بسم الله الرحمن الرحيم

Sudan University of Science and Technology

College of Petroleum Engineering & Technology Petroleum Engineering Department

Numerical Solution by Finite Difference Approach for Homogenous Finite and Infinite Redial Reservoir by Computer Programing

الحلول العددية بإستخدام طريقة الفروقات المحدودة لمكمن محدود وغير محدود متجانس والسريان فيه نصف قطري بإستخدام برمجة حاسوبية

This dissertation is submitted in partial fulfillment of the requirements of degree of B.Sc. in Petroleum Engineering

Prepared by:

- El-Mustafa Mohamed Hamad Mohamed.
- Mohammed Ahmad Hassan Mohammed Ahmad.
- Asia Osman Ali Taha.
- Mohamed Mahdi Al-Abd Al-Hassan

Supervisor:

Assistant professor: Satti Merghani

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استهلال/

قال الله تعالى:

"تُسَبِّحُ لَهُ السَّمَاوَاتُ السَّبْعُ وَالْأَرْضُ وَمَنْ فِيهِنَّ ۚ وَإِنْ مِنْ شَيْءٍ إِلَّا يُسَبِّحُ بِحَمْدِهِ وَلَٰكِنْ لَا تَفْقَهُونَ تَسْبِيحَهُمْ ۗ إِنَّهُ كَانَ حَلِيمًا غَفُورًا"

صدق الله العظيم

الاسراء {٤٤}

Dedication

Every challenge needs self-efforts as well as guidance of elders especially those who are very close to our hearts

Our humble effort we dedicate to our sweet and loving

Parents

Whose affection, love and encouragement make us able to get much success and honor

Along with all hard working and respected Teachers

Also we dedicate this work to our colleagues who helped us by one way or another to accomplish this research.

Acknowledgment

Our first acknowledgment is to our supervisor

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Abstract

One of the most important problem in petroleum industry is to know the pressure distribution in the reservoir with realistic and applicable method, this study gives a pressure description with respect to time and distance.

A mathematical model is developed for a homogenous finite and infinite reservoir, radial and unsteady state flow. The model is written in a dimensionless form.

Numerical solution using the finite difference on implicit approach is applied, and the results are obtained by using Matlab. The results obtained are compared with the analytical solutions indicating the validity of the numerical solutions with an accepted error.

التجريد

واحدة من أكثر المشاكل أهمية في الصناعة النفطية هي معرفة توزيع الضغط في المكمن بطريقة حقيقية و قابلة للتطبيق.

لقد تم تطوير نموذج رياضي لمكمن متجانس محدود وغير محدود و السريان فيه نصف قطري و غير مستقر بصورة لا بعدية.

تم الحصول علي حلول عددية باستخدام طريقة الفروقات المحدودة بالمفاهيم الضمنية .و تم الحصول علي النتائج باستخدام برنامج الماتلاب و أوضحت النتائج التي تم الحصول عليها صلاحية الحل العددي حيث أنها تقارن بالنتائج التحليلية المعروفة بخطأ مقبول.

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Nomenclature

FDE	Finite-difference equations.
$\mathbf{r_e}$	Reservoir radius. (ft)
$\mathbf{r_w}$	Wellbore radius.(ft)
ρ	Density. (Ib./ft^3)
C	Compressibility. (psi^{-1})
∅	Porosity. (Dimensionless)
$\mathbf{B_g}$	Gas formation volume factor. (ft^3/scf)
μ	Viscosity. (cp)
Z	Gas deviation factor. (Dimensionless)
$\mathbf{V_b}$	Bulk volume. (ft^3).
P	Pressure. (psi)
V	Volume. (ft^3)
n	Number of moles. (moles)
z	Vertical direction in radial cylindrical flow geometry coordinates.
θ	Tangential direction in radial cylindrical flow geometry coordinates.

$\mathbf{V}$	Apparent velocity. (ft/day)
(x,y,z)	Rectangular flow geometry (x,y,z) directions
r	Radial direction in radial cylindrical flow geometry coordinates.
A	Total cross-sectional area of the rock in square centimeters. (ft^2)
$\frac{\partial p}{\partial x}$	Pressure gradient in (x) direction for rectangular flow geometry. (psi/ft)
$\mathbf{q_r}$	Volumetric flow rate at radius r. (ft^3/day)
$\mathbf{A_r}$	Cross-sectional area for flow at radial system in radius r. (ft^2).
$\left(\frac{\partial p}{\partial r}\right)_r$	Pressure gradient at radius r for radial cylindrical flow geometry coordinates. (psi/ft)
v_r	Apparent velocity at radius r. (ft/day)
$\mathbf{v_s}$	Velocity component in s directions of a radial coordinate system. (ft/day)
$\mathbf{K_r}$	Permeability at (r) direction in radial cylindrical flow geometry coordinates. (md)
$\mathbf{K_\theta}$	Permeability at (θ) direction in radial cylindrical flow geometry coordinates. (md)
$\mathbf{K_z}$	Permeability at (z) direction in radial cylindrical flow geometry coordinates. (md)

$\frac{\partial P}{\partial r}$	Pressure gradient in (r) direction in radial cylindrical flow geometry coordinates. (psi/ft)
α	Hydraulic diffusivity constant for the reservoir fluid system.
Δt	Time interval. (day)
C_t	Total compressibility. (psi^{-1})
P_D	Dimensionless pressure drop. (dimensionless)
r_{eD}	Dimensionless external radius. (dimensionless)
t_D	Dimensionless time. (dimensionless)
r_D	Dimensionless radius. (dimensionless)
p (r, t)	pressure at radius r and time t.
U	Logarithmic transformation.
NOG	Number of grids.
p_{wf_D}	Dimensionless bottom hole pressure.
P_{i_D}	Dimensionless initial pressure.

Chapter 1

Introduction

Reliable information about in-situ reservoir conditions is important in many phases of petroleum engineering.

There are many ways for getting these information, no single method can give complete unique reservoir description. One of the powerful approaches to obtain values of reservoir properties is the pressure analysis techniques, which can be derived from numerical solution.

1.1. Statement of The Problem

In order to understand the reservoir behavior, it must be expressed in terms of appropriate mathematical equations. These equations constituting the mathematical models of the reservoir are almost always too complex to be solved by analytical methods. Approximations must be made to put the equations in a form that is amenable to solution by digital computers.

Consider The reservoir is cylindrical with a radius (r_e), homogenous and has uniform thickness. The mathematical model which describes this situation will be developed, and solved numerically.

As the reservoir considered to be cylindrical, the grid spacing in reservoir simulation based on radius, this makes grids irregular, this irregularity leads to error on estimating the grid properties. The study addressed this problem to be solved and as well as to reduce the error in calculations.

1.2. Objectives

The main objective of this study is to obtained more representative mathematical discerption or a relationship for pressure in radial reservoir under any time and distance

The sub-objectives are: -

- Best understanding of the mathematical radial homogenous infinite and finite reservoir model.
- To solve the model numerically by using finite difference approach.
- To design computer program by Matlab code for both infinite and finite reservoirs.
- To compare the numerical solution of infinite and finite programs with the analytical solutions.

1.3. Research Structure

Chapter one consists of research introduction; which contains state problem, objectives from the research, and research methodology, chapter two consists of theoretical background and literature review, chapter three consists of methodology which contains all the tools have been used in the solution, chapter four consists of results and discussion to our solution, finally chapter five conclusion and recommendations.

Chapter 2

Literature Review and Theoretical Background

2.1. Introduction

Compared to alternative investigation methods, the advantages of numerical method can be summarized as, the numerical results often can be obtained faster and at lower costs, parameter variations on the computer usually are easily realizable, a numerical simulation often gives more comprehensive information, the reliability of the computations and the possibility of obtaining approximatively solutions via the application of finite-difference methods to the partial differential equations. However, not only the advances in computer technology have had a crucial influence on the possibilities of numerical simulation methods, but also the continuous further development of the numerical algorithms has contributed significantly to this.

To illustrate the different aspects that play a role when employing numerical simulation techniques for the solution of engineering problems. The first step consists in the appropriate mathematical modeling of the processes to be investigated that result in systems of differential equations derived in the framework of continuum mechanics must then be suitably approximated by a discrete problem. the next step consists in the solution of the algebraic equation systems. Here, algorithmic questions and, of course, computers come into play. (Schäfer, 2006)

There are primarily three different approaches available for the solution procedure to obtain the numerical value of unknown parameters or quantities: -

1- **Finite-Volume Methods (FVM):** Mainly employed for the numerical solution of problems in fluid mechanics, which are the basis for the mathematical modelling of continuum mechanical problems, per definition, also are fulfilled for the discrete equations(conservatively)(Schäfer, 2006).
2- **Finite-Element Methods (FEM):** Is widely used primarily for numerical computations in solid mechanics and can be regarded as a standard tool there. (Schäfer, 2006)

3- **Finite-Difference Methods (FDM*):***

Finite-difference equations are used to approximate a differential equation when an analytical solution is unknown, or if the known analytical solution is cumbersome in application. Because complex systems are normally encountered .(Mattax and Dalton, 1990)

2.2. Literature review

The first theoretical study of well equations was given by **Peaceman (1977)** for cell-centered finite difference methods on square grids for single phase flow. Peaceman's study gave a proper interpretation of a well-block pressure, and indicated how it relates to the flowing bottom hole pressure. The importance of his study is that the computed block pressure is associated with the steady state pressure for the actual well at an equivalent radius re. For a square grid with a grid size h, Peaceman derived a formula for re by three different approaches:(Peaceman, 1978)

(1) Analytically by assuming that the pressure in the blocks adjacent to the well block is computed exactly by the radial flow model, obtaining re = 0.208h, (Peaceman, 1978)

(2) numerically by solving the pressure equation on a sequence of grids, deriving re = 0.2h, and (3) by solving exactly the system of difference equations and using the equation for the pressure drop between the injector and producer in a repeated five-spot pattern problem, finding re = 0.1987h. From these approaches, he concluded that re ≈ 0.2h.(Peaceman, 1978).

Peaceman's finite difference well models on square grids have been extended in various directions, including to rectangular grids, anisotropic reservoirs, horizontal wells, and multiphase flows and to incorporating gravity force, skin, and non-Darcy effects. (Peaceman, 1978).

Lee and Milliken (1993): Studied an arbitrary monobore well in a layered system of laterally infinite extent. They combined a semianalytical solution based on slender body theory with a finite difference pressure solution with lateral pressure boundary conditions described by the semianalytical solution.(Lee and Milliken, 1993).

Ding (1996): Introduced a layer potential function to obtain a steady state pressure distribution in the vicinity of the well in three dimensions **(3D)**. Furthermore, he adjusted well block transmissibilities to account for radial flow.(Ding, 1996).

Ewing et al. and Garanzha et al. (1999): Studied the grid pressures obtained from the simulation of single phase flow through an isotropic porous medium using different numerical methods. Furthermore, (non-Darcy) flow well equations are developed for cell-centered finite difference, Galerkin finite element and mixed finite element techniques.(Ewing et al., 1999).

Recently, **Ding and Jeannin (2001):** Developed a multipoint discretization in a curvilinear coordinate system and used the discretization coefficient of an elliptic equation as the well index. This method can reduce the calculation errors for the well dominating well flow. Consequently, good results are obtained with these new approach. This proposed approach can be used for any kind of flexible grid for the near-well modeling.(Ding and Jeannin, 2001).

Wolfsteiner et al (2003): Extended Peaceman's well models to account for different well configurations in heterogeneous porous medium involving horizontal wells, where it is shown –Wolfsteiner's method when applied on horizontal wells- to be capable of approximating the effects of sub-grid heterogeneity in different finite difference models.(Wolfsteiner et al., 2003)

Chen and Yue (2003): Studied steady flow transport through highly heterogeneous porous media driven by extraction wells, they derived a well model by introducing multiscale basis functions that resolve well singularity, they also derived a new homogenization results for green functions in their model, and made numerical experiments for flow transport in both periodic and randomly generated log-normal permeabilities to demonstrate the efficiency and accuracy of their proposed method.(Chen and Yue, 2003).

And **Aarnes (2004):** Proposed a modified mixed multiscale finite element method that can account for radial flow near a well, by solving elliptic flow problems on porous media. The method incorporates the effect of small-scale heterogeneous structures in

the elliptic coefficients into the base functions and produces a detailed velocity field, that can be used to solve phase transport equations at a sub-grid scale.(Aarnes, 2004).

Later **ZHANGXIN CHEN AND YOUQIAN ZHANG (2009):** Presented a systematical derivation of well models for several numerical methods such as: standard finite elements, control volume finite elements, and mixed finite element methods. Their well models have particular applications to groundwater hydrology and petroleum reservoirs.(Chen and Zhang, 2009).

N. Natarajan and G. Suresh Kumar (2010): They proposed an alternative approach to the decomposition method for solving multispecies transport in porous media, coupled with first-order reactions has been proposed. Their numerical solution is based on implicit finite difference method. The task of decoupling the coupled partial differential equations has been overcome in this method. Their proposed approach is very much advantageous because of its simplicity and also can be adopted in situations where non-linear processes are coupled with multi-species transport problems.(Natarajan and Kumar, 2010).

K. RAZMINIA, A. RAZMINIA, R. KHARRAT, D. BALEANU (2014): Solved the diffusivity equation models numerically by using differential quadrature method, their method provided computationally efficient and accurate in differential quadrature analysis of diffusivity equation to overcome the large computation times. This method overcame the difficulties in boundary conditions implementations of second order partial differential equations encountered in such problems.(Razminia et al., 2014).

Finally, **Azizollah Khormali, Seyyed Shahab Tabatabaee Moradi, Dmitry Petrakov (2014):** Determined the pressure distribution in a reservoir in the unsteady state regime of flow by applying Darcy's equation and solving it numerically. The numerical simulation of reservoirs is based on numerical solutions of different partial differential equations (PDEs) representing the multiphase flow of fluids. They obtained Pressure profile in a one dimensional system by solving Darcy's equation explicitly. They investigated the changes in pressure profile in three situations. These situations include section length changes, step time changes and time approach to infinity.(Khormali et al., 2014).

As far as the authors know, however, most of these existing well models have been developed for finite difference methods.

2.3. Theoretical Background

2.3.1. Introduction: -

The information we obtain from a newly discovered field is scanty at best. It is also disjointed to a certain extent, because bits and pieces of information are emanating from different parts of the field, so to integrate these pieces of information as accurately as possible in order to construct a global picture of the system, we need to do a reservoir simulation. As field development progresses, more information becomes available, enabling us to continually refine the reservoir description.

A reservoir simulation study is the only practical laboratory in which we can design and conduct tests to adequately address these questions. From this perspective, reservoir simulation is a powerful screening tool.

There are many factors dictating the choice of reservoir simulation approach they are: The complexity of the problem at hand, the amount of data available, and the study's objectives. Broadly classified, there are two simulation approaches we can take: analytical and numerical.

• The analytical approach, as is the case in classical well test analysis, involves a great deal of assumption in essence, it renders an *exact* solution to an *approximate* problem.

• The numerical approach, on the other hand, attempts to solve the more realistic problem with less stringent assumptions—in other words, it provides an approximate solution to an exact problem.

To do reservoir simulation there is some properties must be known like rock and fluid properties, flow geometry and dimension, number of phase flowing.

2.3.2. Rock properties: -

Porosity, it's a very important property in fluid flow throw reservoir porous, and in flow there is only an effective porosity effect. (Ahmed, 2006)

Effective porosity is a dimensionless quantity, defined as the ratio of interconnected pore volume to the bulk volume. And in the flow equations used in reservoir simulation, porosity appears as one of the parameters that scales the volume of fluids present in the reservoir at any time.(Ahmed, 2006)

Another important property that affect fluid flow throw reservoir porous is **permeability**, it defines by measuring of a rock's ability to transmit fluids. So a hydrocarbon reservoir to be commercial, it must not only be porous, but also permeable, permeability varies widely in naturally occurring reservoirs, from a fraction of a millidarcy to several darcies.(Mattax and Dalton, 1990)

Also the **homogeneity and heterogeneity** of a reservoir system play a big role in the amount of difficulty of a model; homogeneous systems feature uniform spatial distribution (Formation with rock properties that do not change with location in the reservoir) This ideal never actually occurs, but many formations are close enough to this situation that they can be considered homogeneous. Most of the models used for pressure-transient analysis assume the reservoir is homogeneous, and The quality of variation in rock properties with location in a reservoir or formation, while heterogeneous systems exhibit non-uniform distribution (rock properties change with location in the reservoir). For simplicity's sake, we often assume homogeneity in reservoir calculations, even though many reservoirs are heterogeneous. This is where numerical reservoir simulation becomes a very powerful tool, because it allows us to incorporate property variation in the system. (Ahmed, 2006)

Some parameters used in reservoir simulation exhibit directional dependency. A reservoir exhibits **isotropic** property distribution if that property has the same value regardless of the direction in which we measure it. On the other hand, if a property's value does vary with direction, then the reservoir is anisotropic with respect to that property.(Mattax and Dalton, 1990)

2.3.3. Fluid properties: -

Fluid properties, like rock properties, significantly affect fluid flow dynamics in porous media, it is often necessary in reservoir simulation to estimate these properties using correlations and/or equations of state.

For gasses the properties of interest in the gas flow equation are density, compressibility factor, compressibility, formation volume factor and viscosity, the

compressibility factor introduces an important non-linearity, in that it appears in the formation volume factor. Gas viscosity is also strongly dependent on pressure, and needs to be calculated as pressure varies spatially and temporally. Table (2-1) below summarizes the equations and correlations necessary for determining gas properties(Mattax and Dalton, 1990)

Real Gas Law	**PV=ZnRT**
Density	$\boldsymbol{\rho = \frac{PM}{ZRT}}$
Compressibility	$\mathrm{C} = \boldsymbol{\frac{1}{P} - \frac{1}{Z}\left(\frac{\partial Z}{\partial P}\right)}$
Gas Deviation Factor	**Z=f(P,T)**
Formation Volume Factor	$\boldsymbol{B_g = \frac{ZTP_{sc}}{T_{sc}P}}$
Viscosity	$\boldsymbol{\mu = f(P,T)}$

Table 2-1 equations and correlations for real gases.(Mattax and Dalton, 1990)

Oil properties that appear in the governing flow equations for the oil phase are density, compressibility, formation volume factor, viscosity and solubility of gas in oil. In the absence of gas, these oil properties can be treated as constants, because the compressibility of gas-free oil is very small. However, the presence of dissolved gas in oil necessitates the use of appropriate correlations to determine the variation of these properties with pressure and temperature.(Mattax and Dalton, 1990)

Water properties that affecting flow equations for water phase are: density, compressibility, formation volume factor, viscosity and gas solubility. Since gas

solubility in water is very small compared to oil, for most practical cases, we assume constant values for these properties that come into play in the water flow equation.(Mattax and Dalton, 1990)

2.3.4. Reservoir Rock/Fluid Interactions: -

The principal fluids in a petroleum reservoir are water, oil and gas. When they exist as free phases, they are generally immiscible. When these immiscible fluids co-exist in the reservoir pore space, their interactions with one another and with the containing rock control their spatial distribution and movement. The two principal properties used to quantify these interactions are wettability, which pertains to rock-fluid interactions, and interfacial tension, which relates to fluid-fluid interactions.(Mattax and Dalton, 1990)

When two immiscible fluids co-exist in the same pore space, one preferentially adheres to the rock surface. This phenomenon is known as **wetting**, and the fluid that is preferentially attracted is referred to as having a higher wettability index. The parameter which determines the wettability index is called adhesion tension, and it is directly related to interfacial tension. Interfacial tension is a measure of the surface energy per unit area of the interface between two immiscible fluids. The study of surface energy phenomena is very important in recovery processes, in that many EOR processes are based on altering the surface energy so as to favor oil recovery.(Mattax and Dalton, 1990)

Relative permeability also a property results from reservoir rock and fluid interactions, and defined as: When two or more immiscible fluids flow simultaneously through a porous medium, they compete and do not move at equal velocity. This results on the one hand from interactions between the fluids and the rock, and on the other from interactions among the fluids themselves. Although relative permeability is not a fundamental property of fluid dynamics, it is the accepted quantitative parameter used in reservoir engineering. Relative permeability appears prominently in the flow equations used in reservoir simulation. (Ahmed, 2006)

2.3.5. Flow Geometries and Dimensions: -

a) Rectangular flow geometry: - in (x, y, z) directions, that shown in figure (2-1) below, in this

case the streamlines are parallel to the three principal axes (x, y, and z), which are orthogonal. With smaller element of dimensions, (Δx, Δy, Δz) as a control volume to set up and discrete the governing equations**.(**Mattax and Dalton, 1990**).**

b) Radial-cylindrical flow geometry: -

The radial-cylindrical coordinate system is particularly appealing for describing single-well problems. Figure (2-2) shows the principal directions of this flow geometry and its elemental volume.(Mattax and Dalton, 1990)**.**

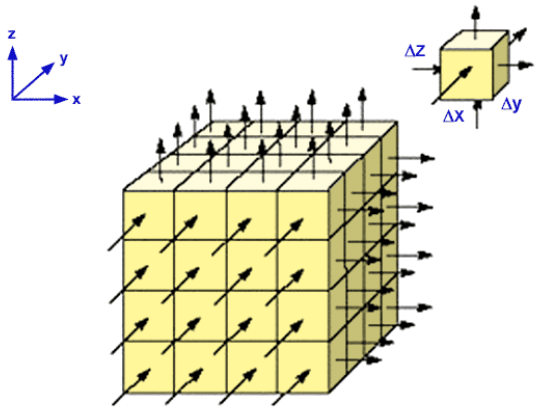

Figure (2-1) Rectangular flow geometry in (x, y, z) directions(Mattax and Dalton, 1990)

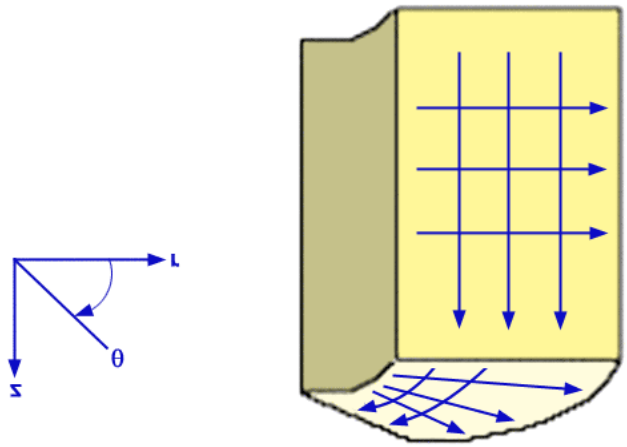

Figure (2-2) the principal directions of radial flow geometry.(Mattax and Dalton, +1990)

The three principal flow directions are radial (r), vertical (z) and tangential (θ). If we assume a reservoir of uniform thickness, then the system becomes two concentric cylinders of the same height. A particle moving in three-dimensional radial-cylindrical flow geometry can be illustrated as in Figure (2-3):

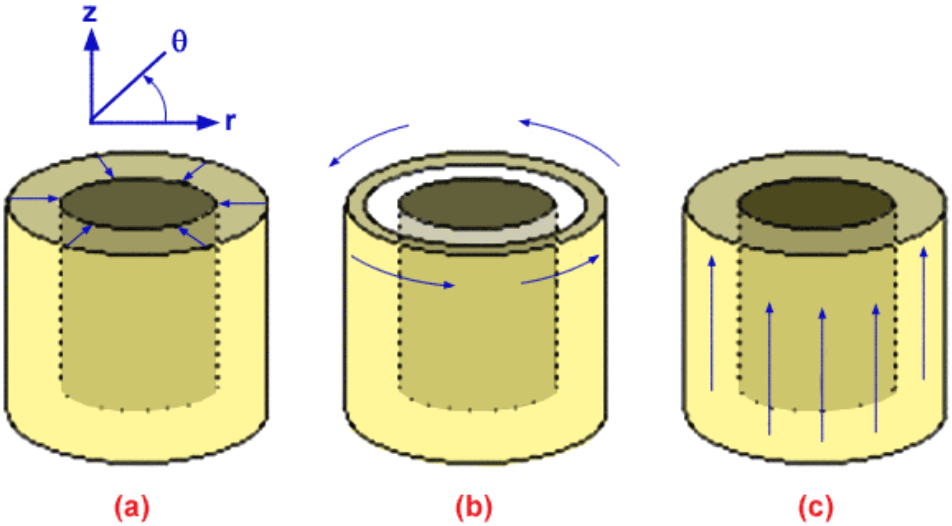

Figure (2-3) moving in three-dimensional radial-cylindrical flow geometry(Mattax and Dalton, 1990)

But in our model we assume the wellbore is cylindrical, and the flow is in one dimension(r-dimension) like in figure (2-4):

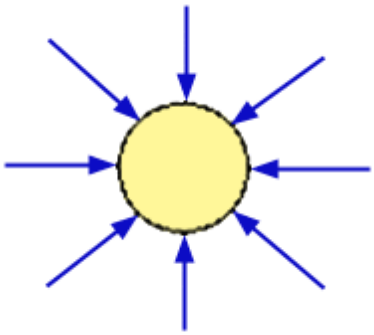

Figure (2-4) flow in one dimension(r-dimension), inside cylindrical wellbore.(Mattax and Dalton, 1990)

2.3.6. Number of flowing fluids in the reservoir: -

The mathematical expressions that are used to predict the volumetric performance and pressure behavior of the reservoir vary in forms and complexity depending upon the number of mobile fluids in the reservoir. There are generally three cases of flowing systems: Single-phase flow (oil, water, or gas), two-phase flow (oil-water, oil-gas, or gas-water)***, and*** three-phase flow (oil, water, and gas)**.** Here us discuss will focus on single phase flow, and we will discuss the equations that control this flow.(Mattax and Dalton, 1990)

A. Single-Phase Flow Equations: -

Single-phase flow in petroleum reservoirs is rare in practice. There are only a limited number of cases dry gas reservoirs, for example where conditions exist for single-phase flow. But we do apply single-phase flow assumptions (predominantly in well test analysis) as a means of simplifying problems and rendering them analytically tractable.

In numerical reservoir simulation, we may relax these types of simplifying constraints because of the more versatile nature of numerical schemes over analytical methods.(Mattax and Dalton, 1990).

Fluid flow equation (Darcy's law): -

The fundamental law of fluid motion in porous media is Darcy's Law. The mathematical expression developed by Henry Darcy in 1856 states the velocity of a homogeneous fluid in a porous medium is proportional to the pressure gradient and inversely proportional to the fluid viscosity.(Ahmed, 2006)

For a horizontal linear system, this relationship is:

$$\boldsymbol{v = \frac{q}{A} = -\frac{k}{\mu}\frac{\partial p}{\partial x}} \qquad \textbf{(2-1)}$$

(v) is the apparent velocity in centimeters per second, (A) is total cross-sectional area of the rock in square centimeters .(μ) is viscosity, and expressed in centipoise units(cp),

and the pressure gradient ($\frac{\partial p}{\partial x}$) is in atmospheres per centimeter(psi), The proportionality constant k is the *permeability* of the rock expressed in Darcy units.

For a horizontal-radial system, this relationship is:

$$\boldsymbol{\upsilon = \frac{q_r}{A_r} = \frac{k}{\mu}\left(\frac{\partial p}{\partial r}\right)_r} \qquad \textbf{(2-2)}$$

Where q_r = volumetric flow rate at radius r

A_r = cross-sectional area to flow at radius

$\left(\frac{\partial p}{\partial r}\right)_r$ = pressure gradient at radius r

ν = apparent velocity at radius r

Darcy's Law applies only when the following conditions exist:

Laminar (viscous) flow, steady-state flow, incompressible fluids, and homogeneous formation.

We usually formulate the differential equations governing fluid flow in porous media based on the continuum assumption, in which we consider a differential element of the system and take balances over a conserved quantity of interest. When the quantity is mass, the resulting equation is the mass balance equation or the continuity equation. Figure (2-5) shows a representative element (control volume) of the reservoir in radial coordinates. (Ahmed, 2006)

Conservation of mass: -

The conservation of mass principle simply says that over a fixed time period,

[Mass in] - [Mass out] = [Net change in mass content]

Applying this principle to the system in Figure (2-5), we obtain the continuity equation (2-3) shown below:

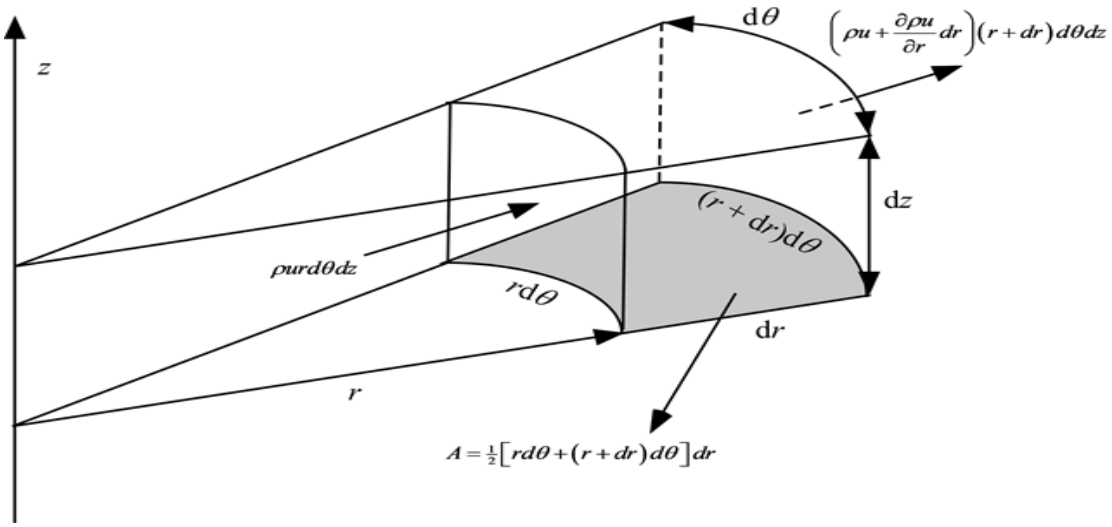

Figure (2-5): control volume of the reservoir in radial coordinates.(Razminia et al., 2014)

$$-\boldsymbol{\nabla}(\overrightarrow{\boldsymbol{\rho v_s}}) = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\emptyset\boldsymbol{\rho}) \qquad \textbf{(2-3)}$$

In the equation above the left side-hand term is referred to as the mass flux term, while the right-side hand term is called the mass accumulation term.

We can rewrite the equation in radial coordinates like:

$$-\boldsymbol{\nabla}(\overrightarrow{\boldsymbol{\rho v_s}}) = -\frac{\mathbf{1}}{\boldsymbol{r}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r}}(\boldsymbol{r\rho v_r}) + \frac{\mathbf{1}}{\boldsymbol{r}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial \theta}}(\boldsymbol{\rho v_\theta}) + \frac{\boldsymbol{\partial}}{\boldsymbol{\partial z}}(\boldsymbol{\rho v_z}) \qquad \textbf{(2-4)}$$

Where: v is the velocity vector,ρ is the density,$\emptyset$ is the porosity,v_s is the velocity component in s directions of a radial coordinate system.

The porosity term in the right-hand-side of the Equation, if treated as a constant, will come out of the differential operator. This is a reasonable assumption for a reservoir with low rock compressibility.

With appropriately defined terms and parameters, Equation (2-4) is general and can be used for any system. To specialize it to porous media, we must invoke Darcy's law. Substituting Darcy's law, written in terms of velocity as:(Mattax and Dalton, 1990)

$$\boldsymbol{\upsilon = \frac{q_r}{A_r} = \frac{k}{\mu}\left(\frac{\partial p}{\partial r}\right)_r} \quad \textbf{(2-5)}$$

We obtain the flow equation for porous media. In a radial coordinate system, this equation becomes:

$$\boldsymbol{\frac{1}{r}\frac{\partial}{\partial r}\left(\rho\frac{K_r}{\mu}\frac{\partial P}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\rho\frac{K_\theta}{\mu}\frac{\partial P}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(\rho\frac{K_z}{\mu}\frac{\partial P}{\partial z}\right) = \frac{\partial}{\partial t}(\rho\emptyset)} \quad \textbf{(2-6)}$$

Where:

P: pressure $\boldsymbol{\rho}$: fluid density **(r,$\boldsymbol{\theta}$, z)**:radial coordinates.

$\boldsymbol{\mu}$: Viscosity. $\boldsymbol{K_r, K_\theta, K_z}$: permeability at (r,$\theta$.z)directions.

The above equation called (**continuity equation**), or flow equation that's control flow in porous media, and that's the general form of it. In the natural reservoirs this equation is controlled by fluids types. In general, reservoir fluids are classified into three groups: Incompressible fluids, slightly compressible fluids, and Compressible fluids.

I. Incompressible fluids:

It's defined as the fluids whose volume (or density) does not change with pressure:

$$\boldsymbol{\frac{\partial V}{\partial P} = 0} \quad \textbf{(2-7)}$$

$$\boldsymbol{\frac{\partial \rho}{\partial P} = 0} \quad \textbf{(2-8)}$$

So at flow equation, for an incompressible fluid density and viscosity are constant, and if we assume that porosity does not vary with pressure, we obtain:

$$\boldsymbol{\rho\frac{K}{\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial P}{\partial r}\right) + \rho\frac{K}{\mu}\frac{\partial}{\partial \theta}\left(\frac{\partial P}{\partial \theta}\right) + \rho\frac{K}{\mu}\frac{\partial}{\partial z}\left(\frac{\partial P}{\partial z}\right) + qsc = 0} \quad \textbf{(2-9)}$$

This equation is written for heterogeneous and anisotropic formations. For such a formation, and without injection or production, equation above can simplify to:

$$\frac{1}{r}\left(\frac{\partial^2 P}{\partial r^2}\right)+\frac{\partial^2 P}{\partial \theta^2}+\frac{\partial^2 P}{\partial z^2}=0 \qquad \textbf{(2-10)}$$

Which known as Laplace equation in radial coordinates.

II. Slightly compressible fluids:

This fluid exhibits small changes in volume, or density, with changes in pressure. And so at flow equation, for a slightly compressible fluid density and viscosity exhibit weak dependence on pressure, Furthermore, for a slightly compressible fluid, we usually assume that compressibility does not vary within the pressure range of interest(Ahmed, 2006).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho\frac{Kr}{\mu}\frac{\partial P}{\partial r}\right)+\frac{\partial}{\partial \theta}\left(\rho\frac{K\theta}{\mu}\frac{\partial P}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(\rho\frac{Kz}{\mu}\frac{\partial P}{\partial z}\right)=V_b\emptyset C\left(\frac{\partial P}{\partial t}\right) \qquad \textbf{(2-11)}$$

For slightly compressible fluids the changes in viscosity with pressure are negligible and they can be treated as constants. Furthermore, if we assume that we are dealing with homogeneous and isotropic porous media with no well, Equation above reduces to a simpler form, which is known as the diffusivity equation at radial form:

$$\frac{1}{r}\left(\frac{\partial^2 P}{\partial r^2}\right)+\frac{\partial^2 P}{\partial \theta^2}+\frac{\partial^2 P}{\partial z^2}=\frac{\emptyset\mu C}{K}\frac{\partial P}{\partial t} \qquad \textbf{(2-12)}$$

$\frac{\emptyset\mu C}{K}=\frac{1}{\alpha}$ (α called hydraulic diffusivity constant for the reservoir fluid system)

C: fluid compressibility **(psi^{-1}).** μ : viscosity **(cp).**

$\emptyset$: Porosity **(dimensionless).** V_b: Bulk volume **(ft^3).**

III. Compressible flow equation:

These are fluids that experience large changes in volume as a function of pressure. All gases are considered compressible fluids; compressible fluid flow involves

additional considerations. The highly compressible nature of gas makes certain gas properties (i.e., viscosity, density, and compressibility factor) strongly dependent on pressure. Since we cannot assume that these properties are constant, they introduce non-linarites to the flow equations. The numerical handling of the flow equations becomes more challenging as the degree of non-linearity increases. (Mattax and Dalton, 1990)

B. Multiphase flow equations: -

Multi-phase flow equations are based on the same principles that govern single-phase flow, except that they must account for interactions between simultaneously flowing phases in porous media. The main parameters that we use to characterize these interactions are relative permeability, saturations and solution gas-liquid ratios. (Mattax and Dalton, 1990).

In this research we will focus on single phase flow equations, with assuming our reservoir is only occupied by oil.

2.4. Mathematical Model

2.4.1. Introduction

Flow in porous media is a very complex phenomenon and as such cannot be described as explicitly as flow through pipes or conduits. in porous media, however, flow is different in that there are no clear-cut flow paths which lend themselves to measurement.

The forms of mathematical relationships that are designed to describe the flow behavior of the reservoir fluids will vary depending upon the characteristics of the reservoirs.

Reservoir properties which we talked about previously in this chapter make the mathematical discerption very hard, that we used assumptions to minimize the difficulty of discerption and solution.

The mathematical model we discussed here fall under this assumptions:

1-Homogeneous and isotropic porous medium.

2-Uniform thickness.

3-Single phase flow, in one dimension.

4-Laminar flow.

5-unsteady state flow.

6-control volume and isothermal system.

7-slithgly compressible fluid.

8-radial flow system and the well in the center of geometry.

9- finite and infinite reservoir.

10-full perforated interval.

2.4.2. Reservoir Initial and Boundary Conditions:

Initial Condition: At the start of production, the pressure in the reservoir is assumed to be at some uniform value, **Pi**.

Boundary condition at infinity: Infinitely far from the well, the pressure will always remain at its initial value, **Pi**.

Boundary condition at finite: The arrival of the pressure disturbance at the well drainage boundary marks the end of the transient flow period and the beginning of the semi (pseudo)-steady state. During this flow state, the reservoir boundaries and the shape of the drainage area influence the wellbore pressure response as well as the behavior of the pressure distribution throughout the reservoir.

Boundary condition at the wellbore: At the wellbore, which is assumed to be infinitely small, the flux must be equal to **Q** (into the well) at all times **t > 0**.

We can therefore formulate the problem in precise mathematical terms as follows:

***Initial condition*:** $$\mathbf{P(r.t=0)=P_i} \quad \textbf{(2-13)}$$

***Boundary condition at wellbore*:** $$\lim_{r\to\infty}\left[\frac{2\pi kh}{\mu}r\frac{\partial p}{\partial r}\right]=Q \quad \textbf{(2-14)}$$

Boundary condition at r = ∞ : $$\lim_{r\to\infty}P(r.t)=P_i \quad \textbf{(2-15)}$$

***Boundary condition at r = re*:** $$\frac{\partial p}{\partial r_e}=0 \quad \textbf{(2-16)}$$

2.4.3. Mathematical Model Derivation:

Consider the flow element shown in Figure 2-6 The element has a width of (dr) and is located at a distance of (r) from the center of the well. The porous element has a differential volume of dV.

According to the concept of the material-balance equation, the rate of mass flow into an element minus the rate of mass flow out of the element during a differential time =Δt must be equal to the mass rate of accumulation during that time interval, or:(Ahmed, 2006)

The mass leaving the volume element during Δt	—	The mass entering the volume element during Δt	=	the mass rate of accumulation during Δt	**(2-17)**

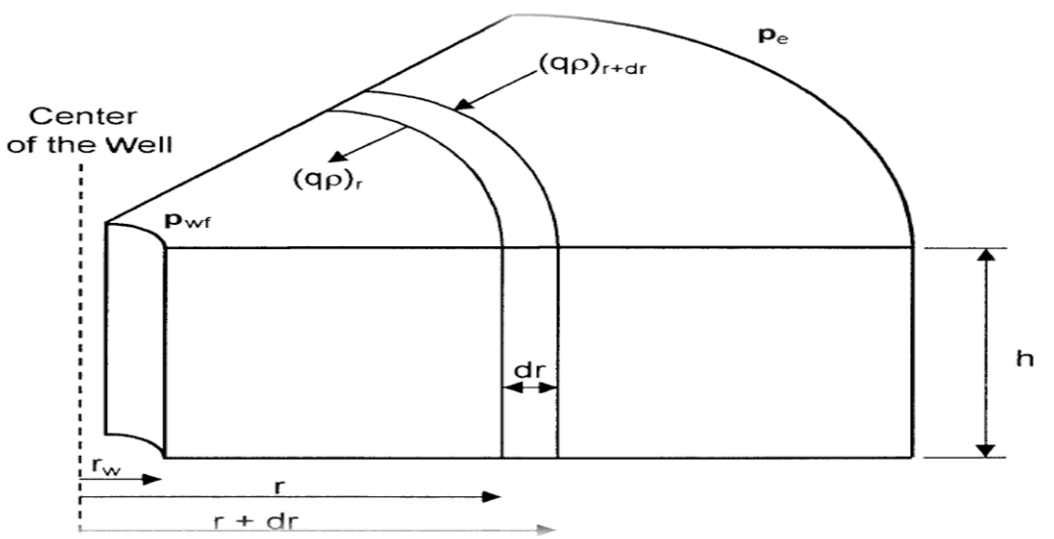

Figure (2-6) Illustration of radial flow.(Ahmed, 2006)

The individual terms of Equation (2-17) are described below:

Mass entering the volume element during time interval Δt

$$(\boldsymbol{mass})_{\boldsymbol{in}} = \Delta \boldsymbol{t}\ [\mathbf{A}\boldsymbol{\rho v}]_{\boldsymbol{r+dr}} \qquad \textbf{(2-18)}$$

ν = velocity of flowing fluid, ft/day.

P = fluid density at (r +dr), (lb./ft^3).

A = Area at (r +dr),(ft^2).

Δt = time interval, (day).

The area of element at the entering side is:

$$\mathbf{A}_{\boldsymbol{r+dr}} = \boldsymbol{2\pi\ (r + dr)\ h} \qquad \textbf{(2-19)}$$

Combining Equation (2-18) with (2-19) gives:

$$(\boldsymbol{mass})_{\boldsymbol{in}} = \boldsymbol{2\pi\ \Delta t(r + dr)h\ [\rho v]_{r+dr}} \qquad \textbf{(2-20)}$$

Mass leaving the volume element: -

Adopting the same approach as that of the leaving mass gives:

$$(\boldsymbol{mass})_{\boldsymbol{out}} = \boldsymbol{2\pi\,\Delta t\, rh\,[\rho v]_r} \quad \textbf{(2-21)}$$

Total Accumulation of Mass: -

The volume of some element with a radius of r is given by:

$$\boldsymbol{V = \pi r^2 h} \quad \textbf{(2-22)}$$

Differentiating the above equation with respect to r gives:

$$\frac{\boldsymbol{dV}}{\boldsymbol{dr}} = \boldsymbol{2\pi rh} \quad \textbf{(2-23)}$$

$$\boldsymbol{dV = (2\pi rh)\,dr} \quad \textbf{(2-24)}$$

Total mass accumulation during Δt $\quad = \boldsymbol{dV[(\rho\phi)_{t+\Delta t} - (\rho\phi)_t]}$ **(2-25)**

Substituting for dV yields:

$$\textbf{Total mass accumulation} = \boldsymbol{(2\pi rh)dr\,[(\rho\phi)_{t+\Delta t} - (\rho\phi)_t]} \quad \textbf{(2-26)}$$

Replacing terms of Equation (2-25) with those of the calculated relationships gives:

$$\boldsymbol{[2\pi\,\Delta t(r+dr)h\,[\rho v]_{r+dr}] - [2\pi\,\Delta t\,rh\,[\rho v]_r] = [(2\pi rh)dr\,[(\rho\phi)_{t+\Delta t} - (\rho\phi)_t]} \quad \textbf{(2-27)}$$

Dividing the above equation by $(2\pi rh)\,dr\,\Delta t$ and simplifying, gives

$$\frac{\boldsymbol{1}}{\boldsymbol{r\,dr}}\boldsymbol{[(r+dr)\,[\rho v]_{r+dr} - r[\rho v]_r]} = \frac{\boldsymbol{1}}{\boldsymbol{\Delta t}}\boldsymbol{[(\rho\phi)_{t+\Delta t} - (\rho\phi)_t]}$$

or $$\frac{\boldsymbol{1}}{\boldsymbol{r}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial r}}\boldsymbol{(-r\rho v)} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\boldsymbol{(\rho\phi)} \quad \textbf{(2-28)}$$

Where:

ϕ = porosity.

ρ = density, (lb./ft^3).

ν= fluid velocity, (ft/day).

Equation (2-28) is called the continuity equation and it provides the principle of conservation of mass in radial coordinates. The transport equation must be introduced into the continuity equation to relate the fluid velocity to the pressure gradient within the control volume dV. Darcy's Law is essentially the basic motion equation, which states that the velocity is proportional to the pressure gradient $\frac{\partial p}{\partial r}$:

$$\nu = (5.615)\,(0.001127)\frac{k}{\mu}\frac{\partial p}{\partial r}$$

$$\mathbf{\nu = 0.006328\frac{k}{\mu}\frac{\partial p}{\partial r}} \qquad \textbf{(2-29)}$$

Where:

K = permeability, (md).

ν =velocity, (ft/day).

Combining Equation (2-28) with Equation (2-29) results in:

$$\mathbf{\frac{0.006328}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}(r\rho)\frac{\partial p}{\partial r}\right] = \frac{\partial}{\partial t}(\rho\phi)} \qquad \textbf{(2-30)}$$

Expanding the right-hand side by taking the indicated derivatives eliminates the porosity from the partial derivative term on the right-hand side:

$$\frac{\partial(\rho\phi)}{\partial t} = \rho\frac{\partial\phi}{\partial t} + \phi\frac{\partial\rho}{\partial t} \qquad \textbf{(2-31)}$$

Porosity is related to the formation compressibility by the following:

$$C_f = \frac{1}{\phi}\frac{\partial \phi}{\partial p} \quad \textbf{(2-32)}$$

Applying the chain rule of differentiation to $\frac{\partial \phi}{\partial t}$:

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p}\frac{\partial p}{\partial t} \quad \textbf{(2-33)}$$

Substituting Equation (2-32) into equation (2-33):

$$\frac{\partial \phi}{\partial t} = C_f\, \phi \frac{\partial p}{\partial t} \quad \textbf{(2-34)}$$

Since

$$\frac{\partial \phi}{\partial p} = C_f \phi \quad \textbf{(2-35)}$$

Density is related to the formation compressibility by the following:

$$C = \frac{1}{\rho}\frac{\partial \rho}{\partial p} \quad \textbf{(2-36)}$$

Applying the chain rule of differentiation to $\frac{\partial \rho}{\partial t}$:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial p}\frac{\partial p}{\partial t} \quad \textbf{(2-37)}$$

Substituting Equation (2-36) into equation (2-37):

$$\frac{\partial \rho}{\partial t} = C\rho \frac{\partial p}{\partial t} \quad \textbf{(2-38)}$$

Since

$$\frac{\partial \rho}{\partial P} = C\rho \qquad \textbf{(2-39)}$$

Substituting Equations (2-30), (2-34) & (2-38) into equation (2-40):

$$\frac{0.006328}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}(r\rho)\frac{\partial p}{\partial r}\right] = C_f\,\phi\frac{\partial p}{\partial t} + C\rho\frac{\partial p}{\partial t} \qquad \textbf{(2-40)}$$

Equation (2-40) is the general partial differential equation used to describe the flow of any fluid flowing in a radial direction in porous media. In addition to the initial assumptions, Darcy's equation has been added, which implies that the flow is laminar. Otherwise, the equation is not restricted to any type of fluid and equally valid for gases or liquids. Compressible and slightly compressible fluids, however, must be treated separately in order to develop practical equations that can be used to describe the flow behavior of these two fluids.

The treatments of the Radial Flow of Slightly Compressible Fluids:

To simplify Equation (2-40), assume that the permeability and viscosity are constant over pressure, time, and distance ranges and assume that the system is homogenous. This leads to:

$$\left[\frac{0.006328\,k\,\rho}{r\,\mu}\right]\frac{\partial}{\partial r}\left[(r)\frac{\partial p}{\partial r}\right] = C_f\,\rho\phi\frac{\partial p}{\partial t} + C\rho\phi\frac{\partial p}{\partial t}$$

(2-41)

The RHS (Right Hand Side) of the above equation:

$$C_f\,\phi\frac{\partial p}{\partial t} + C\rho\frac{\partial p}{\partial t} = \rho\phi(C_f + C)\frac{\partial p}{\partial t} \qquad \textbf{(2-42)}$$

Define total compressibility C_t

$$C_t = C_f + C \qquad \textbf{(2-43)}$$

Combining Equations (2-41) with (2-42) & (2-43), and rearranging gives:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu C_t}{0\ 000264\ \mathrm{k}}\frac{\partial p}{\partial t} \qquad \textbf{(2-44)}$$

k = permeability, (md).

r = radial position, (ft).

p = pressure, (Pisa).

Ct = total compressibility, (psi^{-1}).

t = time, (Days).

Φ = porosity (Dimensionless).

μ = viscosity, (cp).

The term $[\frac{0\ 000264\ \mathrm{k}}{\phi \mu C_t}]$ (Equation 2-44) is called the diffusivity constant and is denoted by the symbol η, or

$$\eta = \frac{0\ 000264\ \mathrm{k}}{\phi \mu C_t} \qquad \textbf{(2-45)}$$

The diffusivity equation can then be written in a more convenient form as:

$$\frac{\partial^2 \mathrm{p}}{\partial \mathrm{r}^2} + \frac{1}{\mathrm{r}}\frac{\partial \mathrm{p}}{\partial \mathrm{r}} = \frac{1}{\eta}\frac{\partial \mathrm{p}}{\partial \mathrm{t}} \qquad \textbf{(2-46)}$$

The diffusivity equation as represented by Equation (2-44) is essentially designed to determine the pressure as a function of time t and position r.

Chapter 3

Methodology

3. 1. Dimensionless Variable Concept

In simulation and well test analysis often makes use of the concept of the dimensionless variables in solving the unsteady-state flow equation. The importance of dimensionless variables is that they simplify the diffusivity equation and its solution by combining the reservoir parameters (such as permeability, porosity, etc.) and thereby reduce the total number of unknowns. To introduce the concept of the dimensionless, consider for example Darcy's equation in a radial form as:(Ahmed, 2006)

$$Q = 0.00708\frac{kh(p_e - p_{wf})}{\mu_o B_o \ln(\frac{r_e}{r_w})} \quad \textbf{(3-1)}$$

Rearrange the above equation to give:

$$\frac{(p_e - p_{wf})}{(\frac{Q_o B_o \mu_o}{0.00708\ kh})} = \ln(\frac{r_e}{r_w}) \quad \textbf{(3-2)}$$

It is obvious that the right hand side of the above equation has no units (i.e., dimensionless) and, accordingly, the left-hand side must be dimensionless. Since the left-hand side is dimensionless, and (pe - pwf) has the units of psi, it follows that the term [Qo Bo μo/(0.00708kh)] has units of pressure. In fact, any pressure difference divided by [Qo Bo μo/(0.00708kh)] is a dimensionless pressure. Therefore, Equation (2-46) can be written in a dimensionless form as:(Ahmed, 2006)

$$P_D = \ln(r_{eD}) \quad \textbf{(3-3)}$$

Where

$$\mathbf{P_D = \frac{(p_i - p(r.t))}{\left(\frac{Q_o B_o \mu_o}{0.00708\ kh}\right)}} \quad \textbf{(3-4)}$$

And:

$$\boldsymbol{r_{eD} = \frac{r_e}{r_w}} \quad \textbf{(3-5)}$$

$$\boldsymbol{r_D = \frac{r}{r_w}} \quad \textbf{(3-6)}$$

$$\boldsymbol{t_D = \frac{0.0002642\ kt}{\phi \mu_o c_t r_w{}^2}} \quad \textbf{(3-7)}$$

Where:

P_D= dimensionless pressure drop.

$\boldsymbol{r_{eD}}$= dimensionless external radius.

t_D= dimensionless time.

$\boldsymbol{r_D}$= dimensionless radius.

t = time, (Days).

p (r, t) = pressure at radius r and time t.

k = permeability, (md).

μ= viscosity, (cp)

3.2. Finite-Difference Approximation

The numerical method of finite differences will be used to approximate solutions to the mathematical model developed previously in chapter 2. Generally, finite-

difference equations are used to approximate a differential equation when an analytical solution is unknown, or if the known analytical solution is cumbersome in application, because complex systems are normally encountered in reservoir modeling, analytical solutions are seldom available. Consequently, numerical methods are usually required. Inherent in the formulation of finite-difference equations are the process of discretization. There are several ways to discretize a given differential equation; however, the subsequent solution will be unique for the selected discretization technique. Therefore, suitable discretization choices must be made to insure accurate approximations. For reservoir modeling, this will include choices concerning grid type, node location, and others. Hydrocarbon reservoir simulation involves the process of obtaining finite-difference equations (FDE) that approximate a given differential equation.

The finite-difference approach gives us a great deal of flexibility in handling the non-linear partial differential equation, in addition to the property distribution in heterogeneous systems for which an analytical solution is not feasible. The governing equations, as well as the boundary conditions used for describing flow in porous media, have only first-order and second-order derivatives.(Hauss, 1988)

the first-order derivative: -

Forward-Difference Approximation:

$$\left|\frac{\partial p}{\partial x}\right|_i = \frac{p_{i+1} - p_i}{\Delta x} \qquad \textbf{(3-8)}$$

Backward-Difference Approximation:

$$\left|\frac{\partial p}{\partial x}\right|_i = \frac{p_i - p_{i-1}}{\Delta x} \qquad \textbf{(3-9)}$$

Central-Difference Approximation:

$$\left|\frac{\partial p}{\partial x}\right|_i = \frac{p_{i+1} - p_{i-1}}{2\Delta x} \qquad \textbf{(3-10)}$$

Second-order derivative: -

To approximate these second-order derivatives, we use central-difference approximation.

$$\left|\frac{\partial^2 p}{\partial x^2}\right|_i = \frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta x^2} \qquad \textbf{(3-11)}$$

3.3. Constructing the Grid

The grid structure is comprised of concentric elemental cylinders of constant thickness (h), bounded at the wellbore by (r_w) and the outer perimeter of the reservoir by (r_e) .The elemental cylinders will be referred to as blocks and will vary continuously in width from the wellbore to the external boundary of the model. The grid variation selected is logarithmic and is developed through a logarithmic transformation of coordinates. The purpose of the transformation is to systematically provide smaller nodal spacing in the vicinity of the wellbore where pressure gradients are higher while providing larger away from the wellbore where gradients are lower. This type of grid has been termed as an "irregular grid" and its primary application is in modeling radial and spherical flow systems; or in general, systems that require local grid refinements. (Hauss, 1988)

$$U = \ln \frac{r}{r_w} \qquad \textbf{(3-12)}$$

Since pressure is approximately a linear function of the logarithm of radius during unsteady state flow, equally spaced nodes within the U-coordinate system should produce approximately equal pressure drops between nodes in the original (r)

and transformed (U) systems Consequently , the uniform increment between nodes (Δu) will be chosen to satisfy.(Hauss, 1988)

$$\Delta u = \frac{u_e - u_w}{m-1} \qquad \textbf{(3-13)}$$

Where the subscripts (e) and (w) are in reference to the perimeter of the reservoir and to the wellbore, respectively. The term M is the total number of grid block and grid points comprising the radial system.

The logarithmic transformation can be included in the reservoir FDE by transforming the equation from the r-coordinate system to the U-coordinate system. Expressing the reservoir equation in terms of the U-coordinate system will provide a systematic approach for developing a FDE with predominately constant spacing which will consequently(Hauss, 1988)

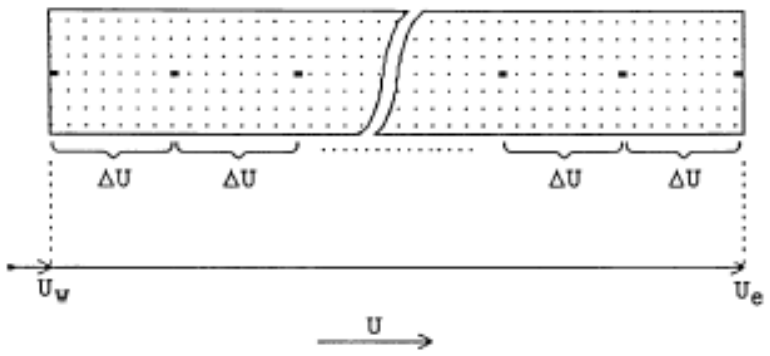

Figure (3-1) constant node spacing, nodes located at u_e and u_w (Hauss, 1988)

Schematic showing the constant node spacing chosen as a result, nodes are located at u_e and u_w

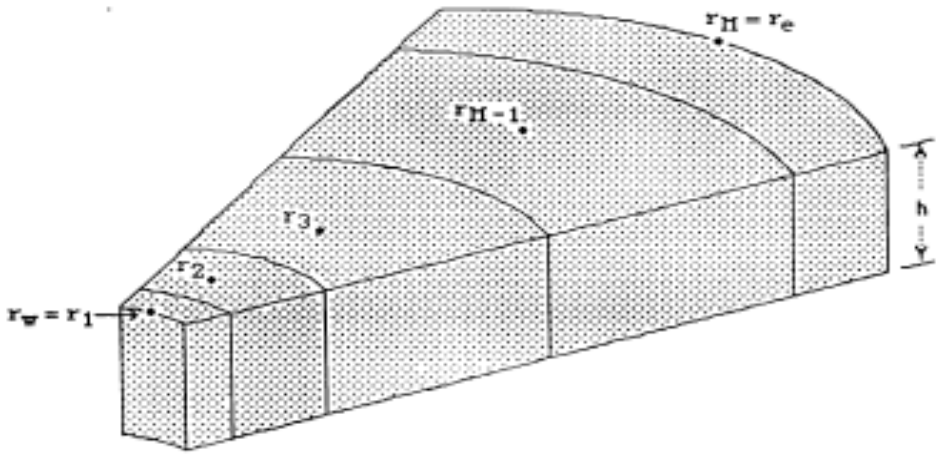

Figure (3-2) actual grid spacing in the cylindrical reservoir(Hauss, 1988)

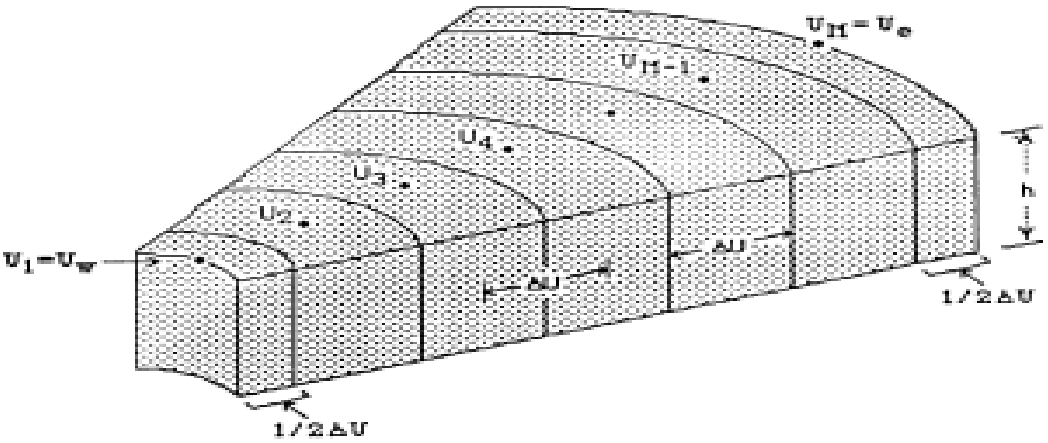

Figure (3-3) transformed grid by local grid refinements(Hauss, 1988)

Illustration of the actual grid Figure (3-2) and the transformed grid Figure (3-3).

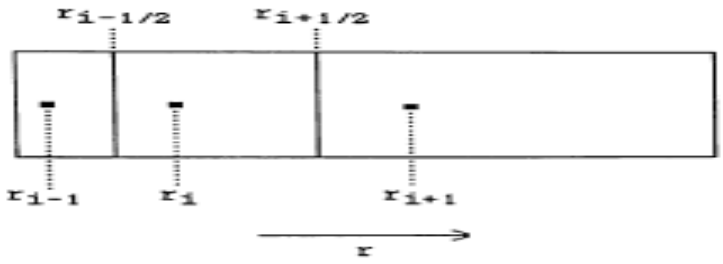

Figure (3-4) adjacent node and interface boundary configuration for the (r) region(Hauss, 1988)

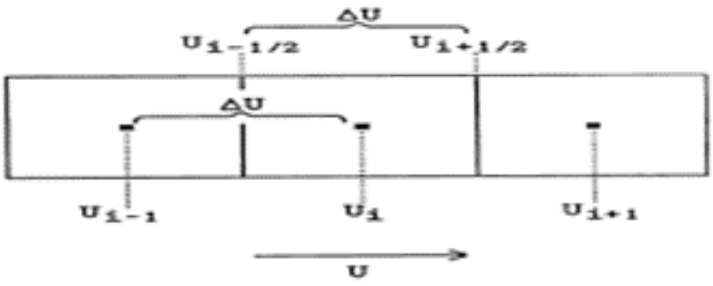

Figure (3-5) adjacent node interface boundary configuration for (U)(Hauss, 1988)

3.4. MATLAB

MATLAB, which is short for Matrix Laboratory, incorporates numerical computation, symbolic computation, graphics, and programming. As the name suggests, it is particularly oriented towards matrix computations, and it provides both state-of-the-art algorithms and a simple, easy to learn interface for manipulating matrices

3.5. Thomas Algorithm

The Thomas algorithm is an efficient way to solving tridiagonal matrix systems. It is a development of Gauss elimination method, used to solve a set of equations by convert them to a tridiagonal matrix system., More information about the general form of tridiagonal matrices is described in Appendix (A).

and figure (3-6) represent the general algorithm in flow chart

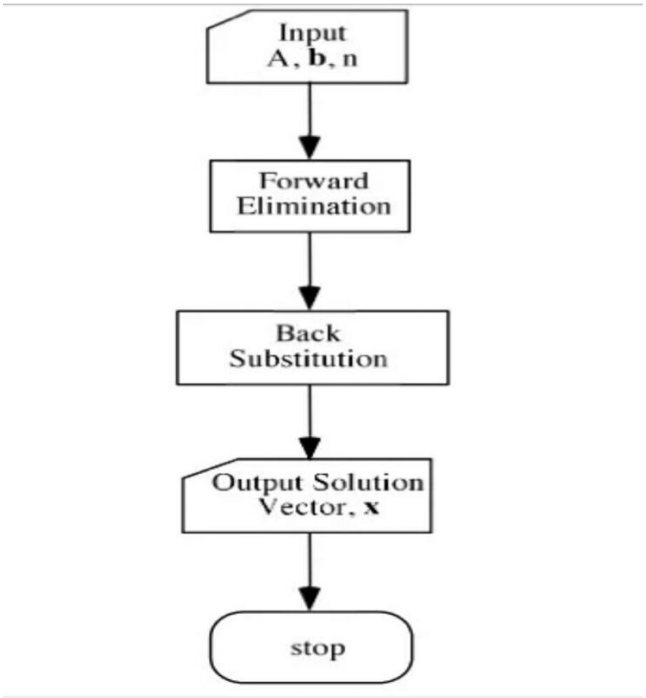

Figure (3-6): the general flow chart of Thomas algorithm

3.6. Analytical Solution

It involves a great deal of assumptions in essence, it renders an exact solution to an approximate problem, to obtain a solution to the diffusivity equation (Equation 2-44), it is necessary to specify an initial condition and impose two boundary conditions. The initial condition simply states that the reservoir is at a uniform pressure (pi) when production begins. The two boundary conditions require that the well is producing at a constant production rate and that the reservoir behaves as if it were infinite in size.(Lee, 1982)

One of the most basic and important problems in petroleum reservoir engineering, and the cornerstone of well-test analysis, is the problem of flow of a single-phase, slightly compressible fluid to a vertical well that is located in an infinite reservoir. This problem can be formulated precisely as follows: -(Lee, 1982)

Geometry: a vertical well that fully penetrates a reservoir which is of uniform thickness (h), and which extends infinitely far in all horizontal directions.(Lee, 1982)

Reservoir Properties: the reservoir is assumed to be isotropic and homogeneous, with constant properties (i.e., permeability, etc.) that do not vary with pressure.

Initial and Boundary Conditions: the reservoir is initially at uniform pressure. Starting at t = 0, fluid is pumped out of the wellbore at a constant rate, Q.

***Initial condition*:** $$\mathbf{P(r.t=0)=P_i} \quad \textbf{(2-13)}$$

***Boundary condition at wellbore*:** $$\lim_{r\to\infty}\left[\frac{2\pi kh}{\mu}r\frac{\partial p}{\partial r}\right]=Q \quad \textbf{(2-14)}$$

Boundary condition at $r=\infty$: $$\lim_{r\to\infty}P(r.t)=P_i \quad \textbf{(2-15)}$$

***Boundary condition at r = re*:** $$\frac{\partial p}{\partial r_e}=0 \quad \textbf{(2-16)}$$

Wellbore diameter: it is assumed that the diameter of the wellbore is infinitely small; this leads to a much simpler problem than the more realistic finite-diameter case.

The basic governing equation for this problem is the diffusion equation in radial coordinates (as it represented in equation 2-41)

$$\frac{1}{r}\frac{\partial}{\partial r}\left[(r)\frac{\partial p}{\partial r}\right]=\frac{\phi\mu C_t}{k}\frac{\partial p}{\partial t}$$

And by introducing the dimensionless concept "because the numerical model will be written on a dimensionless form as will be described in next chapter", and to compare the analytical solution with the numerical one; both must be written in a dimensionless form.

The analytical model can be rewrite in a dimensionless form as follows:

$$\frac{\partial^2 P_D}{\partial {r_D}^2}+\frac{1}{r_D}\frac{\partial P_D}{\partial r_D}=\frac{\partial P_D}{\partial t_D} \quad (3\text{-}14)$$

Van Everdingen and Hurst (1949) proposed an analytical solution to the above equation by assuming: (Lee, 1982)

- Perfectly radial reservoir system
- The producing well is in the center and producing at a constant production rate
- Uniform pressure pi throughout the reservoir before production
- No flow across the external radius re

Van Everdingen and Hurst presented the solution to Equation (3-44) in a form of infinite series of exponential terms and Bessel functions. The authors evaluated this series for several values of $\mathbf{r_{eD}}$over a wide range of values fort_D . Chatas (1953) and Lee (1982) conveniently tabulated these solutions for the following two cases:

(A) Infinite-acting reservoir:

i.e., $\mathbf{r_{eD}} = \infty$, the dimensionless pressure drop function p_D is strictly a function of the dimensionless time t_D, or: (Lee, 1982)

$$\mathbf{p_D = f(t_D)}$$

1-$\mathbf{p_D = \sqrt{t_D/2}}$ **at** $\mathbf{t_D < 0.01}$ **(3-15)**

2-$\mathbf{p_D = 0.5\ln(\ln(t_D) + .080907)}$ **at** $\mathbf{t_D > 100}$ **(3-16)**

3-for $0.02<t_D<1000$:

$$\mathbf{p_D = a_1 + a_2\ln(t_D) + a_3(\ln(t\_D))^2 + a_4(\ln(t\_D))^3 + a_5 t_D + a_6(t_D)^2 + a_7(t_D)^3 + a_7/t_D} \quad \textbf{(3-17)}$$

Where:

$\mathbf{a_1} = 0.8085064$ $\mathbf{a_2} = 0.29302022$ $\mathbf{a_3} = 3.5264177(10^{-2})$

$\mathbf{a_4} = -1.4036304(10^{-3})$ $\mathbf{a_5} = -4.7722225(10^{-4})$ $\mathbf{a_6} = 5.1240532(10^{-7})$

$\mathbf{a_7} = -2.3033017(10^{-10})$ $\mathbf{a_8} = -2.6723117(10^{-3})$

Chatas and Lee tabulated the p_D values for the infinite-acting reservoir

(B) Finite-radial reservoir:

For a finite radial system, the p_D-function is a function of both the dimensionless time and radius, or:(Lee, 1982)

$$p_D = f\left(t_D, , r_{eD}\right)$$

1- For $t_D > 25$ and $r_D^2 >> 1$

$$p_D = \frac{2t_D}{r_{eD}^2} + \ln(r_D) - 0.75 \qquad \textbf{(3-18)}$$

2-for $25 < t_D$ and $0.25\ r_{eD}^2 < t_D$

$$p_D = \frac{0.5 + 2t_D}{r_{eD}^2 - 1} - \frac{r_{eD}^4[3 - 4\ln(r_{eD})] - 2r_{eD}^2 - 1}{4(r_{eD}^2 - 1)^2} \qquad \textbf{(3-19)}$$

Chapter 4

Results and discussion

4.1. The solution of the model

The model that discussed in chapter 2, we are going to solve it by using the tools that presented in chapter 3 as following:

4.1.1. Dimensionless Variable Concept:

By applying equations (3-4), (3-5), (3-6), (3-7):

$$\mathbf{P_D} = \frac{(\mathbf{p_i} - \mathbf{p(r.t)})}{(\frac{\mathbf{Q_o B_o \mu_o}}{\mathbf{0.00708\ kh}})} \quad \textbf{(3-4)}$$

$$\boldsymbol{r_{eD}} = \frac{\boldsymbol{r_e}}{\boldsymbol{r_w}} \quad \textbf{(3-5)}$$

$$\boldsymbol{r_D} = \frac{\boldsymbol{r}}{\boldsymbol{r_w}} \quad \textbf{(3-6)}$$

$$\boldsymbol{t_D} = \frac{\mathbf{0.0002642}\ \boldsymbol{kt}}{\boldsymbol{\phi \mu_o c_t {r_w}^2}} \quad \textbf{(3-7)}$$

into the model on equation (2-44):

$$\frac{\boldsymbol{\partial^2 p}}{\boldsymbol{\partial r^2}} + \frac{\mathbf{1}}{\boldsymbol{r}} \frac{\boldsymbol{\partial p}}{\boldsymbol{\partial r}} = \frac{\boldsymbol{\phi \mu C_t}}{\mathbf{0\ 000264\ k}} \frac{\boldsymbol{\partial p}}{\boldsymbol{\partial t}} \quad \textbf{(2-44)}$$

we can rewrite the equation (2-44):

$$\frac{\mathbf{1}}{\boldsymbol{r}} \frac{\boldsymbol{\partial p}}{\boldsymbol{\partial r}} + \frac{\boldsymbol{\partial^2 p}}{\boldsymbol{\partial r^2}} = \frac{\boldsymbol{\phi \mu C_t}}{\mathbf{k}} \frac{\boldsymbol{\partial p}}{\boldsymbol{\partial t}} \quad \textbf{(4-1)}$$

Multiplying and dividing both side by $\frac{2\pi kh}{q\beta\mu}$

$$\frac{2\pi kh}{q\beta\mu}\left(\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}\right)=\frac{2\pi kh}{q\beta\mu}\left(\frac{\phi\mu C_t}{\mathrm{k}}\frac{\partial p}{\partial t}\right) \quad \textbf{(4-2)}$$

$$\frac{1}{r}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial r}+\left(\frac{\partial^2\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial r^2}\right)=\frac{\phi\mu C_t}{\mathrm{k}}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial t} \quad \textbf{(4-3)}$$

$$\frac{1}{\left(\frac{r_w}{r_w}\right)r}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\left(\frac{r_w}{r_w}\right)\partial r}+\left(\frac{\partial^2\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\frac{{r_w}^2}{{r_w}^2}\partial r^2}\right)=\frac{\phi\mu C_t}{\mathrm{k}}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial t} \quad \textbf{(4-4)}$$

$$\left(\frac{1}{\left(\frac{r}{r_w}\right)}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial\left(\frac{r}{r_w}\right)}+\frac{\partial^2\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\frac{{r_w}^2}{{r_w}^2}\partial r^2}\right)=\frac{\phi\mu C_t}{\mathrm{k}}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial t} \quad \textbf{(4-5)}$$

$$\left(\frac{1}{\left(\frac{r}{r_w}\right)}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial\left(\frac{r}{r_w}\right)}+\frac{\partial^2\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial\left(\frac{r^2}{{r_w}^2}\right)}+\right)=\frac{\phi\mu C_t\,{r_w}^2}{\mathrm{k}}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial t} \quad \textbf{(4-6)}$$

$$\left(\frac{1}{\left(\frac{r}{r_w}\right)}\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial\left(\frac{r}{r_w}\right)}+\frac{\partial^2\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial\left(\frac{r^2}{{r_w}^2}\right)}\right)=\frac{\partial\left(\frac{2\pi kh}{q\beta\mu}p\right)}{\partial\left(\frac{kt}{\phi\mu C_t\,{r_w}^2}\right)} \quad \textbf{(4-7)}$$

$$\frac{1}{r_D}\frac{\partial P_D}{\partial r_D}+\frac{\partial^2 P_D}{\partial {r_D}^2}=\frac{\partial P_D}{\partial t_D} \quad \textbf{(4-8)}$$

The above dimensionless groups (i.e., P_D, t_D, and $\boldsymbol{r_D}$) can be introduced into the diffusivity equation (Equation 2-44) to transform the equation into the following dimensionless form:

$$\frac{\partial^2 P_D}{\partial {r_D}^2}+\frac{1}{r_D}\frac{\partial P_D}{\partial r_D}=\frac{\partial P_D}{\partial t_D} \quad \textbf{(4-9)}$$

4.1.2. Finite-Difference Approximation and Constructing the Grid

The numerical solution of the dimensionless model (equation 4-9) by using the concepts of finite difference approximation and constructing the grids represented at the previous chapter, for finite and infinite reservoir with specific initial and boundary conditions

$$\frac{\partial^2 p_D}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial p_D}{\partial r_D}=\frac{\partial P_D}{\partial t_D} \quad \textbf{(4-10)}$$

But
$$u=\ln\frac{r}{r_w}=\ln r_D \quad \textbf{(4-11)}$$

$$\partial r_D=\partial u e^u \quad \textbf{(4-12)}$$

Rearrange the equation:

$$\frac{1}{e^u}\frac{\partial^2 p_D}{\partial u^2}+\frac{1}{e^{2u}}\frac{\partial P_D}{\partial u}=\frac{\partial P_D}{\partial t_D} \quad \textbf{(4-13)}$$

Using Finite-Difference Approximation:

$$\frac{1}{e^u\Delta u^2}\left[p_{i-1}^{n+1}-2p_i^{n+1}+p_{i+1}^{n+1}\right]+\frac{1}{2e^{2u}\Delta u}\left[p_{i+1}^{n+1}-p_{i-1}^{n+1}\right]=\frac{1}{2\Delta t}\left[p_i^{n+1}-p_i^{n+1}\right] \quad \textbf{(4-14)}$$

$$\frac{2\Delta t}{e^{u}\Delta u^{2}}\left[p_{i-1}^{n+1}-2p_{i}^{n+1}+p_{i+1}^{n+1}\right]+\frac{\Delta t}{e^{2u}\Delta u}\left[p_{i+1}^{n+1}-p_{i-1}^{n+1}\right]-p_{i}^{n+1}=-p_{i}^{n+1} \quad \textbf{(4-15)}$$

$$a_{i}^{n}p_{i-1}^{n+1}+b_{i}^{n}p_{i}^{n+1}+c_{i}^{n}p_{i+1}^{n+1}=d_{i}^{n} \quad \textbf{(4-16)}$$

Where:

$$a=\frac{\Delta u\Delta t-2e^{u}\Delta t}{\Delta u^{2}e^{2u}} \quad \textbf{(4-17)}$$

$$b=\frac{\Delta u\Delta t}{\Delta u^{2}e^{u}}+1 \quad \textbf{(4-18)}$$

$$C=-\left(\frac{2e^{u}\Delta t+\Delta u\Delta t}{\Delta u^{2}e^{2u}}\right) \quad \textbf{(4-19)}$$

$$d=p_{i}^{n} \quad \textbf{(4-20)}$$

Based on initial and boundary conditions for both models:

***Initial condition*:** $P(r.t=0)=P_{i}$ **(2-13)**

***Boundary condition at wellbore*:** $\lim_{r\to\infty}\left[\frac{2\pi kh}{\mu}r\frac{\partial p}{\partial r}\right]=Q$ **(2-14)**

***Boundary condition at r = ∞(Infinity)* :** $\lim_{r\to\infty}P(r.t)=P_{i}$ **(2-15)**

***Boundary condition at r = re(Finite)*:** $\frac{\partial p}{\partial r_{e}}=0$ **(2-16)**

As shown in figure (4-1) and (4-2).

By introducing the dimensionless concept, and finite difference approximation we can rewrite the reservoir condition as:

Initial condition: $\mathbf{t = 0 \quad \rightarrow \quad t_D = 0}$

$$\mathbf{p_D = \frac{k\,h\,\Delta p}{141.5\,q\,\mu\,\beta}}$$

$$\mathbf{P_D(r_D\,.\,0) = 0} \qquad \textbf{(4-21)}$$

$$\mathbf{p_i^n = 0} \qquad \textbf{(4-22)}$$

By applying (4-16) on (4-14) we obtained:

a=0 **b=0** **c= 0** **d=0**

***Boundary condition at wellbore*:** $\mathbf{r = 0 \rightarrow \frac{\partial P_D}{\partial r_D} = 1}$

$$\mathbf{p_i^n - p_{i-1}^n = \Delta u e^u} \qquad \textbf{(4-23)}$$

By applying (4-17) on (4-14) we obtained:

a=0 **b=1** **c=-1** **d=$\Delta u e^u$**

Outer *Boundary condition*

A) ***Boundary condition at infinity*:**

$$\mathbf{r = r_e \rightarrow P_D\,(r_D\,.\,t_D) = 0}$$

$$\mathbf{p_i^n = 0} \qquad \textbf{(4-24)}$$

By applying (4-20) on (4-16) we obtained:

a=0 **b=0** **c= 0** **d=0**

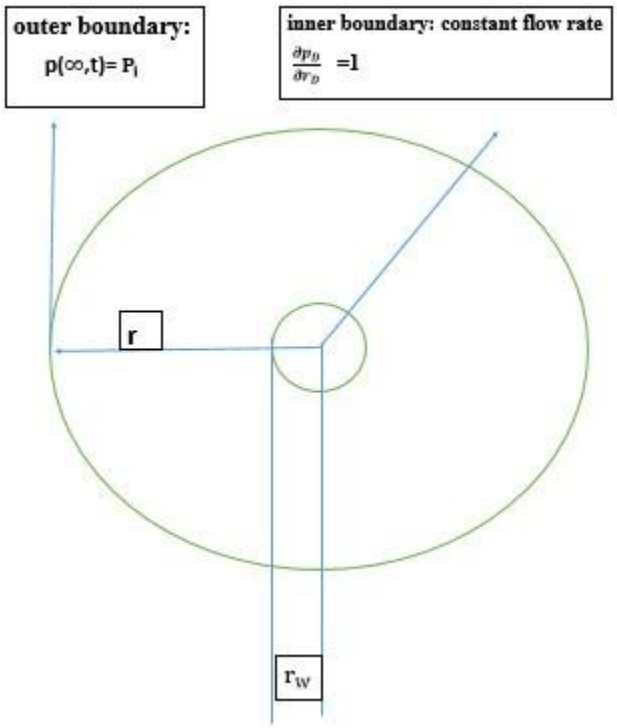

Figure (4-1): inner and outer condition for infinite reservoir

B) **Boundary condition at finite:**

$$\frac{\partial p}{\partial r_e} = 0 \rightarrow \frac{\partial p_D}{\partial r_{eD}} = 0$$

$$p_{i+1}^{n} - p_i^{n} = 0 \qquad \textbf{(4-25)}$$

By applying (4-21) on (4-16) we obtained:

a=1 **b=-1** **c= 0** **d=0**

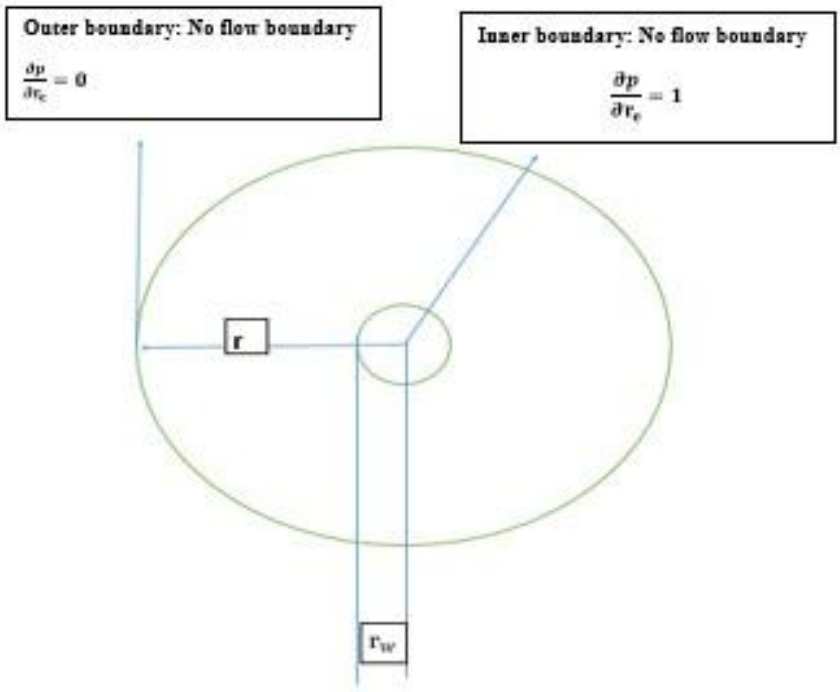

Figure (4-2): inner and outer condition for finite reservoir

Now equation (4-16) has four cases of constants (a, b, c, and d), then can develop a series of equations depending on this cases.

Initial condition:

a=0 b=0 c= 0 d=0

Boundary condition at wellbore

a=0 b=1 c=-1 d=Δue^{u}

Boundary condition at infinity

a=0 b=0 c= 0 d=0

***Boundary condition at finite*:**

a=1 b=-1 c= 0 d=0

4.2. Matlab programming

After solve differential equations numerically and replace the derivatives in the equation with finite difference approximations, this results in a number of algebraic equations that has been solved simultaneously (implicit methods), two self-designed programs (using Matlab codes) programmed by Matlab software are used to solve these sets of equations to obtain the results. The codes have been written for infinite and finite model, to solve difficult and complicated calculations, the first program code called "Infinite program", and the other called "Finite program". Each of this programs will be discussed carefully below:

4.2.1. Infinite program

4.2.1.1. Results of " Infinite program "

Based on boundary condition for Infinite reservoir on Equation (4-20) infinite program has been written based on Thomas algorithm, Image (4-1) shows the user interface of the program, the interface contained two parts; input part (dimensionless time, number of grids, time step, and the name of excel sheet where the output data will be saved on it). and output part (graphical representation of the output data, dimensionless bottom hole pressure, and the output data on excel sheet), The infinite program has been run many times to obtain the results which it’s represented in appendix D (table 1).

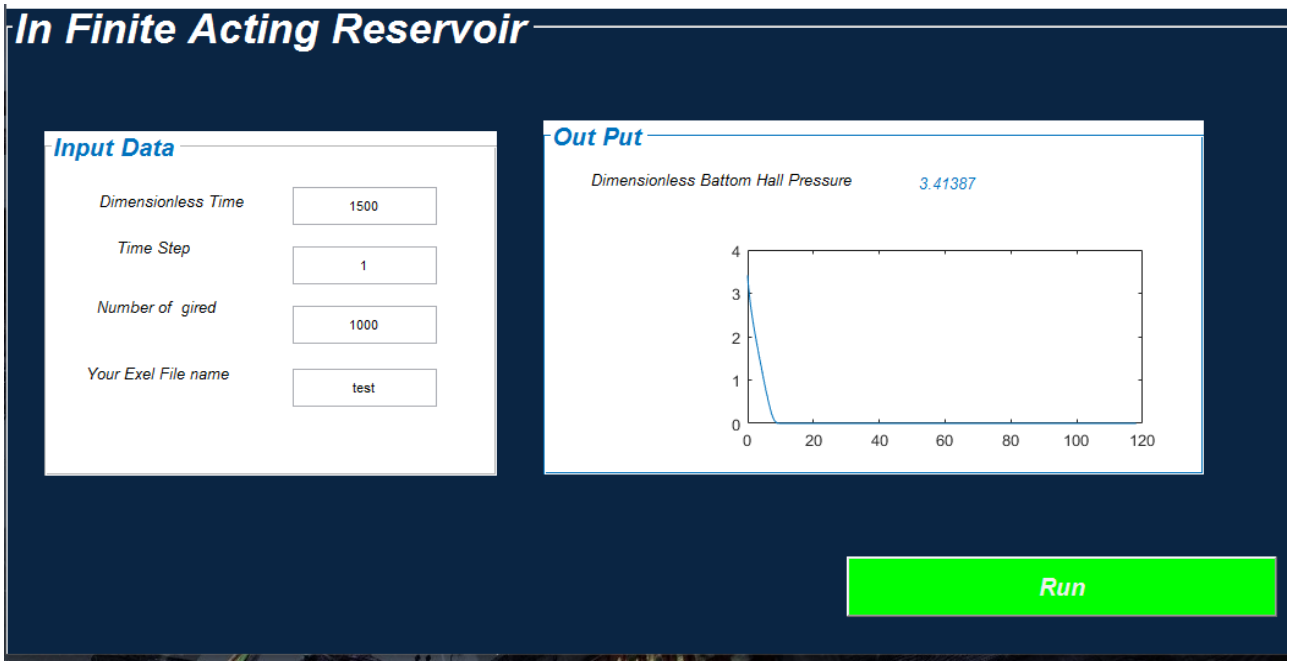

Image (4-1): infinite program user interface

4.2.1.2. Validation of "Infinite program":

check the validity of the solution obtained by comparing with the analytical one -Van Everdingen and Hurst solution-which posted in 'LEE, J. 1982. Well Testing (SPE Textbook Series). Society of Petroleum Engineers, Richardson, TX.'. As previously presented in chapter three in equations (3-14), (3-15), (3-16):

1-$\mathbf{p_D = \sqrt{t_D/2}}$ **at** $\mathbf{t_D < 0.01}$ **(3-14)**

2-$\mathbf{p_D = 0.5\ln(\ln(t_D) + .080907)}$ **at** $\mathbf{t_D > 100}$ **(3-15)**

3-for $0.02<t_D<1000$:

$$\mathbf{p_D = a_1 + a_2 \ln(t_D) + a_3(\ln(t\_D))^2 + a_4(\ln(t\_D))^3 + a_5 t_D + a_6(t_D)^2 + a_7(t_D)^3 + a_7/t_D}$$

(3-16)

These equations have been tested and compared with the "infinite program" as flowing:

1-equation (3-15) has been tested and have an error ($4*10^{-3}$).

2-equation (3-15) at t_D >100 has been tested and have an error ($3*10^{-3}$)..

3-equation (3-16) at t_D <100 has been tested and have an error ($1* 10^{-3}$).

4-The table (4-1) show an error ($2.05*10^{-3}$) from the posted solation on (SPE Textbook Series). (Society of Petroleum Engineers, Richardson, TX) and Figure (4-3) represent the table data graphically "from appendix D table 1".

Infinite Program		**John Lee**	
t_D	p_D	t_D	p_D
1	0.8015	1	0.8019
2	1.0191	2	1.0195

Table (4-1): Comparison with John lee to infinite reservoir.

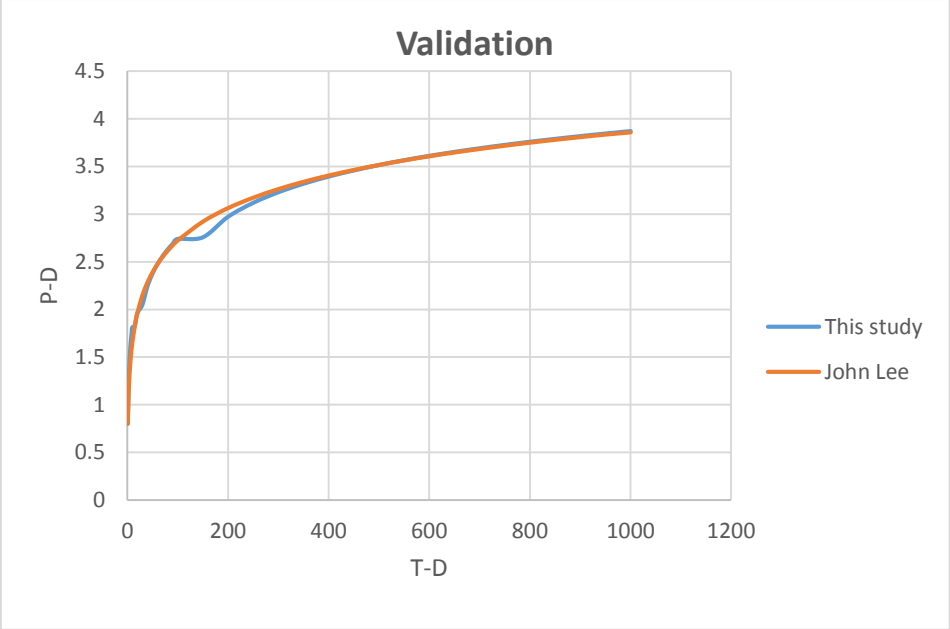

Figure (4-3): Graphically comparison with John lee "from appendix D table 1"

4.2.1.3. Discussion of "Infinite program"

The infinite program has an average error ($2.05*10^{-3}$) from analytical solution, Figure(4-4) shows an infinite reservoir for different dimensionless times, by increase (t_D)the dimensionless bottom hole pressure (P_{wf_D}) increases due to depletion in the reservoir, and reservoir pressure at outer boundary is the initial reservoir pressure (P_{i_D}=0) because it is infinite acting reservoir.

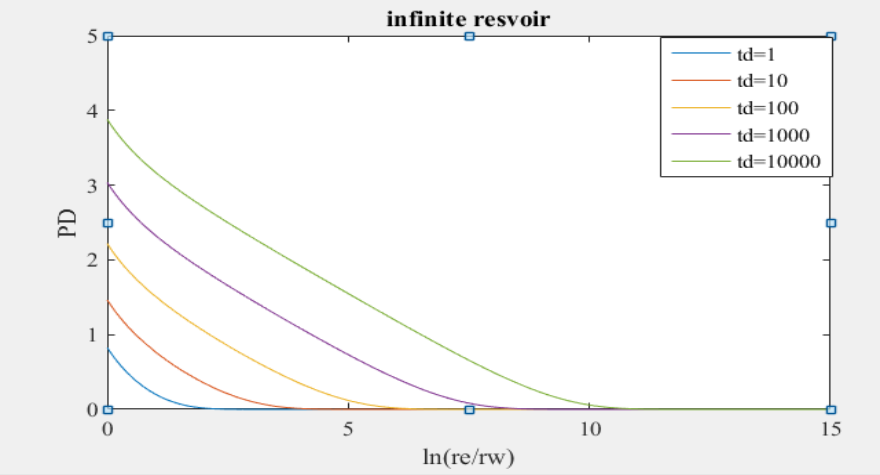

Figure (4-4): describe Infinite reservoir for varies time.

4.2.2. Finite program

4.2.2.1. Result of " Finite program "

Based on boundary condition for finite reservoir on Equation (4-21) finite program has been written based on Thomas algorithm, Image (4-2) shows the user interface of the program, the interface contained two part; input part (dimensionless time, dimensionless radius, number of grids, time step and the name of excel sheet of the output data) and output part (graphical representation of the output data ,dimensionless bottom hole pressure, and the output data on excel sheet) ,the "finite program" has been run many times to obtain the results in which it's represented in "appendix D" (table 2).

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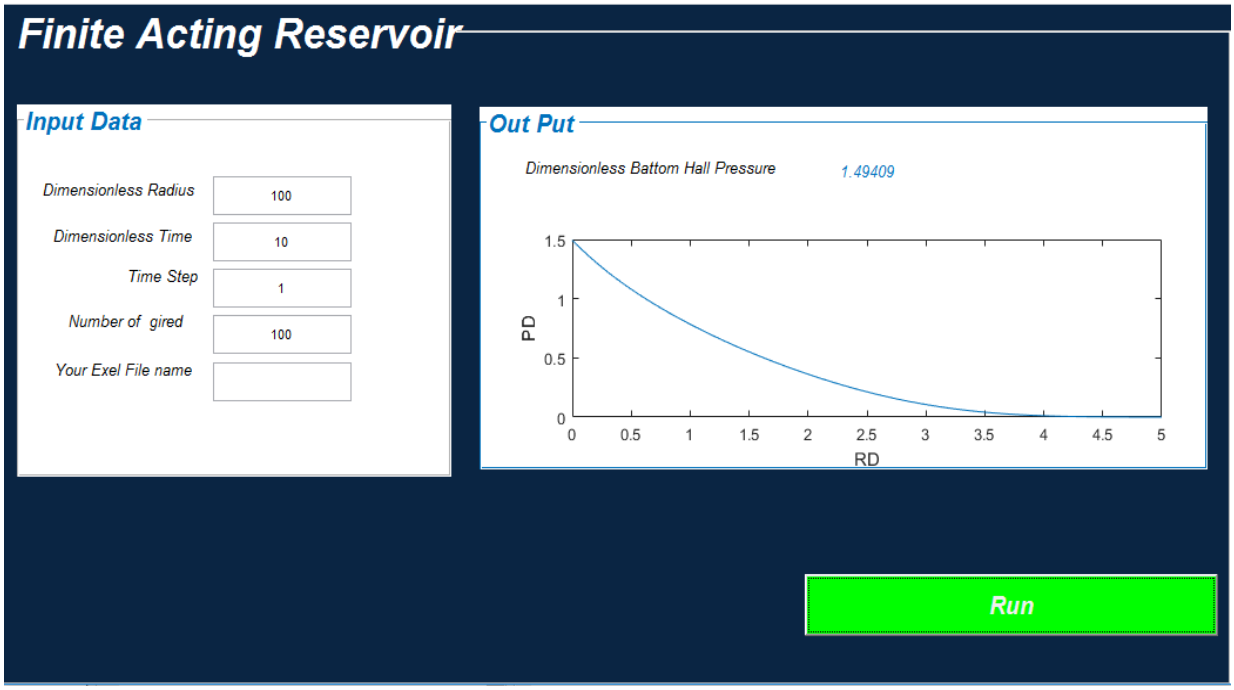

Image (4-2): Finite program user interface.

4.2.2.2. Validation of "Finite program":

check the validity of the solution obtained by comparing with the analytical one - Van Everdingen and Hurst -which posted in 'LEE, J. 1982. Well Testing (SPE

Textbook Series). (Society of Petroleum Engineers, Richardson, TX.' As previously presented in chapter three in equations (3-17), and (3-18):

1- For t_D>25 and $r_D{}^2$>>1

$$\mathbf{p_D=\frac{2t_D}{r_{eD}{}^2}+\ln(r_D)\ -0.75} \qquad (3\text{-}17)$$

2-for 25<$t_D$and 0.25 $r_{eD}{}^2$<t_D

$$\mathbf{p_D = \frac{0.5+2t_D}{r_{eD}{}^2-1} - \frac{r_{eD}{}^4[3-4\ln(r_{eD})]-2r_{eD}{}^2-1}{4(r_{eD}{}^2-1)^2}} \qquad (3\text{-}18)$$

These equations have been tested and compared with the "Finite program" as flowing:

1-For t_D>25 and $r_D{}^2$>>1

equation (3-52) has been tested and have an error $2*10^{-3}$.

2-for 25<$t_D$and 0.25 $r_{eD}{}^2$<t_D

equation (3-53) has been tested and have an error $2.96*10^{-3}$.

3-The table (4-2) show an error ($5.2*10^{-3}$) from posted solation on (SPE Textbook Series). Society of Petroleum Engineers, Richardson, TX) and Figure (4-5) represent the table data graphically "from appendix D table 2".

	John lee		**Finite Program**		
	TD	PD	TD	PD	Error
$r_{eD} = 1.5$	0.06	0.251	0.06	0.2562	0.0052
	0.08	0.288	0.08	0.2892	0.0012

Table (4-2): Comparison with John lee to finite reservoir

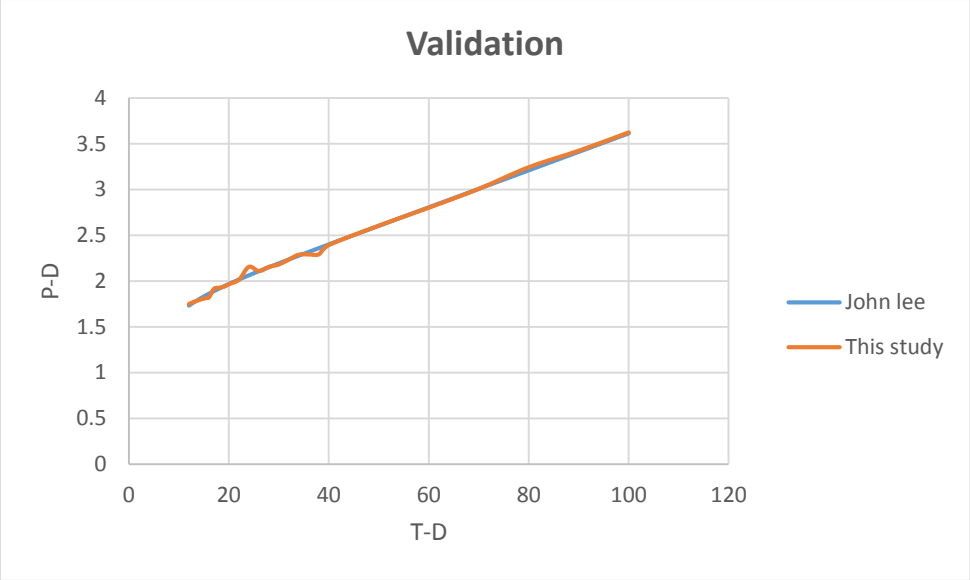

Figure (4-5): Graphically comparison with John lee "from appendix D table 2"

4.2.2.3. Discussion of "Finite program"

The finite program has an average error($5.2*10^{-3}$) from analytical solution Figure (4-6) show an finite reservoir for different dimensionless time ,by increase (t_D)the dimensionless bottom hole pressure (P_{wf_D}) increase due depletion in the reservoir, and reservoir pressure at outer boundary is the initial (P_{i_D}=0) reservoir pressure because it behaved as infinite acting reservoir in large dimensionless Reduces and long time.

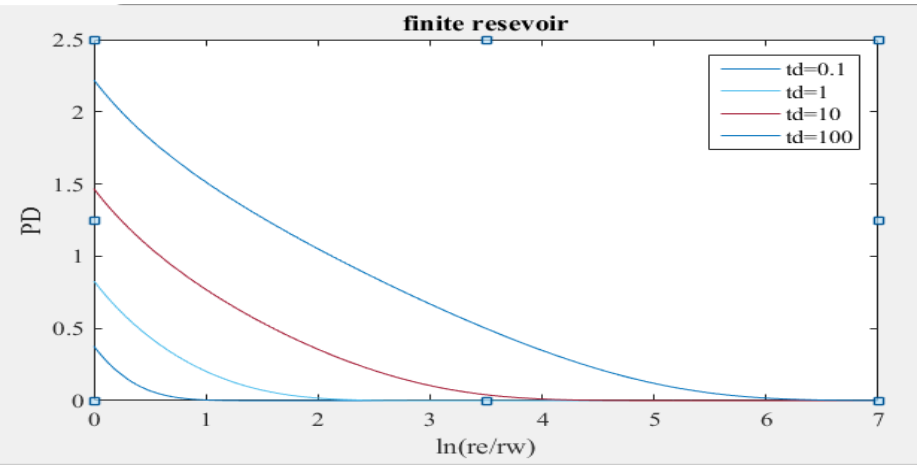

Figure (4-6): describe the finite reservoir for varies times.

Chapter 5

Conclusion and Recommendation

5.1. Conclusion:

The model has been solved under two conditions based on dimensionless concept, and it has accepted error for the two types of reservoir -boundary condition $-2.05*10^{-3}$ for infinite boundary and $5.2*10^{-3}$ for finite boundary.

in implicit method there is stabilizing in results and there isn't any problem with it.

5.2. Recommendation:

We recommended to develop this software by reducing the model assumption:

1- it can be two phase model to be more representative to the reservoir reality (usually the reservoir contains more than one phase).
2- or it can be heterogenic (two dimensions).
3- or even both.

by applying this recommendations, it could be a quite big software based on a dimensionless concept, in next step it could resolved that model(finite&infinite) in two dimensions after rewrite the code in two phase and make the system heterogenic.

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Appendixes

Appendix A:

The Thomas algorithm general form is:

$$\begin{bmatrix} a(1) & c(1) & & & & \\ b(2) & a(2) & c(2) & & & \\ & b(3) & a(3) & c(3) & & \\ & \ldots & \ldots & \ldots & & \\ & & \ldots & \ldots & \ldots & \\ & & & b(n-1) & a(n-1) & c(n-1) \\ & & & & b(n) & a(n) \end{bmatrix} \begin{bmatrix} y(1) \\ y(2) \\ \\ \ldots \\ \\ y(n-1) \\ y(n) \end{bmatrix} = \begin{bmatrix} f(1) \\ f(2) \\ \\ \ldots \\ \\ f(n-1) \\ f(n) \end{bmatrix}$$

$$c_{i=}\begin{cases} \dfrac{c_i}{b_i} & ; \quad i = 1 \\ \dfrac{c_i}{b_i - a_i c_{i-1}} & ; i = 1,2,3, \ldots . n-1 \end{cases}$$

And

$$d_{i=}\begin{cases} \dfrac{d_i}{b_i} & ; \quad i = 1 \\ \dfrac{d_i - a_i d_{i-1}}{b_i - a_i c_{i-1}} & ; i = 1,2,3, \ldots . n-1 \end{cases}$$

The solution is then obtained by back substitution:

$$x_n = d_n$$

$x_i = d_i - c_i x_{i+1}$ i=n-1, n-2,,1

Appendix B

The code of" infinite program "

```
e=ceil(log(re));

 du=ue/nog;

for t=dt:dt:te

z=length(a);

end

g(1)=d(1)/b(1);

n=length(d);

w(1)=c(1)/b(1);

for i=2:n

end

x(n)=g(n);

for i=n-1:-1:1

    x(i)=g(i)-(w(i)*x(i+1));

end

d=x';

d(1,1)=exp(u(1,1))*du;

end

gg=x';
```

```
plot(u,x)

xlabel('RD')

ylabel('PD')

pwf=x(1);

set(handles.pwf,'string',pwf)
```

Appendix C

The code of "finite program"

```
re =str2double(get(handles.re,'string'));

te=str2double(get(handles.te,'string'));

dt =str2double(get(handles.dt,'string'));

nog =str2double(get(handles.nog,'string'));

fn=(get(handles.fn,'string'));

 ue=ceil(log(re));

 du=ue/nog;
```