



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



SUDAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
COLLEGE OF GRADUATE STUDIES

**A COMARATIVE STUDY OF ELLIPSOIDAL AND MEAN SEA LEVEL
HEIGHTS**

دراسة مقارنة بين الإرتفاعات المنسوبة لسطح البحر والجيوديسية

A thesis submitted for partial fulfillment for the requirement of the admission of
the degree of M.Sc. in Geodesy and GIS

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

وَأَلْقَى فِي الْأَرْضِ رَوَاسِي أَنْ تَمِيدَ بِكُمْ وَأَنْهَارًا وَسُبُلًا
لَعَلَّكُمْ تَهْتَدُونَ (15) وَمَلَامَاتٍ وَبِالنَّجْمِ هُمْ يَهْتَدُونَ (16)

صِدْقَ اللَّهِ الْعَظِيمِ

الآيتان (15) و (16) من سورة النحل

Dedication

To my mother...

To my father...

To my mothers...

To my uncles...

To my brothers...

To my friends and colleagues...

To the staff of surveying department...

Abstract

This research aims to compare between heights that measured relative to the mean sea level (Orthometric Heights) and that ones related to the ellipsoid (Ellipsoidal Heights).

By observing 11 points using both methods, the comparison was carried out, it found that the RMSE is 0.023m, and to convert between method to another must add or subtract 1.635m in the study area.

المستخلص

يهدف هذا البحث الي المقارنة بين الإرتفاعات المنسوبة الي متوسط سطح البحر (الإرتفاعات الأورثومتريّة Orthometric Heights) وتلك المأخوذة من الإلييسويد (الإرتفاعات الجيوديسية Ellipsoidal Heights).
تم رصد 11 نقطة (منطقة الشجرة) بالطريقتين وبعد إجراء المقارنة وُجد أن قيمة RMSE تساوي (0.023m) و للتحويل بين الإرتفاعات بالطريقتين يمكن إضافة أو طرح (1.635m) في منطقة الدراسة.

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Chapter One

Introduction

1.1 Overview

The observed differences in elevation between points on the Earth's surface are traditionally obtained using spirit-leveling (and/or its variants such as astrigometric, barometric Leveling, etc.). This is usually referred to as vertical control. Recent advances in Global Positioning System (GPS) Technologies during the last few years and current fully operational status for the Navistar GPS have made it possible to accurately measure ellipsoidal height differences from the observable of the GPS satellites.

GPS Gives three dimensional coordinates either Cartesian coordinates (X, Y, Z) or geodetic coordinates (Φ, λ, h) in a suitable reference.

Thus, height is transferred from a point of known height to another unknown point using fly levelling method. This process is very costly, labor intensive and time consuming. The levelling process need to be done by at least two personnel. In certain situations like a place that hard to reach, as for example forest, hills, and mines areas, levelling methods can be highly dangerous and tedious. With the development of GPS technology and processing techniques, GPS is now commonly used as a tool for horizontal positioning among surveyors. It is also known that, GPS could also be used in height determination. Direct height measurement using GPS derived ellipsoidal heights (h) which refer to ellipsoid surface could be carried out with ease.

Establishment of vertical datum for land related surveying works using conventional levelling technique is a tedious process. With the advent of GPS (Global Positioning System) technology, determination of heights value for vertical datum in engineering works and alike seems feasible.

1.2 Statement of The Problem

In this study, two approaches in height determination using GPS are explored. The two technique involve are the absolute and relative heighting. In the absolute technique, the ellipsoidal heights obtained through GPS observation for the observed. In this technique, only one GPS is needed for the observation. In the other approach, the traditional levelling is the measurement of orthometric height using an optical levelling instrument and a staff to obtain height of unknown points relative to known point (BM).

1.3 Research Objectives

This research aims to

- Compare between the absolute heights that obtained from GPS and other heights from traditional levelling.
- To determine the value of difference between the two methods to be used in the transformation.

1.4 Thesis Layout

This research contains six chapters. Chapter one is the introduction, chapter two covers the figure of the earth, while chapter three describes in details

the levelling and the concept of Mean Sea Level (MSL). Global positioning system is discussed in chapter four. Chapter five describes observation and testing in addition to analysis and results. Conclusion of this work and recommendations for future work are considered in chapter six.

Chapter Two

The Figure of the Earth

2.1 Introduction

Earth is a roughly spherical object with an irregular surface. One of the aims of geodesy is the representation of points on this irregular surface in some co-ordinate systems. One could use, for instance, a Cartesian system, where each point will simply have X, Y and Z co-ordinates referred to three reference axes. Alternatively, the position of a point could be defined by its spherical or spheroidal co-ordinates on a mathematical surface of revolution of known dimensions and orientation with respect to the earth or the position can be defined by its distance measured in orthometric or dynamic units from this surface.

The first scientific method of solving this problem was the spherical idea, in which the earth was regarded as a sphere and the position of each terrestrial point defined by its spherical co-ordinates, Φ and λ , and by its height above sea level. Later it was found that the radius of this sea level surface varied from point to point so that it could not be regarded as spherical. After a famous debate and two equally well known scientific expeditions, it was decided that the sea level surface of the earth took the form of an oblate ellipsoid of revolution.

It was then proposed that the position of a terrestrial point be represented by (Φ, λ, H) , called geodetic co-ordinates. Φ and λ are the spheroidal latitude and longitude of the corresponding point (projected along the normal) on the spheroid, and H is the height of the terrain point above the spheroid, measured along the normal. The necessity to be familiar with the geometry of the spheroid became obvious in order to compute distances, azimuths, etc.

It should be mentioned firstly, however, that the sea level surface is not, in fact, a spheroid as once thought, but is a very irregular surface, called the geoid. One problem of geodesy has been the selection of the spheroid which best fits this irregular geoidal surface.

2.2 Shape of the Earth

The search for the size and the shape of the Earth has a long and interesting history. Although today there is no problem in viewing the earth as an approximately spherical body, this situation does not always exist.

People have been proposing theories about the shape and size of the planet for a couple of thousand years. Early recorded thoughts indicated (e.g. Homer 9th century B.C.) that the earth was flat disk supporting a hemispherical sky. With this view there would be only one horizon with the time and length of day being independent of location.

In the sixth century B.C. Pythagoras suggested that the earth was spherical in shape. This suggestion was made on the basis that a sphere was considered a perfect form and not by deduction from observations. Finally in the fourth century B.C. Aristotle gave arguments that would support the hypothesis that the earth must be spherical in shape. Some specific reasons that were mentioned include:

- The changing horizon as one travels in various directions.
- The round shadow of the earth that was observed in lunar eclipses.
- The observations of a ship at sea where more (or less) of the ship is seen as the ship approaches.

The next developments are now related to the determination of the size of the spherical earth. Although some determinations may have been made before, the first attempt at a precise determination (for the time) is ascribed to Eratosthenes of Egypt. In 230 B.C. Eratosthenes carried out his famous experiment to determine the size of the spherical earth. To do this he made

observations at two cities in Egypt, Alexandria and Syene. The Eratosthenes type experiment was repeated by Ptolemy in the first century B.C.

In the next few centuries little work was done in studies related to the figure of the Earth. In the 17th century, Snellius carried out measurements along a meridian in the Netherlands. For the first time for these purposes he used a triangulation procedure measuring angles with one-minute precision. Combining this measurement with astronomic latitude made at the endpoints of the meridian arc, Snellius determined the size of the spherical Earth using the basic method of Eratosthenes.

The real breakthrough came in 1687 when Sir Isaac Newton suggested that the Earth shape was ellipsoidal in the first edition of his Principia. Years earlier astronomer Jean Richer found the closer he got to the equator, the more he had to shorten the pendulum on his one-second clock. It swung more slowly in French Guiana than it did in Paris. When Newton heard about it, he speculated that the force of gravity was less in South America than in France. He explained the weaker gravity by the proposition that when it comes to the Earth the mass is simply more of it around the equator. He wrote, "The Earth is higher under the equator than at the poles, and that by an excess of about 17 miles". He was pretty close to being right; the actual distance is only about 4 miles less than he thought.

Some supported Newton's idea that the planet bulged around the equator and flattened at the poles, but others disagreed - including the director of the Paris Observatory, Jean Dominique Cassini. Even though he had seen the flattening of the poles of Jupiter in 1666, neither he nor his son Jacques were prepared to accept the same idea when it came to the Earth. It appeared that they had some evidence on their side.

For geometric verification of the Earth model, scientists had employed arc measurements since the early 1500s. First they would establish the latitude

of their beginning and ending points astronomically. Next they would measure north along a meridian and find the length of one degree of latitude along that longitudinal line. Early attempts assumed a spherical Earth and the results were used to estimate its radius by simple multiplication. In fact, one of the most accurate of the measurements of this type, begun in 1669 by the French abbé Jean Picard, was actually used by Newton in formulating his own law of gravitation. However, Cassini noted that close analysis of Picard's arc measurement, and others, seemed to show the length along a meridian through one degree of latitude actually decreased as it proceeded northward. If that was true, then the Earth was elongated at the poles, not flattened.

The argument was not resolved until Anders Celsius, a famous Swedish physicist on a visit to Paris, suggested two expeditions. One group, led by Moreau de Maupertuis, went to measure a meridian arc along the Tornio River near the Arctic Circle, latitude $66^{\circ} 20' N$, in Lapland. Another expedition went to what is now Ecuador, to measure a similar arc near the equator, latitude $01^{\circ} 31' S$. The Tornio expedition reported that one degree along the meridian in Lapland was 57,437.9 Toises, which is about 69.6 miles. A toise is approximately 6.4 ft. A degree along a meridian near Paris had been measured as 57,060 toises, or 69.1 miles. This shortening of the length of the arc was taken as proof that the Earth is flattened near the poles. Even though the measurements were wrong, the conclusion was correct. Maupertuis published a book on the work in 1738, the King of France gave Celsius a yearly pension of 1,000 livres, and Newton was proved right.

Since then there have been numerous meridian measurements all over the world, not to mention satellite observations, and it is now settled that the Earth most nearly resembles an oblate spheroid. An oblate spheroid is an ellipsoid of revolution. In other words, it is the solid generated when an

ellipse is rotated around its shorter axis and then flattened at its poles. The flattening is only about one part in 300. Still the ellipsoidal model, bulging at the equator and flattened at the poles, is the best representation of the general shape of the Earth.

2.3 Geodetic Surfaces

There are three geodetic surfaces which are, the earth surface, the geoid, and the ellipsoid.

2.3.1 The earth's surface

This is the topographic surface of the Earth. It is extremely uneven and not definable mathematically. It is approximately ellipsoidal in shape, the maximum departures from an ellipsoid being of the order of 8.5 km. The earth's surface is important because most of our observations are made on it and most of our points lie on it.

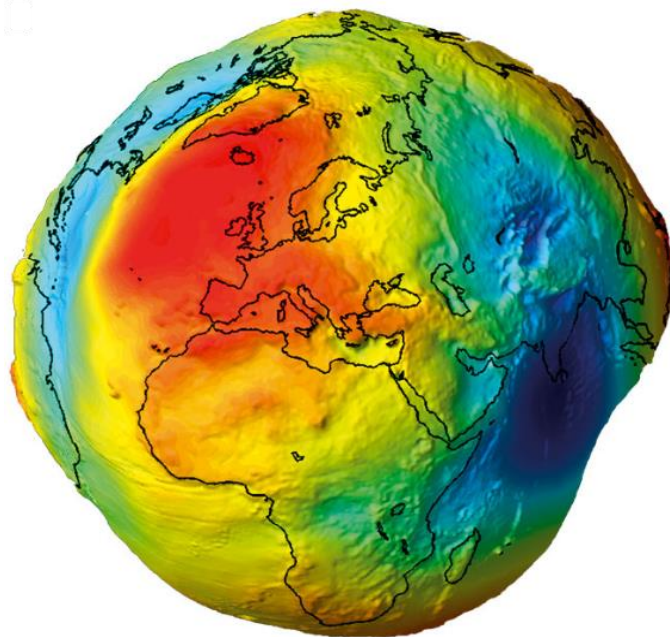


Figure (2.1)The Earth

2.3.2 The Geoid

The geoid is defined as the equipotential surface of the earth's attraction and rotation which coincides, on coverage, with mean sea level in the open

ocean. Another way of defining the geoid (not useful mathematically but nevertheless valuable) is to say that it is the surface which coincides with mean sea level assuming that the sea was free to flow under the land in small frictionless channels. To understand the geoid properly it is essential to understand the meaning of gravity, potential, and equipotential surfaces. It should be pointed out that mean sea level is not quite an equipotential surface owing to non-gravitational forces (such as ocean currents, winds and barometric pressure variations) - hence the use of the word average in the above definition.

The exact shape of the geoid is something which has to be measured. It turns out that the geoid is a fairly even surface. It is approximately ellipsoidal in shape; in fact, a best fitting ellipsoid could be placed in such a way that the maximum departures from that ellipsoid would be of the order of 110m.

The geoid is of fundamental importance in geodesy because many geodetic observations are related to it. For instance, all theodolites are set up with their primary axis along the local direction of gravity, which is perpendicular to the local equipotential surface and almost perpendicular to the geoid. Also it is common practice in land surveying to reduce to sea level (i.e. geoid level) all measured distances. Hence many of our observations, although observed on the earth's surface, are related to the geoid.

2.3.3 The Ellipsoid

An ellipsoid (or spheroid) is simply an ellipse rotated about its minor axis – Figure (2.2); mathematically the shape is an oblate spheroid. Ellipsoids are important in geodesy because they represent the nearest simple mathematical shape to the geoid.

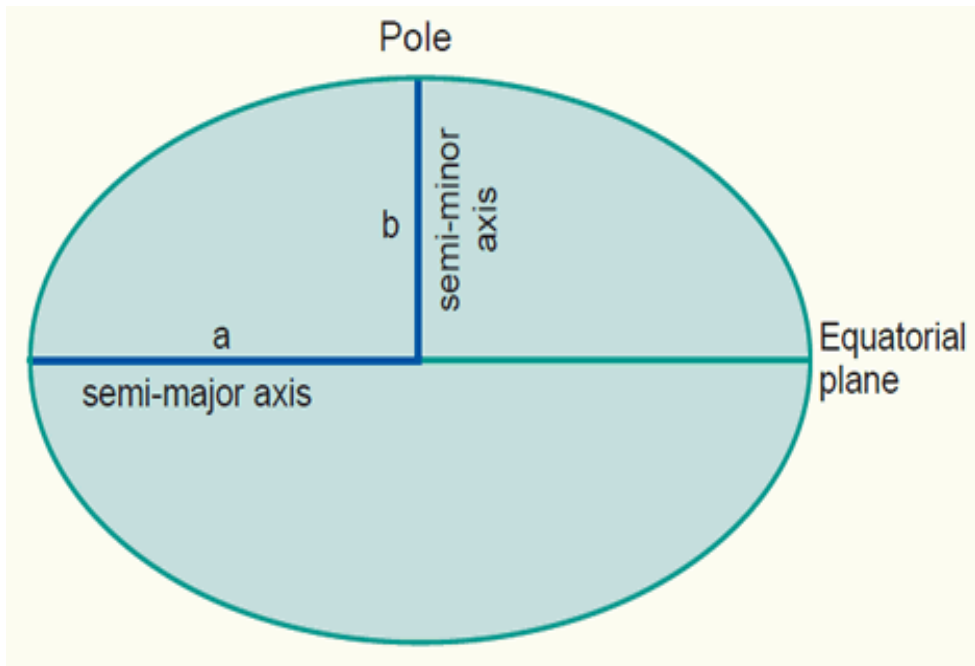


Figure (2.2)The ellipsoid

It should be noted that unlike the earth's surface and the geoid, an ellipsoid does not physically exist. Geodesists use ellipsoids as mathematical models for carrying out computations. There are many different ellipsoids, table (2.1) contains details of a selection of them. Also note that these ellipsoids can be placed in different positions in space.

Table (2.1) Details of some ellipsoids

| Ellipsoid | Semi-major axis (meters) | 1/Flattening |
|---------------------------------------|--------------------------|--------------|
| Airy 1830 | 6,377,563 | 299.33 |
| Everest 1830 | 6,377,276.3 | 300.80 |
| Bessel 1841 | 6,377,397.2 | 299.15 |
| Clarke 1866 | 6,378,206.4 | 294.98 |
| Clarke 1880 | 6,378,249.2 | 293.47 |
| International 1924 | 6,378,388 | 297 |
| Krasovsky 1940 | 6,378,245 | 298.3 |
| International Astronomical Union 1968 | 6,378,160 | 298.25 |
| WGS 72 (1972) | 6,378,135 | 298.26 |

The vertical at any point is the direction of gravity at that point. A normal at any point is the direction of the perpendicular line to an ellipsoid at that point. Note that at any point there are as many normal's as there are ellipsoids, i.e. a normal is not a line like the vertical, which is unique and physically exists.

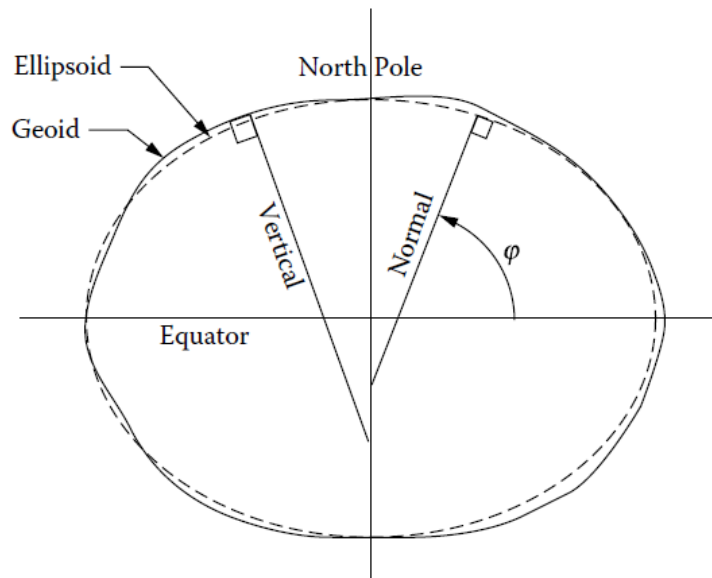
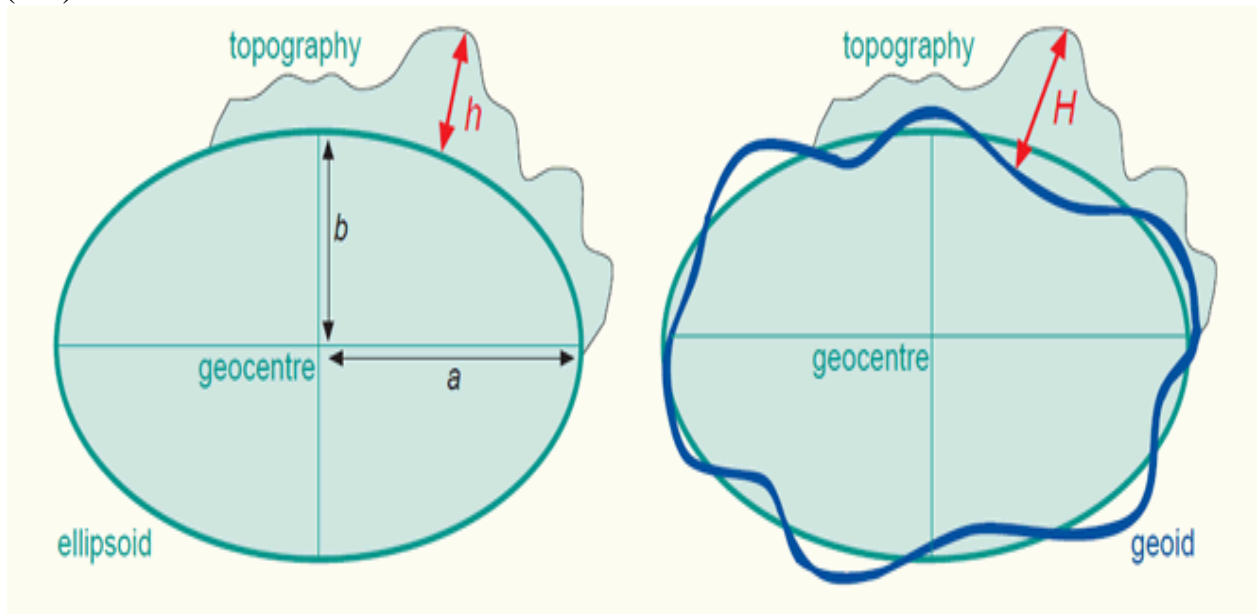


Figure (2.3)The normal and vertical

It should be noted that the height above the geoid is called orthometric height, and the one above the ellipsoid is called ellipsoidal height. Figure (2.4).



(a)

(b)

Figure (2.4) (a) Orthometric (b) ellipsoidal height

The separation between geoid and ellipsoid is called the geoid-ellipsoid separation or the geoid height - Figure (2.5).

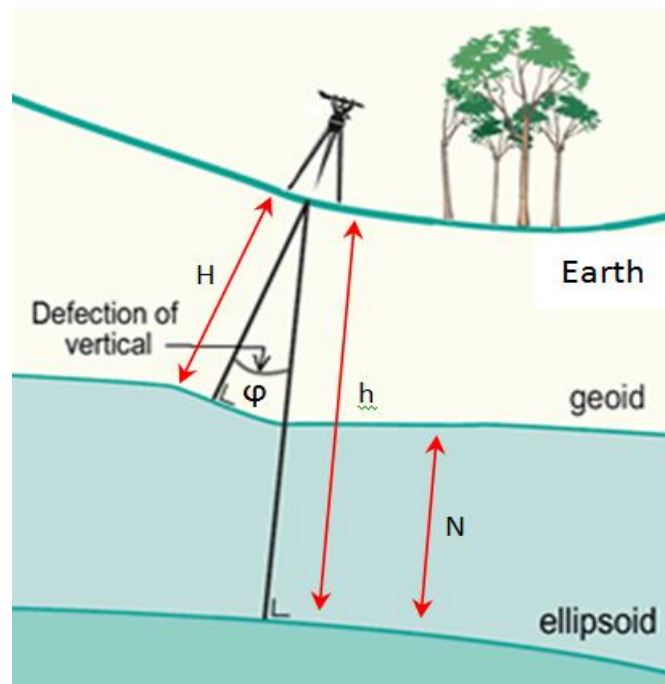


Figure (2.5)The geoid-ellipsoid separation

h is properly called the orthometric height of the point and is usually measured by spirit leveling (after making appropriate corrections for non-parallelism of equipotential surfaces). Hence:

$$H = N + h \cos\phi \dots\dots\dots(2.1)$$

But since, in practice, ϕ is always less than $60''$, $\cos\phi \approx 0.99999996$ and it can be written as:

$$H = N + h \dots\dots\dots(2.2)$$

With an error of less than 0.4mm at the highest point on the Earth.

Chapter Three

Mean Sea level (M.S.L)

3.1 Introduction

The concept of “height” is not simple. There are, for example, a number of different heights that can be defined; most of them are linked to the earth’s gravitational potential. Furthermore, there are a variety of different reference surfaces to which a height may refer. Actually there are two important elements necessary for the definition of height:

- i. The reference surface from which the height originates called vertical datum.
- ii. The physical or geometrical meaningful, measurement, from this reference surface to the point of interest in the earth surface.

As a result of the way these elements are selected, different systems of heights can be defined. The purpose of this chapter is to provide the necessary background information regarding the type of heights and terminology used throughout this thesis. In particular, the focus will be placed on describing the mean sea level(MSL), Orthometric height and ellipsoid height, how they are usually measured and the major error sources occurs that affects their measurement. Discussion will also provide insight into the Problems and challenges encountered when attempting the optimal combination of these Mixed height data.

3.2 Vertical datum definition

A datum (plural datum’s or data) is a reference surface from which measurements are made. In surveying and Geodesy a datum is a reference point on the Earth's surface against which position measurements are made,

and an associated model of the shape of the Earth for computing positions. Horizontal datum's are used for describing a point on the earth's surface, in latitude and longitude or another coordinate system. A vertical datum is defined as reference surface to which the vertical coordinate of point is referred. actually there are different local regional vertical datum's in use for example:

- i. Mean sea level (MSL) Originate for sea level.
- ii. Geoid is the datum for Orthometric height.
- iii. Ellipsoid datum for ellipsoidal height.

In Sudan for example our local official vertical datum is the MSL originates from Alexandria port in Egypt. Here it's important to mention that there is no official global vertical datum for height definition. Global vertical datum is needed for the following reasons:

- i. Accurate elevation models for flood mitigation.
- ii. Accurate elevation models for environmental hazards.
- iii. Enhance aircraft safety and aircraft landing.
- iv. Improve understanding of tectonic movement.
- v. Improve management of natural resources.

3.3 Mean Sea Level (M.S.L) Height

3.3.1 Definition

The general procedure for defining regional vertical datum's is to average sea level observations over approximately 19 years, or more precisely, 18.6 years, which corresponds to the longest tidal component period, for one or more fundamental tide gauge (Fotopoulos, 2003). This average sea level value is known as mean sea level (MSL) and is used as datum for leveling because it was assumed that the geoid and MSL coincided (more or less). This assumption is obviously false, as it is known today that the MSL and the geoid differ by approximately ± 2 meters (Klees and Van Gelderen,

1997). It should be noted that the mean sea level (MSL) is not equipotential surface.

3.3.2 Procedure of establishing M.S.L Datum (B.M)

Figure (3.1) depicts a typical scenario for the establishment of a reference benchmark to define a regional vertical datum. The tide gauge records the instantaneous sea level height HISL and these values are averaged over a long term in order to obtain the mean value of the local sea level MSL. The height of the tide gauge is also measured with respect to a reference benchmark that is situated on land a short distance from the tide gauge station.

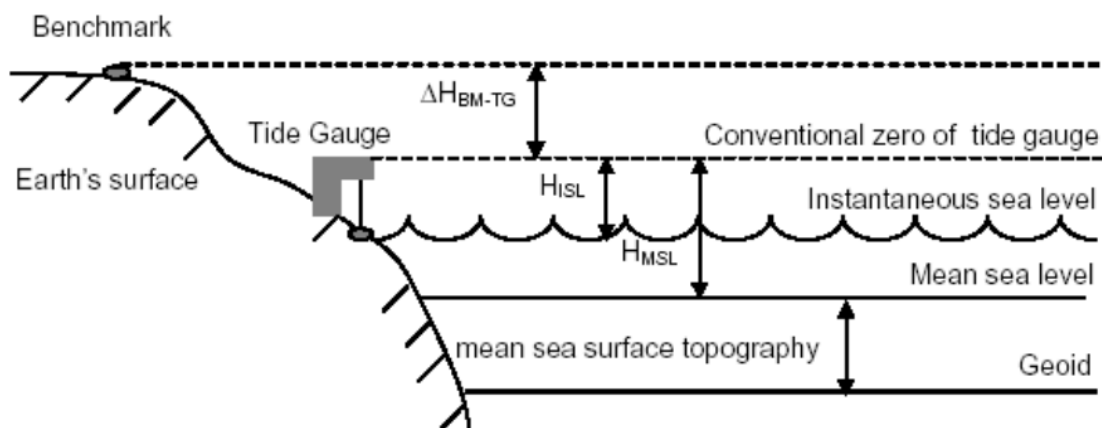


Figure (3.1) Establishment of a reference Benchmark height

Then the height of the reference benchmark above mean sea level H_{BM} is computed by:

$$H_{BM} = H_{MSL} + \Delta H_{BM} - TG \dots \dots \dots (3.1)$$

Leveling begins from this benchmark and reference heights are accumulated by measuring height differences along leveling lines. The accuracy of the reference benchmark height derived in this manner is dependent on the precision of the height difference H_{BM-TG} and the value for mean sea level

HMSL. If one assumes that the value for mean sea level is computed over a sufficiently long period of time which averages out all tidal period components and any higher frequency effects such as currents, then the accuracy depends on HBM-TG (Fotopoulos 2003).

For high accurate work, this tide gage and the nearby bench mark should be checked by GPS or any other geodetic technique. There are some comments in this procedure. In countries where there is no sea to be used as Datum, the used MSL definition has to be a borrowed one from the neighboring country or across many countries.

An accurate determination of MSL at any tidal observation station (TOS) would require observations taken regularly and without any interruption over a complete cycle of 18.67 years as a minimum. It is very interesting to note here, that this 18.67 years cycle condition may not have been met, in many of countries. In one case, it was discovered that a country used only one single observation of the sea surface as its MSL to define the national vertical datum.

As most of the TOSs are located (at least so far) along the coast in shallow waters, the MSLs so Determined at these locations do not represent the true mean sea level.

In case of vertical datum's, at least, every country has used the same surface as zero reference, MSL. However, the realized surface in such cases may not be the same and the selected zero may have many limitations.

Thus, national and regional vertical datum's around the world, which is locally tied to MSL, is significantly different from one another when considered on a global basis. In addition, due to the realization and orthometric height approximations of various vertical datum's, other departures at the meter level or more will be found when comparing

elevations to a global geoid reference. In their report (1990) the Intergovernmental Panel on Climate Change (IPCC) predicted sea level rises of 18cms by 2030 and of 66cms by 2100 this means that the tide gauge need to revised continuously.

It has become clear from different researches that, global sea levels are rising at a rate of approximately 1.8 mm/yr. Thus, the hope of forming a theoretically consistent, national height datum based upon the sea level measurements made at a number of tide gauges, each linked by precise leveling, is so difficult. Figure (3-2) below shows Sea level measurements from 23 long tide gauge records in geologically stable Environments show a rise of around 20 centimeters per century (2mm/year).Source)

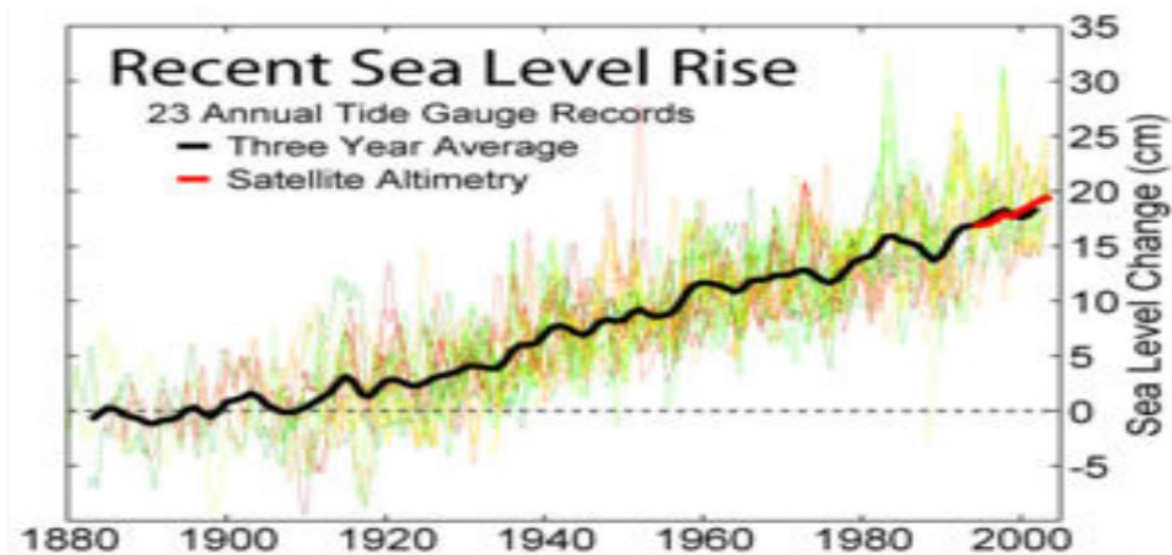


Figure (3-2) shows Sea level measurements

3.3.3 Traditional procedure of finding difference in Height (leveling)

Height differences between points on the Earth's surface have traditionally been obtained through terrestrial differential leveling methods, such as spirit-leveling (and/or barometric leveling, Trigonometric leveling, etc).

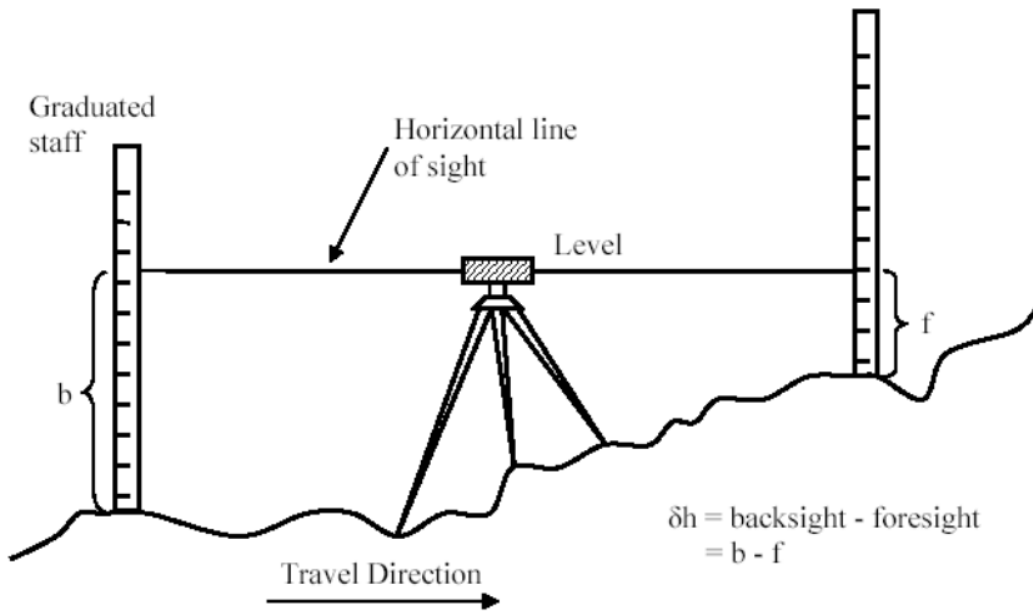
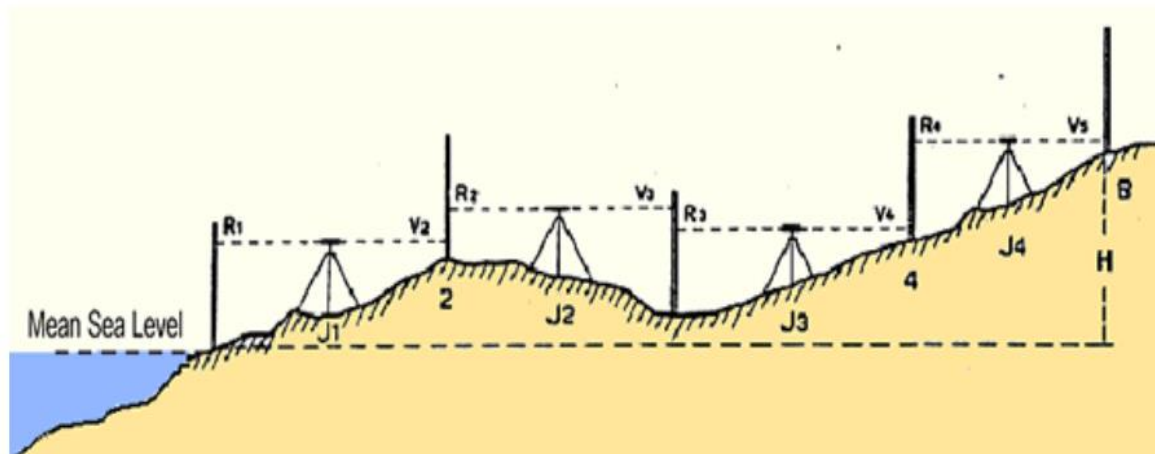


Figure (3.3) Typical setup for precise leveling



Figure(3.4) Geodetic Leveling Procedure Source

It should be noted here that:

- Sea level, does not exactly match the geoid because of the various physical factors such as sea-surface topography at the reference tide gauge.
- Corrected level from point A to point B taking different path may not give the same result due to non parallelism of equipotential surfaces.

3.4 Orthometric Height

Before defining the Orthometric Height, we need to define first, the Geoid, Ellipsoid, deflection of the vertical, and the Geopotential number, as the definition of orthometric height has relation with them.

3.4.1 Geoid

The geoid is an equipotential surface, defined in the Earth's gravity field, which best fits, in a least squares sense, global mean sea level. It should be noted that due to effects such as atmospheric pressure, temperature, prevailing winds and currents, MSL will depart from an equipotential surface by a meter or more. The geoid is a complex, physically-based surface, and can vary by up to 100 meters in height from a geocentric ellipsoid. The perpendicular distance from the geoid to the point on the earth surface is called the vertical.

3.4.2 Ellipsoid

Ellipsoid in geodesy is a mathematical figure formed by revolving an ellipse about its minor axis. It is often used interchangeably with spheroid. Two quantities define an ellipsoid, the length of the semi major axis, a , and the flattening, $f = (a - b)/a$, where b is the length of the semi minor axis. The perpendicular distance from the ellipsoid to the point on the earth surface is called the normal. The ellipsoid does not physically exist but it's mathematically derived. It is the widely used for horizontal co-ordinate systems. There are two type of reference ellipsoids:

- Geocentric ellipsoid, That means the center of the ellipsoid is coincide with the earth center, World Geodetic System (WGS), WGS 66, WGS 72, and WGS84 and Geodetic Reference System 1980 (GRS 80) ellipsoids are example of Earth- centered ellipsoids.
- Non geocentric ellipsoid, that means the Earth center and the ellipsoids are not coincide. Before World War II, all of the ellipsoids

were non-earth centered ellipsoids. Everest and Bessel ellipsoids are example of such type of ellipsoid.

3.4.3 Deflection of the vertical

The deflection of the vertical at any point of observation is the angle between the local vertical and the ellipsoidal normal and can be shown into two orthogonal components:

- The first component in the plane of the local meridian.
- The second component in the plane of the prime vertical.

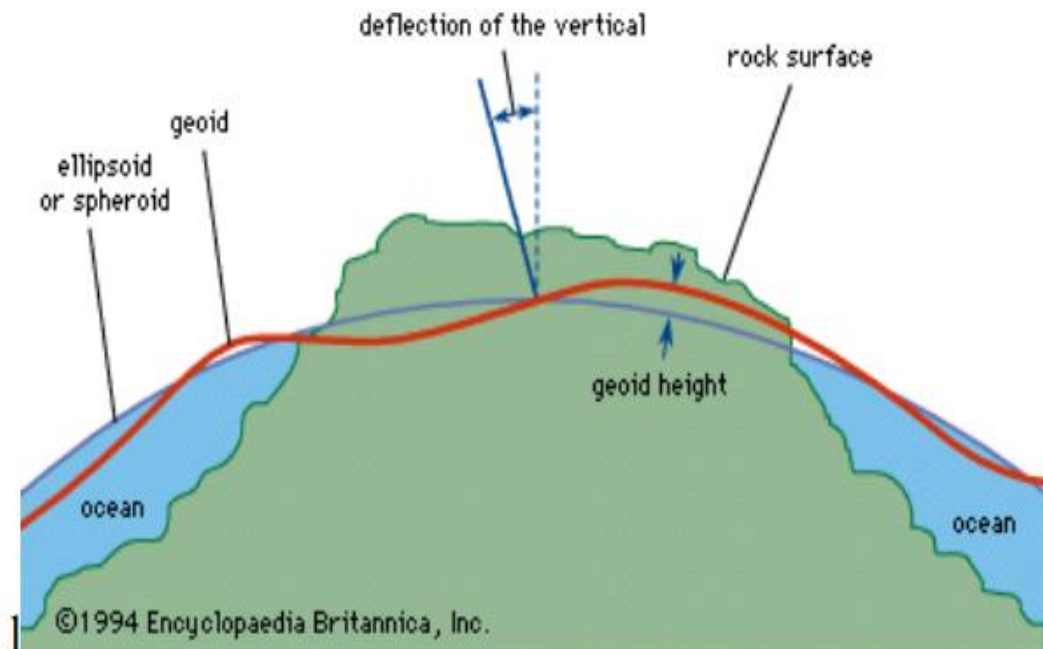


Figure (3.5) Shows in blue the vectors of the deflection of the vertical Source

3.4.4 Method of Measuring Orthometric Heights

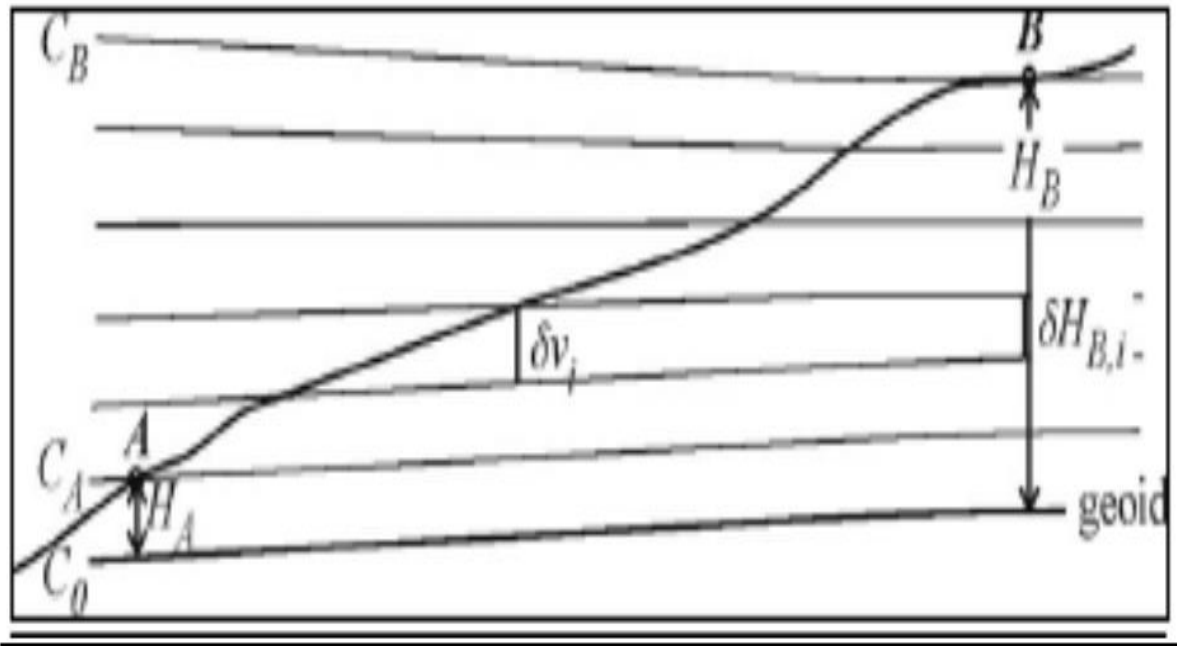


Figure (3.6) Orthometric height difference between two points A and B

It requires knowing a Geopotential number and the average acceleration of gravity along the plumb line but neither of these is measurable. So the orthometric height is measured practically by measuring the geometric height differences δv_i (by differential leveling) if the sum $\delta v_i = \Delta v_{AB}$ (Geometric height difference between station A and B) and if the Orthometric height difference between point A and B is ΔH_{AB} Then (see figure (3-7)).

$$\Delta H_{AB} = \Delta V_{AB} + OC_{AB} \dots \dots \dots (3.2)$$

Where OC is the orthometric correction.

3.4.4.1 Dynamic Height

Dynamic height is not geometrically meaningful, but it's physically meaningful. Measuring dynamic heights is accomplished in a manner similar to that for orthometric heights: geometric height differences observed by differential leveling are added to a correction term that accounts for gravity

3.4.4.2 Normal height

Is defined as vertical distance measured from terrain surface to the ellipsoid measured along the ellipsoidal normal reduced by the height anomaly.

From its definition one finds that a normal height (H) is the ellipsoid height while the normal gravity potential equals the actual geopotential of the point of interest. Figure (3-7) is showing the relation between the orthometric, ellipsoidal height, and the geoid.

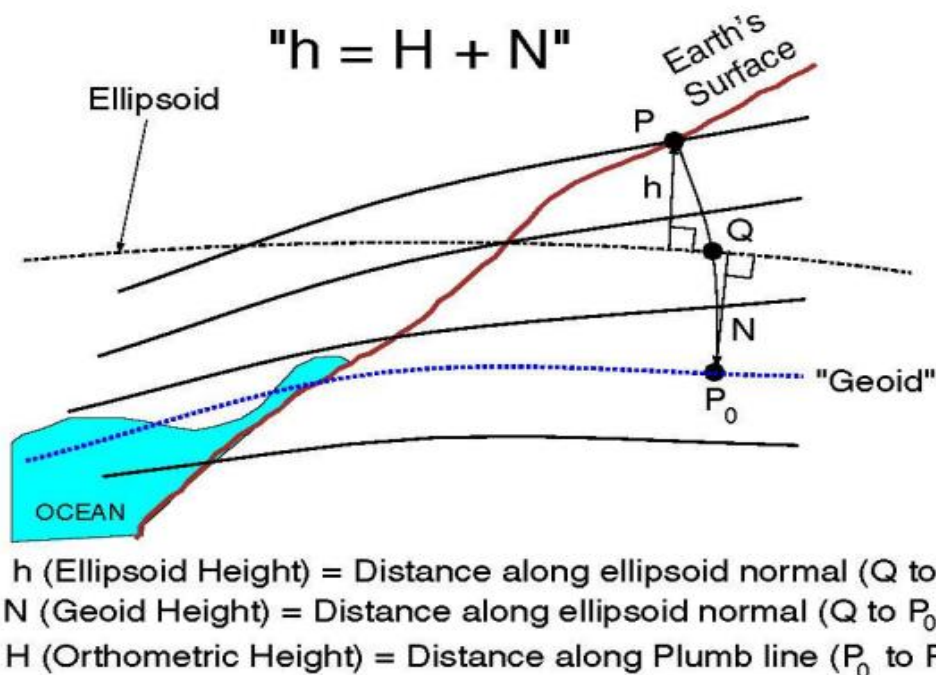


Figure (3.7) Relationship between ellipsoidal and geoid

3.4.4.3 Ellipsoidal Height

Ellipsoidal height is defined as the vertical distance along the ellipsoidal normal from the ellipsoid to the point in the earth surface. The term ellipsoid height is actually a wrong naming for height, because although this is an approximately vertical measurement, it does not give true height because it is not related to a level surface. It does however clearly identify a point in space above or below the ellipsoid surface in a simple geometrical way, which is its purpose; if you need to benefit from the so called ellipsoidal height you should convert it to orthometric height using the well known formula

$$h=H+N.....(3.3)$$

Where h is the ellipsoidal height, H is the orthometric height and N is the geoid ellipsoid separation. In the past (before the advent of GPS) it's difficult to measure the ellipsoid height. now a days GPS give the position of a point in Cartesian coordinate X,Y,Z Which can be transformed to geographical coordinate latitude , longitude , and height (h). See figure(3.8) below.

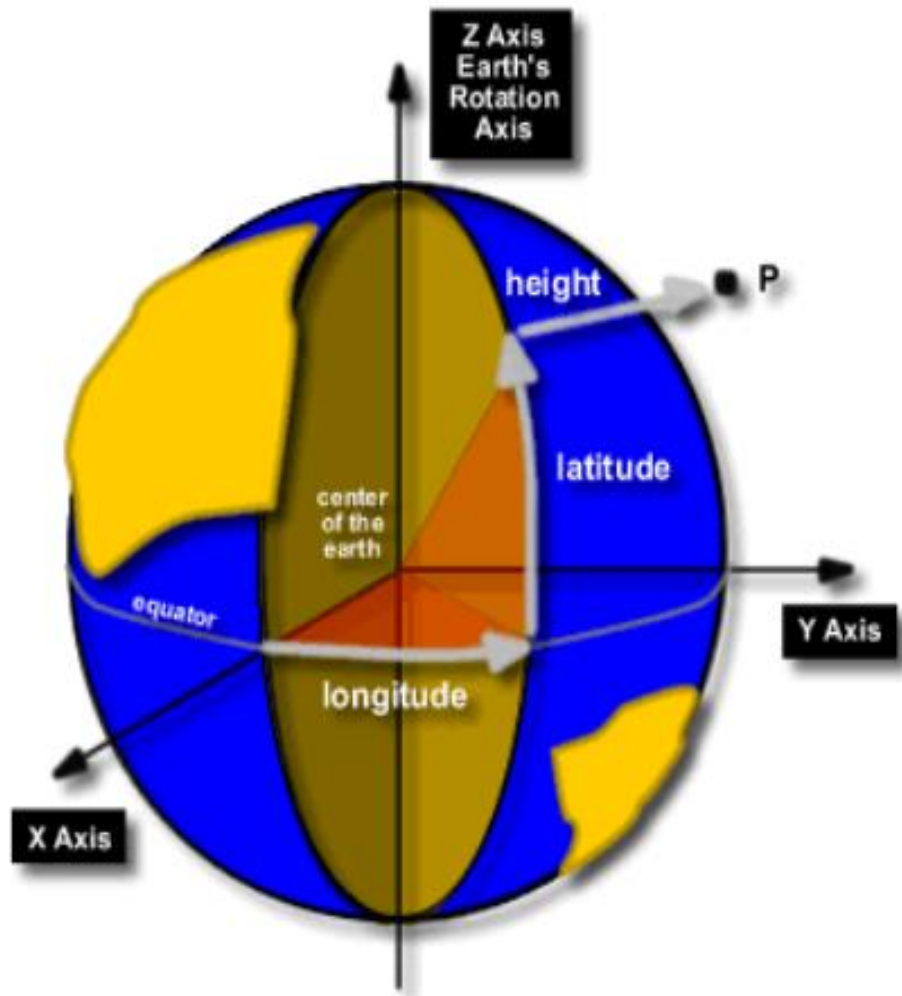


Figure (3.8) WGS-84 Coordinate System

It should be noted here that, for accurate information it's important to mention the reference system for the coordinate. A single point in the earth surface may have different geographic coordinate, due to different reference ellipsoids, also for the grid coordinate e.g. the UTM coordinate its important to mention the reference ellipsoid, because different ellipsoid can give different UTM coordinate for one point, this means that if you said that this point has an ellipsoidal height of say 400 meters this is a meaning less unless you mention the reference ellipsoid. We can not close these topics without

mention that: The axes of the WGS84 Cartesian system and, hence, all lines of latitude and longitude in the WGS84 datum are not stationary with respect to any particular country. Due to: 1- Tectonic plate motion, different parts of the world move relative to each other with velocities of the order of ten centimeters per year. These Temporal effects may require an epoch to be designated with any set of absolute station coordinates.

The epoch of the WGS 84 (G730) reference frame, for example, is 1994.0 while the epoch associated with the WGS 84 (G873) reference frame is 1997.0.

The application of plate motion model is time dependent. For example, a station on a plate that moves at a rate of 5 cm/year may not require this correction if the epoch of the coordinates is less than a year in the past. If, however, these same coordinates are used over a 5-year period, 25 cm of horizontal displacement will have accumulated in that time and application of this correction may be advisable, depending on the accuracy requirements of the geodetic survey. Two Tidal effect, Tidal phenomena are another sources of temporal and permanent displacement of a station's coordinates. These displacements can be modeled to some degree. In the most demanding applications (cm-level or better accuracy), these displacements should be handled as outlined in the (International Earth Rotation and Reference Systems Service) IERS Conventions (1996). The results of following these conventions lead to station coordinates in a 'zero-tide' system. In practice, however, the coordinates are typically represented in a 'tide-free' system. This is the procedure followed in the GPS precise ephemeris estimation process. In this 'tide-free' system, both the temporal and permanent displacements are removed from a station's coordinates. Note that many practical geodetic surveying algorithms are not equipped to account for these tidal effects. Often, these effects are completely ignored or allowed to

'average-out.' This approach may be adequate if the data collection period is long enough since the majority of the displacement is diurnal and semi-diurnal. Moreover, coordinates determined from GPS differential (baseline) processing will typically contain whatever tidal components are present in the coordinates of the fixed (known) end of the baseline. If decimeter level or better absolute accuracy's required, careful 27 considerations must be given to these station displacements since the peak absolute, instantaneous effect can be as large as 42cm.

Chapter Four

The Global Positioning System GPS

4.1 Introduction

The work of ministry of defense in America started the design of global position system by transit system or (Sat – Now) in order to avoid defects as in complete coverage on satellite in addition to in accurate navigation operations. So the system is renewed to tread those damages for political needs.

The GPS system is controlled through military and civilian services in different fields in life which important to land, sea and air.

The first satellite has launched in 1978. This system depends on a net consisted of 24 satellites circumvolve on orbit highly around the globe.

Drawing These satellites are distributed in their circumvolution in specific time. The user has an access of four satellites easily around the world.

This system deserved money spent, the satellites circumvolve highly in order to avoid defects and difficulties face global direction at station. In addition, the system reflects high accuracy of less than centimeter.

4.2 Principles of Operation

Broken down to the simplest terms, the satellites orbiting above the Earth simply broadcast their location and the current time. The receivers listen to several satellites (how many will be discussed below), and from the broadcasts determine what time it is and where the receivers are located. The principles, of course, require much more detail, but this the essence. Each satellite broadcasts two signals consisting of carrier waves that undergo phase changes that occur in a defined pattern at very precise rates and at exact times. A receiver generates a copy of the phase-change pattern and

moves it back and forth in time, attempting to correlate it with signals it receives. If the signal it is trying to correlate with is being received, at some point the received pattern and the internally generated pattern will match. The correlator circuit will then generate a large output. This pattern match and associated correlator output constitute lock-on to a satellite, and provides a pattern generator in the receiver that is working exactly in step with the received signal. Knowing how much this generator was shifted in time tells the receiver when the signal arrived at the receiver with respect to its own internal clock. If the receiver could determine how its clock was adjusted with respect to true GPS time, it would then know exactly how long it took the signal from the satellite to reach the receiver. When the receiver multiplies this time by the speed of light, it knows how far it is from the satellite. In addition to transmitting a specific phase-change pattern that is unique for each satellite, additional data is also added to the signal. This data comprises the Navigation Message. It includes the current time to the nearest second, and the information needed to compute the location of the satellite at the time of transmission. Using this information, the receiver can set its clock to the correct second, and compute the current position of the satellite. It knows how far it is from the satellite, and where the satellite is. Using simple geometry, the receiver now knows it is somewhere on the surface of a sphere centered on that satellite with radius equal to the distance from the satellite. Let's look at the process in an end-to-end scenario. Several satellites are broadcasting their patterns each one unique which are arriving at the antenna of a receiver. Each pattern arrives at a different time determined by the relative distance between the receiver and the satellite sending the pattern. The receiver searches for specific satellites by generating and shifting the pattern for each satellite that might be broadcasting. Once matches are found, the receiver can compute the

distance, called pseudorange, to each satellite. If the receiver's clock is precisely coordinated to GPS time, the receiver could immediately compute its position using simple algebra. Unfortunately, the receiver's clock is usually not set exactly to GPS time. Thus, the pseudorange consists not only of the time it took the signal to travel to the receiver, but also an amount that represents how far the receiver clock and GPS time differ. This is called clock offset, and represents a fourth unknown .

Clock offset could be either positive or negative since the receiver clock could be either ahead of or behind GPS time. Pseudorange is measured in units of time. Because we know that the signal traveled to the receiver at the speed of light (about 300,000,000 meters per second), we can convert it to a distance simply by multiplying it by that number. Similarly, clock offset is measured in units of time and can also be converted to distance as well. This distance or time error is common to all of the pseudoranges since the receiver uses the same clock to measure all pseudoranges. When a receiver acquires a satellite, the receiver monitors the navigation message from the satellite. Part of the data contained in the navigation message is the current GPS time, expressed in seconds. GPS time is the number of seconds since midnight between January 5 and 6, 1980. Thus, the receiver is able to set its own time indication to the exact whole second (the receiver computes fractions of a second later). Another part of the navigation message is a set of numbers called the ephemeris, that together describe the satellite's orbit in space, and where the satellite is in that orbit at a particular time. The receiver computes the exact location of the satellite in space from the ephemeris and the current time. The result is a set of x, y and z coordinates where the satellite was when the signal was transmitted. These values tell the position of the satellite with respect to a coordinate system defined by the World Geodetic System 1984 (referred to as WGS84). The origin of this coordinate

system is near the Earth's mass center, and its z axis matches the mean spin axis of the Earth. +z is towards the North pole; +x emerges from the Earth on the Greenwich meridian at the equator (just south of Ghana, and west of Gabon, in the Atlantic Ocean). The +y axis emerges at the equator on the 90° East meridian (at a point in the Indian Ocean southeast of Sri Lanka and west of Sumatra), thus defining a right-hand coordinate system. At this point the receiver has the location of each satellite, and the pseudorange to that satellite. Using appropriate math the receiver computes its position (x, y and z) and clock offset (Δt). To understand how this works, let's look at it graphically. To make it easier to visualize, we will use a two-dimensional solution. The three-dimensional solution works exactly the same, but with the added z factor. Refer to Figure (4.1) Points A, B and C are the locations of three satellites in the x, y coordinate space of the diagram. Radii d_1 , d_2 and d_3 represent the pseudoranges we measured from each satellite (here shown in the distance form). Thus, we would define our position as located on the intersection of the three circles centered on each satellite with radius equal to the respective pseudoranges. But the three circles do not meet at a point. They intersect to form a triangle with arcs for sides (in some cases, they could even miss each other entirely).

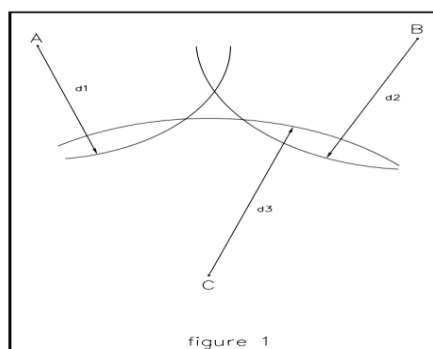


Figure (4.1) Showing Three satellites locations

Now refer to Figure (4.1) . In this case we have added a small amount, t , to each pseudorange. The result is that we have adjusted each pseudorange by the same amount, t , causing the circles to meet at a point. The coordinates of this point represent our position, and t represents our clock offset. As a result of this process, we not only know our position, but we also know the correct time (fractions of a second) within the resolution of our code pattern shifter. Time resolution is typically to fractions of microseconds, resulting in a time determination that is more accurate than about any other method generally available. In fact, GPS receivers designed specifically to adjust atomic clocks yield time determinations that match UTC to within 10 nanoseconds.

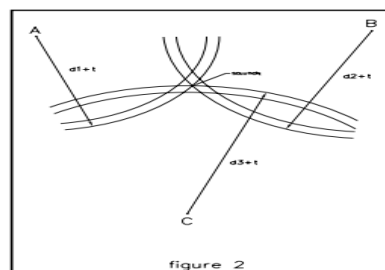


Figure (4.2) Showing Three satellites locations and time

An interesting element of the position determination process becomes apparent here. Note that if the receiver is a long distance from the antenna, the satellite signals must travel that distance inside a cable to reach the electronic circuits that measure the pseudorange. As a result, the measured pseudorange increases by the time required to travel the distance represented by the cable length. In addition, signals tend to travel slower than the speed of light inside cables (in some cables, at less than two thirds the speed of light). This factor also increases the time for the signal to reach the receiver. However, since the signals from all satellites travel this same distance, the effect is to add the same amount of time delay to all signals. Now recall that pseudorange is the sum of the time it takes the signal to travel to the receiver, and the clock offset of the receiver. Of course the distance from each satellite to the antenna is unique. The extra distance from the antenna to

the receiver, and the clock offset of the receiver, are exactly the same in all measurements. When the receiver computes the solution, these two constant terms become merged into a single value referred to as clock offset. This causes the x, y and z position coordinates to be referenced to the common point of the antenna. Thus, using a longer or shorter antenna cable will not affect the position determined, but will only affect the computed clock offset. Further, there exists a single point within the antenna where the antenna detects each signal. This point is called the antenna phase center. Since it typically is displaced from the mounting point of the antenna by several centimeters, manufacturers of precision surveying receivers routinely publish the location of the phase center with respect to some convenient point accessible on the housing of the antenna. This lets surveyors measure the distance between a station being surveyed and the antenna housing, and later relate that measurement to the antenna phase center, thus Allen Osborne Associates Principles of GPS.

Adjusting the survey results for the actual distance between the station and the antenna phase center. Another point of interest specifically to those doing very high precision surveys, such as geodetic surveys, is that the phase center of an antenna is often different for different frequencies. The GPS system transmits two frequencies called L1 and L2. Thus, published phase center displacements for an antenna may give values for both L1 and L2 to allow the surveyor's post-processing software to compensate for the difference.

4.3 The GPS Signal

All GPS satellites broadcast on the same two frequencies. The primary signal is broadcast on what is referred to as L1 frequency, which is 1,575.42 MHz . The signals are broadcast using spread-spectrum techniques, which allow many signals to coexist on the same frequency, and for receivers to

detect and separate the different signals from each other. The L1 signal is modulated with two information signals called C/A (for Coarse and Acquisition code) and P (for Precise code). In addition, the satellites also broadcast a copy of the P code on the second frequency called L2, which is 1,227.60 MHz . (Note that later satellites may add additional codes to L1 and/or L2, and may also add extra frequencies.) Spread-spectrum modulation basically consists of the carrier signal being repeatedly inverted, that is, having its phase shifted 180°, in a specific pattern. The C/A code pattern is generated by a hardware signal generator consisting of a pair of 10-bit shift registers with feedback connections in them, whose outputs are combined by an exclusive-OR gate. The resulting digital sequence is referred to as a Pseudo-Random Number, or PRN, sequence. The generator produces a pattern that is exactly 1,023 bits long, which then repeats. By starting both the shift registers at a defined starting point, and by combining the resulting outputs with a phase shift between them (that is, the output of one register is delayed by some number of bits from the output of the other), several unique codes can be generated. The GPS system defines 36 specific phase shifts to be used, resulting in 36 unique codes (called Gold Codes) that could be transmitted by satellites. Since the satellite number is represented in the navigation message by only 5 data bits, only 32 of these 36 codes are actually used. The others are reserved for other uses, such as ground transmitters. The bit rate of the generator used to modulate the carrier is referred to as the “chipping” rate, and each bit is referred to as a “chip.” For GPS satellites, the C/A code chipping rate is 1.023 MHz . Since the code is 1,023 bits long before it repeats, the code repeats every 1 msec. To receive a spread-spectrum signal, the receiver must know the desired PRN sequence. It generates its own copy of the sequence, and applies it to the output of a down-converter and detector. The receiver then shifts the pattern in time

looking for a match with what appears to be noise coming from the detector. The match is made in a circuit called a correlator, which produces an output that corresponds to the degree of match between the two signals. When the receiver's code matches the received signal, there is a large rise in the magnitude of the correlate's output. To search for a transmitter, the receiver first adjusts the internally generated pattern in time, chip-by-chip, until an indication of matchup has occurred, then shifts the pattern by fractions of a chip until the correlator output is maximized. At this point the internal pattern generator is generating a code in exact step (at least to the resolution of the pattern shifter) with the received signal. For greater precision in determining the time it takes a signal to travel to the receiver, a second signal is generated and transmitted on the same frequency. This second carrier is 90° ahead of the carrier with the C/A code, but it is of a lower amplitude. It is modulated with a PRN sequence called the P (for Precise) code. The P code has a chipping rate of Principles of GPS Allen Osborne Associates Page 12 February, 1997 10.23 MHz, so it is 10 times the rate, and thus, precision, of the C/A code. In addition, the P-code sequence is much longer than the C/A code - it does not repeat over a complete week. This makes it harder to acquire without the initial time setting afforded by the C/A code being acquired first (in fact, this is why the C/A code has the term "acquisition" in its name). A feature called Anti-Spoofing (A-S) can be activated by the US DoD to prevent the intentional deception of receivers by use of a phony, or "spoofing" transmitter. The result of A-S being turned on is that the P code is hidden by an encryption scheme. The P code thus encrypted is called Y code. Figure (4.3) shows how the C/A code and P (or Y) code modifies the carrier sine wave. While the P-code carrier also is phase shifted 180° by its bit pattern, since it is lower in amplitude than the C/A code carrier and 90° out of phase with it, the effect of combining the

two carriers is for the output signal from the satellites to appear to shift by about 70° when P-code bits change.

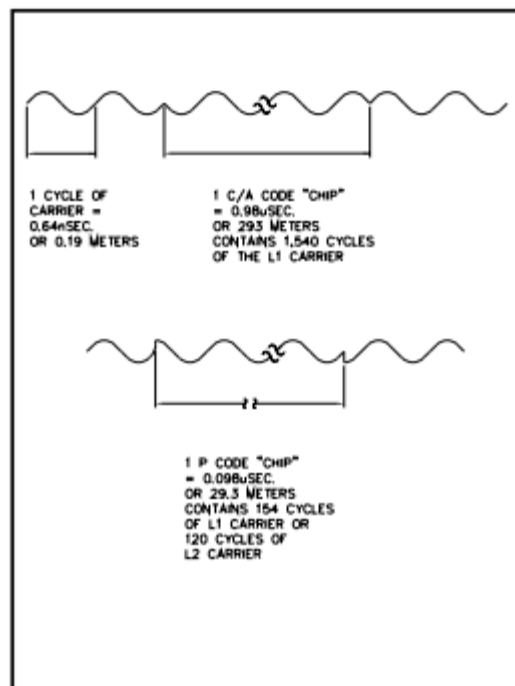


Figure (4.3) Shows how the C/A code and P code

Data being sent on the carrier is represented by either inverting or not inverting the PRN code, so that at the receiver the correlator will generate either a positive or negative correlation output. The data rate is usually much slower than the chipping rate so that it does not interfere with the integration that is done as part of the correlation process. Data on the GPS signals, called the Navigation Message, are modulated at a nominal 50 bits per second rate.

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Each satellite contains multiple atomic clocks, and the carrier and modulation signals are timed precisely to the clocks. Thus, at exactly the start of a second as defined by the master GPS timing, each satellite's signal is crossing zero or passing an integral multiple of one of the phase changes,

and its modulation (both C/A and P codes) are also starting bits. In fact, the L1 and L2 frequencies have been chosen so that they relate to each other and to the chipping rates in a coherent manner. The basic timing is provided by the 10.23 MHz frequency of the P code. L1 is exactly 154 times this frequency, and L2 is exactly 120 times this frequency, so a single P chip consists of 154 cycles of the L1 or 120 cycles of the L2 carrier. The C/A chip rate is composed of 10 P chips, and the actual navigation message data rate is defined as exactly 20 copies of the 1023-bit C/A pattern. Thus, a properly working satellite has all the elements of the signal locked to one reference frequency, and the phase of the carrier, the phase of the chips and the data all align with transitions occurring on 0° boundaries of the unmodulated carriers. The coherency of the transmitted signal provides yet another method of determining pseudorange; this is referred to as carrier phase. Once a receiver has determined its own clock offset, it can determine the actual start of a second. The received signal will differ from this point in time by some integral number of carrier cycles, plus a fractional part of a cycle. Just as with the C/A code, this offset is due to the time it takes the signal to travel from the satellite to the receiver. The fractional cycle can usually be determined to about 1 part in 1000 using current technology. The integer number of cycles, however, is subject to some ambiguity, but can be determined using a process called ambiguity resolution. Since the wavelength of the carrier is about 0.19 meters, resolving this to 0.1% (one part in one thousand) yields a pseudorange measurement that has a resolution of about 0.2 mm (this is less than $8/1000''$). For purposes of illustration, the P code chip length (the equivalent of wavelength) is about 29 meters, and the C/A code chip length is about 290 meters. Assuming the same 0.1% resolution on these as on the carrier waves, this implies

resolution of 2.9 cm for P code (a little over 1”), and 29 cm for the C/A code (about 11”).

4.4 The Navigation Message

The data modulated onto the C/A and P codes consist of several types of information. The data are packaged into 30-bit words that consist of 24 data bits and 6 parity bits. Words are grouped together into groups of 10 called a sub frame. Each sub frame is thus 300 bits long, of which 240 bits are data and 60 bits are parity. Sub frames take 6 seconds to transmit at 50 bps. There are 5 sub frames defined, numbered 1 through 5. The satellite transmits a set of all 5 sub frames in 30 seconds, then begins to transmit another set. The contents of the sub frames change over time as noted below. Figure (4.4) illustrates the navigation message and its components.

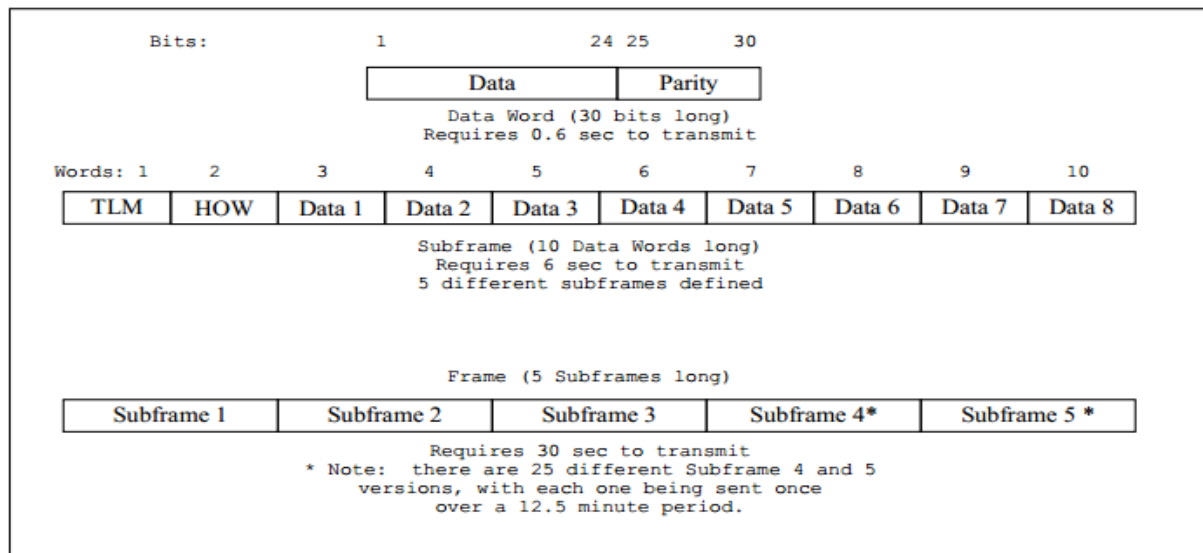


Figure (4.4) illustrates the navigation message and its components. Each sub frame starts with a pair of data elements called the Telemetry Word (TLM) and the Handover Word (HOW). The TLM provides a

standard bit-pattern preamble that can be used to detect the start of a sub frame, plus administrative status information such as data upload status. The HOW contains the GPS system time (referred to as Z-count) that corresponds to the start of the next sub frame, and an identification of which sub frame number this is. The TLM and HOW together take up the first two words of every sub frame. All information discussed below about the 5 sub frames relates only to the remaining 8 data words. Sub frame 1 contains information that can be used to compute a correction term for the satellite's clock. Even though the satellites have multiple atomic clocks on them, the clocks do drift. This drift is monitored by the Control Segment stations, and a second order curve fit is made to it. The coefficients of the expression describing this curve are reported in sub frame 1 so that the users can compute the current modeled clock error and thus improve their own navigation.

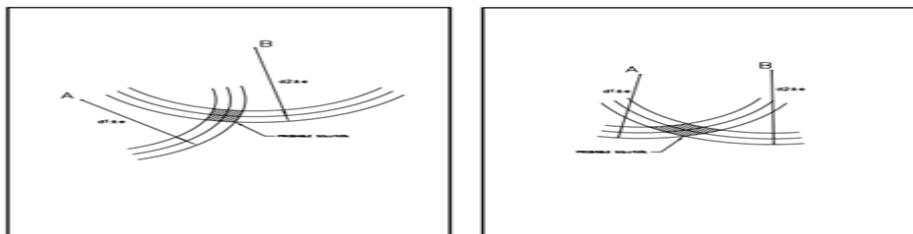
4.5 GPS Errors

The process of transmitting, receiving and detecting the GPS signal is a physical process which, like any other physical process, contains sources for errors. Some of the errors are obvious: the satellite clock is not exactly correct, even when the broadcast correction terms are used to adjust it. The location of the satellite in space is not necessarily correct since it is determined by observations made on the ground, and the ephemeris values only yield a solution accurate to about 30 cm. And the receiver computing its own position can only resolve the received signals to some specific precision determined by the wavelength of either the carrier (for carrier phase measurements) or the code bit length (for code pattern matching), and the resolution of the code or phase shifter in the receiver. Further limitations occur in the receiver based on the precision of the computations, where mathematical processes may truncate or round values rather than carrying

them out to their last possible decimal place. Some other error sources appear when we look at the physical process of a signal traveling through space to the receiver. For example, the signal is transmitted by a satellite traveling at a high rate of speed in space. Since it is unlikely that the receiver is traveling at the same direction and speed, there will be a Doppler shift of the signal that affects the effective wavelength of both the code and carrier waves. And the signal must travel through the ionosphere above the Earth, which has an effect of shifting the signal and bending its path. It also travels through the troposphere (the lower layer of the atmosphere where most weather occurs), which also affects the signal's path and speed. As the signal nears the receiver, some of it may reflect off the ground, water, or buildings, water towers, signs, etc., located near the receiver, and reach the antenna after traveling a greater distance than the signal that arrives directly from the satellite (a phenomenon called multipath). Finally, the receiver is also prone to errors that can be detected. The clock is unstable, causing individual ticks to occur not truly regularly, but with some "wobble" between them. Most receivers compute the pseudorange to a satellite several times per second, and average the measurements. If the clock in the receiver is "noisy," that is, not ticking at a uniform rate, the time during which each individual measurement is made can vary. This would result in each observation being made over a time span that is unique, and the resulting average could contain errors caused by this effect. In addition, all receivers detect noise along with the real signals, and that affects the received signals, further degrading them. Satellites closer to the horizon tend not only to be weaker, and thus more prone to noise, but their signals are more prone to multipath. For this reason that some receivers allow the user to set an elevation mask, that is, an angle below which satellites will not be tracked. The result of all of these and other error sources is that the computed pseudorange is an estimate with a possible

error whose magnitude can be computed using standard statistical methods. So when we compute our location with respect to a satellite, instead of finding ourselves on a sphere as described earlier (shown as the circular arcs in the two dimensional model of figures (4.1) and (4.2), we really find ourselves in a space located Principles of GPS Allen Osborne Associates.

The actual geometry of the satellites in the sky also has an effect on the accuracy of results. Returning to a two dimensional representation for ease of illustration, let's see how the geometry affects our results. Figure (4.5) (a) shows a case where we are only considering two satellites. Instead of a single arc, the distance from each satellite is shown as three arcs. The center, bolder arc is the same one we computed before representing the computed pseudorange. The two lighter arcs surrounding the center arc represent the magnitude of the error we estimate. Thus, the true range from the satellite is really somewhere in between the two outer arcs for each satellite, and therefore we are located somewhere inside the shaded area where these two spaces intersect. Now look at Figure (4.5) (b), in this example the errors and ranges are exactly the same, but the two satellites are located closer together. Note how the geometry of the satellites causes the area where we might be located to grow. Obviously, if the two satellites were at the same point in the sky, the presence of the second satellite would not help our position determination at all.



(a)

(b)

Figure (4.5) (a) Show two satellites (b) Show two arcs

The effect of this geometry on our overall error computation results in what is called a Dilution of Precision, or DOP. If we had such a thing as an ideal geometry, with satellites in all possible directions, we would have a DOP value of 1.0. In a more realistic situation, with 6 to 12 satellites visible, and all of them above the horizon, the DOP value rises. When true position and clock offset are computed, the error on these values and others computed from them can be determined by multiplying the composite error of the observations by the appropriate DOP.

The overall DOP term is called Geometric Dilution of Precision, or GDOP. GDOP may be broken down into two components: one related to the receiver's position (Position DOP, or PDOP) and one related to the time determination (Time DOP, or TDOP). While PDOP is related to the satellite geometry, as discussed above, TDOP is strictly dependent on the time bases in the receiver and all the satellites. Thus, it is a function only of the number of satellites being tracked. GDOP, PDOP and TDOP are related orthogonally by

$$G_{DOP2} = P_{DOP2} + T_{DOP2} \dots \dots \dots (4.1)$$

The PDOP value is further found to have two components: horizontal (HDOP) and vertical (VDOP), again related to PDOP orthogonally. Here one of the limitations on the GPS system becomes apparent. The horizontal component is basically affected by how the satellites are dispersed in azimuth about the receiver. If all satellites are bunched up in a single direction, the HDOP will be larger than if the same number of satellites were evenly spaced around the horizon. But consider VDOP. It is dependent on the elevation of the satellites, as you might expect. But since satellites below the horizon cannot typically be seen by a receiver, by necessity all satellites that we use are bunched up in the space from the horizon upwards. This simple fact explains why receiver manufacturers will specify larger error

values for their receivers for vertical position determinations than they will for horizontal positions. For completeness, here is how PDOP, HDOP and VDOP relate:

$$P_{DOP2} = H_{DOP2} + V_{DOP2} \dots \dots \dots (4.2)$$

One final error source must be noted. Since the original design of the GPS system was for the US military, there was concern that it might be used by an adversary to guide weapons. For this reason, there was a decision to put an intentional error source into the transmitted signals that would limit the availability of the system to users. This was done by such factors as altering the satellite's clock, or by altering the broadcast clock correction terms or other ephemeris terms. A user computing position using the C/A code from a fixed location would find the position solution moving over time such that the resulting position determinations would have about 100 meters 2d RMS variation. At the same time, an authorized user with access to the appropriate technology could still use the P code and navigate as accurately as before. This process is referred to as selective availability, or SA. Don't confuse it with anti-spoofing (A-S), which is the encryption of the P code into a Y code. While the SA makes real-time position determination less precise, the use of post processing with one or more reference receivers can remove most of the effects. Real-time differential GPS tracking can similarly remove these effects.

However, their effect will be seen in the corrections, and will be accounted for in the user receiver when the position is computed since the user receiver will at that time use the same incorrect position of the satellite to convert pseudoranges into positions. Thus, all errors originating on the satellite (including those caused intentionally by selective availability, or SA) will be removed in the DGPS process.

In practice, there are errors which differential techniques cannot correct for, and even errors which are introduced by the process (such as errors in the reference receiver's observations that are not common to all satellites). But the overall effect is quite significant. Just using the pseudorange derived from the C/A code, a stationary receiver will generally compute its position as a point which moves over time by more than 100 meters. When a nearby reference station's corrections are applied, that range of movement will generally be much less than 10 meters, and even less than 2 meters when the receivers are relatively close. This real-time process makes position determinations quite accurate in most navigation modes (but still not as accurate as post-processing for surveying applications). The above description describes what could be referred to as code differential GPS, since the observations and associated corrections are made from pseudoranges derived from the C/A or P code on the satellite carrier. Another form of differential can be developed that looks at the carrier phase. The higher resolution of the carrier wave makes the application of such corrections yield position solutions that vary by only a few centimeters. This form of differential is referred to as Real Time Kinematic, since it allows the performance of the ambiguity resolution process used for kinematic navigation to be performed in real time with immediate results.

4.6 Component of Global Position System (GPS)

This system consists of three basic units:

- i. Space segment: for active satellites.
- ii. Control segment: for observing the time and measurement.
- iii. User segment: different kinds of services.

4.6.1 Space segment

This segment consists of 21 satellites; in addition to 3 satellites are reserves. They are in 6 orbits having an inclination about 55° , 4 satellites in every

orbit at 20200 km high above sidereal circumvolution around globe is 12 hours.

The first satellite has launched to the space from block II in 1989 live 7.5 years. Every satellite sends two signals:

$$L_1 = 150 * 10.23 \text{ MHZ} = 1575.42 \text{ MHZ} \dots \dots \dots (4.3)$$

$$L_2 = 120 * 10.23 \text{ MHZ} = 1227.60 \text{ MHZ} \dots \dots \dots (4.4)$$

These carriers are navigation information of the system. Navigation signal modulated on waves (L_1, L_2) is called pseudo random noise (PRN).

The wave L_1 carries both navigation signals (P code and C/A code) and navigation message.

The wave L_2 carries only P-code in addition to navigation message. The distinction between satellites is completed by space vehicle number or number of the satellite itself.

4.6.2 Control segment

It consists of master control station and many monitor stations in the world. The control segment has to send signals to satellites.

The basic control station founded in Colorado and monitor stations distributed around globe as shown below.

It is from west to east Hawaii and co-ordinate ($19^{\circ}46'$ North - $155^{\circ}30'$ West), Colorado Springs ($38^{\circ}51'$ North - $140^{\circ}49'$ West), ascension ($08^{\circ}00'$ South - $13^{\circ}00'$ East), dip ($07^{\circ}20'$ South - $72^{\circ}26'$ East).

This segment characterized with Monitoring of the system continuously at their position of satellites and renewing navigate information.

These stations receive all signals and compute the pseudo range for all satellites and send signals back to the space segment.

4.6.3 User segment

It includes the reception equipments to receive signals from satellites.

Chapter Five Measurements and Results

5.1 Measurements

Study area was chosen to be ELSHAJARA which is located at south of Khartoum city, and between 446560.693 and 448660.595 easting and between 1717761.614 and 1719153.665 in northing (WGS-84 UTM Zone 36N).



Figure (5.1) Study area – ELSHAJARA

5.1.1 Comparison Points

Eleven bench marks well distributed in the study area were observed using GPS and ordinary levelling, as shown on figure (5.2) below.



Figure (5.2) Comparison points

5.1.2 Absolute Method Measurements

GPS R8 was used to determine locations, the observed coordinates in the table below after using Trimble Business Center for the processing.

Ellipsoid: WGS-84

Datum: WGS-84

Projection: UTM-Zone 36N

Table (5.1) Coordinates of check Points using absolute method

| Point | Easting (m) | Northing (m) | Ellipsoidal Height (m) |
|-------|-------------|--------------|------------------------|
| BM01 | 447175.539 | 1718297.481 | 380.382 |
| BM02 | 447571.402 | 1717971.323 | 379.909 |
| BM03 | 448071.009 | 1717985.491 | 380.115 |
| BM04 | 448620.520 | 1717761.614 | 379.978 |
| BM05 | 448660.595 | 1718293.761 | 380.126 |
| BM06 | 448548.530 | 1719009.213 | 380.058 |
| BM07 | 447890.672 | 1718915.436 | 379.920 |
| BM08 | 447388.663 | 1718842.356 | 379.703 |
| BM09 | 446947.043 | 1719153.665 | 379.720 |
| BM10 | 446560.693 | 1718821.818 | 379.796 |
| BM11 | 446631.099 | 1718329.924 | 379.739 |

5.1.3 Relative Method Measurements

Digital level Leica DNA 03 was used to carry out the ordinary levelling for the check points to obtain their orthometric heights, starting from known point (BM= 382.318m). Figure (5.3) showing the bench mark status.

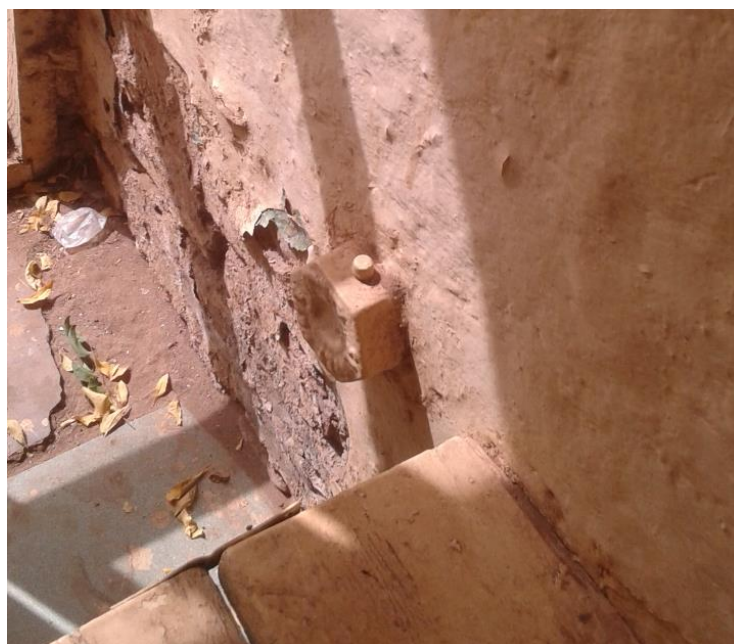


Figure (5.3) BM Status

Table (5.2) Ordinary levelling measurements (Relative method)

| BS | IS | FS | Rise | Fall | RL | Remarks |
|--------|----|--------|--------|--------|----------|---------|
| 1.0531 | | | | | 382.318 | BM00 |
| 1.3133 | | 1.4712 | | 0.4181 | 381.8999 | 1 |
| 1.4537 | | 1.8625 | 0.3401 | 0.5492 | 381.3507 | 2 |
| 1.4914 | | 1.1136 | 0.3388 | | 381.6908 | 3 |
| 1.2381 | | 1.1526 | | | 382.0296 | 4 |
| 1.1691 | | 1.2499 | | 0.0118 | 382.0178 | BM01 |
| 1.1219 | | 1.3355 | | 0.1664 | 381.8514 | 5 |
| 1.3701 | | 1.426 | | 0.3041 | 381.5473 | 6 |
| 1.3741 | | 1.3708 | 0.0362 | 0.0007 | 381.5466 | 7 |
| 1.3417 | | 1.3379 | | | 381.5828 | 8 |
| 1.3595 | | 1.3747 | | 0.033 | 381.5498 | 9 |
| 1.4213 | | 1.4341 | 0.0009 | 0.0746 | 381.4752 | 10 |
| 1.4012 | | 1.4204 | 0.0693 | | 381.4761 | 11 |
| 1.4216 | | 1.3319 | 0.0159 | | 381.5454 | 12 |
| 1.3062 | | 1.4057 | | | 381.5613 | BM02 |
| 1.3879 | | 1.4329 | 0.4914 | 0.1267 | 381.4346 | 13 |
| 1.5497 | | 0.8965 | | | 381.926 | 14 |
| 2.1323 | | 1.831 | 0.2587 | 0.2813 | 381.6447 | 15 |
| 1.3194 | | 1.8736 | | | 381.9034 | 16 |
| 1.2577 | | 1.4554 | | 0.136 | 381.7674 | 17 |
| 1.311 | | 1.2906 | | 0.0329 | 381.7345 | 18 |
| 1.4357 | | 1.4212 | 0.1287 | 0.1102 | 381.6243 | 19 |
| 1.2988 | | 1.307 | 0.1277 | | 381.753 | BM03 |
| 1.4846 | | 1.1711 | 0.019 | | 381.8807 | 20 |
| 1.3692 | | 1.4656 | | | 381.8997 | 21 |
| 1.4117 | | 1.3998 | 0.0219 | 0.0306 | 381.8691 | 22 |
| 1.4547 | | 1.3898 | | | 381.891 | 23 |
| 1.438 | | 1.584 | 0.0134 | 0.1293 | 381.7617 | 24 |
| 1.3528 | | 1.4246 | | | 381.7751 | 25 |
| 1.3338 | | 1.4681 | | 0.1153 | 381.6598 | 26 |
| 1.4805 | | 1.3437 | 0.0948 | 0.0099 | 381.6499 | BM04 |
| 1.4301 | | 1.3857 | | | 381.7447 | 27 |
| 1.5109 | | 1.5005 | 0.055 | 0.0704 | 381.6743 | 28 |
| 1.3649 | | 1.4559 | 0.0078 | | 381.7293 | 29 |
| 1.4342 | | 1.3571 | 0.0795 | | 381.7371 | 30 |
| 1.2856 | | 1.3547 | 0.0953 | | 381.8166 | 31 |
| 1.4284 | | 1.1903 | | | 381.9119 | 32 |
| 1.353 | | 1.5455 | 0.0712 | 0.1171 | 381.7948 | BM05 |
| 1.334 | | 1.2818 | | | 381.866 | 33 |

| | | | | | | |
|--------|--|--------|--------|--------|----------|------|
| 1.3628 | | 1.4873 | 0.0255 | 0.1533 | 381.7127 | 34 |
| 1.3773 | | 1.3373 | | | 381.7382 | 35 |
| 1.447 | | 1.4561 | | 0.0788 | 381.6594 | 36 |
| 1.3839 | | 1.4651 | | 0.0181 | 381.6413 | 37 |
| 1.3814 | | 1.4569 | 0.0219 | 0.073 | 381.5683 | 38 |
| 1.4214 | | 1.3595 | 0.0128 | | 381.5902 | 39 |
| 1.4472 | | 1.4086 | 0.0839 | | 381.603 | 40 |
| 1.405 | | 1.3633 | | | 381.6869 | BM06 |
| 1.4804 | | 1.4672 | 0.3373 | 0.0622 | 381.6247 | 41 |
| 1.3536 | | 1.1431 | | | 381.962 | 42 |
| 1.0409 | | 1.4723 | | 0.1187 | 381.8433 | 43 |
| 1.4752 | | 1.4751 | | 0.4342 | 381.4091 | 44 |
| 1.5643 | | 1.506 | 0.1541 | 0.0308 | 381.3783 | 45 |
| 1.3762 | | 1.4102 | | | 381.5324 | 46 |
| 1.568 | | 1.4885 | 0.1099 | 0.1123 | 381.4201 | 47 |
| 1.3978 | | 1.4581 | | | 381.530 | BM07 |
| 1.4142 | | 1.4237 | | 0.0259 | 381.5041 | 48 |
| 1.3509 | | 1.4503 | | 0.0361 | 381.468 | 49 |
| 1.5597 | | 1.4749 | 0.1845 | 0.124 | 381.344 | 50 |
| 1.3349 | | 1.3752 | | | 381.5285 | 51 |
| 1.4101 | | 1.397 | | 0.0621 | 381.4664 | 52 |
| 1.4311 | | 1.5323 | | 0.1222 | 381.3442 | BM08 |
| 1.4119 | | 1.5141 | | 0.083 | 381.2612 | 53 |
| 1.4637 | | 1.4522 | | 0.0403 | 381.2209 | 54 |
| 1.4835 | | 1.5043 | 0.0724 | 0.0406 | 381.1803 | 55 |
| 1.1893 | | 1.4111 | 0.0129 | | 381.2527 | 56 |
| 1.4302 | | 1.1764 | | | 381.2656 | 57 |
| 1.4303 | | 1.4353 | 0.0577 | 0.0051 | 381.2605 | 58 |
| 1.4566 | | 1.3726 | | | 381.3182 | BM09 |
| 1.4368 | | 1.4776 | 0.0929 | 0.021 | 381.2972 | 59 |
| 1.491 | | 1.3439 | 0.0536 | | 381.3901 | 60 |
| 1.3934 | | 1.4374 | | | 381.4437 | 61 |
| 1.6333 | | 1.5068 | 0.0336 | 0.1134 | 381.3303 | 62 |
| 1.319 | | 1.5997 | | | 381.3639 | 63 |
| 1.4407 | | 1.4702 | 0.208 | 0.1512 | 381.2127 | 64 |
| 1.2821 | | 1.2327 | | | 381.4207 | BM10 |
| 1.5348 | | 1.5627 | 0.1213 | 0.2806 | 381.1401 | 65 |
| 1.3849 | | 1.4135 | | | 381.2614 | 66 |
| 1.558 | | 1.6012 | 0.3728 | 0.2163 | 381.0451 | 67 |
| 1.4621 | | 1.1852 | | | 381.4179 | 68 |
| 1.435 | | 1.6128 | 0.0545 | 0.1507 | 381.2672 | 69 |

| | | | | | | |
|--------|--|--------|--------|--------|----------|------|
| 1.3408 | | 1.3805 | 0.0349 | | 381.3217 | 70 |
| 1.3075 | | 1.3059 | | | 381.3566 | BM11 |
| 1.759 | | 1.3864 | 0.5402 | 0.0789 | 381.2777 | 71 |
| 1.394 | | 1.2188 | 0.032 | | 381.8179 | 72 |
| 1.1263 | | 1.362 | | | 381.8499 | 73 |
| 1.4966 | | 1.555 | 0.451 | 0.4287 | 381.4212 | 74 |
| 1.4631 | | 1.0456 | 0.446 | | 381.8722 | 75 |
| | | 1.0171 | | | 382.3182 | BM00 |

Computational Check:

$$\sum BS - \sum FS = \sum Rise - \sum Fall = Last RL - First RL$$

$$= 0.0002m$$

Thus, the comparison points became had two heights, one from the absolute method by GPS (Ellipsoidal height) and the other from the relative approach by the ordinary levelling (Orthometric height), as in table (5.3).

Table (5.3) Showing Heights in Ellipsoidal & (MSL)

| Point | Easting (m) | Northing (m) | Ellipsoidal Height (m) | Orthometric Height (MSL) (m) |
|-------|-------------|--------------|------------------------|------------------------------|
| BM01 | 447175.539 | 1718297.481 | 380.382 | 382.018 |
| BM02 | 447571.402 | 1717971.323 | 379.909 | 381.561 |
| BM03 | 448071.009 | 1717985.491 | 380.115 | 381.753 |
| BM04 | 448620.520 | 1717761.614 | 379.978 | 381.650 |
| BM05 | 448660.595 | 1718293.761 | 380.126 | 381.795 |
| BM06 | 448548.530 | 1719009.213 | 380.058 | 381.687 |
| BM07 | 447890.672 | 1718915.436 | 379.920 | 381.530 |
| BM08 | 447388.663 | 1718842.356 | 379.703 | 381.344 |
| BM09 | 446947.043 | 1719153.665 | 379.720 | 381.318 |
| BM10 | 446560.693 | 1718821.818 | 379.796 | 381.421 |
| BM11 | 446631.099 | 1718329.924 | 379.739 | 381.357 |

5.1.4 Heights Differences between two methods

From both heights, differences were extracted, and the result is:

Table (5.4) Showing Differences between heights in Ellipsoidal & (MSL)

| Point | Ellipsoidal Height (m) | Orthometric Height (MSL) (m) | Difference (m) |
|-------|------------------------|------------------------------|----------------|
| BM01 | 380.382 | 382.018 | 1.636 |
| BM02 | 379.909 | 381.561 | 1.652 |
| BM03 | 380.115 | 381.753 | 1.638 |
| BM04 | 379.978 | 381.650 | 1.672 |
| BM05 | 380.126 | 381.795 | 1.669 |
| BM06 | 380.058 | 381.687 | 1.629 |
| BM07 | 379.920 | 381.530 | 1.610 |
| BM08 | 379.703 | 381.344 | 1.641 |
| BM09 | 379.720 | 381.318 | 1.598 |
| BM10 | 379.796 | 381.421 | 1.625 |
| BM11 | 379.739 | 381.357 | 1.618 |

5.1.5 Root Mean Square Error (RMSE) Computation

Then, RMSE has been derived for the mean from the equation

$$RMSE = \sqrt{\frac{\sum(\bar{m} - d_i)^2}{n - 1}}$$

Table (5.5) Computation of RMSE for the mean

| Point | Ellipsoidal Height h | Orthometric Height H | Diff D | V=Mean-D | V ² |
|-------|-------------------------|-------------------------|-----------|------------|----------------|
| BM01 | 380.382 | 382.018 | 1.636 | 0.00072727 | 5.28926E-07 |
| BM02 | 379.909 | 381.561 | 1.652 | 0.01672727 | 0.000279802 |
| BM03 | 380.115 | 381.753 | 1.638 | 0.00272727 | 7.43802E-06 |
| BM04 | 379.978 | 381.65 | 1.672 | 0.03672727 | 0.001348893 |
| BM05 | 380.126 | 381.795 | 1.669 | 0.03372727 | 0.001137529 |
| BM06 | 380.058 | 381.687 | 1.629 | -0.0062727 | 3.93471E-05 |
| BM07 | 379.92 | 381.53 | 1.61 | -0.0252727 | 0.000638711 |
| BM08 | 379.703 | 381.344 | 1.641 | 0.00572727 | 3.28017E-05 |
| BM09 | 379.72 | 381.318 | 1.598 | -0.0372727 | 0.001389256 |
| BM10 | 379.796 | 381.421 | 1.625 | -0.0102727 | 0.000105529 |
| BM11 | 379.739 | 381.357 | 1.618 | -0.0172727 | 0.000298347 |

$$RMSE = 0.023m$$

5.2 Analysis

- From the computed root mean square error RMSE, it is obvious that the quality of the computed differences between the two systems that resulted from the applied measurements.
- The average difference is 1.635m, which is the average conversion value between the two systems.
- The maximum difference between the average and differences is 0.037, thus, if the transformation applied by adding 1.635 to the ellipsoidal heights, the error will not exceed 0.04m in the orthometric heights.
- Difference range from 1.598m to 1.672m.
- The mean of the differences is 1.635m.

Chapter Six

Conclusion and Recommendations

6.1 Conclusion

After applying the processing and comparing the results, the following can be concluded:

- The difference between ellipsoidal and orthometric height's in the study area ranges from 1.598m to 1.672m in the study area.
- The computed root mean square error is 0.023m.
- The average difference is 1.635m.
- Discrepancies between a WGS84 ellipsoid, and the geoid vary with location.
- In high accuracy works, it prefers to use relative method (Orthometric heights).

6.2 Recommendations

For the coming researches, it recommended that:

- Expansion the study area and increasing number of check points.
- Creating a geoid model for the study area.
- Checking of transformation formula or application to convert between the two systems.

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