



Sudan University of Science and Technology



Collage of Graduate studies

**Super Conducting Hopping Mechanism and Flux  
Quantization on the Basis of Generalized Statistical  
Physical Model**

آلية القفز في الموصل الفائق و تكميم الفيض علي أساس نموذج الفيزياء  
الإحصائية المعممة

A thesis Submitted for fulfillment for the Requirements of the  
Degree of Doctor Philosophy in Physics

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# Dedication

To my Lovely parents.

## **Acknowledgement**

I would like to express my special appreciation and thanks to my advisor Professor Dr Mubarak Dirar Abd-Allah , you have been a tremendous mentor for me. I would like to thank you for encouraging my research and for allowing me to grow as a research scientist. Your advice on both research as well as on my career have been invaluable.

A special thanks is extent to Sudan University of Science and Technology for giving me this chance for higher studies.

Words cannot express how grateful I am to husband for all of the sacrifices that you've made on my behalf.

Also, I am also very grateful to my, brothers, sisters , and friends, for their good humor and support throughout the production of this project.

## Abstract

Super conductivity is one of the most important physic phenomenons that has a wide variety of applications in technology. It was successfully described by BCS theory. But recently some materials that act as a superconductor at higher temperatures, changes critical temperature due to the effect of pressure or isotope mass. This cannot be explained by BCS model. Some researchers suggest hopping mechanism for explaining high temperature super conductor's behavior. But unfortunately the models proposed are complex and incomplete. The aim of this work is to use the generalized statistical physical model proposed by some researchers was used to explain the conditions that lead to hopping when kinetic energy exceeds coulomb potential, which agrees with that proposed by Hubbard model. The hopping also proved to take place below critical temperature, while it does not exist above this critical temperature. The hopping is also destroyed when magnetic field exceeds certain critical value. The flux is also quantized with in the frame work of this models. The results obtained by this model agree with observations and previous models. Thus one can conclude that generalized statistical model can successfully describe some superconducting phenomena.

## مستخلص

يعتبر التوصيل الفائق من أحد أهم الظواهر الفيزيائية التي لها تطبيقات واسعة في المجال التقني. حيث كانت توصف بنظرية باردين وكوبر وشيفر بنجاح. ولكن مؤخرًا أصبحت بعض المواد ذات التوصيل الفائق عند درجات الحرارة العالية تتغير درجاتها الحرجة بتأثير الضغط وكتلة النظير. وهذه لا يمكن تفسيرها في ضوء نظرية باردين وكوبر وشيفر. لذا اقترح بعض الباحثين آلية القفز لتفسير سلوك الموصلات الفائقة عند درجة الحرارة العالية ولكن لسوء الحظ فإن هذه النماذج المقترحة معقدة وغير مكتملة.

يهدف هذا البحث لاستخدام نموذج معمم للفيزياء الاحصائية، تم اقتراحه بواسطة بعض الباحثين لتفسير الظروف المؤدية للقفز عندما تزيد طاقة الحركة عن جهد كولوم، وهذا ما يتفق مع نموذج هبرد المقترح. وتعتمد طريقة البحث على استخدام التحليل الرياضي المرتكز على قوانين الاحصائية المعممة مع قيود الدرجة الحرجة للحصول على شروط القفز. وقد ثبت ان القفز يحدث عند درجات حرارة اقل من الدرجة الحرجة في حين لا يحدث عند درجات الحرارة الأعلى من هذه الدرجة الحرجة. كما أن القفز لا يحدث عندما تزيد شدة المجال المغنطيسي عن قيمة حرجة معينة. كما أن الفيض يكون مكما في اطار هذا النموذج. كل هذه النتائج التي تم الحصول عليها من هذا النموذج تتوافق مع التجارب ومع النماذج السابقة. وعليه يمكن استنتاج ان نموذج الفيزياء الاحصائية المعمم يمكن ان يفسر بعض ظواهر الاحصائية المعممة.

## Contents

<b>Subject</b>	<b>page</b>
Dedication	I
Acknowledgement	II
Abstract( in English)	III
Abstract(in Arabic)	IV
Contents	V
<b>Chapter one</b>	
<b>Introduction</b>	
1.1The History of Superconductivity	1
1.2Research Problem	2
1.3Literature Review	2
1.4Aim of the Work	3
1.5Thesis Layout	3
<b>Chapter Two</b>	
<b>Superconductivity</b>	
2.1 Introduction	4
2.2 Superconductor Material Properties	4
2.3 Energy Gap	6
2.4 Type I and II Superconductors	7
2.5 Cooper Pairs	9
2.6 London Equations	12
2.7 Josephson Effect	15
2.8 Flux Quantization	22
2.9 Hubbard Model and Conduction	26
2.10 Exact Digitalization	29
2.11Hubbard Model for Superconductivity and Mott Insulator	30
2.12 Coherence Length	32
2.13 High-Temperature Superconductors	35
2.14 Qualitative Explanation of The Spin-Fluctuation	42

Mechanism	
<b>Chapter Three Literature Review</b>	
3.1 Introduction	44
3.2 Effect of Magnetic Field on Super Conduction Complex Resistance According to Quantum Mechanics.	44
3.3 Quantum Effect of Magnetic Field in Destroying Superconductivity	48
3.4 New Derivation of Simple Josephson Effect Relation Using New Quantum Mechanical Equation	51
3.5 Complex Quantum Resistance Model	53
3.6 Using The Tight Binding Approximation in Deriving the Quantum Critical Temperature Superconductivity Equation	59
3.7 Summary and Critique	67
<b>Chapter Four Hopping Mechanisms and Flux Quantization</b>	
4.1 Introduction	68
4.2 Generalized Statistical model Laws Super Conductor	68
4.3 Super conductivity On the basic of generalized statistical Model	72
4.4 Discussion	76
4.5 Conclusion	77
<b>Reference</b>	<b>78</b>

# Chapter One

## Introduction

### 1.1 The History of Superconductivity:

Superconductivity was discovered in 1911 in the Leiden laboratory of Heike Kamerlingh Onnes. He noticed that the resistivity of Hg metal vanished abruptly at about 4K. Although phenomenological models with predictive power were developed in the 1930's and 40's, the microscopic mechanism underlying superconductivity was not discovered until 1957 by Bardeen Cooper and Schrieffer (BCS). Superconductors have been studied intensively for their fundamental interest and for the promise of technological applications which would be possible if a material which super conducts at room temperature were discovered. Until 1986, critical temperatures ( $T_c$ 's) at which resistance drops to zero were always less than about 23K. In 1986, Bednorz and Mueller published a paper, subsequently recognized with the 1987 Nobel Prize, for the discovery of a new class of materials which currently include members with  $T_c$  of about 135K [1, 2, and 3]. Super conductivity (SC) is a phenomenon of exactly zero electrical resistance and expulsion of magnetic flux fields occurring in certain materials when cooled below characteristic critical temperature.

Like ferromagnetism and atomic spectral lines, superconductivity is a quantum mechanical phenomenon. It is characterized by the Meissner effect, the complete ejection of magnetic field lines from the interior of the superconductor as it transitions into the superconducting state. The occurrence of the Meissner effect indicates that superconductivity cannot be understood simply as the idealization of perfect conductivity in classical physics [4, 5].



The electrical resistance of a metallic conductor decreases gradually as temperature is lowered. In ordinary conductors, such as copper or silver, this decrease is limited by impurities and other defects. Even near absolute zero, a real sample of a normal conductor shows some resistance. In a superconductor, the resistance drops abruptly to zero when the material is cooled below its critical temperature. An electric current flowing through a loop of superconducting wire can persist indefinitely with no power source [6, 7].

In 1986, it was discovered that some cuprate ceramic materials have a critical temperature above 90 K ( $-183\text{ }^{\circ}\text{C}$ ) [5]. Such a high transition temperature is theoretically impossible for a conventional superconductor, leading the materials to be termed high-temperature superconductors. Liquid nitrogen boils at 77K, and superconductor at higher temperatures than this facilitates many experiments and applications that are less practical at lower temperatures [11, 12].

## **1.2 Research Problem :**

The intensive research in scaled to appearance of many SC phenomena that cannot be explained theoretically. The research problem is related to the fact that (SC) model used to describe their phenomenology are in complete and complex [13, 14, 15].

## **1.3 Literature Review:**

The SC phenomenology which can not be explained by ordinary theory opens windows for new theoretical models [16, 17, 18, 19 and 20]. Some of them try to explain conduction mechanism, while others try to explain pressure effect, isotope effect and phase diagram [21, 22, 23 and 24]. Unfortunately these models are complex and in complete.

#### **1.4 Aim of the work:**

The useful applications of SC in technology needs completes SC theory that can lead to discovery of new materials.

One needs to use new statistical model and Schrodinger equation to explain some SC phenomena.

#### **1.5 Thesis Layout:**

The thesis consists of four chapters. Chapters one and two are the introduction and theoretical back ground in (SC) chapters three and four are devoted for literature review and contribution respectively.

# Chapter Two

## Superconductivity

### 2.1 Introduction:

This chapter describes basic properties of a superconductor beside phenomenology of superconductors, theoretical background, including London equation beside to flux quantization and Hubbard model [20, 21].

### 2.2 Superconductor Material Properties:

Superconductivity is characterized by Zero resistance, Meissner effect, magnetic flux quantization, Josephson effects, and energy gap.

We shall briefly describe these properties [17, 20, and 21].

#### 2.2.1 Zero Resistance:

The phenomenon of superconductivity was discovered in 1911 by Kamerling Onnes [1], who measured extremely small electric resistance in mercury below. A certain critical temperature  $T_c$  ( $\approx 4.2\text{k}$ ) [see Fig (2.1)]. This zero resistance property can be confirmed by a never-decaying super current ring [17].

#### 2.2.2 Magnetic Properties:

Substances that become superconducting at finite temperatures will be called superconductor in the present text if a superconductor below  $T_c$  is placed under a weak magnetic field it repels the magnetic flux (field)  $B$  completely [20,21].

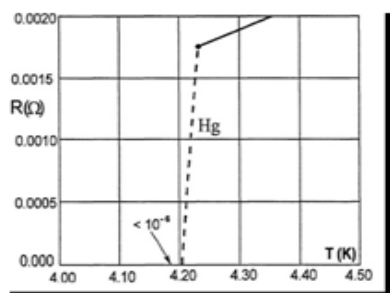


Fig2.1 Resistance versus temperature, after kamerling onnes

This is called the Meissner effect, and it was discovered by Meissner and Chosen field in 1933 [2].

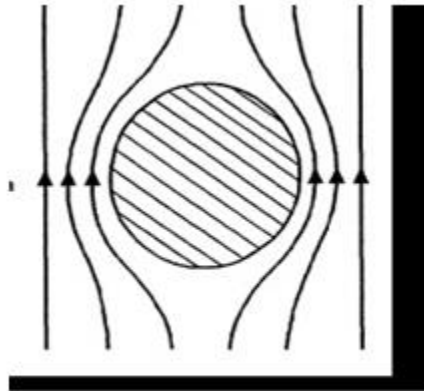


fig:2.2 A superconductor expels amagnetic

The Meissner effect can be demonstrated dramatically by a floating magnet as shown in Fig: 2.3. A small bar magnet above  $T_c$  simply rests on Super conductor dish. If temperature  $T_c$  is lowered below the magnet will float as indicated. The gravitational force exerted on the magnet is balanced by the magnetic pressure (part of electromagnetic stress tensor) due to the in homogeneous magnetic field (B-field) surrounding the magnet which is represented by the magnetic flux lines. The later more refined experiments reveal that the (B-field) penetrates into the superconductor within a very thin surface layer.

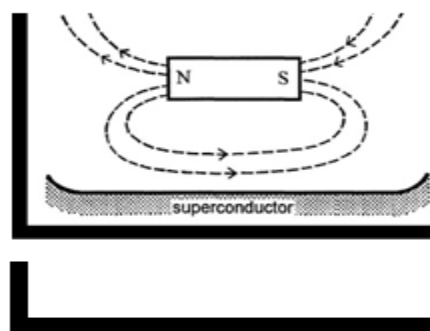


fig:2.3 A floating magnet

When the external field is applied parallel to the boundary, the (B-field) falls off exponentially

$$B(x) = B(0)e^{\frac{-x}{\lambda}} \quad (2.2.1)$$

$\lambda$  is called a penetration depth, and it is on the order of 500 in most superconductors. At lowest temperatures it is a small value on macroscopic scale. This allows us to speak of the superconductor as being perfectly diamagnetic. Penetration depth  $\lambda$  plays a very important role in the description of the magnetic properties [30, 31, and 23].

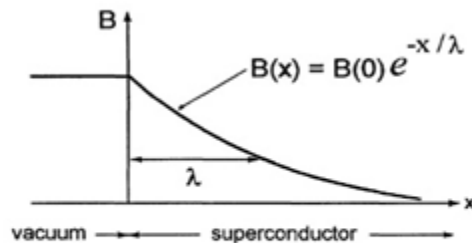


Fig:2.4 Penetration of the magnetic field in to superconductor slab

### 2.3 Energy Gap:

If a continuous band of the excitation energy is separated by a finite gap  $\epsilon_g$  from the discrete ground-state energy level as shown in Fig. 2.5 this gap can be detected by photo-absorption [9]. Quantum tunneling [10]. Heat Capacity [11] and other experiments. The energy gap at  $T_C$  turns out to be temperature-dependent. The energy gap as determined from the tunneling experiments [12] is shown in Fig. 2.6. The energy gap is zero at and reaches a maximum value  $\epsilon_g$  as temperature approaches toward.

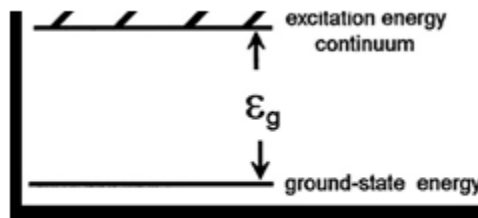


Fig:2.5 Excitation-energy spectrum with a gap

In true thermodynamic equilibrium, there can be no currents, super or normal. Thus we must deal with a non-equilibrium condition when discussing the basic properties of superconductors, such as zero resistance,

flux quantization, and Josephson effects. All of these arise from the super currents that dominate the transport and magnetic phenomena. When superconductors used to form a circuit with a battery and a steady state is established, all currents passing the superconductor are super currents. Normal currents due to the motion of electrons and other charged particles do not show up because no voltage difference can be developed in a homogeneous superconductor

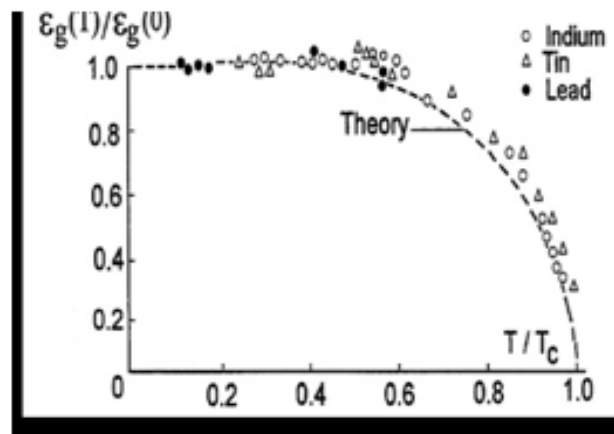


Fig:2.6: The energy gap  $\epsilon_g(T)$  versus temperature

## 2.4 Type I and II Superconductors:

A type I superconductor is the one that in which the (B-field) remains zero inside the superconductor until suddenly the superconductivity is destroyed.

The field where this happens is called the critical field,  $H_C$ [6].

The way magnetization  $M$  changes with  $H$  in a type I superconductor, as shown, the magnetization obeys.

$$M = -\mu_0 H \quad (2.4.1)$$

Where

$$B = \mu_0 H - M = (\mu_0 + x)H = 0$$

For all fields less than  $H_C$  inside the superconductor, and then becomes zero

(or very close to zero) for fields above  $H_c$  many superconductors, however, behave differently.

In a type II superconductor there are two different critical fields, denoted  $H_{c1}$ , the lower Critical field and  $H_{c2}$  the upper critical field.

For small values of applied field  $H$  again Meissner effect leads to  $M = -\mu_0 H$  and there is no magnetic flux density inside the sample,  $B = 0$ . However in a type II superconductor once the field exceeds  $H_{c1}$ , magnetic flux does start to enter the superconductor and hence  $B \neq 0$ , and  $M$  is closer to zero than the full V Meissner effect value of  $-\mu_0 H$ . Upon increasing the field  $H$  further the magnetic flux density gradually increases, until finally at  $H_{c2}$  the superconductivity is destroyed and  $M = 0$ . As a function of the temperature the critical fields vary, and they all approach zero at the critical temperature  $T_c$ . The typical phase diagrams of type I and type II superconductors as a function of  $H$  and  $T$ , are shown in Fig. (2.7).

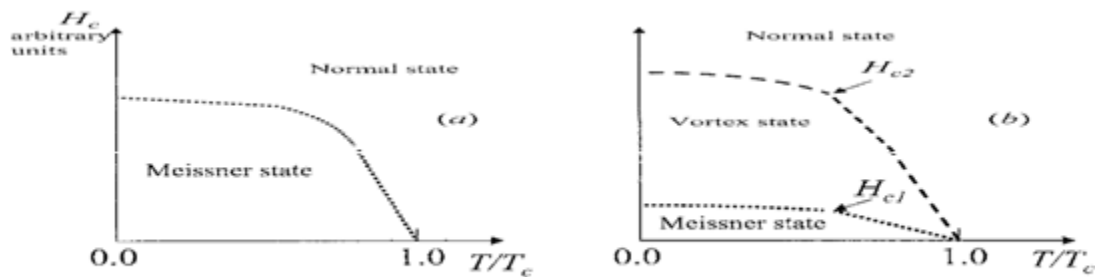


Fig.2.7: The H-T phase diagram of bulk type I (a) and type II (b)

In type II superconductors the phase below  $H_{c1}$  is normally denoted the Meissner state, while the phase between  $H_{c1}$  and  $H_{c2}$  is the vortex or Abrikosov state.

The physical explanation of the thermodynamic phase between  $H_{c1}$  and  $H_{c2}$  was given by Abrikosov. He showed that the magnetic field can enter the superconductor in the form of vortices, each vortex consists of a region of circulating super current around a small central core which has essentially become normal metal. The magnetic field is able to

pass through the sample inside the vortex cores, and the circulating currents serve to screen out the magnetic field from the rest of the superconductor outside the vortex.

It turns out that each vortex carries a fixed unit of magnetic flux

$$\varphi_0 = \frac{h}{2e}$$

where  $h$  is Planck constant and  $e$  is the electron charge. And hence, if there are a total of  $N_v$  vortices in a sample of total area,  $A$  then the average magnetic flux density,  $B$  is

$$B = \frac{N_v}{A} \cdot \frac{h}{2e} \quad (2.4.2)$$

It is instructive to compare this result for the number of vortices per unit area,

$$\frac{N_v}{A} = \frac{2eB}{h} \quad (2.4.3)$$

With the similar expression derived earlier for the density of vortices in rotating super fluid  $^4\text{He}$ . There is in fact a direct mathematical analogy between the effect of a uniform rotation at angular frequency in a neutral super fluid, and the effect of a magnetic field,  $B$  in a superconductor [6, 7].

## 2.5 Cooper Pairs:

We consider a metal at  $T \sim 0$ . All states inside the Fermi sphere are filled with electrons while all states outside are empty

In 1956 Cooper studied [18]. what would happen if two electrons were added to the filled Fermi sphere with equal but opposite moment  $\vec{p}_1 = -\vec{p}_2$

Whose magnitude was slightly larger than the Fermi momentum  $p_f$ . Assuming that a weak attractive force existed he was able to show that the electrons form a bound system with an energy less than twice the Fermi energy,

$$E_{\text{pair}} < 2E_f.$$



The mathematics of Cooper pair formation will be outlined in [18]. What could be the reason for such an attractive force First of all one has to realize that the Coulomb repulsion between the two electrons has a very short range as it is shielded by the positive Ions and the other electrons in the metal. So the attractive force must not be strong if the electrons are several lattice constants apart. Already in 1950, Froehlich and, independently, Bardeen had suggested that a dynamical lattice polarization may create a weak attractive potential. Before going into details Let us look at a familiar example of attraction caused by the deformation of a medium: a metal ball is placed on an elastic membrane and deforms the membrane such that a potential well is created. A second Ball will feel this potential well and will be attracted by it. So effectively, the deformation of the elastic Membrane causes an attractive force between the two balls which would otherwise not notice each other. This visualization of a Cooper-pair is well known in the superconductivity community but it has the disadvantage that it is a static picture.

Suppose you are cross-country skiing in very deep snow. You will find this quite cumbersome; there is a lot of resistance. Now you discover a track made by you another skier, a Lope, and you will immediately realize that it is much more comfortable to ski along this track than in any other direction. The Lope picture can be adopted for our electrons. The first Electron flies through the lattice and attracts the positive ions. Because of their inertia they cannot follow immediately, the shortest response time corresponds to the highest possible lattice vibration frequency [19]. This is called the Debye frequency  $\omega_D$ . The maximum lattice deformation lags behind the electron by a distance

$$d = v_f \frac{2\pi}{\omega_D} \sim 100 - 1000 \text{nm}$$

Obviously, the lattice deformation attracts the second electron because there is an accumulation of positive charge [8, 9, and 10].

The attraction is strongest when the second electron moves right along the track of the First one and when it is a distance  $d$  behind it, see Fig. 21. This explains why a Cooper pair is a much extended object, the two electrons may be several 100 to 1000 lattice constants apart. For a simple cubic lattice, the lattice constant is the distance between adjacent atoms.

In the example of the cross-country skiers or the electrons in the crystal lattice, intuition suggests. That the second partner should preferably have the same momentum,  $\vec{p}_2 = \vec{p}_1$  although opposite moment  $\vec{p}_2 = -\vec{p}_1$  are not so bad either. Quantum theory makes a unique choice only electrons of opposite

Moment form a bound system, a Cooper pair. The quantum theoretical reason is the Pauli principle but there exists probably no intuitive argument why electrons obey the Pauli exclusion principle and thus extreme individualists while other particles like the photons in a laser or the atoms in super fluid helium do just the opposite and behave as extreme Conformists. One may get used to quantum theory but certain mysteries and strange feelings will remain. The binding energy of a Cooper pair turns out to be small,  $10^{-4}$ ,  $10^{-3}$ , so low temperatures are needed to preserve the binding in spite of the thermal motion. According to Heisenberg's Uncertainty Principle a weak binding is equivalent to a large extension of the composite system, in this case the above -mentioned  $d = 100 - 1000$  nm. As a consequence, the Cooper pairs in a superconductor overlap each other. In the space occupied by a Cooper pair there is about a million other Cooper pairs. The situation is totally different from other composite systems like atomic nuclei or atoms which are tightly bound objects and well-separated from another. The strong overlap is an important prerequisite of the BCS

theory because the Cooper pairs must change their partners frequently. In order to provide a continuous binding [12].

## 2.6 London Equations:

The London brother proposed a simple theory to explain the Meissner effect, the London equations provided an early simple model for describing experimental result. In 1935 theory of London brothers provides the first and second London equations, which relate the electric and magnetic fields  $E$  and  $B$ , respectively, in a superconductor to the current density  $J$  [1, 4].

$$E = \mu_0 \lambda_L^2 \frac{dJ}{dt} \quad (2.6.1)$$

$$B = -\mu_0 \lambda_L^2 \nabla \times J \quad (2.6.2)$$

Where  $\mu_0$  is the permeability of vacuum, the constant of proportionality in these expressions is the London penetration depth  $\lambda_L$ , where  $n_s$  the density of superconducting electron is,  $m$  is the electron mass.

### 2.6.1 Derivation of first London equation:

A potential difference applied along a conducting wire produces an electric field  $E$ , and hence the force  $F$  on any electron is given by:

$$F = eE = m \frac{dv}{dt} \quad (2.6.3)$$

Where  $v$  stands for its velocity, electron undergo successive periods of acceleration interrupted by collision and during the average time relaxation time (scattering on defect  $s$ )  $\tau$  between collisions. The velocity is given by which called the drift velocity. The negative sign means that the electrons move in a direction opposite to that of the electric field [35, 36, 38 and 39].

When the electron is assumed to move in a resistive medium, which have frictional force proportional to the velocity, the electron equation of motion is given by.

$$v = -\frac{eE}{m}\tau \quad (2.6.4)$$

This called the drift velocity. The negative sign means that the electrons move in a direction opposite to that of the electric field. When the electron is assumed to move in a resistive medium, which have, frictional force proportional to the velocity, the electron equation of motion is given by:

$$m\frac{dV}{dt} = eE - m\frac{V}{\tau} \quad (2.6.5)$$

Where the frictional force is given by:

$$F = ma, V = V_0 + a\tau = 0 + a\tau = a\tau$$

$$F = \frac{mV}{\tau}$$

For steady state in normal metal, no acceleration exists. i.e.

$$\frac{dV}{dt} = 0$$

Therefore

$$V = \frac{eE}{m}\tau \quad (2.6.6)$$

Hence the current density is given by:

$$J = nev = \frac{ne^2\tau}{m}E = \sigma E \quad (2.6.7)$$

Where  $n$  the density of electrons, is electrical conductivity.

In the two fluid models one has the temperature dependent expression for the super  $n_s$  and normal  $n_n$  electrons densities respectively,

$$n_n(T) + n_s(T) = n$$

The total electron density  $n$  is independent of temperature and at  $T = 0$  one have  $n_n(0) = 0(0)$  and  $n_s(0) = n$ , and the simple theory predicts the following temperature dependences:

$$n_s(T) = n\left(\frac{T}{T_c}\right)^4 \quad (2.6.8)$$

Where  $T_c$  is the critical temperature?

For superconductor below  $T_c$  the resistive force is zero, in this case equation (2.6.8) becomes

$$\frac{dv}{dt} = \frac{eE}{m} \quad (2.6.9)$$

Taking the derivative of  $J$  in equation (2.6.10) with respect to time:

$$E = m \frac{dv}{dt} = eE \quad (2.6.10)$$

$$\frac{dJ}{dt} = ne \frac{dv}{dt} = \frac{ne^2}{m} E \quad (2.6.11)$$

The term  $\frac{m}{n_s e^2} = \Lambda$  is phenomenological parameter equation (2.6.11) can thus be rewritten as:

$$E = \frac{d}{dt}(AJ) = \Lambda \frac{dJ}{dt} \quad (2.6.12)$$

This equation is known as the first London equation.

### 2.6.2 Second London equation:

This equation is concerned with time-dependent fields, and is important For Meissonier effect. The electric current density is given quite generally by:

$$J = nqv \quad (2.6.13)$$

Where  $n$  is concentration of carriers of charge  $q$  In the presence of a magneticfield described by the vector potential  $A$ , the velocity  $v$  is related to the total momentum  $P$  by:

$$p = mv + \frac{q}{c} \Lambda \quad v = \frac{1}{m} \left( p - \frac{q}{c} \Lambda \right) \quad (2.6.14)$$

Where  $m$  is the mass,  $c$  the speed of light in vacuum Thus equation (2.6.13) can be written as

$$J = \frac{nq}{m} p - \frac{nq^2}{me} \Lambda \quad (2.6.15)$$

In the superconducting state, the total momentum  $p$  is zero, although it not equal to zero in normal state .i.e.  $p = 0$ , and equation (2.6.15) reduces to:

$$J = -\frac{nq^2}{mc}A \quad (2.6.16)$$

For electrons,  $q = e$ ,  $n = n_s$  then:

$$J = -\frac{n_s e^2}{mc}A \quad (2.6.17)$$

The vector potential is related to the magnetic field by

$$B = \nabla \times A \quad (2.6.18)$$

Equation (2.6.17) can be rewritten with the aid of (2.6.3) to be

$$J = -\frac{c}{4\pi\lambda_L^2}A \quad (2.6.19)$$

This equation is known as the second London equation.

Equation (2.6.17) can be expressed in another way by taking the curl of both sides and using equation (2.6.18) to obtain

$$\begin{aligned} \nabla \times J &= -\frac{c}{4\pi\lambda_L^2}(\nabla \times A) = -\frac{c}{4\pi\lambda_L^2}B \\ B &= -c\Lambda \nabla \times J \end{aligned} \quad (2.6.20)$$

Where

$$\Lambda = \frac{m}{n_s e^2} = \frac{4\pi\lambda_L^2}{e^2}$$

A phenomenological parameter is a phenomenological parameter.

Equation (2.6.20) is another form of the second equation of London [1, 2, and 5].

## 2.7 Josephson Effect:-

If one takes two SC separated by an oxide Layer quantum tunneling takes place. Consider two metals separated by an insulator, as in the Finger below

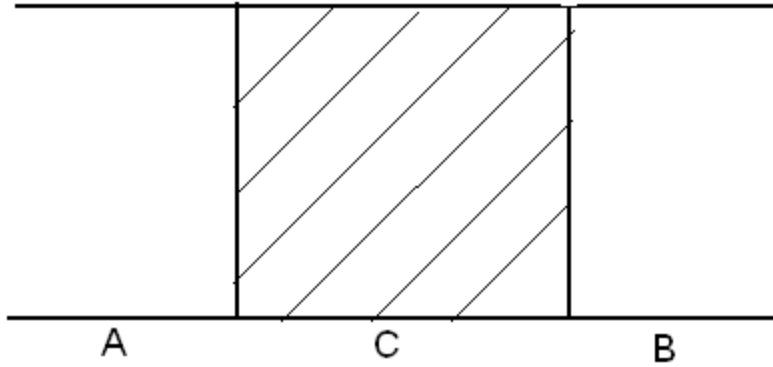


Fig:2.7: Two metals A and B separated a thin layer of insulator C

The insulator normally acts as a barrier to the flow of conduction electrons from one metal to the other. If the barrier is sufficiently thin (less than 10 or 20 Å), there is a significant probability that an electron, which impinges on the barrier, will pass from one metal to the other this is called tunneling. The concept that particles can tunnel through potential barriers is as old as quantum mechanics [1, 2, 3, 4].

When both metals are normal conductors, the current-voltage relation of sandwich or tunneling junction is home at low voltages, with the current directly proportional to the applied voltage. Discovered that if one of the metals becomes superconducting the current-voltage characteristic changes from the straight line to curve [5,6,7].

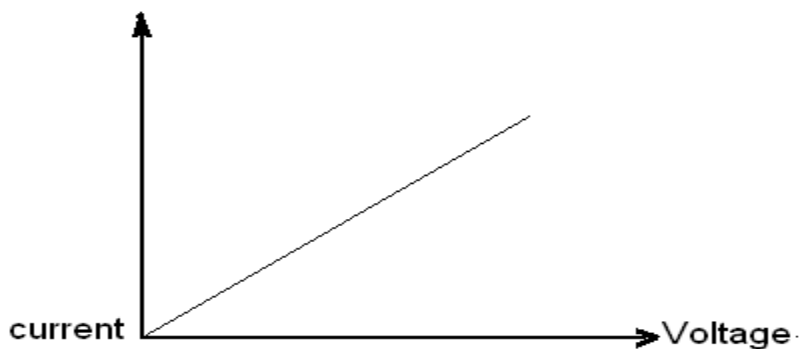


Fig:2.9: Linear current-voltage relation for junction of normal metals

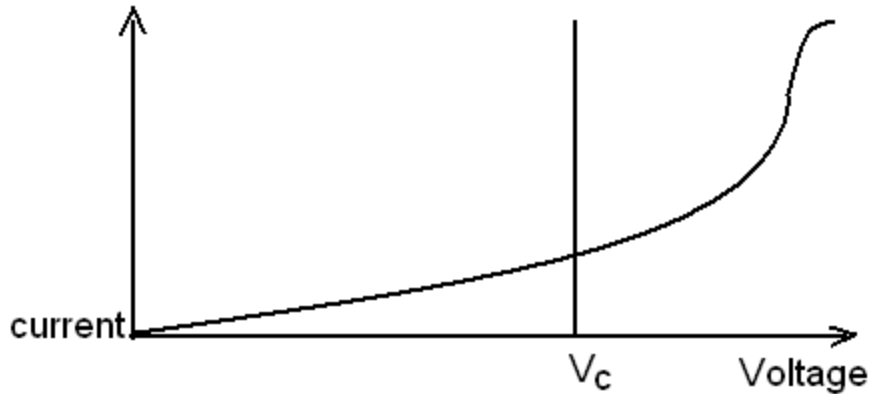


Fig:2.10: Current-voltage relation with one metal normal and the other metal superconducting

In the superconductors there is an energy gap centered at the Fermi level at absolute zero no current can flow until the voltage is

$$V = \frac{E_g}{2e} = \frac{\Delta}{e}$$

Where

$$\Delta = \frac{E_g}{2}$$

The energy gap  $E_g$  corresponds to the break-up of electrons in the superconducting state, with the formation of two electrons and a hole in the normal state.

The current starts when  $V = \Delta$ . At temperatures different from zero there is a small current flow even at low voltage, because of electrons in the superconductor that are thermally excited across the energy gap.

Under suitable conditions, remarkable effect can be observed associated with the tunneling of superconducting electron pairs from a superconductor through a layer of an insulator into another superconductor. The effects of pair tunneling are quite unlike single particle tunneling and include.

### 2.7.1 DC Josephson Effect:

Let  $\psi_1$  be probability amplitude of electron pairs on one side of a junction, and let  $\psi_2$  be the amplitude the other side. For the simplicity, let



both superconductors be identical. For the present, we suppose that they are both at zero potential.

The time dependent Schrödinger equation  $i\hbar \frac{\partial \psi}{\partial t} = H\psi$  applied to the two amplitudes gives:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 \quad , \quad i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1 \quad (2.7.1)$$

Here  $\hbar T$  represents the effect of the electron-pair coupling or transfer interaction across the insulator; thus the dimensions of a rate or frequency. It is a measure of the leakage of  $\psi_1$  in to region 2, and  $\psi_2$  in to the region 1. If the insulator is very thick,  $T$  is zero and there is no pair tunneling.

Let

$$\psi_1 = n_1^{1/2} e^{i\theta_1} \quad , \quad \psi_2 = n_2^{1/2} e^{i\theta_2} \quad (2.7.2)$$

Where  $n_1$  is the electron density in the region 1,  $\theta_1$  the phase angle  $n_2$

Is the electron density in the region 2,  $\theta_2$  is the phase angle.

Then:

$$\frac{\partial \psi_1}{\partial t} = \frac{1}{2} n_1^{-1/2} e^{i\theta_1} \frac{\partial n_1}{\partial t} + i\psi_1 \frac{\partial \theta_1}{\partial t} = -T\psi_2 \quad (2.7.3)$$

With the use of equation (2.7.1) in the form

$$i\hbar \frac{\partial \psi_1}{\partial t} = -\hbar T \psi_2$$

Similarly

$$\frac{\partial \psi_2}{\partial t} = \frac{1}{2} n_2^{-1/2} e^{i\theta_2} \frac{\partial n_2}{\partial t} + i\psi_2 \frac{\partial \theta_2}{\partial t} = -T\psi_1 \quad (2.7.4)$$

Multiplying equation (2.7.3) by

$$n_1^{-\frac{1}{2}} e^{i\theta_1} \quad \text{With } \delta \equiv \theta_2 - \theta_1$$

One obtains:

$$\frac{\partial n_1}{2 \partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -iT(n_1 n_2)^{\frac{1}{2}} e^{i\delta} \quad (2.7.5)$$

Multiplying equation (2.7.4) by

$$n_2^{-1/2} e^{i\theta_2} \text{ With } \delta \equiv \theta_2 - \theta_1$$

One obtains:

$$\frac{\partial n_2}{2 \partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -iT(n_1 n_2)^{\frac{1}{2}} e^{i\delta} \quad (2.7.6)$$

New equating the real and the imaginary parts of equation (2.7.5) and equation (2.7.6) similarly one gets:

$$\frac{\partial n_1}{\partial t} = 2T(n_1 n_2)^{\frac{1}{2}} \sin \delta \quad \frac{\partial n_2}{\partial t} = -2T(n_1 n_2)^{\frac{1}{2}} \sin \delta \quad (2.7.7)$$

$$\frac{\partial \theta_1}{\partial t} = -T \left( \frac{n_2}{n_1} \right)^{1/2} \cos \delta \quad \frac{\partial \theta_2}{\partial t} = -T \left( \frac{n_2}{n_1} \right)^{\frac{1}{2}} \quad (2.7.8)$$

If  $n_1 \approx n_2$  as for identical superconductors 1 and 2, it follows from equation (2.7.8)

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} \frac{\partial}{\partial t} (\theta_2 - \theta_1) = 0 \quad (2.7.9)$$

From equation (2.7.7) it is clear that:

$$\frac{\partial \theta_2}{\partial t} = - \frac{\partial n_1}{\partial t} \quad (2.7.10)$$

The current flow from 1 to 2 is proportional to  $\frac{\partial n_2}{\partial t}$ , the same thing from 2 to 1 is proportional to  $-\frac{\partial n_1}{\partial t}$ . Therefrom one can conclude from equation (2.7.7) that the current  $J$  of the superconductor pairs across the junction depends on the phase difference  $\delta$  as

$$J = \frac{\partial n_1}{\partial t} - \frac{\partial n_2}{\partial t} = T(n_1 n_2)^{\frac{1}{2}} \sin \delta$$

$$J = J_0 \sin \delta = J_0 \sin(\theta_2 - \theta_1) \quad (2.7.11)$$

Where  $J_0$  is proportional to the transfer interaction  $T$ . The current  $J_0$  is the maximum zero-voltage current that can be passed through the junction.

With no applied voltage a DC current will flow across the junction, [figer :( 3.7)] with a value between  $J_0$  and  $-J_0$  according to the value of the phase difference  $\theta_2 - \theta_1$ . This is the DC Josephson Effect.

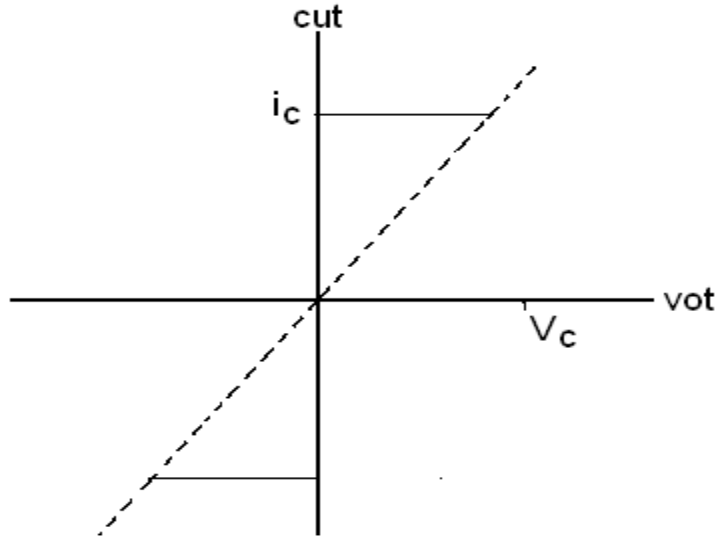


fig:2.11:Current-voltage characteristic of a Josephson junction

DC Currents flow under zero applied voltage up to a critical current  $i_c$ ; this is the DC Josephson Effect. At voltage above  $V_c$  the junction has a finite  $V\hbar$ , this AC Josephson Effect.

### 2.7.2 AC Josephson Effect:

Let voltage  $V$  be applied across the junction. This can be done because the junction is an insulator. An electron pair experiences a potential energy difference  $qV$  on passing across the junction where  $-eV$  and a pair on the other side at  $eV$ . the equations of motion that replaces (2.7.1) is:

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 - eV \psi_1 \quad i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1 - eV \psi_2 \quad (2.7.12)$$

Proceeding as above to find in place of (3.7.5) the equation:

$$\frac{1}{2} \frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = \frac{i e n_1 V}{\hbar} - i T (n_1 n_2)^{\frac{1}{2}} e^{i\delta} \quad (2.7.13)$$

Taking the real parts on sides one gets:

$$\frac{\partial n_1}{2 \partial t} = 2 T (n_1 n_2)^{\frac{1}{2}} \sin \delta \quad (2.7.14)$$

The imaginary contribution also gives:

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T \left( \frac{n_2}{n_1} \right)^{\frac{1}{2}} \cos \delta \quad (2.7.15)$$

Which differs from equation (2.7.8) by the term  $\frac{eV}{\hbar}$ .

Similarly as equation (2.7.13) the equation (2.7.6) for  $n_2$  takes the form:

$$\frac{1}{2} \frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -i \frac{eV n_2}{\hbar} - iT(n_1 n_2)^{\frac{1}{2}} e^{i\delta} \quad (2.7.16)$$

Hence equation real and imaginary parts one gets:

$$\frac{\partial n_2}{2 \partial t} = -2T(n_1 n_2)^{\frac{1}{2}} \sin \delta \quad (2.7.17)$$

$$\frac{\partial \theta_2}{\partial t} = -\frac{eV}{\hbar} - T \left( \frac{n_2}{n_1} \right)^{\frac{1}{2}} \cos \delta \quad (2.7.18)$$

From (2.7.15) and (2.7.18) with  $n_1 \approx n_2$  one nave:

$$\frac{\partial(\theta_2 - \theta_1)}{\partial t} = \frac{\partial \delta}{\partial t} = -\frac{2eV}{\hbar} \quad (2.7.19)$$

By integration of (2.7.19) that with a DC voltage across the junction the relative phase of the probity amplitudes vary as:

$$\delta(t) = \delta(0) - \frac{2eV}{\hbar} t \quad (2.7.20)$$

$$J = J_0 \sin \left[ \delta(0) - \frac{2eV}{\hbar} t \right] \quad (2.7.21)$$

The current oscillates with frequency:

$$\omega = \frac{eV}{\hbar} t \quad (2.7.22)$$

Which says that a photon of energy  $\hbar\omega = 2eV$  is emitted or absorbed when an electron pair crosses the barrier. By measuring the voltage and the frequency. It is possible to obtain a very precise value of  $\frac{e}{\hbar}$ . [1, 2, 7, 9].

### 2.7.3 Macroscopic Long-Ran Quantum Interference:

A DC magnetic field applied through a superconducting circuit containing two junctions causes the maximum super current to show

interference effects as a function of magnetic field intensity. This effect can be utilized in sensitive magnetometers.

## 2.8 Flux Quantization:

When a type II superconductor is immersed in an intermediate magnetic field to transfer it into a mixed state, the bulk of the material is superconducting, but it is threaded by thin filaments of normal material. The vortex lines are oriented parallel to the external magnetic field and they serve as paths for the magnetic flux lines of the external field. A current circulates around the perimeter of each vortex line. These currents shield the bulk of the superconductor from the magnetic field in the filaments, the flow of this current has the character of a vortex and that is why that the filaments were called as vortex lines.

Increasing the magnetic field would not cause an increase of the flux associated with each vortex line, instead. It will cause an increase in the number of vortex lines threading the superconductor. The stronger the external field the more densely will pack the vortex lines. The ends of the vortex lines at surface of a superconducting (type – II) material in the mixed state have been made visible by dusting the surface with powdered iron.

The vortex lines are packed in the form of heaps having regular pattern on the surface. Knowing the magnetic field intensity and the number of vortex lines per square cm, it was found that the amount of flux associated with each vortex line has a fixed value related to Planck's constant, and the electric charge of the electron.

The quantum of flux

$$\varphi_0 = \frac{h}{2e} = 2.07 \times 10^{-7} \text{ Tm}^2$$

In general the flux

$$\varphi = n\varphi_0$$

Where

$$n = 1, 2, 3 \dots$$

This result confirms the significance of electron pairs in the composition of the Superconducting state. Flux quantization is a beautiful example of a long range quantum effect, in the instance of a ring the coherence of the superconducting state extends over the ring [9,10,11,12].

The electromagnetic field is an example of a boson field. The electric field Intensity  $E(r)$  acts qualitatively as field amplitude. The energy density may be

Written as, in a semi classical approximation,

$$\frac{E^*(r)E(r)}{4\pi} \approx n(r)h\omega \quad (2.8.1)$$

Where  $n(r)$  is the number of photons of frequency  $\omega$  per unit volume.

Assume that the total number of photon in the volume is large in comparison with unity. Then

$$E(r) \approx (4\pi h\omega)^{\frac{1}{2}} n(r)^{\frac{1}{2}} e^{i\theta(r)} \quad (2.8.2)$$

$$E^*(r) \approx (4\pi h\omega)^{\frac{1}{2}} n(r)^{\frac{1}{2}} e^{-i\theta(r)} \quad (2.8.3)$$

Where  $\varphi(r)$  the phase of the field.

Now introduce similar particle probability amplitudes into the description of particle bosons, where a particle is an electron pair (the analogy with photon is not exact, but it is helpful).

The ground state of superconductor is made up of weakly-bound electron pairs, called cooper pairs. An electron pair will act as a boson, although a single electron is fermions. The arguments that follow apply specifically to boson gas with a very large number of bosons in the same orbital. we can then treat the boson probability amplitude as a classical quantity, just as the electromagnetic field is used for photons. The arguments do not apply to a

metal in the normal state because an electron in normal state acts as single unpaired fermions.

First how that a charged boson gas obeys the London equation in theorem

$$\nabla \times \mathbf{J} = -\frac{e^2}{4\pi\lambda_L^2} \mathbf{B} = -\frac{1}{\mu_0\lambda_L^2} \mathbf{B} \quad (2.8.4)$$

Let  $\psi(\mathbf{r})$  be the particle probability amplitude. Suppose that the Concentration

$$n = \langle \psi(\mathbf{r}) | \psi(\mathbf{r}) \rangle = \text{constant}$$

At absolute zero  $n$  is one-half of the concentration of electrons in the conduction band, for  $n$  refers to pairs. Then

$$\psi(\mathbf{r}) = n^{1/2} e^{i\theta(\mathbf{r})} \quad (2.8.5)$$

The phase  $\theta(\mathbf{r})$  is important for what follows: make the good approximation that  $\psi(\mathbf{r})$  is classical amplitude rather than a quantum field operator. The velocity of a particle is:

$$\mathbf{v} = \frac{1}{m} \left( \mathbf{p} - \frac{q}{c} \mathbf{A} \right) = \frac{1}{m} \quad (2.8.6)$$

The particle flux is given by

$$\begin{aligned} n\mathbf{v} &= \langle \psi(\mathbf{r}) | \mathbf{v} | \psi(\mathbf{r}) \rangle = \psi^*(\mathbf{r}) \left( -i\hbar\nabla - \frac{q}{c} \mathbf{A} \right) \frac{\psi(\mathbf{r})}{m} \\ &= \psi^*(\mathbf{r}) \left( -i\hbar\nabla - \frac{q}{c} \mathbf{A} \right) \frac{n^{1/2} e^{i\theta(\mathbf{r})}}{m} \\ &= \frac{\psi^*}{m} \left( -i\hbar n^{1/2} e^{i\theta(\mathbf{r})} \nabla \theta(\mathbf{r}) - \frac{q}{c} \mathbf{A} e^{i\theta(\mathbf{r})} n^{1/2} \right) \\ &= \frac{n^{1/2} e^{-i\theta(\mathbf{r})}}{m} \left( -i\hbar n^{1/2} e^{i\theta(\mathbf{r})} \nabla \theta(\mathbf{r}) - \frac{q}{c} \mathbf{A} e^{i\theta(\mathbf{r})} n^{1/2} \right) \\ &= \frac{n}{m} \left( \hbar \nabla \theta - \frac{q}{c} \mathbf{A} \right) \end{aligned}$$

So that the electric density in the ring (which is a multiply -connected region) is:

$$\mathbf{J} = nq\mathbf{v} = \psi^*(\mathbf{r})\psi(\mathbf{r})q\mathbf{v} = nq \left( \hbar \nabla \theta - \frac{q}{c} \mathbf{A} \right) \frac{1}{m} \quad (2.8.7)$$

Taking the curl of both sides one obtains:

$$\nabla \times \mathbf{J} = \frac{nq}{m} (\hbar \nabla \times \nabla \theta(r) - \frac{Q}{C} \times \mathbf{A})$$

From equation (2.8.4)  $\mathbf{B} = \nabla \times \mathbf{A}$ , with use of the fact that the curl of the gradient of a scalar is identically zero, i.e.  $\nabla \times \nabla \theta = 0$  The above equation [2.6.2]. is one form of the London equation

$$\nabla \times \mathbf{J} = \frac{nq^2}{mc} \mathbf{B} \quad (2.8.8)$$

The quantization of the magnetic flux through a ring is a dramatic consequence of the equation of the electric current density  $\mathbf{J}$  above. Let us take a close path  $C$  through the interior of the superconducting material, well away from the surface. The Meissonier effect tells us that  $\mathbf{B}$  and  $\mathbf{J}$  are zero in the interior. Now from equation (2. 7.11)

$$\mathbf{J} = 0 \text{ if } \hbar \nabla \theta_c = q\mathbf{A} \quad (2.8.9)$$

However, we have

$$\oint \nabla \theta \times d\mathbf{L} = \theta_2 - \theta_1 \quad (2.7.10)$$

Hence:

$$c\hbar \oint_C \nabla \theta \times d\mathbf{L} = \oint_C \mathbf{A} \cdot d\mathbf{L} \quad (2.8.11)$$

For the change of phase ongoing once around the ring, the boson probability amplitude is measurable in the classical approximation, so that

must be singing and

$$\theta_2 - \theta_1 = 2\pi S \quad (2.8.12)$$

Where  $S$  is an integer . Also have been the stocks theorem and the fact  $\text{curl } \mathbf{A} = \mathbf{B}$ :

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \oint_C (\text{curl } \mathbf{A}) \cdot d\boldsymbol{\sigma} = \oint_C \mathbf{B} \cdot d\boldsymbol{\sigma} = \varphi \quad (2.7.13)$$



## 2.9 Hubbard Model and Conduction:

Motivated by the successes of production and annihilation in quantizing field, encourages some physics to utilize the same framework to try to solve some long standing problems of condensed matter physic. This trial is not surprising as far as the system of fields consist of a large number of mediators carrying the field force, which is similar to condensed matter which consists of huge number of particles. The physics based on the notion of production operators utilized in solid state physics is called Hubbard model [16, 17].

### 2.9.1 Non- Interacting Electrons:

The Hamiltonian of anon-interacting fermions on a lattice of L sites labeled by i,j takes the form [17].

$$H_0 = \sum_{ij} t_{ij} c_i^\dagger c_j \quad (2.9.1)$$

Localized at site j and described by the winner wave function  $\phi_j$ . These Operators satisfy the anti-commutation relations

$$\{c_i^\dagger, c_j\} = \delta_{ij} \quad (2.9.2)$$

The  $t_{ij}$  are coefficient defined to be

$$\begin{aligned} t_{ij} &= \langle \phi_i | [\hbar^2 \nabla^2 / 2m + V_0] | \phi_j \rangle \\ &= \int \phi_i(x) \left| -\frac{\hbar^2 \nabla^2}{2m} + V_0 \right| \phi_j \end{aligned} \quad (2.9.3)$$

Where  $V_0$  stands for the crystal field interaction for practical considerations  $t_{ij}$  is non zero, only when i,j are nearest neighbors, in which case it is usually denoted by t. Thus Hoin (3.1) can be reduced to the form

$$\bar{H}_0 = -t \sum_{ij} [c_i^\dagger c_j + c_j^\dagger]$$

Assuming the periodic boundary conditions, one can writ

$$c_{k+} = \frac{1}{\sqrt{L}} \sum_k e^{ijk} c_j^+ \quad (2.9.4)$$

$$H_0 = \sum_k \epsilon_k c_k^+ c_k \quad (2.9.5)$$

Where

$$\epsilon_k = -2t \cos ka \quad (2.9.6)$$

With  $L$  allowed  $k$  values in the first Brillion zone. The term here stands for the distance between two sites.

If the number of electrons  $N$  is equal to the number of sites  $L$ , i.e.  $N = L$ ,

In this situation each allowed  $k$  state can be occupied by two and spins. Hence the ground state of  $H_0$  is constructed by filling the lower half of the band which is denoted by continuous line. Since half of the band is filled, the situation,  $N = L$ , is called half filling. This state is known as a Fermi sea state, usually denoted by  $|\text{FS}\rangle$ , where

$$|\text{FS}\rangle = \prod_{k \leq k_F} c_{k\uparrow}^1 c_{k\downarrow}^1 |0\rangle$$

With  $|0\rangle$  standing for the vacuum state in which the lattice is empty,  $k_F$  is the maximum occupied  $k$  state known as Fermi wave vector.

The fact that  $k$  states are occupied by two electrons comes from the fact that the number of allowed values of  $k$  is  $L$ . But since the occupied states are these for which  $\epsilon_k < 0$  [specially the  $k$  axis], only half of the  $k$  states between are empty. Thus the occupied  $k$  states are  $\frac{L}{2}$  [10, 11]. But as far as the number of electrons are

$$N = L = 2 \times (L/2) = 2 \times$$

Number of  $k$  states. Hence each  $k$  state is occupied by two electrons of spins up and down. The Fermi sea state is the Eigen state of the Hamiltonian, thus

$$H_0 |\text{FS}\rangle = E_0 |\text{FS}\rangle \quad (2.9.7)$$

The total energy  $E_0$  is given by The Fermi sea state is the Eigen state of the

Hamiltonian,

thus

$$H_0 |FS\rangle = E_0 |FS\rangle \quad (2.9.8)$$

$$E_0 = \sum_{|k| < \frac{\pi\sigma}{2}} \varepsilon_k n_{k\sigma} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2 \times (-2t \cos ka) \times \frac{dk}{2\pi} \quad (2.9.9)$$

The integral here is multiplied by 2 since each k state is occupied by two electrons, divide k by  $2\pi$  since the system is confined the first Brillion zone. If his system is excited an electron from one of the state is  $|k| < \pi/2$  Jump to higher state  $k + q$  such that  $|k + q| > \pi/2$ , leaving a hole in the state k. if the electron spin is up  $\uparrow$  this new state a denoted by  $|\psi_n\rangle$  where

$$\hat{H}_0 |\psi_n\rangle = \hat{H}_0 (c_{K+q\uparrow}^+ c_{k\uparrow} |F_S\rangle) = \varepsilon_k^{\text{ph}}(q) \psi_n \quad (2.9.10)$$

The energy of state  $\psi_n$  is given by

$$\varepsilon_k^{\text{ph}}(q) = \varepsilon_{k+q} - \varepsilon_k \quad (2.9.11)$$

In this half-filled band one has a metal as far as the filled valence band and the empty conduction band overlap. The ground state of non-interacting electrons can be obtained by occupying the lowest states by maximum number of electrons as possible [17].

This is done by allowing each k state to be occupied by two electrons with  $\uparrow$  and  $\downarrow$  Spins But since the number of allowed state sink space earedL, and since

$$L = N = \text{number of electrons}$$

Hence only  $L/2$  states are occupied, where

$$2 \text{ electron} \times \left(\frac{L}{2}\right) \text{ states} = L = N$$

Thus half-filled

Such occupation to minimize energy in the k – space can be performed by four possible ways in the real space, where four possible occupation of a single site.

The above situation represents the case when the electron interaction is

neglected. When the electron-electron interaction is considered, one has to add to Hamiltonian the term

$$V = \frac{1}{2} \sum V_{\mu\nu\beta\alpha} C_{\mu}^{\dagger} C_{\nu}^{\dagger} C_{\alpha} C_{\beta} \quad (2.9.12)$$

Where the index  $\alpha \equiv \{\sigma, i\}$  described the site  $i$  and spin  $\sigma$ . the two particle interactions is described by the term.

$$V_{\mu\nu\beta\alpha} = \int \psi_{\beta}^*(x) \psi_{\nu}^*(x^-) (|x - x^-|) \psi_{\beta}(x) \psi_{\alpha}(x^-) dx dx^- \quad (2.9.13)$$

In metals the coulomb potential is screened and takes the form

$$V(r) = \frac{e^{k_0 r}}{r} K_0 = K_{TF} \quad (2.9.14)$$

When all indices correspond to the same site  $j$ , Pauli Principle forces

$$\mu = \alpha = \uparrow, \beta = \nu = \downarrow$$

## 2.10 Exact Digitalization:

The digitalization process aims to find the space which constitutes the Eigen vectors of the Hamiltonian. This comes from the fact that the Hamiltonian matrix is diagonal in the Hamiltonian Eigen vectors space. In this case the Eigen vector is the energy Eigen value while the corresponding coefficient is the energy of the state. This toy model consists of two sites labeled 0, 1. In this case the Hubbard model written explicitly (in unit which  $t = 1$ ) as:

$$\begin{aligned} H &= H_t + H_U \\ &= -(C_{0\uparrow}^{\dagger} C_{1\uparrow} + C_{1\uparrow}^{\dagger} C_{0\uparrow} + C_{0\downarrow}^{\dagger} C_{1\downarrow} + C_{1\downarrow}^{\dagger} C_{0\downarrow}) + (n_{0\uparrow} n_{0\downarrow} + n_{1\uparrow} n_{1\downarrow}) \end{aligned} \quad (2.10.1)$$

The first term stands for the kinetic hopping process while the second one represents the field contribution [27, 28].

### 2.10.1 Strong Correlations and Spin Physics

As one already sees in previous section, the ground state of the two sites Hubbard model is a singlet with energy

$$E_s^- = \frac{U}{2} - \sqrt{\frac{U^2}{4} + 4t^2}$$

The ground state wave function

$$|E_s^- \rangle = 4|\psi_2 \rangle + (U + \sqrt{U^2 + 16})|\psi_3 \rangle$$

Since the first excited state is at  $E_t = 0$  the splitting between these two state for large  $U \gg t$  is

$$-J = E_s - E_t = \frac{U}{2} - \frac{\sqrt{U^2}}{4} + \sqrt{4t^2} \approx \frac{4t^2}{U} \quad (2.10.2)$$

Therefore the singlet state is slightly below  $(-\frac{4t^2}{U})$  the triplet state. This indicates that in large  $U$  limit, the low-energy physics of Hubbard model is given by spin fluctuations which are anti ferromagnetic (AF) (singlet has lower energy). This observation in a two site Hubbard model is indeed very general and it can be shown using a unitary Trans formation that the Hubbard model at large  $U$  limit can be mapped into the so called  $t - J$  model, where there are AF spin fluctuations along with hopping restricted to subspace with no double occupancy [17, 18].

## **2.11 Hubbard Model for Superconductivity and Mott Insulator:**

One of the most widely used models to describe HTSC is the Hubbard model. The electric conduction in this model is performed by hopping of valence electrons from site to site. Moreover, this model is used in most applications of condensed matter and many body systems. The Hubbard model is particularly applied to strongly correlated systems, including electronic correlated. An electronic system is said to be correlated when the electrons within it are not free. In the free electron model the conduction electrons move freely within the metal forming a free electron

Fermi gas, which obeys Pauli Exclusion Principle [20, 21, and 24].

The electron state can be described by using the winner Eigen functions

(Basis)  $\phi_n(r - R_i)$  which describe electron localized at  $r$  on the atom at the position. In the standard Hubbard model each atom has only one electron non-degenerate orbital state. The actual atom, however, can have more than one orbit and more than two electrons in the corresponding state. This assumption stems. From the fact that the electrons in other states do not play significant role at low temperature, hence the Hamiltonian of the system takes the form [16]. From the fact that the electrons in other states do not play significant role at low temperature, stands for the electron number operator. To simplify treatment one can use  $U$  to describe coulomb repulsion, and  $t$  to represent kinetic hopping. Hence the quantity  $U/t$  determines whether coulomb repulsion or hopping dominates. However, it is not possible to determine the relative importance of the two in relation to the Pauli Exclusion Principle, which depends on the probability of finding two electrons on the same site. This quantity can somewhat accounted for by the electron density per spin

$$\frac{1}{2N} \sum c_{ik}^+ c_{jk} \quad (2.11.1)$$

Each atom has space for at most two electrons,  $n$  can be in the range from 0 to 1. Hubbard model is affected by three factors, one-site repulsions hopping and Pauli Exclusion Principle. When each atom has one electron, one has a perfect half-filled band. To put an extra electron on that system this needs to overcome coulomb repulsion. When  $U$  is big the band will split into two sub bands with a gap in the middle. For half filled band, i.e.  $n = 0.5$  the Fermi energy will cross the energy gap. Thus the lower band is filled and stands for the valence band while the upper band is empty and behaves as a conduction band. Therefore the electron to become free needs very large energy to cross the energy gap  $E_g$  to be free where

$$E_g \propto \Delta E \propto U \rightarrow \infty$$

In this case, the material becomes an insulator called Mott insulator. If  $n$  is not perfectly equal to 0.5 the Fermi energy will cross either the upper or the lower Hubbard band. This can be understood in terms of Fermi Dirac statistics where.

$$n = \frac{1}{[e^{\beta(E-E_F)} + 1]}$$

$$e^{\beta(E-E_F)} = \frac{1}{n} - 1$$

$$E - E_F = \ln\left(\frac{1-n}{n}\right)$$

$$E_F = E + \ln\left(\frac{n}{1-n}\right) \quad (2.11.2)$$

For zero filling  $n = 0$

$$E_F + E + \ln 0 = E + \ln e^{-\infty}$$

$$E_F = E - \infty \rightarrow \infty \quad (2.11.3)$$

Since the zero energy is at the interface of upper and lower band.

Thus  $E_F$  becomes far below the interface. Therefore  $E_F$  crosses the lower band. Hence there is no gap between conduction and valence band, as far as  $E_F$  separates conduction and valence band. As a result the material is converted into metal.

$$E_F = E + \ln \infty = E + \ln e^{\infty} = E + \infty \rightarrow \infty$$

For complete filling  $n = 1$

Hence  $E_f$  crosses the upper band. Again the material becomes a conductor as far as  $E_f$ , does not cross an energy gap. Thus electrons can easily become conduction electrons.

## 2.12 Coherence Length:

The coherence length is a measure of the distance within which the gap parameter cannot change drastically in a spatially varying magnetic field. The London equation is a local equation: it relates the

current density at a point so to the vector potential. However, the coherence length is a measure of the range over which average  $A$  to obtain  $J$ . Any spatial variation in the state of an electronic system requires extra kinetic energy. It is reasonable to restrict the spatial variation of  $J(r)$  in such way that the extra energy is less than the stabilization energy of the superconducting state. A suggestive argument (based on the uncertainty principle) for the coherence length at absolute zero follows. The electron motion can be described by the plane wave  $\psi(x) = e^{ikx}$  considering that the electron have two states one characterized by  $k$ , and the other is characterized by  $k + q$ . Then the wave function of the electron in the superposition of two states is as follows:

$$\phi(x) = 2^{-\frac{1}{2}}(e^{i(k+q)x} + e^{ikx}) \quad (2.12.1)$$

The probability density as associated with single plane wave is uniform in space where:

$$\psi^* \psi = e^{-ikx} e^{ikx} = 1 \quad (2.12.2)$$

Where the probability  $\psi^* \psi$  is modulated by the wave vector  $q$ . probability  $|\phi|^2$  of two states

$$\begin{aligned} &= \phi^* \phi \\ &= \frac{1}{2}(e^{i(k+q)x} + e^{ikx})(e^{i(k+q)x} + e^{ikx}) \\ &= \frac{1}{2}(2 + e^{-iqx} + e^{iqx}) = 1 + \cos qx \end{aligned} \quad (2.12.3)$$

The kinetic energy of the wave  $\psi(x)$  at a single state  $k$  is:

$$H = \frac{p^2}{2m} + v \quad (2.12.4)$$

For free electrons:

$$v = 0 \quad p = \frac{\hbar}{i} \frac{d}{dx} \quad H\psi = E\psi, \psi(x) = e^{ikx} \quad (2.12.5)$$

$$E = \frac{\hbar^2}{2m} k^2 \quad (2.12.6)$$



The kinetic energy of the wave function  $\varphi$  is:

$$\langle E \rangle = \int \varphi^* H \varphi dx \quad (2.12.7)$$

For

$$V = 0 \quad H = \frac{P^2}{2m} = \frac{\hbar^2}{2m} \frac{d^2}{dx^2}$$

Then:

$$\begin{aligned} \langle E \rangle &= \int \varphi^* \left( -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \right) \varphi dx \\ &= \frac{1}{2} \frac{\hbar^2}{2m} [(k+q)^2 + k^2] \int \varphi^* \varphi dx \\ &= \frac{1}{2} \frac{\hbar^2}{2m} (k^2 + 2kq + k^2) \\ &= \frac{1}{2} \frac{\hbar^2}{2m} (k^2 + kq) \end{aligned} \quad (2.12.8)$$

Where neglected  $q^2$  on the assumption that  $q \ll k$  comparing (2.11.6) and (2.11.8). The increase of energy requires for modulation is  $\frac{\hbar^2 k q}{2m}$ . If this increase exceeds the energy gap  $E_g$ , superconductivity will be destroyed. The critical value  $q_0$  of the modulation wave vector is defined by

$$\frac{\hbar^2 q_0}{2m} k_F = E_g \quad (2.12.9)$$

Define an intrinsic coherence length  $\lambda_0$  related to the critical modulation by

$$\lambda_0 = \frac{2\pi}{q_0}$$

Since  $\hbar k = P = mV$ ; then from (2.11.9) one obtains:

$$\begin{aligned} \lambda_0 &= \frac{2\pi \hbar^2}{2m E_g} k_F \\ &= \frac{\pi \hbar}{E_g} v_F \end{aligned} \quad (2.12.10)$$

Where  $v_F = \frac{\pi \hbar k_F}{m}$  the electron velocity at the Fermi surface on the BCS

theory, a similar result can be obtained i.e.

$$\lambda_0 = \frac{2\hbar v_F}{\pi E_g} \quad (2.12.11)$$

The intrinsic coherence length  $\lambda_0$  is characteristic of a pure superconductor. In impure materials and in alloys the coherence length  $\lambda_c$  is shorter than  $\lambda_0$  this maybe understood qualitatively: impure material, the electron Eigen functions already have wiggles in them. Construct a given localized variation of current density with less energy from wave functions with wiggles then from smooth wave functions.

### **2.13 High-Temperature Superconductors:**

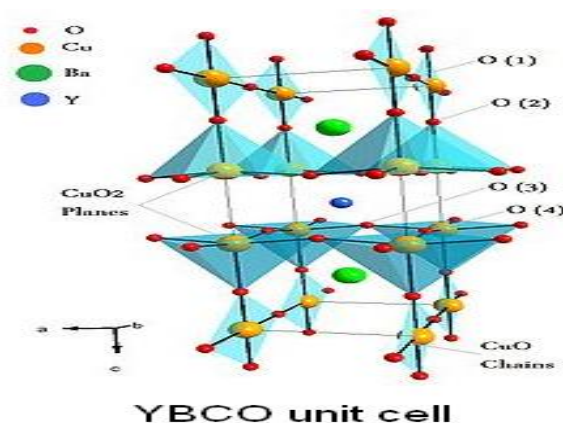
High temperature superconductors (HTC) are materials that behave as superconductors at unusually [29] high temperatures. The first high  $-T_c$  superconductor was discovered in 1986 by IBM researchers Georg Bednorz and K. Alex Muller [30,31,32]. Who were awarded the 1987 Nobel Prize in Physics "for their important break-through in the discovery of superconductivity in ceramic materials" [33]. Whereas "ordinary" or metallic superconductors usually have transition temperatures (temperatures below which they are superconductive) below 30 K ( $-243.2$  °C), and must be cooled using liquid helium in order to achieve superconductivity, HTS have been observed with transition temperatures as high as 138 K ( $-135$  °C), and can be cooled to superconductivity using liquid nitrogen [2]. Until 2008, only certain compounds of copper and oxygen (so-called "cuprite's") were believed to have HTS properties, and the term high-temperature superconductor was used interchangeably with cuprate superconductor for compounds such as bismuth strontium calcium copper oxide (BSCCO) and yttrium barium copper oxide (YBCO). Several iron-based compounds are now known to be superconducting at high temperatures [5, 6, and 7].

In 2015, hydrogen sulfide ( $\text{H}_2\text{S}$ ) under extremely high pressure (around 150 giga scales) was found to undergo superconducting transition near 203 K ( $-70^\circ\text{C}$ ), the highest temperature superconductor known to date [8, 9, 10]. For an explanation about  $T_c$  (the critical temperature for superconductivity), see Superconducting phase transition and the second bullet item of BCS theory Successes of the BCS theory

### **2.13.1 Crystal Structures of High-Temperature Copper Oxides:**

Copper HTC are one of the matrices having interesting proportion. The structure of high- $T_c$  copper oxide or cuprate superconductors are often closely related to perovskite structure, and the structure of these compounds has been described as a distorted, oxygen deficient multi-layered perovskite structure. One of the properties of the crystal structure of oxide superconductors is an alternating multi-layer of  $\text{CuO}_2$  planes with superconductivity taking place between these layers. The more layers of  $\text{CuO}_2$ , the higher  $T_c$ . This structure causes a large anisotropy in normal conducting and superconducting properties, since electrical currents are carried by holes induced in the oxygen sites of the  $\text{CuO}_2$  sheets. The electrical conduction is highly an isotropic, with a much higher conductivity parallel to the  $\text{CuO}_2$  plane than in the perpendicular direction. Generally, critical temperatures depend on the chemical compositions, cation substitutions and oxygen content. They can be classified as super stripes; i.e., particular realizations of super lattices at atomic limit made of superconducting atomic layers, wires, dots separated by spacer layers that give multiband and multiage superconductivity [15, 16, and 18].

### 2.13.2 Y Ba CuO Superconductors:-



The first superconductor found with  $T_c > 77$  K (liquid nitrogen boiling point) is yttrium barium copper oxide ( $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ ); the proportions of the three different metals in the  $\text{YBa}_2\text{Cu}_3\text{O}_7$  superconductor are in the mole ratio of 1 to 2 to 3 for yttrium to barium to copper, respectively. Thus, this particular superconductor is often referred to as the 123 superconductor.

The unit cell of  $\text{YBa}_2\text{Cu}_3\text{O}_7$  consists of three pseudo cubic elementary perovskite unit cells. Each perovskite unit cell contains a Y or Ba atom at the center: Ba in the bottom unit cell, Y in the middle one, and Ba in the top unit cell. Thus, Y and Ba are stacked in the sequence [Ba–Y–Ba] along the c-axis. All corner sites of the unit cell are occupied by Cu, which has two different coordination's, Cu(1) and Cu(2), with respect to oxygen. There are four possible crystallographic sites for oxygen: O(1), O(2), O(3) and O(4) [23,24,25]. The coordination polyhedral of Y and Ba with respect to oxygen is different. The tripling of the perovskite unit cell leads to nine oxygen atoms, whereas  $\text{YBa}_2\text{Cu}_3\text{O}_7$  has seven oxygen atoms and, therefore, is referred to as an oxygen-deficient perovskite structure. The structure has a stackin of different layers (CuO) (BaO)(CuO<sub>2</sub>)(Y)(CuO<sub>2</sub>)(BaO)(CuO). One of the key features of the unit cell of Y  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  (YBCO) is the presence of two layers of CuO<sub>2</sub>. The role of the Y plane is to serve as a space between two CuO<sub>2</sub>

planes. In YBCO, the Cu–O chains are known to play an important role for superconductivity.  $T_c$  is maximal near 92 K when  $x \approx 0.15$  and the structure are orthorhombic. Superconductivity disappears at  $x \approx 0.6$ , where the structural transformation of YBCO occurs from orthorhombic to tetragonal [21].

### 2.13.3 Bi-, T<sub>L</sub>- and Hg-Based High-T<sub>c</sub> Superconductors

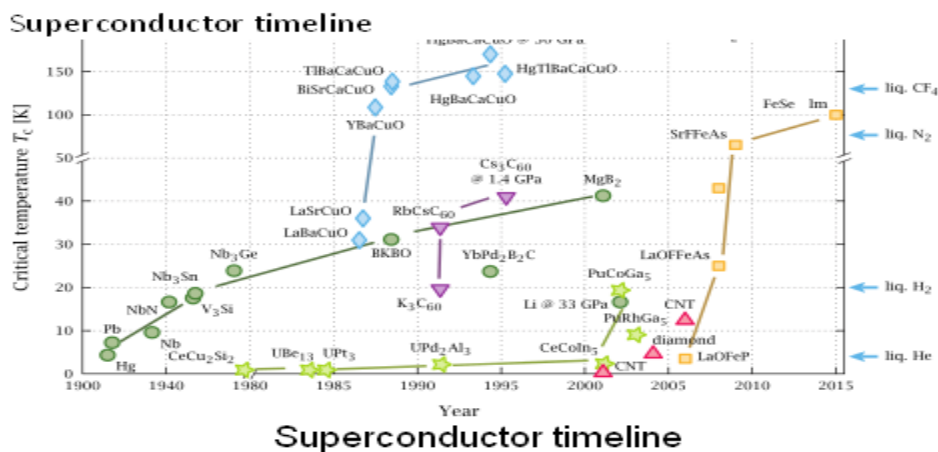
The crystal structure of Bi-, T<sub>L</sub>- and Hg-based high-T<sub>c</sub> superconductors are very similar [25]. Like YBCO, the perovskite-type feature and the presence of CuO<sub>2</sub> layers also exist in these superconductors. However, unlike YBCO, Cu–O chains are not present in these superconductors. The YBCO superconductor has an orthorhombic structure, whereas the other high-T<sub>c</sub> superconductors have a tetragonal structure. The Bi–Sr–Ca–Cu–O system has three superconducting phases forming a homologous series as Bi<sub>2</sub>Sr<sub>2</sub>Ca<sub>n-1</sub>Cu<sub>n</sub>O<sub>4+2n+x</sub> ( $n = 1, 2$  and  $3$ ). These three phases are Bi–2201, Bi–2212 and Bi–2223, having transition temperatures of 20, 85 and 110 K, respectively, where the numbering system represent number of atoms for Bi, Sr, Ca and Cu respectively [26]. The two phases have a tetragonal structure which consists of two sheared crystallographic unit cells. The unit cell of these phases has double Bi–O planes which are stacked in a way that the Bi atom of one plane sits below the oxygen atom of the next consecutive plane. The Ca atom forms a layer within the interior of the CuO<sub>2</sub> layers in both Bi–2212 and Bi–2223; there is no Ca layer in the Bi–2201 phase. The three phases differ with each other in the number of CuO<sub>2</sub> planes; Bi–2201, Bi–2212 and Bi–2223 phases have one, two and three CuO<sub>2</sub> planes, respectively. The c axis lattice constants of these phases' increases with the number of CuO<sub>2</sub> planes (see table below). The coordination of the Cu atom is different in the three

phases. The Cu atom forms an octahedral coordination with respect to oxygen atoms in the 2201 phase, whereas in 2212, the Cu atom is surrounded by five oxygen atoms in a pyramidal arrangement. In the 2223 structure, Cu has two coordination's with respect to oxygen: one Cu atom is bonded with four oxygen atoms in square planar configuration and another Cu atom is coordinated with five oxygen atoms in a pyramidal arrangement [27].

### 2.13.2 Magnetic Properties

All known high- $T_c$  superconductors are Type – II superconductors. In contrast to Type – I superconductors, which expel all magnetic fields due to the Meissner effect, Type – II superconductors allow magnetic fields to penetrate their interior in quantized units of flux, creating "holes" or "tubes" of normal metallic regions in the superconducting bulk called vortices. Consequently, high- $T_c$  superconductors can sustain much higher magnetic fields.

#### Ongoing research



The question of how superconductivity arises in high-temperature superconductors is one of the major unsolved problems of theoretical condensed matter physics. The mechanism that causes the electrons in these crystals to form pairs is not known [5]. Despite intensive research and many

promising leads, an explanation has so far eluded scientists. One reason for this is that the materials in question are generally very complex, multi-layered crystals improving the quality and variety of samples also gives rise to considerable research, both with the aim of improved characterization of the physical properties of existing compounds, and synthesizing new materials, often with the hope of increasing  $T_C$ . Technological research focuses on making HTS materials in sufficient quantities to make their use economically viable and optimizing their properties in relation to applications [40, 41, 42].

### **2.13.3 Possible Mechanism**

There have been two representative theories for high-temperature or unconventional superconductivity. Firstly, weak coupling theory suggests superconductivity emerges from anti ferromagnetic spin fluctuations in a doped system [43,44,45,46]. According to this theory, the pairing wave's function of the cuprate HTS should have a  $dx^2 - y^2$  symmetry. Thus, determining whether the pairing wave function has d - wave symmetry is essential to test the spin fluctuation mechanism. That is, if the HTS order parameter (pairing wave function) does not have d-wave symmetry, and then a pairing mechanism related to spin fluctuations can be ruled out. (Similar arguments can be made for iron-based superconductors but the different material properties allow a different pairing symmetry.) Secondly, there was the interlayer coupling model, according to which a layered structure consisting of BCS - type (s-wave symmetry) superconductors can enhance the superconductivity by itself [58]. By introducing an additional tunneling interaction between each layer, this model successfully explained the anisotropic symmetry of the order parameter as well as the emergence of the HTS. Thus, in order to solve this unsettled problem, there have been

numerous experiments such as photoemission spectroscopy, specific heat measurements, etc. Up to date the results were ambiguous; some reports supported the d symmetry for the HTS whereas others supported the s symmetry. This muddy situation possibly originated from the indirect nature of the experimental evidence, as well as experimental issues such as sample quality, impurity scattering, twinning, etc. [26, 27].



**Fig (2.12) Junction experiment supporting the d symmetry:**

The Meissner effect or a magnet levitating above a superconductor (cooled by liquid nitrogen)

An experiment based on flux quantization of a three-grain ring of  $\text{YBa}_2\text{Cu}_3\text{O}_7$  (YBCO) was proposed to test the symmetry of the order parameter in the HTS. The symmetry of the order parameter could best be probed at the junction interface as the Cooper pair's tunnel across a Josephson junction or weak link [69]. It was expected that a half-integer flux, that is, a spontaneous magnetization could only occur for a junction of d symmetry superconductors. But, even if the junction experiment is the strongest method to determine the symmetry of the HTS order parameter, the results have been ambiguous. J. R. Kirtley and C. C. Tsuei thought that the ambiguous results came from the defects inside the HTS, so that they designed an experiment where both clean limit (no defects) and dirty limit (maximal defects) were considered simultaneously [70]. In the experiment, the spontaneous magnetization was clearly observed in YBCO, which



supported the d symmetry of the order parameter in YBCO. But, since YBCO is orthorhombic, it might inherently have an admixture of s symmetry. So, by tuning their technique further, they found that there was an admixture of s symmetry in YBCO within about 3% [71]. Also, they found that there was pure  $dx^2 - y^2$  order parameter symmetry in the tetragonal  $Tl_2Ba_2CuO_6$  [72].

## 2.14 Qualitative Explanation of the Spin-Fluctuation

### Mechanism:

Main article: Resonating valence bond theory despite all these years, the mechanism of high- $T_c$  superconductivity is still highly controversial, mostly due to the lack of exact theoretical computations on such strongly interacting electron systems. However, most rigorous theoretical calculations, including phenomenological and diagrammatic approaches, converge on magnetic fluctuations as the pairing mechanism for these systems. The qualitative explanation is as follows:

In a superconductor, the flow of electrons cannot be resolved into individual electrons, but instead consists of many pairs of bound electrons, called Cooper pairs. In conventional superconductors, these pairs are formed when an electron moving through the material distorts the surrounding crystal lattice, which in turn attracts another electron and forms a bound pair. This is sometimes called the "water bed" effect. Each Cooper pair requires a certain minimum energy to be displaced, and if the thermal fluctuations in the crystal lattice are smaller than this energy the pair can flow without dissipating energy. This ability of the electrons to flow without resistance leads to superconductivity. In a high- $T_c$  superconductor, the mechanism is extremely similar to a conventional superconductor, except, in this case, phonons virtually play no role and their role is replaced

by spin-density waves. Just as all known conventional superconductors are strong phonon systems, all known high  $-T_c$  superconductors are strong spin-density wave systems, within close vicinity of a magnetic transition to, for example, an antiferromagnetic. When an electron moves in a high  $-T_c$  superconductor, its spin creates a spin-density wave around it. This spin-density wave in turn causes a nearby electron to fall into the spin depression created by the first electron (water-bed effect again). Hence, again, a Cooper pair is formed. When the system temperature is lowered, more spin density waves and Cooper pairs are created, eventually leading to superconductivity. Note that in high  $-T_c$  systems, as these systems are magnetic systems due to the Coulomb interaction, there is a strong Coulomb repulsion between electrons. This Coulomb repulsion prevents pairing of the Cooper pairs on the same lattice site. The pairing of the electrons occurs at near-neighbor lattice sites as a result. This is the so-called  $d$  - wave pairing, where the pairing state has a node (zero) at the origin [50,51,52]

# Chapter Three

## Literature Review

### 3.1 Introduction:

Many attempts were made to cure setback of scThis chapter is concerned with some of these attempts [54, 55, 56, 57, 58, 60, 61]. The attempts exhibited have mainly concerned electrical and magnetic properties

### 3.2 Effect of Magnetic Field on Super conduction Complex

#### Resistance according to Quantum Mechanics

In the work of Dirar [57].Plasma equation was used to describe electron motion easily. This is since electrons are charged .For pressure exerted by the gas plasma equation becomes:

$$mn \frac{dv}{dt} = -\nabla P + F \quad (3.2.1)$$

But for pressure exerted by the medium on the electron gas, the equation become:

$$mn \frac{dv}{dx} = \nabla P + F = \nabla P - \nabla V \quad (3.2.2)$$

In one dimensions, the equation becomes:

$$mn \frac{dv}{dx} \frac{dx}{dt} = \frac{d(nKT)}{dx} - \frac{dnv}{dx}$$
$$mn \frac{v dv}{dx} = \frac{d}{dx} [nKT - nv]$$

Thus in integration both sides by assuming n to be constant, or independent ofk, yields:

$$\frac{n}{2} mv^2 = nKT - nV + c$$

$$\frac{1}{2}mv^2 + V - KT = \frac{c}{n} = \text{constant} = E$$

This constant of motion stands for energy, thus:

$$E = \frac{p^2}{2m} + V - KT \quad (3.2.3)$$

Multiplying by  $\Psi$  yields:

$$E\Psi = \frac{p^2}{2m}\Psi + V\Psi - KT\Psi \quad (3.2.4)$$

According to the wave nature of particles:

$$\begin{aligned} \psi &= Ae^{\frac{i}{\hbar}(px-ET)} \\ i\hbar \frac{\partial \psi}{\partial t} &= E\psi \\ -\hbar^2 \nabla^2 \psi &= p^2 \psi \end{aligned} \quad (3.2.5)$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi - KT\psi \quad (3.2.6)$$

The time in dependent equation becomes:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi - KT\psi = E\psi \quad (3.2.7)$$

Consider the case when these electrons wave subjected to constant crystal field  $V_0$ . This assumption is quite natural as far as particles are distributed homogenously. Thus equation (3.2.7) becomes:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V_0\psi - k\tau = E\psi \quad (3.2.8)$$

One can suggest the solution to be

$$\psi = Ae^{iKx} \quad (3.2.9)$$

A direct substitution yield:

$$\left( \frac{\hbar^2}{2m} K^2 + V_0 - KT \right) \psi = E\psi$$

Therefore:

$$K = \frac{\sqrt{2m(E + KT - V_0)}}{\hbar} \quad (3.2.10)$$

This wave number is related to the momentum according to the relation:

$$p = mv = \hbar K = \sqrt{2m(E + KT - V_0)} \quad (3.2.11)$$

This relation can be used to find the quantum resistance R of a certain material. According to classical laws:

$$R = \frac{V}{t} \quad (3.2.12)$$

For harmonic oscillator:  $T = V$

For electron accelerated by potential .The work done is related to the potential V and kinetic energy k according to the relation:

$$W = V = \frac{1}{2}mv^2 \quad (3.2.13)$$

But since the current is gives by.

$$I = nevA \quad (3.2.14)$$

$$R = \frac{mv^2}{2nevA} = \frac{mv}{2neA} = \frac{p}{2neA} \quad (3.2.15)$$

From (3.2.12) and (3.2.13):

$$R = \frac{\sqrt{2m(E + KT + V)}}{2neA} \quad (3.2.16)$$

Splitting R to real part  $R_s$  and imaginary part  $R_i$

$$R = R_s + jR_i \quad (3.2.17)$$

According to equation (3.2.16) R Becomes pure imaginary, when:

$$E + KT - V_0 < 0$$

$$KT < V_0 - E$$

$$T < \frac{(V_0 - E)}{K} \quad (3.2.18)$$

Thus the critical temperature is given by:

$$T_c = \frac{V_0 - E}{K}$$

Thus equation (3.2.16) becomes:

$$R = \frac{\sqrt{2mk(T - T_c)}}{2enA} \quad (3.1.19)$$

Which r enquires

$$V_0 > E \text{ For } T < T_c$$

Thus R becomes

$$R = \frac{i\sqrt{2mk(T - T_c)}}{2enA} \quad (3.2.20)$$

$$R = jR_i \quad (3.2.21)$$

Using Equation (3.2.16) yields

$$R_s = 0 \quad (3.2.22)$$

Thus the superconductivity resistance vanishes for all T less than the critical value.

When an external magnetic field of flux density B is applied, the total medium field is given by.

$$B_m = B - B_i \quad (3.2.23)$$

Where  $B_i$  is the internal flux density. The corresponding potential applied on electrons or charges is given by  $V_m$ , thus the total potential in equation (3.2.7) becomes.

$$V = V_0 \pm V_m \quad (3.2.24)$$

$$V_m = m_L \left( \frac{e\hbar}{2m} \right) B_m = C_0^{-1} B_m$$

When the net magnetic potential opposes the crystal field

$$V = V_0 - V_m = V_0 - C_0^{-1} B_m \quad (3.2.25)$$

In this case one can rewrite the expression of R in equation (3.2.16) to be

$$R = \frac{\sqrt{2mK(E + kT - V_0 + V_m)}}{2enA} = \frac{\sqrt{2mk(T - T_c + T_m)}}{2enA}$$

$$R = \frac{\sqrt{2mk(T + T_m - T_c)}}{2enA} \quad (3.2.26)$$

Consider now the case when

$$T_m \geq T_C \quad (3.2.27)$$

According to equation (3.2.23), (3.2.25) and (3.2.27) this critical value is given by

$$B_c = c_0(2mKT_C) + B_{ic} \quad (3.2.28)$$

In this case the term under the square root is positive always. This means that, it

$$R = R_s + jR_i \quad (3.2.29)$$

$$R_i = 0 \quad R_s \neq 0 \quad (3.2.30)$$

This means that the superconductivity is destroyed when applying an external magnetic field having strength to satisfy equation (3.2.1).

In this work resistance is considered as sum of real superconducting part and imaginary part as equation (3.2.17) shows. The effect of subjecting superconducting to magnetic field is studied where the magnetic field inside superconducting is considered as consisting of external and internal field (see equation (3.2.23)). Equation (29) shows that when an external magnetic field exceeds a certain critical value given by (28), the superconducting state is destroyed since  $R_s \neq 0$  as equation (30) indicates.

### **3.3 Quantum Effect of magnetic field in destroying superconductivity:**

In the work of Azakaria [54]. Another direct approach can also be found by considering the pressure exerted by thermal particle. In this case (3.3.1) the Hamiltonian becomes:

$$\hat{H} = \frac{\hat{P}}{2m} + KT - V \quad (3.3.1)$$

For spin repulsive force:

$$V = -V_0$$

Thus:

$$\hat{H} = \frac{\hat{P}}{2m} + KT - V_0 \quad (3.3.2)$$

Thus the average energy which is equal to the classical energy is given by:

$$\hat{H} = \left\langle \frac{\hat{p}^2}{2m} \right\rangle + KT - V_0 \quad (3.3.3)$$

Resistance for harmonic oscillator where,

$$\begin{aligned} x &= x_0 e^{i\omega t}, \quad v = i\omega x, \quad T = \frac{1}{2} m |v|^2 = \frac{1}{2} m \omega^2 x^2, \quad V = \frac{1}{2} K x^2 = \frac{1}{2} m \omega^2 x^2 \\ &= T, \quad H = T + V = 2V, \quad eV_E = V = \frac{H}{2} \end{aligned}$$

Where  $V_e$  is the potential, thus

$$V_e = \frac{H}{2e}$$

Using the quantum definition of [10]:

$$\begin{aligned} R &= \frac{V_e}{I} \\ R &= R_+ + R_- \end{aligned} \quad (3.3.4)$$

Where one split Rot positive and negative one.

When:

$$E_0 + KT - V_0 < 0 \quad (3.3.5)$$

$$R_- = \frac{E_0 + KT - V_0}{2eI}, \quad R_+ = 0 \quad (3.3.6)$$

From equation (3.3.5) and (3.3.6) the superconductivity resistance  $R_S$  Vanishes i.e:

$$R_+ = R_S = 0 \quad (3.3.7)$$

When:

$$\begin{aligned} KT &< V_0 - E_0 \\ T &< \frac{V_0 - E_0}{K} \end{aligned} \quad (3.3.8)$$

Thus the critical temperature is given by:

$$T_C = \frac{V_0 - E_0}{K} \quad (3.3.9)$$

Again for  $T_C$  to be positive  $V_0 > E_0$



Thus for:

$$T < T_C \quad (3.3.10)$$

$$R_{SC} = R_+ = 0 \quad (3.3.11)$$

In the case when external magnetic field of flux density  $B_m$  and potential  $V_m$  resulting from both external and internal to magnetic field are given by:

$$B_m B - B_I \quad (3.3.12)$$

$$V_m = V - V_i \quad (3.3.13)$$

Where  $B_i$  and  $V_i$  stands for the internal magnetic density and potential respectively .when the magnetic field attract electron. The Hamiltonian and the average energy in equation (3.3.1), (3.3.2) and (3.3.3) are given.

$$\hat{H} = \frac{\hat{P}^2}{2m} + KT + V_m - V_0$$

$$\langle \hat{H} \rangle = E_0 + KT + V_m - V_0 \quad (3.3.14)$$

Thus according to equation (3.3.4) the quantum resistance is given by:

$$R = \frac{KT + E_0 - V_0 + V_m}{2eL} = R_+ + R_- \quad (3.3.15)$$

In view of equation (3.3.9) and by denoting  $V_m$  to be

$$V_m = KT_m = m_l \left( \frac{e\hbar}{2m} \right) B_m = C m^{-1} B_m \quad (3.3.16)$$

Equation (3.3.15) reads

$$R = \frac{k(T + T_m - T_C)}{2eL} \quad (3.3.17)$$

When

$$T_m > T_C \quad (3.3.18)$$

Thus the critical  $V_m$  and Bare given by (3.3.18), (3.3.12) to be

$$B_C = B_{mC} + B_{ic} = c_m V_{mc} + B_{ic} = c_m kT_C + B_{ic} \quad (3.3.19)$$

In this case R is positive always. No matter what the value of T is therefore

$$R = R_+ + R_- = \frac{k(T + T_m - T_C)}{2eL} \quad (3.3.20)$$

Thus,

$R_+ \neq 0$  Always, when condition (3.3.18) is satisfied.

This work is concerned with applying external magnetic field. the external magnetic field incorporates itself in the resistance relation (3.3.15) via the medium potential terms as shown by equation (3.3.12) and (3.3.13).The quantum resistance consist of positive superconducting part beside negative part according to equation (3.3.15).The critical magnetic flux density is given by equation (3.3.19).The superconducting resistance does not vanish and the superconducting state is destroyed. When the magnetic flux density exceeds this critical value.

### **3.4 New Derivation of simple Josephson Effect relation using new quantum mechanical equation:**

In the paper of Rashida, etal [55], the Newtonian energy  $E$  is a sum of kinetic and potential energy, i.e:

$$E = \frac{1}{2}mv^2 + V = \frac{p^2}{2m} + V \quad (3.4.1)$$

Where  $m, v, p$  are the mass, velocity momentum respectively .According to a theorem of Bloch [7].in such superconductors momentum  $P$  is zero.

$$p = \frac{mv^2}{2} + \frac{qA}{c}$$

Hence equation (3.4.1)

$$E = V \quad (3.4.2)$$

This is related to the fact that in Josephson Effect the tunneling potential is considered to be larger than kinetic term squaring both sides' yields:

$$E^2 = V^2 \quad (3.4.3)$$

Multiplying both sides by  $\psi$ , one gets:

$$E^2\psi = V^2\psi \quad (3.4.4)$$

The wave function of a free particle  $s$  given by:

$$\psi = Ae^{\frac{i}{\hbar}(p_x - E_t)} \quad (3.4.5)$$

Differentiating both sides with respect x and t

$$-\hbar^2 \frac{\partial^2}{\partial t^2} = E^2 \psi \quad (3.4.6)$$

Similarly

Substitute (3.4.6) in (3.4.4) to get

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = V^2 \psi \quad (3.4.7)$$

### 3.4.1 Josephson Effect Equation:

In Josephson Effect electron have small kinetic energy compared to the potential. Thus Schrödinger Equation (3.4.7) in which kinetic term is neglected is suitable for describing the Josephson Effect. To derive Josephson Effect equation, consider the solution.

$$\psi = D \sin(\alpha t + \varphi) \quad (3.4.8)$$

The tunneling potential is constant inside a superconductor, thus

$$V = V_0 \quad (3.4.9)$$

From (3.4.9), one can differentiate with respect to time twice to get

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= \alpha D \cos(\alpha t + \varphi) \\ \frac{\partial^2 \psi}{\partial t^2} &= -\alpha^2 D \sin(\alpha t + \varphi) = -\alpha^2 \psi \end{aligned} \quad (3.4.10)$$

Substituting (3.4.10) in (3.4.11) in (3.4.8) to obtain:

$$\begin{aligned} +\hbar^2 \alpha^2 \psi &= V_0^2 \psi \\ \alpha^2 &= \frac{V_0^2}{\hbar^2} \\ \alpha &= \pm \frac{V_0}{\hbar} \end{aligned} \quad (3.4.11)$$

By Substituting (3.4.11) in (3.4.8) and choosing a negative single, that is in dealing with the change in potential energy one gets

$$\psi = D \sin\left(\frac{eV_0}{\hbar^2} t + \varphi\right) \quad (3.4.12)$$

$$\begin{aligned}
J &= e \frac{\partial n}{\partial t} = e \frac{\partial |\psi|^2}{\partial t} = 2e|\psi| \frac{d|\psi|}{dt} \\
&= 2eD \sin(\alpha t + \varphi) \left( -\frac{e}{\hbar} V_0 \right) \cos(\alpha t + \varphi) \quad (3.4.13) \\
&\quad -2 \frac{e^2 D V_0}{\hbar} \sin \theta \cos \theta \\
&\quad \theta = \varphi - \frac{e}{\hbar} V_0 t
\end{aligned}$$

By using mathematical identity

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

One can rewrite Equation (3.4.13) to be

$$\begin{aligned}
J &= \frac{e^2 D V_0}{\hbar} \sin \left( 2\varphi - \frac{2eV_0 t}{\hbar} \right) = A \sin \left( 2\varphi - \frac{2eV_0 t}{\hbar} \right) \quad (3.4.13) \\
2\varphi &= \delta(0)
\end{aligned}$$

The current density is given by

$$J = J_0 \sin \left( \delta(0) - \frac{2eV}{\hbar} t \right) \quad (3.4.14)$$

This is the Josephson Effect equation:

The new energy equation based on Newtonian, mechanics is used to derive anew quantum Equation (3.4.8). This new quantum equation is based on Newtonian energy with no kinetic term. This derivation resembles simple derivation of Schrödinger equation except the fact that the kinetic term is neglected this equation is used to drive simple Josephson current density equation. This equation (3.4.16) is the same as the old one, but derived using simple arguments.

### 3.5 Complex Quantum Resistance Model:

In the work of Asma Altambori [56]. Plasma equation describes the electron motion. This is since the electrons be behaves as ionized particles. For pressure exerted by the gas plasma equation becomes:

$$m n \frac{dv}{dt} = -\nabla p + F \quad (3.5.1)$$

But for pressure exerted by the medium on the electron gas, the equation become:

$$mn \frac{dv}{dt} = \nabla p - \nabla v \quad (3.5.2)$$

In one dimension, the equation becomes:

$$mn \frac{dv}{dx} \frac{dx}{dt} = \frac{d(nKT)}{dx} - \frac{dnv}{dx}$$

$$mn \frac{v dv}{dx} = \frac{d(nKT)}{dx} - \frac{dnv}{dx}$$

Where  $V$  is the potential for one particle

$$mn \frac{d(1/2v^2)}{dx} = \frac{d}{dx} [nKT - nV]$$

Thus in integrating both sides by assuming to be constant, or in –dependent of  $K$ , yields:

$$\frac{n}{2} mv^2 = nKT - nV + c$$

$$\frac{1}{2} mv^2 + V - KT = \frac{C}{n} = \text{constant} = E$$

This constant of motion stands for energy, thus:

$$E = \frac{p^2}{2m} + v - KT \quad (3.5.3)$$

Multiplying by, yields:

$$E\psi = \frac{p^2}{2m}\psi + V\psi - KT\psi \quad (3.5.4)$$

According to the wave nature of particles:

$$\psi = Ae^{\frac{i(px-Et)}{\hbar}}$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi$$

$$-\hbar^2 \nabla^2 \psi = p^2 \psi \quad (3.5.5)$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi - KT\psi \quad (3.5.6)$$

The time in dependent equation becomes:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi - KT\psi = E\psi \quad (3.5.7)$$

Consider the case when these electrons wave subjected to constant crystal field  $V_0$ .this assumption is quite natural as far as particles are distributed homogenously. Thus equation (3.5.7) becomes:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V_0\psi - KT\psi = E\psi \quad (3.5.8)$$

One can suggest the solution to be:

$$\psi = Ae^{ikx} \quad (3.5.9)$$

A direct substitution yields

$$\left( \frac{\hbar^2}{2m} K^2 + V_0 - KT \right) \psi = E\psi$$

Therefore:

$$K = \frac{\sqrt{2m(E + KT - V_0)}}{\hbar} \quad (3.5.10)$$

This wave number K is related to the momentum according to the relation:

$$p = mv = \hbar K = \sqrt{2m(E + KT - V_0)} \quad (3.5.11)$$

This relation can be used to find the quantum resistance R of a certain material. According to classical laws:

$$R = \frac{V}{I} \quad (3.5.12)$$

For electrons accelerated by the potential. The wave done is related to the potential and Kinetic energy K according to the relation

$$w = V = \frac{1}{2} mv^2 \quad (3.5.13)$$

But since the current I is gives by.

$$I = nevA \quad (3.5.14)$$

$$R = \frac{mv^2}{2nevA} = \frac{mv}{2neA} = \frac{p}{2neA} \quad (5.5.15)$$

From (3.5.12) and (3.5.13):

$$R = \frac{\sqrt{2m(E + KT - V_0)}}{2neA} \quad (3.5.16)$$

Splitting  $R$  to real part  $R_s$  and imaginary part  $R_i$

$$R = R_s + R_i \quad (3.5.17)$$

According to equation (3.5.16)  $R$  becomes pure imaginary, when

$$\begin{aligned} E &= KT - V_0 < 0 \\ KT &< V_0 - E \\ T &< \frac{(V_0 - E)}{K} \end{aligned} \quad (3.5.18)$$

Thus the critical temperature is given by:

$$T_c = \frac{V_0 - E}{K}$$

This requires:

$$V_0 > E$$

In this case (see equation (3.5.17)).

$$\begin{aligned} R &= jR_i \\ R_s &= 0 \end{aligned} \quad (3.5.19)$$

Thus the superconductivity resistance  $R_s$  becomes zero beyond a certain critical temperature given by equation (3.5.17). which require binding energy to dominate.

### 3.5.1 Energy gap and photon absorption:

Such that the magnetic flux density inside the medium is given by:

$$B_m = B_e(N_u - N_d) \quad (3.5.20)$$

Where  $B_e$  is the magnetic flux density of one electron, If photon beams was absorbed this will change  $B_m$  by the transition of electrons from ground state to the excited state. If the number of incident photons is  $N_p$  the new internal flux density is given by:

$$B_m = B_e(N_u - N_d + 2N_p) \quad (3.5.21)$$

This change the potential of electrons to be  $V_m$  and split the energy levels to be

$$V_m = m_L \left( \frac{e\hbar}{2m} \right) B_m \quad (3.5.22)$$

Thus the energy gap is given by:

$$E_g = \frac{e\hbar}{2m} B_m \quad (3.5.23)$$

Here one assumes that any electron is affected by the magnetic field of this pinning gas. When electrons are affected by internal magnetic field there distance in equation (3.5.16) and by the definition of  $T_c$  in equation (3.5.19) is given by:

$$R = \frac{\sqrt{2m(E + KT - V_0)}}{2neA} = R_s + jR_i \quad (3.5.24)$$

Where

$$KT_m = V_m \quad (3.5.25)$$

The superconductivity is destroyed when,

$$T_m \geq T_c \quad (3.5.26)$$

Thus

$$V_m \geq KT_c \quad (3.5.27)$$

Since  $V_m$  is proportional to  $B_m$  according to equation (3.5.23) the energy gap corresponds to the minimum voltage that destroys superconductivity.

Thus,

$$E_g = C_m V_{mg} \quad (3.5.28)$$

But according to equation (3.5.28) the minimum magnetic energy that can destroy superconductivity is,

$$V_{mg} = KT_c \quad (3.5.29)$$

Thus equation (3.5.29) indicates that the energy gap takes the form

$$E_g = C_m KT_c \quad (3.5.30)$$



It is very interesting to note that this expiration for  $E_g$  conforms to the well known ordinary relation. In this model the photon plays a double role. When it is incident and absorbed by the superconductivity it increases the internal field by causing more electron with spin down to be in an excited state. This increase in the internal field  $B_m$  causes splitting of energy levels by the amount

$$\Delta E = g m_g \mu_\beta H_m \quad (3.5.31)$$

Cooper pairs are bound together by attractive force. The minimum energy which destroy the pairs and superconducting is called energy gap. It is clear that energy gap in equation (3.5.29) is related to energy splitting; in addition to the fact that the increase of magnetic field and magnetic potential above energy gap destroy SC. This means that according to Cooper version energy gap represents binding energy. It is very important to note that the relation (3.5.31) for energy gap which relates it to the critical temperature agrees with the ordinary one.

The effect of external magnetic field on superconducting can be explained by Hubbard model by assuming that the external field increase Fermi energy. This increase can be explained on the basis of the relation between Fermi energy and free carriers concentration it is known that magnetic energy increases electrons energy. This can enable more electrons to be free by entering conduction band (CB) from the valence band (VB). According to hopping and Mott model, the Fermi level. By increasing external, field Fermi level can be assumed in the lower continuous band. Thus no gap exists between (CB) and (VB) which are separated by Fermi level. Upon increasing external  $B$ , the free electrons and holes increases thus  $E_F$  move up word till it becomes inside energy gap between the lower band and upper banding this case the lower band which becomes (VB) and the upper band which becomes (CB). Thus an energy gap is produced between (VB) and

(CB) ,which prevent electrons from hopping easily. Thus the material becomes an insulator.

### 3.6 Using the tight binding approximation in deriving the quantum critical temperature superconductivity equation

#### 3.6.1 Plasma equation:

In the work of R.Abd Elhai [ 58 ]. For fluid of particle of mass  $m$ , number density  $n$ , velocity  $v$ , force  $F$ , and pressure  $p$  the equation of motion is given by:

$$mn \left[ \frac{\partial v}{\partial t} + v \cdot \nabla v \right] = F - \nabla p \quad (3.6.1)$$

If  $F$  is a field force then

$$F = n \nabla V$$

Where  $V$  is the potential of one particle in one dimension

$$\begin{aligned} mn \left[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right] &= -n \nabla V - \nabla p = -n \frac{dV}{dx} - \frac{dp}{dx} \\ dv &= \frac{\partial v}{\partial t} dt + \frac{\partial v}{\partial x} dx \\ \frac{dv}{dt} &= \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \frac{dx}{dt} = \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \end{aligned} \quad (3.6.2)$$

Thus according to Equation (3.6.1) in one dimension

$$mn \frac{dv}{dt} = -n \frac{dV}{dx} - \frac{dp}{dx} \quad (3.6.3)$$

#### 3.6.2 Schrodinger Temperature Dependent Equation:

Anew expression of energy can be found from the plasma equation to do this one can use (3.6.3) to get

$$nm \frac{dv}{dx} \frac{dx}{dt} = -n \frac{dV}{dx} - \frac{dp}{dx}$$

Multiplying both sides by  $dx$  and integrating yields

$$mn \int v dv = -n \int dV - \int dp$$

Considering the pressure to be  $p = \gamma nKT$  in general, thus

$$mn \frac{v^2}{2} = -nV - P = -nV - \gamma nKT$$

Hence

$$m \frac{v^2}{2} + V + \gamma KT = \text{const}$$

This constant conserved quantity looks like the ordinary energy beside the ordinary thermal energy term  $\gamma KT$ .

$$E = \frac{p^2}{2m} + V + \gamma KT \quad (3.6.4)$$

To find Schrodinger equation for it, consider the ordinary wave function

$$\Psi = Ae^{\frac{i}{\hbar}(px - Et)}$$

Differentiating both sides by  $t$  and  $x$  yields

$$\begin{aligned} \frac{\partial \Psi}{\partial t} &= -\frac{i}{\hbar} E \Psi \rightarrow i\hbar \frac{\partial \Psi}{\partial t} = E \Psi \\ \frac{\partial^2 \Psi}{\partial x^2} &= -\frac{p^2}{\hbar^2} \Psi \rightarrow -\hbar^2 \nabla^2 \Psi = p^2 \Psi \end{aligned} \quad (3.6.5)$$

Multiplying both sides of equation (3.6.3) by  $\Psi$  yields

$$E \Psi = \frac{p^2}{2m} \Psi + V \Psi + \gamma KT \Psi$$

Substituting Equation (3.6.4), one gets

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi + \gamma KT \Psi$$

This equation represents Schrödinger equation when thermal motion is considered. The solution for time free potential can be

$$\begin{aligned} \Psi &= e^{-\frac{i}{\hbar}(Et)} u \rightarrow \frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar} E \Psi \\ Eu &= -\frac{\hbar^2}{2m} \nabla^2 u + Vu + \gamma KT u \end{aligned} \quad (3.6.6)$$

For constant potential, the solution can be

$$u = e^{ikx}, V = V_0$$

Inserting this solution in equation (3.6.6) yields

$$Eu = \frac{\hbar^2 K^2}{2m} u + V_0 u + \gamma K T u$$

$$E = \frac{\hbar^2 K^2}{2m} + V_0 + \gamma K T$$

If one set the kinetic term to be  $E_0 = \frac{\hbar^2 K^2}{2m}$

One can thus write the energy in the form

$$E = E_0 + V_0 + \gamma K T \quad (3.6.7)$$

This quantum energy expiration involves a thermal term beside kinetic and potential term.

### 3.6.3 Quantum Resistance:

The resistance, per unit length ( $L = 1$ ) per unit area ( $A = 1$ ) can be found from the ordinary definition of  $Z$ . The resistance  $z$  is defined to be the ratio of the potential,  $u$ , to the current per unit area, i.e.

$$z = \frac{u}{I} = \frac{u}{JA} = \frac{u}{J} = \frac{u}{nev} = \frac{mu}{nep} \quad (3.6.8)$$

With  $n$  and  $v$  standing for the free hole or electron density and charge respectively, while  $p$  represent the momentum of electron of mass  $m$ , where  $p = mv$ .

Where the wave function takes the form

$$\Psi = Ae^{iKx} \quad (3.6.9)$$

This selection of  $\Psi$  comes from the fact that the resistance property comes from the motion of the free charge. The potential  $u$  is related to the Hamiltonian  $H$  through the relation

$$H = eu$$

Thus for freely moving charge one gets:

$$\hat{H} = eu = \frac{1}{2}mv^2 = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

In view of equation (3.6.9) and according to the correspondence principle  $V$  take the form

$$u = \frac{\langle \hat{H} \rangle}{e} = \frac{\int \bar{\Psi} \hat{H} \Psi dx}{e} = \frac{\int \bar{\Psi} \hat{p}^2 \Psi dx}{2me} = \frac{\hbar^2 K^2}{2me} \int \bar{\Psi} \Psi dx = \frac{\hbar^2 K^2}{2me} \quad (3.6.10)$$

While  $p$  becomes

$$p = \langle \hat{p} \rangle = \int \bar{\Psi} \hat{p} \Psi dx = \hbar k \int \bar{\Psi} \Psi dx = \hbar k \quad (3.6.11)$$

Thus inserting equation (3.6.10), (10) in (3.6.7) one obtains

$$Z = \frac{m \hbar^2 k^2}{2me^2 \hbar k n} = \frac{\hbar k}{2e^2 n} = \left( \frac{h}{2\pi} \right) \left( \frac{2\pi}{\lambda} \right) \frac{1}{2e^2 n}$$

$$z = \frac{h}{2\lambda e^2 n} = \frac{hf}{2f\lambda e^2 n} = \frac{hf}{2e^2 n v} = \frac{hf\sqrt{\mu\varepsilon}}{2e^2 n} = \hbar\omega \frac{\sqrt{\mu\varepsilon}}{2e^2 n} \quad (3.6.12)$$

$$v = \lambda f = \frac{1}{\sqrt{\mu\varepsilon}} \quad (3.6.13)$$

$$eu = \frac{1}{2} mv^2 = k \quad (3.6.14)$$

The expression for  $z$  can also be found by inserting Equation (3.6.14) in to get

$$z = \frac{u}{j} = \frac{u}{nev} = \frac{mv^2}{2ne^2 v} = \frac{mv}{2ne^2} = \frac{p}{2ne^2} = \frac{h}{2\lambda ne^2}$$

$$z = \frac{hf}{2\lambda fe^2 n} = \frac{hf}{2e^2 n v} = \frac{hf\sqrt{\mu\varepsilon}}{2e^2 n} = \frac{\hbar\omega\sqrt{\mu\varepsilon}(1+x)}{2e^2 n} \quad (3.6.15)$$

It is important to note that his quantum resistance expression resembles the one found by T Sui [3] where one uses De Broglie hypothec  $\omega$ esis [4], i.e.  $p = h/\lambda$

### 3.6.4 Calculation OF Electric Susceptibility:

Consider holes in a conductor having resistive force  $F_r$ , magnetic force  $F_m$  and pressure force  $F_p$  beside the electric force  $F_e$  the equation motion then becomes [3]:

$$F = F_r + F_m + F_{eF_p}$$

Where

$$F_p = -\nabla p, F_r = -\frac{mv}{\tau}, F_m = Bev, F_e = eE = eE_0 e^{i\omega t}$$

$P, x, m, v, \tau, B, e$  and  $E$  stands for the pressure, displacement, mass, velocity, relaxation time, magnetic flux density, electron charge and electric field intensity respectively. Thus the equation of motion takes form.

$$m\ddot{x} = -\frac{mv}{\tau} + Bev + eE - \nabla p \quad (3.6.16)$$

The solution of this equation can be suggested to be:

$$\begin{aligned} x &= x_0 e^{i\omega t} \\ v &= v_0 e^{i\omega t} \\ E &= E_0 e^{i\omega t} \end{aligned} \quad (3.6.17)$$

Inserting (3.6.17) in (3.6.16) yields

$$-m\omega^2 x = \left( -\frac{mv_0}{E_0 \tau} + \frac{Bev_0}{E_0} - \frac{kT\nabla n}{E_0} + e \right) E \quad (3.6.18)$$

$$\begin{aligned} X &= \frac{\left( -\frac{mv_0}{E_0 \tau} + \frac{Bev_0}{E_0} - \frac{kT\nabla n}{E_0} + e \right) E}{m\omega^2} \\ p &= \varepsilon_0 X E = +eN x \end{aligned} \quad (3.6.19)$$

Thus inserting Equation (3.6.18) in (3.6.19) yields

$$\begin{aligned} \varepsilon_0 X E &= eN \frac{\left( -\frac{mv_0}{E_0 \tau} + \frac{Bev_0}{E_0} - \frac{kT\nabla n}{E_0} + e \right) E}{m\omega^2} \\ x &= \frac{eN}{m\omega^2 \varepsilon_0 E_0} \left( \frac{mv_0}{\tau} - Bev_0 - kT\nabla n - eE_0 \right) \end{aligned} \quad (3.6.20)$$

The electric flux density assumes the following relation

$$D = \varepsilon E = \varepsilon_0 E + \chi \varepsilon_0 E = \varepsilon_0 (1 + \chi) E = P + \varepsilon_0 E$$

The electric permittivity is given by

$$\varepsilon = \varepsilon_0 (1 + \chi) \quad (3.6.21)$$

The electric permittivity is thus given according to equation (20) to be.

$$\varepsilon = \varepsilon_0 (1 + \chi)$$

$$= \varepsilon_0 \left[ 1 + \frac{eN}{m\omega^2 E_0} \left( \frac{mv_0}{\tau} - Bev_0 + KT\nabla n - eE_0 \right) \right] \quad (3.6.22)$$

The resistance Z can be found by inserting (3.6.22) in (3.6.15) to get:

$$Z = \frac{\hbar\omega}{2ne^2} \sqrt{\mu\varepsilon_0 \sqrt{1 + \frac{eN}{m\omega^2 \varepsilon_0 E_0} \left( kt\nabla n + \frac{mv_0}{\tau} - Bev_0 - eE_0 \right)}} \quad (3.6.23)$$

$$z = \frac{\hbar\omega}{2ne^2} \sqrt{\mu\varepsilon_0 \sqrt{\frac{m\omega^2 \varepsilon_0 E + eN \left( kt\nabla n + \frac{mv_0}{\tau} - Bev_0 - eE_0 \right)}{m\omega^2 \varepsilon_0 E_0}}}$$

Thus the critical temperature is given by

$$T_c = \frac{(Be\tau - m)v_0}{\tau K\nabla n} + \frac{(e - m\omega^2 \varepsilon_0)E_0}{EnK\nabla n} \quad (3.6.24)$$

If the internal field B result from  $N_0$  atoms each having average flux density  $\mu B$  then [5].

$$B = \mu_B N_0 \quad (3.6.25)$$

Therefore  $T_c$  can take the form

$$T_c = \frac{(\mu_B N_0 e\tau - m)v_0}{\tau K\nabla n} + \frac{(e - m\omega^2 \varepsilon_0)E_0}{eNK\nabla n} \quad (3.6.26)$$

### 3.6.5 Tight binding critical temperature and energy gap:

In tight binding model [5] the energy of electrons in the crystal is given by

$$\varepsilon = \varepsilon_0 + \alpha_1 + 2\gamma \cos ka \quad (3.6.27)$$

Where  $\varepsilon_0$  is the energy in the absence of crystal field, while the other terms describe the effect of the crystal field. The energy  $\varepsilon_0$  can split into two terms the kinetic part which can describe the thermal motion in the form  $\frac{f_0}{2}KT$  beside the potential term  $-V_0$  for attractive force or bounded particle.

Thus one can write

$$\varepsilon_0 = \frac{\hbar^2 k_0^2}{2m} + \frac{f_0}{2}KT - V_0 \quad (3.6.28)$$

$$E = \frac{\hbar^2 k_0^2}{2m} + \gamma KT + V$$

$$\varepsilon_0 = \frac{f_0}{2} KT - V_0 - \alpha_0$$

$$\alpha_0 = \frac{\hbar^2 K^2}{2m}$$

$f_0$  Degrees of freedom.

The terms describing the effect of the crystal force are

$$\begin{aligned}\alpha_1 &= \langle \phi_m | \hat{H}_{\text{cry}} | \phi_m \rangle \\ \gamma &= \langle \phi_j | \hat{H}_{\text{cry}} | \phi_m \rangle \\ \alpha &= \alpha_0 + \alpha_1\end{aligned}\tag{3.6.29}$$

In view of Equation (3.6.27) and (3.6.28)

$$\varepsilon = \frac{f_0}{2} KT - V_0 + \alpha + 2\gamma \cos ka\tag{3.6.30}$$

Here  $H_{\text{cry}}$  stands for the crystal force Hamiltonian part, while  $\phi_m$  and  $\phi_j$  are the states of particles located at the site  $m$  and  $j$  respectively.

The superconductor is characterized by the existence of energy gap can be understood here in two ways .if the electrons or holes are not free. This requires  $E$  to negative. Thus equation (3.6.28) and (3.6.27) needs

$$\varepsilon = \frac{f_0}{2} KT - V_0 + \alpha + 2\gamma \cos ka < 0\tag{3.6.31}$$

Or the max value of  $\varepsilon$  where  $\cos ka = -1$  is less than zero, i.e.

$$\begin{aligned}\varepsilon_{\text{max}} &= \frac{f_0}{2} KT - V_0 + \alpha + 2\gamma \cos ka < 0 \\ \frac{f_0}{2} KT &\leq V_0 - \alpha + 2\gamma\end{aligned}\tag{3.6.32}$$

For constant attractive crystal force

$$\begin{aligned}H_{\text{cry}} &= -V_{\text{cry}} \\ \alpha_1 &= \langle \phi_m | H_{\text{Cry}} | \phi_m \rangle = -\langle \phi_m | V_{\text{CRY}} | \phi_m \rangle = -V_{\text{Cry}} \delta_{mm} \\ \gamma &= \langle \phi_j | -V_{\text{Cry}} | \phi_m \rangle = -V_{\text{Cry}} \langle \phi_j | \phi_m \rangle = -V_{\text{cry}} \delta_{jm=0}\end{aligned}\tag{3.6.33}$$



$$\frac{f_0}{2}KT \leq V_0 - \alpha$$

Thus the critical temperature is given by

$$\frac{f_0}{2}KT = V_0 - \alpha \quad (3.6.34)$$

Substituted equation (3.6.34) beside equation (3.6.33) in equation (3.6.31) one gets

$$\varepsilon = \frac{f_0}{2}KT - \frac{f_0}{2}kT_c \quad (3.6.35)$$

The energy gap  $\Delta$  s equal to the difference between zero energy in conduction band and the negative energy in the valence band. Thus

$$\Delta = 0 - \varepsilon = \frac{f_0}{2}KT_c - \frac{f_0}{2}kT$$

Since this relation holds for  $T < T_c$  one can neglect  $T$  since it is small to get

$$\Delta = \frac{f_0}{2}KT_c$$

Equation (3.6.31) can also be utilized to get the forbidden energy states which characterizes superconductors, where

$$\cos ka = \frac{\varepsilon - \frac{f_0}{2}KT + V_0 - \alpha}{2\gamma}$$

The energy is forbidden when  $\cos ka \geq 1$

$$\frac{\varepsilon - \frac{f_0}{2}KT + V_0 - \alpha}{2\gamma} \geq 1$$

$$\varepsilon - \frac{f_0}{2}KT + V_0 - \alpha \geq 2\gamma$$

$$\frac{f_0}{2}KT + \alpha - \varepsilon - V_0 \leq -2\gamma$$

$$\frac{f_0}{2}KT \leq \varepsilon + V_0 - 2\gamma - \alpha$$

Thus the critical temperature

$$\frac{f_0}{2}KT_c = \varepsilon + V_0 - 2\gamma - \alpha \quad (3.6.36)$$

The forbidden energy is thus related to the critical temperature through the relation

$$\varepsilon = \frac{f_0}{2}KT_c - V_0 + 2\gamma + \alpha \quad (3.6.37)$$

If the particle has a 4-degree of freedom, 3-translational and one vibration.

$$\varepsilon = 2kT_c - V_0 + 2\gamma + \alpha \quad (3.638)$$

In view of equation (3.6.33) and (3.6.29), since Plank constant is very small and for very small crystal field and for bound force  $\varepsilon \approx 2kT_c$ , since the energy gap  $\Delta$  is the difference between bound valence energy  $E$ . Thus minimum free conduction electron energy zero. Thus

$$\Delta = E - 0 = 2kT_c \quad (3.6.39)$$

Which shows linear relation between  $\Delta$  and  $T_c$ , thus it resembles the empirical relation. Where the energy gap is found to be  $\Delta \sim 1.75kT_c$  [6].

### 3.7 Summary and Critique

Many Attempts were made to be the defects of HTSC [41, 42, 43, 44, 45, 46, 47, and 48]. These attempts are either complex or incomplete. In the models discussed temperature dependent Schrodinger equation was used successfully. But none of them tries to speak about hopping mechanism and flux quantization.

# Chapter Four

## Hopping Mechanisms and Flux Quantization

### 4.1 Introduction:

Superconductor (SC) is one of the most important phenomena in modern physics. It is based on Barden, Cooper and Schrieffer theory (BCS).

But at high temperature above 135, this theory suffers. It cannot explain how the resistivity abruptly drops to zero below critical temperature  $T_c$ .

Beside the explanation of the so-called pseudo gap, and isotope and pressure effects. In addition to the phase transition from insulating to superconductivity state [1, 2].

This model is proposed to cure some of these drawbacks.

This new model can explain why the resistance drops to zero below  $T_c$  abruptly, beside the expression for isotope and pressure effect. It also gives an expression which is mathematically simple and is in conformity with experimental results such as the phase change from insulator to superconductor. The work of this model [5].

### 4.2 Generalized Statistical model Laws Super Conductor:

The generalized statistical model was proposed by some authors to solve some of the long-standing problems in material science [91]. According to this version, the number of particles having local energy  $E$  and moving in a medium having average energy  $\bar{E}$  is given by.

$$n = n_0 e^{\frac{-E}{\bar{E}}} \quad (4.2.1)$$

This expression can be used to try to describe the hopping mechanism [6].

To do this considers electrons hopping in superconductor having uniform coulomb attractive field. In this attractive the potential is negative. Thus the energy is given by.

$$\bar{E} = -V_c \quad (4.2.2)$$

Assume now the local energy is equal to kinetic energy T beside potential energy which result again from attractive Coulomb force. Therefore.

$$E = T + V = T - V_c \quad (4.2.3)$$

Inserting equation (4.2.2) in equation (4.2.1) yields.

$$n = n_0 e^{\frac{E}{V_c}} \quad (4.2.4)$$

It is well known that hopping process takes place when the kinetic energy exceeds coulomb potential, i.e.

$$T \gg V_c \quad (4.2.5)$$

Thus according to equation (4.2.3) and (4.2.4)

$$E \gg 0 \quad (4.2.6)$$

$$n = n_0 e^{\frac{E}{V_c}} \rightarrow \text{large} \quad (4.2.7)$$

This means that hopping take place as far as n which reflects hopping probability is large.

In superconductivity, when coulomb attraction dominate, it follows that

[See equation (4.2.3)]

$$V_c \gg T \quad (4.2.8)$$

Hence

$$\begin{aligned} E = T - V_c &= -|E| \\ |E| &\rightarrow \infty \end{aligned} \quad (4.2.9)$$

A direct substitution of (4.2.9) in (4.2.4) yields

$$n = n_0 e^{-\frac{|E|}{V_c}} = n_0 e^{-\infty} = 0 \quad (4.2.10)$$

This means that when coulomb attraction dominates, no hopping takes place when the coulomb potential dominates. This conforms to what is written in the literature [7].

The SC hopping mechanism can also be studied by considering a large coulomb repulsive potential resulting from the repulsion between the electron which needs to hop and the electron which exists in the atom to which the electron needs to hop,

In this case the local repulsive energy becomes

$$E = V \quad (4.2.11)$$

The average uniform energy can be assumed to result from thermal energy and average attractive positive ionic field. In this case the average energy become

$$\bar{E} = KT - V_0 \quad (4.2.12)$$

Substituting (4.2.11) and (4.2.12) in equation (4.2.1) yields

$$n = n_0 e^{\frac{V}{V_0 - KT}} \quad (4.2.13)$$

Hopping takes place when the probability is large. This requires

$$V_0 > KT$$

$$KT < V_0 \quad (4.2.14)$$

Thus the critical temperature  $T_c$  is given by

$$V_0 = KT_c \quad (4.2.15)$$

Thus hopping is large when

$$T < T_c \quad (4.2.16)$$

Which is ordinary SC condition since coulomb repulsion  $V$  is large.

Thus

$$e^{\frac{V}{K(T_c - T)}} \rightarrow \infty \quad (4.2.17)$$

Hence according to equation (4.2.13)

$$n_0 \rightarrow 0 \quad (4.2.18)$$

To make  $n$  finite

But when

$$T > T_c \quad (4.2.19)$$

$$KT > KT_c$$

$$KT > V_0 \quad (4.2.20)$$

Thus from (4.2.13)

$$C = V_0 - KT < 0$$

$$C = -|C| \quad (4.2.21)$$

Thus equation (4.2.13) reads

$$n = n_0 e^{\frac{-V}{|C|}} \quad (4.2.22)$$

For very large repulsive force

$$V \rightarrow \infty$$

$$e^{\frac{-V}{|C|}} \rightarrow 0$$

Thus from (4.2.18) and (4.2.22) equation (4.2.13) gives

$$n \rightarrow 0 \quad (4.2.23)$$

Thus no hopping takes place when

$$T > T_c \quad (4.2.24)$$

Thus SC is destroyed

Another attractive approach can be suggested by assuming the electron moving in uniform repulsive electron field, and subjected to coulomb repulsive potential. In the case

$$E = V \quad \bar{E} = V_0 \quad (4.2.25)$$

$$n = n_0 e^{-\frac{V}{V_0}} \quad (4.2.26)$$

In this case hopping can take place when no local field exist or when it is very small. In this case

$$V = 0 \quad (4.2.27)$$

This can forms with the fact that the electric local field E vanishes inside SC

$$n = n_0 e^{-0} = n_0 \quad (4.2.28)$$

The electrons are driven here by the uniform field.

But when very large external magnetic field is applied, such that the local field becomes extremely large, i.e.

$$V = V_m \rightarrow \infty \quad (4.2.29)$$

In this case equation (4.2.22) gives

$$n = n_0 e^{-\infty} \rightarrow 0 \quad (4.2.30)$$

Thus no hopping takes place and the conductivity is destroyed due to the existence of external magnetic field.

### 4.3 Super conductivity on the basic of generalized statistical

#### Model:

The generalized statistical model, which was proposed by some authors, generalized the statistical distribution laws by using plasma equation [92].

According to the model, Maxwell distribution law can be generalized to be written in the form.

$$n = n_0 e^{\frac{-E}{\bar{E}}} \quad (4.3.1)$$

With n standing for the number of particles  $\bar{E}$  represent the average field over the whole sample, While E stands for the local field. It is well known that the SC is characterized by infinite conductivity. In this work one can assume the conductivity to be in the form

$$\sigma = \frac{ne\tau}{m} \quad (4.3.2)$$

$\tau$  is the relaxation time. The infinite conductivity can result from infinite relaxation time or very large number of charges. Let us choose the latter possibility. It is well known at inside SC the electric field vanishes

$$E = 0 \quad (4.3.3)$$

Thus

$$\sigma = \frac{j}{E} = \frac{j}{0} \rightarrow \infty \quad (4.3.4)$$

If one assumes that the only uniform field is the electric one, it follows that

$$\bar{E} = V(\text{electric}) = 0 \quad (4.3.5)$$

One can assume also the existence of local coulomb attractive force. This means that

$$E = -V_c \quad (4.3.6)$$

Inserting (4.3.5) and (4.3.6) in (4.3.1) yields

$$n = n_0 e^{\frac{V_c}{0}} = n_0 e^{\infty} \rightarrow \infty \quad (4.3.7)$$

Thus according to equation (4.3.2) and (4.3.7)

$$\sigma = \infty \quad (4.3.8)$$

This means that the conductivity is infinite

Consider now an external magnetic field of potential  $V_m$  enters the SC. Thus according to equation (4.3.5)

$$\bar{E} = V_m \quad (4.3.9)$$

Thus inserting (4.3.6) and (4.3.9) in equation (4.3.1)

$$n = n_0 e^{\frac{V_c}{V_m}} \neq \infty \quad (4.3.10)$$

i.e.,  $n$  is finite. Thus (4.3.8) according to equation (4.3.2)

$$\sigma = \frac{ne\tau}{m} \neq \infty \quad (4.3.11)$$

Hence the conductivity is finite and the SC converted to ordinary conductor.

The effect of magnetic field in destroying super conductivity can also be found by considering Local repulsive electric coulomb field of potential  $V$ . Thus

$$E = V \quad (4.3.12)$$

Consider now the average uniform field is result from coulomb attraction between hopping electron and the host atom in which the electron exists before hopping. Assume also that this electron requires uniform thermal energy this means that the uniform energy is given by



$$\bar{E} = KT - V_0 \quad (4.3.13)$$

Substitute (4.3.13) and (4.3.12) in (4.3.1) to get

$$n = n_0 e^{\frac{V}{V_0 - KT}} \quad (4.3.14)$$

The hopping process take place, when

$$n \rightarrow \text{large} \quad (4.3.15)$$

This requires

$$V \rightarrow \text{large} \quad (4.3.16)$$

And

$$\begin{aligned} V_0 &> KT \\ KT &< V_0 \end{aligned} \quad (4.3.17)$$

If one write

$$V_0 = KT_c \quad (4.3.18)$$

Thus hopping take place when

$$\begin{aligned} KT &< KT_c \\ T &< T_c \end{aligned} \quad (4.3.19)$$

Which agree with SCdefinition but if a uniform magnetic field of potential  $V_m$  is applied such that the uniform energy becomes [see equation (4.3.13), (4.3.18)]

$$\bar{E} = KT - KT_c + V_m \quad (4.3.20)$$

Thus equation (4.3.14) reads

$$n = n_0 e^{\frac{V}{KT_c - KT - V_m}} \quad (4.3.21)$$

When

$$\begin{aligned} V_m &\geq KT_c \\ KT_m &\geq KT_c \end{aligned} \quad (4.3.22)$$

Thus

$$c = K(T_c - T_m - T) < 0$$

When

$$c = -|c| \quad (4.3.23)$$

$$T_m > T_c \quad (4.3.24)$$

For all values of T i.e.

$$V_m > KT_c \quad (4.3.25)$$

The minimum magnetic potential which destroyed SC is

$$V_{mc} = KT_c \quad (4.3.26)$$

This conforms to the expression of the energy gap

Substituting (4.3.23) in (4.3.21)

$$n = n_0 e^{\frac{-V}{|c|}} \rightarrow 0 \quad (4.3.27)$$

When

$$V \rightarrow \infty$$

When coulomb potential is large the flux quantization can be obtained by considering the local energy to be consisting of imaginary term representing energy loss in hopping process the energy loss may result from coulomb repulsion or magnetic interaction which decreases electron total energy.

In this case the local energy E can be written as [5]

$$E = E_1 + iE_2 \quad (4.3.28)$$

Therefore from (4.3.1) and (4.3.28)

$$n = n_0 e^{\frac{(E_1 + iE_2)}{E}} = n_0 e^{\frac{E_1}{E}} \left[ \cos \frac{E_2}{E} + i \sin \frac{E_2}{E} \right] \quad (4.3.29)$$

Since n is real it follows that the imaginary part vanishes,

$$\sin \frac{E_2}{E} = 0$$

This requires

$$\frac{E_2}{E} = \left( n + \frac{1}{2} \right) \pi \quad (4.3.30)$$

Where  $E_2$  may be assumed to represent energy loss due to the interaction of external field B with diamagnetic momentum.

Where

$$\mu = iA \quad (4.3.31)$$

Thus the magnetic energy losses is given by

$$E_2 = \mu \cdot B = iAB = (fe)\phi \quad (4.3.32)$$

Assuming electrons as harmonic oscillator are can write

$$\bar{E} = hf \quad (4.3.33)$$

Thus inserting (4.3.33),(4.3.32) in (4.3.30) yields

$$\frac{(fe)\phi}{hf} = 2\pi\left(n + \frac{1}{2}\right)$$

Thus the flux is given by

$$\phi = \frac{2\pi\left(n + \frac{1}{2}\right)h}{e} \quad (4.3.34)$$

This means that the flux is quantized

#### 4.4 Discussion:

The conductivity of SC in equation (4.3.2) is found to be infinite when the uniform electric field vanishes and the local field is attractive coulomb field as shown by equation (4.3.5) and (4.3.6) beside the GSM expression(4.3.1) for n.

The GSM can explain the SC mechanism by hopping process. In the first approach a verge uniform energy can be assumed to be an attractive coulomb potential [see equation (4.2.2)], while the local field energy, as proposed by Hubbard model is the kinetic and potential energy. The Probability n is large, and hopping takes place as shown by equations (4.2.7) when the kinetic energy is Large.

These agree with that proposed by Hubbard model. But when the kinetic energy is less than the potential no hopping takes place, since the probability vanishes [as equation (4.2.10)].

When thermal energy is considered as contributing to the uniform energy [see equation (4.2.12)].

For repulsive Coulomb force shown by relation (4.2.11)

Hopping is possible if T exceeds a critical value  $T_c$  shown by equation (4.2.15). This strikingly agrees with observation. It is also very interesting to note that when T is greater than  $T_c$  , the probability n vanishes [see equation (4.2.23)] and no hopping take place. Hopping can also exist when

no local field exist as shown by equation (4.2.28) which conforms with the fact the inside SC

The electric field vanishes. When very strong magnetic field is applied SC is destroyed as shown by equation (4.2.29) and (4.2.30).

This agree with the fact that very strong external magnetic field should of course exceeds critical values above which SC is destroyed.

By assuming the uniform energy to result from thermal beside coulomb attraction, and considering the local energy to be repulsive coulomb energy [see equation (4.3.12) m (4.3.13), (4.3.14)]

The presence of external magnetic field destroyed sc as shown is equation [(4.3.20), (4.3.26)].

This agrees with observation it is very interesting to note that the critical magnetic energy value shown in equation (4.3.26) is typical to the energy gap  $E_g$  above which the sc is destroyed. Using the expression for number of particles inGSM, the flux is shown to be quantized with an ordinary expression.

#### **4.5 Conclusion:**

It can also successfully is describe some magnetic properties of superconductors.

The generalized statistical physical model can successfully describeSC hopping conduction mechanism proposed by Hubbard.

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