)\right]^2$$

$$= 4g^2\frac{m_f^2}{m_w^2}\left(k.k \acute{-} m_f^2\right)^2$$

Utilizing our trace technology we get

$$k\acute{k} = \gamma_\mu k^\mu \gamma_\nu k^\nu$$

$$Tr(\gamma_\mu\gamma_\nu) = 4g_{\mu\nu}$$

$$Tr\left[k.k + m_f(\acute{k} \acute{-} k) - m_f^2\right] =$$

$$\left[Tr(\gamma_\mu\gamma_\nu).\acute{k}^\mu - Tr(\gamma^\nu k_\nu)m_f - Tr(1)m_f^2\right]$$

$$= 4g_{\mu\nu}\acute{k}^{\acute{\mu}}k_{\nu} - 4m_f^2$$

$$= 4\left(k.\acute{k} - m_f^2\right)$$

In the center of mass we have

$$m_h^2 = \left(k + \acute{k}\right)^2 = k^2 + \acute{k}^2 + 2k.\acute{k}$$

$$k.\acute{k} = \frac{1}{2}\left(m_h^2 - 2m_f^2\right)$$

Substitute the above equation into the equation of matrix elements

$$\sum |M|^2 = 4g^2 \frac{m_f^2}{m_w^2}\left(\left(\frac{1}{2}m_h^2 - m_f^2\right) - m_f^2\right)^2$$

$$= 4g^2 \frac{m_f^2}{m_w^2}\left(\left(\frac{1}{2}m_h^2 - m_f^2\right) - m_f^2\right)^2$$

$$= g^2 \frac{m_f^2}{m_w^2} m_h^4 \left(1 - 4\frac{m_f^2}{m_h^2}\right)\left(1 - 4\frac{m_f^2}{m_h^2}\right)$$

$$= \frac{g^2}{m_w^2} m_f^2 m_h^4 \left(1 - \frac{4m_f^2}{m_h^2}\right)^2$$

Thus the decay rate can be written as

$$\Gamma = \frac{1}{2m_h}\rho \sum |M|^2$$

$$= \frac{1}{2m_h}\frac{m_h}{8\pi}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{1/2} \times \frac{8G_f}{\sqrt{2}} m_f^2 m_h^4 \left(1 - \frac{4m_f^2}{m_h^2}\right)^1$$

$$= \frac{4}{8\pi}\frac{G_f}{\sqrt{2}} m_f^2 m_h^4 \left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}}$$

Therefore the Higgs decay rate into two fermions is given by

$$\Gamma_{(h\to ff)} = N_c \frac{4}{8\pi}\frac{G_f}{\sqrt{2}} m_f^2 m_h^4 \left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}}$$

$$where\ N_c = \begin{cases} 1\ for\ leptons \\ 3\ for\ quarks \end{cases}$$

In this section we shall calculate in details the Higgs decay rate into gauge bosons and fermions (H → WW) and $(H \to ff)$ by using the standard model theory, then we summarized our numerical calculations in the table below.

We use the following equations for our calculations:

$$\Gamma(H \to WW) = \frac{{m_h}^3 G_F}{8\sqrt{2}\,\pi}\left(1 - \frac{4{m_W}^2}{{m_h}^2}\right)^{\frac{1}{2}}\left(1 - 4\frac{{m_W}^2}{{m_h}^2} + 12\frac{{m_W}^4}{{m_h}^4}\right), \qquad (4\text{-}1)$$

$$\Gamma(H \to ZZ) = \frac{{m_h}^3 G_F}{16\sqrt{2}\,\pi}\left(1 - \frac{4{m_Z}^2}{{m_h}^2}\right)^{\frac{1}{2}}\left(1 - 4\frac{{m_Z}^2}{{m_h}^2} + 12\frac{{m_Z}^4}{{m_h}^4}\right) \qquad (4\text{-}2)$$

And

$$\Gamma(H \to ff) = \frac{3{m_f}^2\, m_h G_F}{4\sqrt{2}\,\pi}\left(1 - 4\frac{{m_f}^2}{{m_h}^2}\right)^{\frac{3}{2}}. \qquad (4\text{-}3)$$

As can be seen from above equations, in order for the higgs to decay to gauge bosons (W and Z) its mass should be bigger or equal twice the mass of these particles at least and for the higgs to decay into any fermions its mass should be also bigger or equal twice the mass of top particle.

We have also computed the Higgs decay into bottom quark $H \to bb$ with $m_h = 200\ GeV$ and we get

$$\Gamma(H \to bb) = 6.2\ \times 10^{-3} GeV$$

This means that the Higgs decay to bb is very small and can be neglected in our calculation only top is the dominanet contribution becuase its mass is bigger compare to other quarks and leptons . And we have double checked for the Higgs decay to tau lepton and found the decay is also too small compare to top quark.

All higgs decay rates are modified by electroweak (EW) and Quantum Chromodynamics (QCD) corrections. QCD corrections are important for Higgs decay into$H \rightarrow \overline{\psi}\,\psi$.

(3-7) Higgs decay to gluons.

The next process is a one-loop process. We could naively think that it's decay rate must be very low compared to the tree-level ones, but that is not exactly true. Due to the very heavy top quark mass, this diagram generates a high enough decay rate that necessarily must be taken in consideration. We shall see at the end of our computation that for a massless quark this diagram does not contribute.

$$\mathrm{H(p_1)} \rightarrow \mathrm{g(p_2)g(p_3)} \tag{3.1}$$

first diagram:

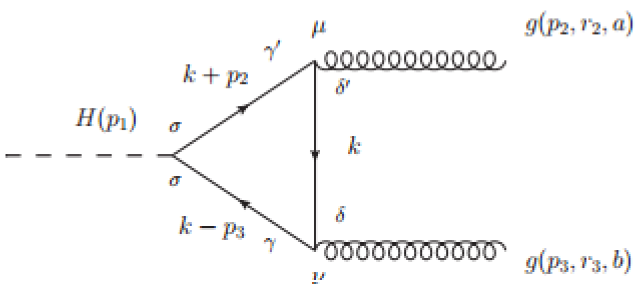

Figure 4: Higgs decay to gluons, first diagram.

The transition amplitude of the first diagram is given by:

$$\mathcal{M}_{(1)} = (-i) g_s^2 \frac{m}{v}\, \epsilon_{\mu,r2}^{a}\, \epsilon_{v,r3}^{b} \left(\frac{\lambda^a}{2}\right)_{\delta'\gamma'} \left(\frac{\lambda^b}{2}\right)_{\gamma\delta} \delta_{\delta\delta'}\delta_{\gamma'\delta}\delta_{\delta\gamma} \times$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{T_r\{\gamma^{\mu}(k+p_2+m)(k-p_3+m)\gamma^{\nu}(k+m)\}}{(k^2-m^2)[(k+p_2)^2-m^2][((k-p_3)^2-m^2)]} \tag{3.2}$$

Here m = m_t , the top quark mass. Let's first analyze the colour trace:

$$\left(\frac{\lambda^a}{2}\right)_{\delta'\gamma'} \left(\frac{\lambda^b}{2}\right)_{\gamma\delta} \delta_{\delta\delta'}\delta_{\gamma'\delta}\delta_{\delta\gamma} = \frac{1}{4} Tr\{\lambda^a \lambda^b\} = \frac{1}{2}\delta_{ab} \tag{3.3}$$

The spinor trace is a little more complicated:

$T_r\{\gamma^\mu(k+p_2+m)(k-p_3+m)\gamma^\nu(k+m)\}$

$= \ 4m\big(p_3^\mu p_2^\nu + \ 4k^\mu k^\nu - \ 2k^\mu p_3^\nu + 2p_2^\mu k^\nu - p_2^\mu p_3^\nu + g^{\mu\nu}(m^2 - \ p_2 p_3) - \ g^{\mu\nu}k^2\big) \equiv \ 4mN^{\mu\nu}$

(3.4)

Before we try to perform the integral, we shall use the Feynman parameterization to simplify the denominator:

$$\frac{1}{\mathrm{ABC}} = \int_0^1 dx \int_0^1 dy \int_0^1 dz\, \delta(x\ +\ y\ +\ z\ -\ 1)\frac{2}{[\mathrm{Ax+By+Cz}]^3} \qquad (3.5)$$

We have, A = k^2-m^2, B=$(k-p_3)^2-m^2$ and C= $(k+p_2)^2-m^2$, so the denominator

D ≡ Ax + By + Cz

can be written as it follows (as a first order approximation we shall consider on-shell gluons):

$$D = (k^2-m^2)x + (k^2+p_2^2-m^2+2kp_2)y + (\mathrm{k}^2 + \ \mathrm{p}_1^2 - \ \mathrm{m}^2 - \ 2\mathrm{kp}_3)z$$

$$= (k^2-m^2)(x+y+z) + 2(kp_2)y - 2(kp_3)z$$

$$= k^2-m^2 \ + 2(kp_2)y - 2(kp_3)z$$

$$= (k+p_2y-p_3z)^2 + 2(p_2p_3)yz - m^2 \qquad (3.6)$$

We define $a^2 \equiv \ m^2 \ \ - 2(p_2p_3)\mathrm{yz}$, therefore, we can write D in the simplified form : $D = (k+p_2y-p_3z)^2 - a^2$

In terms of the Feynman parameters, our integral becomes:

$$\mathrm{I}^{\mu\nu} \ \equiv \ \int \frac{d^4k}{(2\pi)^4}\int_0^1 dy \int_0^{1-y} dz \frac{8mN^{\mu\nu}}{[\ (k+p_2y-p_3z)^2-a^2]^3} \qquad (3.7)$$

Making a variable shift from k to $k+p_2y-p_3z$, $\mathrm{I}^{\mu\nu}$ takes the form:

$$\mathrm{I}^{\mu\nu} \ \equiv \ \int \frac{d^4k}{(2\pi)^4}\int_0^1 dy \int_0^{1-y} dz \frac{8mN'^{\mu\nu}}{[\ k^2-a^2]^3} \qquad (3.8)$$

Where the new numerator is:

$$N'^{\mu\nu} = 4(k - p_2y + p_3z)^{\mu}(k - p_2y + p_3z)^{\nu} - 2(k - p_2y + p_3z)^{\mu}p_3^{\nu} + 2p_2^{\mu}(k - p_2y + p_3z)^{\nu} + p_3^{\mu}\, p_2^{\nu} - p_2^{\mu}p_3^{\nu} + g^{\mu\nu}(m^2 - p_2p_3) - g^{\mu\nu}(k - p_2y\, p_3z)^2 \tag{3.9}$$

Knowing that all terms that are lineal in k^{μ} vanish when integrated (k^{μ} is an odd function) we can discard them from N0μν, so what we have left is:

$$N'^{\mu\nu} = 4k^{\mu}k^{\nu} - g^{\mu\nu}k_2 + p_3^{\mu}p_2^{\nu}(1 - 4yz) + p_2^{\mu}p_3^{\nu}(-1 - 4yz + 2y + 2z)$$

$$+p_3^{\mu}\, p_2^{\nu}(4z^2 - 2z) + p_2^{\mu}p_3^{\nu}(4y^2 - 2y) + g^{\mu\nu}(m^2 - p_2p_3 + 2p_2\, p_3yz) \tag{3.10}$$

There are a couple terms that are apparently ultraviolet divergent, such as $4k^{\mu}k^{\nu} - g^{\mu\nu}k^2$ so we need to employ dimensional regularization to perform the four-momentum integral. We will also use the same technique to calculate the finite integrals. The scheme used here is the MS, so we take the identity matrix trace in D space-time dimensions to be 4 (T r{ID} = 4). Now let us define the following integral:

$$J\,(D, \alpha, \beta\,, a^2) \equiv \int \frac{d^Dk}{(2\pi)^D} \frac{(k^2)^{\propto}}{(k^2 - a^2)^{\beta}} \tag{3.11}$$

where D is the number of space-time dimensions. We can easily show that:

$$J\,(D, \alpha, \beta\,, a^2) = \frac{i}{(4\pi)^{D/2}}\,(a^2)^{D/2}(-a^2)^{\propto-\beta}\,\frac{\Gamma(\beta-\alpha-D/2)\Gamma(\alpha+D/2)}{\Gamma(\beta)\Gamma(D/2)} \tag{3.12}$$

All terms that do not depend on the four momentum k^{μ} in the numerator give rise to finite integrals, thus in this case we can directly take D as 4; so $J\,(4, 0, 3\,, a^2)$) takes the simple form:

$$J\,(4, 0, 3\,, a^2) = \frac{i}{32\pi^2}\frac{1}{a^2} \tag{3.13}$$

Due to Lorentz symmetry, we find the following property:

$$\int \frac{d^Dk}{(2\pi)^D} \frac{(k^2)^{\propto}k^{\mu}k^{\nu}}{(k^2 - a^2)^{\beta}} = \frac{g^{\mu\nu}}{D}\, J\,(D, \alpha + 1, \beta\,, a^2) \tag{3.14}$$

Using this property we are now able to integrate the terms $4k^{\mu}k^{\nu} - g^{\mu\nu}k^2$ from $N'^{\mu\nu}$

$$\int \frac{d^D k}{(2\pi)^D} \frac{4k^\mu k^\nu - g^{\mu\nu} k^2}{(k^2-a^2)^3} = \left(\frac{4}{D} - 1\right) g^{\mu\nu} J\,(D, 1, 3\,, a^2)$$

$$= \left(\frac{4}{D} - 1\right) g^{\mu\nu} \frac{i}{(4\pi)^{D/2}} (a^2)^{D/2} (-a^2)^{-2} \frac{\Gamma(2-D/2)\Gamma(1+D/2)}{\Gamma(3)\Gamma(D/2)}$$

$$= \left(\frac{4}{D} - 1\right) g^{\mu\nu} \frac{i}{(4\pi)^{D/2}} (a^2)^{D/2} \frac{4}{D} \Gamma(2 - D/2) \tag{3.15}$$

Taking D = 4 + 2ϵ with ϵ ≪ 1 we find:

$$\left(\frac{4}{D} - 1\right) \frac{D}{4} = -\frac{\epsilon}{2} \tag{3.16}$$

$$\Gamma(2 - D/2) = \Gamma(-\epsilon) = -\frac{1}{\epsilon} - \gamma_E + O(\epsilon^2) \tag{3.17}$$

where γ_E is the Euler-Mascheroni constant. Substituting this result in our integral the pole of the Gamma function disappears therefore the ultraviolet divergence disappears. We can now take the limit → 0 to obtain:

$$\int \frac{d^D k}{(2\pi)^D} \frac{4k^\mu k^\nu - g^{\mu\nu} k^2}{(k^2-a^2)^3} = \frac{i}{32\pi^2} g^{\mu\nu} = \frac{i}{32\pi^2} \frac{a^2}{a^2} g^{\mu\nu} \tag{3.18}$$

We obtain the following expression for $I^{\mu\nu}$:

$$I^{\mu\nu} = \frac{8im}{32\pi^2} \int_0^1 \int_0^{1-y} \frac{dydz}{-a^2} \begin{bmatrix} p_2^\mu\, p_2^\nu (4y^2 - 2y) + p_3^\mu\, p_3^\nu (4z^2 - 2z) + p_3^\mu\, p_2^\nu (1 - 4yz) \\ +p_2^\mu\, p_3^\nu (-4yz + 2y + 2z - 1) g^{\mu\nu} (4p_2\, p_3 yz - 4p_2\, p_3\,) \end{bmatrix} \tag{3.19}$$

Now let us remember that we have considered on-shell gluons, therefore we can apply the transversality condition to eliminate terms from $I^{\mu\nu}$, thus keeping in mind that $\epsilon^a_{\mu,r_i} p_i^\mu = 0$ with i=2,3, then the only remaining tensorial structure is the following:

$$I^{\mu\nu} = \frac{8im}{32\pi^2} \int_0^1 \int_0^{1-y} \frac{dydz}{-a^2} \left[p_3^\mu\, p_2^\nu (1 - 4yz) + g^{\mu\nu} (4p_2\, p_3 yz - 4p_2\, p_3\,) \right] \tag{3.20}$$

Rearranging terms we can write the following:

$$I^{\mu\nu} = \frac{8im}{32\pi^2} \int_0^1 \int_0^{1-y} \frac{dydz}{-a^2} \left[p_3^\mu\, p_2^\nu + g^{\mu\nu} p_2\, p_3 \right] (1 - 4yz) \tag{3.21}$$

To simplify our notation let us define the following:

$$\int_0^1 \int_0^{1-y} dydz \frac{1-4yz}{-a^2} \equiv \mathrm{C} \tag{3.22}$$

Now we can write $I^{\mu\nu}$ in a simple compact form:

$$I^{\mu\nu} = \frac{8im}{32\pi^2} C \left[p_3^{\mu}\, p_2^{\nu} + g^{\mu\nu} p_2\, p_3 \right] \tag{3.23}$$

Finally, we write the transition amplitude $M_{(1)}$:

$$M(1) = (-i) g_s^2 \frac{m_t}{2v} \epsilon^a_{\mu,r_2} \epsilon^a_{v,r_3} \delta_{ab} I^{\mu\nu} \tag{3.24}$$

Second diagram:

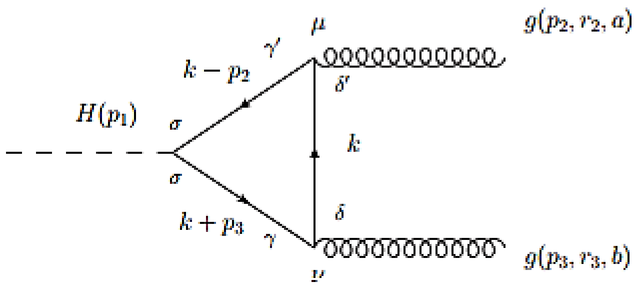

Figure 4: Higgs decay to gluons, second diagram.

The transition amplitude of the first diagram is given by:

$$\mathcal{M}_{(1)} = (-i) g_s^2 \frac{m}{2v} \epsilon^a_{\mu,r2} \epsilon^b_{v,r3} \delta_{ab} \int \frac{d^4k}{(2\pi)^4} \frac{T_r\{\gamma^{\mu}(k+m)\gamma^{\upsilon}(k+p_3+m)(k-p_2+m)\}}{(k^2-m^2)[(k+p_2)^2-m^2][((k-p_3)^2-m^2)]} \tag{3.25}$$

Computing the spinor trace, and D in terms of the Feynman parameters we find:

$$T_r\{\gamma^{\mu}(k+m)\gamma^{\upsilon}(k+p_3+m)(k-p_2+m)\}$$

$$= 4m\left(p_3^{\mu} p_2^{\nu} + 4k^{\mu}k^{\nu} + 2k^{\mu}p_3^{\nu} - 2p_2^{\mu}k^{\nu} - p_2^{\mu}p_3^{\nu} + g^{\mu\nu}(m^2 - p_2 p_3 - g^{\mu\nu}k^2)\right) \equiv 4mN^{\mu\nu} \tag{3.26}$$

and also, the following integral:

$$J^{\mu\nu} \equiv \int \frac{d^4k}{(2\pi)^4} \int_0^1 dy \int_0^{1-y} dz \frac{8mN^{\mu\nu}}{[(k-p_2y+p_3z)^2-a^2]^3} \tag{3.27}$$

Performing the parameter shift $k \to k - p_2 y + p_3 z$, $J^{\mu\nu}$ takes the form:

$$J^{\mu\nu} \equiv \int \frac{d^4k}{(2\pi)^4} \int_0^1 dy \int_0^{1-y} dz \frac{8mN'^{\mu\nu}}{[\,k^2 - a^2]^3} \tag{3.28}$$

with the non zero contributing terms of $M'^{\mu\nu}$:

$$M'^{\mu\nu} = p_2^{\mu} p_2^{\nu}(4y^2 - 2y) + p_3^{\mu} p_3^{\nu}(4z^2 - 2z) + p_3^{\mu} p_2^{\nu}(1 - 4yz) + p_2^{\mu} p_3^{\nu}(-4yz + 2y + 2z - 1) + g^{\mu\nu}(2p_2\, p_3 yz + m^2 - p_2 p_3)` = N'^{\mu\nu} \tag{3.29}$$

So we find that $I^{\mu\nu} = J^{\mu\nu}$, therefore the amplitude of the second diagram is exactly the same as the first one $M_{(1)} = M_{(2)}$; the total squared amplitude is then given by:

$$|M|^2 = 4\left|M_{(1)}\right|^2 \tag{3.30}$$

The sum over spins and gluon colours gives:

$$\sum_{a,b} \delta_{ab}\delta_{ab} = \sum_a \delta_{aa} = 8\, ; \sum_{r_2,r_3} \epsilon^{a*}_{\rho,r_2} \epsilon^{b}_{\mu,r_2} \epsilon^{b*}_{\sigma,r_3} \epsilon^{b}_{\upsilon,r_3} = g_{\mu\rho} g_{\sigma\upsilon} \tag{3.31}$$

We obtain the simple formula:

$$\sum_{r_2,r_3} \left|M_{H\to gg}\right|^2 = g_s^4 \, \frac{8m^2}{\upsilon^2} \, I^{\mu\nu} I^*_{\mu\nu}\, ; \qquad I^{\mu\nu} I^*_{\mu\nu} = \frac{m^2\,(p_2 p_3)^2 |C|^2}{8\,\pi^4} \tag{3.32}$$

The squared amplitude than reads:

$$\sum_{r_2,r_3} \left|M_{H\to gg}\right|^2 = g_s^4 \, \frac{m^4\,(p_2 p_3)^2}{\upsilon^2\,\pi^4} |C|^2 \tag{3.33}$$

Let’s compute now the integral C explicitly:

$$C = \int_0^1 \int_0^{1-y} dy\, dz\, \frac{1 - 4yz}{-a^2} = \int_0^1 \int_0^{1-y} dy\, dz\, \frac{1 - 4yz}{2p_2\, p_3 yz\; - m^2} = \frac{1}{2p_2\, p_3} \int_0^1 \int_0^{1-y} dy\, dz \frac{1 - 4yz}{yz\; - \dfrac{m^2}{2p_2\, p_3}}$$

$$= \frac{1}{2p_2\, p_3} \left[-2 + (4n - 1)\left(Li_2\left(\frac{-2}{\sqrt{1-4n}-1}\right) + \left(Li_2 \frac{-2}{\sqrt{1-4n}+1} \right)\right) \right]$$

$$\equiv \frac{1}{2p_2\, p_3}\, D(n) = \frac{n}{m^2}\, D(n)$$

were we have defined n ≡ $m^2/2p_2\, p_3$. Taking the limit $\lim_{\mathrm{m}\to 0}\, n\, D(n)$ n we observe that the result is zero,

therefore, if we consider massless quarks as usual, except for the top quark, we only have one contribution, as we mentioned at the beginning. Moving on, in the center of mass the four-momenta are given by:

$$p_1^\mu = (\mathcal{M}_H, 0), p_2^\mu = (p, \vec{p}), p_3^\mu = (p, -\vec{p})$$

We can easily find that:

$$\mathcal{M}_H = 2p \longrightarrow p^2 = \frac{1}{4}\, \mathcal{M}_H^2 \longrightarrow p_2 p_3 = 2p^2 = \frac{1}{2}\, \mathcal{M}_H^2$$

Therefore we can write the squared transition amplitude as:

$$\sum_{r_2, r_3} \left|\mathcal{M}_{H\to gg}\right|^2 = \frac{4\mathcal{M}_H^4}{v^2} \left(\frac{\alpha_s}{\pi}\right)^2 n^2 |D(n)|^2$$

The phase space integral is easy to compute:

$$\int dQ_s = \frac{1}{2} \int \frac{1}{(2\pi)^2} \frac{p}{4\sqrt{s}}\, d\Omega_{CM} = \frac{1}{16\pi}$$

Note that we have included the symmetry factor 1/2 in the phase space integral because this time we are dealing with identical final state particles. Thus, the decay width of the process is given by (n = $n = m^2/\mathcal{M}_H^2$:

$$\Gamma(H \longrightarrow gg) = \frac{\mathcal{M}_H^3}{8\pi\, v^2} \left(\frac{\alpha_s}{\pi}\right)^2 n^2 |D(n)|^2$$

Chapter V

Discussion and Conclusion

Discussion:

As depicted in figure 4.1 the Higgs decay rate into W gauge boson increase with Higgs mass increases. And the decay rate is almost zero with Higgs mass less than twice the mass of W gauge boson and the same applies to Higgs decaying into Z boson. We observe similar behavior for the Higgs decay to top quark.

We found numerically that the Higgs decay to tau lepton is 10 smaller than Higgs decaying to bottom quark $H \to \tau\tau \ll 10\ H \to bb$; such effect can be safely ignored.

In particle physics and nuclear physics, the **branching fraction** for decay is the fraction of particles which decay by an individual decay mode with respect to the total number of particles which decay and can be define as follow:

$$Br(H) = \frac{\Gamma(H \to any\ particle)}{\Gamma(H \to All)} \qquad (4\text{-}4)$$

The branching ratios of the SM Higgs boson are shown in figure 4.2. The main decay channel by far in the Higgs mass range is $H \to WW$ with $Br \sim 80\%$ followed by the decay into ZZ and tt with $Br \sim 2\%$.

The next process is a one-loop process. We could expect that its decay rate must be very small compared to the tree-level ones, but that is not the case. Because top quark mass is heavy, this diagram produces a high enough decay rate that necessarily must be taken into account as highlighted in figure 4.3.

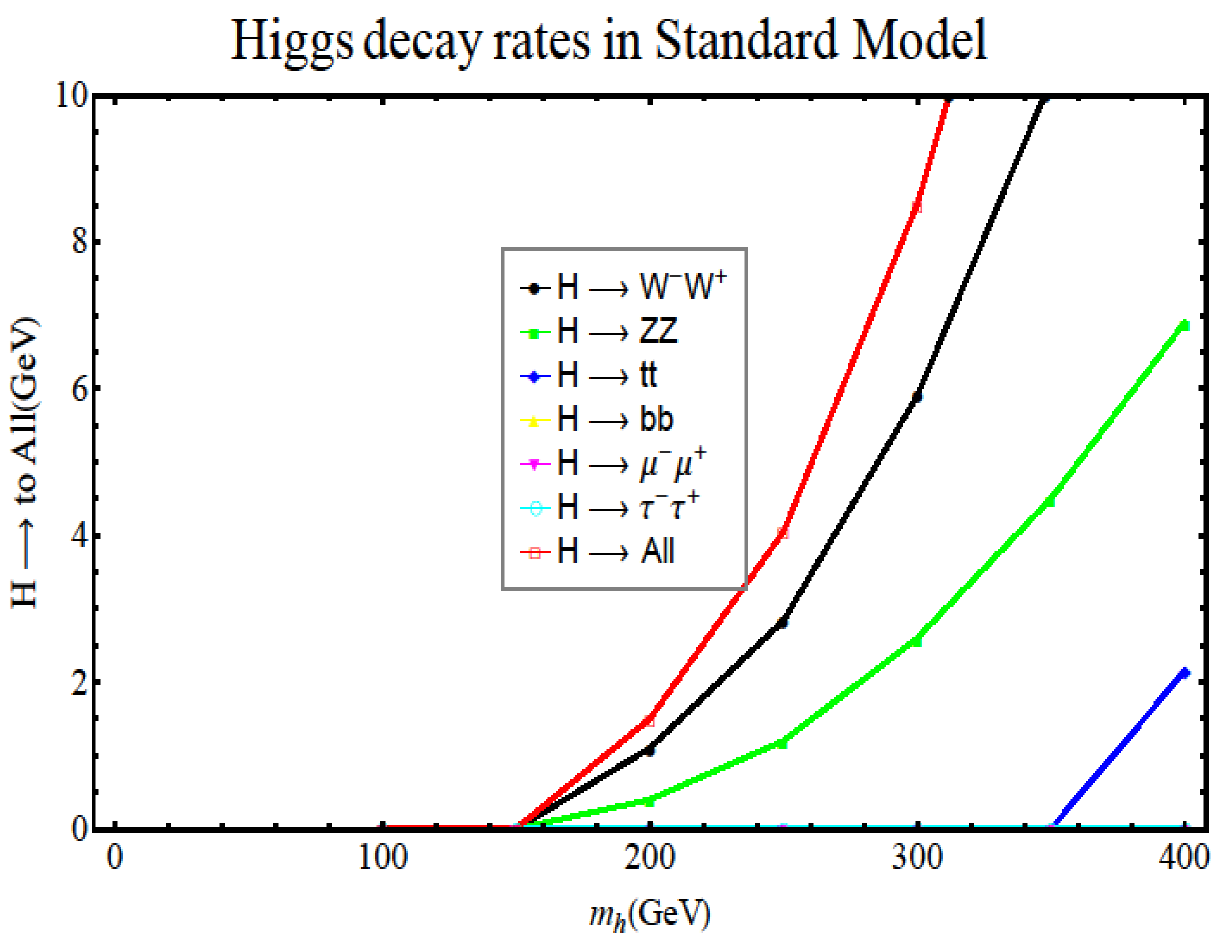

Fig.4.1. Higgs Decay rates as function of Higgs mass in the standard model.

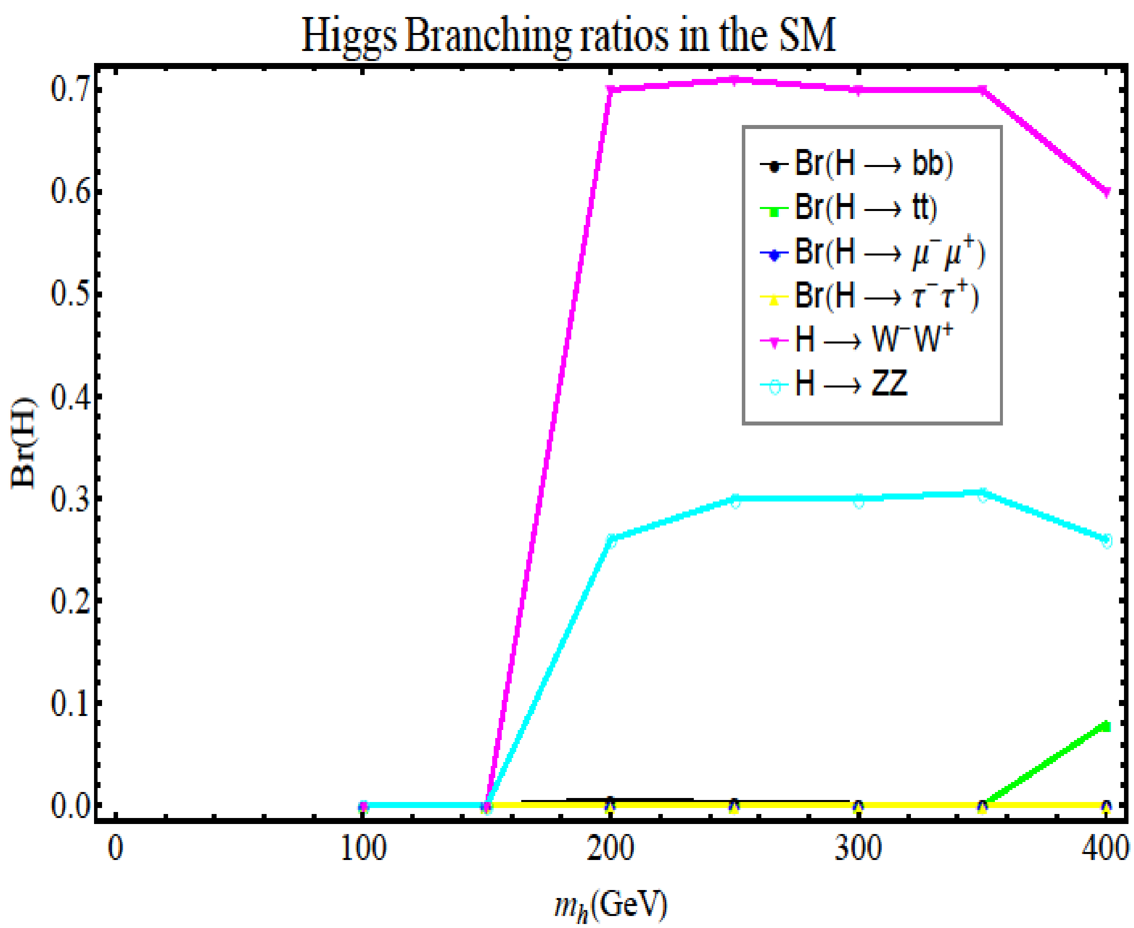

Fig.4.2. Higgs Branching ratio as function of Higgs mass in the standard model.

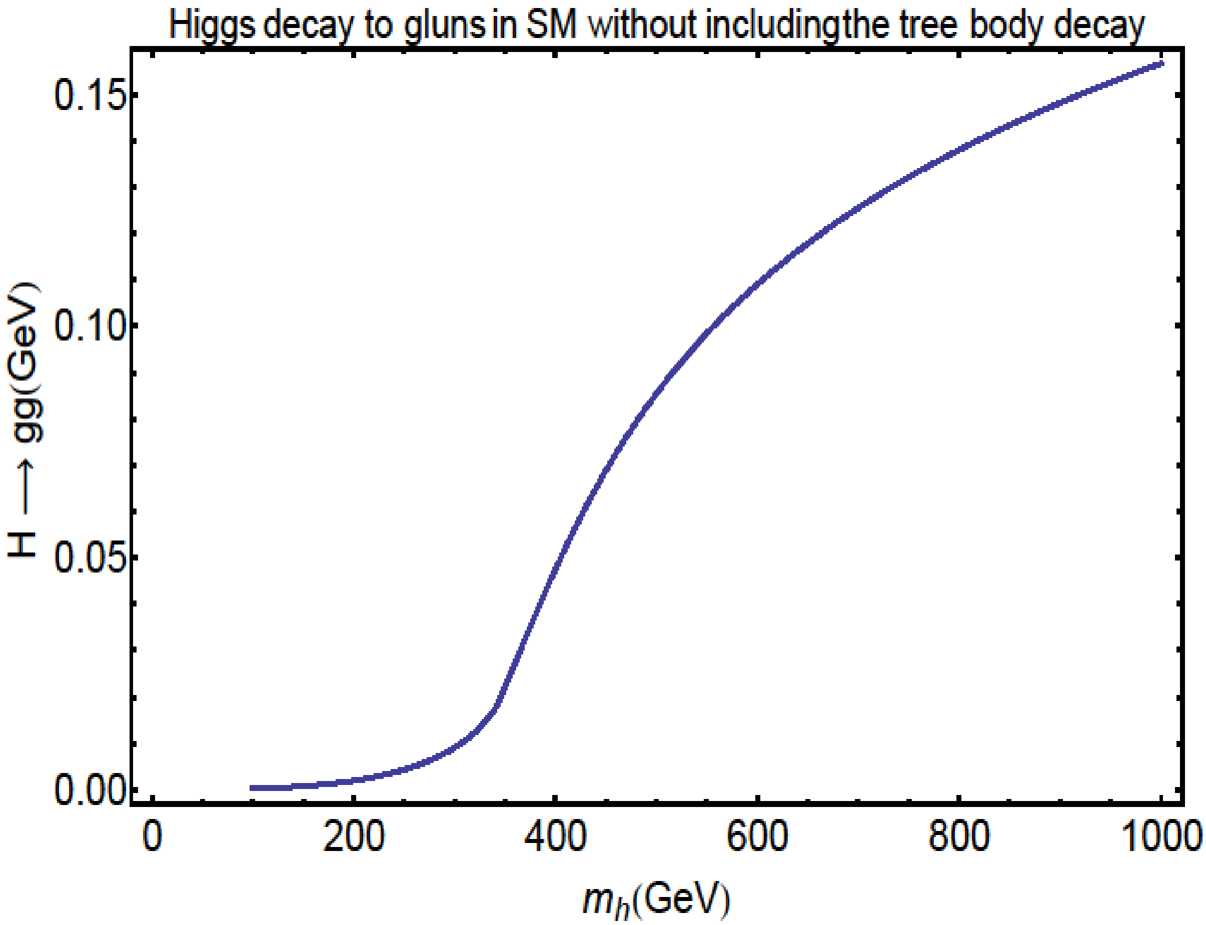

Fig.4.3. Higgs decay rate to gluons as function of Higgs mass in the standard model.

Conclusion:

In this thesis we calculated the decay rates of the Higgs in the Standard Model of particle physics, as well as the branching ratio of the Higgs gauge boson in details at tree level. We find that the dominant decay channel is the Higgs decaying into WW gauge boson with $Br \sim 80\%$ this result hold for $m_h \geq 142\ GeV$.

The total decays widths of the Higgs bosons and the various branching ratios in the SM are discussed.

This work could be extended by including the effects from the loop contribution such as Higgs decaying into photons which appear only at loop level.

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