بِسۡمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيمِ

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College of Graduate Studies

A research submitted for the partial Fulfilment for the Degree of M.Sc. in Geodesy & Geographic Information System:

A Comparative Study Between Three Different Adjustment Techniques as Applied to Part of the Old Sudanese Triangulation Networks

دراسة مقارنة بين ثلاث تقنيات ضبط مختلفة لجزء من شبكات التثليث السودانية القديمة

Prepared by:

Ahmed Elzein Eltaj Osman

Supervisor:

Dr.Khalaf Alla Mohmed Badi

Des 2017

﴿وَءَايَةٞ لَّهُمُ ٱلَّيۡلُ نَسۡلَخُ مِنۡهُ ٱلنَّهَارَ فَإِذَا هُم مُّظۡلِمُونَ ٣٧ وَٱلشَّمۡسُ تَجۡرِي لِمُسۡتَقَرّٖ لَّهَاۚ
ذَٰلِكَ تَقۡدِيرُ ٱلۡعَزِيزِ ٱلۡعَلِيمِ ٣٨ وَٱلۡقَمَرَ قَدَّرۡنَٰهُ مَنَازِلَ حَتَّىٰ عَادَ كَٱلۡعُرۡجُونِ ٱلۡقَدِيمِ
٣٩ لَا ٱلشَّمۡسُ يَنۢبَغِي لَهَآ أَن تُدۡرِكَ ٱلۡقَمَرَ وَلَا ٱلَّيۡلُ سَابِقُ ٱلنَّهَارِۚ وَكُلّٞ فِي فَلَكٖ
يَسۡبَحُونَ ٤٠ ﴾

سورة يس ,٣٧-٤٠

Abstract

Triangulation in survey is the process of determining the control points by measuring angles and distances. The adjustment process of triangulating a network can be carried out by many methods, one of these methods is variation of coordinates using distance, angle and azimuth observation equation. The objective of this study was to use the variation of coordinates method by distance observation equation only. The network has been adjusted was a part of an old triangulation in Sudan, three techniques of adjustment were implemented, the first technique is using plane distance without weights, and in the second one is using weights on plane distance and in the last technique some of the distances were too long. Therefore, the curvature correction was added to the distance and a new adjustment was performed. The results emphasized that the technique of adding curvature gives better adjusted values. Nonetheless, the three techniques gave more or less same values, but the difference from the original coordinates were tremendously large. This shows that the new technique is likely to approximate the true value of the coordinates.

المستخلص

شبكة المثلثات تستخدم لتحديد نقاط الضبط بقياس الزوايا و المسافات بين النقطة المعلومة و النقطة التي يراد تحديدها. تتم عملية ضبط شبكة المثلثات بعدد من الطرق منها طريقة فرق الإحداثيات باستخدام معادلات الرصد للمسافات و الزوايا و الانحرافات. الهدف من هذه الدراسة هو عملية ضبط لجزء من شبكة المثلثات القديمة الموجدة في السودان, باستخدام طريقة فرق الاحداثيات, و باستخدام معادلة الرصد للمسافات فقط, في المرة الاولى تكون معادلة التثليث بدون وزن و المسافات محسوبة على سطح مستوي, ثم يتم ادخال الوزن للمعادلة وحساب الاحداثيات, واخيرا وجد ان المسافات بين بعض النقاط طويلة جدا لذلك تم ادخال مسافات تاخذ في الاعتبار انحناء الارض,ومنهم يتم حساب الضبط للاحداثيات. النتائج تظهر ان الافضل هي الطريقة الاخيرة التي تم ادخال انحناء الارض في المسافت بين النقاط. و لكن حقيقة الطرق الثلاثة معظمها متساوية في القيم , لكن الفرق بين القيم المضبوطة و البيانات المرصودة نجدة كبيرا, و هذا يظهر ان التقنيات الحديثة هي المفضلة لتعطي قيم دقيقة للاحداثيات.

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Chapter One
Introduction

1.1 Forward:

Since the 1960s, spatial geodetic methods have allowed orientation of the classical networks with respect to the global geocentric reference system, and control of scale and systematic distortions. To describe the design of these networks, the measurement and computation techniques applied, the accuracy achieved, and the orientation with respect to the Earth's body (geodetic datum). Having served (and serving) as a basis for many applications in surveying and mapping, they are still of relevance and now in a state of transition to the global 3D reference frame. Horizontal control networks have been realized by trigonometric (triangulation) points, which in principle should be distributed evenly over the country. One distinguishes between different orders of trigonometric points, from first-order or primary (station separation 30 to 60 km) to second-order (about 10 km) to fourth- or even fifth-order (down to 1 to 2 km) stations, where the state of the networks' coverage strongly depends on the development of the respective region or country. The maximum distance between first-order points was determined by terrestrial measurement methods, which required indivisibility between the network stations. Consequently, first- and partly also second order stations were established on the top of hills and mountains; observation towers (wooden or steel constructions with heights of 30 m and more) were erected especially in at areas. The stations have been permanently marked by underground and surface monuments (stone plates, stone or concrete pillars, bolts in hard bedrock). Eccentric marks have been set up in order to aid in the recovery and verification of the center mark [11].

1.2 Statement of the Problem:

All measurements contain errors and with global positioning systems, total station instruments, digital metric cameras and satellite imaging systems now generating vast quantities" of data, adjustment for errors is crucial to accurate interpretation. Adjustment computations provides a complete. Up to date treatment of every aspect of least squares adjustment. The most rigorous procedure available for computing adjustments to measured data. Prior to the development of electronic distance measuring equipment and the global positioning system, triangulation was the preferred method for extending horizontal control over long distances. The positions of widely spaced stations were computed from measured angles and a minimal number of measured distances called baselines [1, 12].

1.3 Research Objectives:

The main objective of this study is adjustment and detecting the error in the triangulation network using statistical methods, to adjust distance and coordinates for the station.

1.4 Methodology:

The methodology will be used in this study consisting of three phases. In phase one chose the data from the old triangulation network in Sudan from "General Authority for Sudanese Survey". In phase two processing to adjustment the distance and coordinate using adjustment by distance observation equation. Phase Three is compare between different techniques into the Trilateration.

1.5 Expected Output:

Based on data collection and the process in them, that be able to extract the high resolution of triangulation network. This certainly will be helpful to make the high resolution for Triangulation Geodetic Network.

1.6 Layout:

The research will be document in six Chapters; Chapter One will introduce the research problem and the objectives. Chapter Two will discuss the concept of triangulation Network, design and classification. Chapter Three will reflect the adjustment for triangulation network. The research methodology will be discussed in Chapter Four. Chapter five will list and analysis the output results of the research work. Conclusions and recommendations, will be summarized in Chapter six.

Chapter two
The Concept of Horizontal Network

2.1 Introduction:

Triangulation is the surveying technique in which unknown distances between stations may be determined by trigonometric applications of a triangle or triangles. In triangulation, one side called the baseline and at least two interior angles of the triangle must be measured. When all three interior angles are measured, accuracy of the calculated distances is increased and a check provided against any measurement error.

The most basic use of triangulation can be found in surveys of the public domain. Although the use of electronic measuring instruments has eliminated most requirements for this type of triangulation, the 1973 Manual of Surveying Instructions made the following statement:

Triangulation may be used in measuring distances across water or over precipitous slopes. The measured base should be laid out so as to adopt the best possible geometric proportions of the sides and angles of the triangle. If it is necessary to determine the value of an angle with a precision of less than the least reading of the Vernier, the method of repetition should be employed.

A complete record of the measurement of the base, the determination of the angles, the location and direction of the sides, and other essential details is entered in the field tables, together with a small diagram to represent the triangulation. In the longer and more important triangulations, all of the stations should be occupied, if possible, and the angles should be repeated and checked to a satisfactory closure; the latter may be kept within 0'20" by careful use of the one-minute transit.

In line practice the chainmen are frequently sent through for taped measurement over extremely difficult terrain, but with the length of the interval verified by triangulation. This is done to ensure the most exact determination of the length of the line while also noting the intervening topographic data [11].

2.2 Horizontal Geodetic Network:

Horizontal control networks have been realized by trigonometric (triangulation) points, which in principle should be distributed evenly over the country. One distinguishes between different orders of trigonometric points, from first-order or primary (station separation 30 to 60 km) to second-order (about 10 km) to fourth- or even fifth-order (down to 1 to 2 km) stations, where the state of the networks coverage strongly depends on the development of the respective region or country. The maximum distance between first-order points was determined by terrestrial measurement methods, which required inter visibility between the network stations. Consequently, first- and partly also second order stations were established on the top of hills and mountains; observation towers (wooden or steel constructions with heights of 30 m and more) were erected especially in flat areas. The stations have been permanently marked by underground and surface monuments (stone plates, stone or concrete pillars, bolts in hard bedrock). Eccentric marks have been set up in order to aid in the recovery and verify cation of the center mark [12].

2.2.1 The Horizontal Datum:

The horizontal datum provides the basis for establishing the national geodetic coordinate system and for calculating the geodetic coordinates of each point in the national horizontal control network. It includes a set of initial data, i.e., the geodetic longitude and latitude of the initial point and the geodetic azimuth from the initial point to its adjacent point in the national geodetic control network. (The initial point is the geodetic origin in classical geodetic survey.) The extension of the horizontal datum is realized by the horizontal control network formed by a series of control points. Coordinates of the control points are computed from the geodetic origin and obtained by classical geodetic methods such as traversing, triangulation, and so on. In modern geodetic survey, the horizontal datum is usually realized by 3-D datum obtained from the GPS method [3,12].

2.3 Geodetic Triangulation:

To measure terrain, surface features, position coordinates, heights, and gravity values at points on the Earth's surface, there need to be corresponding reference points or surfaces. Namely geodetic datums, to which surveying and mapping results are referred. Geodetic datums consist chiefly of coordinate datums (including classical horizontal datums and three-dimensional coordinate datums), vertical datums, sounding datums, as well as gravity datums. Geodetic datums provide initial data for all kinds of surveying and mapping work and serve as the foundation for determining the geometric shape and spatial–temporal distribution of geospatial information. Classical horizontal and vertical datums are realized by classical geodetic methods. Due to their limited controlling area, these two datums can only be used as regional datums and are usually applicable countrywide. The three-dimensional coordinate datums and gravity datums can be used as both global and regional datums. The datums are represented by the position coordinates, heights, and gravity values at a series of control points. [1]. The Geodetic networks consist of monument control points that provide the reference frames for positioning and gravity-field determination [12]. Many type of figures used in triangulation and strength factors as explanation below:

(A) Simple quadrilateral The simple quadrilateral is the best figure, and it should be employed wherever possible. It combines maximum strength and progress with a minimum of essential geometrical conditions when approximately equilateral or square and therefore the square quadrilateral is the perfect figure.

(B) Four sided central point figure with one diagonal. When one diagonal of the quadrilateral is obstructed, a central point, which is visible from the four comers can be inserted. This figure requires the solution of two side equations and five angle equations, and hence adds to the labor of adjusting.

(C) Four sided central point figure without diagonal. At times, neither diagonal can be made visible and the figure becomes a simple four sided central point quadrilateral with a strength factor of 0.64. The central point in this case should be carefully located to maintain the strength of the RJ chain of triangles. An excellent location is near one side line and about midway along it. If too near the side line, however, refraction errors may be almost the same for the closely adjacent lines, and furthermore, the R2 value will be so large as to be of little value as a check on lengths computed through the RJ triangles.

(D) sided central point figure. This is a simple and usually very strong figure. It is often used to compensate for a great variation in length of the side lines of adjacent quadrilaterals, and to quickly change the direction of the scheme.

(E) Five sided figure with four diagonals. This figure may be considered as a four sided central point figure with one diagonal, in which the central point falls outside the figure. It is used to afford a check when either a diagonal or a side line is obstructed.

This figure can often be used by the observing party when a side line of a quadrilateral is found to be obstructed.

(F) Five sided figure with three diagonals. This figure is similar to the four-sided central-point figure, (C), except that the central point falls outside the figure.

(G) Five sided central-point figure with two diagonals. This figure is an overlap of a central-point quadrilateral and a simple quadrilateral, and is the most complicated figure employed. It has been used to carry the scheme over difficult or convex areas. This figure can generally be made very strong.

(H) Five and six sided central point figures without diagonals. Any polygon with a central point, having separate chains of triangles on either side of the central point, will give a double determination of length, since it is permissible to carry the two lengths through the same triangle provided different combinations of distance angles are employed [3].

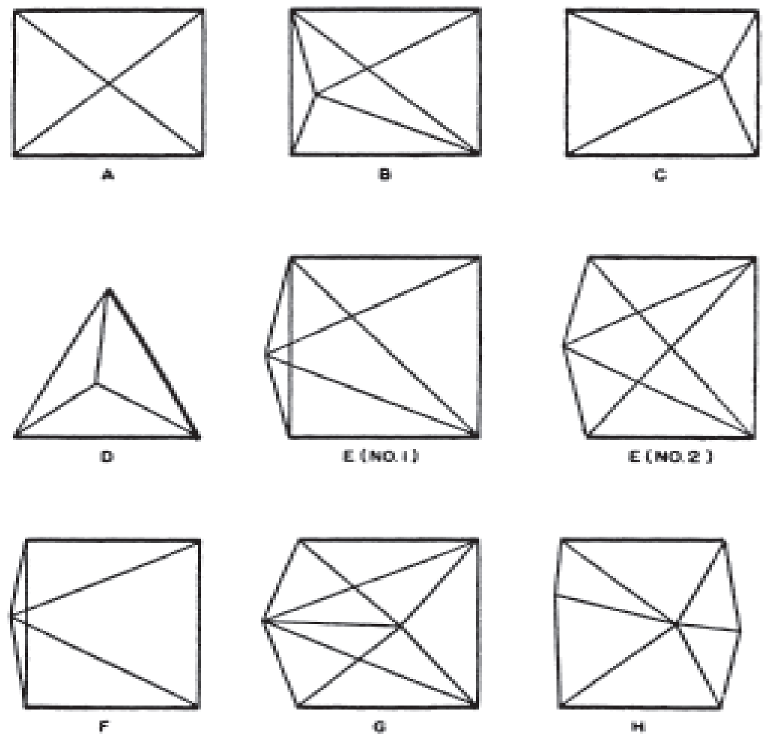

Figure (2.1): type of figures used in triangulation and strength factors

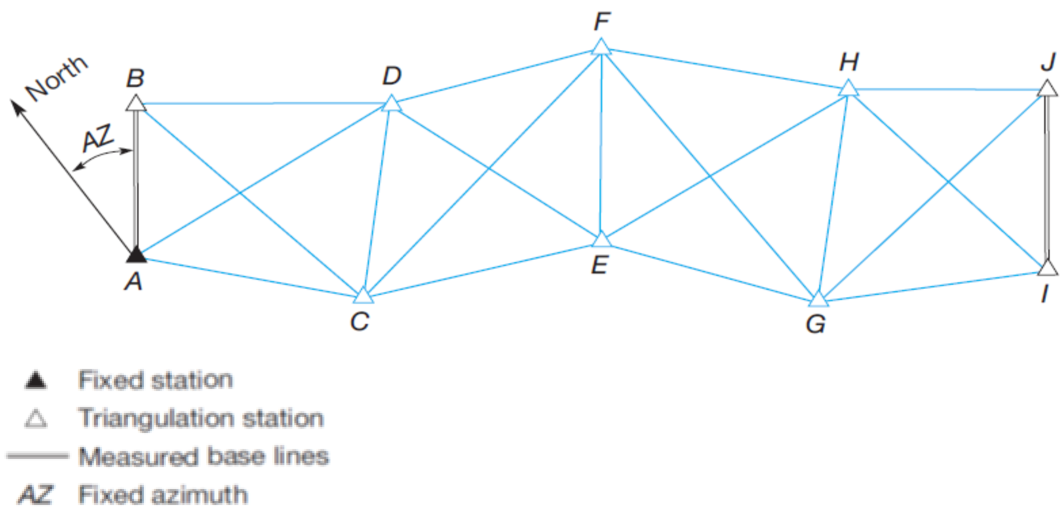

Figure (2.2): Chain of quadrilaterals

2.4 Classification:

In the design stage of a geodetic network one has to decide on its configuration, that is the point location and the types of observations, and on the distribution of observational work respectively the precision of the measurements. In the adjustment stage one has to decide on an optimal datum, using all available information of a relative and an absolute nature. This is valid, not only for new planned networks, but also when existing networks are extended. The different optimization problems are usually classified into different orders, a classification which has provided useful in the last years in despite of some weaknesses.

The datum problem is a search for an optimal datum or coordinate system and is called the zero-order design problem. The first order design problem is to be understood as the configuration problem, where the positions of the points and the observation plan have to be optimized, provided that the precision of the observations is known a priori. The weight problem, which is the optimal distribution of observational work in a fixed configuration, is called the second-order design problem. A further class is the third-order design problem, which is defined as the optimal improvement of an existing network or an existing design by insertion of additional [1].

Points and/or additional observations. There are proposals to introduce a further class of design, in which for deformation networks the optimal time difference between the observation epochs has to be found.

But this proposal does not fit the hitherto existing classification which is obvious if one regards the free elements in the formula of a least-squares adjustment by variation of coordinates.

2.4.1 Zero Order Design:

Many problems in physical science involve the estimation of a number of unknown parameters which bear a linear (or linearized) relationship to a set of experimental data. The data may be contaminated by (systematic or random) errors, insufficient to determine the unknowns, redundant, or all of the above and consequently, questions as existence, uniqueness, stability, approximation and the physical description of the set of solutions are all of interest.

In econometrics, for instance, the problem of insufficient data is discussed under the heading of "multi collinearity" and the consequent lack of determinability of the parameters from the observations, is known there as the "identification problem". In geophysics, where the physical interpretation of an anomalous gravitational field involves deduction of the mass distribution which produces the anomalous field, there is a fundamental non uniqueness in potential field inversion, such that, for instance, even complete, perfect data on the earth's surface cannot distinguish between two buried spherical density anomalies having the same anomalous mass but different radii. Also in geodesy one is confronted with similar problems. In physical geodesy, for instance, the fact that the data are generally measured only at discrete points, leaves one with the problem of determining a continuous unknown function from a finite set of data. And in geometric geodesy the non-uniqueness in coordinate system definitions, plays a fundamental role when identifying, interpreting, qualifying and comparing results from geodetic network adjustments. All the above mentioned problems are very similar and even formally equivalent if they are described in terms of a linear model E{y} A x, with rank mx1 mxn nx1 A < n. And these problems of solving systems of linear equations with arbitrary size and degeneracy are readily handled via the concept of a generalized inverse [3,4].

2.4.2 First Order Design:

The first-order triangulation chain is a national primary network, used to build a precise framework of a unified coordinate system throughout the country to control the establishment of the second- and lower-order triangulation networks and provide data for studying the size and shape of the Earth and geodynamics. Mapping control is not the direct objective—accuracy has more importance in this case.

The first-order triangulation chain runs along the meridian and the parallel as shown in Fig.2.3. The triangulation chain between the intersections is called the chain section; the circle formed by the east–west and north–south chain sections is called the chain loop; many chain loops form the chain system. The chain section is approximately 200 km long and is usually formed by single triangles and may also include some geodetic quadrilaterals or mid-point polygons. The average side length of triangles in the chain ranges from 20 to 25 km, any arbitrary angle of triangles is not less than 40°, and the distance angles of the geodetic quadrilaterals or mid-point polygons should be greater than 30°. Computed by the triangle closure, the mean square error of angle observation should not be greater than ∓ 0. 7”.

The initial side length at the crossing of the chain sections should be determined with a relative accuracy of no less than 1/350,000. The astronomical longitude, latitude, and azimuth are measured at the two endpoints of the initial side and the former two are also measured at a point in the center of the chain. The determined mean square error of the astronomical longitude, latitude, and azimuth should be less than ∓0.3’’, ∓0.3”, and ∓0.5”, respectively. All points with measured astronomical longitude and latitude will provide data for computation of the deflection of vertical. As astronomical surveying is involved in the plans for network establishment the national horizontal control network is also called the astrogeodetic network [1,3].

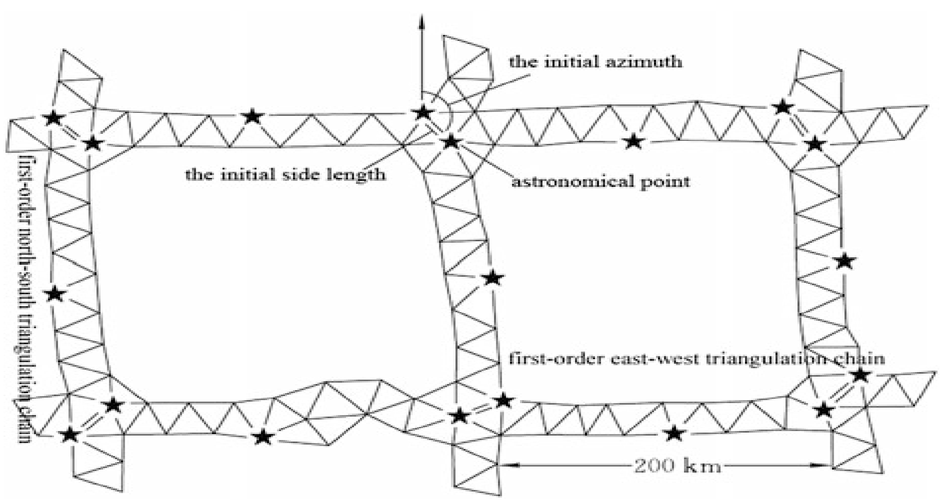

Figure (2.3): First-order triangulation chain with astronomical points

2.4.3 Second Order Design:

Set within the area circled by the first-order triangulation chain loop, the second order network is the overall basis for densification of the third- and fourth-order networks, as shown in Fig. 2.4. The average side length of the second-order network is 13 km and the density of such a network basically satisfies the needs of the 1:50,000 scale mapping. The second-order network, together with the first-order chain, belongs to the national high-order network. Hence, accuracy should be the primary concern whereas density is secondary. The mean square error of angle observation computed through the triangle closure should be less than 100. An initial side and azimuth are to be determined at the center of the network. For larger chain loops, the initial azimuth should be measured as well. Angles of triangles in the network should be no less than 30. The second-order network on either side of the first-order triangulation chain should be connected with the first-order chain to form a continuous triangulation network.

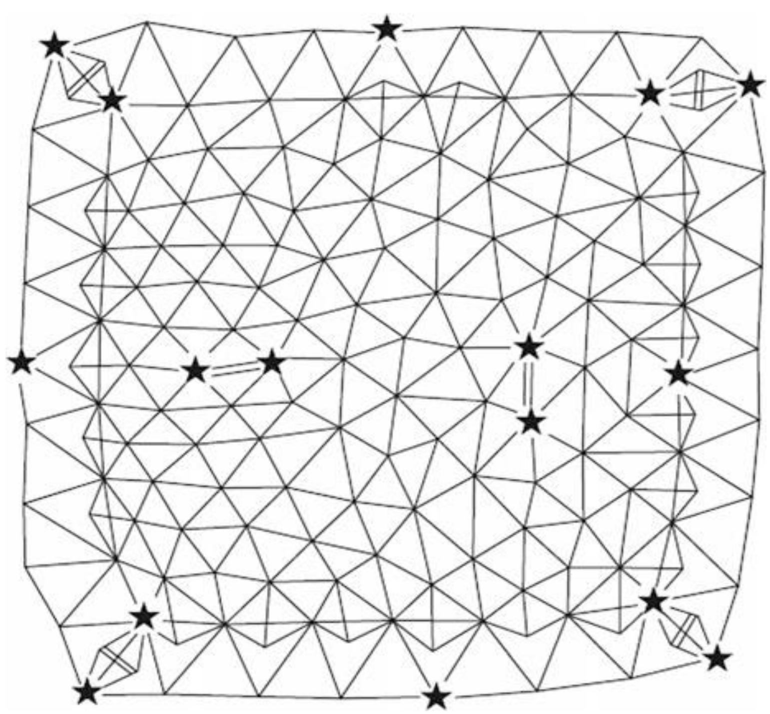

Figure (2.4): Second-order continuous network

2.4.4 Third Order Design:

National third- and fourth-order triangulation networks (points) can be further densified on the basis of the second-order network, as illustrated in Figs. 2.5 and 2.6. They are foundational to the mapping control survey and their density should accord with the mapping scale. The average side length of the third-order triangulation network is 8 km and the controlling area of each point is roughly 50 km^2, which can basically meet the needs of 1:25,000 scale mapping. The average side length of the fourth-order network is 4 km and the controlling area of each point is around 20 km2, which can meet the needs of 1: 10,000 and 1: 50,000 scale mapping. At each point of the third- and fourth-order networks there will be stations set for observation. The mean square error of angle observation computed through triangle closure should be less than $\mp$1.8" and $\mp$2.5" for the third- and fourth-order networks, respectively[4].

Note: The Sudan triangulation network is constraining of first, second and third order only.

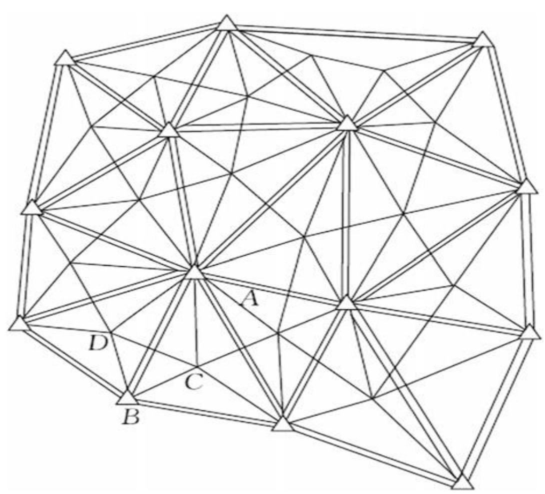

Figure (2.5): Network densifying through point inserting

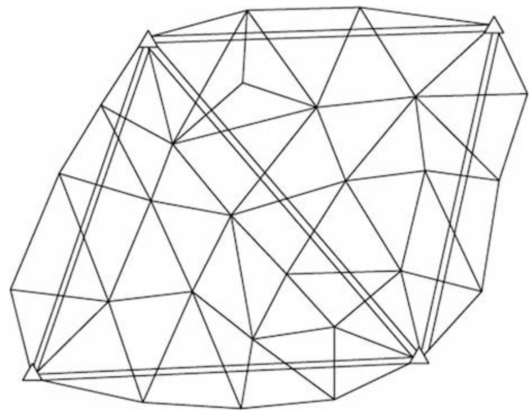

Figure (2.6): Network densifying through sub-network inserting

2.5 Numerical Methods in Network Design

Although the history of computer aided optimization of geodetic networks is rather short. probably less than twenty years. a vast number of different design strategies have been devised. All the practically useful methods share the disadvantage of needing to use expensive (in terms of computer storage and number of computer operations) numerical techniques in order to obtain the desired solution. Clearly the economy (if not the effectiveness) of any design procedure is largely dependent upon the numerical methods employed.

2.5.1 Optimal Design Problems:

In most of the network design problems discussed in this paper it is assumed that the network quality requirements are known. i.e. the quality of the final product has been decided upon. The design procedure then involves solving for the optimum network. In other words, finding a set of observations that will satisfy these requirements with the minimum cost. Note that there are two other classes of optimal design problem that will not be addressed here in the same detail: given a specified cost find the network that will have the highest quality and given a maximum cost find the network that best approximates a specified quality. It is usual to consider the quality of a network under three headings:

(i) precision. (ii) reliability and (iii) accuracy.

which are measures of the sizes of random, gross and systematic errors that may be expected (with specified probabilities) in the final network. The author is not aware of any work that has been carried out in the third category. Most attention is paid to the first of these because the greatest success has been in designing networks to meet specified precision criteria. The reader should not deduce from this that precision is any more or less important than accuracy or reliability [12,3].

2.5.2 Network Design Strategies:

Virtually all of the design strategies developed to date can be considered as belonging to one of two categories: computer simulation or analytical design. In computer simulation a solution to the design problem is postulated and the design and cost criteria computed. Should either of these criteria not be fulfilled a new solution is postulated (usually by slightly altering the original postulate) and the criteria recomputed. The procedure is repeated until a satisfactory (unlikely to be the optimum) network is found. Decisions on which networks to postulate are usually made manually by a skilled geodesist and are based on his past experience of the quality and cost of particular network configurations. Most practical computer aided design of geodetic networks is carried out in this way.

In contrast the so-called "analytical" methods offer specific algorithms for the solution of particular design problems. Once set in motion such an algorithm will automatically produce a network that will satisfy the user quality requirements and that will, in some mathematical sense, be optimum. So far almost all of the advances in analytical methods have been in finding solutions of the second order design problem is defined in Grafarend (1974) to be the determination of the observation weights given the required precision and proposed network configuration. The results can often be used to aid the solution of the first and third order problems (determination of network configuration and additional observations) by deciding not to make observations which are required with extremely low precision (on the grounds that they hardly contribute to the network precision) but care must be taken to maintain reliability [3,12].

Chapter Three
Adjustment for Triangulation network

3.1 Introduction:

Horizontal surveys, those are covering a large area, must compute the symmetric effect for the Earth curvature. One way this can be accomplished is to do the computations using coordinates from a mathematically rigorous map projection system such as the plane coordinate system (SPCS), universal transverse Mercator (UTM) system, or a local plane coordinate system that accounts rigorously for Earth curvature [4].

Three-dimensional geodetic network adjustment is developed for traditional surveying observations, including differential leveling, slant distances, and vertical angles.

It should be noted that if plane coordinates are used, the numbers are usually rather large. Consequently, when they are used in mathematical computations, errors due to rounding and truncation can occur. This can be prevented by translating the origin of the coordinates prior to adjustment, a process that involves simply subtracting a constant value from all coordinates. Then after the adjustment is finished, the true origin is restored by adding the constants to the adjusted values.

Horizontal surveys are performed for the purpose of determining precise relative horizontal positions of points. They have traditionally been accomplished by trilateration, triangulation, and traverse. These traditional types of surveys involve making distance, direction, and angle observations. As with all types of surveys, errors will occur in making these observations, and thus they must be analyzed and, if acceptable, adjusted [4,5,12].

3.2 Distance Observation Equation:

In adjusting trilateration surveys using the parametric least squares method, observation equations are written that relate the observed quantities and their inherent random errors to the most probable values for the x and y coordinates (the parameters) of the stations involved. the following distance equation can be written for any observation l_{ij}:

$$l_{ij} + v_i = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{3.1}$$

In Equation (3.1), l_{ij} is the observed distance of a line between stations I and J, v_i the residual in the observation l_{ij} , x_i and y_i the most probable coordinate values for station I, and x_j and y_j the most probable coordinate values for station J.Equation (3.1) is a nonlinear function involving the unknown parameters x_i, y_i , x_j , and y_j , which can be rewritten as:

$$f(x_i , y_i , x_j , y_j) = l_{ij} + v_i \tag{3.2}$$

Where

$$f(x_i , y_i , x_j , y_j) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

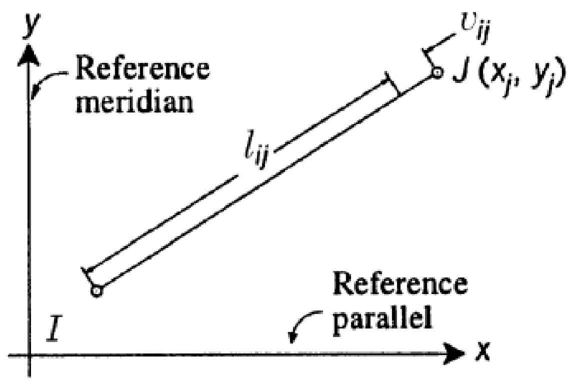

Figure (3.1): Observation of a distance

Equation (3.2) can be linearized and solved using a first-order Taylor series approximation. The linearized form of Equation (3.2) is:

$$F(x_i , y_i , x_j , y_j) = F(x_{i0} , y_{i0} , x_{j0} , y_{j0}) + \left(\frac{\partial F}{\partial x_i}\right) dx_i + \left(\frac{\partial F}{\partial y_i}\right) dy_i + \left(\frac{\partial F}{\partial x_j}\right) dx_j + \left(\frac{\partial F}{\partial y_j}\right) dy_j = l_{ij} + v_i \tag{3.3}$$

where $(\partial F/\partial x_i)_0$, $(\partial F/\partial yi)_0$, $(\partial F/\partial x_j)_0$, and $(\partial F/\partial y_j)_0$ are the partial derivatives of F with respect to x_i, y_i , x_j , and y_j , respectively, evaluated with the approximate coordinate values x_{i0}, y_{i0} , x_{j0} , and y_{j0}; x_i, y_i , x_j , and y_j the unknown parameters; and dx_i , dy_i , dx_j , and dy_j the corrections to the approximate coordinate values such that

$$x_i = x_{i0} + dx_i \qquad y_i = y_{j0} + dy_i \tag{3.4}$$
$$x_j = x_{j0} + dx_j \qquad y_j = x_{j0} + dy_j$$

The evaluation of partial derivatives is straightforward and will be illustrated with $\partial F/\partial xi$. Equation (3.2) can be rewritten as

$$F\left(x_i, y_i, x_j, y_j\right) = \left[\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2\right]^{1/2} \tag{3.5}$$

Taking the derivative of Equation (3.5) with respect to x_i yields:

$$\frac{\partial F}{\partial x_i} = \frac{1}{2}\left[\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2\right]^{-1/2}\left[2\left(x_j - x_i\right)(-1)\right] \tag{3.6}$$

Simplifying Equation (3.6) yields

$$\frac{\partial F}{\partial x_i} = \frac{-(x_j-x_i)}{\sqrt{(x_j-x_i)^2+(y_j-y_i)^2}} = \frac{x_j-x_i}{D_{ij}} \tag{3.7}$$

Employing the same procedure, the remaining partial derivatives are

$$\frac{\partial F}{\partial y_i} = \frac{y_j-y_i}{D_{ij}} \qquad \frac{\partial F}{\partial x_j} = \frac{x_j-x_i}{D_{ij}} \qquad \frac{\partial F}{\partial x_j} = \frac{y_j-y_i}{D_{ij}} \tag{3.8}$$

If Equations (3.7) and (3.8) are substituted into Equation (3.3) and the results substituted into Equation (3.2), the following prototype linearized distance observation equation obtained is [5,6]

$$\left(\frac{y_i-y_j}{D_{ij}}\right)_0 dx_i + \left(\frac{x_i-x_j}{D_{ij}}\right)_0 dy_i + \left(\frac{x_j-x_i}{D_{ij}}\right)_0 dx_j + \left(\frac{y_j-y_i}{D_{ij}}\right)_0 dy_j = k_i + v_i \tag{3.9}$$

where $(\cdot)_0$ is evaluated at the approximate parameter values, $k_l = l_{ij} - D_{ij}$, and

$$D_{ij} = f\left(x_i, y_i, x_j, y_j\right) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

3.2.1 Trilateration:

Even though the geometric figures used in trilateration are many and varied, they are equally adaptable to the observation equation method in a parametric adjustment. The distance from three station with known coordinate for points (A, B, C) to the unknown point (Z), since the unknown point has two unknown coordinate and there are three observations, this yields one redundant observation that is, the coordinates of station (Z) could be determined using any two of the three observations. But all three observations can be used simultaneously and adjusted by the method of least squares to determine the most probable value for the coordinates of the station [5,6].

The observation equations are developed by substituting into prototype equation (3.9). For example, the equation for distance AZ is formed by interchanging subscript I with A and subscript J with Z in Equation (3.9). In a similar fashion, an equation can be created for each observed distance using the following subscript substitutions:

I	J
A	Z
B	Z
C	Z

When one end of the observed line is a control station, its coordinates are fixed, and thus those terms can be dropped in prototype equation (3.9). This can be thought of as setting the dx and dy corrections for the control station equal to zero. In this example, station Z always takes the position of J in the prototype equation, and thus only the coefficients corresponding to dx_j and dy_j are used. Using the appropriate substitutions, the following three linearized observation equations result:

$$\begin{aligned} \frac{x_{z0}-x_A}{D_{AZ}}dx_z+\frac{y_{z0}-y_A}{D_{AZ}}dy_z &= (l_{AZ}-D_{AZ})+v_{AZ} \\ \frac{x_{z0}-x_B}{D_{BZ}}dx_z+\frac{y_{z0}-y_B}{D_{BZ}}dy_z &= (l_{BZ}-D_{BZ})+v_{BZ} \\ \frac{x_{z0}-x_C}{D_{CZ}}dx_z+\frac{y_{z0}-y_C}{D_{CZ}}dy_z &= (l_{CZ}-D_{CZ})+v_{CZ} \end{aligned} \qquad (3.10)$$

In Equation (3.10)

$$D_{AZ} = \sqrt{(x_{z0} - x_A)^2 + (y_{z0} - y_A)^2}$$

$$D_{BZ} = \sqrt{(x_{z0} - x_B)^2 + (y_{z0} - y_B)^2}$$

$$D_{CZ} = \sqrt{(x_{z0} - x_C)^2 + (y_{z0} - y_C)^2}$$

l_{AZ} , l_{BZ} , and l_{CZ} are the observed distances; the v's are residuals; x_{z0} and y_{z0} and are initial coordinate values for station Z . Equations (3.10) can be expressed in matrix form as:

$$\boldsymbol{JX} = \boldsymbol{K} + \boldsymbol{V} \tag{3.11}$$

where J is the Jacobian matrix of partial derivatives, X the matrix of unknown corrections dx_z and dy_z, K the matrix of constants (i.e., the observed lengths, minus their corresponding lengths computed from the initial approximate coordinates), and V the residual matrix. Equation (3.11) in expanded form is:

$$\begin{bmatrix} \frac{x_{z0}-x_A}{D_{AZ}} & \frac{y_{z0}-y_A}{D_{AZ}} \\ \frac{x_{z0}-x_B}{D_{BZ}} & \frac{y_{z0}-y_B}{D_{BZ}} \\ \frac{x_{z0}-x_C}{D_{CZ}} & \frac{y_{z0}-y_C}{D_{CZ}} \end{bmatrix} \begin{bmatrix} dx_z \\ dy_z \end{bmatrix} = \begin{bmatrix} l_{AZ} - D_{AZ} \\ l_{BZ} - D_{BZ} \\ l_{CZ} - D_{CZ} \end{bmatrix} + \begin{bmatrix} v_{AZ} \\ v_{BZ} \\ v_{CZ} \end{bmatrix} \tag{3.12}$$

The Jacobian matrix can systematically be formed using the following steps:
Step 1: Head each column with an unknown value.
Step 2: Create a row for every observation.
Step 3: Substitute in the appropriate coefficient corresponding to the column into each row.
Once Equation (3.12) is created, the corrections of dx_z and dy_z, and thus the most probable coordinate values, x_z and y_z can be computed using Equation

observation equation of course, to obtain the final adjusted values, the solution must be iterated [8,9].

Finally, to find the correction x, y coordinate can use the least square matrix to find the adjusted coordinate as equation:

$$x = (J^T J)^{-1} J^T K \tag{3.13}$$

3.3 Triangulation:

Prior to the development of electronic distance measuring equipment and the global navigation satellite systems, triangulation was the preferred method for extending horizontal control over long distances. The positions of widely spaced stations were computed from observed angles and a minimal number of observed distances called baselines. This method was used extensively by the National Geodetic Survey in extending much of the national network. Triangulation is still used by many surveyors in establishing horizontal control, although surveys that combine trilateration (distance observations) with triangulation (angle observations) are more common. In this chapter, methods are described for adjusting triangulation networks using the least squares method. A least squares triangulation adjustment can use condition equations or observation equations written in terms of either azimuths or angles. In this chapter the observation equation method is presented. The procedure involves a parametric adjustment where the parameters are coordinates in a plane rectangular system such as state plane coordinates. In the examples, specific types of triangulations known as intersections, resections, and quadrilaterals are adjusted [8,9].

3.3.1 Azimuth Observation Equation:

The azimuth equation in parametric form is

$$\text{azimuth} = \alpha + C \tag{3.14}$$

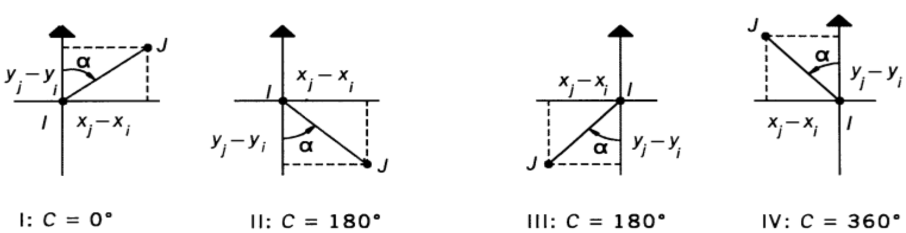

Figure (3.2): Relationship between the azimuth and the computed angle, α.

where $\alpha = \tan^{-1}\left[\frac{x_j - x_i}{y_j - y_i}\right]$, xi and y_i are the coordinates of the occupied station I, x_j and y_j are the coordinates of the sighted station J, and C is a constant that depends on the quadrant in which point J lies as shown in Figure (3.2).

3.3.2 Linearization of the Azimuth Observation Equation:

Referring to Equation (3.14), the observation equation for an observed azimuth of line IJ is:

$$\tan^{-1}\frac{x_j - x_i}{y_j - y_i} + C = D_{AZ} + v_{AZ} \qquad (3.15)$$

where D_{AZ} is the observed azimuth from station I to station J, vA_z the residual in the observed azimuth, x_i and y_i the most probable values for the coordinates of station I, x_j and y_j the most probable values for the coordinates of station J, and C. Equation (3.15) is a nonlinear function involving variables x_i, y_i , x_j , and y_j that can be rewritten as:

$$F\left(x_i , y_i , x_j , y_j\right) = D_{AZ} + v_{Az} \qquad (3.16)$$

Where

$$F\left(x_i , y_i , x_j , y_j\right) = \tan^{-1}\left[\frac{x_j - x_i}{y_j - y_i}\right] + C$$

nonlinear equations such as (3.16) can be linearized and solved using a first-order Taylor series approximation. The linearized form of Equation (3.16) is:

$$F\left(x_i , y_i , x_j , y_j\right) = F\left(x_i , y_i , x_j , y_j\right)_0 + \left(\frac{\partial F}{\partial x_i}\right) dx_i + \left(\frac{\partial F}{\partial y_i}\right) dy_i + \left(\frac{\partial F}{\partial x_j}\right) dx_j + \left(\frac{\partial F}{\partial y_j}\right) dy_j \qquad (3.17)$$

where $(\partial F/\partial x_i)_0$, $(\partial F/\partial yi)_0$, $(\partial F/\partial x_j)_0$, and $(\partial F/\partial y_j)_0$ are the partial derivatives of F with respect to x_i, y_i , x_j , and y_j that are evaluated at the initial approximations, and dx_i , dy_i , dx_j , and dy_j are the corrections applied to the initial approximations after each iteration such that:

$$
\begin{aligned}
x_i &= x_{i_0} + dx_i \\
y_i &= y_{i_0} + dy_i \\
x_j &= x_{j_0} + dx_j \\
y_j &= y_{j_0} + dy_j
\end{aligned}
\tag{3.18}
$$

To determine the partial derivatives of Equation (3.17) requires the prototype equation for the derivative of tan^{-1}u with respect to x, which is

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx} \tag{3.19}$$

Using Equation (3.19), the procedure for determining the ∂F/∂xi is demonstrated as follows:

$$
\begin{aligned}
\frac{\partial F}{\partial x_i} &= \frac{1}{1+\left[(x_j - x_i)/(y_j - y_i)\right]^2}\frac{-1}{y_j - y_i} \\
&= \frac{-1\,(y_j - y_i)}{(x_j - x_i)^2 + (y_j - y_i)^2}
\end{aligned}
\tag{3.20}
$$

By employing the same procedure, the remaining partial derivatives are:

$$\frac{\partial F}{\partial y_i} = \frac{y_j - y_i}{D_{ij}} \qquad \frac{\partial F}{\partial x_j} = \frac{x_j - x_i}{D_{ij}} \qquad \frac{\partial F}{\partial x_j} = \frac{y_j - y_i}{D_{ij}} \tag{3.21}$$

Where

$$D_{ij}{}^2 = (x_j - x_i)^2 + (y_j - y_i)^2$$

If Equations (3.19) and (3.20) are substituted into Equation (3.17) and the results then substituted into Equation (3.16), the following prototype azimuth equation is obtained:

$$\left(\frac{y_i - y_j}{D_{ij}{}^2}\right)_0 dx_i + \left(\frac{x_i - x_j}{D_{ij}{}^2}\right)_0 dy_i + \left(\frac{x_j - x_i}{D_{ij}{}^2}\right)_0 dx_j + \left(\frac{y_j - y_i}{D_{ij}{}^2}\right)_0 dy_j = k_{AZ} + v_{Az} \tag{3.22}$$

Both:

$$k_{AZ} = D_{AZ} - \left[tan^{-1}\left(\frac{x_j - x_i}{y_j - y_i}\right) + C\right] \quad and \qquad D_{ij}{}^2 = (x_j - x_i)^2 + (y_j - y_i)^2$$

are evaluated using the approximate coordinate values of the unknown parameters.

3.3.3 Angle Observation Equation:

Figure 3.3 illustrates the geometry for an angle observation. In the figure, B is the back sight station, F is the foresight station, and I is the instrument station. As shown in the figure, an angle observation equation can be written as the difference between two azimuth observations, and thus for clockwise angles:

$$\angle BIF = \mathrm{Az}_{IF} - \mathrm{Az}_{IB} = \tan^{-1}\frac{x_f - x_i}{y_f - y_i} - \tan^{-1}\frac{x_b - x_i}{y_b - y_i} + D = \theta_{bif} + \nu_\theta \quad (3.23)$$

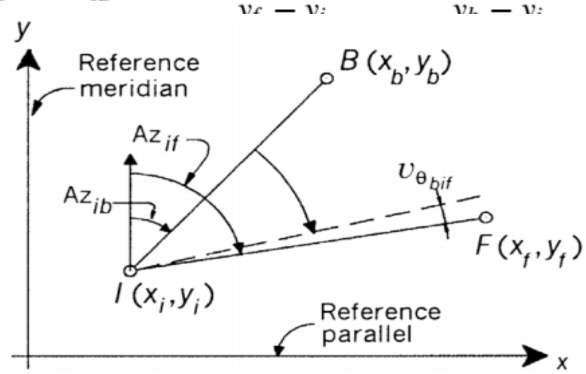

Figure (3.3): Relationship between an angle and two azimuths.

where θ_{bif} is the observed clockwise angle, ν_θ the residual in the observed angle, x_b and y_b the most probable values for the coordinates of the back sight station B, $x_i$and y_i the most probable values for the coordinates of the instrument station I, x_f and y_f the most probable values for the coordinates of the foresight station F, and D a constant that depends on the quadrants in which the back sight and foresight occur [6,9]. This term can be computed as the difference between the C terms from Equation (3.13) as applied to the back sight and foresight azimuths; that is,

$$D = C_{if} - C_{ib}$$

Equation (3.23) is a nonlinear function of x_b, y_b, x_i , y_i , x_f , and y_f that can be rewritten as:

$$F\left(x_b, y_b, x_i, y_i, x_f, y_f\right) = \theta_{bif} + v_\theta \quad (3.24)$$

Where:

$$F\left(x_b, y_b, x_i, y_i, x_f, y_f\right) = \tan^{-1}\frac{x_f - x_i}{y_f - y_i} - \tan^{-1}\frac{x_b - x_i}{y_b - y_i} + D$$

Equation (3.24) expressed as a linearized, first-order Taylor series expansion is

$$F\left(x_b, y_b, x_i, y_i, x_f, y_f\right) = F\left(x_b, y_b, x_i, y_i, x_f, y_f\right)_0 + \left(\frac{\partial F}{\partial x_b}\right)_0 dx_b + \left(\frac{\partial F}{\partial y_b}\right)_0 dy_b$$

$$+ \left(\frac{\partial F}{\partial x_i}\right)_0 dx_i + \left(\frac{\partial F}{\partial y_i}\right)_0 dy_i + \left(\frac{\partial F}{\partial x_f}\right)_0 dx_f + \left(\frac{\partial F}{\partial y_f}\right)_0 dy_f \quad (3.25)$$

where ∂F/∂x_b, ∂F/∂y_b, ∂F/∂x_i, ∂F/∂y_i, ∂F/∂x_f , and ∂F/∂x_f are the partial derivatives of F with respect to x_b, y_b, x_i , y_i , x_f , and x_f , respectively. Evaluating partial derivatives of the function F and substituting into Equation (3.25), then substituting into Equation (3.24), results in the following equation:

$$\left(\frac{y_i - y_b}{IB^2}\right)_0 dx_b + \left(\frac{x_b - x_i}{IB^2}\right)_0 dy_b + \left(\frac{y_b - y_i}{IB^2} - \frac{y_f - y_i}{IF^2}\right)_0 dx_i$$

$$+ \left(\frac{x_i - x_b}{IB^2} - \frac{x_i - x_f}{IF^2}\right)_0 dy_i + \left(\frac{y_f - y_i}{IF^2}\right)_0 dx_f + \left(\frac{x_i - x_f}{IF^2}\right)_0 dy_f = k_\theta + v_\theta \quad (3.26)$$

Where:

$$k_\theta = \theta_{bif} - \theta_{bif_0} \qquad \theta_{bif_0} = \tan^{-1}\left(\frac{x_f - x_i}{y_f - y_i}\right)_0 - \tan^{-1}\left(\frac{x_b - x_i}{y_b - y_i}\right)_0 + D$$

$$IB^2 = (x_b - x_i)^2 + (y_b - y_i)^2 \qquad IF^2 = \left(x_f - x_i\right)^2 + \left(y_f - y_i\right)^2$$

are evaluated at the approximate values for the unknowns. Formulating the linearized angle observation equation can be thought of as the difference in two linearized azimuth equations. Using Equation (3.20) as a guide, the difference between the foresight and back sight azimuth is:

$$\left(\frac{y_i - y_f}{IF^2}\right)_0 dx_i + \left(\frac{x_f - x_i}{IF^2}\right)_0 dy_i + \left(\frac{y_f - y_i}{IF^2}\right)_0 dx_f + \left(\frac{x_i - x_f}{IF^2}\right)_0 dy_f$$

$$- \left[\left(\frac{y_i - y_b}{IB^2}\right)_0 dx_i + \left(\frac{x_b - x_i}{IB^2}\right)_0 dy_i + \left(\frac{y_b - y_i}{IB^2}\right)_0 dx_b + \left(\frac{x_i - x_b}{IB^2}\right)_0 dy_b\right] = k_{bif} + v_\theta$$

In formulating the angle observation equation, remember that (I) is always assigned to the instrument station, B the back sight, and F the foresight station.

Chapter Four
Methodology

4.1 Introduction:

As mention before in chapter one the objective of this study to adjust the Triangulation network by three techniques and compare between these, many steps were followed which are summarized in four main steps as shown in the chart below (Figure 4.1). Step one is the data collection where the data has been acquired from the old triangulation network in Sudan. Step two is the transformation of the data from geodetic coordinates to projected coordinates. Step three is adjusting this triangulation using distance observation equation by three techniques and then in last step a comparison was made between these results and also a comparison is made with the original coordinates.

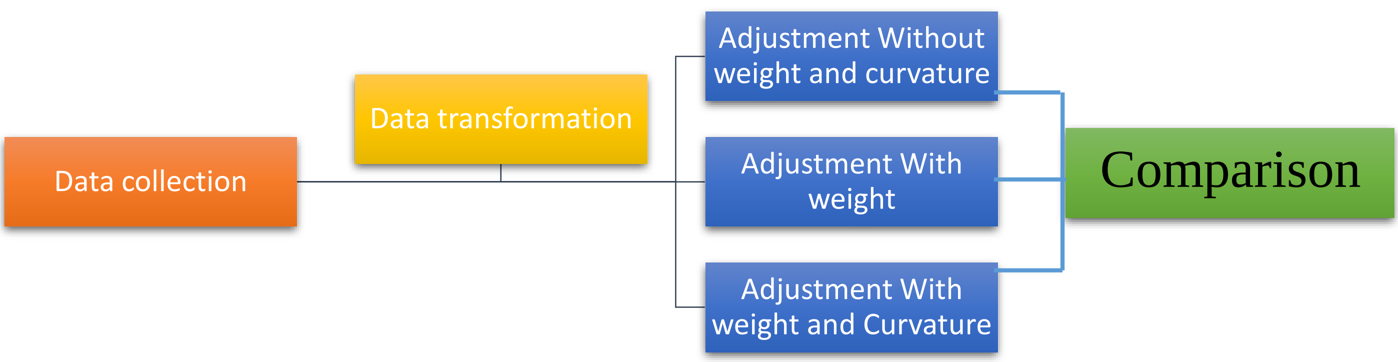

Figure (4.1) Main Steps for research

4.1.1 Data collection:

In 1952 the project was finished in Sudan and finally a triangulation network was built by "General Authority for Sudanese Survey" to cover all the Sudan by triangulating a network. This triangulation data was recorded so as to make an ideal reference for any further projects. This reference included many volumes, the first volume covers the field records and important reports besides detailed information about the history of the Thirties meridian triangulation. Volume two included the first and second order of triangulation for specific parts in Sudan. The data used in this research is from volume two and precisely in the first order triangulation. The Sabaloka base line was chosen beside the points around this base, the total number of points used is eleven points (G212, G213, G214, G215, G216, G217, G218, G219, G220, G221, G222), the Coordinates of these points are illustrated in table (4.1) [7].

Table (4.1): Study points' coordinates.

Name	φ	λ
G212	15°58'40.150"	32°14'49.944"
G213	15°42'58.189"	32°24'48.930"
G214	16°10'30.625"	32°35'55.768"
G215	15°54'9.846"	32°41'36.253"
G216	16°14'52.314"	32°40'38.867"
G217	16°14'31.862"	32°40'51.838"
G218	16°9'49.097	32°45'20.063"
G219	16°18'44.205"	32°49'7.664"
G220	16°06'15.071"	32°52'55.940"
G221	16°5'33.434"	33°2'19.769"
G222	16°20'25.432"	33°10'57.319"

4.1.2 Data transformation:

The original data from the old triangulation network in Sudan is in projected coordinates system and to adjust this triangulation; a transformation must be followed to get the results in geodetic coordinates system. For the Transverse Mercator projection, the spheroid using in this study is Clarke 1880s, Adindan (Sudan) is the datum using in this study and zone 36 in north, the ellipsoidal points φ, λ have been mapped into E, N points through by the following series expansions [4,5]:

$$\begin{aligned} y =\; & B(\varphi) + \frac{t}{2} N \cos^2\varphi\, \ell^2 + \frac{t}{24} N \cos^4\varphi\,(5 - t^2 + 9\eta^2 + 4\eta^4)\,\ell^4 \\ & + \frac{t}{720} N \cos^6\varphi\,(61 - 58\,t^2 + t^4 + 270\,\eta^2 - 330\,t^2\eta^2)\,\ell^6 \\ & + \frac{t}{40\,320} N \cos^8\varphi\,(1385 - 3111\,t^2 + 543\,t^4 - t^6)\,\ell^8 + \ldots \end{aligned} \tag{4.1}$$

$$\begin{aligned} x =\; & N \cos\varphi\, \ell + \frac{1}{6} N \cos^3\varphi\,(1 - t^2 + \eta^2)\,\ell^3 \\ & + \frac{1}{120} N \cos^5\varphi\,(5 - 18\,t^2 + t^4 + 14\,\eta^2 - 58\,t^2\eta^2)\,\ell^5 \\ & + \frac{1}{5040} N \cos^7\varphi\,(61 - 479\,t^2 + 179\,t^4 - t^6)\,\ell^7 + \ldots \end{aligned} \tag{4.2}$$

4.1.3 Adjustment by distance observation equation:

As mentioned in chapter three, in geometry, distance observation equation is the process of determining absolute or relative locations of points by measuring distances, using the geometry of circles, spheres or triangles.

In this research the distance observation equation has been used to adjust the triangulation network which contains eleven points as mentioned above, the adjustment was made for each point from the observed point to a specific point, for example the distances between the point G212 and the points (G213, G214, G215) were observed, and these distances were used to adjust the point G212.

The approximate coordinates for each points they use the coordinates from the triangulation network original data, and use this coordinates to compute the plan distance by equation (4.2), and the observed distance, it given from the triangulation data.

$$L_{AZ} = \sqrt{(x_A - x_Z)^2 + (y_A - y_Z)^2} \tag{4.2}$$

Hence, the coordinates and the distance were used in equation (4.3) to compute the Jacobian matrix (4.4), after that, the observed distance was used to compute the distance and to find the (K) in equation (3.9).

$$\frac{x_Z - x_A}{D_{AZ}} \tag{4.3}$$

$$J = \begin{bmatrix} \frac{x_{z0} - x_A}{D_{AZ}} & \frac{y_{z0} - y_A}{D_{AZ}} \\ \frac{x_{z0} - x_B}{D_{BZ}} & \frac{y_{z0} - y_B}{D_{BZ}} \\ \frac{x_{z0} - x_C}{D_{CZ}} & \frac{y_{z0} - y_C}{D_{CZ}} \end{bmatrix} \tag{4.4}$$

Finally, by using Jacobian matrix and (K) the residual was found and used to get the coordinates using the least square in distance observation equation as shown in equation (4.5)

$$x = (J^T J)^{-1} J^T K \tag{4.5}$$

To process all these points in distance observation equation it was ideal to use the MATLAB software, it makes three codes for the distance observation equation; one without a weight and curvature, distance observation equation with weight and also a distance observation equation with weight and curvature.

4.1.4 Adjustment by distance observation equation with the weight:

The first process used the least square of distance observation equation without a weight (4.5), the next process will insert the weight matrix to the equation as illustrated in equation (4.6):

$$x = (J^T WJ)^{-1} J^T WK \tag{4.6}$$

The weight matrix is variance covariance matrix where the diagonal is the variance and the covariance represent another element on the matrix, this covariance is zero because there is no relation between a distance and another, and the variance would be the inverse of length between the two points as described in equation (4.7):

$$\sigma^2 = \frac{1}{D_{AZ}} \tag{4.7}$$

The length (IJ) used in this equation is the observed distance.

4.1.5 Adjustment by distance observation equation with weight and curvature:

Surveyors usually measure distances along a sloping path between two ground stations, then they correct these observed distances to one with the same curvature as the ellipsoid at the mean sea level (Geoid). Correction for refraction, curvature, slope, and height above the mean sea level are usually applied.

The distance between points in the triangulation was more than three kilometers, and verily for such long distance an equation of the curvature (4.8) must be used to correct the earth curve.

$$S = s' - \frac{NS'}{R} \tag{4.8}$$

Where R is the average radius of curvature along the line, using the earth radius for Adindan (Sudan) is 6378249.145m, N is the separation between the ellipsoid and the Geoid, it is supposed (1) in all points, (S) is the ellipsoid distance and the (S’) is the geodetic distance, the geodetic distance is supposing as the observed distance.

Then the curvature distance has been used instead of the observed distance in the least square on the distance observation equation.

4.1 Comparison:

Ultimately, the comparison between the three techniques was made and origin data of the distance observation equation to find which on is more accurate. The results chapter will discuss the comparison between these techniques.

Chapter Five
Computation and Results

5.1 Data collection:

As mentioned in chapter four the data was collected from the "Sudan surveying department" for first order triangulation, the data contain the distance from each observed points, table (5.1) illustrative those observations and the computed distance from the equation (4.3).

Table (5.1): observation and computed data.

From	**To**	**Distance (Meters) From log**	**Computed distance**
G 212	G 213	33997.901	33986.452
	G 214	43500.084	43484.802
	G 215	48490.627	48473.264
G 213	G 212	33997.901	33997.538
	G 214	54523.798	54503.886
	G 215	36398.702	36385.197
G 214	G 212	43500.084	43499.539
	G 213	54523.798	54518.301
	G 215	31799.193	31787.052
	G 216	11636.005	11631.573
	G 217	11502.525	11498.153
	G 218	16812.179	16805.700
	G 219	27985.231	27974.403
	G 220	31312.230	31300.039
G 215	G 212	48490.627	48491.438
	G 213	36398.702	36401.632
	G 214	31799.193	31795.745
	G 216	38226.919	38212.169
	G 220	30087.327	30075.513
G 216	G 214	11636.005	11634.638
	G 215	38226.919	38221.571
	G 218	12514.670	12509.790
	G 219	16703.578	16697.051
	G 220	27058.754	27048.120
G 217	G 214	11502.525	11501.904
	G 218	11790.222	11785.626
	G 219	16639.865	16633.356
G 218	G 214	16812.179	16815.961
	G 216	12514.670	12516.999
	G 217	11790.222	11790.772
	G219	17782.135	17775.149

From	To	Distance (Meters) From log	Computed distance
G 219	G 214	27985.231	27979.495
	G 216	16703.578	16694.290
	G 217	16639.865	16636.215
	G 218	17782.135	17780.913
	G 220	24003.369	23993.843
	G 221	33827.523	33814.015
	G 222	38999.694	38984.156
G 220	G 214	31312.230	31311.038
	G 215	30087.327	30084.343
	G 216	27058.754	27054.106
	G 219	24003.369	24004.367
	G 221	16804.589	16797.859
	G 222	41407.842	41391.324
G 221	G 219	33827.523	33823.617
	G 220	16804.589	16801.158
	G 222	31432.050	31419.536
G 222	G 219	38999.694	38986.904
	G 220	41407.842	41396.133
	G 221	31432.050	31429.109

Figuer (5.1) illustrates the shape of the triangulation and points' distribution with the observed lines between those points.

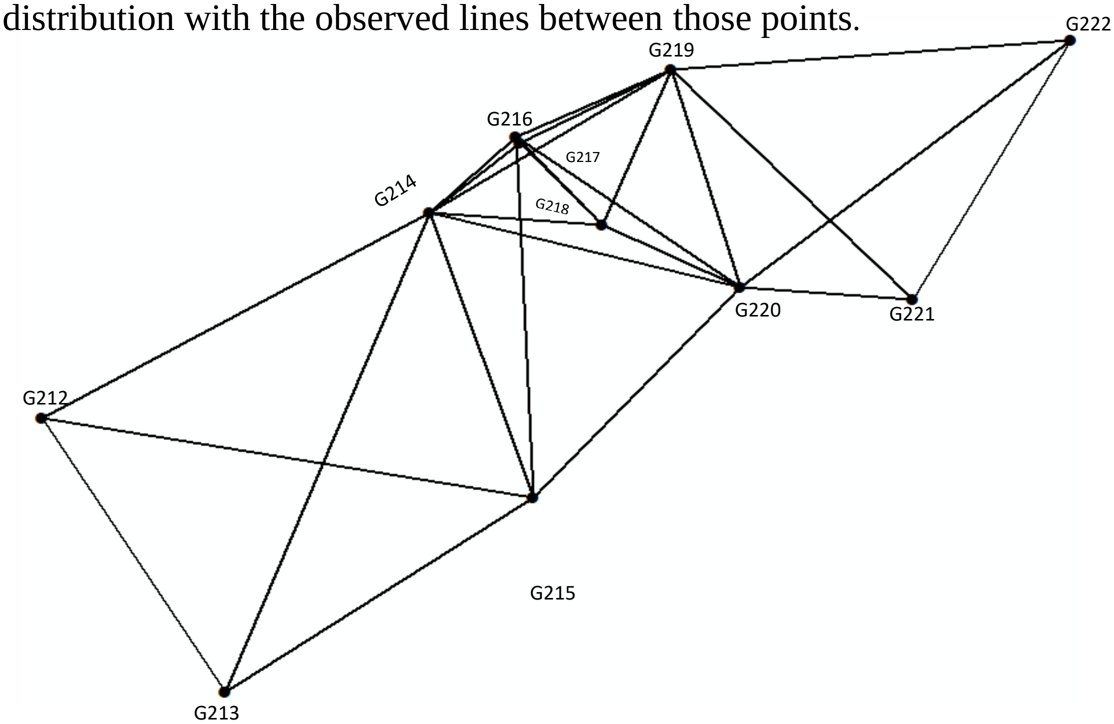

Figure (5.1): The shape of triangulations with observation.

5.2 Data transformation:

The original data was in geographical coordinates and it had to be transformed into projected coordinates (UTM) by the equations (4.1) and (4.2), table (5.2) represents the result of the transformation.

Table (5.2): The transformation of coordinate from geographic to projected coordinate

Name	$lat(\varphi)$	$Lon(\lambda)$	E	N
G212	15°58'40.150"	32°14'49.944"	419444.850	1766471.926
G213	15°42'58.189"	32°24'48.930"	437169.147	1737473.163
G214	16°10'30.625"	32°35'55.768"	457113.959	1788196.709
G215	15°54'9.846"	32°41'36.253"	467180.103	1758045.599
G216	16°14'52.314"	32°40'38.867"	465533.222	1796222.263
G217	16°14'31.862"	32°40'51.838"	465917.274	1795593.275
G218	16°9'49.097	32°45'20.063"	473869.131	1786894.483
G219	16°18'44.205"	32°49'7.664"	480642.600	1803328.474
G220	16°06'15.071"	32°52'55.940"	487403.257	1780306.788
G221	16°5'33.434"	33°2'19.769"	504152.090	1779024.329
G222	16°20'25.432"	33°10'57.319"	519502.480	1806438.792

5.3 Adjustment by distance observation equation:

The adjustment was made by using distance observation equation, in first technique. The results displayed in the table (5.3) with the origin coordinates and the residual between the adjusted coordinates and the origin coordinates.

Table (5.3): The adjustment coordinates by distance observation equation.

Point ID	E	N	Adjusted E	Adjusted N	Residual E	Residual N
G212	419444.850	1766471.926	419426.793	1766473.751	18.057	-1.826
G213	437169.147	1737473.163	437156.225	1737461.721	12.922	11.442
G214	457113.959	1788196.709	457103.968	1788204.003	9.991	-7.294
G215	467180.103	1758045.599	467177.301	1758036.265	2.801	9.334
G216	465533.222	1796222.263	465530.549	1796229.128	2.673	-6.865
G217	465917.274	1795593.275	465912.396	1795596.044	4.878	-2.769
G218	473869.131	1786894.483	473863.468	1786890.319	5.663	4.164
G219	480642.600	1803328.474	480637.106	1803339.044	5.494	-10.570
G220	487403.257	1780306.788	487399.449	1780302.984	3.808	3.804
G221	504152.090	1779024.329	504150.371	1779014.263	1.718	10.066
G222	519502.480	1806438.792	519514.701	1806437.314	-12.220	1.478

5.4 Adjustment by distance observation equation with weight:

The second technique of adjustment, using distance observation equation with adding the weight in the equation and the result for this process it will shown in table (5.4) with the residual between this adjustment coordinate and the origin coordinate.

Table (5.4): the adjustment coordinate by distance observation equation with weight.

Point ID	*E*	*N*	Adjusted *E*	Adjusted *N*	Residual *E*	Residual *N*
G212	419444.850	1766471.926	419426.733	1766473.841	18.117	-1.916
G213	437169.147	1737473.163	437156.403	1737462.685	12.744	10.478
G214	457113.959	1788196.709	457104.125	1788202.590	9.834	-5.881
G215	467180.103	1758045.599	467177.456	1758036.074	2.647	9.525
G216	465533.222	1796222.263	465532.171	1796230.991	1.051	-8.728
G217	465917.274	1795593.275	465912.618	1795595.991	4.656	-2.716
G218	473869.131	1786894.483	473863.531	1786890.557	5.599	3.926
G219	480642.600	1803328.474	480639.049	1803338.399	3.550	-9.925
G220	487403.257	1780306.788	487399.700	1780303.541	3.557	3.247
G221	504152.090	1779024.329	504152.404	1779013.182	-0.314	11.146
G222	519502.480	1806438.792	519516.647	1806435.789	-14.167	3.003

5.5 Adjustment by distance observation equation with weight and Curvature:

The last technique of adjustment by distance observation equation. Bending will be compute the curvature in the computed distance, then the computed distance will be replaced by the curvature distance in each point, the table (5.6) show the new adjusted coordinates for the points. And table (5.5) illustrates the curvature distance.

Table (5.6): the adjustment coordinate by distance observation equation with weight and curvature

Point ID	*E*	*N*	Adjusted *E*	Adjusted *N*	Residual *E*	Residual N
G212	419444.850	1766471.926	419426.725	1766473.842	18.125	-1.917
G213	437169.147	1737473.163	437156.398	1737462.681	12.749	10.482
G214	457113.959	1788196.709	457104.121	1788202.593	9.837	-5.883
G215	467180.103	1758045.599	467177.454	1758036.070	2.648	9.529
G216	465533.222	1796222.263	465532.168	1796230.993	1.053	-8.730
G217	465917.274	1795593.275	465912.616	1795595.992	4.657	-2.717
G218	473869.131	1786894.483	473863.529	1786890.555	5.602	3.928
G219	480642.600	1803328.474	480639.047	1803338.403	3.552	-9.929
G220	487403.257	1780306.788	487399.699	1780303.540	3.559	3.248
G221	504152.090	1779024.329	504152.404	1779013.178	-0.314	11.150
G222	519502.480	1806438.792	519516.654	1806435.787	-14.174	3.005

Table (5.5) the curvature distance points

From	To	Computed distance	Computed distance with curvature
G 212	G 213	33986.452	33986.446
G 212	G 214	43484.802	43484.795
G 212	G 215	48473.264	48473.256
G 213	G 212	33997.538	33997.532
G 213	G 214	54503.886	54503.877
G 213	G 215	36385.197	36385.191
G 214	G 212	43499.539	43499.533
G 214	G 213	54518.301	54518.293
G 214	G 215	31787.052	31787.047
G 214	G 216	11631.573	11631.571
G 214	G 217	11498.153	11498.151
G 214	G 218	16805.700	16805.698
G 214	G 219	27974.403	27974.399
G 214	G 220	31300.039	31300.034
G 215	G 212	48491.438	48491.430
G 215	G 213	36401.632	36401.627
G 215	G 214	31795.745	31795.740
G 215	G 216	38212.169	38212.163
G 215	G 220	30075.513	30075.508
G 216	G 214	11634.638	11634.637
G 216	G 215	38221.571	38221.565
G 216	G 218	12509.790	12509.788
G 216	G 219	16697.0518	16697.049
G 216	G 220	27048.120	27048.116
G 217	G 214	11501.904	11501.902
G 217	G 218	11785.626	11785.624
G 217	G 219	16633.356	16633.353
G 218	G 214	16815.961	16815.958
G 218	G 216	12516.999	12516.997
G 218	G 217	11790.772	11790.770
G 218	G 219	17775.149	17775.146
G 219	G 214	27979.495	27979.490
G 219	G 216	16694.290	16694.287
G 219	G 217	16636.215	16636.212
G 219	G 218	17780.913	17780.910
G 219	G 220	23993.843	23993.840
G 219	G 221	33814.015	33814.009
G 219	G 222	38984.156	38984.149
G 220	G 214	31311.038	31311.033
G 220	G 215	30084.343	30084.339
G 220	G 216	27054.106	27054.102
G 220	G 219	24004.367	24004.363
G 220	G 221	16797.859	16797.856
G 220	G 222	41391.324	41391.318
G 221	G 219	33823.617	33823.612

From	To	Computed distance	Computed distance with curvature
G 221	G 220	16801.158	16801.156
G 221	G 222	31419.536	31419.532
G 222	G 219	38986.904	38986.898
G 222	G 220	41396.133	41396.126
G 222	G 221	31429.109	31429.104

5.6 Comparison between the three types:

The comparison between the distance observation equation adjustment techniques by according of origin coordinates data. Using the residual to comparison in each points between the three techniques, the table (5.6) shows the residual of coordinates for techniques in X Axis, and table (5.7) displays the residual of coordinates for techniques in Y Axis. The table (5.8) shows the variation between residuals in the three techniques in X axis and Y axis, the figure (5.2) and (5.3) illustrate the chart for differences in residual in the three techniques for points in X and Y axis respectively.

Table (5.6): residual in three techniques in East Axis

Pint ID	Residual in equation with No weight & Curvature (E axis)	Residual in equation with weight (E axis)	Residual in equation with weight & Curvature (E axis)
G212	18.057	18.117	18.125
G213	12.922	12.744	12.749
G214	9.991	9.834	9.837
G215	2.801	2.647	2.648
G216	2.673	1.051	1.053
G217	4.878	4.656	4.657
G218	5.663	5.599	5.602
G219	5.494	3.550	3.552
G220	3.808	3.557	3.559
G221	1.718	-0.314	-0.314
G222	-12.220	-14.167	-14.174

Table (5.6): residual in three techniques in North Axis

Pint ID	Residual in equation with No weight & Curvature (N axis)	Residual in equation with weight (N axis)	Residual in equation with weight & Curvature (N axis)
G212	-1.826	-1.916	-1.917
G213	11.442	10.478	10.482
G214	-7.294	-5.881	-5.883
G215	9.334	9.525	9.529
G216	-6.865	-8.728	-8.730
G217	-2.769	-2.716	-2.717
G218	4.164	3.926	3.928
G219	-10.570	-9.925	-9.929
G220	3.804	3.247	3.248
G221	10.066	11.146	11.150
G222	1.478	3.003	3.005

Table (5.7): the variation between the residual in the three techniques.

Name	Variation between third and first techniques		Variation between third and second techniques	
	E	N	E	N
G212	0.068	-0.091	0.008	-0.001
G213	-0.173	-0.960	0.005	0.005
G214	-0.153	1.411	0.004	-0.002
G215	-0.153	0.195	0.001	0.004
G216	-1.620	-1.866	0.002	-0.002
G217	-0.221	0.052	0.002	-0.001
G218	-0.061	-0.236	0.002	0.002
G219	-1.942	0.641	0.002	-0.004
G220	-0.249	-0.556	0.002	0.001
G221	-2.033	1.085	0.000	0.004
G222	-1.953	1.527	0.007	0.001

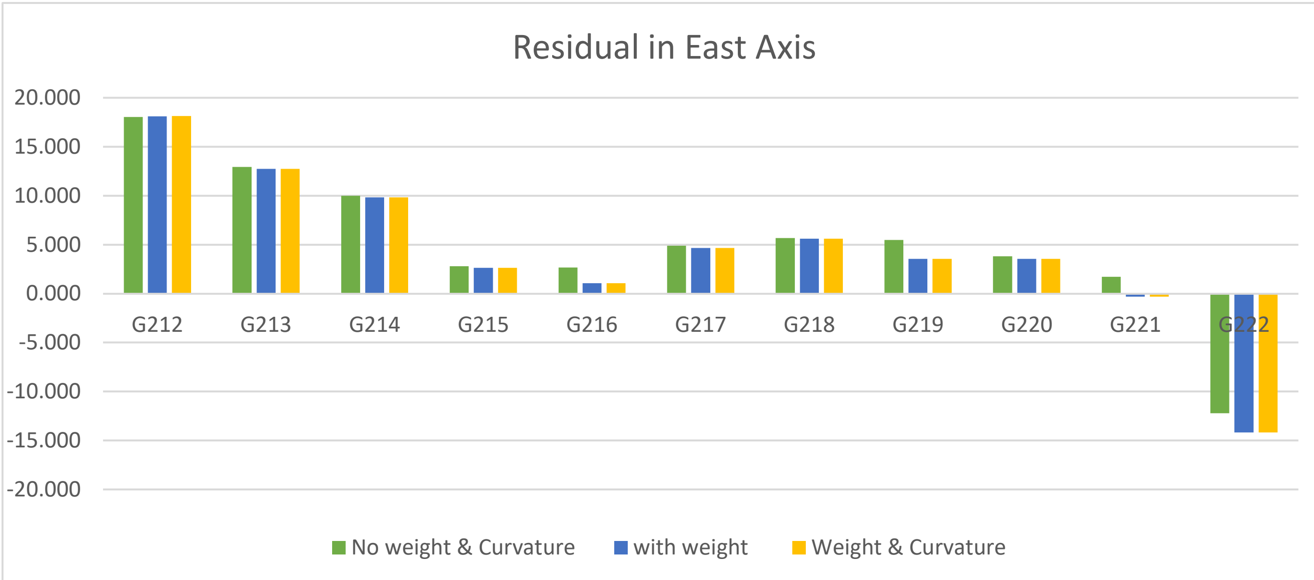

Figure (5.2): the different between residuals of coordinates in three techniques in East axis

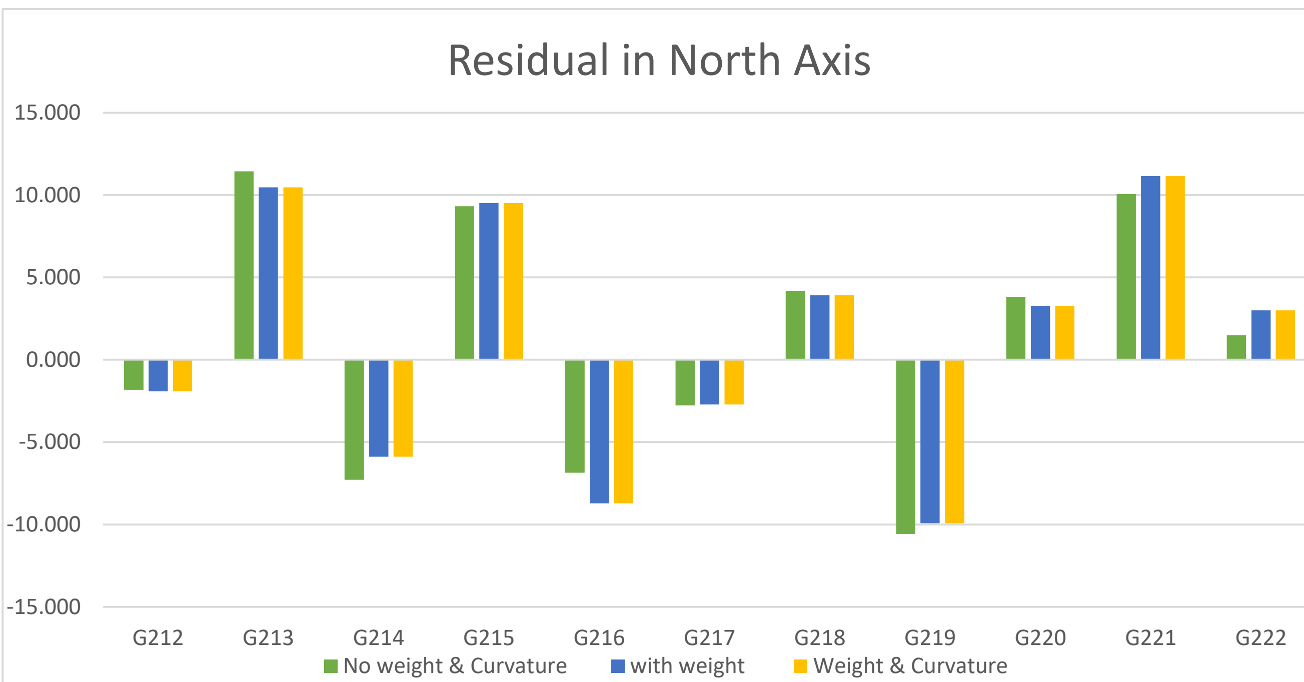

Figure (5.3): the different between residuals of coordinates in three techniques in North axis

5.7 Results and Discussions:

From Tables (5.6) and (5.7) it was found that variations in the East coordinates are large and that due to fact the network is expended to East more than to North and the curvature is smaller.

The difference between the first and second techniques is small, and given at maximum 2 meters, compared between second and third techniques is given at maximum 0.008 meters in the E and N axis, and can be ignored, and that be clear in the table (5.8).

But the variation between the origin coordinate and adjustment coordinates is large in almost of points, given at maximum 18 meters, and that mean the original coordinate for the old triangulation network in Sudan is weak and must be adjusted. That display in figures (5.2) and (5.3).

The adjustment is preformed using MATLAB software, when the old adjustment network used the logarithm tables, there for we can conclude that the old network must be re adjustment using modern techniques of adjustment.

Chapter six
Conclusion and Recommendation

6.1 Conclusion:

According to test carried out in this study it could be concluded that:

- In this study we carried out three techniques of adjustment.
- It was found these techniques give equal value in most of points, exactly in the second and third techniques.
- The variations between the three techniques are small and can be ignored when computing the adjustment of triangulation networks.
- The variations between the origin coordinates and the adjustment coordinate is large and may even reach 18 meters in some points.
- It was found that variations in the north direction is larger than east direction.
- Geoid ellipsoid separation consider ignore throughout the adjustment to network.

6.2 Recommendations:

The Following recommendation can help researchers in the future:

- The old triangulation networks should be adjusted or re adjusted, that is very clear from the result obtained.
- The variation of coordinates can extend to include the azimuth and triangulation methods.
- The comparison can use the Azimuth, angle and the distance observation equation to get the better result.
- The separation between the Geoid and the ellipsoid should be computed for each point in the network to get better result.

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