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LINEAR PROGRAMMING BASED OPTIMAL POWER FLOW

التدفق الأمثل للقدرةِ الكهربائيةِ بِناءاً على إستخدام البرمجةِ الخطيةِ

A Thesis submitted to Sudan University of Science and Technology in partial fulfillment for the requirements of the degree of M.Sc. in Electrical Engineering (Power)

Researcher:

Momen Ahmed Dahab Abdulatif

Supervisor:

Dr. Elfadil Zakaria Yahia

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الآيه

قال تعالى:

(قَالُواْ سُبْحَانَكَ لاَ عِلْمَ لَنَا إِلاَّ مَا عَلَّمْتَنَا إِنَّكَ أَنتَ الْعَلِيمُ الْحَكِيمُ)

صدق الله العظيم

سورة البقرة الآيه 32

DEDICATION

I dedicate all this work to my parents.

Specially the person who carried me for nine months, the person who raised me with nap less, the person who sacrificed with everything to made me Engineer.

Momen Dahab.

My beloved mother

Muna Zyada Hussien

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ABSTRACT

The Optimal Power Flow Problem is a large and complicated non-linear optimization problem in power system transmission, design, planning and operation subjected to various types of constraints. This research focuses on the understanding of Optimal Power flow (OPF) using Linear programming (LP) optimization method, firstly, the OPF problem is discussed in a literature manner in view of the historical review, problem formulation and the different methods that used in order to solve the OPF problem, then a detailed illustration of LP as an optimization tool, likewise, Linear programming Optimal Power Flow (LPOPF) using Piecewise linear approach and the full AC Incremental LP method illustration and mathematical formulation was presented, moreover a conceptual review of reactive power pricing and a proposed formulation of including the VAR cost function to the objective function was presented, lastly, a brief illustration about the Locational Marginal Prices (LMPs) and an implementation of both methods using *POWERWORLD* Simulator and Microsoft Excel on the 6-bus test system using step by step procedure and the IEEE 30 bus system was made, and then a comparison between both methods before and after the OPF and before and after the inclusion of the VAR cost function was presented.

مُسْتَخْلَص

التدفق الأمثل للقدرةِ الكهربائيةِ عباره عن معضلةٍ ضخمةٍ، معقدةٌ و غير خطيةٍ تُواجَه في تصميم، تخطيط و تشغيل منظومات القدرةِ الكهربائيةِ. هذا البحث يركز على توضيح و شرح التدفق الأمثل للقدرةِ الكهربائيةِ عن طريق إستخدام البرمجةِ الخطيةِ، أولاً، تم التطرق لِمعضلةِ تدفق القدرةِ الكهربائيةِ الأمثل بِمراجعات أدبيةٍ و تاريخيةٍ للمعضله وطرق الإستمثال الرياضي المختلفةِ المستخدمه في حل مشكلةِ التدفق الأمثل للقدرةِ الكهربائيةِ ومن ثم تمت صياغة المشكلةِ رياضياً. تم شرح طريقة البرمجه الخطيه كأداة في علم الإستمثال الرياضي بإسهاب، أيضاً تم شرح الصياغةِ الرياضيةِ للطرق المبنيةِ على البرمجةِ الخطيةِ في حل معضلة التدفق الأمثل للقدرةِ الكهربائيةِ (Piecewise Linear approach and Incremental LP method) بالإضافه إلى توضيح نظري لتسعير تدفق القدرةِ غير الفعالةِ مع مقترح لتضمين دالة التكلفة لتدفق وتوليد القدرةِ غير الفعالةِ إلى الدالة الرئيسية (دالة تكلفة القدرة الفعالة)، و أخيراً، شرح مبسط لتأثير إضافة أو إزالة حمل في منظومة القدرةِ الكهربائيةِ على التكلفةِ العامه لتوليد و سريان القدرةِ (Locational Marginal Prices). وبإستخدام برنامج المحاكاة (*POWERWORLD* Simulator) والبرنامج متعدد المهام (Microsoft Excel)، تم تطبيق الطريقتين أعلاه على منظومتين، الأولى تتكون من ستة قضبان توصيل و هنا تم شرح طريقة تطبيق تدفق القدرةِ الأمثل بإستخدام الطريقتين بإسهاب مع إتباع الخطوات بالتفصيل، المنظومةُ الثانيةُ تتكون من ثلاثين قضيب للتوصيل وأيضا تم عليها تطبيق كلتي الطريقتين، و من ثم تمت المقارنةُ بين الطريقتين و بين قبل و بعد تضمين دالة التكلفةِ لتدفق القدرةِ غير الفعالةِ للدالةِ الرئيسيةِ.

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CHAPTER FIVE

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LIST OF ABBREVIATIONS

AC	Alternating Current.
AVR	Automatic Voltage Regulator.
BP	Break Point.
ED	Economic Dispatch.
FACTS	Flexible Alternating Current Transmission System.
GSF	Generation Shift Factor.
IC	Incremental Cost.
I/O	Input/output.
IEEE	Institute of Electrical and Electronics Engineers.
LMP	Locational Marginal Price.
LP	Linear Programming.
LPOPF	Linear Programming Optimal Power Flow.
MVA	Mega Volt-Ampere.
MVAR	Mega Volt-Ampere Reactive.
MW	Mega Watt.
N-R	Newton-Raphson.
OPF	Optimal Power Flow.
PF	Power Flow.
PF0	Initial Power Flow.
PW	Piecewise.
TLR	Transmission Loading Relief.
VAR	Volt-Ampere Reactive.

CHAPTER ONE

INTRODUCTION

1.1 Introduction:

Before the invention of the optimal power flow, the economic dispatch (ED) was used to determine the optimum (best) way to share the real load between several thermal generating units having a total capacity greater than the generation required [1]. Best or optimum way incomes the scheduling of these units to meet the minimum generation cost with respect to a constraint that the total generation must equal to total demand plus losses.

Till the early of 1960s, and when the use of the network being close to their limit, line overloading became a real problem and threatening to the economical dispatched power systems, therefore a more constraints were introduced to insure the security of the system and then the optimal power flow (OPF) was presented. An optimal power flow is defined in [1] as "the determination of the complete state of the power system with an optimum operation within security constraints". Optimum for the minimum fuel cost and security for operating at that optimal point without a violation of any constraint, these constraints may be represented as the real and reactive power generation limits, bus voltage limits, transformer tab ratios, phase shift limits, transmission line limits and possibly the emission constraints and this made the problem larger and more complicated. However, this is solved by using an optimization mathematical tool plus power flow calculation.

Optimization is defined in [2] as "the process of minimizing or maximizing an objective functional", this process is done through a mathematical optimization tool such as linear optimization, non-linear optimization, and many other techniques. Linear optimization is done through linear programming (LP) method, LP is one of the most powerful optimization methods due to its ability to solve linear and non-linear objective functions through linearization and its ability to handle the inequality constraints very easily [3].

1.2 Objectives:

- Illustration of LP OPF understanding.
- To Obtain an optimum secured system.

1.3 Statement of the Problem:

Before stating the problem, the system is assumed to be all thermal power system network and running at the normal operating conditions with constant loads and constant losses.

The optimal power flow (OPF) problem is a combination between economic dispatch (ED) and power flow (PF) therefore the ED and PF are solved simultaneously [3], the power flow problem is to determine the unknown parameters of all three types of buses; slack or reference bus, P-V or voltage regulated buses and P-Q or load buses, the total losses are part of the PF calculation and the ED problem is solved using an optimization tool, in the ED problem, in addition to the power balance constraints and the real power generation limits constraints, reactive power limits, other reactive power sources limits such as synchronous condensers, capacitor banks and FACTS devices, bus voltage limits, transmission line limits and transformer tab ratio and phase shift limits are employed and hence the problem is to minimize the total operating cost subject to all of these constraints.

1.4 The Proposed Solution:

Starting with a base power flow calculation and substituting the results into the ED objective function [3] where it is a polynomial in output power, usually in degree 2 in ($/hr.), the power flow problem is solved using N-R power flow solution [4], linearizing the objective function and linearizing the constraints, setting the variables limits and using the simplex LP optimization method to minimize the objective function, a new variables are calculated, substituting theses variables into the power flow as new set points and run the power flow calculation. Repeating this process until there is no change in variables of the power flow or LP and thus the problem is solved.

1.5 The Aim of this research:

The Aim of this research is to illustrate the understanding of Linear Programming optimal power flow theoretically and mathematically, what is optimization? What is LP optimization? How to implement LP optimization in OPF? What is the benefits of running the system in an optimal secured way? In addition to make sure that the reader can get the full understanding of LPOPF and how to Implement LPOPF into any system.

1.6 Research Methodology:

The optimal power flow in general will be discussed in a literature manner and a quick historical review of the OPF and the optimization techniques that used in order to solve the OPF problem, the anticipated linear programming OPF will be introduced theoretically and mathematically including the concept of reactive power pricing and the locational marginal pricing and then an implementation in a simple power system network using two methods: Piecewise linear approach and incremental LP method and both are solved using step by step procedure in order to illustrate the understanding through incorporation of an LP solver (Microsoft Excel 2016) and *POWERWORLD* Simulator, then an implementation on the IEEE 30 bus system will be introduced. Finally, a discussion and a comparison before and after LPOPF using both methods and before and after the addition of the VAR cost function to the objective function.

1.7 Thesis Layout:

- CHAPTER II Literature Review: a brief review about the power flow problem, the ED dispatch problem and the OPF problem in a literature manner.
- CHAPTER III Linear Programming Optimization: a detailed illustration of LP optimization methods such as the graphical method and the simplex method with illustrative examples.
- CHAPTER IV Linear Programming Optimal Power Flow: illustration and mathematical formulation of the piecewise linear

approach, the full AC incremental method to solve the LPOPF, reactive power pricing and the locational marginal pricing (LMP).

- CHAPTER V Implementation: implementation of piecewise linear approach and full AC incremental LPOPF using *POWERWORLD* Simulator and Microsoft Excel for the six-bus system example of [3] in detailed illustration, and then for the IEEE 30 bus system.
- CHAPTER VI Conclusion and Future work: a conclusion of the research and a suggestion about that additional studies could be applied to this research.

CHAPTER TWO

LITERATURE REVIEW

2.1 The Power Flow Problem:

The power flow problem is to identify the unknown parameters of the power system network parts; the system is assumed to be operating under balanced condition and represented by a single line diagram. The power system network contains hundreds of buses and branches with impedances specified in per-unit on a common MVA base.

The formulation of the network equations in the nodal admittance form results in a complex linear simultaneous equation in terms of node (bus) currents, thus the resulting equations become non-linear and must be solved through iterative techniques, the iterative techniques that used to solve the power flow equation are:

- Gauss-Seidel method.
- Newton-Raphson method.
- Decoupled power flow solution.
- Fast Decoupled power flow solution.

Power flow (load flow) studies are very important for power system analysis and design, it is important for planning and operation such as optimization studies, sensitivity analysis, economic studies, voltage stability, transient stability and contingency analysis.

As stated earlier power flow problem is to define the unknown parameters, these parameters are classified depending on the type of the buses in the network. Four quantities are associated with each bus:

- Bus voltage magnitude |V|.
- The voltage Phase angle δ.
- Generator real power P.

- Generator reactive power Q.

The types of buses are classified into:

- **The Slack bus**: also, known as the swing bus, it is taken as the reference bus which takes the differences between the generated power and loads that caused by the losses in the network. In this bus, the voltage magnitude and the phase angle are specified, the real and reactive power to be calculated.
- **Load buses**: also, known as the P-Q buses where the real and reactive powers are specified, the voltage magnitude and the phase angle to be calculated.
- **Regulated buses**: also, known as the P-V buses and voltage controlled buses, these buses are the generator buses where the real power and voltage magnitude are specified and the limits of the generator reactive power are specified, the real power and the phase angle to be calculated [4]. See [4] for a detailed information.

2.2 Economic Dispatch:

Optimal Dispatch or Economic Dispatch [3], [5], [6] is a process of determination the scheduling of generating units to minimize the total operating cost subject to a constraint that the total generation must equal to total demand plus losses.

ED problem is a non-linear optimization problem subject to equality and inequality constraints, the non-linearity came from the input-output (I/O) generation cost function, the equality constraint is the power balance constraint and the inequality constraint is the generation capacity limits constraint.

Economic dispatch problem dates back form 1920s or even earlier since the idea of scheduling the generators to minimize the total operating cost became in mind. In 1930, various methods were used to find the most economic form for the network: “the base load method” and “the best point loading method”.

The most economic results are gained by the use of the equal incremental method in the early of 1930s. The effect of losses is considered in the ED in

1940s and a method of combining the incremental fuel costs with the incremental transmission losses and the refinement of the loss formula was the next challenge till the appearance of the use of the coordination equations [5] and defining a more accurate economic dispatch for the system considering the system losses and used till this day. For a more detailed comprehensive survey see [6], this paper covered more than 112 references about ED and OPF.

2.3 The Optimal Power Flow:

2.3.1 Introduction:

The Optimal Power Flow (OPF) problem is first discussed by Carpentier in 1962 and took more than three decades to become a successful algorithm that could be applied in everyday use, the (OPF) problem is large and complicated non-linear optimization problem, it's a combination between the economic dispatch and the power flow solution which they are solved simultaneously [3], [5].

The objective of the OPF is to find an optimum secured system, optimum for minimizing total generation cost and total losses, secured for all operating parts that must run at their limits such as generators, bus-bars, transformers and transmission lines.

Optimal power flow results in an optimal active and reactive power generated and bought at each bus, the bus (nodal) pricing is very important in the electricity market. These bus prices known as the locational marginal prices (LMPs), the basic definition of the LMP is the marginal increase in cost to the system to supply one additional MW of load at a bus in the system. The LMP values are affected by generator bid prices, transmission system congestion, the losses on the system and the electrical characteristics of the system [3], [7].

2.3.2 The Objective Function:

The OPF problem is an optimization problem, consists of an objective function and constraints, usually, in OPF the optimization process is for minimizing the objective function, the objective function in OPF problem could be for:

- Minimization of the real power operating cost.
- Minimization of real and reactive power operating cost.
- Minimization of real and reactive power transmission losses.
- Environmental effects minimization by the addition of the emission variables and constraints.

The General form of the OPF objective function:

$$\text{Min } f\,(\underline{x}, \underline{u})$$

Subject to:

$$\omega(\underline{x}, \underline{u}) = 0$$

$$g(\underline{x}, \underline{u}) \geq 0$$

Where:

$\underline{x}$ ≡ a vector of the controlled variables such as the generator bus real power, the generator bus voltage magnitude, the transformer taps ratios and reactive power compensation devices. Note that the slack bus variables are not included.

$\underline{u}$ ≡ a vector of the dependent variables such as the slack bus real and reactive power, the generator bus reactive power (in case of real power only OPF), the load bus voltage magnitude and the flow in transmission lines.

$\omega(\underline{x}, \underline{u})$ ≡ the conventional ED power balance equality constraint that total generation must equal to the total load plus losses.

$g(\underline{x}, \underline{u})$ ≡ the set of the inequality constraints such as all generators real and reactive power limits, all bus voltage limits, transformer tap ratio limits, other reactive power sources limits (shunt devices) and transmission line flow limits.

2.3.3 OPF Optimization Methods:

In order to solve the OPF objective function, there are several methods that can be used to solve the OPF problem, these methods are classified into two main parts, conventional methods and intelligent methods:

i. Conventional Methods:

- The Gradient methods [5].
- The Hessian-based method.
- The Newton-based method [5].
- The Linear Programming method [3], [5].
- The Quadratic Programming method [3].
- The Interior point method [3], [5].

ii. Intelligent Methods:

- Artificial Neural Networks method.
- Fuzzy Logic.
- Evolutionary Programming.
- Ant Colony.
- Particle Swarm Optimization (PSO) methods.

[8] made a detailed review about these methods history, definitions, merits and demerits, this paper guides the reader to many papers discussing the OPF optimization methods.

CHAPTER THREE

LINEAR PROGRAMMING OPTIMIZATION

3.1 Introduction:

Linear programming [9], [10] is a mathematical tool used to solve the optimization problems, it has the capability to solve linear objective functions and constraints and non-linear objective functions and constraints through linearization and it has the capability to easily handle the inequality constraints where this is one of Linear programming‘s powerful features [3].

There are several LP techniques that might be used to solve the optimization problems such as the Graphical method, the Standard (Canonical) form solution and the Simplex method, the last one is the most widely used due to speed and simplicity.

3.2 The linear programming is summarized mathematically as:

$$\text{Minimize: } c^T x \qquad (3.1)$$

$$\text{Subjected to: } Ax = b \qquad (3.2)$$

$$x \geq 0$$

$$x \in \mathcal{R}^n$$

Where:

c ≡ the $n \times 1$ vector of cost coefficients.

x ≡ the $n \times 1$ vector of the unknown variables.

A ≡ the $m \times n$ matrix of cost coefficients.

b ≡ the right-hand side $m \times 1$ vector.

3.3 The Graphical method:

Solving the following classical problem using LP Graphically:

Minimize: $-x_1 - 3x_2$

Subjected to: $x_1 + x_2 \leq 6$

$-x_1 + 2x_2 \leq 8$

$-2x_1 + 3x_2 \geq 0$

$x_1, x_2 \geq 0$

After drawing each set of constraints, the following figure is presented:

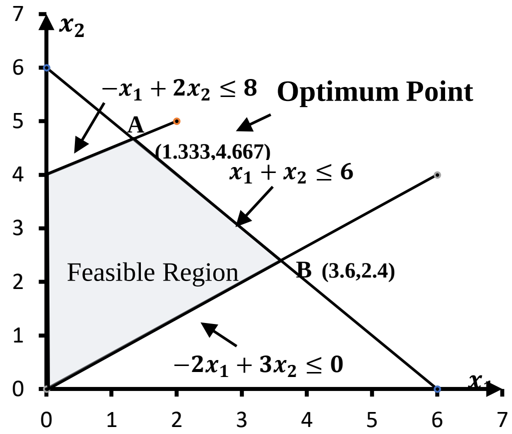


Figure (3.1): LP graphically

This graph is representing the graphical linear programming optimization, from the graph, the linear constraints are bounded an area, this area is called the feasible region because in this region the optimal solution can be found within satisfaction of all constraints.

The intersection of the three constraints forms a two points, A and B, and the optimum solution is within these two points.

For point A, Z = -15.333, and for point B, Z = -10.8, therefore the optimal solution at point A when x_1 = 1.333 and x_2 = 4.667 [9], [11].

3.4 The standard form solution:

In order to solve LP problems, problem should be formulated in a standard form, the standard form as in [3] is built to minimize not to maximize. This type of solution searches for the basic feasible solution and then for the optimal basic feasible solution by setting a number of sets and searching through these sets until the optimal solution to be found.

The first step of the solution is converting all inequality constraints to equality constraints by adding a slack variable, for all greater than or equal, we will subtract a slack variable and for all less than or equal we will add a slack variable:

$$\text{a-} \begin{cases} \sum_j a_{ij}x_{ij} \geq b_i & (3.3) \\ \sum_j a_{ij}x_{ij} - s_i = b_i & (3.4) \end{cases}$$

$$\text{b-} \begin{cases} \sum_j a_{ij}x_{ij} \leq b_i & (3.5) \\ \sum_j a_{ij}x_{ij} + s_i = b_i & (3.6) \end{cases}$$

Returning to the mathematical representation of the LP:

$$\text{Minimize: } c^T x$$

$$\text{Subjected to: } Ax = b$$

$$x \geq 0$$

$$x \in \mathcal{R}^n$$

The second step is dividing the [A] matrix into basic and non-basic variables, and dividing the $\underline{x}$ and $\underline{c}$ vectors into basic and non-basic variables as well. Hence:

$$A \equiv [A_B | A_N]$$

$$x \equiv \begin{bmatrix} x_B \\ x_N \end{bmatrix}$$

$$c \equiv \begin{bmatrix} c_B \\ c_N \end{bmatrix}$$

Where:

$A_B \equiv$ non-singular $n \times n$ submatrix called the basis and contains the basic constraint coefficients.

$A_N \equiv$ the non-basic variables submatrix contains the slack variables coefficients.

$x_B \equiv$ the unknown vector of the basic variables.

$x_N \equiv$ the unknown vector of the non-basic variables.

$c_B \equiv$ the cost coefficients of the basic variables.

$c_N \equiv$ the cost coefficients of the non-basic variables.

3.4.1 Defining the basic feasible region:

To define the basic feasible solution, and then the optimal solution, a trial of all combinations of the basic and non-basic variables of the [A] matrix must be made in order to find the optimal set of variables. To find the number of the trial combinations:

$$C_m^n = \frac{n!}{(n-m)!\,m!} \tag{3.7}$$

Where:

$m \equiv$ the number of rows and $n \equiv$ the number of columns of the [A] matrix. And, to find the unknown vector [x], from [A][x] =[b], $\therefore [x] = [A]^{-1}[b]$

This method is not useful for big problems, if we have an [A] matrix consists of 5 rows and 10 columns, the number of the trial sets is 252, and this is very big and usually problems are larger and has more complexity, therefore the Simplex method is presented.

3.5 The Simplex Method:

Invented by George Dantzig in 1947, the simplex method [3], [9], [12] procedure is to move from one basic feasible solution to another with the lower cost.

Starting with the basic LP problem:

$$\text{Minimize: } z = \mathrm{c}^{\mathrm{T}}\mathrm{x}$$

$$\text{Subjected to: } \mathrm{Ax} = \mathrm{b}$$

$$\mathrm{x} \geq 0$$

$$\mathrm{x} \in \mathcal{R}^n$$

Dividing the [A] matrix, [c] and [x] into basic and non-basic parts, hence:

Minimize: $z = c_B^T x_B + c_N^T x_N$ (3.8)

Subjected to: $[A_B \ \ A_N] \begin{bmatrix} x_B \\ x_N \end{bmatrix} = \mathrm{b}$ (3.9)

$\therefore A_B \, x_B + \ A_N \, x_N = \mathrm{b}$

$\therefore A_B \, x_B = \mathrm{b} - \ A_N \, x_N$

$\therefore x_B = {A_B}^{-1}(\mathrm{b} - \ A_N \, x_N)$

$\therefore z = c_B^T {A_B}^{-1}(\mathrm{b} - \ A_N \, x_N) + c_N^T x_N = c_B^T {A_B}^{-1} b - c_B^T {A_B}^{-1} A_N \, x_N + c_N^T x_N$

$\therefore z = c_B^T {A_B}^{-1} b - (c_B^T {A_B}^{-1} A_N + c_N^T) x_N$

Or $z = c_B^T {A_B}^{-1} b + (c_N^T - c_B^T {A_B}^{-1} A_N) x_N$

Let: $c_N^T - c_B^T {A_B}^{-1} A_N = {r_N}^T$

$\therefore$ the objective function become:

Minimize: $z = c_B^T {A_B}^{-1} b + {r_N}^T x_N$ (3.10)

Subjected to: $x_B = {A_B}^{-1}(\mathrm{b} - \ A_N \, x_N)$ (3.11)

$x_B, x_N \geq 0$

Where ${r_N}^T \equiv$ the reduced cost row in the LP Tableau.

In this form, the objective function become a function of the non-basic variables and the basic variables become a function of the non-basic variables.

3.5.1 The LP Tableau:

The LP tableau consists of the elements of the [A] matrix plus the right-hand side vector [b] and the reduced cost row, it is designated as the [Y] matrix. Unlike the standard form solution where slack variables are taken as the non-basic variables, in the [Y] matrix the slack variables are taken as the basic variables.

In the [Y] matrix the basic part is an identity matrix, and the inverse of any identity matrix equals to the identity matrix itself $\therefore A_B = {A_B}^{-1}$ and by this, ${A_B}^{-1}$ can be eliminated from the reduced cost equation to become:

$$
{r_N}^T = c_N^T - c_B^T A_N \tag{3.12}
$$

The general form of the [Y] matrix is:

$$
\begin{array}{cccc|ccc|c|}
x_1 & x_2 & x_3 & x_n & s_1 & s_2 & s_3 & \text{b} \\
a_{11} & a_{12} & a_{13} & a_{1n} & 1 & 0 & 0 & b_1 \\
a_{21} & a_{22} & a_{23} & a_{2n} & 0 & 1 & 0 & b_2 \\
a_{31} & a_{32} & a_{33} & a_{3n} & 0 & 0 & 1 & b_3 \\
\hline
r_1 & r_2 & r_3 & r_n & 0 & 0 & 0 & -c_B^T b \\
 & \text{Non-Basic} & & & & \text{Basic} & &
\end{array}
$$

3.5.2 Pivoting:

Pivoting is used to move from one basic solution to another by changing the set of basic variables.

3.5.3 Pivoting steps:

- Identify the pivot element y_{ij}:

And this is made through identifying the row element i and the column element j, to identify the column element j, we choose the most negative value at the reduced cost row and locate this value at which column, and then j is identified.

And the row element i is identified through the epsilon test:

$$
\epsilon = minimum \left\{ \frac{b_i}{y_{ij}} : y_{ij} > 0 \right\} \tag{3.13}
$$

Here a division of each element at the right-hand side vector [b] by the corresponding element of the identified column results in a different set of values, locate the most minimum value at which row and then the i element is identified.

- Normalize the row of the pivot to make $y_{ij} = 1$.
- Make all the elements of the pivot column equal to zero except y_{ij}.

3.5.4 The Simplex Algorithm:

- Start with a basic feasible solution.
- Formulate the [A] matrix and the right-hand side vector [b], calculate the reduced cost and then formulate the [Y] matrix.
- If the reduced cost $r_i \geq 0$ stop, otherwise:
- Identify the pivot element by finding the most negative cost to identify j and use ε test to determine the variable that should leave the basis i.
- Pivot on element y_{ij}, repeat until $r_i \geq 0$.

In case of greater than or equal (≥) constraints, the Simplex Big M method must be used to obtain the optimal solution, and in case of a negative right hand side value, multiply the constraint equation by -1 and change the sign of the inequality, if it is less than or equal (≤) then it must be changed into great than or equal (≥) and vice versa.

In both LP OPF solution methods the Simplex algorithm is used, and problem still huge, where for example in the full AC OPF, the number of the slack variables depends on the number of the inequality constraints and the full AC OPF has numerous number of inequality constraints; could be thousands, and thus a huge [Y] matrix will exist, how much will take to pivot on each non-basic variable? Therefore, the OPF problem is a very big and complicated problem.

CHAPTER FOUR

LINEAR PROGRAMMING OPTIMAL POWER FLOW

4.1 Introduction:

As stated in Chapter two, the OPF problem is a combination between ED and PF calculation in which by calculating the dependent and control variables of the objective function through the power flow calculation and solve the optimization problem as same as solved through the ED, several methods are used to solve this problem such as non-linear methods, linear methods…etc., as stated in [13] the non-linear methods are suffering from some difficulties, lambda iteration and Newton based methods have been found to converge very fast but has difficulties in handling the inequality constraints, the gradient method is suffering from both convergence speed and inequality constraints, but these drawbacks did not exist in LP methods.

Linear programming as stated earlier is a very useful technique to be used, where it has no difficulties with both inequality constraints or convergence speed as observed in the previous chapter.

In OPF problem, two different methods of solution using LP optimization can be used, the Piecewise (PW) Linear approach method and the full AC Linear Programming method, in the Piecewise approach the linearization is done through approximating the input-output (I/O) cost function [5] (the objective function) by straight line segments; in the full AC incremental LP method the linearization is done through the first order Taylor series expansion and solves the OPF problem through either the decoupled set of AC equation or the full AC power flow equations. Before the formulation of both methods, the general formulation of the OPF problem is presented.

4.2 The General Formulation of the Optimal Power Flow:

4.2.1 The Economic Dispatch Formulation:

The ED solves the following problem:

- Minimize the generation cost function:

$$min \sum_{i=1}^{n} F_i(P_{\text{gen}_i}) \tag{4.1}$$

Where: $F_i(P_{\text{gen}_i}) = a + bP_{\text{gen}_i} + cP_{\text{gen}_i}{}^2$ and a, b and c are cost coefficients.

- Subjected to the equality constraint:

$$\sum_{i=1}^{N} P_{\text{gen}_i} = P_{\text{Total load}} + P_{\text{Total losses}} \tag{4.2}$$

- Subjected to the inequality constraint:

$$P_{\text{gen}_i}^{\text{min}} \geq P_{\text{gen}_i} \geq P_{\text{gen}_i}^{\text{max}} \text{ , for } i = 1,2,3, \ldots, n \tag{4.3}$$

The ED formulation in a compacted form:

$$f(\underline{P_{\text{gen}}}, \underline{u})$$

Subject to:

$$\omega\left(\underline{P_{\text{gen}}}, \underline{u}\right) = 0$$

$$g\left(\underline{P_{\text{gen}}}, \underline{u}\right) \geq 0$$

Where:

$$\underline{P_{\text{gen}}} = \begin{bmatrix} P_{\text{gen}_1} \\ \vdots \\ P_{\text{gen}_n} \end{bmatrix}, and\ \underline{u} \equiv P_{Total\ load}, P_{Total\ losses}, P_{\text{gen}_i}^{\text{min}} \text{ and } P_{\text{gen}_i}^{\text{max}}$$

$$\omega = \sum_{i=1}^{N} P_{gen_i} = P_{Total\ load} + P_{Total\ losses}, \qquad g = P_{\text{gen}_i}^{\text{min}} \leq P_{\text{gen}_i} \leq P_{\text{gen}_i}^{\text{max}}$$

4.2.2 The Optimal Power Flow Formulation combining the Economic dispatch and the Power Flow:

- The objective function:

$$\min \sum_{i=1}^{n} F_i\left(P_{\text{gen}_i}\right), \text{Same as ED}$$

- Subjected to the equality constraint:

$$\sum_{i=1}^{N} P_{\text{gen}_i} = P_{\text{Total load}} + P_{\text{Total losses}}, \text{Same as ED}$$

- Subjected to the inequality constraints:

$$P_{\text{gen}_i}^{\min} \le P_{\text{gen}_i} \le P_{\text{gen}_i}^{\max}$$

$$Q_{\text{gen}_i}^{\min} \le Q_{\text{gen}_i} \le Q_{\text{gen}_i}^{\max}$$

$$P_{ij}^{\min} \le P_{ij} \le \ P_{ij}^{\max}$$

$$\text{Or}, S_{ij}^{\min} \le S_{ij} \le \ S_{ij}^{\max}$$

$$V_i^{\min} \le V_i \le V_i^{\max}, \text{for } i = 1,2,3, \dots, n$$

Where P_{gen_i}, Q_{gen_i}, V_i, P_{ij} and S_{ij} are the real generated power at generator i, the reactive generated power at generator i, the voltage at bus i, the real power flow at line ij and the complex or the apparent power flow at line ij respectively. These variables are calculated through the power flow solution [4].

4.2.3 The Power Flow Equation:

$$\frac{P_{\text{gen}_i} - jQ_{\text{gen}_i}}{V_i^*} = V_i \sum_{\substack{j=0 \\ i \ne j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i \ne j}}^{n} y_{ij} V_j \tag{4.4}$$

$$\therefore P_{\text{gen}_i} - jQ_{\text{gen}_i} = V_i^* \left[V_i \sum_{\substack{j=0 \\ i \ne j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i \ne j}}^{n} y_{ij} V_j \right] \tag{4.5}$$

$$\therefore P_{gen_i} = \Re\left\{V_i^*\left[V_i\sum_{\substack{j=0\\i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0\\i\neq j}}^{n} y_{ij}V_j\right]\right\} \tag{4.6}$$

$$\text{And } Q_{gen_i} = -\Im\left\{V_i^*\left[V_i\sum_{\substack{j=0\\i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0\\i\neq j}}^{n} y_{ij}V_j\right]\right\} \tag{4.7}$$

$$P_{ij} = \Re\left\{V_i\left[(V_i - V_j)y_{ij} + V_i^2 y_{\text{shunt}_{ij}}\right]^*\right\} \tag{4.8}$$

$$S_{ij} = \text{abs}\left\{V_i\left[(V_i - V_j)y_{ij} + V_i^2 y_{\text{shunt}_{ij}}\right]^*\right\} \tag{4.9}$$

Where:

$y_{ij} \equiv$ the ij term of the admittance matrix.

$V_i^* \equiv$ the conjugate value of the complex voltage at bus i.

$y_{\text{shunt}_{ij}} \equiv$ the shunt charging admittance to ground of line ij.

Therefore, the OPF equality constraint is written as:

- The equality constraint:

$$\left(P_{\text{gen}_i} - P_{\text{load}_i}\right) - j\left(Q_{\text{gen}_i} - Q_{load_i}\right) = V_i^*\left[V_i\sum_{\substack{j=0\\i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0\\i\neq j}}^{n} y_{ij}V_j\right] \tag{4.10}$$

$$P_{\text{gen}_i} - P_{\text{load}_i} = \Re\left\{V_i^*\left[V_i\sum_{\substack{j=0\\i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0\\i\neq j}}^{n} y_{ij}V_j\right]\right\} \tag{4.11}$$

$$Q_{\text{gen}_i} - Q_{\text{load}_i} = -\Im\left\{V_i^*\left[V_i\sum_{\substack{j=0\\i\neq j}}^{n} y_{ij} - \sum_{\substack{j=0\\i\neq j}}^{n} y_{ij}V_j\right]\right\} \tag{4.12}$$

As observed, the equality constraint changed from total generation must equal to total load plus total losses as in ED, into total generation minus total load at bus i must equal to the power flow into bus i, because the power flow calculation results in generation output equal to total load plus losses as required, therefore there is no need to calculate the losses or the generator incremental losses as was in conventional ED.

The OPF formulation in a compacted form:

$$f(\underline{P_{\mathrm{gen}}}, \underline{u})$$

Subject to: $$\omega\left(\underline{P_{\mathrm{gen}}}, \underline{u}\right) = 0$$

$$g\left(\underline{P_{\mathrm{gen}}}, \underline{u}\right) \geq 0$$

Where:

The vector $\underline{u}$ now is containing the generator cost function parameters plus all the power flow solution parameters such as the generator real and reactive power limits, the admittance matrix, the fixed voltages of the P-V busses, the reference bus fixed voltage magnitude and phase angle.

$\omega\left(\underline{P_{\mathrm{gen}}}, \underline{u}\right) = 0$ representing the power flow admittance matrix equations.

$g\left(\underline{P_{\mathrm{gen}}}, \underline{u}\right) \geq 0$ containing all inequality constraints limits, such as generator real and reactive power limits, power flow transmission equations and bus voltage limits.

This formulation is implemented in the full AC optimal power flow, the Piecewise linear approach OPF is as well iterates between the ED and the PF but it differs from the full AC OPF in several aspects. However, in the next section the PW LPOPF (real power OPF) is introduced.

4.3 Linear Programming Optimal Power flow using Piecewise Linear Approach:

As in [14] “the piecewise approach can fit an arbitrary curve convexly to any desired accuracy with a sufficient number of segments”. In LPOPF the piecewise approach is used to fit the non-linear I/O cost curve (figure 4.1.a) into fixed straight line segments (figure 4.1.b) and therefore the objective function becomes linear objective function.

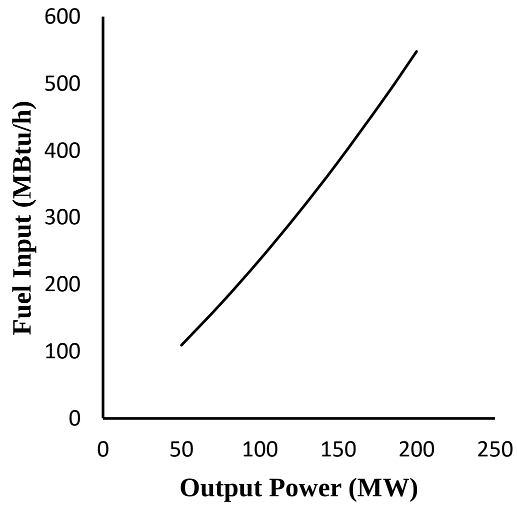


Figure (4.1.a): I/O cost curve.

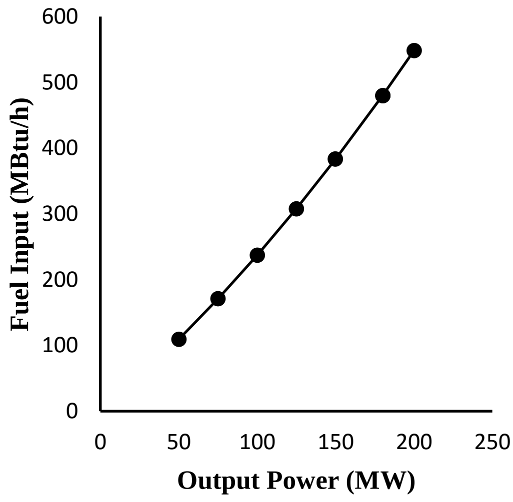


Figure (4.1.b): PW I/O cost curve.

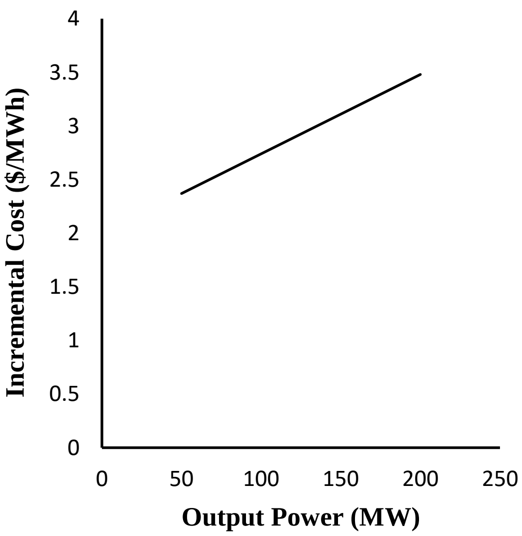


Figure (4.1.c): I/O IC curve.

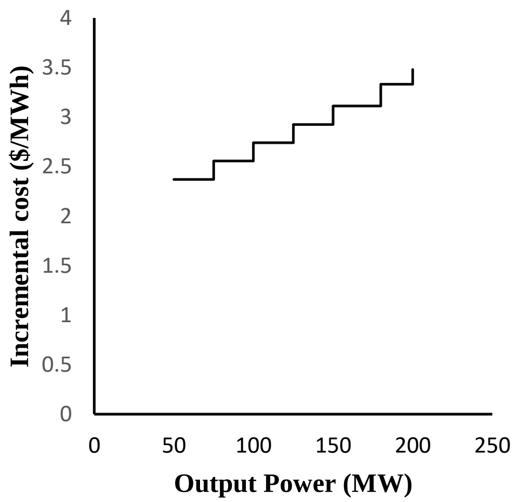


Figure (4.1.d): PW I/O IC curve.

4.3.1 Formulation of the Piecewise Linear Programming:

The first step of formulating the piecewise LP OPF objective function is by converting the I/O cost curve into straight line segments through break points:

$$\text{The break point step} = \frac{\text{Max. limit} - \text{Min. Limit}}{\text{No. of the desired segments}} \tag{4.13}$$

The cost curve is representing the relation between the fuel input in (MBtu/h) and the output power in MW, the above figure is a plot for a unit having a capacity limits from 50MW to 200MW (Figure 4.1.a), the above equation is used to change the relation into linear relation by converting the I/O curve into six straight line segments (Figure 4.1.b), the same process is used to convert the I/O incremental cost curve (Figure 4.1.c) into straight line segments (figure 4.1.d), each segment can be represented as P_{i1}, P_{i2}, P_{i3},…,P_{in}, and each segment will have a limit which is given by:

$$\text{Segment Limit} = \text{BP}_{i+1} - \text{BP}_i \tag{4.14}$$

And each segment will have a slope designated as s_{i1}, s_{i2}, s_{i3}….s_{in}, the slope of the generator cost curve segments is given by:

$$s_{ij} = \frac{F_i\left(P_{ij}^{max}\right) - F_i(P_{ij}^{min})}{P_{ij}^{max} - P_{ij}^{min}} \tag{4.15}$$

∴The linearized objective function is:

$$F_i(P_i) = F_i\left(P_i^{min}\right) + s_{i1}P_{i1} + s_{i2}P_{i2} + s_{i3}P_{i3} + \cdots + s_{in}P_{in} \tag{4.16}$$

Where:

$$F_i\left(P_i^{min}\right) = a + bP_i^{min} + c(P_i^{min})^2$$

For the new values of the generation power P_i:

$$P_i = P_i^{min} + P_{i1} + P_{i2} + P_{i3} + \cdots + P_{in} \tag{4.17}$$

4.3.2 Optimal Power Flow Problem Formulation using Piecewise LP method:

Minimize: $F_i(P_i) = \sum F_i\left(P_i^{min}\right) + \sum_{\substack{i=1 \\ j=1}}^{n} s_{ij} P_{ij}$ (4.18)

Subjected to: $\sum_{\substack{i=1 \\ j=1}}^{n} P_{ij} =$ Total generation + losses - $\sum_{i=1}^{n} P_i^{min}$ (4.19)

$$P_{ij} \leq P_{ij}^{max}$$

$$P_{ij} \geq 0$$

Where:

$P_{load} \equiv$ total load of the system.

$P_{loss} \equiv$ total transmission losses.

$\sum_{\substack{i=1 \\ j=1}}^{n} P_{ij} =$ Total generation + losses - $\sum_{i=1}^{n} P_i^{min} \equiv$ power balance equality constraint.

$P_{ij} \leq P_{ij}^{max}$ and $P_{ij} \geq 0 \equiv$ the inequality constraints for each segment.

4.3.3 Solution Algorithm for Piecewise LP OPF:

- Start with a base power flow solution.
- Linearize the objective function using equation (4.13), (4.14) and (4.15).
- Set the control variables limits (the equality and inequality constraints).
- Formulate the problem in an LP solver and solve.
- Substitute the LP results into the power flow as new set points and run a power flow solution.
- No change in variables and no transmission overloads, stop. Otherwise:
- Set the new variables limits.

- No change in variables but transmission overload, use the generation shift factors to relief the overloading.
- Add the new transmission constraints.
- Repeat until there is no change in variables of power flow or LP.

In this method the control variables are the real powers only, where the iteration process between the power flow and LP are just for the real powers and the reactive power (voltages) are adjusted through the AVR [4], but PW linear approach may go beyond than (real OPF) such as in [15], however, this method has a very fast rate of convergence but solution may vary with respect to the number of segmentation [3], therefore the number of segments must be specified correctly to meet the most accurate approximation to the non-linear objective function in order to get the most optimal solution.

The full AC optimal power flow more complicated where in addition to generation real power limits and transmission limits, the reactive power limits and the bus voltage limits are employed as observed in section 4.2.1.

4.4 The Full AC Linear Programming Optimal power flow- The Iterative LP Method:

The full ACOPF iterative LP method or the incremental LP method as in [13] is formulated by “linearizing the nonlinear objective function and constraints of the OPF AC power flow formulation around the current operating point using a first order Taylor series expansion in order to create a convex LP problem”, and since the real and reactive power constraints are not well represented by linear functions, a suggested solution to this drawback is presented in [13] as “The real and reactive power equality constraints, however, are not well represented by linear functions. In order for the linearized problem to accurately model the nonlinear problem, the movement of each variable must be restricted to a small region during each iteration, and the problem must be re-linearized after each iteration”, this small region is suggested as a window as stated in [3] “This smaller set of limits can be referred to as a window within which the variables are allowed to move on any LP execution. At the end of that execution, the limits of the window are moved but always stay within the limits. Thus, the LP solves one small region about a starting point, then re-linearizes about the solution and solves another LP within a small region about the solution". This method possesses speed and flexibility during calculation and produces reliable results for all types of systems which is called the trust region method, however, the adjustment of the window size and implementation of trust region method is not included in this research, for more information see [13].

4.4.1 Problem Formulation:

In the full AC power flow using Newton-Raphson method [4], the following problem is solved:

$$\begin{bmatrix} \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_2} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_2}{\partial \delta_n} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix} & \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_n}{\partial V_2} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_2}{\partial V_n} & \cdots & \frac{\partial P_n}{\partial V_n} \end{bmatrix} \\ \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_n}{\partial \delta_2} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_2}{\partial \delta_n} & \cdots & \frac{\partial Q_n}{\partial \delta_n} \end{bmatrix} & \begin{bmatrix} \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_n}{\partial V_2} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_2}{\partial V_n} & \cdots & \frac{\partial Q_n}{\partial V_n} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_n \\ \Delta V_2 \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} P_{\text{scheduled}_2} - P_2(V,\delta) \\ \vdots \\ P_{\text{scheduled}_n} - P_n(V,\delta) \\ Q_{\text{scheduled}_2} - Q_2(V,\delta) \\ \vdots \\ Q_{\text{scheduled}_n} - Q_n(V,\delta) \end{bmatrix} \tag{4.20}$$

Equation (4.20) in a compacted form:

$$[\mathcal{J}] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P_{\text{gen}} \\ \Delta Q_{\text{gen}} \end{bmatrix} \tag{4.21}$$

Where:

$\mathcal{J} \equiv$ the Jacobean matrix.

ΔP & $\Delta Q \equiv$ are the change in power due to the change of voltage magnitudes ΔV and their phase angles $\Delta\delta$.

In the Incremental LP method and since using the first order Taylor series expansion, the optimization process will be written in terms of ΔP_{gen}, ΔQ_{gen}, ΔV and $\Delta\delta$ where:

$$F_i(P_{\text{gen}_i}) = F_i(P_{\text{gen}_i}) + F_i(P_{\text{gen}_i})' (P_{\text{scheduled}_i} - P_{\text{gen}_i}(V,\delta)) \tag{4.22}$$

The LP OPF should be started by a base power flow solution, here the power flow solution is designated as power flow zero (PF0) and the values of the base power flow solution are designated as:

$$P_{\text{gen}}^{0},\ Q_{\text{gen}}^{0},\ V^{0} \text{ and } \delta^{0}$$

The linearized objective function of the incremental LPOPF is:

$$\min \sum_{i=1}^{n} \left[F_i\left(P_{\text{gen}_i}^{0}\right) + \frac{\mathrm{d}F_i\left({P_{\text{gen}_i}}^{0}\right)}{\mathrm{d}{P_{\text{gen}_i}}^{0}} \Delta P_{\text{gen}_i} \right] \tag{4.23}$$

Where:

$F_i\left(P_{\text{gen}_i}^{0}\right) \equiv$ the objective function in terms of the base PF solution values.

$\frac{\mathrm{d}F_i({P_{\text{gen}_i}}^{0})}{\mathrm{d}{P_{\text{gen}_i}}^{0}} \equiv$ the incremental cost function in terms of the base PF solution.

$\because$ $F_i\left(P_{\text{gen}_i}^{0}\right)$ is considered to be as constant, it can be eliminated from the objective function, therefore the linearized objective function becomes:

$$\min \sum_{i=1}^{n} \left[\frac{\mathrm{d}F_i({P_{\text{gen}_i}}^{0})}{\mathrm{d}{P_{\text{gen}_i}}^{0}} \Delta P_{\text{gen}_i} \right] \tag{4.24}$$

In order to linearize the real and reactive power equality constraints, the constraints of the power flow solution are formulated similar to the expression of the N-R method except that all variables are included even the slack bus variables, and there is no need for the inversion of the Jacobean matrix to calculate $\Delta\delta_i$ and ΔV_i since the LP optimization is responsible of calculating these values [3]. The linearized real and reactive power equality constraints are:

$$\begin{bmatrix} \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_n}{\partial \delta_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_1}{\partial \delta_n} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix} & \begin{bmatrix} \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_n}{\partial V_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_1}{\partial V_n} & \cdots & \frac{\partial P_n}{\partial V_n} \end{bmatrix} \\ \begin{bmatrix} \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_n}{\partial \delta_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_1}{\partial \delta_n} & \cdots & \frac{\partial Q_n}{\partial \delta_n} \end{bmatrix} & \begin{bmatrix} \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_n}{\partial V_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_1}{\partial V_n} & \cdots & \frac{\partial Q_n}{\partial V_n} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_n \\ \Delta V_1 \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} P_{\text{scheduled}_1} - P_1(V,\delta) \\ \vdots \\ P_{\text{scheduled}_n} - P_n(V,\delta) \\ Q_{\text{scheduled}_1} - Q_1(V,\delta) \\ \vdots \\ Q_{\text{scheduled}_n} - Q_n(V,\delta) \end{bmatrix} \tag{4.25}$$

Where $\Delta\delta_1$, ΔV_1 , ΔP_{load_i} and ΔQ_{load_i} are taken as constants and equal to zero.

The inequality constraints are formulated as:

- The generator real power limits:

$$P_{\mathrm{gen}_i}{}^{\min} - P_{\mathrm{gen}_i}{}^{0} \le \Delta P_{\mathrm{gen}_i} \le P_{\mathrm{gen}_i}{}^{\max} - P_{\mathrm{gen}_i}{}^{0} \quad (\forall \text{ generators } i)$$

- The generator reactive power limits:

$$Q_{\mathrm{gen}_i}{}^{\min} - Q_{\mathrm{gen}_i}{}^{0} \le \Delta Q_{\mathrm{gen}_i} \le Q_{\mathrm{gen}_i}{}^{\max} - Q_{\mathrm{gen}_i}{}^{0} \quad (\forall \text{ generators } i)$$

- The bus voltage magnitude limits:

$$V_i^{\min} - V_i^{0} \le \Delta V_i \le V_i^{\max} - V_i^{0} \quad (\forall \text{ buses } i)$$

- The phase angle limits:

$$\delta_i^{\min} - \delta_i^{0} \le \Delta \delta_i \le \delta_i^{\max} - \delta_i^{0} \ (\forall \text{ buses } i)$$

- Transformer tap ratio limits:

$$t_{ij}{}^{\min} - t_{ij}{}^{0} \le \Delta t_{ij} \le t_{ij}{}^{\max} - t_{ij}{}^{0} \ (\forall \text{ transformer } ij)$$

4.4.2 Full ACOPF Incremental LP method General Formulation:

$$\min \sum_{i=1}^{n} \left[\frac{\mathrm{d}F_i(P_{\mathrm{gen}_i}{}^{0})}{\mathrm{d}P_{\mathrm{gen}_i}{}^{0}} \Delta P_{\mathrm{gen}_i} \right]$$

Subject to:

$$\sum_{i=1}^{n} \frac{\partial P_i(V,\delta)}{\partial V_i} \Delta|V_i| + \sum_{i=1}^{n} \frac{\partial P_i(V,\delta)}{\partial \delta_i} \Delta|\delta_i| + \sum_{i=1}^{n} \frac{\partial P_i}{\partial t_{ij}} \Delta t_{ij} = \Delta P_{\mathrm{gen}_i}$$

$$\sum_{i=1}^{n} \frac{\partial Q_i(V,\delta)}{\partial V_i} \Delta|V_i| + \sum_{i=1}^{n} \frac{\partial Q_i(V,\delta)}{\partial \delta_i} \Delta|\delta_i| + \sum_{i=1}^{n} \frac{\partial Q_i}{\partial t_{ij}} \Delta t_{ij} = \Delta Q_{\mathrm{gen}_i}$$

$$\sum_{i=1}^{N_{\mathrm{gen}}} P_{\mathrm{gen}_i}{}^{0} + \Delta P_{\mathrm{gen}_i} = \sum_{i=1}^{N_{\mathrm{gen}}} P_{\mathrm{gen}_i} + P_{\mathrm{loss}}$$

$$\sum_{i=1}^{N_{\mathrm{gen}}} \mathrm{Q}_{\mathrm{gen}_i}{}^{0} + \Delta \mathrm{Q}_{\mathrm{gen}_i} = \sum_{i=1}^{N_{\mathrm{gen}}} \mathrm{Q}_{\mathrm{gen}_i} + \mathrm{Q}_{\mathrm{loss}}$$

$$P_{\mathrm{gen}_i}{}^{\min} - P_{\mathrm{gen}_i}{}^{0} \le \Delta P_{\mathrm{gen}_i} \le P_{\mathrm{gen}_i}{}^{\max} - P_{\mathrm{gen}_i}{}^{0} \quad (\forall \text{ generators } i)$$

$${Q_{\mathrm{gen}_i}}^{\min} - {Q_{\mathrm{gen}_i}}^{0} \leq \Delta Q_{\mathrm{gen}_i} \leq {Q_{\mathrm{gen}_i}}^{\max} - {Q_{\mathrm{gen}_i}}^{0} \ \ (\forall \text{ generators } i)$$

$${V_i}^{\min} - {V_i}^{0} \leq \Delta V_i \leq {V_i}^{\max} - {V_i}^{0} \ \ (\forall \text{ buses } i)$$

$${\delta_i}^{\min} - {\delta_i}^{0} \leq \Delta \delta_i \leq {\delta_i}^{\max} - {\delta_i}^{0} \ (\forall \text{ buses } i)$$

$${t_{ij}}^{\min} - {t_{ij}}^{0} \leq \Delta t_{ij} \leq {t_{ij}}^{\max} - {t_{ij}}^{0} \ (\forall \text{ transformer } ij)$$

$$\Delta V_{i_{\mathrm{ref.}}},\ \Delta \delta_{i_{\mathrm{ref.}}},\ \Delta P_{\mathrm{load}_i} \text{and } \Delta Q_{\mathrm{load}_i} = 0$$

Where:

t_{ij}≡ Transformer tap ratio in case of a transformer between bus i and j.

4.5 Reactive Power Pricing:

Reactive power plays an important role in real power transfer and effects power system operation in numerous ways [16], [17]. Pricing of reactive power is very important for the deregulated electric industry both financially and operationally, financially through improving the economic efficiency of the system which is reactive power has an operation cost same as the real power, operationally the system efficiency and reliability will be improved by the reduction of the total transmission losses and the improvement of the voltage profile of the network [18].

As observed in the last section, the incremental LPOPF is optimizing both real and reactive powers through the linearized objective function but a pricing procedure for reactive power is not considered and there is no reactive power representative in the objective function. In this section, the inclusion of reactive power cost function in the objective function and a pricing procedure are introduced.

Reactive power costing is composed of two components, fixed costs or investment costs and variable costs, the variable costs are the operating costs (operation costs and maintenance costs) and opportunity costs, opportunity cost is resulting from reduction of the active power generation [19]. The costing of other reactive power sources such as FACTS devices, capacitor banks, synchronous condensers and transformers are considered as well and named as the explicit costs of these sources [16].

The pricing procedure of other reactive power sources such as shunt devices, condensers and transformers is illustrated in [16] and [20] which is not included in this research.

4.5.1 Reactive Power Cost Allocation:

The conventional reactive power cost function which is based on empirical approximation is:

$$\text{Cost } Q_i = \text{profit rate} * b * Q_i^2 \quad (4.26)$$

Where:

Profit rate≡ the profit rate of the real power and usually ranged from 5% to 10%, in this research the profit rate is taken as 5% or 0.05, this equation only considers the operating cost of reactive power [19].

Another approach is introduced in [21] to overcome the inaccuracies with the conventional method and it is based on the triangular relationship between the real and reactive powers, this method is criticized in [19] in which that it is mainly depend on the real power cost and the investment cost of generators is essentially based on the optimal solution for active power solution and using the same formula for reactive power costing will lead to calculation of wrong fixed costs for reactive power. Another approach is introduced in [22] in which that a, b and c constants are approximated to be 10% of those for the cost of real power, also this approach has a limitation which is valid for a special range of reactive power production as observed in [19].

In [19] a new approach is proposed which is covers all investment, operation and opportunity costs by considering the cost of the maximum generation power (P_{max}), the cost of generation when producing both real and reactive powers ($P_{max} - \Delta P$) and the cost of the reduction of the active power due to the production of reactive power (the opportunity cost, ΔP) figure (4.2), the cost of reactive power is then given by:

$$\mathrm{Cost}(\mathrm{Q}_i) = \frac{\mathrm{P}_{\max} - \Delta\mathrm{P}}{\mathrm{P}_{\max}}\,\mathrm{cost}(\mathrm{P}_{\max}) - \mathrm{cost}(\Delta\mathrm{P}) \tag{4.27}$$

The amount of Q_i is generated in terms of the real power reduction and it is calculated through:

$$\mathrm{Q}_i = \sqrt{{P_{max}}^2 - {P_i}^2} \tag{4.28}$$

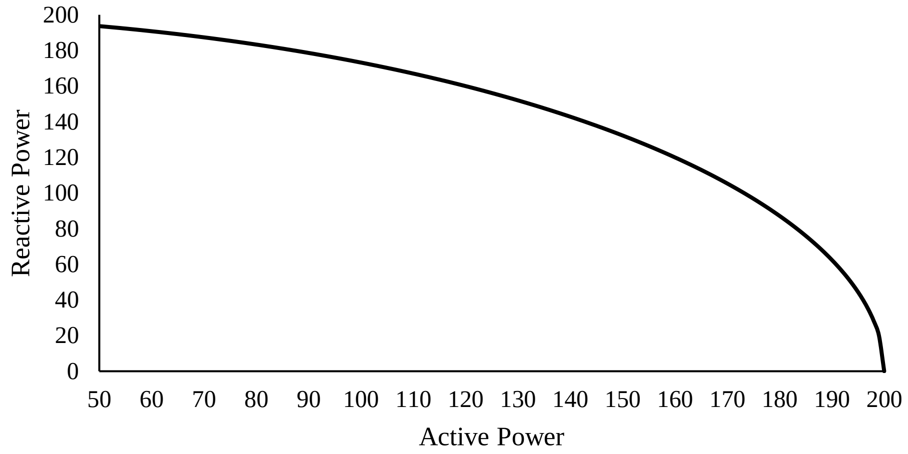


Figure (4.2): Reduction of real power due to production of reactive power

And

$$\Delta\mathrm{P} = P_{max} - P_i \tag{4.29}$$

The amount of Q_i is calculated as a function of P_i by the use of equation (4.11) and the cost of Q_i is calculated in function of P_i by using equation (4.10), the results are interpolated by the use of Newton-Gregory polynomial to be fitted into quadratic polynomial form as:

$$\mathrm{F}_i(\mathrm{Q}_i) = \mathrm{a} + \mathrm{b}\,\mathrm{Q}_i + \mathrm{c}\,\mathrm{Q}_i^2 \tag{4.30}$$

Another approach is proposed in [23] in a manner similar to the proposed method in [19] with a slight difference which is the cost of reactive power is equal to the cost of the reduced real power due to generation of reactive power:

$$\mathrm{Cost}(\mathrm{Q}_i) = \mathrm{Cost}(\Delta\mathrm{P}) \tag{4.31}$$

Where: $\mathrm{Cost}(\Delta\mathrm{P}) = \mathrm{Cost}\ (\mathrm{P}_{\mathrm{rated}}) - \ \mathrm{Cost}\ (\mathrm{P}_i)$ (4.32)

And by the use of the same technique of [19] the final quadratic cost function of reactive power is obtained.

In this research, the conventional reactive power operating cost function with a profit rate of 0.05 is employed:

$$\mathrm{F}_i(\mathrm{Q}_i) = 0.05\mathrm{b}\mathrm{Q}_i{}^2 \tag{4.33}$$

4.5.2 The Inclusion of reactive power cost function to the objective function:

Linearizing equation (4.33) using Taylor series expansion:

$$\mathrm{F_i}\left(\mathrm{Q}_{\mathrm{gen}_i}\right) + \frac{\mathrm{dF_i}\left(\mathrm{Q}_{\mathrm{gen}_i}\right)}{\mathrm{dQ}_{\mathrm{gen}_i}}\Delta\mathrm{Q}_{\mathrm{gen_i}} \tag{4.34}$$

Then the objective function of the incremental LPOPF becomes:

$$\min\sum_{i=1}^{n}\left[\frac{\mathrm{d}F_i(P_{\mathrm{gen}_i}{}^0)}{\mathrm{d}P_{\mathrm{gen}_i}{}^0}\Delta P_{\mathrm{gen}_i} + \frac{\mathrm{d}F_i(Q_{\mathrm{gen}_i}{}^0)}{\mathrm{d}Q_{\mathrm{gen}_i}{}^0}\Delta Q_{\mathrm{gen}_i}\right] \tag{4.35}$$

Subject to:

$$\sum_{i=1}^{n}\frac{\partial P_i(V,\delta)}{\partial V_i}\Delta|V_i| + \sum_{i=1}^{n}\frac{\partial P_i(V,\delta)}{\partial \delta_i}\Delta|\delta_i| + \sum_{i=1}^{n}\frac{\partial P_i}{\partial t_{ij}}\Delta t_{ij} = \Delta P_{\mathrm{gen}_i}$$

$$\sum_{i=1}^{n}\frac{\partial Q_i(V,\delta)}{\partial V_i}\Delta|V_i| + \sum_{i=1}^{n}\frac{\partial Q_i(V,\delta)}{\partial \delta_i}\Delta|\delta_i| + \sum_{i=1}^{n}\frac{\partial Q_i}{\partial t_{ij}}\Delta t_{ij} = \Delta Q_{\mathrm{gen}_i}$$

$$\sum_{i=1}^{N_{\mathrm{gen}}} P_{\mathrm{gen}_i}{}^0 + \Delta P_{\mathrm{gen}_i} = \sum_{i=1}^{N_{\mathrm{gen}}} P_{\mathrm{gen}_i} + P_{\mathrm{loss}}$$

$$\sum_{i=1}^{N_{\mathrm{gen}}} \mathrm{Q}_{\mathrm{gen}_i}{}^0 + \Delta\mathrm{Q}_{\mathrm{gen}_i} = \sum_{i=1}^{N_{\mathrm{gen}}} \mathrm{Q}_{\mathrm{gen}_i} + \mathrm{Q}_{\mathrm{loss}}$$

$$P_{\mathrm{gen}_i}{}^{\min} - P_{\mathrm{gen}_i}{}^0 \le \Delta P_{\mathrm{gen}_i} \le P_{\mathrm{gen}_i}{}^{\max} - P_{\mathrm{gen}_i}{}^0 \ \ (\forall \text{ generators } i)$$

$$Q_{\mathrm{gen}_i}{}^{\min} - Q_{\mathrm{gen}_i}{}^0 \le \Delta Q_{\mathrm{gen}_i} \le V_i{}^{\max} - V_i{}^0 \ \ (\forall \text{ buses } i)$$

$$\delta_i{}^{\min} - \delta_i{}^0 \le \Delta\delta_i \le \delta_i{}^{\max} - \delta_i{}^0 \ (\forall \text{ buses } i)$$

$$t_{ij}^{\min} - t_{ij}^{0} \le \Delta t_{ij} \le t_{ij}^{\max} - t_{ij}^{0} \ (\forall \text{ transformer } ij)$$

$$\Delta V_{i_{\text{ref.}}},\ \Delta \delta_{i_{\text{ref.}}}, \Delta P_{\text{load}_i} \text{ and } \Delta Q_{\text{load}_i} = 0$$

In incremental LP method, reactive power is already optimized therefore the inclusion of reactive power to the objective function is for improving the optimization process, if the influence of this inclusion is favorably i.e. improving the optimization process for the real power, then it can be included, if the influence is unfavorably then reactive power cost function must not be included (Prof. Wollenberg).

4.6 The Locational Marginal Price (LMP):

The basic definition of LMP is the marginal increase in cost to the system to supply 1 additional MW of load at bus j [3]. The LMP value is the same as the Lagrange multiplier of the conventional ED and non-linear OPF, the LMP values are differ in AC OPF due to transmission losses and limits, if the line is congested (at their limit) then the LMP values at each bus will have different magnitudes as illustrated in [3].

4.6.1 The LMP At No Line Congestion:

$$\text{LMP}=\text{LMP}_{\text{ref.}} - \frac{\partial \text{P}_{\text{loss}}}{\partial \text{P}_i} \text{LMP}_{\text{ref.}} \tag{4.36}$$

Where:

$\text{LMP}_{\text{ref.}} \equiv$ the LMP at reference bus and can be calculated by: $\frac{\partial F_i(P_{ref.})}{\partial P_{ref.}}$.

$\frac{\partial \text{P}_{\text{loss}}}{\partial \text{P}_i} \equiv$ the incremental loss at bus i see [24] .

4.6.2 The LMP At a Congested Line:

$$\text{LMP}=\text{LMP}_{\text{ref.}} - \frac{\partial \text{P}_{\text{loss}}}{\partial \text{P}_i} \text{LMP}_{\text{ref.}} - \sum_{\ell=1}^{Nll} \mu_\ell a_{\ell i} \tag{4.37}$$

Where:

$\mu_\ell \equiv$ the Lagrange multiplier for line l.

$$\mu_\ell = \frac{\partial F_{ref}\left(P_{ref.}\right)}{\partial P_{ref.}}\left(1 - \frac{\partial \mathrm{P}_{\mathrm{loss}}}{\partial \mathrm{P}_i}\right)\left(\frac{1}{a_{\ell i}}\right) - \frac{\partial f_i(P_i)}{\partial P_i}\left(\frac{1}{a_{\ell i}}\right) \tag{4.38}$$

$a_{\ell i} \equiv$ the line flow sensitivity factor.

$Nll \equiv$ number of lines at limit.

LMP calculation is very important in OPF, where LMPs gives an indication of how much increase or decrease of the total system cost in case of addition or removal of load in a specific bus. Derivations of equation (4.36), (4.37) and (4.38) are available in [3].

Solution algorithm for the full AC incremental LPOPF is same as algorithm for the piecewise LPOPF in section 4.3.3 except that in step 2 the linearization process is done through equation (4.24) and (4.25).

CHAPTER FIVE

IMPLEMENTATION

5.1 Introduction:

In this chapter, implementation of Piecewise LPOPF and full AC iterative LPOPF are introduced, a step by step procedure using *POWERWORLD* Simulator and Microsoft Excel for the 6-bus test system of [3], likewise, an implementation of both methods for the IEEE 30 bus system and a comparison between both methods are presented.

5.2 Systems Description:

5.2.1 The 6-bus System:

The system is consisted of 6-buses, 3 generating units and 11 transmission lines, bus 1 is the slack (reference bus), bus 2 and bus 3 are the P-V buses, bus 4, 5 and 6 are the load (P-Q) busses. The impedances are in per-unit on a base of a 100 MVA and bus voltage limits are from 1.07pu. to 0.95pu. The power flow input data and generation cost functions are available in [3] and Appendix B.

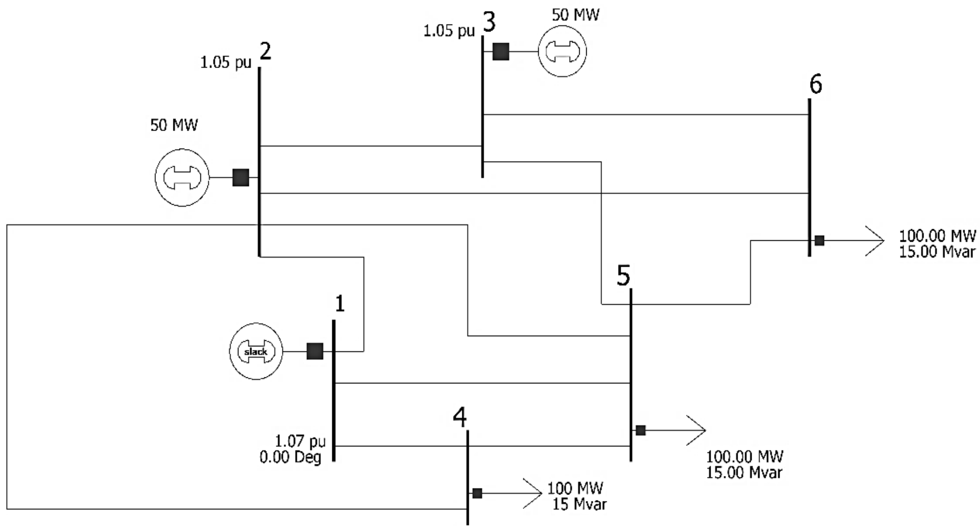


Figure (5.1): Single line diagram of the 6-bus system.

5.2.2 The IEEE 30 Bus System:

This system is part of the American Power Service Cooperation Network which is being made available to the electric utility industry as a standard test case for evaluating various analytical methods and computer programs for the solution of power system problems [4]. This system consists of 30 buses, 6 generating units and 41 transmission lines, bus 1 is the slack bus, bus 2, 5, 8, 11 and 13 are the P-V buses, capacitor banks are existing on bus 2 and bus 10, tap changing transformers are existing between bus 4-12, 6-9, 6-10 and 28-27, the impedances are in per-unit on a base of a 100 MVA and bus voltage limits are from 1.1pu. to 0.9pu, the power flow data for the base power flow study are different from the data for economic and optimization studies where for the power flow studies, generators on bus 5, 8, 11 and 13 are synchronous condensers, and for the economic and optimization studies are generating units which generates both real and reactive powers. Data are available in Appendix C.

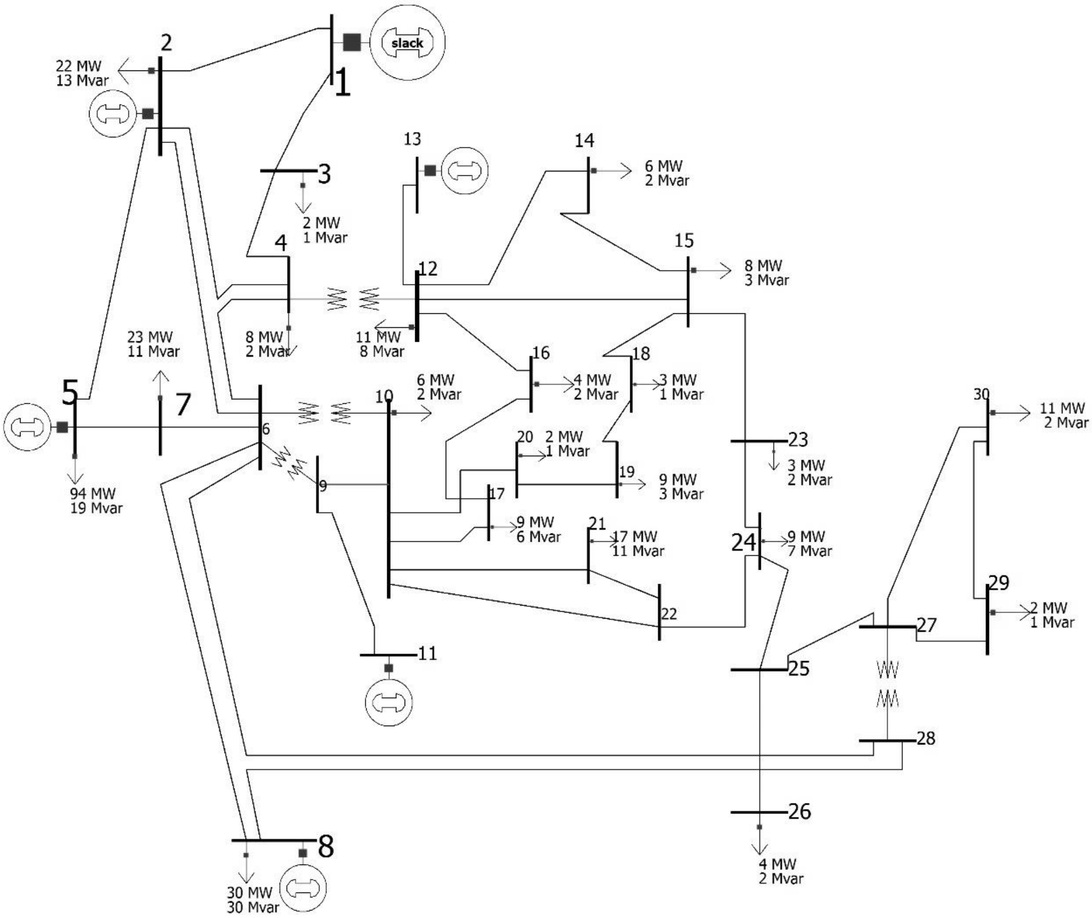


Figure (5.2): Single line diagram of the IEEE 30 bus System

5.3 Implementation of Piecewise LPOPF in the 6-bus system:

- **Step one:** Start with a base power flow solution:

Using solution algorithm of section (4.3.3) for solving PW LPOPF, the first step of solution is starting with a base power flow solution. Results of the initial power flow using *POWERWORLD* Simulator are:

5.3.1 Initial power flow results:

Table (5.1): Initial Power Flow Results.

Bus No.	Generation MW	Generation MVAR	Bus PU Volt	Angles Radians
1	212.96	-10.76	1.07	0
2	50	21.76	1.05	-0.13
3	50	19.02	1.05	-0.16
4	0	0	1.02721	-0.15
5	0	0	1.02212	-0.18
6	0	0	1.02458	-0.21
Total Gen.	312.96 MW	30.02 MVAR		
Total losses	12.96 MW	-14.98 MVAR		

Table (5.2): Line flows and losses of the initial PF.

From Bus	To Bus	MW Flow	Lim MW	MW Loss
1	2	62.18	100	3.6
1	4	82.8	100	3.02
1	5	67.98	100	3.25
2	3	14.76	60	0.1
2	4	28.86	60	0.42
2	5	21.94	60	0.45
2	6	43.01	60	1.17
3	5	12.43	60	0.2
3	6	52.23	60	0.55
4	5	8.21	60	0.14
5	6	6.53	60	0.05
				Total Losses 12.95 MW

- **Step two:** Linearize the objective function:

Linearization of the objective function is the second step of the solution algorithm, using equations (4.13), (4.14) and (4.15) in order to linearize the objective function, the I/O generation cost functions and generation limits of unit 1, 2, and 3 respectively are:

$$F_1(P_1) = 213.1 + 11.669P_1 + 0.00533P_1^2 \quad (5.1)$$
$$F_2(P_2) = 200 + 10.333P_2 + 0.00889P_2^2 \quad (5.2)$$
$$F_3(P_3) = 240 + 10.833P_3 + 0.00741P_3^2 \quad (5.3)$$

Unit 1 limits: $50 \leq P_1 \leq 200$ MW.

Unit 2 limits: $37.5 \leq P_2 \leq 150$ MW.

Unit 3 limits: $45 \leq P_3 \leq 180$ MW.

As stated in section (4.3.1), the first step of formulating the piecewise LP OPF objective function is by converting the I/O cost curve into straight line segments through break points through equation (4.13):

$$\text{The break point step} = \frac{Max.\ limit - \text{Min. Limit}}{\text{No. of the desired segments}}$$

The number of the desired segments are chosen to be 3, therefore:

The BP step for unit 1:

$$\frac{200 - 50}{3} = 50$$

∴ the BPs for unit 1 are:

Table (5.3): BPs of Unit 1.

Unit	Break point 1	BP2	BP3	BP4
1	50	100	150	200

Note that during calculation in this case and the IEEE 30 bus case, when identical break point values are used, solution did not converge with the power flow solution and circles infinitely, therefore a change of one or two BP steps is compulsory in order to obtain the final solution. In this case, BP No. 3 is changed to be equal to 160 as used in [5], and hence:

Table (5.4): BPs of the three units.

Unit	Break point 1	BP2	BP3	BP4
1	50	100	160	200
2	37.5	70	130	150
3	45	90	140	180

The next step is obtaining the limit of each segment using equation (4.14):

Segment 1 of unit 1 P_{11}, segment 2 of unit 1 P_{12}, segment 3 of unit 1 P_{13} limits are: $P_{11} = 100 - 50 = 50$, $P_{12} = 60$ and $P_{13} = 40$. And for unit 2 and 3:

Table (5.5): Segment values of unit 2 and 3.

Unit	Segment	Limit
	P_{21}	32.5
2	P_{22}	60
	P_{23}	20
	P_{31}	45
3	P_{32}	50
	P_{33}	40

Next, the calculation of generation cost segments slope using equation (4.15):

Table (5.6): Segment slopes of unit 1.

Unit 1		
Pi	F(Pi)	Slope
50	809.875	12.4685
100	1433.3	13.0548
160	2216.588	13.5878
200	2760.1	

Table (5.7): Segment slopes of unit 2.

Unit 2		
Pi	F(Pi)	Slope
37.5	599.989063	11.288675
70	966.871	12.111
130	1693.531	12.8222
150	1949.975	

Table (5.8): Segment slopes of unit 3.

Unit 3		
Pi	F(Pi)	Slope
45	742.49025	11.83335
90	1274.991	12.5373
140	1901.856	13.2042
180	2430.024	

And therefore, the linearized objective functions for unit 1, unit 2 and unit 3 using equation (4.16) are:

$$F_1(P_1) = 50 + 12.4685P_{11} + 13.0548P_{12} + 13.5878P_{13} \quad (5.4)$$

$$F_2(P_2) = 37.5 + 11.2886P_{21} + 12.111P_{22} + 12.8222P_{23} \quad (5.5)$$

$$F_3(P_3) = 45 + 11.8333P_{31} + 12.5373P_{32} + 13.2042P_{33} \quad (5.6)$$

$\because F_i\left(P_i^{min}\right)$ for all units are considered to be constants, the generalized objective function including all units is:

Objective function

$$= 12.4685P_{11} + 13.0548P_{12} + 13.5878P_{13} + 11.2886P_{21} + 12.111P_{22} + 12.8222P_{23} + 11.8333P_{31} + 12.5373P_{32} + 13.2042P_{33} \quad (5.7)$$

- **Step three:** Set the control variable limits:

Starting with the equality constraint where total generation must equal to total load plus losses:

$$\sum_{\substack{i=1 \\ j=1}}^{n} P_{ij} = \text{Total generation + losses - } \sum_{i=1}^{n} P_i^{min} \quad (5.8)$$

$$\therefore P_{11} + P_{12} + P_{13} + P_{21} + P_{22} + P_{23} + P_{31} + P_{32} + P_{33} = 180.46 \quad (5.9)$$

Next the inequality constraints that each segment must be at their limit:

$P_{ij} \leq P_{ij}^{max}$, $P_{ij} \geq 0$ therefore:

Table (5.9): Segment limits for the three units.

P_{ij}		P_{ij}^{max}
P_{11}	$\leq$	50
P_{12}	$\leq$	60
P_{13}	$\leq$	40
P_{21}	$\leq$	32.5
P_{22}	$\leq$	60
P_{23}	$\leq$	20
P_{31}	$\leq$	45
P_{32}	$\leq$	50
P_{33}	$\leq$	40

- **Step four:** Formulate the problem in an LP solver and solve:

The used LP solver is Microsoft Excel 2016 which is responsible of obtaining the unknow variables (P_{11} to P_{33}), for a detailed illustration of how to use Excel to solve LP problems see Appendix A.

Before optimizing the objective function, the total cost of the initial power flow solution was 4478.9062 $/hr. and total system losses were 12.96 MW. After solving the LP program, a new generation schedule is obtained:

Table (5.10): Segments values of the initial LP results.

Segment	Min. MW	Solution	Max. MW
P_{11}	0	42.96	50
P_{12}	0	0	60
P_{13}	0	0	40
P_{21}	0	32.5	32.5
P_{22}	0	60	60
P_{23}	0	0	20
P_{31}	0	45	45
P_{32}	0	0	50
P_{33}	0	0	40

Table (5.11): Initial LP results after using equation (4.17)

P_1	P_2	P_3	LP results
92.96	130	90	
Total cost 4312.432 $/hr.			

- **Step five:** Substitute the LP results into the power flow as new set points and run a power flow solution:

After substituting the LP results into *POWERWORLD* Simulator, the power flow results are:

Table (5.12): PF results.

P_1	P_2	P_3	PF results
87.44	130	90	

- **Step six:** No change in variables and no transmission overloads stop, otherwise use step seven which setting the new variables limits, and because of P_1 of the LP solution is not equal to P_1 of the PF, another solution must be found.

The new variable limit is the new power balance equality constraint that reflects the new value of losses due to the new generation scheduling, therefore:

$$P_{11} + P_{12} + P_{13} + P_{21} + P_{22} + P_{23} + P_{31} + P_{32} + P_{33} = 307.44 - 132.5 = 174.95 \quad (5.10)$$

After solving the LP program with the new equality constraint:

Table (5.13): Iteration 2 LP results.

P_1	P_2	P_3	LP results
87.44	130	90	
Total cost 4242.7 $/hr.			

The PF results:

Table (5.14): Iteration 2 PF results.

P_1	P_2	P_3	PF results
87.44	130	90	

Table (5.15): Line flow and losses after convergence of LP and PF.

From Bus	To Bus	MW Flow	Lim MW	MW Loss
1	2	8.68	100	0.1
1	4	42.78	100	0.91
1	5	35.99	100	0.96
2	3	5.26	60	0.01
2	**4**	**64.65**	**60**	**1.9**
2	5	29.83	60	0.81
2	6	38.83	60	0.96
3	5	28.54	60	0.89
3	**6**	**66.72**	**60**	**0.85**
4	5	4.62	60	0.05
5	6	-3.73	60	0.01
				Total losses 7.44 MW

And here, LP results and PF results are equal after 2 iterations and hence the convergence is reached, total cost is 4242.84 $/hr. and total losses are 7.45 MW which is the most least operation cost can be reached. Back to step six: no change in variables? Yes, No transmission overloads? No, there are overloading in line 2-4 and line 3-6, moving to step eight.

- **Step eight:** No change in variables but transmission overload, use the generation shift factors to relief the overloading:

And here a new inequality constraint is introduced in which that the flow of line 2-4 and line 3-6 must be less than or equal to the flow limits. In order to hold each line to their limit, the generation shift factors (GSF) (transmission loading relief TLR factors) must be used, see [3].

- **Step nine:** Add the new transmission constraints:

The new flow constraints for line 2-4 and line 3-6 are modeled as:

$$f_{2-4} = f_{2-4}^0 + a_{2-4,1}(P_1 - P_1^0) + a_{2-4,2}(P_2 - P_2^0) + a_{2-4,3}(P_3 - P_3^0) \leq 60 \qquad (5.11)$$

$$f_{3-6} = f_{3-6}^0 + a_{3-6,1}(P_1 - P_1^0) + a_{3-6,2}(P_2 - P_2^0) + a_{3-6,3}(P_3 - P_3^0) \leq 60 \qquad (5.12)$$

Where:

$f_{2-4}^0 \equiv$ the initial power flow in line 2-4.

$f_{3-6}^0 \equiv$ the initial power flow in line 3-6.

$a_{2-4,1} \equiv$ the GSF factor which is the sensitivity of line 2-4 overloading on generation at bus 1 and always equal to zero for the slack bus.

$a_{2-4,2}, a_{2-4,3} \equiv$ the sensitivity of line 2-4 overloading on generation at bus 2 and 3. As well for $a_{3-6,1}$, $a_{3-6,2}$ and $a_{3-6,3}$.

$f_{2-4}^0 =$ 28.86 MW, $a_{2-4,1} = 0$, $a_{2-4,2} = 0.325$, $a_{2-4,3} = 0.239$, $P_2^0 = 50$ MW, $P_2 = 37.5 + P_{21} + P_{22} + P_{23}$, $P_3^0 = 50$, $P_3 = 45 + P_{31} + P_{32} + P_{33}$.

$\therefore$ the new flow constraint for line 2-4 is:

$$28.86 + 0.325(37.5 + P_{21} + P_{22} + P_{23} - 50) + 0.239(45 + P_{31} + P_{32} + P_{33} - 50) \leq 60 \qquad (5.13)$$

Or $0.325(P_{21} + P_{22} + P_{23}) + 0.239(P_{31} + P_{32} + P_{33}) \leq 36.3975$ (5.14)

Similarly, for line 3-6:

$$-0.005(P_{21} + P_{22} + P_{23}) + 0.371(P_{31} + P_{32} + P_{33}) \leq 9.5625 \qquad (5.15)$$

After the addition of the new flow constraints, the LP results are:

Table (5.16): LP results after the addition of the new flow constraints.

P_1	P_2	P_3	LP results
106.4138457	129.1477862	71.87836814	
Total cost 4254.9 $/hr.			

The PF Results:

Table (5.17): PF results of generator 1, 2 and 3.

P_1	P_2	P_3	PF results
106.65	129.16	71.91	

Table (5.18): Line flow and losses.

From Bus	To Bus	MW Flow	Lim MW	MW Loss
1	2	16.06	100	0.23
1	4	48.66	100	1.12
1	5	41.97	100	1.28
2	3	11.71	60	0.06
2	4	60.01	60	1.63
2	5	30.12	60	0.82
2	6	43.13	60	1.18
3	5	23.54	60	0.6
3	6	59.99	60	0.7
4	5	5.91	60	0.08
5	6	-1.24	60	0
				Total losses 7.72 MW

Repeating step six:

$$P_{11} + P_{12} + P_{13} + P_{21} + P_{22} + P_{23} + P_{31} + P_{32} + P_{33} = 175.22 \qquad (5.16)$$

After solving the LP program with the new equality constraint:

Table (5.19): Iteration 2 LP results after the overloading relief.

P_1	P_2	P_3	LP results
106.6938	129.14778	71.878	
Total cost 4258.462 $/hr.			

And hence the convergence is reached. Note that the total operating cost and total losses are increased after overloading relief from 4242.84 $/hr., 7.44 MW to 4258.488 $/hr., 7.72 MW due to the new generation scheduling.

The OPF solution is found after four iterations and therefore, this is an indication of the PW LPOPF speed in obtaining the optimal solution.

5.3.2 Results of PW LPOPF in a Compacted Form:

- Results of the initial PF and results of the OPF:

Table (5.20): Results of the initial power flow PF0.

Bus No.	Generation MW	Generation MVAR	Bus PU Volt	Angles Radians
1	212.96	-10.76	1.07	0
2	50	21.76	1.05	-0.13
3	50	19.02	1.05	-0.16
4	0	0	1.02721	-0.15
5	0	0	1.02212	-0.18
6	0	0	1.02458	-0.21
Total Gen	312.96	30.02		
Total losses	12.96	-14.98		
Total Cost	4478.9062 $/hr.			

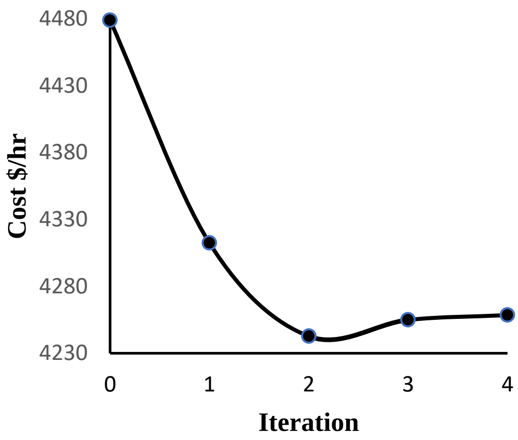
Table (5.21): LPOPF results by incorporating *POWERWORLD* Simulator and Microsoft Excel.

Bus No.	Generation MW	Generation MVAR	Bus PU Volt	Angles Radians
1	106.69	17.22	1.07	0
2	129.15	-16.43	1.05	-0.03
3	71.88	12.29	1.05	-0.05
4	0	0	1.02412	-0.08
5	0	0	1.02193	-0.11
6	0	0	1.02492	-0.11
Total Gen	307.72	13.08		
Total losses	7.72	-31.92		
Total Cost	4258.487 $/hr.			

- Reduction of total operating cost and transmission losses:

Table (5.22): Reduction of total and losses cost during iterations.

Iteration	Total Cost $/hr.	Losses Mw
0	4478.9	12.96
1	4312.4	7.44
2	4242.7	7.72
3	4254.9	7.72
4	4258.49	7.72



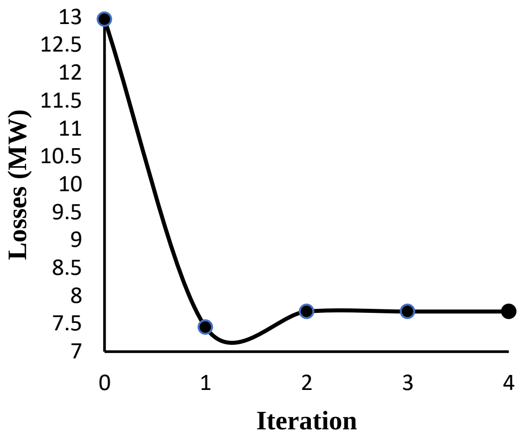


Figure (5.3): Reduction of total cost. Figure (5.4): Reduction of losses.

Note that *POWERWORLD* Simulator uses LP method in order to solve the OPF problems by using the same technique.

- Results of OPF using *POWERWORLD* Simulator directly:

Table (5.23): Result of LPOPF using *POWERWORLD* Simulator

Bus No.	Generation MW	Generation MVAR	Bus PU Volt	Angles Radians
1	106.71	17.21	1.07	0
2	129.1	-16.42	1.05	-0.03
3	71.9	12.28	1.05	-0.05
4	0	0	1.02412	-0.08
5	0	0	1.02193	-0.11
6	0	0	1.02492	-0.11
Total Gen	307.71	13.07		
Total losses	7.71	-31.93		
Total Cost	4258.35$/hr.			

5.4 Implementation of Full AC Incremental LP Method in the 6-bus System:

By the use of same steps that used in order to obtain the PW LPOPF, the incremental LPOPF is solved.

- **Step one:** Start with a base PF solution:

Results of the base PF are in table 5.1.

- **Step two:** Linearize the objective function and linearize constraints:

As illustrated in the previous chapter, linearization of the objective function is done through equation (4.24) and linearization of constraints through equation (4.25) knowing that the incremental LPOPF is formulated through the increments of $P_{gen}, Q_{gen}, |V_i|$ and δ_i which is ΔP_{gen}, ΔQ_{gen}, ΔV and $\Delta\delta$. Linearizing equation (5.3), (5.4) and (5.6):

$$\frac{\partial F_1({P_1}^0)}{\partial {P_1}^0} = 11.669 + 0.01066{P_1}^0 \tag{5.18}$$

$$\frac{\partial F_2({P_2}^0)}{\partial {P_2}^0} = 10.333 + 0.01778{P_2}^0 \tag{5.19}$$

$$\frac{\partial F_3({P_3}^0)}{\partial {P_3}^0} = 10.833 + 0.01482{P_3}^0 \tag{5.20}$$

Where: ${P_1}^0, {P_2}^0$ and ${P_3}^0$ are the initial power flow results PF0, therefore:

$$\frac{\partial F_1({P_1}^0)}{\partial {P_1}^0} = 11.669 + 0.01066 \times 212.96 = 13.94$$

$$\frac{\partial F_2({P_2}^0)}{\partial {P_2}^0} = 10.333 + 0.01778 \times 50 \ = \ 11.222$$

$$\frac{\partial F_3\left({P_3}^0\right)}{\partial {P_3}^0} = 10.833 + 0.01482 \times 50 = 11.574$$

∴ The linearized objective function using equation (4.24) is:

$$\frac{\partial F_1({P_1}^0)}{\partial {P_1}^0}\Delta P_1 + \frac{\partial F_2({P_2}^0)}{\partial {P_2}^0}\Delta P_2 + \frac{\partial F_3({P_3}^0)}{\partial {P_3}^0}\Delta P_3 \tag{5.21}$$

$$= 13.94\Delta P_1 + 11.222\Delta P_2 + 11.574\Delta P_3 \tag{5.22}$$

From equation (4.25):

$$\Delta P = [\boldsymbol{J}]\begin{bmatrix}\Delta\delta \\ \Delta V\end{bmatrix} \tag{5.23}$$

Therefore:

$$\Delta P_1 = \begin{bmatrix}\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_6} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6\end{bmatrix} \tag{5.24}$$

$$\Delta P_2 = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_1} & \cdots & \frac{\partial P_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial P_2}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \tag{5.25}$$

$$\Delta P_3 = \begin{bmatrix} \frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \tag{5.26}$$

The linearized objective function becomes:

$$\begin{aligned} \text{Minimize: } & 13.94 \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_6} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \\ + \quad & 11.222 \begin{bmatrix} \frac{\partial P_2}{\partial \delta_1} & \cdots & \frac{\partial P_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial P_2}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \\ + \quad & 11.574 \begin{bmatrix} \frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \end{aligned} \tag{5.27}$$

Therefore, the unknown variables that to be calculated through the LP solution are $\Delta\delta_2$ to $\Delta\delta_6$ and ΔV_2 to ΔV_6.

The linearized real and reactive power balance constraints are:

$$\begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_6} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_6} \\ \frac{\partial P_2}{\partial \delta_1} & \cdots & \frac{\partial P_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial P_2}{\partial V_6} \\ \frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6} \\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_6} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_6} \\ \frac{\partial Q_2}{\partial \delta_1} & \cdots & \frac{\partial Q_2}{\partial \delta_6} & \frac{\partial Q_2}{\partial V_1} & \cdots & \frac{\partial Q_2}{\partial V_6} \\ \frac{\partial Q_3}{\partial \delta_1} & \cdots & \frac{\partial Q_3}{\partial \delta_6} & \frac{\partial Q_3}{\partial V_1} & \cdots & \frac{\partial Q_3}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} = \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} \tag{5.28}$$

The linearized power balance constraints of the initial formulation:

$$
\begin{array}{c}
\begin{array}{c|cccccccccccc|}
 & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\
P_1 & 13.244 & -4.747 & 0.000 & -5.313 & -3.184 & 0.000 & 7.972 & -1.567 & 0.000 & -0.495 & 1.470 & 0.000 \\
P_2 & -4.165 & 25.360 & -4.270 & -8.720 & -3.280 & -4.910 & -2.627 & 10.270 & -0.670 & -4.010 & -0.860 & -1.260 \\
P_3 & 0.000 & -4.210 & 18.080 & 0.000 & -3.440 & -10.430 & 0.000 & -0.950 & 4.840 & 0.000 & -1.460 & -1.560 \\
Q_1 & -0.652 & 1.645 & 0.000 & 0.509 & -1.502 & 0.000 & 12.768 & -4.521 & 0.000 & -5.172 & -3.115 & 0.000 \\
Q_2 & 2.811 & -9.780 & 0.700 & 4.120 & 0.880 & 1.290 & -3.892 & 24.560 & -4.060 & -8.490 & -3.210 & -4.790 \\
Q_3 & 0.000 & 0.990 & -4.080 & 0.000 & 1.490 & 1.600 & 0.000 & -4.010 & 17.580 & 0.000 & -3.360 & -10.180
\end{array}
\end{array}
\; X \;
\begin{vmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \Delta\delta_6 \\ \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \\ \Delta V_5 \\ \Delta V_6 \end{vmatrix}
=
\begin{vmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{vmatrix}
$$

- **Step three:** Set the control variables limits:

And from section (4.4.2), the inequality constraints are:

$$P_{\text{gen}_i}{}^{\min} - P_{\text{gen}_i}{}^{0} \le \Delta P_{\text{gen}_i} \le P_{\text{gen}_i}{}^{\max} - P_{\text{gen}_i}{}^{0} \quad (\forall \text{ generators } i)$$

$$Q_{\text{gen}_i}{}^{\min} - Q_{\text{gen}_i}{}^{0} \le \Delta Q_{\text{gen}_i} \le Q_{\text{gen}_i}{}^{\max} - Q_{\text{gen}_i}{}^{0} \quad (\forall \text{ generators } i)$$

$$V_i{}^{\min} - V_i{}^{0} \le \Delta V_i \le V_i{}^{\max} - V_i{}^{0} \quad (\forall \text{ buses } i)$$

$$\delta_i{}^{\min} - \delta_i{}^{0} \le \Delta \delta_i \le \delta_i{}^{\max} - \delta_i{}^{0} \quad (\forall \text{ buses } i)$$

The real and reactive power limits:

Table (5.24): Real power limits.

Real Power limits	Min.	Max.
Unit 1	50	200
Unit 2	37.5	150
Unit 3	45	180

Table (5.25): Reactive power limits.

Reactive Power limits	Min.	Max.
Unit 1	-100	150
Unit 2	-100	150
Unit 3	-100	120

ΔP_{gen}, ΔQ_{gen}, ΔV and $\Delta\delta$ limits:

Table (5.26): ΔP limits.

ΔP limits	Min.	Max.
Unit 1	-162.96	-12.96
Unit 2	-12.5	100
Unit 3	-5	130

Table (5.27): ΔQ limits.

ΔQ limits	Min.	Max.
Unit 1	-89.24	160.76
Unit 2	-121.76	128.24
Unit 3	-119.02	100.98

Table (5.28): Bus voltage limits.

V limits	
Min.	Max.
0.95	1.07

Different values can be used for $\Delta\delta$ limits such as from -45 to 45 or from -90 to 90, in this solution, from -56.7 to 56.7 degrees are used.

And the new values of P_{gen} and Q_{gen} are:

$$P_{\mathrm{gen\ i\ new}} = {P_{\mathrm{gen}_i}}^0 + \Delta P_{\mathrm{gen}_i} \tag{5.29}$$

$$Q_{\mathrm{gen\ i\ new}} = {Q_{\mathrm{gen}_i}}^0 + \Delta Q_{\mathrm{gen}_i} \tag{5.30}$$

The Final Problem form to be formulated in Microsoft Excel:

Minimize:

$$13.94\begin{bmatrix}\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_6} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\,11.222\begin{bmatrix}\frac{\partial P_2}{\partial \delta_1} & \cdots & \frac{\partial P_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial P_2}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\,11.574\begin{bmatrix}\frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

Subjected to:

$$\begin{bmatrix}\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_6} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_6}\\ \frac{\partial P_2}{\partial \delta_1} & \cdots & \frac{\partial P_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial P_2}{\partial V_6}\\ \frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6}\\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_6} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_6}\\ \frac{\partial Q_2}{\partial \delta_1} & \cdots & \frac{\partial Q_2}{\partial \delta_6} & \frac{\partial Q_2}{\partial V_1} & \cdots & \frac{\partial Q_2}{\partial V_6}\\ \frac{\partial Q_3}{\partial \delta_1} & \cdots & \frac{\partial Q_3}{\partial \delta_6} & \frac{\partial Q_3}{\partial V_1} & \cdots & \frac{\partial Q_3}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix} = \begin{bmatrix}\Delta P_1\\ \Delta P_2\\ \Delta P_3\\ \Delta Q_1\\ \Delta Q_2\\ \Delta Q_3\end{bmatrix}$$

$$212.96 + \Delta P_1 + 50 + \Delta P_2 + 50 + \Delta P_3 = 312.96\ \mathrm{MW}$$

$$-10.76 + \Delta Q_1 + 21.76 + \Delta Q_2 + 19.02 + \Delta Q_3 = 30.02\ \mathrm{MVAR}$$

$$
\begin{array}{rcccr}
-162.96 & \leq & \Delta P_1 & \leq & -12.96 \\
-12.5 & \leq & \Delta P_2 & \leq & 100 \\
-5 & \leq & \Delta P_3 & \leq & 130 \\
-89.24 & \leq & \Delta Q1 & \leq & 160.76 \\
-121.76 & \leq & \Delta Q2 & \leq & 128.24 \\
-119.02 & \leq & \Delta Q3 & \leq & 100.98 \\
-0.1 & \leq & \Delta V2 & \leq & 0.02 \\
-0.1 & \leq & \Delta V3 & \leq & 0.02 \\
-0.07721 & \leq & \Delta V4 & \leq & 0.04279 \\
-0.07212 & \leq & \Delta V5 & \leq & 0.04788 \\
-0.07458 & \leq & \Delta V6 & \leq & 0.04542 \\
-0.86 & \leq & \Delta\delta 2 & \leq & 1.12 \\
-0.83 & \leq & \Delta\delta 3 & \leq & 1.15 \\
-0.84 & \leq & \Delta\delta 4 & \leq & 1.14 \\
-0.81 & \leq & \Delta\delta 5 & \leq & 1.17 \\
-0.78 & \leq & \Delta\delta 6 & \leq & 1.2 \\
50 & \leq & P_1 & \leq & 200 \\
37.5 & \leq & P_2 & \leq & 150 \\
45 & \leq & P_3 & \leq & 180 \\
-100 & \leq & Q_1 & \leq & 150 \\
-100 & \leq & Q_2 & \leq & 150 \\
-100 & \leq & Q_3 & \leq & 120
\end{array}
$$

$$P_1,\ P_2,\ P_3 \geq 0$$

- **Step four:** Formulate the problem in an LP solver and solve:

And by formulating the above equations in Microsoft Excel, the optimal solution can be found. Note that ΔV_1 and $\Delta\delta_1$ must not set to equal to zero, they must set as non-constraint variables to circulate freely in order to obtain the final solution, during solution and when they are forced to equal to zero, solution circles infinitely and convergence will not be reached. Therefore, the LP program must calculate the values of ΔV_1 and $\Delta\delta_1$ as non-restricted variables, but, actually the values of ΔV_1 and $\Delta\delta_1$ are equal to zero where P_1 and Q_1 are calculated through the PF solution, but in LP formulation they are set to be as non-restricted variables in order to solve the OPF problem correctly.

i. **The First Iteration: LP results:**

Table (5.29): Variables result after the first LP solution.

Variable	Min.	Solution	Max.
$\Delta\delta_2$	-0.86	1.120	1.120
$\Delta\delta_3$	-0.83	1.150	1.150
$\Delta\delta_4$	-0.84	-0.840	1.140
$\Delta\delta_5$	-0.81	-0.810	1.170
$\Delta\delta_6$	-0.78	-0.780	1.200
ΔV_2	-0.1	0.020	0.020
ΔV_3	-0.1	0.020	0.020
ΔV_4	-0.077	-0.077	0.043
ΔV_5	-0.072	-0.072	0.048
ΔV_6	-0.075	-0.075	0.045

Table (5.30): Variables result after the first LP solution.

Variable	Min.	Solution	Max.
ΔP_1	-163	-95.225519	-12.96
ΔP_2	-12.5	67.927	100.000
ΔP_3	-5	27.298	130.000
ΔQ_1	-89.24	61.885	160.760
ΔQ_2	-121.8	-57.120	128.240
ΔQ_3	-119	-4.765	100.980

And by the use of equation (5.29) and (5.30):

Table (5.31): LP results for real power.

P_1	117.73448
P_2	117.92748
P_3	77.29804
Total generation	312.96

Table (5.32): LP results for reactive power.

Q_1	51.124887
Q_2	-35.35973
Q_3	14.254848
Total generation	30.02

The objective function value, equation (5.22):

$$13.93915\times-95.2255187+11.222\times67.9274787+11.574\times27.29804 = -249.133$$

- **Step five:** Substitute the LP results into the power flow as new set points and run a power flow solution:

PF results:

Table (5.33): Generation results.

Unit	MW	MVAR
1	112.99	37.25
2	117.93	-35.36
3	77.3	14.25
Total Generation	308.22	16.14

Table (5.34): Bus voltages & phase angles.

Bus No.	PU Volt	Angle in Radians
1	1.07	0
2	1.03027	-0.03
3	1.03561	-0.05
4	1.01064	-0.08
5	1.00863	-0.11
6	1.00896	-0.11

Table (5.35): Line flows and total losses.

From Bus	To Bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
1	2	19.65	9.31	21.75	100	0.45	-3.5
1	4	50.6	18.72	53.95	100	1.31	0.91
1	5	42.74	9.22	43.72	100	1.39	-1.28
2	3	9.39	-7.14	11.8	60	0.05	-6.16
2	4	57.78	-7.93	58.32	60	1.59	1.11
2	5	28.91	-3.09	29.08	60	0.79	-1.79
2	6	41.04	-4.37	41.27	60	1.11	-2.02
3	5	24.44	-2.48	24.57	60	0.67	-3.78
3	**6**	**62.2**	**15.74**	**64.16**	**60**	**0.77**	**1.78**
4	5	5.47	-6.23	8.29	60	0.07	-8.02
5	6	-1.35	-2.71	3.03	60	0	-6.1
					Total losses	8.2	-28.85

Therefore, the reduced total cost using equations (5.1), (5.2) and (5.3):

$$F_1(P_1) + F_2(P_2) + F_3(P_3)$$
$$= 213.1 + 11.669 \times 112.99 + 0.00533 \times 122.99^2 + 200 + 10.333 \times 117.93 + 0.00889 \times 117.93^2 + 240 + 10.833 \times 77.3 + 0.00741 \times 77.3^2 = 4263.5031 \text{ \$/hr}.$$

- **Step seven:** set the new control variables limits:

Here, the new variables to be substituted in the LP program are the new total generation values for both real and reactive powers, the new voltage magnitudes, the new phase angles and the new Jacobean matrix.

After eight iterations, PF and LP results are equal, but as observed there is overloading in line 2-4 and line 3-6 and therefore the GSF factors must be used, moving to step eight:

- **Step eight:** No change in variables but transmission overload, use the generation shift factors to relief the overloading:

A new inequality constraint will be added to the LP program:

$$\sum_{i=1}^{N_{\text{gen}}} a_{\ell i} P_i \leq f_\ell^{\max} + \sum_{i=1}^{N_{\text{gen}}} a_{\ell i} P_i^{\;0} - f_\ell^{\;0} \tag{5.31}$$

For line 2-4:

$$a_{2-4,2} = 0.327, a_{2-4,3} = 0.245, P_2 = 127.01\text{MW}, P_3 = 77.93\text{MW}, f_\ell^{\max} = 60\text{MW}, f_\ell^{\;0} = 28.86\text{MW}, P_2^{\;0} = 50\text{MW}, P_3^{\;0} = 50\text{MW}$$

$\therefore$ 0.327×127.01+0.245×77.93 ≤60+(0.327×50+0.245×50)-28.86

$\therefore$ The new transmission constraint for line 2-4 is:

$$60.7 \leq 59.77$$

For line 3-6:

$$a_{3\text{-}6,2} = -0.0045, a_{3\text{-}6,3} = 0.372, P_2 = 127.01\text{MW}, P_3 = 77.93\text{MW}, f_\ell^{\max} = 60\text{MW}, f_\ell^{\;0} = 52.23\text{MW}, P_2^{\;0} = 50\text{MW}, P_3^{\;0} = 50\text{MW}$$

$\therefore$ -0.0045×127.01+0.372×77.93 ≤60+(-0.0045×50+0.372×50)-52.23

$\therefore$ The new transmission constraint for line 3-6 is:

$$28.45 \leq 26.167$$

After twelve iterations, the optimal solution is found, where total operating cost is reduced from **4478.906 \$/hr.** to **4258.032 \$/hr.,** total transmission losses for real power from **12.96 MW** to **7.62 MW** and total transmission losses for reactive power from **-14.98 MVAR** to **-32.82 MVAR.**

5.4.1 Final Solution of Incremental and PW LPOPF in a Compacted from:

Table (5.36): Results of the initial power flow PF0.

Bus No.	Generation MW	Generation MVAR	Bus PU Volt	Angles Radians
1	212.96	-10.76	1.07	0
2	50	21.76	1.05	-0.13
3	50	19.02	1.05	-0.16
4	0	0	1.02721	-0.15
5	0	0	1.02212	-0.18
6	0	0	1.02458	-0.21
Total Gen	312.96	30.02		
Total losses	12.96	-14.98		
Total Cost	4478.9062 $/hr.			

Table (5.37): Incremental LPOPF results by incorporating *POWERWORLD* Simulator and Microsoft Excel.

Bus No.	Generation MW	Generation MVAR	Bus PU Volt	Angles Radians
1	110.01	7.18	1.07	0
2	125.83	-10.8	1.05732	-0.03
3	71.78	15.81	1.05982	-0.06
4	0	0	1.02962	-0.09
5	0	0	1.02867	-0.11
6	0	0	1.03377	-0.11
Total Gen	307.62	12.19		
Total losses	7.62	-32.81		
Total Cost	4258.032 $/hr.			

Table (5.38): PW LPOPF results by incorporating *POWERWORLD* Simulator and Microsoft Excel.

Bus No.	Generation MW	Generation MVAR	Bus PU Volt	Angles Radians
1	106.69	17.22	1.07	0
2	129.15	-16.43	1.05	-0.03
3	71.88	12.29	1.05	-0.05
4	0	0	1.02412	-0.08
5	0	0	1.02193	-0.11
6	0	0	1.02492	-0.11
Total Gen	307.72	13.08		
Total losses	7.72	-31.92		
Total Cost	4258.487 $/hr.			

Where from table (5.98) and table (5.99), the full AC incremental LPOPF is better than the PW LPOPF in all aspects.

5.4.2 Results of Incremental LPOPF Graphically:

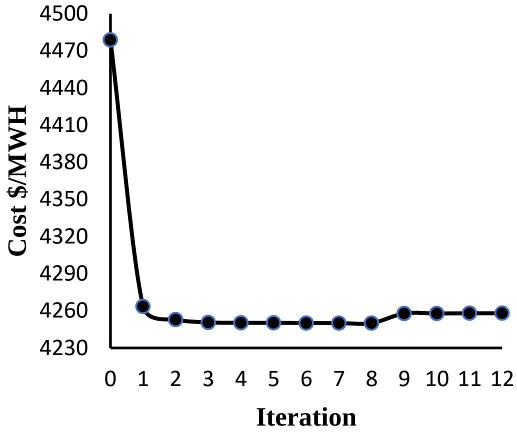


Figure (5.5): Total cost reduction.

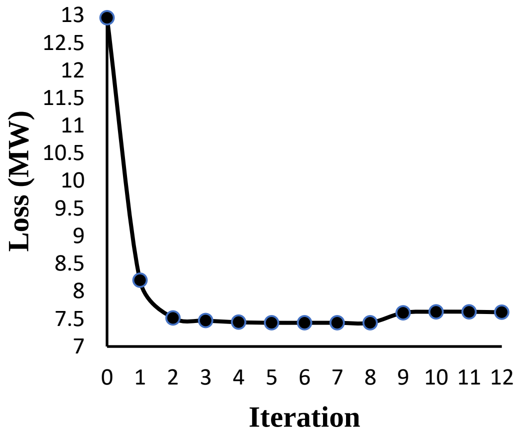


Figure (5.6): Total loss reduction (MW).

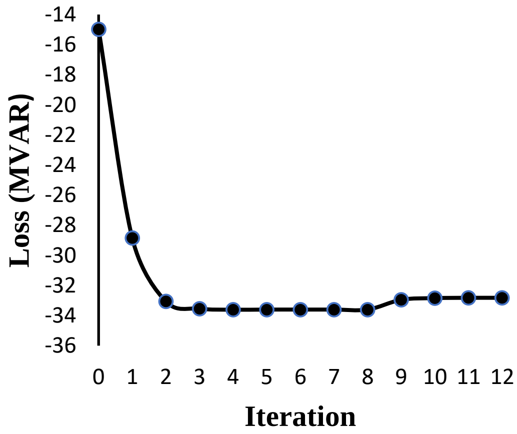


Figure (5.7): Total loss reduction (MVAR).

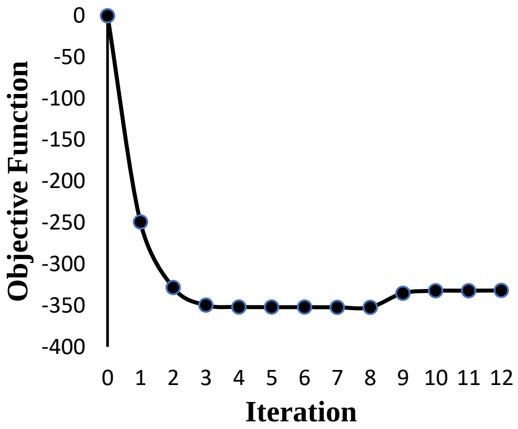


Figure (5.8): Objective function.

5.5 Reactive Power Pricing for the 6-bus System and Including the VAR cost Function in the Objective Function:

The equation that will be used in order to calculate the total operating cost for reactive power is:

$$\mathrm{F}_i(\mathrm{Q}_i)=0.05\mathrm{b}\mathrm{Q}_i^2 \qquad (5.32)$$

The linearized version of equation (5.31) is:

$$\frac{\partial \mathrm{F}_i(\mathrm{Q}_i)}{\partial \mathrm{Q}_i} = 0.1\mathrm{b}\mathrm{Q}_i \qquad (5.33)$$

∴ For unit 1, 2 and 3:

$$\frac{\partial \mathrm{F_1(Q_1)^0}}{\partial Q_1{}^0} = 0.1 \times 11.669 \times -10.76$$

$$= -12.555 \tag{5.34}$$

$$\frac{\partial \mathrm{F_2(Q_2)^0}}{\partial Q_2{}^0} = 0.1 \times 10.333 \times 21.76$$

$$= 22.485 \tag{5.35}$$

$$\frac{\partial \mathrm{F_3(Q_3)^0}}{\partial Q_3{}^0} = 0.1 \times 10.833 \times 19.02$$

$$= 20.6 \tag{5.36}$$

And from equation (4.34), the new part to be added is:

$$\frac{\partial \mathrm{F_1(Q_1)^0}}{\partial Q_1{}^0}\Delta \mathrm{Q_1} + \frac{\partial \mathrm{F_2(Q_2)^0}}{\partial Q_2{}^0}\Delta \mathrm{Q_2} + \frac{\partial \mathrm{F_3(Q_3)^0}}{\partial Q_3{}^0}\Delta \mathrm{Q_3} \tag{5.37}$$

Where:

$$\Delta \mathrm{Q_1} = \begin{bmatrix} \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_6} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \tag{5.38}$$

$$\Delta \mathrm{Q_2} = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_1} & \cdots & \frac{\partial Q_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial Q_2}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \tag{5.39}$$

$$\Delta \mathrm{Q_3} = \begin{bmatrix} \frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} \tag{5.40}$$

∴ The term to be added to the objective function is:

$$-12.555\begin{bmatrix}\frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_6} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\quad 22.485\begin{bmatrix}\frac{\partial Q_2}{\partial \delta_1} & \cdots & \frac{\partial Q_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial Q_2}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\quad 20.6\begin{bmatrix}\frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

Therefore, the new objective function is:

$$\text{Minimize:}\qquad 13.94\begin{bmatrix}\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_6} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\quad 11.222\begin{bmatrix}\frac{\partial P_2}{\partial \delta_1} & \cdots & \frac{\partial P_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial P_2}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\quad 11.574\begin{bmatrix}\frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$-\quad 12.555\begin{bmatrix}\frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_6} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\quad 22.485\begin{bmatrix}\frac{\partial Q_2}{\partial \delta_1} & \cdots & \frac{\partial Q_2}{\partial \delta_6} & \frac{\partial Q_2}{\partial V_1} & \cdots & \frac{\partial Q_2}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

$$+\quad 20.6\begin{bmatrix}\frac{\partial Q_3}{\partial \delta_1} & \cdots & \frac{\partial Q_3}{\partial \delta_6} & \frac{\partial Q_3}{\partial V_1} & \cdots & \frac{\partial Q_3}{\partial V_6}\end{bmatrix}\begin{bmatrix}\Delta\delta_1\\ \vdots\\ \Delta\delta_6\\ \Delta V_1\\ \vdots\\ \Delta V_6\end{bmatrix}$$

Subjected to:

$$\begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_6} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_6} \\ \frac{\partial P_2}{\partial \delta_1} & \cdots & \frac{\partial P_2}{\partial \delta_6} & \frac{\partial P_2}{\partial V_1} & \cdots & \frac{\partial P_2}{\partial V_6} \\ \frac{\partial P_3}{\partial \delta_1} & \cdots & \frac{\partial P_3}{\partial \delta_6} & \frac{\partial P_3}{\partial V_1} & \cdots & \frac{\partial P_3}{\partial V_6} \\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_6} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_6} \\ \frac{\partial Q_2}{\partial \delta_1} & \cdots & \frac{\partial Q_2}{\partial \delta_6} & \frac{\partial Q_2}{\partial V_1} & \cdots & \frac{\partial Q_2}{\partial V_6} \\ \frac{\partial Q_3}{\partial \delta_1} & \cdots & \frac{\partial Q_3}{\partial \delta_6} & \frac{\partial Q_3}{\partial V_1} & \cdots & \frac{\partial Q_3}{\partial V_6} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \Delta\delta_6 \\ \Delta V_1 \\ \vdots \\ \Delta V_6 \end{bmatrix} = \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \\ \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix}$$

$$212.96 + \Delta P_1 + 50 + \Delta P_2 + 50 + \Delta P_3 = 312.96 \text{ MW}$$

$$-10.76 + \Delta Q_1 + 21.76 + \Delta Q_2 + 19.02 + \Delta Q_3 = 30.02 \text{ MVAR}$$

-162.96	≤	ΔP_1	≤	-12.96
-12.5	≤	ΔP_2	≤	100
-5	≤	ΔP_3	≤	130
-89.24	≤	ΔQ_1	≤	160.76
-121.76	≤	ΔQ_2	≤	128.24
-119.02	≤	ΔQ_3	≤	100.98
-0.1	≤	ΔV_2	≤	0.02
-0.1	≤	ΔV_3	≤	0.02
-0.07721	≤	ΔV_4	≤	0.04279
-0.07212	≤	ΔV_5	≤	0.04788
-0.07458	≤	ΔV_6	≤	0.04542
-0.86	≤	$\Delta\delta_2$	≤	1.12
-0.83	≤	$\Delta\delta_3$	≤	1.15
-0.84	≤	$\Delta\delta_4$	≤	1.14
-0.81	≤	$\Delta\delta_5$	≤	1.17
-0.78	≤	$\Delta\delta_6$	≤	1.2
50	≤	P_1	≤	200
37.5	≤	P_2	≤	150
45	≤	P_3	≤	180
-100	≤	Q_1	≤	150
-100	≤	Q_2	≤	150
-100	≤	Q_3	≤	120

$$P_1,\ P_2,\ P_3 \geq 0$$

Following the same algorithm that used in last section, the final optimal solution after the addition of the reactive power cost function:

LP results:

Table (5.39): Variables result of the optimal solution.

Variable	Min.	Solution	Max.
ΔP_1	-163	-97.960062	-12.96
ΔP_2	-12.5	71.2137638	100
ΔP_3	-5	21.7262979	130
ΔQ_1	-89.24	33.0168704	160.8
ΔQ_2	-121.8	-41.937472	128.2
ΔQ_3	-119	-6.5993981	101

Table (5.40): Real power results.

P_1	114.999938
P_2	121.213764
P_3	71.7262979
Total generation	307.94

Table (5.41): Reactive power results.

Q_1	22.2568704
Q_2	-20.177472
Q_3	12.4206019
Total generation	14.5

PF results:

Table (5.42): Generation results.

Unit	MW	MVAR
1	115	22.26
2	121.21	-20.18
3	71.73	12.42
Total Generation	307.94	14.5

Table (5.43): Bus voltages & phase angles.

Bus No.	PU Volt	Angle in Radians
1	1.07	0
2	1.0433	-0.03
3	1.0441	-0.06
4	1.01973	-0.09
5	1.0171	-0.11
6	1.01883	-0.11

Table (5.44): Line flows and total losses.

From Bus	To Bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
1	2	19.9	2.3	20.03	100	0.36	-3.74
1	4	51.29	13.8	53.12	100	1.26	0.68
1	5	43.81	6.1	44.24	100	1.4	-1.27
2	3	11.33	-5.7	12.69	60	0.06	-6.23
2	4	57.45	-3.6	57.57	60	1.52	0.91
2	5	29.32	-1.6	29.37	60	0.79	-1.88
2	6	42.63	-3.2	42.75	60	1.17	-1.98
3	5	22.99	-1.9	23.07	60	0.58	-4.05
3	6	60.01	14.8	61.81	60	0.71	1.41
4	5	5.96	-6.4	8.73	60	0.08	-8.14
5	6	-0.77	-3.4	3.52	60	0	-6.22
					Total losses	7.93	-30.51

The objective function value is **-1808.34**.

The reduced cost for real power is **4263.777** $/hr.

The reduced cost for reactive power is **582.93**$/hr.

As observed, the optimal results before the addition of the reactive power cost function was better in all aspects except that the voltage profile is improved:

Table (5.45): Before and after the addition of the VAR cost function.

	Before	After
Total real power cost	4258 $/hr.	4263.777 $/hr.
Total reactive power cost	159.9 $/hr.	362.2 $/hr.
Total real power losses	7.62 MW	7.94 MW
Total reactive power losses	-32.81 MVAR	-30.51 MVAR

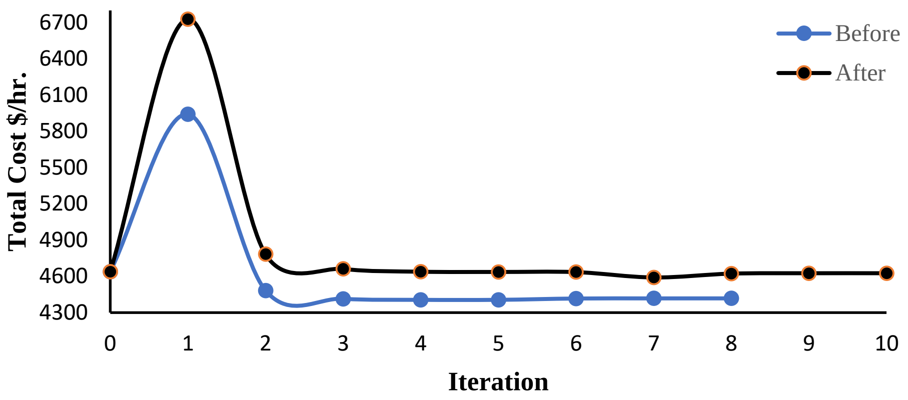


Figure (5.9): Total cost reduction.

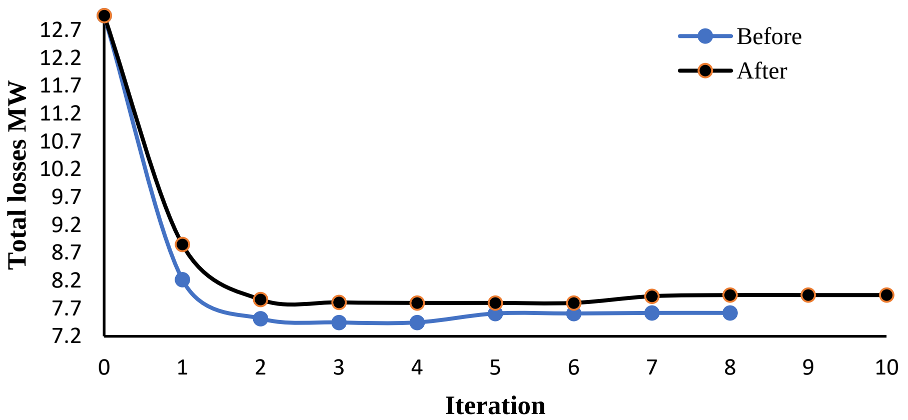


Figure (5.10): Loss reduction MW.

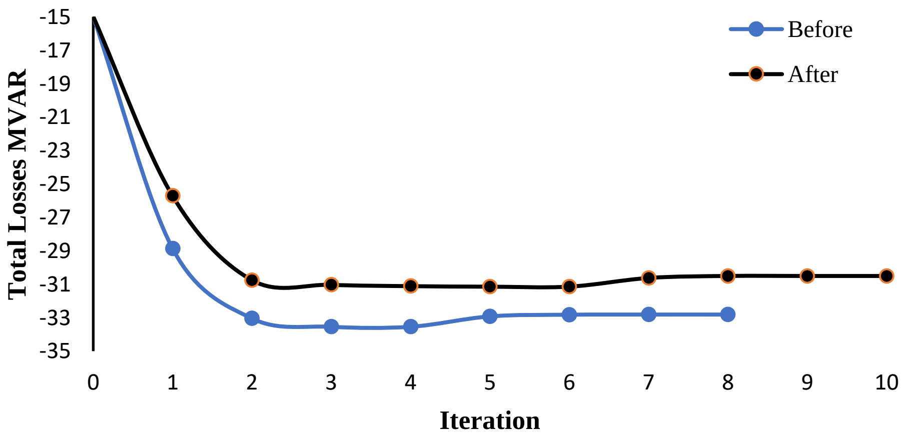


Figure (5.11): Loss reduction MVAR.

Table (5.46): Voltage profile Before and after.

	Voltage profile	
Bus No.	Before	After
1	1.07	1.07
2	1.05732	1.0433
3	1.05982	1.0441
4	1.02962	1.01973
5	1.02867	1.0171
6	1.03377	1.01883

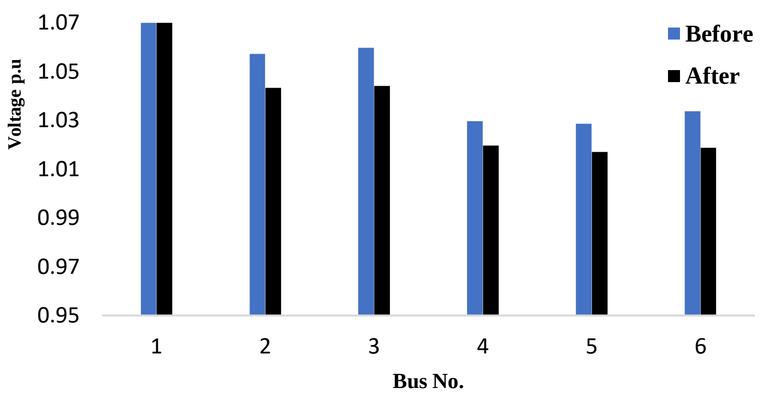


Figure (5.12): Voltage profile before and after.

In this case, the effect of including the VAR cost function in the optimization process is unfavorable where optimization results before the inclusion was better and therefore in this case, the VAR function must not be added to the objective function while it can be used for pricing purposes only. Note that the addition of the VAR function can improve the optimization process in other systems.

5.6 Calculation of The Locational Marginal Prices:

Using equation (4.37) and equation (4.38):

$$\text{LMP}=\text{LMP}_{\text{ref.}}-\frac{\partial \text{P}_{\text{loss}}}{\partial \text{P}_i}\text{LMP}_{\text{ref.}}-\sum_{\ell=1}^{Nll}\mu_\ell a_{\ell i}$$

$$\mu_\ell=\frac{\partial F_{ref}(P_{ref.})}{\partial P_{ref.}}\left(1-\frac{\partial \text{P}_{\text{loss}}}{\partial \text{P}_i}\right)\left(\frac{1}{a_{\ell i}}\right)-\frac{\partial f_i(P_i)}{\partial P_i}\left(\frac{1}{a_{\ell i}}\right)$$

$$\text{LMP}_{\text{ref.}}=\frac{\partial F_1(P_1)}{\partial P_1}=11.669+0.01066\times 110.01=12.842\ \$/\text{MWH}$$

$\text{LMP}_2=12.57\$/\text{MWH}$, $\text{LMP}_3=11.897\$/\text{MWH}$.

Table (5.47): The LMP calculation.

Bus No.	LMP $_{ref.}$	$\frac{\partial P_{loss}}{\partial P_i}$	$1-\frac{\partial P_{loss}}{\partial P_i}$	$a_{2\text{-}4}$	$\mu_{2\text{-}4}$	$a_{3\text{-}6}$	$\mu_{3\text{-}6}$		LMP \$/MWH
1	12.841	0	0	0	0	0	0	0	12.841
2	12.841	-0.0164	1.0164	0.3215	1.499	-0.007	3.772	0.4555	12.596
3	12.841	-0.0357	1.0357	0.2324	1.499	0.3720	3.772	1.7519	11.548
4	12.841	-0.0634	1.0634	-0.382	1.499	-0.002	3.772	-0.583	14.239
5	12.841	-0.0782	1.0782	0.1181	1.499	0.0125	3.772	0.2243	13.621
6	12.841	-0.0674	1.0674	0.2399	1.499	-0.357	3.772	-0.987	14.694

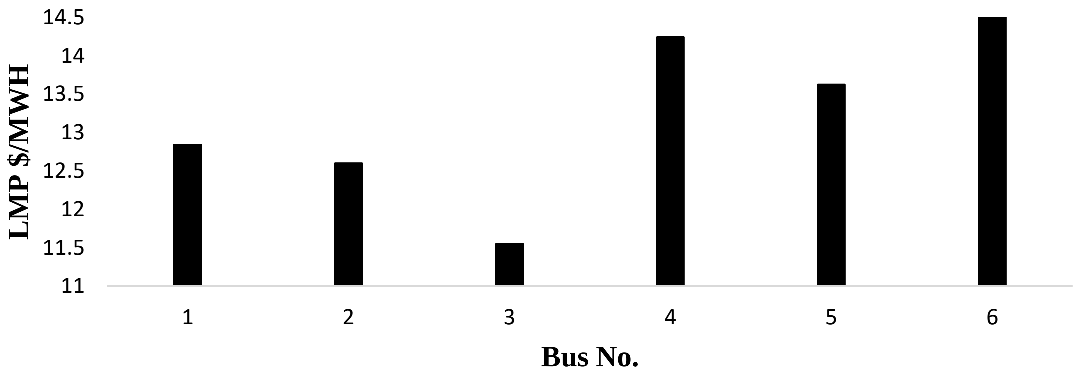


Figure (5.13): The LMP values at each bus.

Based on the basic definition of the LMP, where in case of adding a 1 MW of load to the system for example at bus-3, the marginal increase of the total operating cost is approximately 11.55 dollars and for bus-6 is approximately 14.7 dollars which is the most expensive bus. Therefore, LMP calculation is very important for planning, operation and future studies of electrical power systems.

5.7 Implementation on the IEEE 30-bus System:

5.7.1 Initial power flow results:

Table (5.48): Initial PF results for the IEEE 30-bus system.

Bus No.	Generation MW	Generation MVAR	Generation Min MW	Generation Max MW	Generation Initial Cost
1	260.95	-16.53	50	200	614.37
2	40	49.56	20	80	252
3	0	36.94	15	50	0
4	0	37.22	10	35	0
5	0	16.18	10	30	0
6	0	10.63	12	40	0
Total Generation	300.95	134			
Total Load	283.4	126.2			
Total Losses	17.55	7.8			
Total Cost	875.256 $/hr.	591.8 $/hr.			

Table (5.49): Voltage magnitudes and phase angles.

Bus No.	Min	BUS PU Volt	Max.	Angle (Radians)
1	0.9	1.06	1.1	0
2	0.9	1.043	1.1	-0.09
3	0.9	1.02071	1.1	-0.13
4	0.9	1.01173	1.1	-0.16
5	0.9	1.01	1.1	-0.25
6	0.9	1.01023	1.1	-0.19
7	0.9	1.00236	1.1	-0.22
8	0.9	1.01	1.1	-0.21
9	0.9	1.0509	1.1	-0.25
10	0.9	1.04511	1.1	-0.27
11	0.9	1.082	1.1	-0.25
12	0.9	1.0571	1.1	-0.26
13	0.9	1.071	1.1	-0.26
14	0.9	1.04226	1.1	-0.28
15	0.9	1.03767	1.1	-0.28
16	0.9	1.04437	1.1	-0.27
17	0.9	1.03988	1.1	-0.28
18	0.9	1.02814	1.1	-0.29
19	0.9	1.02563	1.1	-0.29
20	0.9	1.02972	1.1	-0.29
21	0.9	1.03271	1.1	-0.28
22	0.9	1.03324	1.1	-0.28
23	0.9	1.02716	1.1	-0.28
24	0.9	1.02156	1.1	-0.29
25	0.9	1.01732	1.1	-0.28
26	0.9	0.99964	1.1	-0.29
27	0.9	1.02323	1.1	-0.27
28	0.9	1.0068	1.1	-0.2
29	0.9	1.00339	1.1	-0.29
30	0.9	0.99191	1.1	-0.31

Table (5.50): Line flows and losses.

From bus	To bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
1	**2**	**173.2**	**-21.09**	**174.5**	**130**	**5.18**	**9.69**
1	3	87.7	4.57	87.85	130	3.11	6.97
2	4	43.6	3.9	43.79	65	1.02	-0.79
2	5	82.4	1.75	82.4	130	2.95	8
2	6	60.3	0.44	60.34	65	1.95	1.97
3	4	82.2	-3.6	82.29	130	0.86	1.59
4	6	72.2	-16.35	73.98	90	0.63	1.29
4	12	44.2	14.24	46.44	65	0	4.69
5	7	-14.8	11.69	18.83	70	0.17	-1.63
6	7	38.1	-2.97	38.23	130	0.38	-0.55
6	8	29.6	-8.14	30.67	32	0.11	-0.53
6	9	27.7	-8.17	28.9	65	0	1.63
6	10	15.8	0.16	15.84	32	0	1.28
6	28	18.7	-0.04	18.67	32	0.06	-1.12
8	28	-0.5	-0.39	0.67	32	0	-4.35
9	10	27.7	5.91	28.34	65	0	0.8
9	11	0	-15.71	15.71	65	0	0.47
10	17	5.3	4.42	6.92	32	0.01	0.04
10	20	9	3.71	9.75	32	0.08	0.18
10	21	15.8	10.01	18.69	32	0.11	0.24
10	22	7.6	4.6	8.9	32	0.05	0.11
12	13	0	-10.49	10.49	65	0	0.14
12	14	7.9	2.4	8.22	32	0.07	0.15
12	15	17.9	6.8	19.14	32	0.22	0.43
12	16	7.2	3.35	7.99	16	0.05	0.11
14	15	1.6	0.65	1.71	16	0.01	0.01
15	18	6	1.6	6.23	16	0.04	0.08
15	23	5	2.91	5.82	16	0.03	0.06
16	17	3.7	1.44	3.97	16	0.01	0.03
18	19	2.8	0.62	2.85	16	0	0.01
19	20	-6.7	-2.79	7.28	16	0.02	0.03
21	22	-1.8	-1.43	2.32	32	0	0
22	24	5.7	3.06	6.51	16	0.05	0.07
23	24	1.8	1.25	2.2	16	0.01	0.01
24	25	-1.2	2.02	2.35	16	0.01	0.02
25	26	3.5	2.37	4.26	16	0.04	0.07
25	27	-4.8	-0.37	4.77	16	0.02	0.05
28	27	18.1	5.03	18.75	65	0	1.29
27	29	6.2	1.67	6.41	16	0.09	0.16
27	30	7.1	1.66	7.28	16	0.16	0.31
29	30	3.7	0.61	3.75	16	0.03	0.06

5.7.2 Piecewise LPOPF results:

By following the same algorithm that used in solving the 6-bus system, the IEEE 30-bus system is solved, the I/O curve is fitted into 6 straight line segments through 7 break-points:

Table (5.51): BPs of all units.

Unit No	Min MW	Max MW	BP_1	BP_2	BP_3	BP_4	BP_5	BP_6	BP_7
1	50	200	50	75	100	125	150	180	200
2	20	80	20	30	40	50	60	73	80
3	15	50	15	20.8	25.8	31.6	37.5	44.5	50
4	10	35	10	14.2	18.36	22.6	26.76	31	35
5	10	30	10	13.3	16.6	20	23.3	27.6	30
6	12	40	12	16.7	21.4	26.1	30.8	36	40

Table (5.52): Segment slopes for all units.

Uni1 1			Unit 2			Unit 3		
Pi	F(Pi)	Slope Si	Pi	F(Pi)	Slope	Pi	F(Pi)	Slope
50	109.25	2.4625	20	42	2.625	15	29.0625	3.2375
75	170.8125	2.6475	30	68.25	2.975	20.8	47.84	3.9125
100	237	2.8325	40	98	3.325	25.8	67.4025	4.5875
125	307.8125	3.0175	50	131.25	3.675	31.6	94.01	5.31875
150	383.25	3.221	60	168	4.0775	37.5	125.390625	6.125
180	479.88	3.406	73	221.0075	4.4275	44.5	168.265625	6.90625
200	548		80	252		50	206.25	
Unit 4			Unit 5			Unit 6		
Pi	F(Pi)	Slope	Pi	F(Pi)	Slope	Pi	F(Pi)	Slope
10	33.33	3.45086	10	32.5	3.5825	12	39.6	3.7175
14.2	47.823612	3.520248	13.3	44.32225	3.7475	16.7	57.07225	3.9525
18.36	62.4678437	3.589968	16.6	56.689	3.915	21.4	75.649	4.1875
22.6	77.689308	3.659688	20	70	4.0825	26.1	95.33025	4.4225
26.76	92.9136101	3.729408	23.3	83.47225	4.2725	30.8	116.116	4.67
31	108.7263	3.7978	27.6	101.844	4.44	36	140.4	4.9
35	123.9175		30	112.5		40	160	

Table (5.53): Segment limits for all units.

P11	≤	25	P21	≤	10	P31	≤	5.8
P12	≤	25	P22	≤	10	P32	≤	5
P13	≤	25	P23	≤	10	P33	≤	5.8
P14	≤	25	P24	≤	10	P34	≤	5.9
P15	≤	30	P25	≤	13	P35	≤	7
P16	≤	20	P26	≤	7	P36	≤	5.5
P41	≤	4.2	P51	≤	3.3	P61	≤	4.7
P42	≤	4.16	P52	≤	3.3	P62	≤	4.7
P43	≤	4.24	P53	≤	3.4	P63	≤	4.7
P44	≤	4.16	P54	≤	3.3	P64	≤	4.7
P45	≤	4.24	P55	≤	4.3	P65	≤	5.2
P46	≤	4	P56	≤	2.4	P66	≤	4

After performing PW LPOPF, the optimal solution results are:

Table (5.54): PW LPOPF results by incorporating *POWERWORLD* Simulator and Microsoft Excel.

Bus No.	Generation MW	Generation MVAR	Generation Min MW	Generation Max MW
1	191.4	-3.48	50	200
2	50	32.69	20	80
3	20.8	27.51	15	50
4	10	27.7	10	35
5	10	15.4	10	30
6	12	8.82	12	40
Total Generation	294.2	108.64		
Total Load	283.4	126.2		
Total Losses	10.8	-17.56		
Real & Reactive power Costs	802.866 $/hr.	304.484 $/hr.		
Total Cost	1107.35 $/hr.			

Table (5.55): Line flows and losses.

From bus	To bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
1	2	124	-9	125	130	2.65	2.1
1	3	67	5.51	67.3	130	1.83	2.26
2	4	35.8	3.7	36	65	0.69	-1.8
2	5	65.5	3.38	65.6	130	1.88	3.48
2	6	48.7	1.82	48.7	65	1.27	-0.1
3	4	62.8	2.05	62.8	130	0.5	0.55
4	6	56.1	-8.7	56.7	90	0.37	0.36
4	12	33.8	14	36.6	65	0	2.88
5	7	-9.8	8.42	12.9	70	0.08	-1.9
6	7	32.9	-0.2	32.9	130	0.28	-0.9
6	8	20.7	-0.1	20.7	32	0.05	-0.8
6	9	19.8	-8.2	21.4	65	0	0.89
6	10	13.3	0.05	13.3	32	0	0.9
6	28	16.5	1.41	16.5	32	0.05	-1.2
8	28	0.6	-1.7	1.78	32	0	-4.4
9	10	29.8	5.7	30.3	65	0	0.91
9	11	-10	-15	17.9	65	0	0.6
10	17	4.7	4.55	6.54	32	0.01	0.03
10	20	8.7	3.79	9.51	32	0.08	0.17
10	21	16.1	9.9	18.9	32	0.11	0.24
10	22	7.8	4.53	9.01	32	0.05	0.11
12	13	-12	-8.6	14.7	65	0	0.27
12	14	8	2.33	8.37	32	0.08	0.16
12	15	18.6	6.63	19.8	32	0.23	0.45
12	16	7.9	3.25	8.53	16	0.06	0.13

Table (5.55): Line flows and losses (continued).

From bus	To bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
14	15	1.8	0.57	1.85	16	0.01	0.01
15	18	6.3	1.51	6.49	16	0.04	0.09
15	23	5.7	2.72	6.28	16	0.04	0.07
16	17	4.3	1.32	4.52	16	0.01	0.04
18	19	3.1	0.53	3.12	16	0.01	0.01
19	20	-6.4	-2.9	7.05	16	0.02	0.03
21	22	-1.6	-1.5	2.19	32	0	0
22	24	6.2	2.87	6.81	16	0.05	0.08
23	24	2.4	1.05	2.64	16	0.01	0.02
24	25	-0.2	1.63	1.64	16	0	0.01
25	26	3.5	2.37	4.26	16	0.04	0.07
25	27	-3.7	-0.8	3.78	16	0.02	0.03
28	27	17	5.26	17.8	65	0	1.16
27	29	6.2	1.67	6.41	16	0.09	0.16
27	30	7.1	1.66	7.28	16	0.16	0.3
29	30	3.7	0.61	3.75	16	0.03	0.06

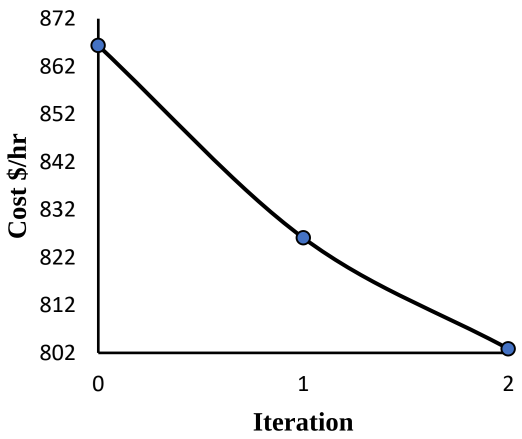


Figure (5.14): Reduction of total cost.

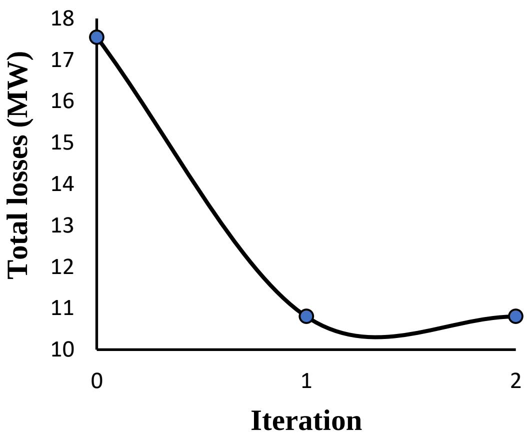


Figure (5.15): Reduction of total losses.

5.7.3 Locational marginal prices:

In this case and due to line congestion (no overloaded line being forced at their limit), equation (4.37) is used in order to calculate the LMPs of the system:

Table (5.56): LMP calculation.

Bus No.	LMP Ref.	$\frac{\partial P_{loss}}{\partial P_i}$	LMP $/MWH
1	3.4355	0	3.43550
2	3.4355	-0.022	3.51108
3	3.4355	-0.0496	3.60590

Table (5.56): LMP calculation (continued).

Bus No.	LMP Ref.	$\frac{\partial P_{loss}}{\partial P_i}$	LMP $/MWH
4	3.4355	-0.0639	3.65503
5	3.4355	-0.0942	3.75912
6	3.4355	-0.0763	3.69763
7	3.4355	-0.0909	3.74779
8	3.4355	-0.0808	3.71309
9	3.4355	-0.0769	3.69969
10	3.4355	-0.0773	3.70106
11	3.4355	-0.0769	3.69969
12	3.4355	-0.0627	3.65091
13	3.4355	-0.0626	3.65056
14	3.4355	-0.0802	3.71103
15	3.4355	-0.086	3.73095
16	3.4355	-0.0764	3.69797
17	3.4355	-0.0806	3.71240
18	3.4355	-0.0981	3.77252
19	3.4355	-0.1011	3.78283
20	3.4355	-0.0958	3.76462
21	3.4355	-0.0886	3.73989
22	3.4355	-0.0882	3.73851
23	3.4355	-0.0967	3.76771
24	3.4355	-0.1019	3.78558
25	3.4355	-0.0968	3.76806
26	3.4355	-0.1171	3.83780
27	3.4355	-0.0853	3.72855
28	3.4355	-0.0827	3.71962
29	3.4355	-0.1155	3.83230
30	3.4355	-0.1365	3.90445

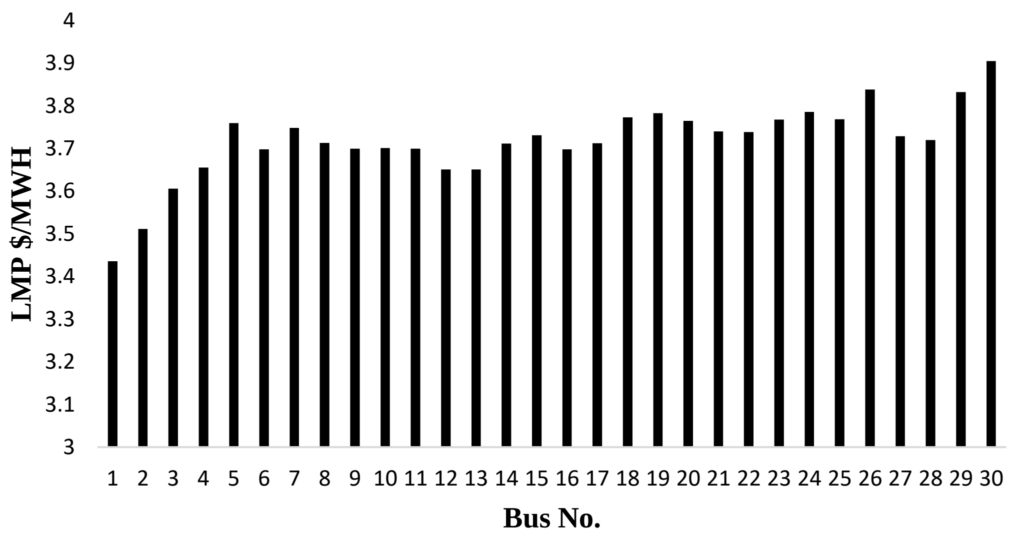


Figure (5.16): LMP values at each bus.

5.7.4 Incremental LPOPF results:

- Before the addition of the VAR cost function:

Table (5.57): Incremental LPOPF results before adding the VAR cost function by incorporating *POWERWORLD* Simulator and Microsoft Excel.

Bus No.	Generation MW	Generation MVAR	Generation Min MW	Generation Max MW
1	147.78	7.8	50	200
2	80	-3.83	20	80
3	24.86	30.21	15	50
4	13.82	38.97	10	35
5	10.27	16.03	10	30
6	15.26	10.83	12	40
Total Generation	291.99	100.01		
Total Load	283.4	126.2		
Total Losses	8.59	-26.19		
Real power cost	824.497 $/hr.			
Reactive power cost	355.951 $/hr.			
Total operating cost	1180.448 $/hr.			

Table (5.58): Line flows and losses.

From bus	To bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
1	2	89.8	4.91	89.91	130	1.39	-1.67
1	3	58	2.9	58.08	130	1.36	0.53
2	4	36.2	-2.65	36.33	65	0.69	-1.81
2	5	62.8	-1.85	62.86	130	1.72	2.8
2	6	47.6	-5.46	47.94	65	1.22	-0.27
3	4	54.2	1.18	54.25	130	0.37	0.16
4	6	50.4	-13.18	52.07	90	0.31	0.12
4	12	31.4	11.76	33.57	65	0	2.39
5	7	-8.2	6.56	10.52	70	0.06	-1.96
6	7	31.3	1.38	31.37	130	0.25	-0.99
6	8	17.4	-9.97	20.02	32	0.04	-0.79
6	9	19.1	-9.2	21.22	65	0	0.86
6	10	12.9	-0.38	12.95	32	0	0.84
6	28	15.7	-0.32	15.72	32	0.04	-1.22
8	28	1.1	-0.22	1.15	32	0	-4.47
9	10	29.4	5.34	29.87	65	0	0.87
9	11	-10.3	-15.4	18.51	65	0	0.63
10	17	4.3	4.49	6.2	32	0.01	0.03
10	20	8.5	3.76	9.29	32	0.07	0.16
10	21	16	9.86	18.8	32	0.11	0.24
10	22	7.8	4.5	8.97	32	0.05	0.11
12	13	-15.3	-10.42	18.48	65	0	0.41
12	14	8.1	2.31	8.46	32	0.08	0.16
12	15	19	6.65	20.17	32	0.23	0.46

From bus	To bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
12	16	8.3	3.32	8.96	16	0.07	0.14
14	15	1.9	0.55	1.94	16	0.01	0.01
15	18	6.5	1.53	6.72	16	0.04	0.09
15	23	5.9	2.71	6.51	16	0.04	0.08
16	17	4.8	1.38	4.95	16	0.01	0.04
18	19	3.3	0.54	3.34	16	0.01	0.01
19	20	-6.2	-2.87	6.84	16	0.01	0.03
21	22	-1.6	-1.58	2.25	32	0	0
22	24	6.1	2.82	6.72	16	0.05	0.07
23	24	2.7	1.03	2.88	16	0.01	0.02
24	25	0	1.68	1.68	16	0	0.01
25	26	3.5	2.36	4.26	16	0.04	0.06
25	27	-3.5	-0.7	3.59	16	0.01	0.03
28	27	16.8	5.14	17.58	65	0	1.1
27	29	6.2	1.66	6.41	16	0.08	0.16
27	30	7.1	1.65	7.28	16	0.16	0.3
29	30	3.7	0.6	3.75	16	0.03	0.06

Table (5.58): Line flows and losses (continued).

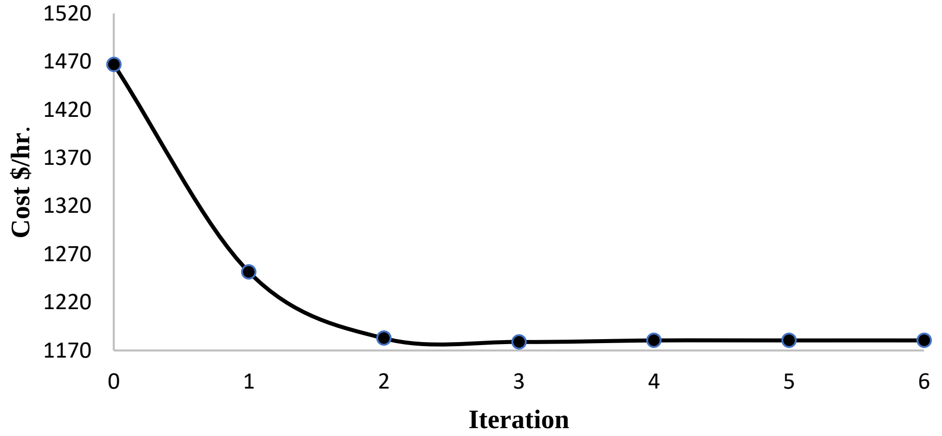
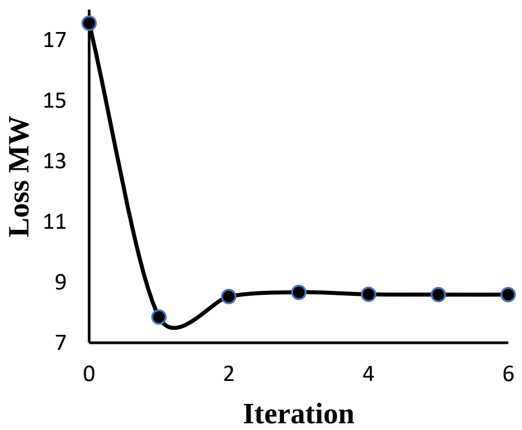


Figure (5.17): Total cost reduction.



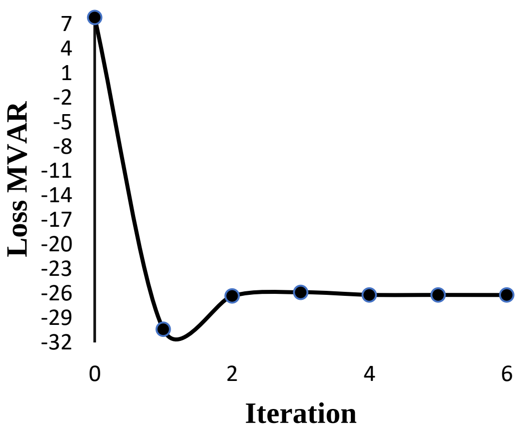


Figure (5.18): MW loss reduction. Figure (5.19): MVAR loss reduction.

Table (5.59): LMP calculation.

Bus No.	LMP Ref.	$\frac{\partial P_{loss}}{\partial P_i}$	LMP Calc.
1	3.108	0	3.108
2	3.108	-0.0238	3.211
3	3.108	-0.0438	3.280
4	3.108	-0.0564	3.329
5	3.108	-0.0806	3.429
6	3.108	-0.0683	3.371
7	3.108	-0.081	3.417
8	3.108	-0.0728	3.385
9	3.108	-0.0701	3.375
10	3.108	-0.0716	3.378
11	3.108	-0.0691	3.374
12	3.108	-0.0529	3.327
13	3.108	-0.0521	3.325
14	3.108	-0.0705	3.381
15	3.108	-0.0774	3.401
16	3.108	-0.0689	3.372
17	3.108	-0.0742	3.387
18	3.108	-0.0905	3.440
19	3.108	-0.0941	3.450
20	3.108	-0.0892	3.434
21	3.108	-0.0828	3.413
22	3.108	-0.0824	3.411
23	3.108	-0.0891	3.435
24	3.108	-0.0959	3.453
25	3.108	-0.0928	3.438
26	3.108	-0.113	3.501
27	3.108	-0.0823	3.403
28	3.108	-0.0748	3.391
29	3.108	-0.1127	3.497
30	3.108	-0.1337	3.561

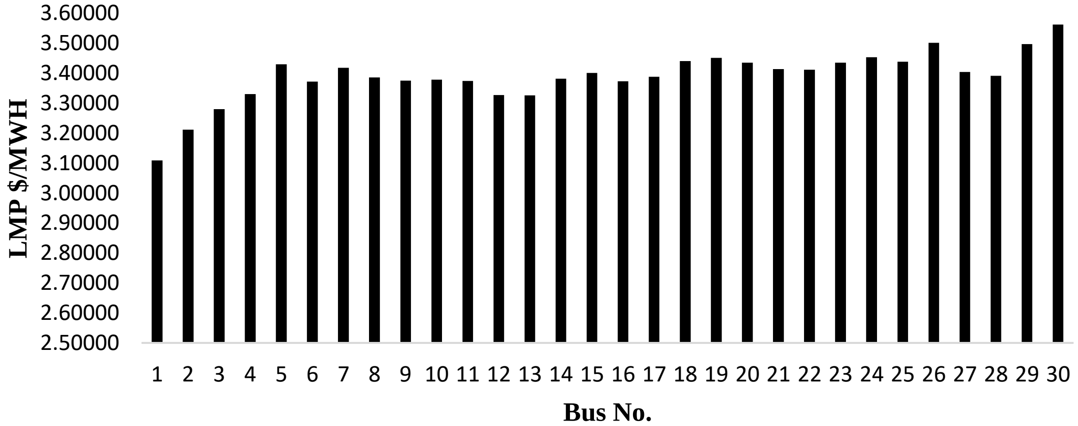


Figure (5.20): LMP values at each bus.

- After the addition of the VAR cost function:

Table (5.60): Incremental LPOPF results after adding the VAR cost function by incorporating *POWERWORLD* Simulator and Microsoft Excel.

Bus No.	Generation MW	Generation MVAR	Generation Min MW	Generation Max MW
1	149.83	11.34	50	200
2	80	3.03	20	80
3	24.67	30.28	15	50
4	15.64	26.65	10	35
5	10	18	10	30
6	12	12.32	12	40
Total Generation	292.14	101.62		
Total Load	283.4	126.2		
Total Losses	8.74	-24.58		
Real power cost	823.515 $/hr.			
Reactive power cost	246.235 $/hr.			
Total operating cost	1069.75 $/hr.			

Table (5.61): Line flows and losses.

From bus	To bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
1	2	91.1	5.9	91.31	130	1.43	-1.53
1	3	58.7	5.43	58.96	130	1.41	0.71
2	4	36.8	-0.19	36.79	65	0.72	-1.72
2	5	63.2	-0.2	63.2	130	1.75	2.93
2	6	48	-1.85	48.03	65	1.24	-0.2
3	4	54.9	3.51	55.01	130	0.38	0.21
4	6	49.5	-8.08	50.15	90	0.29	0.07
4	12	33.5	11.31	35.36	65	0	2.68
5	7	-8.1	8.16	11.48	70	0.07	-1.92
6	7	31.2	-0.14	31.19	130	0.25	-0.97
6	8	15.8	0.58	15.77	32	0.03	-0.83
6	9	20	-10.56	22.62	65	0	0.99
6	10	13.4	-0.83	13.4	32	0	0.91
6	28	15.6	1.15	15.68	32	0.04	-1.19
8	28	1.4	-1.94	2.37	32	0	-4.39
9	10	30	5.72	30.54	65	0	0.91
9	11	-10	-17.26	19.95	65	0	0.74
10	17	4.9	4.15	6.39	32	0.01	0.03
10	20	8.8	3.58	9.49	32	0.08	0.17
10	21	16.1	9.94	18.92	32	0.11	0.24
10	22	7.8	4.55	9.05	32	0.05	0.11
12	13	-12	-11.97	16.95	65	0	0.35
12	14	8	2.43	8.39	32	0.08	0.16
12	15	18.5	7.03	19.83	32	0.23	0.45
12	16	7.7	3.64	8.54	16	0.06	0.13
14	15	1.8	0.67	1.88	16	0.01	0.01

Table (5.61): Line flows and losses (continued).

From bus	To bus	MW	MVAR	MVA	Lim MW	MW Loss	MVAR Loss
15	18	6.2	1.72	6.48	16	0.04	0.08
15	23	5.6	3.03	6.39	16	0.04	0.07
16	17	4.2	1.71	4.5	16	0.01	0.03
18	19	3	0.73	3.09	16	0.01	0.01
19	20	-6.5	-2.68	7.03	16	0.02	0.03
21	22	-1.5	-1.5	2.13	32	0	0
22	24	6.3	2.94	6.92	16	0.05	0.08
23	24	2.4	1.36	2.74	16	0.01	0.02
24	25	-0.1	2.07	2.07	16	0.01	0.01
25	26	3.5	2.37	4.26	16	0.04	0.07
25	27	-3.7	-0.31	3.68	16	0.01	0.03
28	27	17	4.78	17.62	65	0	1.12
27	29	6.2	1.67	6.41	16	0.09	0.16
27	30	7.1	1.66	7.28	16	0.16	0.3
29	30	3.7	0.6	3.75	16	0.03	0.06

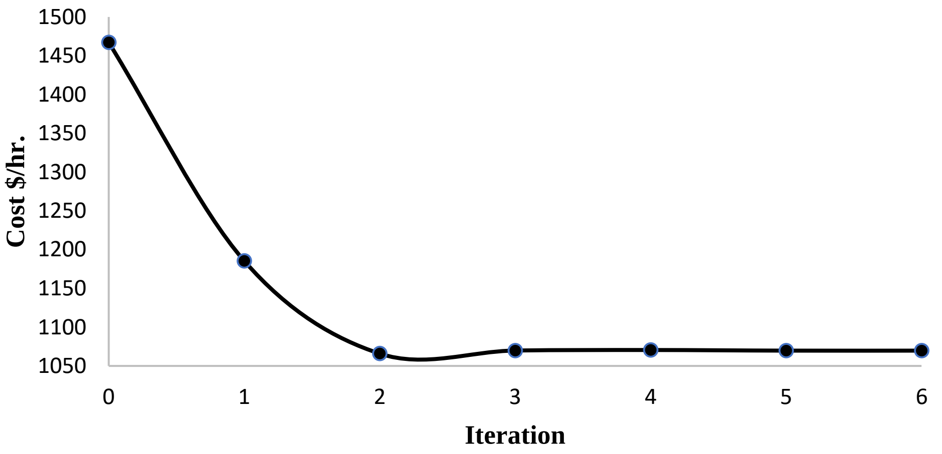
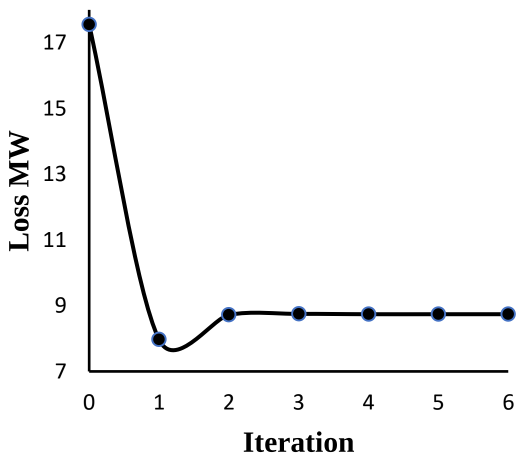


Figure (5.21): Total cost reduction.



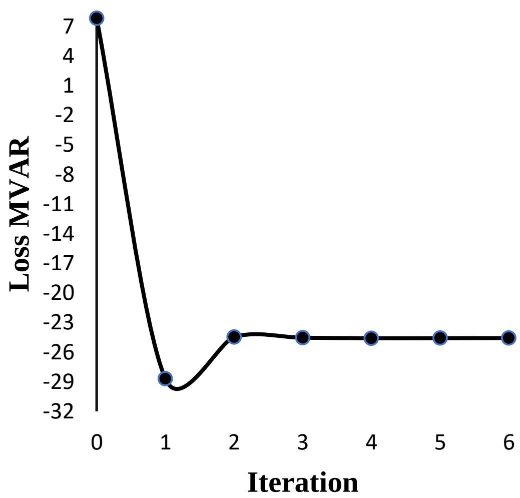


Figure (5.22): MW loss reduction. Figure (5.23): MVAR loss reduction.

Table (5.62): LMP calculation

Bus No.	LMP Ref.	$\frac{\partial P_{loss}}{\partial P_i}$	LMP Calc.
1	3.124	0	3.124
2	3.124	-0.025	3.229
3	3.124	-0.047	3.300
4	3.124	-0.06	3.351
5	3.124	-0.083	3.452
6	3.124	-0.071	3.393
7	3.124	-0.083	3.441
8	3.124	-0.072	3.407
9	3.124	-0.073	3.398
10	3.124	-0.074	3.401
11	3.124	-0.072	3.396
12	3.124	-0.061	3.354
13	3.124	-0.06	3.352
14	3.124	-0.078	3.410
15	3.124	-0.084	3.428
16	3.124	-0.074	3.398
17	3.124	-0.078	3.412
18	3.124	-0.096	3.467
19	3.124	-0.099	3.477
20	3.124	-0.093	3.460
21	3.124	-0.085	3.437
22	3.124	-0.085	3.436
23	3.124	-0.094	3.462
24	3.124	-0.099	3.480
25	3.124	-0.093	3.464
26	3.124	-0.113	3.528
27	3.124	-0.081	3.427
28	3.124	-0.076	3.414
29	3.124	-0.112	3.523
30	3.124	-0.133	3.590

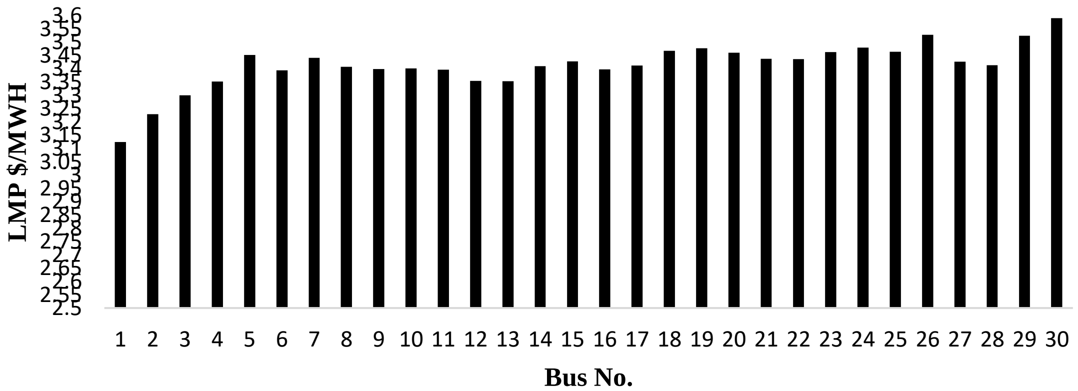


Figure (5.24): LMP values at each bus.

5.7.5 Comparison between LPOPF results before and after the inclusion of reactive power cost function graphically:

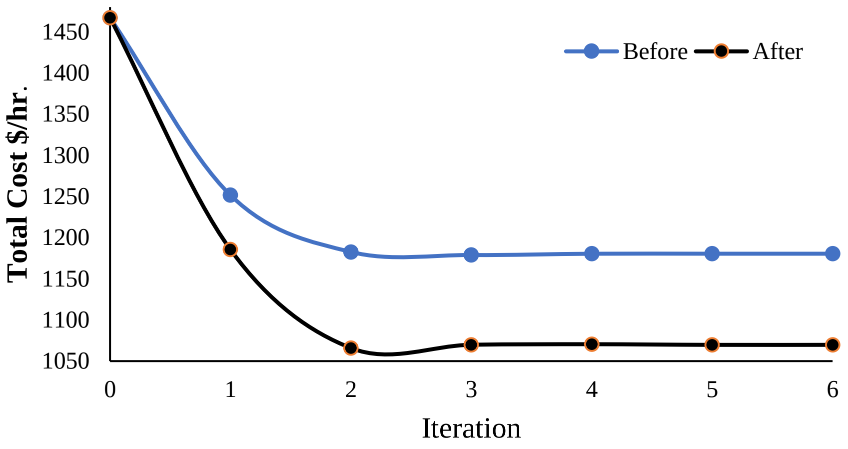


Figure (5.25): Total cost reduction.

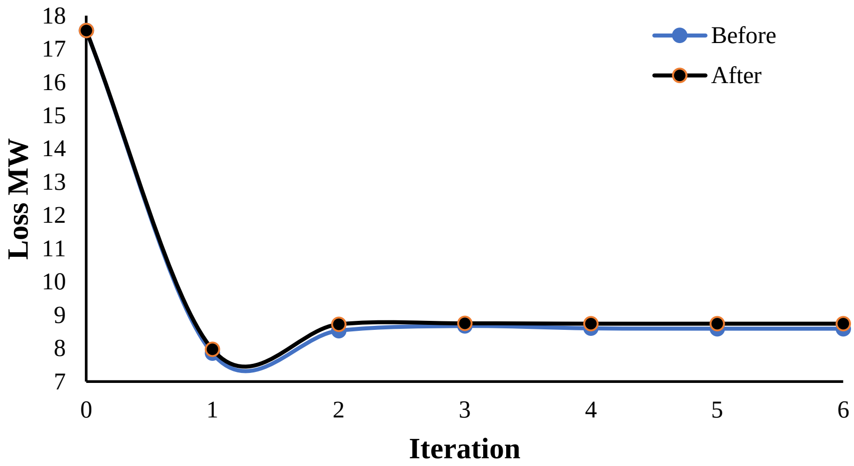


Figure (5.26): MW loss reduction.

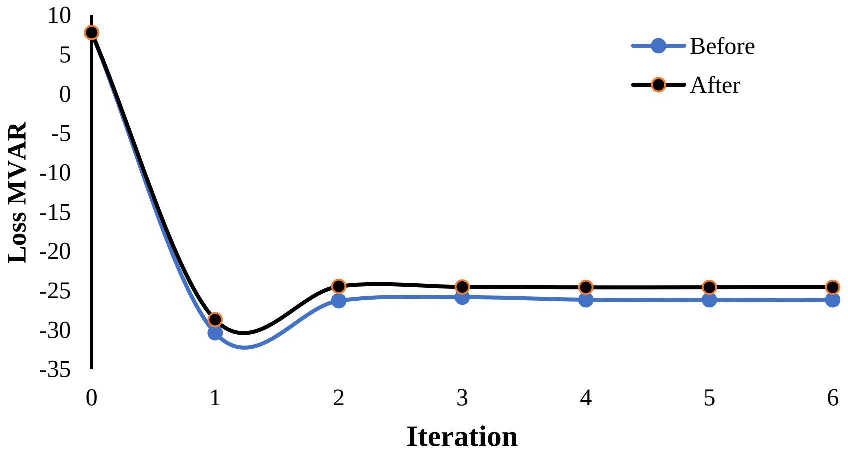


Figure (5.27): MVAR loss reduction.

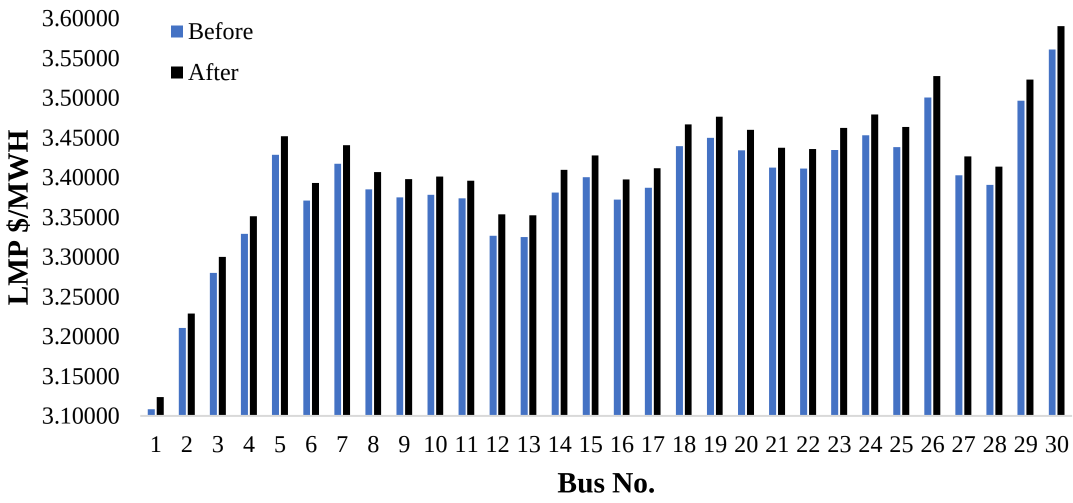


Figure (5.28): LMP values at each bus

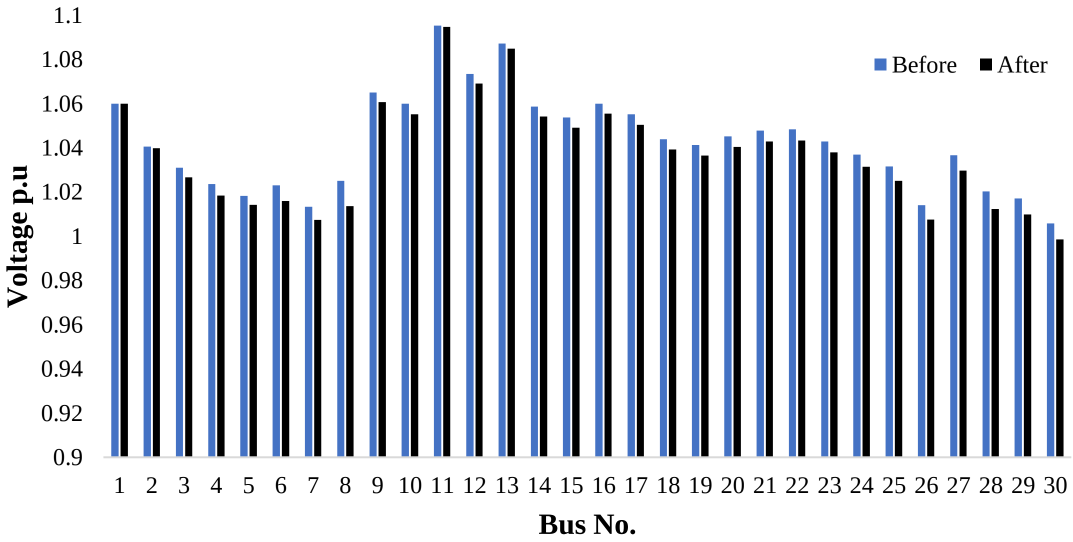


Figure (5.29): Voltage profile.

Unlike the 6-bus case, the inclusion of the VAR cost function into the objective function improved the optimization process in different aspects, as observed, total operating cost is reduced by **110.7 $/hr.** from the first optimal solution and by **397.3 $/hr.** from the base case, likewise the voltage profile has improved by a considerable amount than of the first optimal solution and thus system security is improved after including of the VAR cost function.

And as observed the LMP values are increased at each bus and hence adding a new load at any bus will be more expensive than the first optimal solution, likewise, the first optimal solution has an advantage on total real and reactive power losses.

CHAPTER SIX

CONCLUSION AND FUTURE WORK

6.1 Conclusion:

This research illustrated an important study in power system design, planning, operation and optimization, as the name implies “Linear Programming Based Optimal Power Flow”, LP optimization is used in order to solve the OPF problem, its explained in detail with illustrative examples for both graphical and simplex methods.

Two different methods based on LP are used through this research, the Piecewise Linear Approach and full Incremental LP method, PW linear approach is used in order to optimize the real power only while the full AC incremental LP method for optimizing both real and reactive powers, both methods are implemented on the 6-bus and the IEEE 30 bus test systems through incorporating *POWERWORLD* Simulator and Microsoft Excel 2016, it is observed that the PW method has a very fast rate of convergence and simple formulation compared with the full AC method, but the full AC method has an advantage of all optimization goals aspects such as total operating cost and total losses.

Reactive power pricing is found to be very important in power system operation and optimization studies, the conventional VAR cost function is used for pricing the reactive power, a proposed formulation is presented by including the VAR cost function to the objective function of the incremental LP method and through simulation, the influence of this addition is found to be unfavorably for the 6-bus system while satisfactory for the 30-bus system, but in both systems and after the inclusion, their voltage profiles are improved.

Research has shown that the way of formulating the LPOPF problem in Microsoft Excel is simple and provides accurate results with fast rate of convergence.

6.2 Future Work:

This research focused on the understanding and the mathematical formulation of LPOPF but OPF studies may go beyond than this such as:

- **Security constraint OPF (SCOPF):** this analysis for improving system security, it is OPF plus contingency analysis, by adding new contingency constraints for the worst cases to the OPF constraints in case of all system parts outages.
- **Voltage stability analysis:** voltage stability analysis can be added to this research to identify the weakest bus in the system in order to find the optimal placement for the shunt compensation device.
- **Series and parallel compensation:** as observed in the 6-bus system, two lines are run at their limits (at critical values) and this is very dangerous for system security, therefore series compensation must be applied in order to improve system security, likewise, in the IEEE 30 bus system, bus 11 and bus 13 are running near to their limits and therefore a shunt compensation must be applied in order to improve system security.

REFERENCES

[1] J. Carpentier, "Optimal Power Flows," *International Journal of Electrical Power & Energy Systems,* vol. I, no. 1, pp. 3-15, 1979.

[2] M. E. EL-Hawary and G. S. Christensen, Optimal Economic Operation of Electric Power Systems, 1st ed., Academic Press Inc., 1979.

[3] A. J. Wood, B. F. Wollenberg and G. B. Sheblé, Power Generation, Operation and Control, 3rd ed., John Wiley & Sons, Inc., 2013.

[4] H. Sadaat, Power System Analysis, 2nd ed., The McGraw-Hill Companies.Inc, 1999.

[5] A. J. Wood and B. F. Wollenberg, Power Generation, Operation and Control, 2nd ed., John Wiley & Sons, Inc., 1996.

[6] H. H. Happ, "Optimal Power Dispatch - A Comprehensive Survey," *IEEE Transactions on Power Apparatus and Systems,* vol. 96, no. 3, pp. 841-854, 1977.

[7] J. Frame, "Locational marginal pricing," *Power Engineering Society Winter Meeting, 2001. IEEE,* pp. 377-382, 2001.

[8] R.V.Amarnath And N. V. Ramana, "State of Art in Optimal Power Flow Solution Methodologies," *Journal of Theoretical and Applied Information Technology,* vol. 30, no. 2, pp. 128-154, 2005.

[9] M. S. Bazaraa, J. J. Jarvis and H. D.Sherali, Linear Programming and Network Flows, 4th ed., John Wiley & Sons, Inc, 2010.

[10] N. S. Rau, Optimization Principles: Practical Applicationsto the Operation and Markets of of the Electric Power Industry, 1st ed., John Wiley & Sons, Inc., 2003.

[11] J.Emmanuel,"Youtube,"2015.[Online].Available: https://www.youtube.com/watch?v=pQp4GZfoxTo. [Accessed 25 January 2017].

[12] S.Dahman, "*POWEWORLD*®,"2014.[Online]. Available: https://www.powerworld.com/training/online-training/market modeling/linear-programming. [Accessed 16 January 2017].

[13] B. F. Wollenberg and M. Giacomoni, "Linear Programming Optimal Power Flow Utilizing a Trust Region Method," *IEEE Transaction in Power Systems-North American Power Symposium (NAPS),* 2010.

[14] O. Alsac, J. Bright and B. S. M. Prais, "Further Developments in LP-Based Optimal Power Flow," *IEEE Transactions on Power Systems,* vol. VI, no. 3, pp. 697-711, 1990.

[15] B.Stott and J.L.Marinho, "Linear Programming For Power-System Network Security Applications," *IEEE Transactions on Power Apparatus and Systems,* Vols. PAS-98, no. 3, pp. 837-848, 1979.

[16] J. W. Lamont and J. Fu, "Cost Analysis of Reactive Power Support," *IEEE Transactions on Power Systems,* vol. 14, no. 3, pp. 890-898, 1999.

[17] J. H. Chow, F. F. Wu and J. A. Momoh, Applied Mathematics For Restructured Electric Power Systems: Optimization, Control, and Computational Intelligence, Springer Science+Business Media, Inc., 2005.

[18] S. Hao and A. Papalexopoulos, "Reactive Power Pricing and Management," *IEEE Transactions on Power Systems*, vol. 12, no. 1, pp. 95-104, 1997.

[19] S. Hasanpour, R. Ghazi and M. H. Javidi, "A new approach for cost allocation and reactive power pricing in a deregulated environment," *Springer-Verlag,* no. 91, pp. 27-34, 2008.

[20] Y.Dai, X.Liu, Y.Ni, F.Wen, Z.Han, C.Shen and F.F.Wu, "A cost allocation method for reactive power service based on power flow tracing," *Electric Power Systems Research 64,* pp. 59-65, 2002.

[21] Y. Zhao, M.R. Irving and Y. Song, "A Cost Allocation and Pricing Method for Reactive Power Service in the New Deregulated Electricity Market Environment," *IEEE/PES Transmission and Distribution Conference & Exhibition: Asia and Pacific Dalian, China*, 2005.

[22] K.Abinaya, D.Danalakshmi and S.Kannan, "Cost Analysis of Reactive Power Using Marginal Cost Theory in Electricity Markets," *International Conference on Circuit, Power and Computing Technologies [ICCPCT],* pp. 79-83, 2014.

[23] A. Kumar and R. Kumar, "Reactive Power Cost Analysis with Generators and Condensers in Deregulated Power System," *International Conference*

on Informatics, Environment, Energy and Applications, vol. 82, pp. 56-62, 2015.

[24] H. H. Happ, "Optimal Power Dispatch," *IEEE Transactions on Power Apparatus and Systems*, Vols. PAS-93, no. 3, pp. 820 - 830, 1974.

[25] R. Belmans, G. Deconinck and Z. Qiu, "A literature survey of Optimal Power Flow problems in the electricity market context," *Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES*, 2009.

APPENDICES

APPENDIX A

LINEAR PROGRAMMING USING MICROSOFT EXCEL SOLVER

Starting with the Objective Function:

$$\text{Minimize: } Z = -3x_1 - 5x_2$$

$$\text{Subjected to: } x_1 \leq 4$$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

We do need to find the optimum values for x_1 and x_2 that satisfies all constraints and to minimize the objective function.

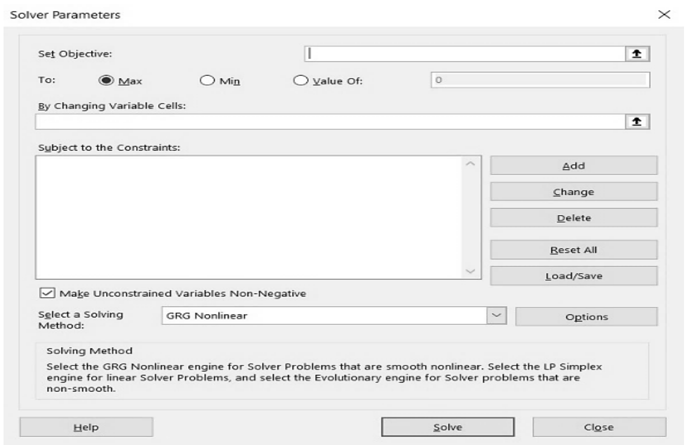
Step 1: Formulating this problem into Excel as:

Optimum Solution			**Objective Function**			**Constraints**		
$\boldsymbol{x_1}$	**0**		$\boldsymbol{3x_1 + 5x_2 =}$	0		$x_1 \leq$	4	
$\boldsymbol{x_2}$	**0**					$x_2 \leq$	6	
$\boldsymbol{3x_1 + 2x_2}$	**0**					$3x_1 + 2x_2 \leq$	18	
						$x_1, x_2 \geq$	0	

The unknown values are in column **B**, therefore the objective function in Excel can be written as **3*B2+5*B3** and as in the above figure it is equal to zero because we didn't ran the solver yet.

Step 2: Using the LP solver:

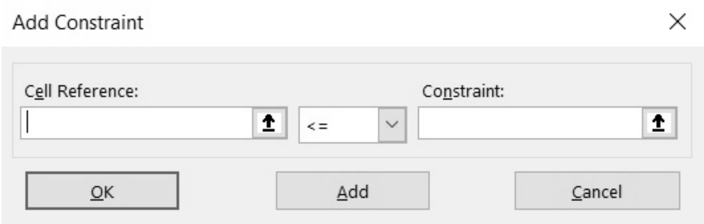
Open Data ribbon and select analysis/solver, if the solver did not appear in the analysis tap go to file/options/add-ins, select Excel Add-ins/solver add-in and press OK and the solver appears as:



1. Set the objective function in **set objective** by highlight the objective cell and choose which you want to maximize or minimize.

2. In **by Changing the Variable Cells**, you will highlight the unknown victor where it is in column **B** in our example, therefore highlighting **B2** and **B3.**

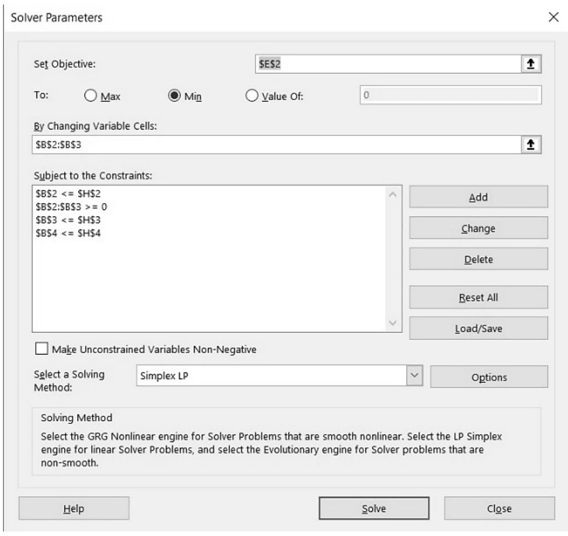
3. In **subject to the constraints** select Add and a small window will appear:



Set all variables constraints and press OK.

4. In **select a solving method** select Simplex LP.

Finally, the Solver will appear as:



After pressing **solve** the optimal solution for minimizing the OF will be introduced by filling all the empty cells:

Optimum Solution			**Objective Function**			**Constraints**		
x_1	2		$3x_1 + 5x_2 =$	-36		$x_1 \leq$	4	
x_2	6					$x_2 \leq$	6	
$3x_1 + 2x_2$	18					$3x_1 + 2x_2 \leq$	18	
						$x_1, x_2 \geq$	0	

∴ The optimal solution when $x_1 = 2$ and $x_2 = 6$.

APPENDIX B

POWER FLOW AND ECONOMIC INFORMATION FOR THE 6-BUS SYSTEM

Bus Data:

Bus type	Nom. kV	Radians	Min.	Max.	Load MW	Load MVAR	G Shunt MW	B Shunt MVAR
Slack	230	0	0.95	1.07	0	0	0	0
P-V	230	0	0.95	1.07	0	0	0	0
P-V	230	0	0.95	1.07	0	0	0	0
P-Q	230	0	0.95	1.07	100	15	0	0
P-Q	230	0	0.95	1.07	100	15	0	0
P-Q	230	0	0.95	1.07	100	15	0	0

Generator Data:

Bus No.	Gen MW	Gen MVAR	Set Volt	Min MW	Max MW	Min MVAR	Max MVAR
1	0	0	1.07	50	200	-100	150
2	50	0	1.05	37.5	150	-100	150
3	50	0	1.05	45	180	-100	120

Branch Data:

From Bus	To Bus	*R*	*X*	*B*	Lim MW
1	2	0.1	0.2	0.04	100
1	4	0.05	0.2	0.04	100
1	5	0.08	0.3	0.06	100
2	3	0.05	0.25	0.06	60
2	4	0.05	0.1	0.02	60
2	5	0.1	0.3	0.04	60
2	6	0.07	0.2	0.05	60
3	5	0.12	0.26	0.05	60
3	6	0.02	0.1	0.02	60
4	5	0.2	0.4	0.08	60
5	6	0.1	0.3	0.06	60

Economic Data:

Gen. No.	*a*	*b*	*c*
1	213.1	11.669	0.00533
2	200	10.333	0.00889
3	240	10.833	0.00741

APPENDIX C

POWER FLOW AND ECONOMIC INFORMATION FOR THE IEEE 30-BUS SYSTEM

Bus Data:

Bus type`	Nom kV	Angle rad.	Load MW	Load MVAR	G Shunt MW	B Shunt MVAR
Slack	132	0	0	0	0	0
P-V	132	0	21.7	12.7	0	0
P-Q	132	0	2.4	1.2	0	0
P-Q	132	0	7.6	1.6	0	0
P-V	132	0	94.2	19	0	0
Bus	132	0	0	0	0	0
P-Q	132	0	22.8	10.9	0	0
P-V	132	0	30	30	0	0
Bus	1	0	0	0	0	0
P-Q	33	0	5.8	2	0	19
P-V	11	0	0	0	0	0
P-Q	33	0	11.2	7.5	0	0
Bus	11	0	0	0	0	0
P-Q	33	0	6.2	1.6	0	0
P-Q	33	0	8.2	2.5	0	0
P-Q	33	0	3.5	1.8	0	0
P-Q	33	0	9	5.8	0	0
P-Q	33	0	3.2	0.9	0	0
P-Q	33	0	9.5	3.4	0	0
P-Q	33	0	2.2	0.7	0	0
P-Q	33	0	17.5	11.2	0	0
Bus	33	0	0	0	0	0
P-Q	33	0	3.2	1.6	0	0
P-Q	33	0	8.7	6.7	0	4
Bus	33	0	0	0	0	0
P-Q	33	0	3.5	2.3	0	0
Bus	33	0	0	0	0	0
Bus	132	0	0	0	0	0
P-Q	33	0	2.4	0.9	0	0
P-Q	33	0	10.6	1.9	0	0

Branch Data:

From Bus	To Bus	Branch Type	R	X	B	Lim MW	Tap Ratio
1	2	Line	0.0192	0.0575	0.0528	130	-
1	3	Line	0.0452	0.1652	0.0408	130	-
2	4	Line	0.057	0.1737	0.0368	65	-
2	5	Line	0.0472	0.1983	0.0418	130	-
2	6	Line	0.0581	0.1763	0.0374	65	-
3	4	Line	0.0132	0.0379	0.0084	130	-
4	6	Line	0.0119	0.0414	0.009	90	-
4	12	Transformer	0	0.256	0	65	0.932
5	7	Line	0.046	0.116	0.0204	70	-
6	7	Line	0.0267	0.082	0.017	130	-
6	8	Line	0.012	0.042	0.009	32	-
6	9	Transformer	0	0.208	0	65	0.978
6	10	Transformer	0	0.556	0	32	0.969
6	28	Line	0.0169	0.0599	0.013	32	-
8	28	Line	0.0636	0.2	0.0428	32	-
9	10	Line	0	0.11	0	65	-
9	11	Line	0	0.208	0	65	-
10	17	Line	0.0324	0.0845	0	32	-
10	20	Line	0.0936	0.209	0	32	-
10	21	Line	0.0348	0.0749	0	32	-
10	22	Line	0.0727	0.1499	0	32	-
12	13	Line	0	0.14	0	65	-
12	14	Line	0.1231	0.2559	0	32	-
12	15	Line	0.0662	0.1304	0	32	-
12	16	Line	0.0945	0.1987	0	16	-
14	15	Line	0.221	0.1997	0	16	-
15	18	Line	0.1073	0.2185	0	16	-
15	23	Line	0.1	0.202	0	16	-
16	17	Line	0.0524	0.1923	0	16	-
18	19	Line	0.0639	0.1292	0	16	-
19	20	Line	0.034	0.068	0	16	-
21	22	Line	0.0116	0.0236	0	32	-
22	24	Line	0.115	0.179	0	16	-
23	24	Line	0.132	0.27	0	16	-
24	25	Line	0.1885	0.3292	0	16	-
25	26	Line	0.2544	0.38	0	16	-
25	27	Line	0.1093	0.2087	0	16	-
28	27	Transformer	0	0.396	0	65	0.968
27	29	Line	0.2198	0.4153	0	16	-
27	30	Line	0.3202	0.6027	0	16	-
29	30	Line	0.2399	0.4533	0	16	-

Shunt Capacitor Data:

Bus No.	MVAR
10	19
24	4

Generator Data:

Bus No.	Gen MW	Gen MVAR	Set Volt	Min MW	Max MW	Min MVAR	Max MVAR
1	0	-16.5266	1.06	50	200	-20	200
2	40	49.56483	1.043	20	80	-20	100
5	0	36.93597	1.01	15	50	-20	80
8	0	37.21866	1.01	10	35	-15	60
11	0	16.17982	1.082	10	30	-10	50
13	0	10.63062	1.071	12	40	-15	60

Economic Data:

Gen No.	*a*	*b*	*c*
1	0	2	0.00375
2	0	1.75	0.0175
5	0	1	0.0625
8	0	3.25	0.00834
11	0	3	0.025
13	0	3	0.025