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Interference Mitigation in Multiuser Multiple Input Multiple Output Long Term Evolution-Advanced Systems

تخفيف التداخل في الهوائيات متعددة المداخل والمخارج ومتعددة المستخدمين في
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الاستهلال

بسم الله الرحمن الرحيم

قال تعالى

قال تعالى ((اقرأ باسم ربك الذي خلق (١) خلق الانسان من علق (٢) اقرأ وربك الأكرم (٣) الذي علم بالقلم (٤) علم الانسان ما لم يعلم (٥)))

صدق الله العظيم

الايات من (١-٥) سورة العلق

DEDICATION

I dedicate this work to my family for their endless love, support and encouragement.

Thanks for always being there for us.

Acknowledgement

First, all praises and thanks for God since He has given me wisdom, health and all the necessities that I need for all these years.

I would like to express my sincere thank to my adviser Dr. Fath Elrahman Ismael Khalifa Ahmed to his excellent advise and continuous support during the work of this thesis. Without his guidance and encouragement, several breakthroughs in this thesis would be impossible to be achieved.

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I am especially indebted to my parents for their love, sacrifice and support. They are my teachers after I came to this world and have set great example for me about how to live, study and work.

Abstract

Wireless data usage is increasing at a phenomenal rate and driving the need for continued innovations in wireless data technologies to provide more capacity and higher quality of service. 3GPP LTE advanced is an evolving standard targeting 4G wireless system, LTE-Advanced introduces new functionalities such as Carrier aggregation, enhanced use of multi-antenna techniques Multiple-Input Multiple-Output (MIMO) antenna technology can have a multiplicative effect on LTE's data capacity and better spectral efficiency. The predicted enormous capacity gain of MIMO is nonetheless significantly limited by interference, it consider as a problem which will degrade the system performance arises in transmission quality and the system capacity.

This research studies the interference problem in LTE Advanced MIMO as one of the challenges facing 4G systems, Smart antenna technology offer significantly improved solution to reduce this interference level and improve system capacity. With this technology, each user's signal is transmitted and received only in the direction of that particular user. Smart antenna technology attempts to address this problem via advanced signal processing technology called beam-forming ,which uses adaptive algorithms to cancel the interference signals by increasing the gain in a chosen direction. Therefore, it would improve the system performance.

In this thesis least mean square (LMS) and recursive least square (RLS) Adaptive Beamforming algorithms methods are studied and analyzed to update weights of the smart antenna to form narrower beams towards the desired user and nulls towards interfering users, considerably improving the signal-to-interference-plus-noise ratio. The system performance of MU-MIMO case was evaluated in different scenarios, with and without beamforming techniques in terms of Bit-Error-Rate (BER) and signal-to-interference-plus-noise ratio (SINR). For the simulation purpose we use MATLAB software package. The achieved results shows that both the two algorithms offers a significantly improved solution to reduce interference levels and improve the system capacity but the RLS algorithm has faster convergence rate and much better performance than LMS algorithm.

المستخلص

يتزايد استخدام البيانات اللاسلكية بمعدل هائل مما جعل الحاجة ماسة إلى الابتكارات المتواصلة في تكنولوجيايات البيانات اللاسلكية لتوفير قدر أكبر من القدرات وجودة أعلى من الخدمة. إن الجيل الثالث لنظام التطورات المتلاحقة على المدى البعيد المتقدم عبارة عن معيار منطور يستهدف نظاما لاسلكيا من الجيل الرابع. التطور طويل الامد المتقدم يقدم وظائف جديدة مثل تجميع الناقل وتعزيز استخدام تقنيات الهوائيات متعددة الادخال والايخارج والذي يؤدي الي تحسين ساعات بيانات التطور طويل الامد وله كفاءة طيفية افضل، و لكن هذه الساعات العالية المتوقعة لنظام الهوائيات متعددة الادخال والايخارج محددة بالتداخل بشكل كبير، حيث تعتبر مشكلة حقيقية تؤدي الي تقليل اداء النظام المتمثل في جودة وساعات النقل .

هذا البحث يدرس مشكلة التداخل في نظام التطور طويل الامد المتقدم متعددة الادخال والايخارج كواحدة من التحديات التي تواجهه انظمة الجيل الرابع. تقنيات الهوائيات الزكية تقدم حل لمشكلة التداخل بصورة ملحوظة وتحسن من سعة النظام وبأستخدام هذه التقنية اشارة المستخدم سترسل وتستقبل في اتجاه ذلك المستخدم فقط , الهوائيات الزكية تقوم بحل هذه المشكلة من خلال تقنية متقدمة لمعالجة الاشارة تسمى تشكيل الشعاع والذي يستخدم الخوارزميات المتكيفة لالغاء اشارات التداخل وذلك بزيادة الكسب في الاتجاه المحدد مما يحسن اداء النظام. في هذه الاطروحة تمت دراسة وتحليل خوارزميات مربع متوسط القيمة الاقل ومربع القيمة الاقل المتكررة المتكيفة لتحديث اوزان الهوائيات الزكية لتكوين شعاع ضيق تجاه المستخدم المطلوب وعدم تكوين شعاع تجاه المستخدمين الغير مرغوب فيهم وهذه العملية تؤدي الي تحسين نسبة الاشارة الي التداخل والضجيج لحد كبير. تم تقييم اداء النظام لعدة مستخدمين متعددة المداخل والمخارج في سيناريوهات مختلفة باستخدام تقنية تشكيل الشعاع وبدونها بالنسبة لمعدل الخطأ في البتات ونسبة الاشارة الي التداخل و الضجيج, ولغرض المحاكاة تم استخدام برنامج الماتلاب . توضح النتائج ان خوارزميات مربع متوسط القيمة الاقل ومربع القيمة الاقل المتكررة كلاهما تؤديان الي تقليل مستوي التداخل و تحسين سعة النظام بصورة ملحوظة , ولكن خوارزمية مربع القيمة الاقل المتكررة لها معدل تقارب اسرع واداء افضل من خوارزمية مربع متوسط القيمة الاقل.

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List of Abbreviations

3GPP	Third Generation Partnership Project
4G	Fourth Generations
AMS/UE	Advanced Mobile Station/User Equipment
BER	Bit-Error-Rate
CMA	Constant Modulus Algorithm
CSI	Channel State Information
FNBW	First Null Beam Width
HPBW	Half Power Beam Width
IMT	International Mobile Telecommunication
LMS	Least Mean Square
LTE	Long Term Evolution
LS-CMA	Least Square Constant Modulus Algorithm
MIMO	Multiple-Input Multiple-Output
MIMO IFC	MIMO Interference Channels
MSE	Mean Square Error
MU-MIMO	Multi User MIMO
OFDM	Orthogonal Frequency Division Multiplexing
RLS	Recursive Least Square
S/N	Signal-to-Noise Power Ratio
SINR	Signal-to-Interference and Noise Ratio
SIR	Signal-to-Interference Ratio
SNR	Signal to Noise Ratio
SU-MIMO	single user MIMO
UE	User Equipment

ULA	Uniform Linear Array
WiMAX	Worldwide interoperability for Microwave Access
WLAN	Wireless Local Area Network

LIST OF SYMBOLS

T_x	Transmit antenna
R_x	Receive antenna
$y(t)$	Array output
$X(t)$	Matrix signal vector
$E(k)$	Error
$d^*(t)$	Reference signal
$S(t)$	The signal
$W(t)$	The Signal Weight
$N(t)$	The Noise Signal
$I(t)$	The Interference Signal
$d(t)$	desired signal
$u(t)$	undesired interfering signals
d	Inter-element spacing
e	Error signal
N	Number of Array Elements
R	Array Correlation Matrix
r	Array Correlation vector
m	<i>number of</i> transmit antennas
n	<i>number of</i> receive antennas
H	transmission matrix
α	forgetting factor
μ	step size parameter

Chapter One

Introduction

1.1 Preface

In recent years, an enormous increase in telecommunication traffic has been experienced by wireless communication systems, due to a significant growth in the number of users as well as to the development of new high bit rate applications. It is foreseen that in the near future this trend will be confirmed. This challenging scenario involves not only the well established market of cellular systems, but also the field of emerging wireless technologies, such as Worldwide interoperability for Microwave Access (WiMAX) for wireless metropolitan area networks, and Wireless Fidelity (Wi-Fi) for wireless local area networks, mobile ad-hoc networks , wireless mesh networks and Long Term Evolution (LTE) which is specified by Third generation partnership project (3GPP) standards structured as Releases .

Demand for higher data rate lead to the introduction of LTE-advanced to achieve this high data rate, it is essential to increase the transmission bandwidth than that supported by a single carrier[1].

Enhanced MIMO concept is another important aspect of LTE-Advanced. It is categorized into two forms based on the number of users, transmitting the stream of data to a single user is termed as single user MIMO (SU-MIMO) whereas if the transmission takes place simultaneously to multiple users are termed as multi user MIMO (MU-MIMO).

Multiple input multiple output (MIMO) techniques support multiple antennas at the transmitter and at the receiver.

The predicted enormous capacity gain of MIMO is significantly limited by interference between antenna elements which will degrade the system performance arises in transmission quality and the system capacity. The aim of MIMO is to achieve different kinds of gains namely: spatial diversity and spatial multiplexing[13]. Spatial multiplexing allows to increase the capacity by transmitting different streams of data simultaneously in parallel from different antennas. Spatial diversity can be used to increase the robustness of communication in fading channels by transmitting multiple replicas of the transmitted signal from different antennas. Thus MIMO can be used to improve the cell capacity. Furthermore beamforming can be used to shape the antenna beam in the direction of certain UEs. It allows to increase throughput and link range without additional bandwidth or transmit Power. Multiple-Input Multiple-Output (MIMO) is a key technique in any modern cellular system. Base stations and terminals are therefore equipped with multiple antenna elements intended to be used in transmission and reception to make MIMO capabilities available at both the downlink and the uplink and MIMO is a very useful tool towards increasing the spectral efficiency of the wireless transmission. Enhanced MIMO is considered as one of the main aspects of LTE-Advanced that will allow the system to meet the IMT Advanced rate requirements established by the ITU-R. Other MIMO configurations already introduced in LTE are expected to continue playing a fundamental role in LTE-Advanced.

1.2 Problem Statement

Using multiple-input multiple-output(MIMO) antennas in LTE-Advanced MU-MIMO (simultaneous data transmission) , the data will be subject to interference

and It is more when compared with SU-MIMO case , this interference reduces the channel capacity and will results in degrade the system performance.

1.3 Proposed Solution

In order to mitigate the problem of interference in MU-MIMO using beamforming, There are several beamforming techniques, in this thesis Least Mean Square(LMS) and Recursive Least Square (RLS) adaptive beamforming Algorithms is applied to the system, The performance will be analyzed through a comparison between the two algorithms.

1.4 Objectives

The objective of this thesis is to make the comparison between the least mean square (LMS) and recursive least squares (RLS) adaptive beamforming algorithms to get the effective way to:

- Reduce the interference caused by multiple antennas in transmitting and receiving site
- Improve the quality of the received signal
- Improve the system performance in term of SINR
- Get high spectral efficiency in a given bandwidth.

1.5 Methodology

To achieve the objectives of this work the research are organized as follows: step one is literature review which includes deep literature survey to have a knowledge of the smart antenna systems and how its work, it was the major step to achieve the objectives of this research, in step two the System modeling was introduced which involves the formulation of least mean square (LMS) and

recursive least square (RLS) adaptive beamforming algorithms and the system performance metrics.

Step three involves simulating the modeled communication system using LMS and RLS beam forming algorithms in MATLAB. Finally the results obtained from the simulation analyzed and compared based on performance analysis criteria.

1.6 Thesis Outlines

The remainder of this thesis is organized as follows:

Chapter Two contain the literature review which consists of fundamentals of LTE MIMO concept, smart antenna beamforming and interference in MIMO systems.

Chapter Three describes the research methodology which presents the smart antenna beamforming techniques, performance metrics for the used algorithms to mitigate the interference and their mathematical representations. Chapter Four include result and discussion for simulation of LMS and RLS smart antenna algorithms and the performance metrics using MATLAB program.

Finally, Chapter Five concludes the thesis and summarizes the results of the work, and shows the scope for future researches in the area of adaptive beamforming algorithms for interference mitigation.

Chapter Two

Literature Review

2.1 LTE MIMO Concept and Smart Antenna Beamforming

With applications including social media, high definition video streaming, mobile banking, and full-featured web browsing, broadband cellular applications provide exciting opportunities for consumers and network operators alike. However, these data-intensive applications also create new bandwidth delivery challenges for mobile operators. In order to expand the available wireless network capacity to meet the demands of data-intensive applications, operators have invested heavily in acquiring radio frequency bandwidth. Even so, RF spectrum remains a finite resource. Therefore, network operators must look at new technologies such as LTE to generate more throughput from existing bandwidth [9] IEEE 802.16m and 3GPP LTE-Advanced are the two evolving standards targeting 4G wireless systems. In both standards, multiple input multiple-output antenna technologies play an essential role in meeting the 4G requirements. The application of MIMO technologies is one of the most crucial distinctions between 3G and 4G. It not only enhances the conventional point-to-point link, but also enables new types of links such as downlink multiuser MIMO. A large family of MIMO techniques has been developed for various links and with various amounts of available channel state information in both IEEE 802.16e/m and 3GPP LTE/LTE-Advanced [10]. Figure 2.1 shows the MIMO concept.

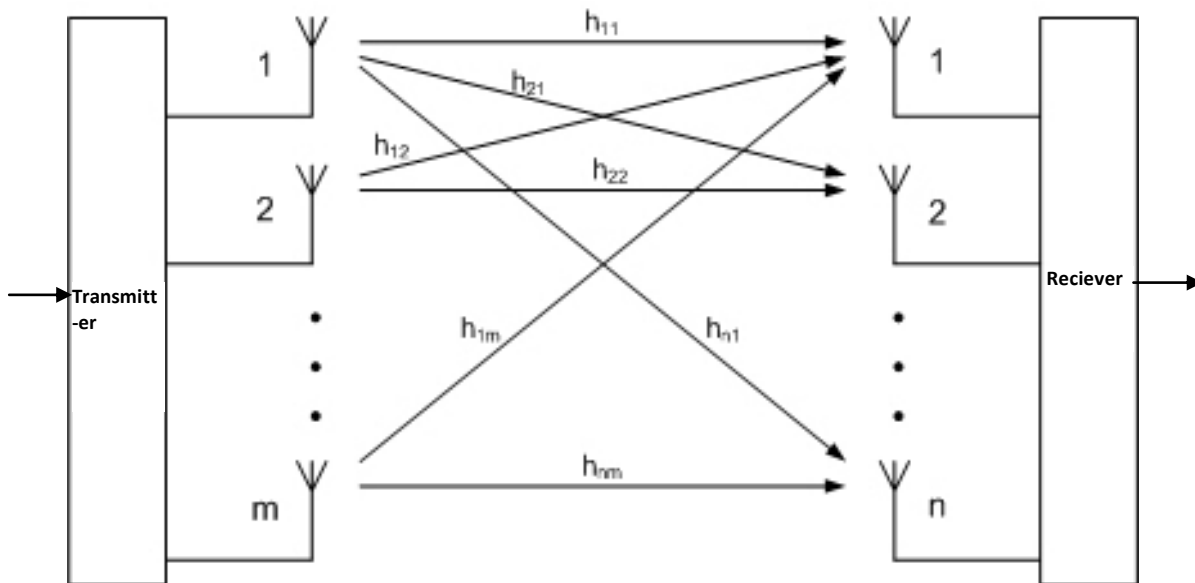


Figure 2.1 general MIMO system channel configuration [11]

2.1.1 Multi-Input Multi-Output (MIMO)

MIMO stands for Multiple-Input Multiple-Output, meaning that MIMO systems use more than one transmit antenna (Tx) to send a signal on the same frequency to more than one receive antenna (Rx).

Although MIMO has been deployed for years in WLAN networks, it is a relatively new feature in commercial wireless networks. MIMO technology is a standard feature of next-generation LTE networks, and it is a major piece of LTE's promise to significantly boost data rates and overall system capacity. However, MIMO also represents a new challenge for network operators.

A MIMO system typically consists of m transmit and n receive antennas as shown in figure 2.1 by using the same channel, every antenna receives not only the direct components intended for it, but also the indirect components intended for the other antennas. A time-independent, narrowband channel is assumed. The direct

connection from antenna 1 to 1 is specified with h_{11} , etc., while the indirect connection from antenna 1 to 2 is identified as cross component h_{21} , etc. From this, a transmission matrix \mathbf{H} is obtained with the dimensions $\mathbf{n} \times \mathbf{m}$ [11]. As illustrated in equation 2.1.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{00} & h_{1m} \\ h_{21} & h_{22} & h_{00} & h_{2m} \\ h_{00} & h_{00} & h_{00} & h_{0m} \\ h_{n1} & h_{n2} & h_{n0} & h_{nm} \end{bmatrix} \quad (2.1)$$

the transmission formula is illustrated in equation 2.2 :

$$\mathbf{Y} = \mathbf{H}\mathbf{x} + n \quad (2.2)$$

Where \mathbf{x} is transmit vector, \mathbf{Y} receive vector is the transmission channel matrix and n is noise vector.

Data to be transmitted is divided into independent data streams. The number of streams M is always less than or equal to the number of antennas; For example, a 4x4 system could be used to transmit four or fewer streams, the capacity C increases linearly with the number of streams M [11]. as given in equation 2.3.

$$C = MB \log_2 \left(1 + \frac{S}{N} \right) \quad (2.3)$$

Where M is the number of streams, B is channel bandwidth and S/N is the signal-to-noise power ratio.

Traditional cellular networks generally provide the best service under line-of-sight conditions. MIMO thrives under rich scattering conditions, where signals bounce around the environment.

Under rich scattering conditions, signals from different Tx take multiple paths to reach the user equipment (UE) at different times, as shown in Figure 2.2. In order to achieve promised throughputs in LTE systems, operators must optimize their networks' multipath conditions for MIMO, targeting both rich scattering conditions

and high SNR for each multipath signal. This optimization process requires accurate measurement of these multipath conditions in order to achieve the best performance for a given environment while avoiding the time and expense of guesswork. With strong measurements, however, an optimized MIMO system can result in massive throughput gains without the expenses associated with adding spectrum or network node ((eNodeB)).[9]

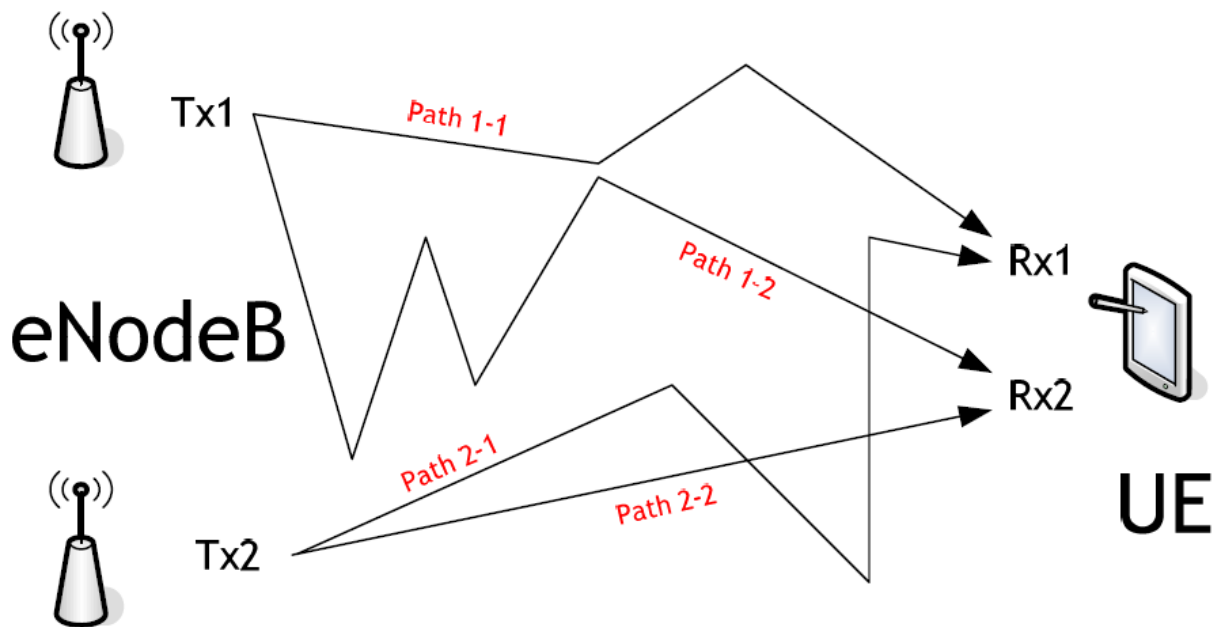


Figure: 2.2 multiple paths from eNodeB to UE in 2x2 MIMO [9]

MIMO technology has its roots in more widely deployed antenna techniques. MIMO builds on Single-Input Multiple-Output (SIMO), also called receive diversity, as well as Multiple-Input Single-Output (MISO), also called transmit diversity. SIMO techniques have been around for decades, while MISO is used in most advanced cellular networks today. Both of these techniques seek to boost signal-to-noise ratio (SNR) in order to compensate for signal degradation [10].

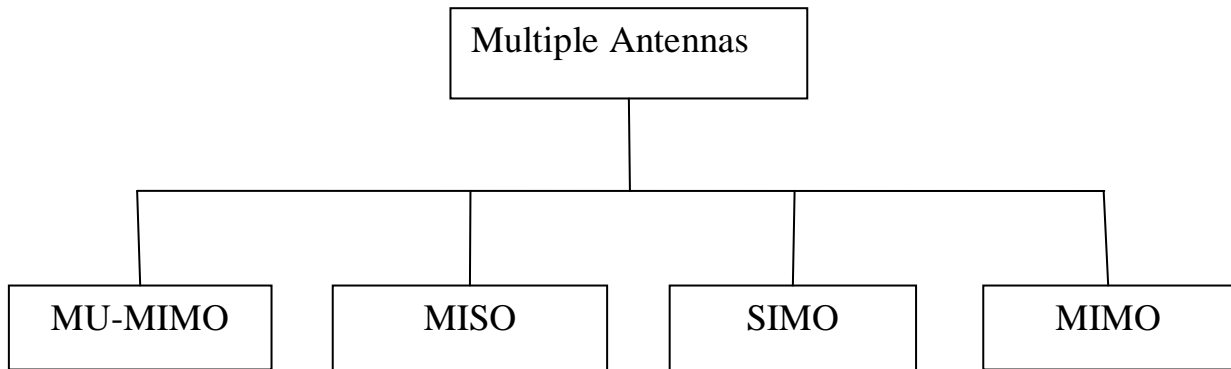


Figure 2.3 multiple antenna configuration

2.1.2 LTE MIMO Features and Techniques

2.1.2.1 LTE Downlink Transmission Modes

This section discusses the basic MIMO features and techniques available in LTE downlink operations. Because network conditions and UE capabilities can vary greatly, MIMO systems must be highly flexible in order to maximize gains in throughput. Since each eNodeB can be configured differently in terms of how it adapts transmissions in real time, it is important to understand the key transmission modes available in LTE, as well as the conditions under which they are most useful. Network operators can then compare scanning receiver measurements to UE-reported data logged by the network to determine if the eNodeB is effectively adapting transmissions to the RF environment. As shown in Table 2.1, LTE Release 8 supports seven different transmission modes based on multiple antenna configuration, with an eighth available in Release 9. These modes are designed to take the best advantage of different channel and multipath conditions and eNodeB antenna configurations, as well as differences in UE capabilities and mobility. Modes 2, 3, 4, and 6 are Single-User MIMO (SU-MIMO) modes. These modes

form the core of LTE's MIMO operations, in which more than one antenna at the eNodeB communicates with more than one antenna at a single UE [9],[4].

The remaining modes are less relevant to current LTE MIMO techniques. Modes 1 and 7 represent non-MIMO based antenna techniques where mode 5 and 8 are early versions of antenna techniques that are expected to be used minimally in early LTE deployments, but more robust versions of these techniques are planned for LTE-Advanced. [10]

Table 2.1 - Downlink Transmission Modes for LTE Release 8⁵

Bold indicates MIMO modes expected to be widely used in early LTE deployments

Transmission Mode	Downlink Transmission Scheme
Mode1	Single Antenna Port (SISO or SIMO)
Mode2	Transmit Diversity
Mode3	Open-Loop Spatial Multiplexing
Mode4	Close-Loop Spatial Multiplexing
Mode5	Multi-User MIMO
Mode6	Close-Loop Rank-1 Spatial Multiplexing
Mode7	Single Antenna Port Beamforming
Mode8	Dual-Layer Beamforming

2.2.1.2 SU-MIMO and MU-MIMO

When the data rate is to be increased for a single UE, this is called Single User MIMO (SU-MIMO)[11], SU-MIMO can be applied in both uplink and downlink. In uplink single user equipment transmits the signals to the base station whereas in downlink the base station transmits the signal to user equipment.

When the individual streams are assigned to various users, this is called Multi User MIMO (MU-MIMO). In the uplink, each user sends the data to a base station at different time but in downlink, base station broadcast the data to all the user

simultaneously[1]. So MU-MIMO is automatically adapted to downlink, figure 2.3 and 2.4 show SU-MIMO and MU-MIMO respectively.

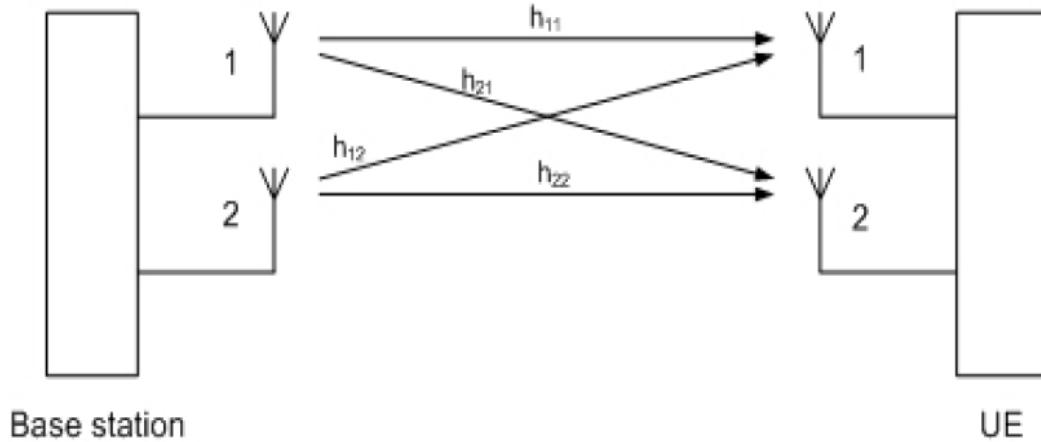


Figure 2.4: Single User MIMO (SU-MIMO)[11]

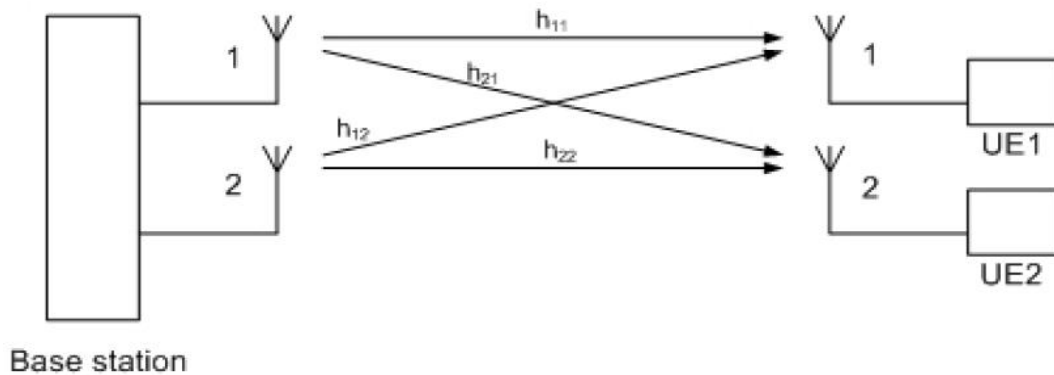


Figure 2.5: Multi User MIMO (MU-MIMO) [11]

2.1.3 Smart Antenna and Beamforming

2.1.3.1 Antenna Array

Usually the beamwidth of a single antenna element pattern is too wide for many wireless terrestrial and space applications, which, in many cases, require high gains towards certain directions [4]. Besides, a narrower HPBW (or FNBW) can be necessary for spatial filtering operations and to increase the gain in the desired

directions as well as to reduce it in the undesired ones. Directional patterns can be obtained by increasing the single antenna element size or by using more radiating elements in order to form an array. In this second case the individual antennas are arranged according to a proper geometrical configuration and the current excitations of each element are synthesized to provide the desired radiation pattern. For practical purposes all radiators belonging to an array are usually chosen as identical, even if, for specific applications, antenna arrays consisting of different elements can be built. By adopting multi-antenna structures, it is possible to generate radiation patterns with multiple main beams, each with a desired beamwidth, while satisfying side-lobe level and null constraints. The synthesis process can be performed by properly selecting the number of array elements, their geometrical configuration and the interelement spacing. Besides, the radiation pattern can be electronically controlled by modifying the phase and/or the amplitude of the current excitations. The total field radiated by an antenna array is determined by the vector addition of the individual elements. [12]

2.1.3.2 Smart Antenna

In truth, antennas are not smart antenna systems are smart. Generally collocated with a base station, a smart antenna system combines an antenna array with a digital signal-processing capability to transmit and receive in an adaptive, spatially sensitive manner. In other words, such a system can automatically change the directionality of its radiation patterns in response to its signal environment. This can dramatically increase the performance characteristics (such as capacity) of a wireless system.

In many applications, the desired information to be extracted from an array of sensors is the content of a spatially propagating signal from a certain direction. The

content may be a message content in the signal, such as in communications applications, or merely the existence of the signal, as in the radar and sonar. To this end, we want to linearly combine the signals from all the sensors in a manner that is with a certain weighting, so as to examine signals arriving from specific angles. This operation is known as beam-forming because the weighting process emphasizes signals from a particular direction while attenuating those from other directions and can be thought of as forming a beam. In this sense, the beam-former is a spatial filter [5].

two major categories of smart antennas regarding the choices for transmit strategy:

- Switched Beam Antennas

Switched beam antenna systems form multiple fixed beams with heightened sensitivity in particular directions. As shown in figure 2.5 ,these antenna systems detect signal strength, choose from one of several predetermined, fixed beams, and switch from one beam to another as the mobile moves throughout the sector. Instead of shaping the directional antenna pattern with the metallic properties and physical design of a single element (like a sectorized antenna), switched beam systems combine the outputs of multiple antennas in such a way as to form finely sectorized (directional) beams with more spatial selectivity than can be achieved with conventional, single-element approaches.[5]

-Adaptive Array Antennas

Adaptive antenna technology represents the most advanced smart antenna approach to date. Using a variety of new signal-processing algorithms, the adaptive system takes advantage of its ability to effectively locate and track various types of signals to dynamically minimize interference and maximize intended signal reception. Both systems attempt to increase gain according to the location of the user;

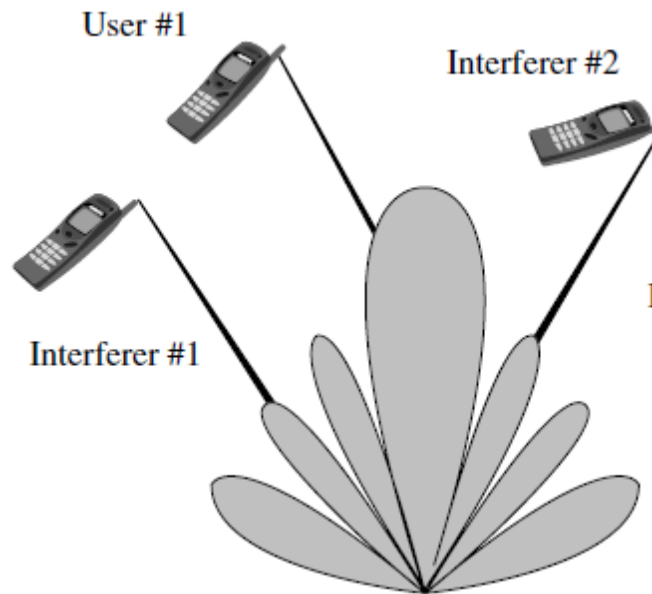


Figure 2.6: Switched Beam Antennas [15]

however, only the adaptive system provides optimal gain while simultaneously identifying, tracking, and minimizing interfering signals as shown in figure 2.6 [5],[14]. The dual purpose of a smart antenna system is to augment the signal quality of the radio-based system through more focused transmission of radio signals while enhancing capacity through increased frequency reuse.

Generally speaking, each approach forms a main lobe toward individual users and attempts to reject interference or noise from outside of the main lobe. [5]

2.1.4 Beamforming

Beamforming which is also called spatial filtering is a signal processing technique used in sensor antenna arrays for directional signal transmission or reception. This beamforming is achieved by combining elements in a phased antenna array in such a way that signals at particular angles face a constructive interference while others face destructive interference.

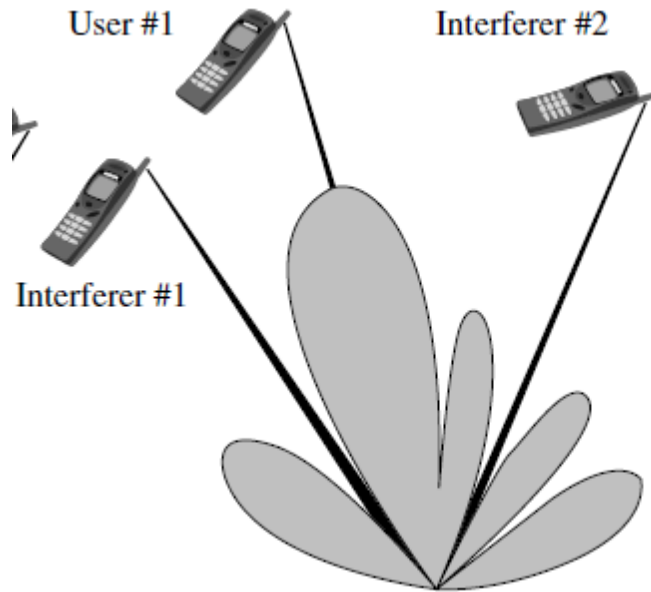


Figure 2.7: Adaptive Array Antennas[15]

To achieve spatial selectivity and to mitigate channel interference in MIMO communication system, beamforming can be used at both the transmitting and receiving ends of the communication system. The improvement by beamforming compared with omnidirectional reception or transmission is known as the receive or transmit gain (or loss). Beamforming can be used for radio or other waves also. It has found many applications in radar, sonar, wireless communications, acoustics, and biomedicine [6], figure 2.7 is General structure of a beamformer.

beamforming techniques can be subdivided in two main groups: fixed beamforming and adaptive beamforming ,in the first case the interference is mitigated but not suppressed and the system can be usually realized at a reasonable cost. Adaptive antennas, instead, require the adoption of complex signal processing algorithms in order to steer the main lobe towards the desired direction and to suppress the undesired sources.

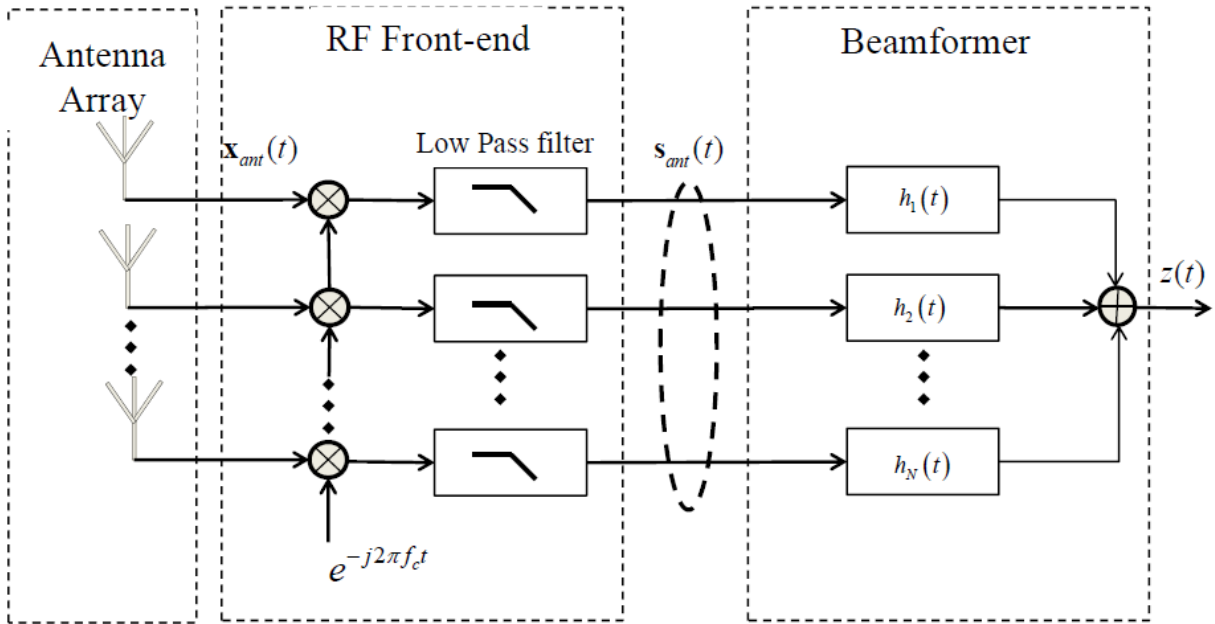


Figure 2.8: General structure of a beamformer [3]

This second approach leads to optimal performance, but is more expensive and needs considerable implementation efforts [13]

1-Fixed Beamforming

Fixed beamforming does not perform any operation on the amplitude weighting of the received signals and can be realized by adopting either an analog approach as an example switched beam, delay and sum or a digital approach like beam-space beamforming.

2-Adaptive Beamforming

Adaptive Beamforming is a technique in which an array of antennas is exploited to achieve maximum reception in a specified direction by estimating the signal arrival from a desired direction (in the presence of noise) while signals of the same frequency from other directions are rejected. This is achieved by varying the

weights of each of the sensors (antennas) used in the array. It basically uses the idea that, though the signals emanating from different transmitters occupy the same frequency channel, they still arrive from different directions. This spatial separation is exploited to separate the desired signal from the interfering signals. In adaptive beamforming the optimum weights are iteratively computed using complex algorithms based upon different criteria[14].

In figure 2.8 beamforming is applied to separate sub-carrier to reduce the interference among the carriers. Then IDFT is applied to convert the symbol from time domain to frequency domain to analyze symbol characteristics. Cyclic prefix is added in order to reduce the inter symbol interference (ISI) and reverse operation takes place in receive beamforming as shown in Figure 2.9

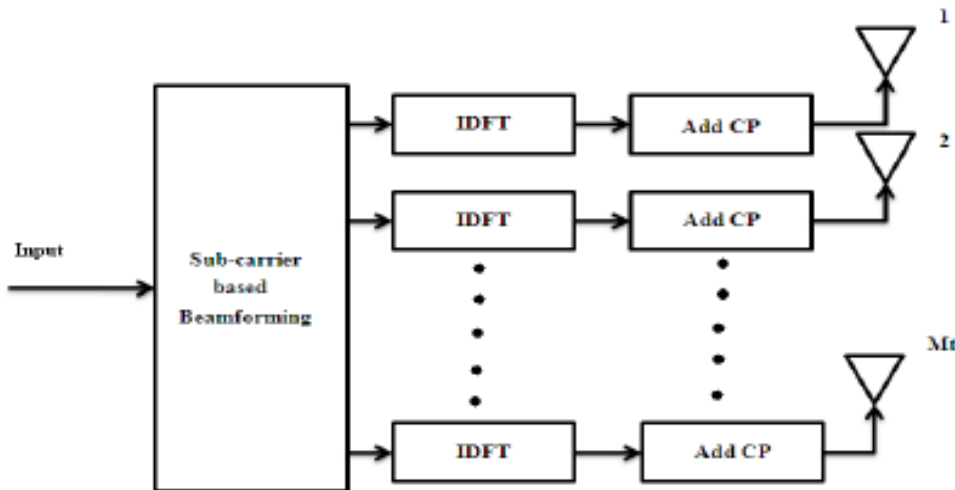


Fig 2.9 transmitting beamforming[1]

Beamforming is generally accomplished by phasing the feed to each element of an array so that signals received or transmitted from all elements will be in phase in a particular direction. The phases (the interelement phase) and usually amplitudes are adjusted to optimize the received signal, the array factor for an N-element equally spaced linear array is given.

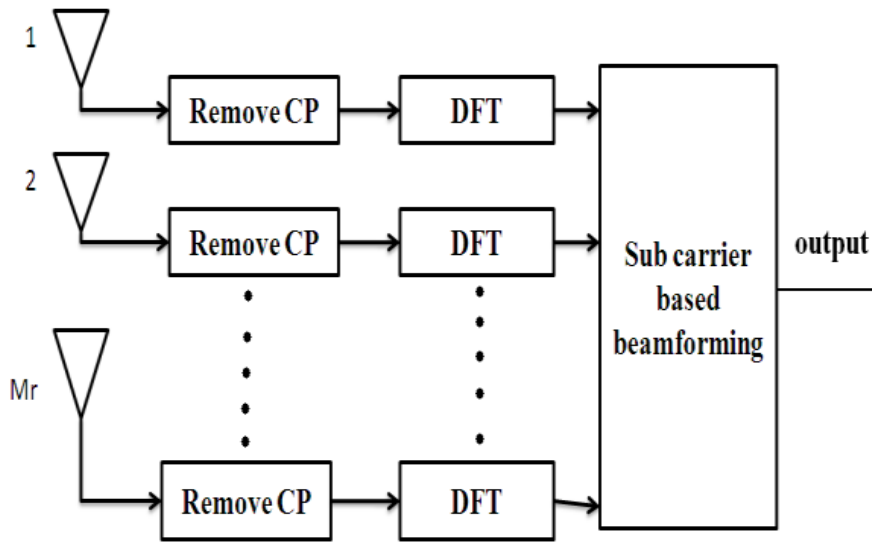


Fig 2.10 receiveing beamforming[1]

$$AF(\phi) = \sum_{n=0}^{N-1} A_n e^{jn(\frac{2\pi d}{\lambda} \cos \phi + \alpha)} \quad (2.4)$$

Where A_n is complex array weight at element n , ϕ is the angle of incidence of electromagnetic plane wave from array axis and λ the wavelength.

Note that variable amplitude excitation is used. The interelement Phase shift is given by:

$$\alpha = \frac{2\pi d}{\lambda_0} \cos \phi_0 \quad (2.5)$$

ϕ_0 is the desired beam direction. At wavelength λ_0 the phase shift corresponds to a time delay that will steer the beam to ϕ_0 . To illustrate different beamforming aspects, let us consider an adaptive beamforming configuration shown below in figure 2.10 below.

The output of the array $y(t)$ with variable element weights is the weighted sum of the received signals $s_i(t)$ at the array elements and the noise $n(t)$ the receivers connected to each element. The weights wme iteratively computed based on the

array output $y(t)$ reference. Signal $d(t)$ that approximates the desired signal, and previous weights. The reference signal is approximated to the desired signal using a training sequence or a spreading code, which is known at the receiver. The format of the reference signal varies and depends upon the system where adaptive beamforming is implemented. The reference signal usually has a good correlation with the desired signal and the degree of correlation influences the accuracy and the convergence of the algorithm.

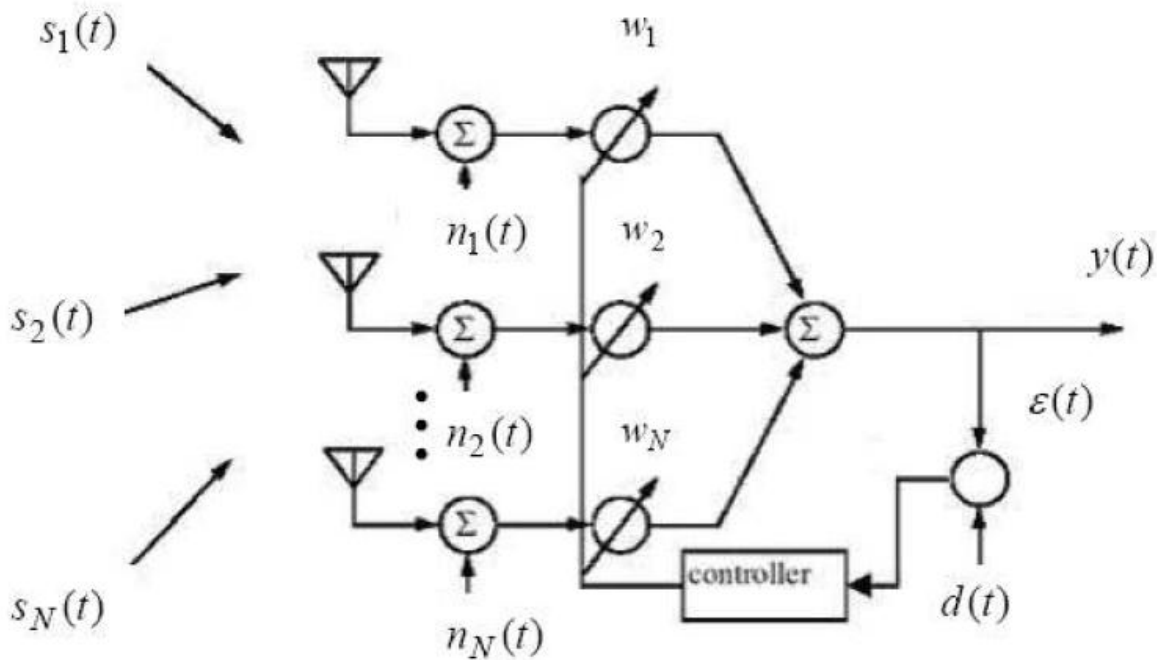


Fig 2.11 : An adaptive array system[14]

The array output $y(t)$ is given by:

$$y(t) = w^H x(t) \quad (2.6)$$

Where H denotes the complex conjugate transpose of the weight vector

$$W^H = [w_0 \quad w_1 \quad w_2 \quad \cdots \quad w_{n-1}]^T \quad (2.7)$$

is matrix of weights and

$$X(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)]^T \quad (2.8)$$

is matrix of signal vector.

In order to compute the optimum weights, the array response vector from the sampled data of the array output has to be known. The array response vector is a function of the incident angle as well as the frequency. The baseband received signal at the N-th antenna is a sum of phase-shifted and attenuated versions of the original signal $S_i(t)$.

$$x_N(t) \cong \sum_{i=1}^N a_N(\theta_i) S_i(t) e^{-j2\pi f_c \tau_N(\theta_i)} \quad (2.9)$$

The $s_i(t)$ consists of both the desired and the interfering signals, f_c is the carrier frequency.

$$a(\theta_i) = [a_1(\theta_i) e^{-j2\pi f_c \tau_1(\theta_i)}, \dots, a_N(\theta_i) e^{-j2\pi f_c \tau_N(\theta_i)}]^T \quad (2.10)$$

$$A(\theta) = [a(\theta_1) \quad a(\theta_2) \quad \dots \quad a(\theta_d)] \quad (2.11)$$

$$S(t) = [s_1(t) \quad s_2(t) \quad \dots \quad s_d(t)]^T \quad (2.12)$$

$$x(t) = A(\theta)S(t) \quad (2.13)$$

With noise

$$x(t) = A(\theta)S(t) + n(t) \quad (2.14)$$

$a(\theta)$ is referred to as the array propagation vector. The beamformer response can be expressed in the vector form as:

$$r(\theta, \omega) = w^H a(\theta, \omega) \quad (2.15)$$

This includes the possible dependency of $a(\theta)$ on ω as well.

To have a better understanding let us re-write $x(t)$ in equation by separating the desired signal from the interfering signals. Let $s(t)$ denote the desired signal arriving at an angle of incidence θ_0 at the array and the $u_i(t)$ denotes the number of undesired interfering signals arriving at angles of incidence θ_i . It must be noted

that, in this case, the directions of arrival are known a priori using a direction of arrival (DOA) algorithm. The output of the antenna array can now be re-written as;

$$x(t) = S(t)a(\theta_0) + \sum_{i=1}^{N_u} u_i(t)a(\theta_i) + n(t) \quad (2.16)$$

Where $S(t)$ denotes the desired signal arriving at angle θ_0 and $u_i(t)$ denotes interfering signals arriving at angle of incidences θ_i respectively $a(\theta_0)$ and $a(\theta_i)$ represents the steering vectors for the desired signal and interfering signals respectively. Therefore, having the above information, adaptive algorithms are required to estimate $s(t)$ from $x(t)$ while minimizing the error between the estimate $s(t)$ and the original signal $s(t)$.

Let $d^*(t)$ represent a signal that is closely correlated to the original desired signal $s(t)$. $d^*(t)$ is referred to as the reference signal, the mean square error (MSE) $\varepsilon^2(t)$ between the beamformer output and the reference signal can now be computed as follows;

$$\varepsilon^2(t) = [d^*(t) - w^H x(t)]^2 \quad (2.17)$$

After taking an expectation on both sides of the equation we get,

$$E\{\varepsilon^2(t)\} = E\{[d^*(t) - w^H x(t)]^2\} \quad (2.18)$$

$$E\{\varepsilon^2(t)\} = E\{d^2(t)\} - 2w^H r + w^H R w \quad (2.19)$$

where $r = E\{[d^*(t)x(t)]\}$ is the cross-correlation matrix between the desired signal and the received signal $R = E[x(t)x^H(t)]$ is the auto-correlation matrix of the received signal also known as the covariance matrix. The minimum MSE can be obtained by setting the gradient vector of the above equation with respect to zero

$$\nabla_w (E\{\varepsilon^2(t)\}) = -2r + 2Rw = 0 \quad (2.20)$$

Therefore the optimum solution for the weight is given by:

$$w_{opt} = R^{-1}r \quad (2.21)$$

This equation is referred to as the optimum Weiner solution. There are several adaptive beamforming techniques like LMS (least mean square) algorithm beamforming, RLS (recursive least square) algorithm beamforming techniques. They are very effective techniques to mitigate the interference.[6]

2.1.5 Interference in MIMO Systems

Multiple input multiple output (MIMO) systems have been shown to have tremendous potential in increasing the average throughput in cellular wireless communication systems. The performance gain in channel capacity, reliability and spectral efficiency in single user (point-to-point) MIMO (SU-MIMO) systems has spurred the inclusion of SU-MIMO in various cellular and other wireless communication standards such as 3GPP high-speed packet access (HSPA) and long term evolution (LTE) where SU-MIMO has successfully demonstrated its ability to enhance the performance of wireless networks. In cellular systems where spectrum scarcity/cost is a major concern, a frequency reuse factor of 1 is desirable [8].

Such systems however have to deal with the additional problem of inter-cell interference which does not exist in isolated point-to-point systems. Interference is being increasingly identified as the major bottleneck limiting the throughput in wireless communication networks. Traditionally, the problem of interference has been dealt with through careful planning and (mostly static) radio resource management. With the widespread popularity of wireless devices following different wireless communication standards, the efficacy of such interference avoidance solutions is fairly limited. Indeed, major standardization bodies are now including explicit interference coordination strategies in next generation cellular communication standards.

A systematic study of the performance of cellular communication systems where each cell communicates multiple streams to its users while enduring/causing interference from/to neighboring cells due to transmission over a common shared resource comes under the purview of MIMO interference channels (MIMO IFC). A K-user MIMO-IFC models a network of K transmitreceive pairs where each transmitter communicates multiple data streams to its respective receiver. In doing so, it generates interference at all other receivers.[On the MIMO Interference Channel][3].

2.2 Related Works

the author of the work in [2] proposes a dynamic Fractional Frequency Reuse (FFR) mechanism that selects the optimal frequency allocation based on the total throughput and user's satisfaction. Particularly, the mechanism divides the cell into two regions (inner and outer) and selects the optimal size as well as the optimal frequency allocation between these regions with main target to maximize the overall throughput and user satisfaction. FFR has its intrinsic drawback of limited frequency selective gain and lower overall spectral efficiency due to large reuse factor at the cell edge.

In [1] to overcome the problem of the interference, beamforming technique is used under the assumption of perfect channel knowledge at the transmitter availability to evaluate the system performance of SU-MIMO and MU-MIMO cases in the different scenarios, with and without beamforming technique in terms of Bit-Error-Rate (BER) and Signal to Noise Ratio (SNR) .

the author of the work in [3] uses methods based on space or space-time processing such as(Space-time correction matrix formation, Spoofing signal

channel coefficients estimation and null steering) to mitigate the interference and this methods assumed that the environment is stationary. This may limit the application of these methods. In many applications the adaptive cases is needed in which changes in the environment are followed.

Another method used the LMS style beamforming technique was discussed ,in [9] which is very effective technique to mitigate the co-channel interference and the performance was analyzed ,it achieved better in term of SNR but the limitation of the LMS is its low convergence. In [5] both the block adaptive which is employs a block of data to estimate the optimum weight vector and sample-by-sample methods are used to update weights of the smart antenna such as least mean square (LMS) algorithm, constant modulus algorithm (CMA), least square constant modulus algorithm(LS-CMA) and recursive least square (RLS) algorithm the work offers a significantly improved solution to reduce interference levels and improve the system capacity.

the performance of the LMS style beamforming technique which is very effective technique to mitigate the co-channel interference was proposed in [6], The comparison between the performance of adaptive beamforming and null steering beamforming was also analyzed , it is observed that to mitigate the co-channel interference , adaptive beamforming technique outperforms the method based on the null steering beamforming. A novel beamforming scheme has been proposed in [7], featuring better adaptive CCI suppression on short signal intervals than the classic preamble-based SMI beamforming methods, The enhanced scheme has turned out to reduce the gap to the SINR attained by an ideal signal combining based on perfectly known spatial covariance, over a large range. The performance gain is independent of the particular structure of the CC.

Chapter Three

Interference Mitigation Using Smart Antenna

3.1 Introduction

Smart antenna technology offers a significantly improved solution to reduce interference levels and improve the system capacity. Each user's signal is transmitted and received by the basestation only in the direction of that particular user. This drastically reduces the overall interference level in the system. A smart antenna system, consists of an array of antennas that together direct different Transmission/reception beams towards each user in the system. This method of transmission and reception is called beamforming and is made possible through smart (advanced) signal processing at the baseband[5]. Beamforming is a signal processing technique used in sensor arrays for directional signal transmission or reception. This is achieved by combining elements in the array in a way where signals at particular angles experience constructive interference and while others experience destructive interference. Beamforming can be used at both the transmitting and receiving ends in order to achieve spatial selectivity. The improvement compared with an omnidirectional reception/transmission is known as the receive/transmit gain (or loss) [9].

This research investigates the performance of two adaptive beamforming algorithms, LMS (Least Mean Square) and RLS(Recursive Least Square) and made a comparison between them through implementation in Matlab.

3.2 The Least Mean Square (LMS) Algorithm

The Least mean square (LMS) is an adaptive algorithm, which uses a gradient-based method of steepest decent. LMS incorporates an iterative procedure that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. Compared to other algorithms LMS algorithm is relatively simple; it does not require correlation function calculation nor does it require matrix inversions.

3.2.1 LMS Algorithm and Adaptive Arrays

Consider a Uniform Linear Array (ULA) with N isotropic elements, which forms the integral part of the adaptive beamforming system as shown in the figure 3.1.

The output of the antenna array $x(t)$ is given by the equation (2.16)

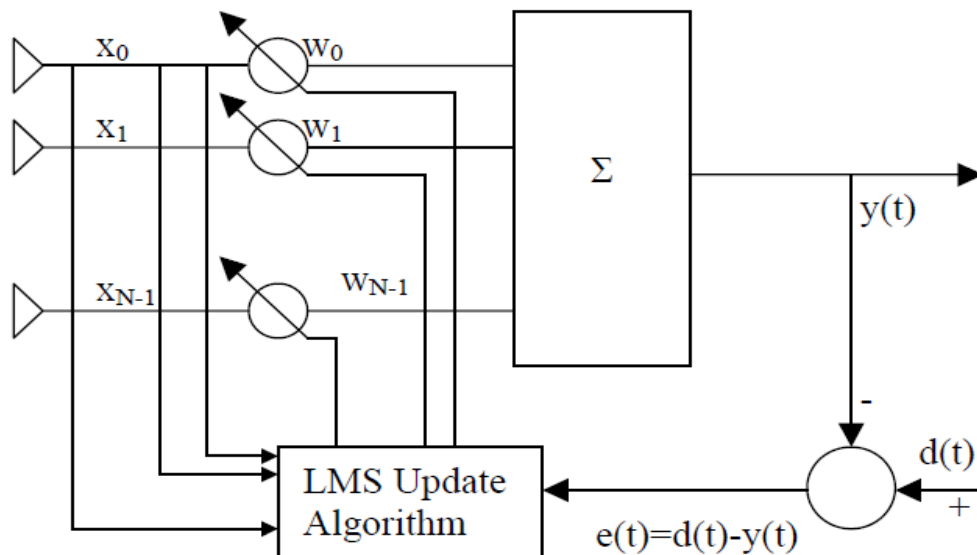


Fig 3.1 LMS Adaptive beamforming network [8]

Therefore it is required to construct the desired signal from the received signal amid the interfering signal and the additional noise $n(t)$.

As shown in the figure (3.1) above the outputs of the individual sensors are linearly combined after being scaled using corresponding weights such that the antenna array pattern is optimized to have maximum possible gain in the direction of the desired signal and nulls in the direction of the interferers. The weights here will be computed using LMS algorithm based on Minimum Squared Error (MSE) criterion. Therefore the spatial filtering problem involves estimation of signal from the received signal (i.e. the array output) by minimizing the error between the reference signal, which closely matches or has some extent of correlation with the desired signal estimate and the beamformer output $y(t)$ (equal to $wx(t)$). This is a classical Weiner filtering problem for which the solution can be iteratively found using the LMS algorithm.

3.2.2 LMS Algorithm Formulation

From the method of steepest descent, the weight vector equation is given by

$$w(n+1) = w(n) + \frac{1}{2}\mu[-\nabla(E\{e^2(n)\})] \quad (3.1)$$

Where μ is the step-size parameter and controls the convergence characteristics of the LMS algorithm, its a real valued positive constant generally between 0 and 1. If μ is chosen to be very small then the algorithm converges very slowly. A large value of μ lead to a faster convergence; $e^2(n)$ is the mean square error between the beamformer output $y(n)$ and the reference signal which is given by:

$$e^2(n) = [d^*(n) - w^h x(n)]^2 \quad (3.2)$$

The gradient vector in the above weight update equation can be computed as

$$\nabla_w(E\{e^2(n)\}) = -2r + 2Rw(n) \quad (3.3)$$

In the method of steepest descent the biggest problem is the computation involved in finding the values r and R matrices in real time. The LMS algorithm on the other

hand simplifies this by using the instantaneous values of covariance matrices r and R instead of their actual values i.e.

$$R(n) = x(n)x^h(n) \quad (3.4)$$

$$r(n) = d^*(n)x(n) \quad (3.5)$$

Therefore the weight update can be given by the following equation,

$$w(n + 1) = w(n) + \mu x(n)[d^*(n) - x^h(n)w(n)] = w(n) + \mu x(n)e^*(n) \quad (3.6)$$

The LMS algorithm is initiated with an arbitrary value $w(0)$ for the weight vector at $n=0$. The successive corrections of the weight vector eventually leads to the minimum value of the mean squared error.

Therefore the LMS algorithm requires three distinct steps in this order:

- 1) The beamformer output $y(n)$ is calculated using the equation (3.7).
- 2) The mean square error $e(n)$ between the beamformer output $y(n)$ and the reference signal $d^*(n)$ is given by the equation (3.8).
- 3) The weight vector is updated by the equation (3.9).

$$y(n) = w^h x(n) \quad (3.7)$$

$$e(n) = d^*(n) - y(n) \quad (3.8)$$

$$w(n + 1) = w(n) + \mu x(n)e^*(n) \quad (3.9)$$

Where $y(n)$ is the output , $e(n)$ is the error and $w(n + 1)$ is the weight.

The figure 3.2 illustrate the flow chart of LMS algorithm

3.3 RLS Algorithm

The recursive least-squares (RLS) algorithm uses a different approach in carrying out the adaptation. Instead of minimizing the mean square error as in the LMS algorithm, the sum of the squared errors of different set of inputs is the subject of

minimization. This algorithm was first derived from the Kalman filter. In contrast to the LMS algorithm, the RLS algorithm uses information from all past input samples and not only from the current tap-input samples, we can recursively calculate the required correlation matrix and the required correlation vector.

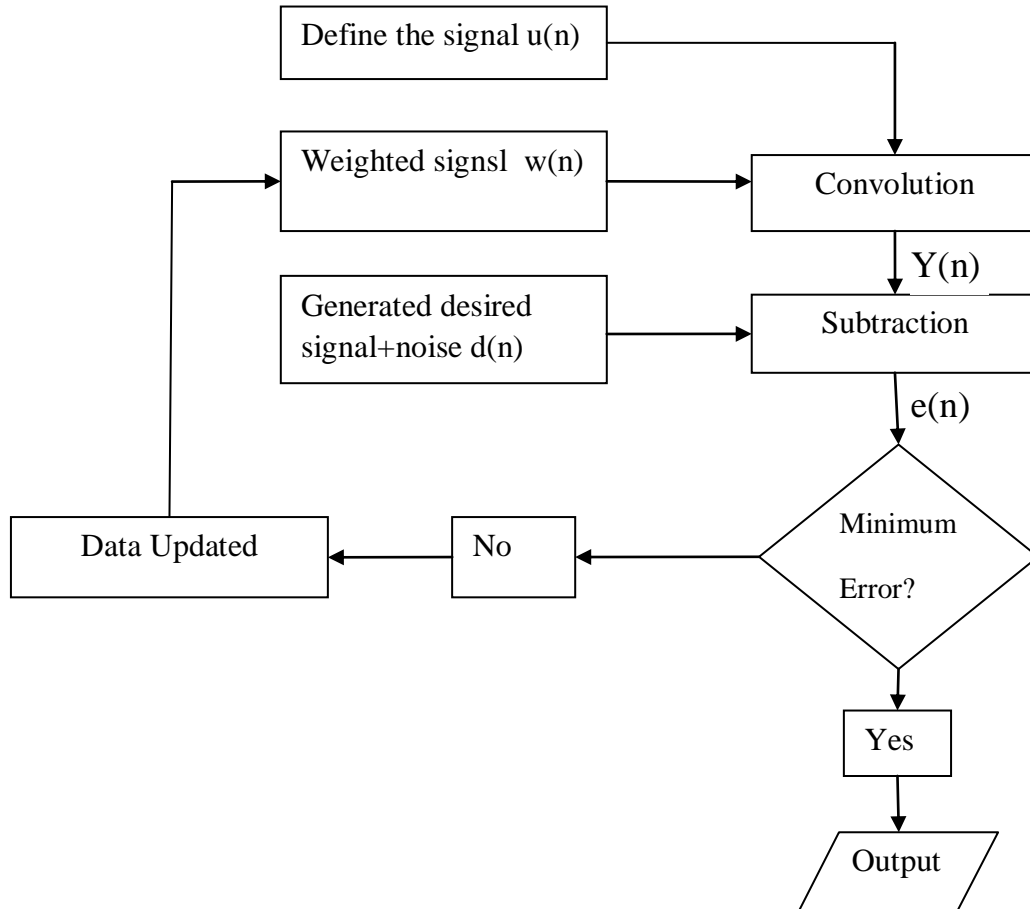


Figure 3.2 flow chart of LMS algorithm

In the following equations, the positive constant forgetting factor λ and regularization δ parameters are set by the user. The forgetting factor is roughly a measure of the memory of the algorithm which determines how quickly the previous data are de-emphasized. Its typical value is close to, but less than one. When $\lambda = 1$, we restore the ordinary least squares algorithm. And the regularization parameter's value is determined by the signal-to-noise ratio (SNR) of the signals,

where: δ =small positive constant for high SNR and large positive constant for low SNR [4]. The vector \mathbf{W} represents the adaptive filter's weight vector and the matrix \mathbf{P} is referred to as the inverse correlation matrix. To start the recursions we must provide initial values for weight vector and the inverse correlation matrix \mathbf{P} , if we have some apriori information about the parameter w this information will be used to initialize the algorithm. Otherwise, the typical initialization is:

$$\mathbf{W}^H(0) = 0 \quad (3.10)$$

The approximate initialization commonly used for matrix \mathbf{P} is

$$\mathbf{P}(0) = \delta^{-1}\mathbf{I} \quad (3.11)$$

The vector $\boldsymbol{\pi}$ is used as an intermediary step to computing the gain vector \mathbf{k} . For each instance of time $n = 1, 2, 3 \dots$,

$$\boldsymbol{\pi}(n) = \mathbf{P}(n-1)\mathbf{u}(n) \quad (3.12)$$

$$\mathbf{k}(n) = \frac{\pi(n)}{\lambda + \mathbf{u}^H(n)\pi(n)} \quad (3.13)$$

This gain vector is multiplied by the a priori estimation error $\xi(n)$ and added to the weight vector to update the weights.

$$\xi(n) = d(n) - \mathbf{W}^H(n-1)\mathbf{u}(n) \quad (3.14)$$

$$\mathbf{W}(n) = \mathbf{W}(n-1) + \mathbf{k}(n)\xi^*(n) \quad (3.15)$$

Once the weights have been updated the inverse correlation matrix is recalculated, and the training resumes with the new input values.

$$\mathbf{P}(n) = \lambda^{-1}\mathbf{P}(n-1) - \lambda^{-1}\mathbf{k}(n)\mathbf{u}^H(n)\mathbf{P}(n-1) \quad (3.16)$$

The RLS algorithm is carried out along the following steps in this order:

1. The intermediary step and the gain vector is calculated using the equations (3.12), (3.13).
2. The value of the estimated error is calculated according to the equation (3.14).
3. The vector weights are updated using equation (3.15).

- Once the weights have been updated the inverse correlation matrix is recalculated by the equation (3.16).

Figure 3.3 shows the RLS algorithm representation while figure 3.4 the flow chart of RLS algorithm in which gain vector is correlated with the error vector and added to previous weights. It shows the relation of algorithm with noise cancellation application. $u(n)$ is a notation for input signal of adaptive filter and $d(n)$ is for desire signal.

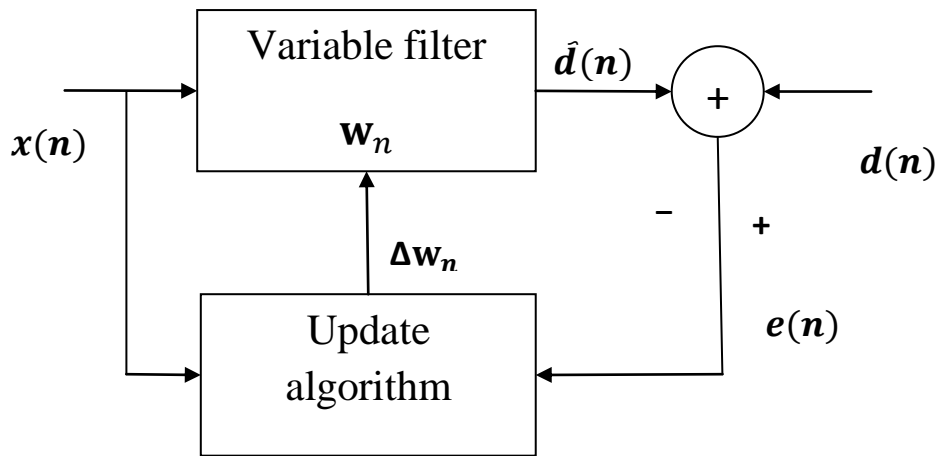


Figure 3.3 Recursive Least Square (RLS) algorithm

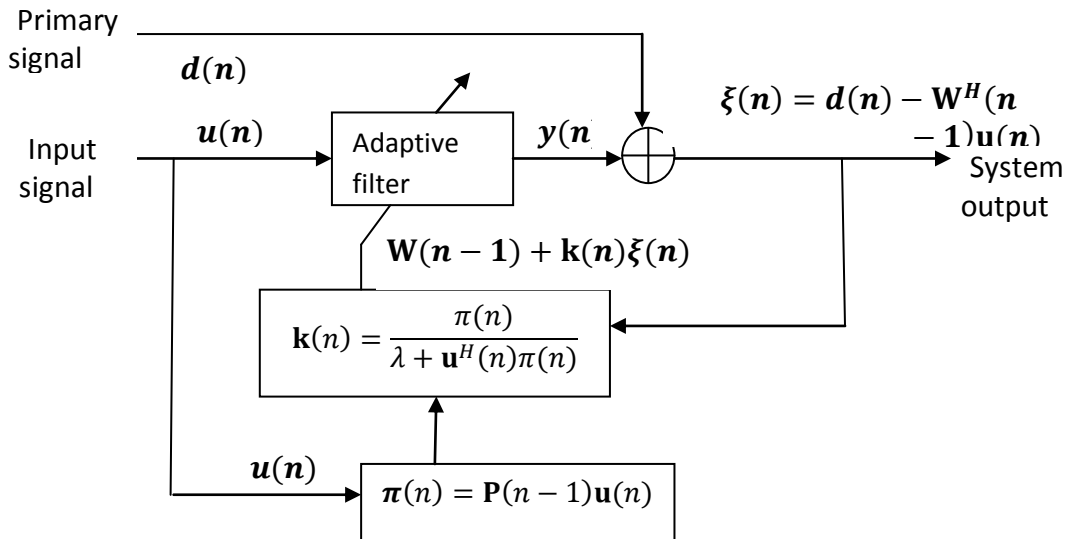


Figure 3.4 flow chart of RLS algorithm

simulations will be designed followed by the comparison between two algorithms in term of SINR and BER to evaluate the performance of the proposed adaptive beamforming algorithms.

3.4 Performance Analysis of RLS MIMO

In this thesis we design the beamformer consist of system model for MIMO with an array of N_t transmitting antennas and the N_r receiving antennas. The received signal can be expressed as

$$\mathbf{Y}(t) = \mathbf{H} \mathbf{W}_n \mathbf{x}(t) + \sum_{m=1}^l H_m W_{nm} x_m(t) + \mathbf{Z}(t), \quad (3.17)$$

where $\mathbf{Y}(t)$ is the received signal, \mathbf{H} is the channel matrix, and \mathbf{W}_n is the transmission weight vector. The received signal is an N dimensional array and it can be written as

$$\mathbf{Y}(t) = [Y_1(t) \quad Y_2(t) \quad \dots \quad Y_l(t)]^T \quad (3.18)$$

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & \dots & h_{1,N_r} \\ \vdots & \ddots & \vdots \\ h_{N_t,1} & \dots & h_{N_t,N_r} \end{bmatrix} \quad (3.19)$$

$$\mathbf{W}_n = [W_{n1} \quad W_{n2} \quad \dots \quad W_{n,N_r}]^T \quad (3.20)$$

h_{nm} is the channel response for mth transmitter and nth receiver. Here $x(t)$ is the source data signal and $Z(t)$ is the AWGN noise with zero mean, m is the number of interferences, \mathbf{H}_m is the channel matrix of the mth interfering signal. In this work we represented conjugate and transpose by the symbols $(\cdot)^*$ and $(\cdot)^T$ respectively.

3.4.1 Antenna weight vector receiver

The simple MIMO system consist of N_t array of transmit antennas and N_r array of receiving antennas. Hence the received signal can be expressed as

$$Y(t) = \mathbf{H} \cdot \mathbf{W}_n \cdot x(t) + Z(t) \quad (3.21)$$

After applying adaptive beamforming the output signal changes to the following form:

$$V(t) = W_p^H \cdot \mathbf{H} \cdot \mathbf{W}_n \cdot x(t) \quad (3.22)$$

Here not considering the noise signal for making the calculation simpler. Also the error signal for qth symbol can be expressed as

$$\hat{\epsilon} = d_p - V_p = d_p - W_p^H \cdot \mathbf{H} \cdot \mathbf{W}_n x(t) \quad (3.23)$$

d_p is the reference signal that can be obtained by projecting the output signal $V(p)$ to the nearest constellation.

RLS algorithm tries to reduce the square error and to get the exact transmitted signal. To achieve this goal, the gradient is obtained by differentiating the square error of the weight vector by the receiver antenna weight,

$$\frac{d\hat{\epsilon}_p^2}{dw_p^*} = -2\hat{\epsilon}_p Y(p) \quad (3.24)$$

Hence the updated equation for weight can be done as follows

$$W_{p+1}(k+1) = W_p(k) - 2\mu\hat{\epsilon}(k)Y(k) \quad (3.25)$$

3.4.2 Capacity

MIMO technology has been shown to improve the capacity of the communication link without the need to increase the transmission power. In a system with N_t transmit antennas and N_r receive antennas, the received SNR can increase in proportion to $N_t \times N_r$, and the signal power is divided among the channels. When CSI is available at the receiver, the Channon formula in equation (2.1) is used to derive the channel capacity which is given by:

$$C = R \cdot \log_2 \left[\det \left(1 + \frac{\gamma}{N_T} \mathbf{H} \mathbf{H}^* \right) \right] \quad \text{bits/s/Hz} \quad (3.26)$$

where $R = \min(N_R, N_T)$ is the rank of the channel matrix \mathbf{H} , γ denotes the Signal to Noise Ratio, $(\cdot)^*$ stand for the conjugate transpose operator.

3.4.3 Bit error rate

The BER is evaluated for communication systems with Rayleigh fading MIMO

channel and additive Gaussian noise. At the receive side, MIMO technology allows for a significant improvement of the BER. Once the MIMO technology is presented, we introduce the multi-user MIMO systems.

We calculate the final BER using QAM modulation scheme

$$\text{BER} = \text{code rate} \times \log_2 M \times (n_{\text{sym}} - \text{train length}) \quad (3.27)$$

where *code rate* is the convolutional coding rate, M is the M-ary number for QAM modulation, n_{Sym} is the number of symbols transmitted and *train Length* is the number of training bits used for the RLS.

3.4.4 Maximum Signal-to-Interference-plus-Noise Ratio

In this performance criterion, the weights can be chosen to directly maximize the output signal to Interference-plus noise ratio (SINR) where the receiver can estimate the strengths of the desired signal and of an interfering signal; weights are adjusted to maximize the ratio. Once again consider where signals from each element are multiplied by a complex weight and summed to form the array output. The output power of the array at any time t is given by the magnitude square of the array output, that is:

$$P(t) = \mathbf{W}^H \mathbf{X}(t) \mathbf{X}^H(t) \mathbf{W} \quad (3.28)$$

And assuming $\mathbf{X}(t)$ is a zero mean stationary processes, then a given \mathbf{W} the mean output power of the array system is obtained by taking the expectation over $\mathbf{X}(t)$.

Let X_s, X_1 and $n(t)$ denote the signal vector due to the desired signal source, unwanted interference, and random noise respectively. The components of signal, interference, and random noise in the output are then obtained by taking the inner product of the weight vector. These are given by:

$$y_s(t) = \mathbf{W}^H X_s(t) \quad (3.29)$$

$$y_1(t) = \mathbf{W}^H X_1(t) \quad (3.30)$$

$$y_s(t) = \mathbf{W}^H \mathbf{n}(t) \quad (3.31)$$

Define the array correlation matrices due to the signal source, unwanted interference, and random noise, respectively, as

$$\mathbf{R}_S(t) = E[X_S(t)X_S^H(t)] \quad (3.32)$$

$$\mathbf{R}_1(t) = E[X_1(t)X_1^H(t)] \quad (3.33)$$

$$\mathbf{R}_n(t) = E[n(t)n^H(t)] \quad (3.34)$$

\mathbf{R} is the sum of these three matrices, that is

$$\mathbf{R} = \mathbf{R}_S + \mathbf{R}_1 + \mathbf{R}_n \quad (3.35)$$

Let P_S , P_1 and P_n denote the mean output power due to the signal source, unwanted interference, and random noise, respectively.

$$P_S = \mathbf{W}^H \mathbf{R}_S \mathbf{w} \quad (3.36)$$

$$P_1 = \mathbf{W}^H \mathbf{R}_1 \mathbf{w} \quad (3.37)$$

$$P_n = \mathbf{W}^H \mathbf{R}_n \mathbf{w} \quad (3.38)$$

Let P_N denote the mean power at the output of the array contributed by random noise and unwanted interference, that is,

$$P_N = P_1 + P_n \quad (3.39)$$

P_N is also referred to as noise plus interference

Substituting (3.37) and (3.38) in (3.39)

$$P_N = \mathbf{W}^H \mathbf{R}_1 \mathbf{w} + \mathbf{W}^H \mathbf{R}_n \mathbf{w} = \mathbf{W}^H (\mathbf{R}_1 + \mathbf{R}_n) \mathbf{w} \quad (3.40)$$

Let \mathbf{R}_N denote the noise array correlation matrix, that is, $\mathbf{R}_N = \mathbf{R}_1 + \mathbf{R}_n$, then P_N the mean noise power at the output of the system can be expressed in terms of weight vector and \mathbf{R}_N as

$$P_N = \mathbf{W}^H \mathbf{R}_N \mathbf{w} \quad (3.41)$$

Let the output signal to interference plus noise ratio (SINR), be defined as the ratio of the mean output signal power to the mean output noise power at the output of the array system, that is,

$$SINR = \frac{P_S}{P_N} \quad (3.42)$$

Substituting (3.36) and (3.41) in (3.42)

$$SINR = \frac{\mathbf{W}^H R_S \mathbf{w}}{\mathbf{W}^H R_N \mathbf{w}} \quad (3.43)$$

To maximize the output SINR, we take the derivative of equation with respect to \mathbf{w} and set it to zero, which gives the following result

$$R_S \mathbf{w} = \frac{\mathbf{W}^H R_S \mathbf{w}}{\mathbf{W}^H R_N \mathbf{w}} R_N \mathbf{w} \quad (3.44)$$

In the case of uncorrelated RLS model , the output SINR In [dB] is estimated as:

$$SINR_{out} (dB) = G + \log_{10}(N) + SNR_{in} (dB) \quad (3.45)$$

Where N is the number of antennas and G is the array Gain achieved by an adaptive array and can be expressed as:

$$G = \log_{10} (N) \quad (3.46)$$

Chapter Four

Results and Discussion

4.1 Simulation parameters

In this chapter, array parameters and adaptive Beamforming algorithms (LMS and RLS) which are described in Chapters 3 are programmed in MATLAB and simulations are made with the assumptions and parameters in table 4.1 with $d = 0.5\lambda$ to compare the performances of the RLS and LMS adaptive Beamforming algorithms .

Table 4.1 Simulation parameters and assumptions

Channel types	-Rayleigh fading channel with mean zero and variance one is used -Zero mean AWGN
SNR Range	0 dB to 30 dB
SIR	15 dB
number of iteration (K)	500
LMS step size parameter (μ)	0.05
RLS forgotten factor (α)	0.01
Coupling	Neglected
spacing between array elements	Uniform
The signals	Narrow band signals and uncorrelated

4.2 Simulation Results

4.2.1 Performance of LMS Algorithm

The figure 4.1 is the number of iterations versus mean square error plot with parameters summarized in TABLE 4.1 for LMS algorithm to show the Error between desired signal and LMS output for linear array of $N = 8$ elements . It is observed that error function value is reduced quickly by increasing the number of iterations.

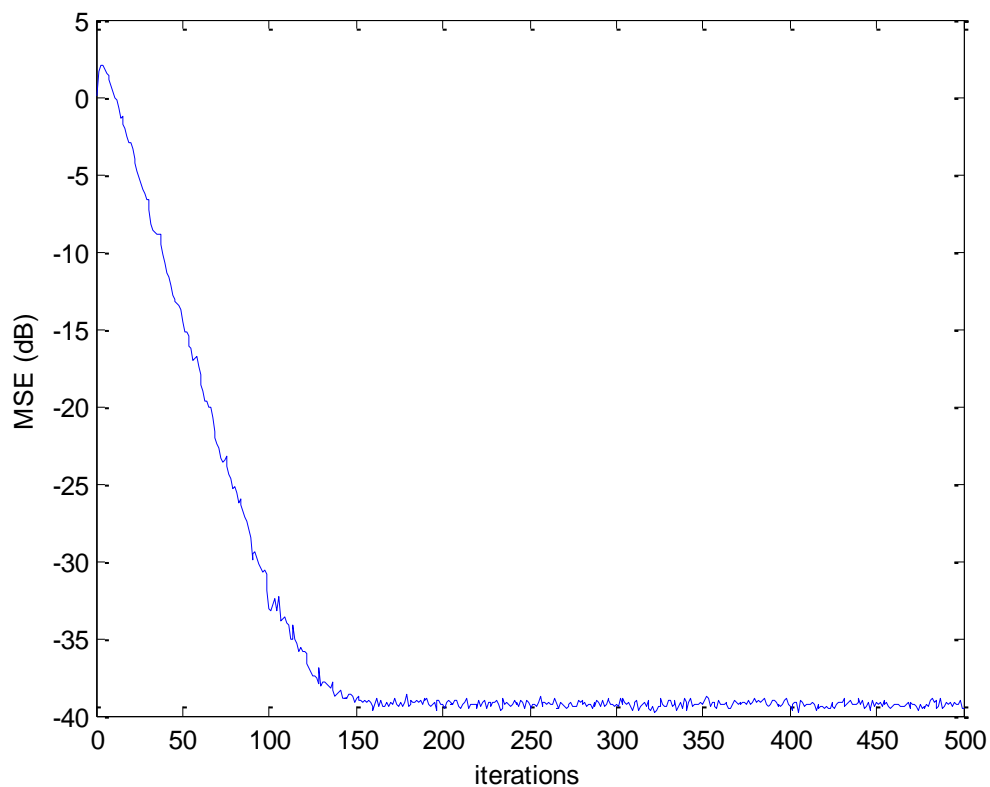


Figure 4.1 Error between desired signal and LMS output

4.2.2 Performance of RLS Algorithm

4.2.2.1 Error Between Desired Signal and RLS Output

The figure 4.2 is the number of iterations versus square error plot ,a simulation run of 500 iterations with parameters summarized in TABLE 4.1 for RLS algorithm to show the Error between desired signal and RLS output for linear array of $N = 8$ elements , From the figure we notice that in RLS algorithm the error function value is reduced quickly by increasing the number iterations and it provides fastest convergence rate and lowest MSE compared to the LMS algorithm.

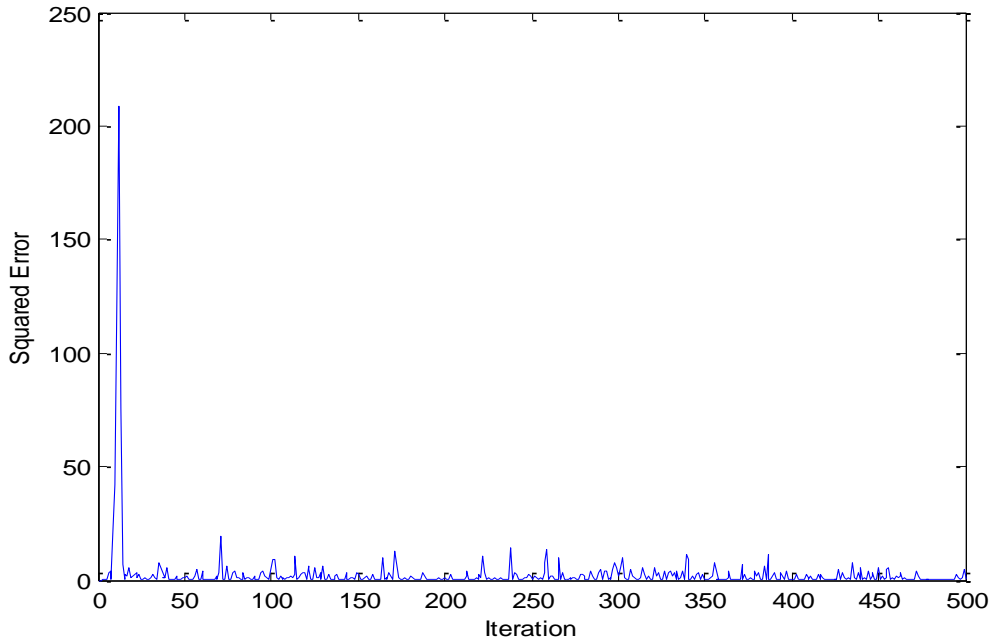


Figure 4.2 Error between desired signal and RLS output

To compare the ability of algorithms to give maximum gain in the direction of source signal and placing the null in the direction of interference signal, simulations were performed using least mean square (LMS) and recursive least square (RLS) adaptive algorithms for 500 iterations According to the results above in figures 4.1

and 4.2, we used the RLS algorithm to evaluate the performance of using beamforming to mitigate the interference in MIMO systems because it has better performance than LMS.

4.2.2.2 MIMO Capacity

In figure 4.3 we compare the average capacities for single-input single-output (SISO) and MIMO with 4 transmitting antennas and 4 receiving antennas systems. It is evident that MIMO systems offer far greater capacity than SISO over a wide range of SNR. Also, the capacity of MIMO system increases with increasing number of SNR.

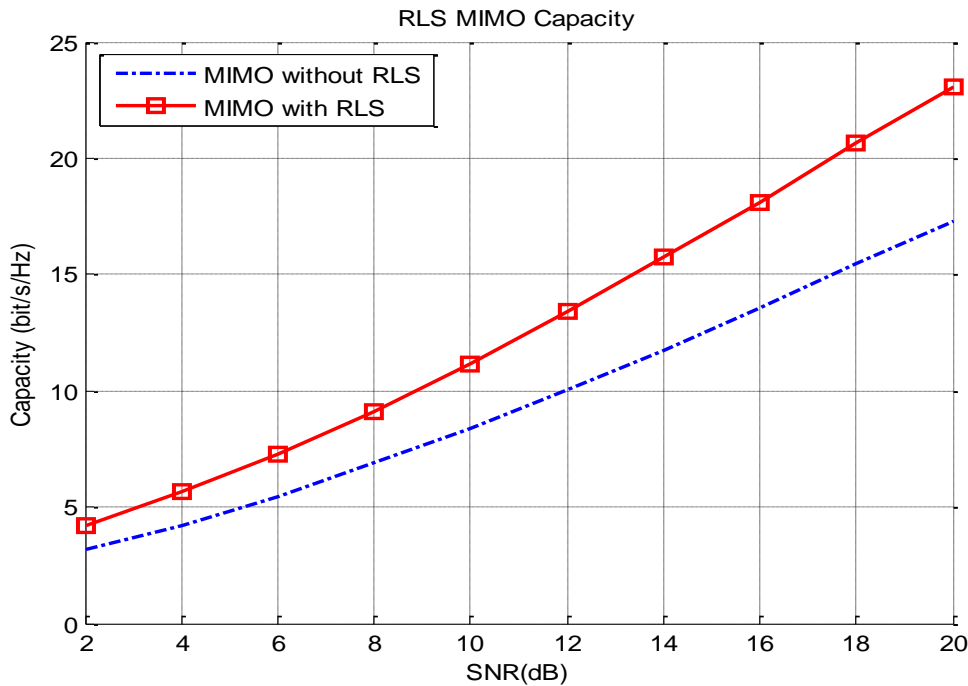


Figure 4.3: RLS MIMO capacity

4.2.2.3 BER vs SNR With and Without RLS

The BER with respect to signal to noise ratio (SNR) in MIMO system with and without RLS is plotted using Matlab in figure 4.4 with 4 transmitting antennas and

4 receiving antennas . the bit error rate is goes on reducing as the signal to noise ratio is from 10 to 30 d B, as shown in figure 4.4 in the case of MIMO with RLS the BER is reducing quickly rather than using MIMO without RLS giving better performance.

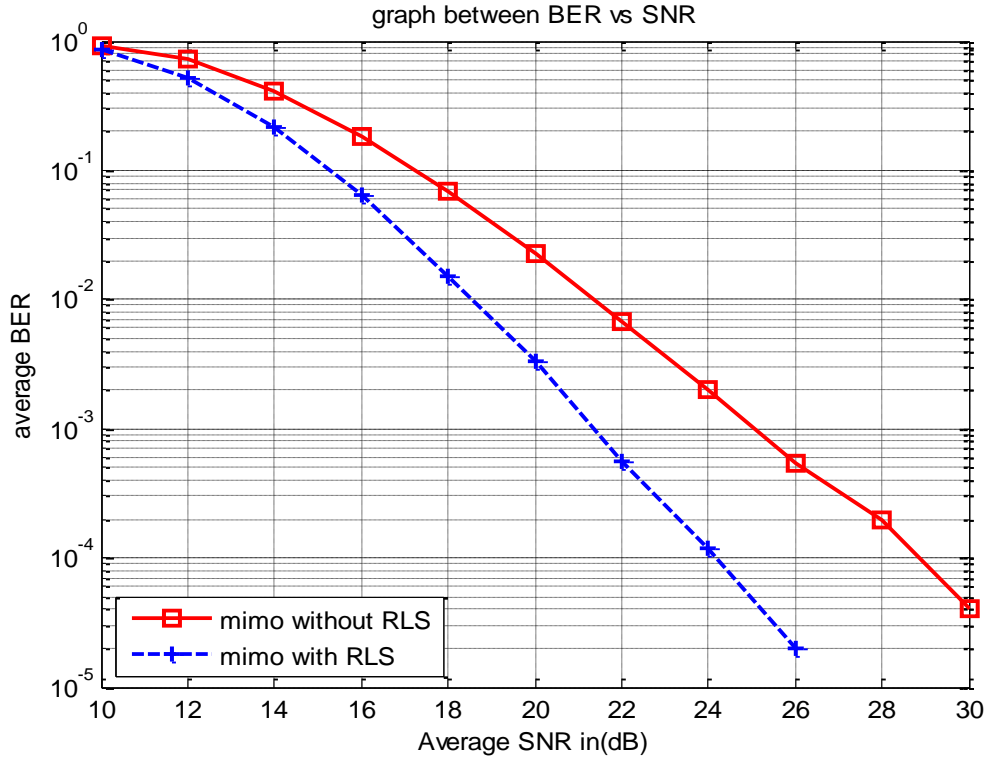


figure 4.4 : graph between BER and SNR with and without RLS

4.2.2.4 BER vs SNR for Different No. of MIMO Elements

The figure 4.5 is BER versus SNR in MIMO system plot with and without RLS using different No. of antenna elements with 4x4 and 8x8 transmitting and receiving antennas. The signal to noise ratio is from 10 to 30 d B, in all cases the bit error rate is decreasing with the increasing of SNR . The simulation result shows that there is an improvement in BER when we use the RLS and as the antenna element goes on increasing from 4 to 8 , the BER performance increase.

It is evident that an increasing number of antennas improves the system performance .

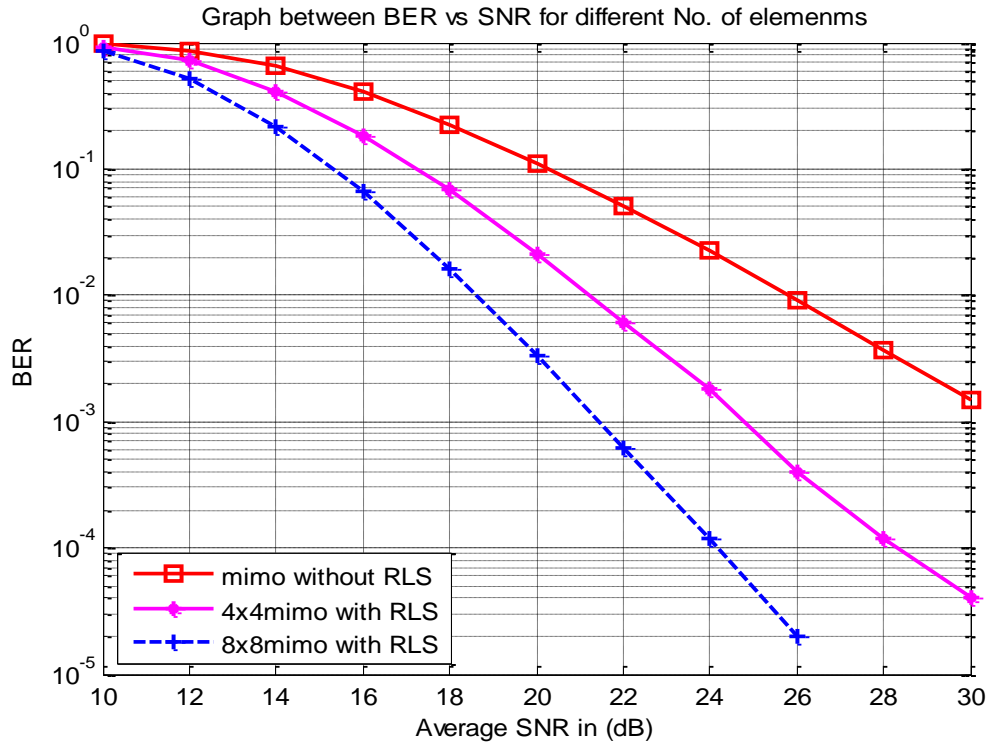


Figure 4.5: graph between BER and SNR for different No. of elements

4.2.2.5 BER vs SINR

The figure 4.6 is a graph plotted between Bit Error Rate (BER) and Signal to interference and Noise Ratio (SINR) for MIMO system with and without RLS algorithm, from the figure we notice that the bit error rate is reducing very quickly and when the SINR reaches 20 db the BER about zero, it is clearly there is improvement when applying RLS algorithm.

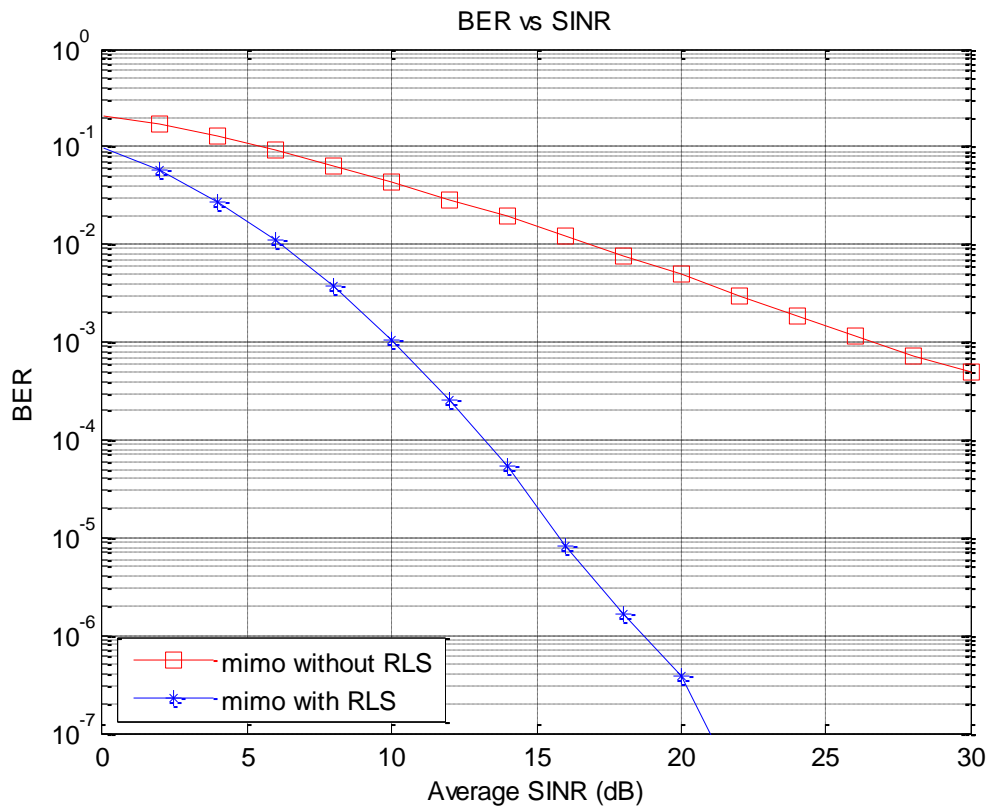


Figure 4.6: BER vs. SINR

Chapter Five

Conclusion and recommendations

5.1 Conclusion

In this thesis work the problem of interference existing in MU-MIMO is addressed and inferred that it greatly reduces the system performance , adaptive Beamforming algorithms for smart antenna system technique is introduced to mitigate the interference problem , Particularly the two algorithms Least Mean Square (LMS) and Recursive Least Square (RLS) adaptive beamforming Algorithms has been investigated and applied to the MIMO system and the performance was analyzed through a comparison between the two algorithms using MATLAB , from the simulations and analyzing the graphs we observed that both LMS and RLS has better capability to form beam towards the user of interest.

Also we notice that LMS has less speed of convergence, less computational cost compared with RLS which has fast convergence rate and a computation complexity in the order of square of M , where M is the number of taps or in this case, the number of elements. the RLS algorithm has much better performance than LMS algorithm.

Using the RLS algorithm we evaluated the performance of using beamforming to mitigate the interference in MIMO systems , first we evaluate the MIMO capacity with and without RLS , the performance in term of BER for different number of antenna elements and BER with respect to SINR are also implemented.

We notice that there is improvement in system capacity when we apply RLS algorithm, The simulation result shows that there is an improvement in the BER

when we use the RLS and as the antenna element goes on increasing from 4 to 8, the BER performance increase, also the result of plotting BER vs. SINR shows that there is an improvement when the RLS algorithm is applied.

5.2 Recommendations

we have investigated the adaptive algorithm performance and the results presented in this work are useful for smart antenna system, there are still some issues that require further research for further improvement like:

- In this thesis uniform linear arrays are considered. Further work can be extended for other array geometries such as: planar and circular array
- The coupling effect in this work is not considered. Thus the work can be extended by including coupling effect.
- In this work, when the angle difference is 0 degree, it is very challenging to differentiate the desire and the interferer signal. Thus, the work can be extended to alleviate such type of problem either by modify the available algorithm or introduce the new one.
- In modern cellular satellite mobile communications systems and in global positioning systems (GPS's), both desired and interfering signals change their directions continuously. Therefore, a fast tracking system is needed for the computation of the optimum weights.

References

- [1] A. Suban ,V. S. Priyanka , and S. Atchaya .” Interference Mitigation in LTE-Advanced MU-MIMO through Beamforming Technique”. *International Journal of Advanced Research in Computer Engineering & Technology (IJARCET)*, Vol.2, pp.1-4, Feb. Feb 2013.
- [2] Sherief Hashima, Hossam Shalaby, Masoud Alghoniemy, and Osamu Muta . “Performance Analysis of Fractional Frequency Reuse Based on Worst Case Signal to Interference Ratio in OFDMA Downlink Systems,” *IEEE 24th International Symposium on Personal, Indoor and Mobile Radio Communications: Fundamentals and PHY Track*, UK, 2013.
- [3] Saeed Daneshmand. “GNSS Interference Mitigation Using Antenna Array Processing.” PHD Thesis, University of Calgary, Canada, 2013.
- [4] Dimitrios Biliou, Christos Bouras, Vasileios Kokkinos, Andreas Papazois, and Georgia Tseliou. “A Performance Study of Fractional Frequency Reuse in OFDMA Networks.” *IEEE Wireless and Mobile Networking Conference*, Slovakia, Sept 2012.
- [5] Shankar Ram. “A study of Adaptive Beamforming Techniques Using Smart Antenna for Mobile Communication.” Master of Technology , National Institute of Technology, Rourkela, 2007.

- [6] Subhankar Chakrabarti. "Study the Effect of Co-Channel Interference in STC MIMO-OFDM System and Mitigation of CCI using Beamforming Technique." Master of Technology , National Institute of Technology, India,2013.
- [7]Thomas Hunziker and Makoto Taromaru. "An Adaptive Beamforming Scheme for Enhanced Cochannel Interference Mitigation on Short Array Signal Intervals".IEEE International Conference on Acoustics Speech and Signal Processings, Kyoto, Japan, 2006.
- [8] L. Surendra, Syed. Shameem, Dr. Habibullah Khan." Performance Comparison of LMS, SMI and RLS Adaptive Beamforming Algorithms for Smart Antennas." *International Journal of Computer Science And Technology*, vol.3,pp. 2-5, Jun.2012.
- [9] brain settler, "Maximizing LTE performance through MIMO optimization." USA. Patent 20876, April 2011.
- [10] David Mazzaresse and Bruno Clerckx, "MIMO Techniques in WiMAX and LTE: A Feature Overview," *IEEE Communications Magazine*, vol.48, pp.86-92, May 2010.
- [11]Rohde & Schwarz, Introduction to MIMO, Application Note 1MA142_0e, Jul 2009.
- [12] Comisso, Massimiliano,"Beamforming Techniques for Wireless Communications in Low-Rank Channels: Analytical Models and Synthesis Algorithms", PHD Tthesis, Università degli Studi di Trieste,Italia,2008.

[13] Tshiteya Dikamba “ Downlink Scheduling in 3GPP Long Term Evolution (LTE)” Master of Science Thesis, Delft university of technology, Netherlands,2011.

[14] Debashis Panigrahi, Abhinav Garg & Ravis. Verma”A study of Beamforming Techniques and Their Blind Approach.” Bachelor degree thesis, National Institute of Technology, Rourkela, 2007.

[15] S.Ranjitha Muthiraj *.”Smart Antenna.”Internet:
<https://www.slideshare.net/ranjithamudhiraj/smart> , Oct 23, 2013*[Nov. 29, 2016].

APPENDEX

LMS CODE

```
clear all
I = 1000;
K = 500;
sigmax = 1; % SNR = sigmax.^2/sigman.^2;
Wo = [1 0.8 0.5 0.4 0.3 0.1]';
sigman = 0.01;
mu = 0.05;
% length of the plant/system
L=length(Wo);
% order of the plant/system
N=L-1;
% MSE vector
MSE=zeros(K,1);
% Minimum MSE vector
MSEmin=zeros(K,1);
for i=1:I,
    X=zeros(L,1);
    W=zeros(L,1);
    % input
    x=randn(K,1)*sigmax;
    % noise
    n=randn(K,1)*sigman;
    for k=1:K,
        X=[x(k)
            X(1:N)];
        % desired signal
        d=Wo'*X;
        % output estimate
        y=W'*X;
        % error signal
        e=d+n(k)-y;
        % new/updated filter
        W=W+mu*conj(e)*X;
        % accumulation of MSE
        MSE(k)=MSE(k)+e^2;
        % accumulation of MSE
        MSEmin(k)=MSEmin(k)+(n(k))^2;
    end
end
end
% sample index
ind=0:(K-1);
MSE = MSE/I;
MSEmin = MSEmin/I;
% Misadjustment computation
M=MSE./MSEmin-1;
plot(10*log10(MSE));
xlabel('iterations');
ylabel('MSE (dB)');
```

RLS code

```
clear all; % clear memory
% Input:
Nr = 100;
dim = 5e2;
Sx = 1;
Sn = 1e-1;
lambda = 0.98;

% Body:
for j=1:Nr
    n=Sn*randn(dim,1); % noise at system output
    x=Sx*randn(dim,1); % input signal
    x11=zeros(dim,1); x12=x11;
    x11(2:dim)=x(1:dim-1); x12(3:dim)=x(1:dim-2);
    d=zeros(dim,1);
    d=-.76*x-x11+x12+.5*x.^2+2*x.*x12-1.6*x11.^2+1.2*x12.^2+.8*x11.*x12+n; ...
    % unknown system output
    w=zeros(9,dim); % initial coefficient vector
    ux1=[x x11 x12 x.^2 x.*x11 x.*x12 x11.^2 x11.*x12 x12.^2]'; % input vectors

    Sd=eye(9);
    for i=1:dim
        elinha(i)=d(i)-w(:,i)'*ux1(:,i)+n(i); % error sample
        psi=Sd*ux1(:,i);
        Sd=(1/lambda)*(Sd-(psi*psi')/(lambda+psi'*ux1(:,i)));
        w(:,i+1)=w(:,i)+elinha(i)*Sd*ux1(:,i); % new coefficient vector
        y(i)=w(:,i+1)'*ux1(:,i); % output sample
        e(i)=d(i)-y(i)+n(i);
    end
    mse(j,:)=e.^2;
end

MSE=mean(mse);

% Output:
figure,
plot(10*log10(MSE));
title(' MSE Curve ');
xlabel('iterations'); ylabel('Square Error');
```

RLS mimo capacity

```
clc

%Shannon capacity

snr=0;

for i = 1:10
```



```

snr = snr +2;
c=(log(1+10^(snr/10)))/log(2);
x(i)=snr;
y(i)=c;
end
figure
hold on
% MIMO without RLS
NR=3;
rand('state',456321)
snr=0;
for i=1:10;
snr=snr+2;
for j=1:10000;
c(j)=(NR*log(1+(10^(snr/10))*abs(normrnd(0,1))))/log(2);
end
yy(i)=mean(c);
xx(i)=snr;
end
plot(xx,yy,'-.','LineWidth',1.5)
hold on
% MIMO with RLS
NR=4;
rand('state',456321)
snr=0;
for i=1:10;
snr=snr+2;

```

```

for j=1:10000;

c(j)=(NR*log(1+(10^(snr/10))*abs(normrnd(0,1)))/log(2));

end

yy(i)=mean(c);

xx(i)=snr;

end

plot(xx,yy,'r-s','LineWidth',1.5)

xlabel('SNR(dB)')

ylabel('Capacity (bit/s/Hz)')

grid on

legend('MIMO without RLS','MIMO with RLS' ,2)

title('RLS MIMO Capacity')

```

BER vs. SNR for DIFF NO.OF Elements

```

clear all

It = 50000;
fprintf('Total number of channel realizations: %d\n', It);

% Number of transmit and receive antennas
M=4;

% SNR range in dB
SNRdBvalues = [10:2:30];

% Selected transmission rate: 2, 4 and 6 bits per transmission
RR = 4;

% initialize variables = number of channel realizations for which the
% selected rate exceeds the maximal achievable rate
Nsic1 = zeros(1,length(SNRdBvalues));
Nsic2 = zeros(1,length(SNRdBvalues));
Nsic3 = zeros(1,length(SNRdBvalues));
Nsic4 = zeros(1,length(SNRdBvalues));

for kk=1:It

    if mod(kk,10000) == 0
        fprintf('Number of channel realizations: %d\n', kk);
    end
end

```

```

end

% generate channel realization
H = (randn(M)+j*randn(M))/sqrt(2);

SNRidx = 0;
for SNRdB = SNRdBvalues

    SNR = 10.^(SNRdB./10); % linear scale
    SNRidx = SNRidx + 1;

    H2 = H(:,2:M);
    Dmmse = inv(H2'*H2 + eye(M-1)/SNR*M);
    SNRmmse = (1./diag(Dmmse))*SNR/M -ones(M-1,1);
    Cmmse = (log2( 1 + SNRmmse(1)));
    Nsic2(SNRidx) = Nsic2(SNRidx) + .5*(1-sign(Cmmse-RR));

    H3 = H(:,3:M);
    Dmmse = inv(H3'*H3 + eye(M-2)/SNR*M);
    SNRmmse = (1./diag(Dmmse))*SNR/M -ones(M-2,1);
    Cmmse = (log2( 1 + SNRmmse(1)));
    Nsic3(SNRidx) = Nsic3(SNRidx) + .5*(1-sign(Cmmse-RR));

    H4 = H(:,4:M);
    Dmmse = inv(H4'*H4 + eye(M-3)/SNR*M);
    SNRmmse = (1./diag(Dmmse))*SNR/M -ones(M-3,1);
    Cmmse = (log2( 1 + SNRmmse(1)));
    Nsic4(SNRidx) = Nsic4(SNRidx) + .5*(1-sign(Cmmse-RR));

end

end

Psic1 = Nsic1/It
Psic2 = Nsic2/It
Psic3 = Nsic3/It
Psic4 = Nsic4/It

% plot
figure(1)
semilogy(SNRdBvalues, Psic2, 'rs-','LineWidth',2)
hold on
semilogy(SNRdBvalues, Psic3, 'm-*','LineWidth',2)
hold on
semilogy(SNRdBvalues, Psic4, 'b--+', 'LineWidth',2)
title('Graph between BER vs SNR for different No. of elemenms')
xlabel('Average SNR in (dB)')
ylabel('BER')
legend('mimo without RLS','4x4mimo with RLS','8x8mimo with RLS',3)
grid
hold off

```

BER & SINR

```
clear all

It = 50000;

% SNR range in dB
SNRdBvalues = [0:2:30];

% variable initialization: BER for each channel iteration
PPostbc      = zeros(1,It);
PPqostbc     = zeros(1,It);
PPsiso       = zeros(1,It);

SNRidx = 0;
for SNRdB=SNRdBvalues
    SNRdB
    SNRidx = SNRidx + 1;    % SNR index
    SNR = 10^(SNRdB/10);   % SNR in linear scale

    % iterations on channel realizations
    for kk=1:It

        % generate channels
        H = (randn(1,4) + j * randn(1,4)) / sqrt(2);    % 1x4 MIMO
channel
        h1 = H(1,1); h2 = H(1,2); h3 = H(1,3); h4= H(1,4); % subchannels

        % equivalent channel
        HH_o = [h1 h2 h3 h4;
                h2 -h1 h4 -h3;
                h3 -h4 -h1 h2;
                h4 h3 -h2 -h1;
                h1' h2' h3' h4';
                h2' -h1' h4' -h3';
                h3' -h4' -h1' h2';
                h4' h3' -h2' -h1'];

        % computation of post-processing SNR
        R = inv(HH_o'*HH_o);
        SNRostbc = SNR/4/R(1,1);
        % BER for uncoded QPSK input
        PPostbc(kk) = 0.5*erfc(sqrt(0.5*SNRostbc));

        % equivalent channel
        HH_qo = [h1 -h2 -h3 h4;
                 h2' h1' -h4' -h3';
                 h3' -h4' h1' -h2';
                 h4 h3 h2 h1];

        % computation of post-processing SNR
        R = inv(HH_qo'*HH_qo );
```

```

SNRqostbc = SNR/4/R(1,1);
% BER for uncoded QPSK input
PPqostbc(kk) = 0.5*erfc(sqrt(0.5*SNRqostbc));

SNRsiso = SNR*norm(h1)^2;
% BER for uncoded QPSK input
PPsiso(kk) = 0.5*erfc(sqrt(0.5*SNRsiso));

end

% average over channel realizations
Postbc(SNRidx) = mean(PPostbc)
Pqostbc(SNRidx) = mean(PPqostbc)
Psiso(SNRidx) = mean(PPsiso)

end

figure(1)

semilogy(SNRdBvalues, Pqostbc, 'r-s', 'linewidth', 2)
hold on

semilogy(SNRdBvalues, Postbc, 'b-*', 'linewidth', 2)
axis([SNRdBvalues(1) SNRdBvalues(end) 10e-8 1])
title('BER vs SINR')
xlabel('Average SINR (dB)')
ylabel('BER')
legend('mimo without RLS', 'mimo with RLS', 3)
grid
hold off

```