Sudan University Of Science & Technology

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Effect Of Friction and Diffusion On Lasing process

تأثير الإحتكاك والإنتشار علي عملية إنتاج الليزر

A Thesis Submitted for Fulfillment Requirement of the Degree of Doctor of Philosophy in Physics

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الآية

بسم الله الرحمن الرحيم

قال تعالي:

(قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنتَ الْعَلِيمُ الْحَكِيمُ)

سورة البقرة الآية (32)

Dedication

This thesis is dedicated To all my family, the symbol of love and giving: To my father , my late mother, my husband who encourage and support me, and My beloved kids: Ebaa and Mohamed, whom I can't force myself to stop loving.

To my friends , All the people in my life who touch my heart.

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Abstract

Quantum mechanics is a physical theory dealing with the behavior of matter and waves on the scale of atoms and subatomic particles.

The aim of this work is to use quantum mechanics to study the effect of friction on optical properties of matter namely lasing process.

The research methodology is based on using quantum postulates and the notion of energy in the presence of friction to derive the quantum laws that determine lasing process in the presence of friction.

In this study Schrodinger equation was accounted for the effect of medium friction on the spatial evolution of the quantum system. By considering electrons as vibrating strings, the solution of harmonic oscillator equation for resistive medium, showed that the electron mass is quantized.

By solving new Schrodinger equation, the wave function is shown to be spatially affected by friction. Therefore the lasing can take place due to the fact that the frictional effect of the incident beam excite atoms to emit coherent photons. The effect of friction was also obtained within the framework of klein - Gordon equation, the wave function shows the possibility of lasing and the amplification factor depends on medium conductivity. This research shows the possibility of lasing in the presence of friction with the framework of quantum laws.

المستخلص

ميكانيكا الكم هي نظرية فيزيائية تهتم بدراسة سلوك المادة والموجات علي المستوي الذري ودون الذري.

الهدف من هذه الدراسة إستخدام ميكانيكا الكم لدراسة تأثير الإحتكاك علي الخواص الضوئية للمادة و بالأخص عملية توليد الليزر . طريقة هذا البحث تعتمد علي إستخدام فرضيات ميكانيكا الكم ومفهوم الطاقة في وجود الإحتكاك لإستنباط قوانين الكم التي تحدد شروط حدوث الليزر في وجود الإحتكاك.

في هذه الدراسة تم استنباط معادلة شرودينجر التي تأخذ في الاعتبار تأثير إحتكاك الوسط علي التغير الأحداثي للمنظومة الكمية. وبإعتبار أن الإلكترونات أوتار متذبذبة تم حل معادلة المهتز التوافقي للوسط الإحتكاكي، وثم وجد أن كتلة الالكترون مكممة.

تم حل معادلة شرودينجر الجديدة أتضح أن الدالة الموجية تتأثر إحداثياً بالاحتكاك. بحيث أن توليد الليزر ناتجاً من حقيقة أن تأثير إحتكاك الحزمة الساقطة، يؤدي إلي إثارة الذرات لإصدار فوتونات متطايرة. ثم تم الحصول علي تأثير الإحتكاك إيضاً في إطار معادلة كلاين – قوردين. حيث توضح الدالة الموجية إمكانية توليد الليزر ويعتمد معامل التضخيم علي موصلية الوسط. هذا البحث يوضح إمكانية توليد الليزر في وجود الإحتكاك في إطار القوانين الكمية.

Contents

Chapter One

Introduction

1-1: Introduction

Quantum mechanics is the theory that describes the dynamics of matter at the microscopic scale. It is the only valid framework for describing the microphysical world[1,2].

It is vital for understanding the physics of solids, lasers, semiconductor and superconductor devices, plasmas, etc. In short, quantum mechanics is the founding basis of all modern physics: solid state, molecular, atomic, nuclear, and particle physics, optics, thermodynamics, statistical mechanics, and so on[3,4,5].

Despite the remarkable successes of quantum equations, but they suffer from noticeable set backs. For example, the quantum equation can not differentiate between the behavior of two particles subjected to the same potential, but one moves in free space and the other moves inside matter. This is in direct conflict whit experimental observations. Thus one needs new quantum equation that differentiates between the two situations[6]. One of the important features of the medium is the friction effects.

Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other[7,8,9]. It is the price for moving too fast. An attempt to induce a rapid change in the state of a system would be accompanied by additional entropy generation and will be encumbered by energy costs[10,11,12]. As an example of friction we can consider a body moving rapidly against a stationary background. Its kinetic energy is dissipated, generating heat and entropy in the environment, the amount of dissipation is proportional to the velocity[13,14,15,16].

The friction is typically modeled by a phenomenological theory within classical mechanics. it is extended to the quantum domain. There are ample examples of quantum frictional phenomena. These include friction observed in micro-mechanical systems at low temperatures, in superfluid theory, and even in quantum cosmology and more[17,18,19,20].

In particular, friction is the term widely used in descriptions of ion collisions. Frictional forces depend in this case on position and their range is comparable with the nuclear radius[21].

Moreover, The real area of contact is made up of a large number of small regions of contact, where atom to atom contact takes place[22,23]. In most real physical systems friction plays an important role. It's precise description requires accounting[24] .

Friction effects in all physical laws including quantum mechanics, unfortunately there is no room for effect of friction in these laws except some isolated attempts made by some researchers.

1-2: Research problem

The research problem is related to the fact that There is no theoretical model describes the effect of friction on the lasing process. It requires modifying Schrodinger equation and Klein – Gordon equation for resistive medium.

1-3: Objective of the Research

The aim of this work is to construct a theoretical model that takes into describe frictional effect due to diffusion and interference by deriving Schrodinger Equation, Klein-Gordon equation for frictional medium , and also explain the effect of friction and diffusion on lasing intensity and amplification factor.

1-4: Research Methodology

The research methodology is based on using quantum postulates and energy in the presence of friction to derive the quantum laws that determine lasing process in the presence of friction.

1-5: Thesis Layout

This work is organized as follows: chapter one is the introduction, and chapter two is theoretical background and literature review. Chapter three is the effect of friction and diffusion on the lasing process while chapter four is devoted for results and discussion.

Chapter Two

Theoretical Background and Literature Review

2-1: Introduction.

This chapter is concerned with the theoretical background and literature review of laser and quantum equations.

The different attempts were made to construct new quantum laws to explain some physical phenomena. Some of them try to modify Schrodinger equation and Klein – Gordon equation. Also there are some researchers have been written in study of amplification of laser. But no one of them concentrate on the effects of friction on momentum term in Schrodinger equation , and no one of them uses this equations to construct a quantum equation that accounts for lasing due to friction . Here in this chapter one tries to mention some of them.

2-2: Laser and light Amplification

Laser plays an important role in ours day life. Thus it is important study The laser properties . The stimulated emission beside spontaneously emission processes are studied here . The laser production is also discussed.

2-2-1: Emission and Absorption of light:

The atom now a day is considered as a system consisting of a central**,** positively charged nucleus surrounded by a number of negatively charged electrons revolving around the nucleus in certain orbits .Each orbit describes energy level .The energy is characterized by a principal quantum number denoted by n . The nearest level to the nucleus is called the ground state and its principal quantum number is equal to one. Each type of atom contains a certain amount of energy levels. If the atom contains additional energy states over and above its ground state it can emit or absorb photons .The absorption takes place when an electron makes a transition from a lower to a higher energy state , with a photon being absorbed in this process. In the emissions process the electrons move from a higher state to a lower one. A photon with energy equal to the energy difference between the two levels E_1 and E_2 is released or absorbed in the emission or absorption process .The frequency f of the photon is related to the energy difference between the two levels E_1 and E_2 according to the relation [25]

$$E_2 - E_1 = hf \qquad (2.2.1)$$

Where h is Planck's constant.

2-2-2: Absorption process

Absorption is the process by which a photon is absorbed by atom, the photon of frequency f passes through an atomic system with energy levels E_1 and E_2 can absorb this photon if

$$hf = E_1 - E_2 \qquad (2.2.2)$$

As a result an electron leave E_1 to E_2. The population of the lower level E_1 will be depleted at a rate proportional both , i.e.

$$\frac{dN_{10}}{dt} = -\beta_{12}\rho N_1 \qquad (2.2.3)$$

Where β_{12} is a constant of proportionality called Einstein coefficient. The produced $\beta_{12}\rho$ can be interpreted as .The probability per unit frequency that transitions are induced by the effect of the field [26,27].

2-2-3: spontaneous emission process

spontaneous emission is the process by which an electron spontaneously without any outside influence, decays from a higher energy level to a lower one after an electron has been raised to the upper level by absorption. The population of the upper level2 decays spontaneously to the lower level1 at a rate proportional to the upper level population N_2 ,i.e. [28].

$$\frac{dN_{20}}{dt} = -A_{21}N_2 \qquad (2.2.4)$$

Where A_{21} is a constant of proportionality

2-2-4: Stimulated emission process

The process is described by the Einstein coefficient E_2 which gives the probability per unit energy density of the radiation field that electrons from the excited state E_2 are forced to return to its ground state E_1 is a photon of energy $hf = E_2 - E_1$ is incident on the atom .The rate of transition of electron from $E_2$to E_1 is given by [29].

$$\frac{dN_{20}}{dt} = -\beta_{21} N_2 \rho \qquad (2.2.5)$$

The emitted and incident photon have the same frequency, direction and are in phase.

Stimulated emission is one of the fundamental processes that led to the development of laser. This is because the coherence of the incident and emitted photon increases the light intensity .

2-2-5: Amplification of light and population inversion

If a light of intensity I_0 is incident on a medium, it's intensity in active medium increases[30]. The intensity, I, of light at a distance Z inside the medium is given by

$$I = I_0 e^{\beta z} \qquad (2.2.6)$$

Where β is called amplification factor.

If a light of radiation density ρ is incident on a medium the rate of electrons leaving level E_1 is given by $\beta_{12} N_1$, While the rate of electron coming to E_1 from E_2 by spontaneous and stimulated emission are given by $A_{12} N_2$ and $\beta_{21} N_2$ respectively [31,32,33]. Thus the rate of change of electrons in level1 E_1 is given by

$$\frac{dN_{10}}{dt} = (-\beta_{12} N_1 + \beta_{21} N_2)\rho + A_{12} N_2 \qquad (2.2.7)$$

Similarly the rate of change of electrons in level2 E_2 is given by

$$\frac{dN_{20}}{dt} = (\beta_{12} N_1 - \beta_{21} N_2)\rho - A_{21} N_1 \qquad (2.2.8)$$

At equilibrium the number of atom N_2 in level E_2 is constant. Thus the rate of change of N_2 vanishes, I.e.

$$\frac{dN_{20}}{dt} = 0$$

Thus a equation (2.2.8) becomes

$$(\beta_{12} N_1 - \beta_{21} N_2)\rho - A_{21} N_1 = 0$$

If

$\beta_{12} = \beta_{21} = \beta$

Then

$\rho\beta(N_1 - N_2) = A_{21}N_1$

On the other hand the rate of electron transition

$\frac{dN_{20}}{dt} = A\Delta Z$

from level2 is equal to the rate of photon emission

$\frac{\Delta I}{hf}A$

Through the area A, I.e

$$-\frac{dN_{20}}{dt}A\Delta Z = \frac{\Delta I}{hf}A \qquad (2.2.9)$$

By reviewing of equation (2.2.9) and neglecting the process of spontaneous emission one gets

$(\beta_{12}N_1 - \beta_{21}N_2)\rho A\Delta Z = \frac{\Delta I}{hf}A$

But since $I = \rho c$, then,

$$(\beta_{12}N_1 - \beta_{21}N_2)\frac{I_0 A\Delta Z}{c} = \frac{\Delta I}{hf}A \qquad (2.2.10)$$

Bearing in mind that

$\beta = \beta_{12} = \beta_{21}$

$$\frac{\Delta I}{\Delta Z} = \frac{dI}{dZ} = \beta(N_2 - N_1)\frac{hfI_0}{c} \qquad (2.2.11)$$

Hence

$$\int\frac{dI}{I} = \beta(N_2 - N_1)\frac{hf}{c}\int \mathrm{dz}\,; \qquad (2.2.12)$$

$\ln I = \beta(N_2 - N_1)\frac{hf}{c}Z + Co$

$$I = I_0 e^{\beta(N_2 - N_1)\frac{hf}{c}Z} \qquad (2.2.13)$$

Comparing (2.2.6) with (2.2.13) one finds that the amplification coefficient β is given by

$$\beta = \beta(N_2 - N_1)\frac{hf}{c} \qquad (2.2.14)$$

Thus according to equation (2.2.13) I increases when the radiation enters more and more inside the medium when

$\beta(N_2 - N_1)\frac{hf}{c} > 0$

this requires that

$$N_2 > N_1 \tag{2.2.15}$$

Which means that the population N_2 of the upper level E_2 should be more than the population N_1 of lower level E_1 . this condition is called population inversion.

2-2-6: Excitation mechanism

The excitation mechanism is a source of energy that excites or "pumps" the atoms in the active medium from a lower to a higher energy state in order to produce a population inversion. In gas lasers and semiconductors lasers the excitation mechanism usually consists of an electrical current flow through the active medium. Solid and liquid lasers most often employ optical pumps, for example, in a ruby laser the chromium atoms inside the ruby crystal may be pumped into an excited state by means of a powerful burst of light from a flash lamp containing xenon gas[34,35,36,37].

2-3: Friction in Quantum Mechanics.

Friction is the price for moving too fast. An attempt to induce a rapid change in the state of a system would be accompanied by additional entropy generation and will be encumbered by energy costs. As an example of friction we can consider a body moving rapidly against a stationary background. Its kinetic energy is dissipated, generating heat and entropy in the environment. The amount of dissipation is proportional to the velocity. Another archtypical case of friction is a driven gas compression process.

For a system that is thermally decoupled from its environment, rapid changes in the piston position that compresses the gas will result in internal heating of the gas and entropy generation. To restore the system to its slow quasi-static compression equivalent, heat has to be removed from the gas. This additional heat is equivalent to extra work against friction.

Friction is typically modeled by a phenomenological theory within classical mechanics. How does it extend to the quantum domain? Can a first principles model of friction be developed? There are ample examples of quantum frictional phenomena. These include friction observed in micro-mechanical systems at low temperatures, in superfluid theory, and even in quantum cosmology and more.

Are such quantum examples of friction essentially classical phenomena that endure into the quantum domain, or is there something distinctly quantum about (at least some kinds of) friction? In this review will argue that the latter is the case—there are distinctly quantum types of friction. Even more so, quantum mechanics generally adds frictional affects above and beyond any classical ones, so that a quantum (and therefore fuller) description of the system will include more friction.

Several important changes occur in the transition into quantum mechanics. First, the physics of an isolated system is strictly reversible and unitary. We will argue below that internal friction still occurs but its analytical description requires a proper extension of the classical concepts of entropy and temperature.

Furthermore, we will point to a new mechanism that creates distinctly quantum internal friction.

Secondly, the quantum description of the system plus its environment allows for entanglement correlations that have no classical analog. We will describe the quantum theory of open systems in some detail, and argue that in the quantum domain it is difficult to separate the system from its environment.

This difficulty has led to misunderstandings and incorrect attempts to characterize friction.

Finally, quantum fluctuations and zero-point energy change the nature of the environment, which leads to additional frictional forces.

friction in both the classical and the quantum domains. We will argue that friction is manifested as two phenomena:

1. External friction amounts to the dissipation of kinetic energy from a small "open system" to its environment, creating entropy and heat. The relevance of kinetic energy is related to the isotropy of the environment, and the asymmetry of a particular direction of (non-zero) velocity.

Irreversibility and the generation of heat and entropy are related to the disparity in the size and time scales of the systems, leading to effective irreversible memoryless dynamics and a large entropy generation. Heuristically, external friction is the attempt by the environment to lower the system to a "symmetric" velocity of zero.

2. Internal friction, which is the generation of excitations (which are then typically dissipated by external friction) due to the disparity between the internal time-scales of the system and the external driving time scale. Heuristically, internal friction is the resistance of the system to rapid change.

Both processes can have quantum contributions. Quantum fluctuations will generally function as an extra dissipative environment, while quantum non-commutativity will ensure that the system cannot perfectly follow external changes.

2-3-1: Internal Friction

Internal friction is induced in a system when its external constrains are changed rapidly. Consider a quantum system with a discrete energy spectrum (we will further assume non-degeneracy for simplicity).

An ensemble will have some average energy

$$E = \sum p_i E_I$$

Where $p_i = p(E_i)$ is the probability to find the system in a certain energy eigenstate.

A rapid change in external constraints corresponds to a change in some external semi-classical field in the system's Hamiltonian. For the paradigmatic case of a gas in a piston the field will be related to the location of the potential barrier confining the gas particles. If the change to the external field is slow enough, the quantum adiabatic theorem assures us that there will be no change in the energy populations and therefore the change in the energy will be due only to the change in energy levels. The energy in such a “quasistatic” process changes minimally in this sense.

Now consider a faster change. The rapid change in energy levels can now lead to population changes, changing the energy beyond the “minimal” quasistatic change of the energy levels themselves. For simplicity, let us assume for now that we start from the ground state. Then we can only lose population density to higher states, so that we can only reach a higher (or equal) energy compared to the quasistatic change. Thus we receive a “resistance” to velocity: when we strive to drive the system quickly we need to invest more work.

This process, by itself, is reversible. Simply reversing the field-change protocol will yield back the original state and the original energy. The evolution is reversible because it is a unitary dynamics.

However, consider appending a non-unitary step to the process. Now we leave the field at its final value for a time, while bringing the system into contact with a heat bath at its original temperature (zero temperature, in this example). This will induce irreversible thermalization and loss of information about the original state. Such thermalization will convert the extra energy required into extra heat in the environment.

This process can be generalized for a finite temperature. Allahverdyan and Nieuwenhuizen proved that, barring level-crossing, for a system initially at a thermal state the minimal work (energy) is reached by a quasistatic process. The derivation is too long to be repeated here, but in general it hinges on realizing that the state’s eigenvalues do not change during unitary evolution, and that for a smooth enough field-change protocol the adiabatic theorem ensures absence of transitions between states. Having an initial thermal state is not required, but it is necessary to have an initial state with decreasing occupations in the energy eigenbasis. Most importantly, the requirement for no level crossing is satisfied for a single varying field parameter, a result known as the non-crossing rule. No level crossing, in turn, allows us to estimate the time scale required for the quasistatic limit as related to the inverse of the minimum energy level gap. We emphasize that if level-crossing does occur, Allahverdyan and Nieuwenhuizen show that the quasistatic protocol may not be the optimal one. A quasistatic timescale for the adiabatic theorem can still be defined in this case.

For a concrete example we turn to the paradigmatic case of ideal gas in a piston. For simplicity, weconsider spinless particles in one dimension and furthermore replace the square-well confining potential with an harmonic well. A change in piston size corresponds to changing the frequency of the harmonic potential. We are therefore dealing with an ensemble of time-dependent harmonic oscillators.

The harmonic oscillator has an adiabatic conserved quantity, $\hat{H}/\omega$ (where $\hat{H}$ is the oscillator's Hamiltonian) .This means that in the "quasistatic limit" of a slow change in frequency ω(t), this quantity will be conserved. A quasistatic change from !i and energy $E_0$to ω_f will therefore lead to a final energy of $\left(\omega_f/\omega_i\right)E_0$. Since there is no heat exchange with any environment, it is only possible to interpret this energy change as work. This is the baseline quasistatic work that a faster change needs to be compared against.

Treating faster, finite-time, frequency change protocol ω(t) requires more mathematical tools. We can begin by considering the system initially at zero temperature. Then, the energy for any later time will be given by

$$E(t) = \frac{\omega(t)Q(t)}{2} \tag{2.3.1}$$

where $Q(t) \geq 1$ is a parameter introduced by Husimi and is related to a quasistatic protocol: it is unity for a quasistatic change and increases the faster the change in frequency is. This makes the effects of non-adiabacity readily apparent but we would like to deal with the more general initial thermal Gibbs state.

To properly consider an initial thermal state, we turn to the concept of a dynamical Lie algebra.

A dynamical Lie algebra is a Lie algebra generated by the Hamiltonian of the system. The idea is to look for a Lie algebra with elements $\{\hat{L}_i\}$ so that any operator within the algebra $\hat{o} = \sum_i h_i \hat{L}_i$ will remain within the algebra under the dynamics generated by commutation with the Hamiltonian, $[\hat{H};\ \hat{o}]$.

This is most easily assured by taking as $\{\hat{L}_i\}$ all the operators within the Hamiltonian and then adding elements by commutation with the Hamiltonian until no new operators are generated. For the harmonic oscillator the Hamiltonian $\hat{H} = {}^{1}/_{(2m)\hat{p}^2} + {}^{1}/_{2m\omega^2\hat{x}^2}$ can considered to be comprised of two time-independent operators $\hat{L}_i$ with time-dependant parameters, $\hat{H} = \sum_i h_i(t)\hat{L}_i$, with $\hat{L}_1 = \hat{p}^2$ and $\hat{L}_2 = \hat{x}^2$. The minimal completion of this algebra is then $\hat{L}_3 = {}^{1}/_{2(\hat{x}\hat{p} + \hat{p}\hat{x})}$; it can be verified that $\{\hat{L}_1, \hat{L}_2, \hat{L}_3\}$ forms a closed S(1,1) Lie algebra under commutation. Alternative dynamical Lie algebras can be constructed by linear superpositions of these three operators. Using the creation and annihilation operators one such algebra is $\{\hat{H}, \hat{\alpha}^2, (\hat{\alpha}^\dagger)^2\}$.

One advantage of using a dynamical Lie algebra is that it allows one to easily write down a state whose form is conserved by the dynamics. Consider the state

$$\hat{\rho}(t) = \frac{1}{z} = e^{\gamma\hat{\alpha}^2} e^{-\beta\hat{H}} e^{\gamma^*(\hat{\alpha}^\dagger)^2} \quad (2.3.2)$$

Although this operator does not lie within the dynamical Lie algebra, its form is conserved in the dynamics. This can be verified by expanding the exponentials and examining the commutations order by order. Note that the thermal Gibbs state is obtained for $\gamma = 0$; this corresponds to a state with no coherences and no position-momentum correlations.

Another advantage of the dynamical Lie algebra is that it allows one to easily identify dynamical invariants of the motion. “Dynamica invariants” are quantities that remain constant, but are explicit functions of time-dependent parameters and of (implicitly time-dependent) expectation values; for the harmonic oscillator, the first such invariant was noted by Lewis. Korsch and Koshual used dynamical algebras to derive the dynamical invariants which lie within the algebra (i.e., invariants which are linear in the expectation values of the algebra's operators), and Sarris and Proto demonstrated that for our state (Eq. (2.3.2)) it is possible to generalize further and derive dynamical invariants that are outside the algebra, consisting of higher powers of the the algebra's expectation values (they actually consider maximum entropy states,

$\exp \hat{\rho} = \exp\left(\sum_n \lambda_n (t)\hat{L}_n\right)$,

But a product form and exponential sum are interchangeable. One such invariant in our case was found to be

$$C = \frac{E^2}{\hbar^2\omega^2} - \frac{4\gamma\gamma^*}{\left(e^{\beta\hbar\omega}-1\right)^2-4\gamma\gamma^*} \qquad (2.3.3)$$

The normalization of the state (Eq.(2.3.2)) requires

$4\gamma\gamma^* < \left(e^{\beta\hbar\omega}-1\right)^2$,

So that $C \leq E^2/\omega^2$, and the maximum occurs only in the thermal state $(\gamma = 0)$

.

We now have the tools to return to analyzing the time-dependent harmonic oscillator under a finite change of frequency. We assume an initial thermal state so that $\gamma = 0$ and the state is diagonal in the energy eigenbasis. A quasistatic change from ω_i to ω_f will maintain the diagonal form, leading to another thermal state with $\gamma = 0$ and some energy E_f, set by the constant C. Now consider what would have happened if we would have changed the field quickly. In general we would not expect the final state to be thermal, so that $|\gamma| > 0$ and the invariance of C (Eq.(2.3.3)) now implies a higher energy.

Reaching a final thermal state may be possible, but would require a specially-chosen protocol and even then will only lead to the same final energy, not a lower one. We conclude that for any initial thermal state a non-quasistatic protocol will generally lead to higher energies (than the quasistatic baseline),

$$E_f = E_f^{quasistatic}$$

E_f as noted above, appending a thermalization stage will convert this extra energy into excess heat.

We have demonstrated that driving the system at a finite rate will require excess energy to make the same change. But has it generated more entropy and increased the system's temperature? To answer this we must first consider what entropy and temperature mean in the quantum domain. The entropy of quantum states is usually considered to be the von Neumann entropy:

$S_{VN} = -tr(\hat{\rho}\log(\hat{\rho}))$

This quantity can be shown to be invariant to unitary transformation, and in particular to the Hamiltonian dynamics at hand. For our system, the von Neumann entropy can be shown to depend only on the dynamical invariant c. This entropy does not increase during the creation of the "internal friction" in this respect it appears that there is no friction!

The paradox can be resolved by realizing that the von Neumann entropy is an idealized entropy, representing the information missing after all possible measurements have been made. This is not a measure that is necessarily relevant. Since we are interested in the dissipation and dynamics of energy it makes sense to consider an entropy that is related to energy. And indeed, if we look at the Shannon energy entropy, $S_E = -\sum_i p_i \log(p_i)$

Where $p_i = p(E_i)$ is the probability of finding the system in the E_i energy state.

A non-quasistatic change always increases it. The two entropies are equal only when the state is diagonal in the energy eigenbasis, i.e., when $\gamma = 0$ and the state is thermal. A finite rate of frequency change will generally lead to the development of coherence $\gamma \neq 0$ and therefore to the deviation from the diagonal form in the energy eigenbase. Since the von Neumann entropy is always smaller than any other Shannon entropy, the Shannon energy entropy will be higher in these cases internal friction indeed generates entropy.

What about increasing the system's temperature? Note that when $\gamma \neq 0, \beta$ no longer serve as the (inverse) temperature. To understand what the temperature is, we have to go back to its thermodynamic definition as the ratio between energy and entropy change. The Shannon energy entropy of our system is the same as that of a thermal oscillator with the same energy:

$$S_E = \left(E + \frac{\hbar\omega}{2}\right) \log\left(\frac{E + \hbar\omega/2}{E - \hbar\omega/2}\right) - \log\left(\frac{1}{E - \hbar\omega/2}\right) \quad (2.3.4)$$

This is a concave monotonically increasing function of energy.

However, are only true from the appropriate (quantum) perspective. We note that these effects are purely due to the fact that the system could not adiabatically follow the change in frequency, which is in turn due to the fact that the system's Hamiltonian does not commute at different times:

$$\left[\hat{H}(t)\hat{H}(t')\right] \neq 0 \quad (2.3.5)$$

which is a purely quantum feature. Quantum internal friction, therefore, stems from the non-Abelian nature of quantum algebra.

In our treatment we separated out the thermalization phase from the driving phase. In realistic cases, however, driven systems will be at least weakly coupled to thermal environments. The two processes will occur simultaneously, implying that any external driving will be converted through dissipation to some quantum friction (although this effect may be negligible).

Furthermore, residual interactions with the environment can also lead to dephasing noise. This will also be the effect of imperfect control over the external field. Since pure dephasing is identical to a weak measurement of the momentary energy, one would expect it to draw the state the momentary energy eigenbasis, thereby approximating the quasistatic process and thus acting as a“quantum lubricant” that reduces friction. This indeed happens in some cases. However, in at least some cases pure dephasing of this sort can decrease efficiency. The effect of dephasing noise in general is still not sufficiently understood, but it does not appear to eliminate quantum internal friction entirely even when it does function as a lubricant.

It should be noted that, formally, there are frequency-change protocols that avoid generating friction (in these solutions Q(t) = 1 at some finite time t_f). Using such protocols, it is seemingly possible to drive the system at a finite rate and still avoid friction. However, although it appears that such processes can occur in arbitrarily short time, that requires an arbitrarily large available energy.

This can be understood in light of the energy-time uncertainty relation: an infinitely fast process would require an infinite variance in energy. An instantaneous frictionless solution is therefore not viable and any finite-period solution will result in dissipative losses to the environment on its points where $Q(t < t_f) > 1$. In at least some cases, frictionless solutions also seem unstable under dephasing noise. In the realistic case of weak coupling, then, some frictional loss is unavoidable (although it may benegligible in practice).

Our results are not limited to the harmonic oscillator - separate analysis reveals that it is valid for spin systems, and holds under continuous coupling to the bath for a three-level system . Since the underlying features that give rise to the phenomena are the non-commutative nature of the Hamiltonian at different times and the irreversible nature of thermalization, there is good reason to believe that this kind of quantum friction would be endemic in all realistic systems.

A constraint on the universal applicability of our results is the assumption of a thermal environment.

So far treated the thermalization and dephasing processes only roughly, so will now amend this lacuna by devoting the next section to examining more carefully how dissipation to the environment occurs.

2-3-2: The origin of the friction

The origin of the van der Waals friction (quantum friction) is closely related to the van der Waals interaction. Let us consider a smooth surface with a neutral atom in close proximity. Despite the fact that a neutral atom has a zero mean electric dipole moment, is has also a nonzero electric dipole moment caused by the fluctuating current density due to quantum and thermal fluctuations .The short-lived electric dipole moment of an atom can induce another dipole moment on a surface or in an atom some distance away. The interaction between two dipole moments results in attraction or repulsion, and is called conservative van der Waals interaction. Any two electrically neutral extended bodies interact with each other in the same way. There are two different regimes that must be distinguished:

- Separation between bodies d is small compared to the wavelength $\lambda \sim c/\omega_{0,c}$

- The speed of the light, ω_0 - a characteristic frequency of the charge fluctuation; in this regime the interactions are determined by the fluctuations in an instantaneous Coulomb field. Retardation effects are negligible.

- If $d > \lambda$ retardation effects must be taken into account.

The interaction between moving bodies is called “dissipative van der Waals interaction”.

Let us suppose that we have two smooth parallel surfaces in proximity but not in contact. They must be separated by a wide band gap to prevent particles from tunneling across it. The surfaces are defined by their electromagnetic reflection coefficients only.

If the surfaces are in relative motion the induced charge will lag slightly behind the fluctuating charge inducing it. This is the origin of the van der Waals friction. It is important that the friction arises in the absence of any roughness. Let us take a closer look at the origin of this friction. If the first body emits electromagnetic waves parallel and antiparallel to the moving direction, then in the rest reference frame of the second body these waves are opposite Dopler-shifted. Due to the frequency

dispersion of the reflection amplitude electromagnetic waves will reflect differently from the surface of the body. The same statement is true for waves emitted by the second body. The surface will emit radiation field due to thermal fluctuation of the current density, but even at low temperature the surface will be surrounded by radiation field created by zero-point quantum fluctuations.

Let us now attempt to describe van der Waals friction with the mechanism of the photon exchange. The forces mediated by photon can be long ranged, since there is no force preventing photon's to leave a surface. That is why a photon is the main candidate to describe quantum friction. Quantum friction originates from two types of different

processes:

- The photons are created in both bodies with opposite momentum, and frequencies obey an equation

$vq = \omega_1 + \omega_2$,

q - the momentum transfer

- The photon is created on one body and annihilated in another body.

The theory of van der Waals friction is controversial, that is why many theories have been created and many different experiments have been carried out, but the obtained results are in sharp contradiction with each other. For example, Volokitin and Persson and Pendry theoretically proved that difference between the Dopler shift of two modes can lead to a frictional force, if the reflectivities of the surfaces depend on frequency. Pendry defends his work, maintaining that he derived this result using different lines of argument. In addition, the quantum-frictional effects have been observed experimentally. However, T. Philbin and U. Leonhardt showed using Lifshitz's theory, that there is no lateral force, that is why no quantum friction between plates moving parallel. To prove their theory Philbin describe experiment that would seem to allow the extraction of unlimited energy from the quantum vacuum if the lateral force exists[38].

2-3-3: friction between two surfaces in relative motion

A straightforward calculation of the van der Waals friction based on the general theory of the fluctuating field developed by Rytov and applied by Lifshitz was carried out by Volokitin and Persson In their study, two semi-infinite solids having smooth parallel surfaces separated by a distance d and moving with velocity v relative to each other were observed. The two coordinate systems K and K′ associated with the first and the second solids. The interaction between the bodies was mediated by the fluctuating electromagnetic field. The fluctuating field between bodies was calculated by introduction a 'random' field into the Maxwell equations and substituting the boundary conditions on the surface of bodies. The relationship between the calculated fields was then determined by Lorentz transformation. The frictional stress that acts on the surface of the body can be calculated from the xz - component of

the Maxwell stress tensor σ_{ij}:

$$\sigma = \frac{1}{8\pi}\int_{-\infty}^{\infty} d\omega[\langle E_z E_x^* \rangle + \langle E_x E_z^* \rangle + \langle B_z B_x^* \rangle + \langle B_x B_z^* \rangle]_z = 0 \qquad (2.3.6)$$

By substituting expressions for electric and magnetic field, changing the integration over ω between the limits −∞ and ∞ to integration only over positive values ω, and neglecting the terms of order$\left(\frac{v}{c}\right)^2$ the following results were obtained:

$$\sigma = \frac{\hbar}{8\pi^2}\int_0^{\infty} d\omega \int d^2q q_x \left\{\frac{\left(1-|R_{1p}|^2\right)\left(1-|R_{\bar{2}P}|^2\right)}{|1-e^{2ipd}R_{1p}R_{\bar{2}P}|^2}\left(n\left(\omega - q_x v\right)\right) - n(\omega) + \right.$$

Rp→Rs+ħ2π30∞dωd2qqxe−2pd×ImR1pR2p−1−e−2pdR1pR2P−nω−qv−nω+

RP→RSq>ωC (2.3.7)

This formula is general and can be easily transformed, e.g. for different distances d,temperature T, sliding velocities ν etc[39].

2-3-4: The frictional drag force between 2D electron systems

Suppose we have electron 2-dimensional systems next to each other. A voltage is applied to the first system, while the second system remains open-circuit. Since no current can flow in the second system, an electric field arises that opposes the frictional drag force from the first system. The frictional drag force between two isolated 2-dimensional electron systems separated by a barrier can be considered as the dissipative Van der Waals friction. The origins of the drag force are Coulomb interaction and an exchange of phonons between the layers. By using a described system it is easy to observe the dependence of the frictional drag force on the separation d , electron density n and temperature T. The idea of such a system was first proposed by Shevchenko and Lozovik and Yudson and few years later Coulomb drag between separated 2D electron gases was discussed by Pogrebinskii and Price and other.

The frictional stress $\sigma = \gamma v$ that acts on the electrons in the first metallic plate due to the current density $J = nev$ in the second plate must be calculated. Since current is not allowed to flow in the plate 1, an electric field E_1 arises and cancels the frictional
stress σ :

$$\gamma = \frac{n_1 e E_1}{v} = \frac{n_1 n_2 e^2}{J_2} = n_1 n_2 e^2 \rho_{12} \qquad (2.3.8)$$

n- carriers concentration per unit area, $\rho = \frac{E_1}{J_2}$ transresistivity, the ratio of the induced electric field in the first plate to the current density in the second plate[40].

2-4: Quantum diffusion:

Quantum diffusion (QD) describes a wave packet spreading in a dissipative environment . Since quantum effects are significant for light particles mainly, QD is very essential for electrons, which on the other hand are very important in physics and chemistry.

QD has been experimentally observed. Studies on electron transport in solids are strongly motivated by the semiconductor industry, exploring nowadays quantum effects on nano-scale. A contemporary review on electron quantum diffusion in semiconductors[41]. Traditionally, QD is theoretically described by means of the models of quantum state diffusion , quantum Brownian motion, quantum drift-diffusion , etc.

If a quantum particle, an electron for instance, moves in vacuum its wave function ψ evolves according to the Schrödinger equation

$$i\hbar\,\partial_t\psi = \left(-\hbar^2\nabla^2/_{2m} + U\right)\psi \qquad (2.4.1)$$

where m is the particle mass and U is an external potential. Since the wave function is complex it can be generally presented in the polar form

$$\psi(r,t) = \sqrt{\rho}\exp\left(iS/_{\hbar}\right)$$

where $\rho(r,t)$ is the probability density to find the quantum particle in a given point r at time t and S(r, t) is the wave function phase.

Introducing this presentation in Eq. (2.4.1) results rigorously in the following two equations , corresponding to the imaginary and real parts, respectively

$$\partial_t\rho + \nabla.(\rho V) = 0 \qquad (2.4.2)$$

$$m\,\partial_t V + mV.\nabla V = -\nabla U - \nabla.\,P_Q/_{\rho} \qquad (2.4.3)$$

Equation (2.4.2) is a continuity equation. Therefore, the velocity $V = \nabla S/_{m}$ represents the flow in the probability space and Eq. (2.4.3) is its hydrodynamic-like force balance. As is seen, the quantum effect is completely included in the quantum pressure tensor

$$P_Q \equiv -\left(\hbar^2/_{4m}\right)\rho\nabla\otimes\nabla\ln\rho \qquad (2.4.4)$$

The description of the quantum evolution by these Madelung equations is identical to the Schrödinger picture and it is a matter of convenience which method will be employed.

Equation (2.4.3) describes the force balance in vacuum. If the quantum particle moves in adissipative environment it will experience also a friction force, which is generally proportional to the particle velocity. Hence, the corresponding generalization of Equation (2.4.3)

$$m\,\partial_t V + mV.V + bV = -\nabla(U + Q) \qquad (2.4.5)$$

where

b is the particle friction constant

$$Q = -\hbar^2\nabla^2\sqrt{\rho} \Big/ 2m\sqrt{\rho}$$

Is the Bohm quantum potential being related to the quantum pressure tensor via the relation.

$\nabla.P_Q = \rho\nabla Q$

Thus, the system of Equations (2.4.2) and (2.4.5) describes the probability spreading in a dissipative environment(QD). A speculative reversal back to a wave function via the Madelung presentation leads to a nonlinear Schrödinger equation . In the case of a free quantum particle $U = 0$ the probability density is Gaussian

$$\rho = \exp^{\left(-r^2/2\sigma^2\right)} \Big/ (2\pi\sigma^2)^{3/2} \qquad (2.4.6)$$

where

$\sigma^2(t)$ is the dispersion of the wave packet.

2-5: Relaxation time and Friction:

For any particle having mass m and velocity v the force Fexerted on it can be described[42] by the equation:

$$m\frac{dv}{dt} = F \qquad (2.5.1)$$

Considering the particle as harmonic oscillator the velocity v is given by :

$$v = v_0 e^{-i\omega_0 t} \qquad (2.5.2)$$

Where

ω_0 is the angular frequency

v_0 is the maximum velocity

From equation (2.5.2)

$$\frac{dv}{dt} = -i\omega_0 v \qquad (2.5.3)$$

Substituting equation (2.5.3) in (2.5.1) yields

$$-i\boldsymbol{\omega_0} vm = F \qquad (2.5.4)$$

if the particle moves in a resistive medium of coefficient γ the

equation of motion becomes

$$F_T = F - F_r \qquad (2.5.5)$$

Where

$$F_r = F_N \gamma \qquad (2.5.6)$$

But

$F_N = \frac{mv}{t}$

$\frac{m}{t} \ll v$

$$F_N = v \tag{2.5.7}$$

According to equation (2.5.2) the force affect the velocity amplitude and frequency. The force also changes the energy of the system.

Assuming that the frictional force affects the frequency only, one can assume

$$v = v_0 e^{-i\omega t} \tag{2.5.8}$$

$$F_T = m\frac{dv}{dt} = -i\omega mv \tag{2.5.9}$$

Substituting equation (2.5.4),(2.4.7) and (2.5.9) in equation (2.5.5) yields

$$-i\omega mv = -i\omega_0 vm - \gamma v \tag{2.5.10}$$

$$im(\omega - \omega_0) = \gamma \tag{2.5.11}$$

If one treat the particle as a harmonic oscillator, the energies for ω_0 ,

And ω are given by:

$$E_0 = \hbar\omega_0 \quad , \quad E = \hbar\omega \tag{2.5.12}$$

This means that E is affected by the frequency only. This conforms to our assumption that F affect the frequency only as far as F affect E.

Thus the energy loss is given by

$$\Delta E = E - E_0 \tag{2.5.13}$$

$$\Delta E = \hbar(\omega - \omega_0) \quad (2.5.14)$$

From equation (2.5.14)

$$(\omega - \omega_0) = \frac{\Delta E}{\hbar} \quad (2.5.15)$$

The energy loss and relaxation time τ of it can be found from uncertainty principle by using the relation:

$$\Delta E = \frac{-i\hbar}{\tau} \quad (2.5.16)$$

Inserting equation (2.5.16) and (2.5.15) in equation (2.5.11) yields

$$\gamma = \frac{m}{\tau} \quad (2.5.17)$$

2-6: Schrodinger and Klein-Gordon Equations

The Schrödinger equation is the fundamental of quantum mechanics and the starting point for any improvement to the description of submicroscopic physical systems[43,44] . Although it cannot be proved or derived strictly, it has associated with it various formulations and derivations[45] .

The Klein- Gordon equation is analog of the Schrödinger equation which tries to make quantum mechanics compatible with special relativity unlike the Schrödinger equation which is compatible only with Galilean relativity. Historically, the Klein-Gordon equation invented by Schrödinger even be for Klein and Gordon in the context of understanding the fine structure of the hydrogen spectrum but was abounded by him as it did not give him the right results[46].

In this chapter we try to obtain the Schrödinger equation for a particle with energy E and momentum p traveling in the x direction, and then we apply the relativistic energy E to obtain the Klein-Gordon equation[47].

2-6-1: Derivation of Schrödinger Equation:

Suppose the wave function for plane wave travelling in the x direction with a well defined energy and momentum that is[48,49] :

$$\psi = Ae^{i/\hbar(px - Et)} \quad (2.6.1)$$

Where

$E = \hbar\omega$

$P = \hbar k$

For a particle moving in a potential energy field we write the energy according to the relation

$$E = \frac{P^2}{2m} + V(x) \qquad (2.6.2)$$

Multiplying the both sides of equation (2.6.2) by ψ, one gets

$$E\psi = \frac{P^2}{2m}\psi + V(x)\psi \qquad (2.6.3)$$

From equation (2.6.1) we see that for the equality to hold the product of energy times the wave function must be equal to the first derivation of the wave function with respect to time multiplied by i$\hbar$, that is[50,51,52]:

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}Ae^{i/\hbar(px-Et)} = -\frac{iE}{\hbar}\psi$$

$$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad (2.6.4)$$

Similarly by examining equation (2.6.4) we see that:

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar}PAe^{i/\hbar(px-Et)}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{i^2}{\hbar^2}P^2Ae^{i/\hbar(px-Et)} = -\frac{1}{\hbar^2}P^2\psi$$

$$P^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \qquad (2.6.5)$$

Inserting equations (2.6.4) and (2.6.5) in equation (2.6.3) hence one get

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x) \qquad (2.6.6)$$

Which is the famous Schrödinger equation.

2-6-2: Derivation of Klein-Gordon Equation:

The Schrödinger equation was motivated by taking a look at the classical relation between energy and momentum of particle, quantization is done by replacing the physical quantities by operators corresponding to them and state or wave function on which they operate. These corresponding operators for the energy and momentum are given by[53,54,55]:

$$P \rightarrow -i\hbar\vec{\nabla} \quad (2.6.7)$$

$$E = i\hbar\frac{\partial}{\partial t} \quad (2.6.8)$$

Assuming the case of a free particle one get the following relation between momentum and energy

$$E = \frac{P^2}{2m} \quad (2.6.9)$$

Multiplying both sides of equation (2.6.9) by ψ, one gets

$$E\psi = \frac{P^2}{2m}\psi \quad (2.6.10)$$

Substituting the operators in (2.6.7) and (2.6.8) to equation (2.6.10), one gets

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \quad (2.6.11)$$

$$\frac{P^2}{2m}\psi = (-i\hbar\nabla)^2\psi = -\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2}$$

$$\frac{P^2}{2m}\psi = -\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2} \quad (2.6.12)$$

Inserting equations (2.6.11) and (2.6.12) in equation (2.6.10), one gets

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2} \quad (2.6.13)$$

Equation (2.6.13) is a Schrödinger equation for a free particle. We could now assume that we could obtain the relativistic version of Schrödinger equation by simply repeating the same procedure with relativistic correlation between momentum and energy[56].

$$E = \sqrt{P^2c^2 + m_0^2c^4} \quad (2.6.14)$$

$$E^2 = P^2c^2 + m_0^2c^4 \quad (2.6.15)$$

Where

E ≡ energy of particle

P ≡ particle momentum

C ≡ speed of light

m_0 ≡ mass of rest electron

Suppose the wave function for plane wave travelling in x direction is given by:

$$\psi = Ae^{i/\hbar(px-Et)} \quad (2.6.16)$$

$E = \hbar\omega$

$P = \hbar k$

Multiplying both sides of equation (2.6.15) by ψ, one gets

$$E^2\psi = P^2c^2\psi + m_0^2c^4\psi \tag{2.6.17}$$

From equation (2.6.16)

$$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}Ae^{i/_{\hbar}(px-Et)}$$

$$\frac{\partial^2\psi}{\partial t^2} = \frac{i^2E^2}{\hbar^2}Ae^{i/_{\hbar}(px-Et)} = -\frac{E^2}{\hbar^2}Ae^{i/_{\hbar}(px-Et)}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\frac{E^2}{\hbar^2}\psi$$

$$E^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial t^2} \tag{2.6.18}$$

Also from equation (2.6.16)

$$\frac{\partial\psi}{\partial x} = \frac{iP}{\hbar}Ae^{\frac{i}{\hbar}(Px-Et)}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{i^2P^2}{\hbar^2}Ae^{\frac{i}{\hbar}(Px-Et)}$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2}\psi$$

$$P^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \tag{2.6.19}$$

Inserting equations (2.6.18) and (2.6.19) in equation (2.6.17), one gets

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = -\hbar^2c^2\frac{\partial^2\psi}{\partial x^2} + m_0^2c^4\psi$$

$$\hbar^2\frac{\partial^2\psi}{\partial t^2} = \hbar^2c^2\frac{\partial^2\psi}{\partial x^2} - m_0^2c^4\psi \tag{2.6.20}$$

Which is the Klein-Gordon equation

The Klein-Gordon equation describes a wide variety of physical phenomena such as in wave propagation in continuum mechanics and in the theoretical description of spin less particles in relativistic quantum mechanics[57].

2-7: Derivation of Schrodinger Equation from Variational Principle:

In this work Schrödinger equation was derived from calculus of variations (variational principle), so the methodology of calculus of variations was used. The variational principle one of great scientific significance as they provide a unified approach to various mathematical and physical problems and yield fundamental exploratory ideas[58].

2-8: Derivation of Klein-Gordon Equation from Maxwell's Electric Wave Equation:

In this study Klein-Gordon equation was derived from Maxwell's equation. Maxwell's equation for electric field was used to derive Einstein energy-momentum relation. This was done by using Plank photon energy relation beside wave solution in insulating no charged matter. Klein-Gordon quantum equation was also derived from the same Maxwell's equation by utilizing resemblance between electric field vector and wave function in the intensity expression. However, the relation between polarization and electron rest mass was also used[59].

2-9: Relativistic Quantum Frictional Equation:

In this model new special relativistic quantum equation was derived by using the formal definition of force. This expression includes mass energy beside potential energy, with energy conserved. The effect of friction on energy lost is found by using uncertainty relation. The special relativistic energy in the presence of friction is found. This relation is used to find new special relativistic quantum equation. Treating particles as vibrating string the mass is quantized[42].

2-10: Modification of Schrödinger Equation in a Media:

A collision phenomenon is one of the oldest quantum mechanical problems. It includes scattering process in which a particle or a beam of particles is scattered by a medium. The scattering quantum theory is very complex [60, 61, 62, 63,64]. Therefore it is very difficult to solve scattering quantum equations without doing certain approximations, or doing special treatments. For example the inelastic scattering process is explained by the so called optical potential in which an imaginary potential is inserted by hand in the energy expression [65,66, 67]. These problems motivate to propose a new quantum mechanical equation for scattering process [68].

In this model new quantum Schrodinger Equations derived by using the expression of the electric decaying wave in a conducting medium. This expression is based on Maxwell's equations.

2-11: Schrodinger Quantum Equation From Classical And Quantum Harmonic Oscillator

New Schrodinger Equation was derived by using Maxwell Equations for damping or non-damping electromagnetic wave, in the presence of friction. this equation reduces to ordinary Schrodinger Equation and shows quantized friction energy[69].

2-12: Quantization of Friction for Non Isolated Systems:

In this model the plasma equation for a fluid having a pressure P is given by:

$$mn[\dot{v} + v.\nabla v] = +F - \nabla P - F_r \qquad (2.12.1)$$

With F, F_r Standing for field and frictional forces respectively, where

$$F = -n\nabla V \qquad (2.12.2)$$

n here is the particle number density and V is the potential per unit particle.

According to this equation the particle energy in the presence of pressure and friction is given by:

$$E = \frac{P^2}{2m} + V + KT - \frac{i\hbar^3}{2\tau m^2 c^2} \qquad (2.12.3)$$

τ here is the relaxation time and T is the absolute temperature.

According to this energy equation the new Schrödinger equation[70] is given by:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - KT\psi - \frac{i\hbar^3}{2\tau m^2 c^2}\psi \qquad (2.12.4)$$

For particle in a box, the friction energy E_2 is quantized, where:

$$E_2 = \frac{n^2\hbar^2}{8L^2\tau m^2 c^2}$$

2-13: Growth rate enhancement of free-electron laser by two consecutive wigglers with axial magnetic field

The study of free-electron laser (FEL) as a high-power tunable source of radiation has been the subject of many papers published by different groups all around the world. The radiation is generated by relativistic electron beam passing through a wiggler. In the conventional FEL configuration, the energy of a relativistic electron beam is transferred into high-frequency coherent radiation. Since the radiation wavelength varies with electron energy, it can be continuously tuned in frequency. The theory of conventional FEL has been studied extensively. Not only can the existence of an axial magnetic field focus on the electron beam, against the self-field, but it can also exploit the resonance between the frequency of the focussing device and the frequency of the wiggler. As a result, the axial magnetic field greatly increases the gain and growth rate in an FEL . The purpose of using a wiggler in an FEL is to impart sufficient transverse oscillatory motion to the electrons of the beam to interact with the radiation that is amplified. Recently, considerable attention has been paid to the interaction of a relativistic electron beam and electromagnetic wave in an FEL having one wiggler. But the study of FEL with two wigglers and magnetized electron beam is comparatively limited. The motivation for this work is to present an analytic the expression for the dispersion relation in an FEL consisting of a uniform axial magnetic field and a two-sectioned helical wiggler having opposite circular polarization. An FEL device operating with two undulators can provide an output linearly polarized field. In optical-Klystron FEL, two undulators having opposite polarization are employed because the output light may be linearly polarized. The particular configuration one has employed is that of a relativistic electron beam propagating along the z direction through an ambient magnetic field composed of two consecutive magnetic wigglers having opposite polarization and a uniform guide field [71].

2-14: Comparison of Growth Rate of Electromagnetic Waves in Pre-bunched Cerenkov Free Electron Laser and Free Electron Laser

The increase in the growth rate by using a pre-bunched electron beam in a Cerenkov free electron laser (CFEL) and Free electron laser (FEL) has been studied by Anuradha Bhasin and Bhupesh Bhatia.

Cerenkov free electron laser (CFEL) is the widely used source of broad- band, high power microwave generation at short wavelengths. In this device, an electron beam passing through wave structure resonantly interacts with wave whose phase velocity equals the drift velocity of electrons and the wave grows at the expense of energy of the beam. Since the electron velocity cannot exceed the velocity of light, a slow wave structure is needed to slow down the phase velocity of electromagnetic modes. In case of Cerenkov free electron laser (CFEL). which employs a slow wave medium to slow down the phase velocity of transverse electric (TE) or transverse magnetic (TM) modes to less than c, the velocity of light so that they can be excited by a moderately relativistic electron beam by the process of cerenkov emission. A Cerenkov free electron laser generally employs two kinds of slow wave structures:

(i) A dielectric whose dielectric constant is $|\varepsilon| > 1$ reduces the phase velocity of the radiation below c. A moderately relativistic electron beam can excite the electromagnetic radiation by cerenkov emission,

(ii) A plasma lining have a dielectric constant $\varepsilon = 1 - \frac{\omega_P^2}{\omega^2}$ can act as a slowing down medium for $\omega_p \gg \omega$ so that $\varepsilon \gg 1$ (where ω_p is the electron plasma frequency and ω is the radiation frequency).

A CFEL consisting of two dielectrically lined parallel plates driven by dense moderately relativistic electron beam has been studied and reported to produce coherent high power radiation from 375 micrometer to 1mm wavelengths.

More recently, a lot of research work has been carried out in studying the free electron laser by pre-bunched electron beams. A high power microwave free electron laser experiment has been performed using pre-bunched electron beam of 35Mev. Here when the electron beam is pre-bunched at a frequency close to an eigen frequency of the cavity, the oscillation build process is speed up and the radiation build time is shortened significantly. Free electron maser experiment with a pre-bunched electron beam has been demonstrated at Tel Aviv University. In this case, they utilize a 1.0A current pre-bunched electron beam obtained from a microwave tube. The electron beam is bunched at 4.87GHz frequency and is subsequently accelerated to 70KeV .

The bunched beam is injected into a planar wiggler ($B_w = 300$gauss , $\lambda_w = 4.4cm$,where B_w is the wiggler field and λ_w is the wiggler wavelength) constructed in a Halbach configuration with 17 periods. A theoretical model for gain and efficiency enhancement in a FEL using pre-bunched electron beam has been developed and studied by Beniwal et al. Sharma and Bhasin have studied the gain and efficiency enhancement in a slow wave FEL using pre-bunched electron beam in a dielectric loaded waveguide. They have found that the growth rate and gain of a slow wave FEL increase with the increase in modulation index and is maximum when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave. In this section, we develop a theoretical model of a pre-bunched CFEL and present the analytical analysis for the excitation of electromagnetic waves by a pre-bunched electron beam in a CFEL. We compare the increase in growth rate with the increase in the modulation index for pre-bunched CFEL with a pre-bunched FEL. The growth rate has been calculated at experimentally known CFEL and FEL parameters. Consider a dielectric loaded waveguide of effective permittivity ε_0 . A pre-bunched relativistic electron beam of density nb0, velocityv_{bz} , relativistic gamma factor $\gamma = \frac{1+ev_b}{mc^2}(1 + \Delta sin\omega_0\tau) \approx \gamma_0(1 + \Delta sin\omega_0\tau)$ [where Δ is the modulation index (its value lie from 0 to 1), mc^2 is the rest mass energy of the electrons, e is the electronic charge, $\omega_0(\approx K_{z0}v_b)$ and K_{z0} bare the modulation frequency and wave number of the pre-bunched electron beam], respectively propagates through the waveguide An electromagnetic signal E_1 is also present in the interaction region [72] .

2-15: Quantum mechanical lasing mechanism

Quantum mechanical laws are used to find the wave function factor β in this form, quantum mechanical nano lasing mechanism.

$$\beta = \frac{2}{c}\sqrt{\frac{P_0 - E_0}{x_0 m}} \tag{2.15.1}$$

x_0, m, P_0, E_0 are the vibration amplitude atomic mass, polarization and external field amplitude respectively. Lasing takes place when

$$P_0 > E_0 \qquad n_m > n_i \ , \tag{2.15.2}$$

where n_m, n_i are the number of emitted and incident photons respectively.

Thus amplification exists when emitted photon exceed incident ones. The expression for amplification factor for population inversion shows lasing can takes place when lattice force F_l , which is related to collision and excitation rate, exceeds the external one F_l,[73].

Where

$$\beta \alpha \left(\frac{F_l - F_e}{F_e}\right) \tag{2.15.3}$$

2-16: Effects of Fields on lasing of Thermally Vibrating Atoms and Electrons in The Presence of Electric and Magnetic Fields

This model was shown The effects of fields on lasing of thermally vibrating atoms and electrons in the presence of electric and magnetic Fields[74] .

Ionized atoms emit radiation according to electromagnetic theory as for as accelerated and oscillating charged particle emit electromagnetic radiation. Two cases are considered here. The first case concerns with ionic crystals vibration while the second one concerns the electrons vibration.

2-17: Phase effect between the Electric Internal Current Field and the External Current Field on Amplification of the Total Field and Intensity of the Electromagnetic Radiation

This work was shown the Phase effect between the Electric Internal Current Field and the External Current Field on Amplification of the Total Field and Intensity of the Electromagnetic Radiation . This work is devoted for searching new mechanisms of amplification of electromagnetic radiation which are related to the phase between the external and the internal fields. It is found that amplification takes place if external and internal fields are in phase. The amplification is also found to be related to the electrical conductivity of the medium, in which imaginary conductivity disappears. Thus new lasing materials, can generate laser[75].

2-18: Derivation of Maxwell's Equation for Diffusion Current and Klein-Gordon Equation beside New Quantum Equation Form Maxwell's Equation for Massive Photon.

Maxwell's equations accounting for diffusion current was derived. Maxwell's equations are used to derive Klein- Gordon equation by replacing the electric field intensity by the wave function. Anew quantum equation which accounts for relativistic rest mass energy beside potential energy as well as medium friction is also derived[76].

2-19: Schrodinger Equation in Presence of Thermal and Resistive Energy

In the work new energy relations was made. The energy of ordinary Schrödinger equation includes kinetic and potential energy .However, there are other energy types which should be considered, for example the energy lost E by friction for oscillating
system[77].

2-20: The Quantum Expression of the Role of Effective Mass in the Classical Electromagnetic Theory Form & in Absence of Binding Energy

this a quantum model proposed to the change of electron mass in Crystal . The conventional expression for the effective mass was introduced to account for the effect of the crystal field on the mass. This definition is based on the expression of energy (E) for a free particle[78].

2-21: The Schrodinger equation with friction from the quantum trajectory perspective

Similarity of equations of motion for the classical and quantum trajectories is used to introduce a friction term dependent on the wavefunction phase into the time-dependent Schrödinger equation.

The term describes irreversible energy loss by the quantum system. The force of friction is proportional to the velocity of a quantum trajectory. The resulting Schrödinger equation is nonlinear, conserves wavefunction normalization, and evolves an arbitrary wavefunction into the ground state of the system (of appropriate symmetry if applicable). Decrease in energy is proportional to the average kinetic energy of the quantum trajectory ensemble. Dynamics in the high friction regime is suitable for simple models of reactions proceeding with energy transfer from the system to the environment.

Examples of dynamics are given for single and symmetric and asymmetric double well Quantum molecular dynamics with dissipation, relevant to many processes in chemistry, physics, and biology, is a Long standing theoretical challenge. Dissipation describes interaction of the actively rearranging "system" with the "bath," representing the environmental degrees of freedom.

With few exceptions, the numerically exact simulations of such quantum processes occurring in condensed phase, have been performed, using path integral Monte Carlo methods for models consisting of a lowdimensional system coupled to a bath of harmonic oscillators. Inclusion of friction directly into the Schrödinger equation may be viewed as a simple way to mimic the effect of energy transfer from the system to the environment while limiting quantum dynamics calculations to the system degrees of freedom. Such picture is simplistic, yet it might be useful for some processes. For example, the quantum transition state theory of dissipative tunneling reproduces measurement of the H/D motion on Pt(111) surface with few adjustable parameters.

The force of friction, often taken for processes in condensed phase as linear in velocity of a particle, is most straightforwardly incorporated into equations of motion of a classical particle, characterized by position x_t and momentum p_t,

$$\frac{dp_t}{dt} = -\frac{dV(x)}{dx}\bigg|_{x=x_t} - \gamma p_t, \qquad \frac{dx_t}{dt} = \frac{p_t}{m} \tag{2.21.1}$$

The trajectory evolves under the influence of an external potential V (x)which is a function of the Cartesian coordinate x; parameter γ denotes the friction coefficient.

The friction-generating term for the time-dependent Schrödinger equation (TDSE) is obtained from the analogy between classical mechanics and the Madelung-de Broglie-Bohm formulation of TDSE. The friction term depends on the phase of the evolving wavefunction. The resulting TDSE is nonlinear; the time-dependent wavefunction conserves normalization, while the total energy of the wavefunction decreases with time to the zero-point energy value. On a finite time-scale in the high friction regime such quantum evolution may represent a reactive system losing energy to the environment.

For simplicity, we consider one-dimensional Schrödinger equation in Cartesian coordinates, $-\infty < x < \infty$, and work in atomic units,

$$\frac{i\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,t) \tag{2.21.2}$$

The wave function $\psi(x, t)$ is considered normalizable. The Hamiltonian of the system is

$$\widehat{H} = -\frac{1}{2m}\frac{\partial}{\partial x^2} + V(x) \quad (2.21.3)$$

The equivalent, hydrodynamic or quantum trajectory (also Bohmian) formulation of the TDSE (2.21.2) is based on a polar form of the wavefunction expressed in terms of real amplitude A(x, t) and phase S(x, t),

$$\psi(x,t) = A(x,t)\exp(\imath S(x,t)). \quad (2.21.4)$$

Using equation (2.20.4), the conventional TDSE (2.21.2) yields a system of two equations:

$$\frac{\partial s}{\partial t} = -\frac{1}{2m}\left(\frac{\partial s}{\partial x}\right)^2 - (V + U) \quad (2.21.5)$$

$$\frac{\partial A^2}{\partial t} = -\frac{1}{m}\frac{\partial s}{\partial x}\frac{\partial A^2}{\partial x} - \frac{A^2}{m}\frac{\partial^2 S}{\partial x^2} \quad (2.21.6)$$

Function U,

$$U(x,t) = -\frac{1}{2mA}\frac{\partial^2}{\partial x^2} \quad (2.21.7)$$

Is the quantum potential entering evolution equations on par with the external classical potential $V(x)$, and formally generating all quantum-mechanical effects. Equation (2.21.6) is an equation of continuity of the wavefunction density $A^2(x, t)$; equation (2.21.5) connects quantum and classical mechanics once the gradient of the wave function phase is associated with the trajectory momentum

$$p(x,t) = \frac{\partial s(x,t)}{\partial x} \quad (2.21.8)$$

Differentiation of equation (2.21.5) defines time-evolution of p(x, t),

$$\frac{\partial p}{\partial t} = -\frac{p}{m}\frac{\partial p}{\partial x} - \frac{\partial}{\partial x}(V + U) \quad (2.21.9)$$

which in the Lagrangian frame-of-reference gives Newton's equation of motion for the quantum trajectory (x_t, p_t),

$$\frac{\partial p_t}{\partial t} = -\frac{\partial}{\partial x}(V + U)\Big|_{x=x_t} \quad (2.21.10)$$

$$\frac{\partial x_t}{\partial t} = \frac{p_t}{m} \quad (2.21.11)$$

An interested reader may find overviews of theory and implementations based on quantum trajectories. By analogy with classical equation (2.21.1), the friction term γp is subtracted from the right-hand-side of Eq. (2.21.10) leading to

$$\frac{\partial p}{\partial t} = -\frac{p}{m}\frac{\partial p}{\partial x} - \frac{\partial}{\partial x}(V + U) - \gamma p \tag{2.21.12}$$

(Other functional forms of friction may be introduced at this step.) Integrating equation (2.21.12) with respect to x using equation (2.21.8), the evolution of S with friction becomes

$$-\frac{\partial s}{\partial t} = \frac{p^2}{2m} + V + U + \gamma s + c(t) \tag{2.21.13}$$

The constant of integration C(t) can be defined on physical grounds: the overall phase of a wave function should not affect its evolution, including wave functions describing eigenstates.

This requirement is satisfied by the choice,

$$C(t) = -\gamma\langle s(x,t)\rangle \tag{2.21.14}$$

Together with equation (2.21. 6) unchanged by friction, the conventional TDSE with friction becomes

$$i\frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,t) + \gamma(s - \langle s\rangle)\psi(x,t) \tag{2.21.15}$$

where

$$S = \arg\psi(x,t) \tag{2.21.16}$$

A similar equation termed the Schrödinger-Langevin equation has been proposed for a Brownian particle. The expression includes random potential term and is obtained from the Langevin equation for the Heisenberg operators. The idea of quantum trajectory evolution with friction has been used for the Caldeira-Leggett equation for the density matrix and, with ad hoc friction, to stabilize the numerical implementation of the quantum trajectory formulation of Eqs. (2.21. 7), (2.21.10), and (2.21.11). Equation (2.21.15) derived above has simple form and, being directly related to the quantum and classical Newton's equations of motion, allows straightforward analysis and interpretation of its solutions.

The TDSE (2.21.15) is nonlinear due to the friction term dependent on the evolving wave function. The time-evolution of $\psi^*(x,t)$

$$-i\frac{\partial}{\partial t}\psi^*(x,t) = (\widehat{H} + \gamma(s - \langle s \rangle)\psi^*(x,t) \qquad (2.21.17)$$

The wavefunction dissipates energy and evolves into the ground state of the system, similar to the imaginary time evolution with the quantum Boltzmann operator, i.e., to thermal cooling of a wavefunction. Unlike the imaginary time evolution, friction does not change the wave function norm:

$$N = \langle \psi(t) | \psi(t) \rangle$$

$$\frac{dN}{dt} = i \int \psi\left[\widehat{H}\psi^* + \gamma(s - \langle s \rangle)\psi^*\right]dx - i \times \int \psi^*\left[\widehat{H}\psi + \gamma(s - \langle s \rangle)\psi\right]dx = 0 \qquad (2.21.18)$$

The total energy of the system, E, decreases with time until the systems comes to rest. Note that with the definition of Eq. (2.21.14) the explicit contribution of the friction term to the total energy is zero,

$$E = \langle \psi(t) \middle| \widehat{H} + \gamma(s - \langle s \rangle \middle| \psi(t) \rangle = \langle \psi(t) \middle| \widehat{H} \middle| \psi(t) \rangle \qquad (2.21.19)$$

Differentiation of Eq. (2.20.16) with respect to time gives integration by parts simplifies the expression to

$$\frac{\partial E}{\partial t} = i \int \psi^* \left(\widehat{H} + \gamma(s - \langle s \rangle)\right) \widehat{H}\psi dx - i \times \int \psi^* \widehat{H}(\widehat{H} + \gamma(s - \langle s \rangle)\psi dx = i\gamma s,H \qquad (2.21.20)$$

$$\frac{\partial E}{\partial t} = -\frac{\gamma}{m}\int \left(\frac{\partial s}{\partial x}\right)^2 |\psi|^2 dx = -\frac{\gamma \langle p^2 \rangle}{m} \qquad (2.21.21)$$

Equation (2.21.21) gives a simple visualization of the wavefunction dynamics with friction. Decrease of the total energy due to friction is proportional to the classical kinetic energy of the system. $k = \langle p^2 \rangle / (2m)$, associated with the momenta of quantum trajectories p defined by Equation (2.20. 8). The total energy stops changing once the systems comes to rest, i.e., the quantum trajectories do not move: p = 0. Formally, any eigenstate is characterized by the zero momentum p, however application of the nonlinear friction term to a non-eigenstate wavefunction adds a mixture of eigenstates at each timestep until the lowest energy state is reached.

Let us verify that dynamics with friction reduces a general wavefunction to the ground state (for symmetric $V(x)$, to the lowest energy state of the same symmetry as $\psi(x,0)$).

Consider the short-time evolution of a wavefunction initially comprised of the ground state, $\emptyset_0$, and the first excited state, $\emptyset_1$, of a harmonic oscillator of unit mass and frequency,

$$\widehat{\mathrm{H}} = \frac{1}{2}\frac{\partial^2}{\partial \mathrm{x}^2} + \frac{\mathrm{x}^2}{2}, \widehat{\mathrm{H}}\emptyset_\mathrm{n} = \frac{1}{2}(\mathrm{n} + \frac{1}{2})\emptyset_\mathrm{n} \ , \qquad \mathrm{n} = 0{,}1 \qquad (2.21.22)$$

The initial wavefunction has a small contribution of φ_1 added to φ_0,

$$\psi(\mathrm{x}, 0) = \frac{1}{\sqrt{1+\mathrm{d}^2}}(\ \varphi_0(\mathrm{x}) + \mathrm{id}\ \varphi_1(\mathrm{x}) \qquad (2.21.23)$$

The phase of $\psi(x, 0)$ is ($\widetilde{\wp}$ denotes the imaginary part)

$$\mathrm{S}(\mathrm{x}, 0) = \int \widetilde{\wp}\ (\psi^{-1}\frac{\partial\psi}{\partial \mathrm{x}})\mathrm{dx} = \arctan\ (\mathrm{d}\sqrt{2\mathrm{x}} \qquad (2.21.24)$$

The constant of integration may be left unspecified because $\langle s \rangle$ is subtracted from S in Eq. (2.21.25). For the wavefunction and phase given by equations (2.21.23) and (2.21.24), $\langle s \rangle = 0$. Expanding the time-dependent solution to Equation (2.21.15) through the first orders in time increment t and coefficient d and projecting the result onto the eigenstates φ_0 and φ_1 we find that the relative population of the excited state φ_1,

$$\eta(\mathrm{t}) = \frac{|\langle \emptyset_1 | \psi(\mathrm{x,t})\rangle|^2}{|\langle \emptyset_0 | \psi(\mathrm{x,t})\rangle|^2} = \mathrm{d}^2(1 - 2\gamma \mathrm{t}) + \mathrm{o}(\mathrm{d}^2\mathrm{t}^2) \qquad (2.21.25)$$

is lower than its initial value, $\eta(0) = d_2$. Therefore, in the course of dynamics with friction a wavefunction evolves to the lowest energy eigenstate (of the symmetry of the system if applicable)[79].

2-22: String Theory

In physics, string theory is a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional objects called strings.

String theory aims to explain all types of observed elementary particles using quantum states of these strings.

In addition to the particles postulated by the standard model of particle physics, string theory naturally incorporates gravity, and so is a candidate for a theory of everything, a self-contained mathematical model that describes all fundamental forces and forms of matter.

Besides this hypothesized role in particle physics, string theory is now widely used as a theoretical tool in physics, and has shed light on many aspects of quantum field theory and quantum gravity[80].

The discovery of string theory as a potential unified theory was something of an accident. In a context unrelated to the unification of forces, researchers in the 1970's wondered what theory one might be able to write down to describe a fundamental quantum string - an object with a finite spatial extent, which could not be described in terms of deeper constituents. It was certainly an interesting new mathematical problem in a physical context.

Such a string would be described classically by giving the location of an object extended like a (straight or curved) line in space at a given time. The string could be closed, like a loop, or open, with two end points.

Just as a particle has an intrinsic mass, a string would have an intrinsic tension. Just as a particle is subject to the laws of special relativity, a string would also be relativistic. Finally, one would have to devise a "quantum mechanics" of strings in analogy with that for point particles. The presence of an intrinsic tension means that string theory possesses an inherent mass scale, a fundamental parameter with the dimensions of mass. This defines the energy scale at which "stringy" effects (effects associated to the oscillation of the string) become important.

Without even doing a calculation, one can predict from experience that a quantum string should have infinitely many, discretely spaced oscillation modes, very much like the string of a musical instrument. All these modes would effectively be localised in the neighbourhood of the string, and would behave like elementary particles with different masses related to the oscillation frequency of the string. Thus a single species of string would produce lots of particle-like excitations.

The details ought to have been relatively straightforward, but several unexpected results emerged. A string is like a collection of infinitely many point particles, constrained to fit together to form a continuous object. Thus it effectively has infinitely many degrees of freedom always a dangerous thing! The mathematics of relativistic strings was fairly straightforward at the level of classical theory, but on attempting to promote it to a quantum theory, researchers discovered that the total number of spacetime dimensions is fixed uniquely to be 26. So, quantum strings could exist only in a world with 25 (rather than 3) spatial dimensions, plus time. The excitement of finding for the first time a mathematical consistency condition that determines the number of spacetime dimensions, rather than treating this number as an experimental input, was somewhat tempered by the absurd value predicted for this number.

Enthusiasm was further dampened by the discovery that even in 26 spacetime dimensions, the string has an additional unpleasant feature. Its spectrum of particle-like excitations includes one particle whose mass is an imaginary number a "tachyon", generally believed to be an unphysical object.

Never easily put off by such obstacles, theorists noticed that the theory held yet another surprise. After the tachyon, the next particle in the spectrum of the oscillating string was a spin-2 particle with vanishing mass. A massless particle can propagate to very large distances, so the force that it mediates is a long-range force[81, 82].

The natural starting point is to consider the action

$$S_{string} = \frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{- \det\left(\partial_\alpha X^\mu \partial_\beta X^v \eta_{\mu v}\right)} \qquad (2.22.1)$$

which is simply the area of the two-dimensional worldvolume that the string sweeps out.

Here $\sigma^{\alpha}, \alpha = 0{,}1$ labels the p spatial and temporal coordinates of the string: τ, σ. Here $\sqrt{\alpha'}$ is a length scale that determines the size of the string.

2-22-1: Open Strings

Strings come in two varieties: open and closed. To date we have tried to develop as many formulae and results as possible which apply to both. However now we must make a decision and proceed along slightly different but analogous roots. Open strings have two end points which traditionally arise at $\sigma = 0$ and $\sigma = \pi$. We must be careful to ensure that the correct boundary conditions are imposed. In particular we must choose boundary conditions so that the boundary value problem is well defined. This requires That

$$\eta_{\mu v}\delta_{\sigma}X^{\mu} = 0 \qquad (2.22.2)$$

at $\sigma = 0{,}1$.

2-22-2: Closed Strings

Now consider a closed string, so that $\sigma \sim \sigma + 2\pi$. The resulting "boundary condition" is more simple:

$\hat{X}^{\mu}(\tau, \sigma + 2\pi) = \hat{X}^{\mu}(\tau, \sigma)$.

This is achieved by again taking n to be an integer. However we now have two independent sets of left and right moving oscillators. Thus the mode expansion is given by

$$X^{\mu} = X^{\mu} + \alpha' P^{\mu}\tau + \sqrt{\frac{\alpha'}{2}}\, i \sum_{n \neq 0} \left(\frac{a_n^{\mu}}{n} e^{-in(\tau+\sigma)} + \frac{\tilde{a}_n^{\mu}}{n} e^{-in(\tau-\sigma)} \right) \qquad (2.22.3)$$

note the absence of the factor of 2 in front of $P^{\mu}\tau$. The total momentum of such a string is calculated as before to give

$$P^{\mu} = \frac{1}{2\pi\alpha'} \int_0^{2\pi} d\sigma \dot{X}^{\mu}$$

$$= \frac{1}{2\pi\alpha'}\int_0^{2\pi} d\sigma P^\mu + \sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0} a_n^\mu e^{-in(\tau+\sigma)} + \frac{\tilde{a}_n^\mu}{n} e^{-in(\tau-\sigma)} = P^\mu \quad (2.22.4)$$

so again p^μ is the spacetime momentum of the string.

Chapter Three

The Effect Of Friction and Diffusion On The lasing process

3-1: Introduction

Friction plays an important role inside ordinary matter. One of the most effect of it is electrical resistance and heat. This chapter is concerned with the effect of friction in describing diminishing of wave intensity due to the decrease of incident beam by inelastic scattering. It also describe quantum lasing mechanism.

3-2: Friction Effect On Momentum Term In Schrodinger Equation

When particle move in frictional medium , the frictional energy and relaxation time τ of it can be found from uncertainty principle by using the relation:

$$E_{\mathrm{f}} = \left(\frac{\hbar}{\tau}\right) \mathrm{i} \tag{3.2.1}$$

Where τ is the relaxation time.

To find the corresponding frictional momentum one can use special relativistic energy – momentum relation. According to this (SR) relation the relativistic frictional energy given by

$$E_{\mathrm{f}} = cP_{\mathrm{f}} \tag{3.2.2}$$

Where c is the speed of light.

Thus using (3.2.1) and (3.2.2) yields:

$$P_{\mathrm{f}} = \frac{E_{\mathrm{f}}}{C} = \left(\frac{\hbar}{c\tau}\right) i \tag{3.2.3}$$

Which is the momentum loss by friction .

For any system moving with velocity v , the momentum is given by

$$\mathrm{P} = \mathrm{mv} \tag{3.2.4}$$

Thus the total momentum for frictional medium is given by

$$\widetilde{\mathrm{P}} = \mathrm{P} - \mathrm{P_f} \tag{3.2.5}$$

For the situation in which there is both a kinetic energy and a potential present, the total energy of the system in Newtonian mechanics (SR for law speed) is given by

$$\mathrm{E} = \frac{(\mathrm{P} - \mathrm{P_f})^2}{2\mathrm{m}} + \mathrm{V}$$

Thus

$$\mathrm{E} = \frac{\mathrm{P}^2}{2\mathrm{m}} - \frac{\mathrm{PP_f}}{\mathrm{m}} + \frac{\mathrm{P_f}^2}{2\mathrm{m}} + \mathrm{V} \tag{3.2.6}$$

Multiplying both sides of equation (3.2.6) by ψ , yields

$$\mathrm{E}\psi = \frac{\mathrm{P}^2}{2\mathrm{m}}\psi - \frac{\mathrm{PP_f}}{\mathrm{m}}\psi + \frac{\mathrm{P_f}^2}{2\mathrm{m}}\psi + \mathrm{V}\psi \tag{3.2.7}$$

Relation (3.2.7) can be used to find Schrodinger equation for particles moving in a resistive medium. This wave function for a free particle of momentum P and E. This wave function is given by

$$\psi = e^{\frac{i}{\hbar}(Px - Et)}$$

$$\frac{\partial \psi}{\partial x} = \frac{iP}{\hbar} e^{\frac{i}{\hbar}(Px - Et)}$$

Thus

$$\nabla \psi = \frac{\partial \psi}{\partial x} = \frac{iP}{\hbar}\psi$$

$$P\psi = \frac{\hbar}{i}\nabla\psi \tag{3.2.8}$$

Differentiating again ω.r.t x gives

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2}e^{\frac{i}{\hbar}(Px-Et)}$$

Hence

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2}\psi$$

$$P^2\psi = -\hbar^2\nabla^2\psi \tag{3.2.9}$$

The wave function can also be differentiated ω.r.t t to get

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}Ee^{\frac{i}{\hbar}(Px-Et)}$$

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

$$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{3.2.10}$$

By Substituting equations (3.2.3) ,(3.8),(3.2.9) and (3.2.10) in equation (3.2.7) the modified Schrodinger equation , yields

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2}{\mathrm{mc\tau}}\nabla\psi - \frac{\hbar^2}{2\mathrm{mC}^2\tau^2}\psi + \mathrm{V}\psi \tag{3.2.11}$$

From relation between the friction and the relaxation time [40] the coefficient of friction γ is given by

$$\gamma = \frac{\mathrm{m}}{\tau} \tag{3.2.12}$$

Thus Schrodinger equation is given by

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2\gamma}{\mathrm{m}^2\mathrm{c}}\nabla\psi - \frac{\hbar^2\gamma^2}{2\mathrm{m}^3\mathrm{C}^2}\psi + \mathrm{V}\psi \tag{3.2.13}$$

In the absence of friction

$$\gamma = 0 \tag{3.2.14}$$

The equation (3.2.13) reduces to ordinary Schrodinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \mathrm{V}\psi \tag{3.2.15}$$

The time independent Schrodinger equation (3.2.13) can be written in the form

$$\mathrm{E}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2\gamma}{\mathrm{m^2c}}\nabla\psi - \frac{\hbar^2\gamma^2}{\mathrm{2m^3C^2}}\psi + \mathrm{V}\psi \tag{3.2.16}$$

3-2-1: Harmonic Oscillator Solution

Electrons moving in a circular orbit around the nucleus is a harmonic oscillator. This is since for such motion

$$\mathrm{F = ma = -m\omega^2 r}$$

$$\mathrm{V} = -\int \mathrm{Fds} = \mathrm{m\omega^2}\int \mathrm{r(2\pi rd\theta)} = \mathrm{m\omega^2(2\pi)^2r^2} \sim \mathrm{Kr^2} \tag{3.2.17}$$

To find solution for harmonic oscillator, consider the wave function is given by

$$\psi = e^{ikx}u(x) \tag{3.2.18}$$

Which can be differentiated equation (3.2.18) respect to position to get

$$\nabla\psi = ike^{ikx}u + e^{ikx}\nabla u$$

$$\nabla^2\psi = -k^2e^{ikx}u + 2ike^{ikx}\nabla u + e^{ikx}\nabla^2 u \tag{3.2.19}$$

substituting equations (3.2.18) and (3.2.19) in equation (3.2.16) yields:

$$\frac{\hbar^2k^2}{2m}u - \frac{ik\hbar^2}{m}\nabla u - \frac{\hbar^2}{2m}\nabla^2 u - \frac{ik\hbar^2}{mc\tau}u - \frac{\hbar^2}{mc\tau}\nabla u - \frac{\hbar^2}{\mathrm{2mC^2\tau^2}}\mathrm{u} + \mathrm{Vu}$$

$$= \mathrm{Eu} \tag{3.2.20}$$

For

$$\frac{ik\hbar^2}{m} = -\frac{\hbar^2}{mc\tau} \tag{3.2.21}$$

The wave number is given by

$$k = \frac{i}{c\tau} \tag{3.2.22}$$

Substituting equations (3.2.21) and (3.2.22) in equation (3.2.20) ,yields :

$$-\frac{\hbar^2}{2m}\nabla^2 u + \left(-\frac{ik\hbar^2}{m} + \frac{ik\hbar^2}{m}\right)\nabla u + \left(\frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 k^2}{m} + \frac{\hbar^2 k^2}{2m}\right)u + \text{Vu} = \text{Eu}$$

Thus

$$-\frac{\hbar^2}{2m}\nabla^2 u + \text{ Vu} = \text{Eu} \tag{3.2.23}$$

Comparing this expression it is clear that this is typical to that of the ordinary harmonic oscillator. For Harmonic Oscillator the potential and energy are given by:

$$\text{V} = \frac{1}{2}kx^2 \tag{3.2.24}$$

$$\text{E} = \left(n + \frac{1}{2}\right)\hbar\omega \tag{3.2.25}$$

Which is the ordinary energy for oscillator in the absence of friction. However the wave function in equation(3.2.18) for k given by equation (3.2.22) takes the form:

$$\psi = e^{-\frac{x}{c\tau}}u(x) \tag{3.2.26}$$

which represents a decaying wave function this means that friction decreases the number of particles.

Consider now the solution

$$u = C_0 e^{\alpha x^2}$$

$$\nabla u = 2\alpha x C_0 e^{\alpha x^2} = 2\alpha x u$$

$$\nabla^2 u = (2\alpha + 4\alpha^2 x^2)u \tag{3.2.27}$$

Substituting equations (3.2.24),(3.2.25) and (3.2.26) in equation (3.2.23) ,yields :

$$-\frac{\hbar^2}{2m}(2\alpha + 4\alpha^2 x^2)u + \frac{1}{2}kx^2 u = \left(n + \frac{1}{2}\right)\hbar\omega u$$

$$-\frac{\hbar^2}{2m}(2\alpha + 4\alpha^2 x^2) + \frac{1}{2}kx^2 = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (3.2.28)$$

Comparing the free terms and coefficients of x^2 on both sides yields:

$$-\frac{2\hbar^2\alpha^2}{m} + \frac{1}{2}k = 0$$

$$\alpha^2 = \frac{m}{4\hbar^2}k$$

$$\alpha = \frac{\sqrt{mk}}{2\hbar} \qquad (3.2.29)$$

Thus

$$-\frac{\hbar^2}{m}\alpha = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\alpha = -\frac{m}{\hbar}\left(n + \frac{1}{2}\right)\omega \qquad (3.2.30)$$

Thus from equation (3.2.29) and (3.2.30) :

$$\frac{\sqrt{mk}}{2\hbar} = -\frac{m}{\hbar}\left(n + \frac{1}{2}\right)\omega$$

$$k = 2m\left(n + \frac{1}{2}\right)^2\omega^2$$

$$m = \frac{k}{2\left(n+\frac{1}{2}\right)^2\omega^2} \qquad (3.2.31)$$

This means that The mass is quantized

3-3: String Quantum Mechanical Lasing Due To Friction

The lasing process is based on the spatial evolution of the photon beam. Thus one needs to modify Schrodinger equation to take care of the effect of friction on the spatial evolution of the wave function.

3-3-1: Laser amplification

Consider the frictional momentum given according to equation (4.3.1), when neglecting rest mass term, to be

$$P_f = \frac{E_f}{c} \tag{3.3.1}$$

$$P_{\mathrm{f}} = -\left(\frac{\hbar}{c\tau}\right) i \tag{3.3.2}$$

$$\mathrm{E}\psi = \frac{\mathrm{P}^2}{2\mathrm{m}}\psi - \frac{\mathrm{PP_f}}{\mathrm{m}}\psi + \frac{\mathrm{P_f}^2}{2\mathrm{m}}\psi + \mathrm{V}\psi \tag{3.3.3}$$

$$P\psi = \frac{\hbar}{i}\nabla\psi \tag{3.3.4}$$

From equations (3.3.3),(3.3.4).

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \frac{\mathrm{i}P_{\mathrm{f}}\hbar}{\mathrm{m}}\nabla\psi + \frac{\mathrm{P_f}^2}{2\mathrm{m}}\psi + \mathrm{V}\psi = E\psi \tag{3.3.5}$$

The wave function is given by

$$\psi = e^{ikx}u(x) \tag{3.3.6}$$

$$\nabla\psi = ike^{ikx}u + e^{ikx}\nabla u \tag{3.3.7}$$

$$\nabla^2\psi = -k^2e^{ikx}u + 2ike^{ikx}\nabla u + e^{ikx}\nabla^2 u \tag{3.3.8}$$

Substituting equations(3.3.2), (3.3.7)and (3.3.8) in equation (3.3.5) , yields:

$$\frac{\hbar^2k^2}{2m}u - \frac{ik\hbar^2}{m}\nabla u - \frac{\hbar^2}{2m}\nabla^2 u + \frac{ik\hbar^2}{mc\tau}u + \frac{\hbar^2}{mc\tau}\nabla u + \frac{\hbar^2}{2\mathrm{mC^2\tau^2}}\mathrm{u} + \mathrm{Vu} = \mathrm{Eu} \tag{3.3.9}$$

This equation can be made reduced to Schrodinger equation by assuming

$$\frac{ik\hbar^2}{m} = \frac{\hbar^2}{mc\tau}$$

Thus

$$k = -\frac{i}{c\tau} \qquad (3.3.10)$$

Which can be written

$$-\frac{\hbar^2}{2m}\nabla^2 u + \left(\frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 k^2}{m} - \frac{\hbar^2 k^2}{2m}\right)u + Vu = \mathrm{Eu}$$

$$-\frac{\hbar^2}{2m}\nabla^2 u + Vu = \left(E + \frac{\hbar^2 k^2}{m}\right)u \qquad (3.3.11)$$

From equations (3.3.6) and (3.3.10)

$$\psi = e^{\frac{1}{c\tau}x} u(x) \qquad (3.3.12)$$

Thus the number of electrons are given by

$$n = |\psi|^2 = \psi\bar{\psi}$$

$$n = \left(e^{\frac{1}{c\tau}x} u\right)\left(e^{\frac{1}{c\tau}x} u\right) = e^{\frac{2}{c\tau}x} u^2 \qquad (3.3.13)$$

We assume that n is the number of excited electrons, then the number of emitted photons n_p is proportional to it. Hence

$$I = n_p c = cu^2 e^{\frac{2x}{c\tau}} \qquad (3.3.14)$$

Comparing with laser amplification condition

$$I = I_0 e^{\beta x} \qquad (3.3.15)$$

$$\beta = \frac{2}{c\tau}$$

This lasing can take place.

The fact that

$$\beta \sim \frac{1}{\tau}$$

Is related to the fact that smaller relaxation time τ means very small distances between adjacent atoms. which means existence of more intensive excited states.

For harmonic oscillator

$$-\frac{\hbar^2}{2m}\nabla^2 u + V = E_0\psi \tag{3.3.16}$$

Where

$$\mathrm{V} = \frac{1}{2}k_0 x^2 \tag{3.3.17}$$

$$E_0 = (n+\tfrac{1}{2})\hbar\omega \tag{3.3.18}$$

Thus comparing equation (3.3.11) and(3.3.18) yields

$$E_0 = \left(n+\frac{1}{2}\right)\hbar\omega = E + \frac{\hbar^2 k^2}{2m} \tag{3.3.19}$$

For

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \mathrm{V}\psi \tag{3.3.20}$$

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2\gamma}{\mathrm{m}^2\mathrm{c}}\nabla\psi - \frac{\hbar^2\gamma^2}{2\mathrm{m}^3\mathrm{C}^2}\psi + \mathrm{V}\psi \tag{3.3.21}$$

But comparing equations (3.3.3),(3.3.4),(3.3.11),(3.3.20) and (3.3.21) yields

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \tag{3.3.22}$$

Let:

$$\psi = f(x)u(x) \tag{3.3.23}$$

To get

$$i\hbar\frac{\partial f}{\partial t} = Ef \tag{3.3.24}$$

The solution for f given

$$f = f_0 e^{-\frac{iEt}{\hbar}} = e^{-i\omega t} \tag{3.3.25}$$

The periodicity condition for harmonic oscillator yields

$$f(t+T) = f(t) \tag{3.3.26}$$

$$e^{-i\omega t} = 1 \tag{3.3.27}$$

$\cos \omega T + i \sin \omega T = 1$

Thus

$\cos \omega T = 1$

$\sin \omega T = 0$

Hence

$$\omega T = 2n_0 \pi \tag{3.3.28}$$

Thus

$$\omega = \frac{2\pi n_0}{T} = 2\pi f_0 n_0 = n_0 \omega_0 \tag{3.3.29}$$

$n_0 = 1,2,3, \ldots$

From (3.3.25) and (3.3.29)

$$E = \hbar\omega = n_0 \hbar \omega_0 \tag{3.3.30}$$

Thus from (3.3.19) and (3.3.30)

$$\frac{\hbar^2 K^2}{2m} = \frac{P^2}{2m} = E_0 - E = \left(n + \frac{1}{2}\right)\hbar\omega - \hbar\omega = \left(n - \frac{1}{2}\right)\hbar\omega$$

$$= \left(n - \frac{1}{2}\right)\hbar_0 \omega_0 \tag{3.3.31}$$

3-4: Derivation of Klein – Gordon Equation For Frictional Medium

According to Klein – Gordon equation :

The wave function of a free particle is given by:

$$\psi = e^{\frac{i}{\hbar}(Px-Et)} \tag{3.4.1}$$

Differentiating (3.4.1) with respect to t yields:

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \tag{3.4.2}$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = E^2\psi \tag{3.4.3}$$

Differentiating (3.4.1) with respect to position gives

$$\frac{\hbar}{i}\nabla\psi = P\psi \tag{3.4.4}$$

$$-\hbar^2\nabla^2\psi = P^2\psi \tag{3.4.5}$$

For frictional medium harmonic model[80] propose that

$$\psi = e^{-\frac{i}{\hbar}\left(E-i\hbar\frac{\sigma}{\varepsilon}\right)t} \tag{3.4.6}$$

Where the quantities σ and ε are respectively the conductivity and permittivity

From equation (3.4.6)

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}\left(E - i\hbar\frac{\sigma}{\varepsilon}\right) \tag{3.4.7}$$

From equation (4.4.7)

$$i\hbar\left[\frac{\partial}{\partial t} + \frac{\sigma}{\varepsilon}\right]\psi = E\psi \tag{3.4.8}$$

Where the quantities σ and ε are respectively the conductivity and permittivity

The energy operator becomes

$$\hat{H} = i\hbar\frac{\partial}{\partial t} + i\hbar\frac{\sigma}{\varepsilon} \tag{3.4.9}$$

But from Klein – Gordon equation

$$E^2\psi = c^2P^2\psi + m_0^2c^4\psi \qquad (3.4.10)$$

Thus the energy operator takes the form:

$$\hat{H}\psi = E\psi \qquad (3.4.11)$$

Inserting equation(3.4.9) in equation (3.4.11) the energy eigen equation becomes

$$i\hbar\frac{\partial\psi}{\partial t} + i\hbar\frac{\sigma}{\varepsilon}\psi = E\psi \qquad (3.4.12)$$

From equations (3.4.4), (3.4.10) and (3.4.12). one gets

$$\left(i\hbar\frac{\partial}{\partial t} + i\hbar\frac{\sigma}{\varepsilon}\right)^2\psi = c^2\left(\frac{\hbar}{i}\nabla\right)^2\psi + m_0^2c^4\psi \qquad (3.4.13)$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} - 2\hbar^2\frac{\sigma}{\varepsilon}\frac{\partial\psi}{\partial t} - \hbar^2\frac{\sigma^2}{\varepsilon^2}\psi = -c^2\hbar^2\nabla^2\psi + m_0^2c^4\psi$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} - 2\hbar^2\frac{\mu\sigma}{\mu\varepsilon}\frac{\partial\psi}{\partial t} + c^2\hbar^2\nabla^2\psi = \hbar^2\frac{\sigma^2}{\varepsilon^2}\psi + m_0^2c^4\psi \qquad (3.4.14)$$

For very poor conductor or insulator

$\sigma \to 0$

Thus one gets

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + c^2\hbar^2\nabla^2\psi = m_0^2c^4\psi \qquad (3.4.15)$$

Consider a photon moving inside a medium. It is equation can be solved by suggesting the solution

$$\psi = Ae^{i(\alpha x - \beta t)} \qquad (3.4.16)$$

To get:

$$\frac{\partial^2\psi}{\partial t^2} = i^2\beta^2\psi = -\beta^2\psi$$

$$\nabla^2\psi = i^2\alpha^2\psi = -\alpha^2\psi \qquad (3.4.17)$$

For a photon moving in free space or insulator one substitute (3.4.17) in (3.4.15) to get

$$\hbar^2\beta^2 - c^2\hbar^2\alpha^2 = m_0^2c^4 \qquad (3.4.18)$$

But the wave equation for free particle is

$$\psi = Ae^{\frac{i}{\hbar}(Px-Et)} \qquad (3.4.19)$$

A direct comparison of equation (3.4.19) with (3.4.16) gives:

$$P = \hbar\alpha \quad , \quad E = \hbar\beta \qquad (3.4.20)$$

Inserting (3.4.20) in (4.4.18) gives

$$(E^2 - c^2P^2)\psi = (m_0^2c^4)\psi$$

$$E^2 = c^2P^2 + m_0^2c^4 \qquad (3.4.21)$$

Which is the ordinary energy – momentum relativistic relation. For a photon in a conductor however, substituting (3.4.18) in (3.4.14) yields

$$(\hbar^2\beta^2 + 2\hbar c^2\mu\sigma Ei - c^2\hbar^2\alpha^2)\psi = \left(\frac{\hbar^2\sigma^2}{\varepsilon^2} + m_0^2c^4\right)\psi \qquad (3.4.22)$$

Using relation (3.4.20) in equation (3.4.22) yields

$$E^2 + 2\hbar c^2\mu\sigma Ei - c^2P^2 = \frac{\hbar^2\sigma^2}{\varepsilon^2} + m_0^2c^4 \qquad (3.4.23)$$

Using relation (3.4.21), one can simplify (3.4.22) to get

$$2\hbar c^2\mu\sigma Ei = \frac{\hbar^2\sigma^2}{\varepsilon^2}$$

$$E = -\frac{\hbar\sigma}{2c^2\mu\varepsilon^2}i \qquad (3.4.24)$$

According to special relativistic energy – momentum the energy given by

$$E = cP \tag{3.4.25}$$

Inserting equation (3.4.25) in equation (3.4.24) yields

$$P = -\frac{\hbar\sigma}{2c^3\mu\varepsilon^2}i \; = \; -\frac{\hbar\sigma}{2c\varepsilon}i \tag{3.4.24}$$

From equation (3.4.20)

$$\hbar\alpha = -\frac{\hbar\sigma}{2c\varepsilon}i \tag{3.4.25}$$

$$\alpha = -\frac{\sigma}{2c\varepsilon}i \tag{3.4.26}$$

Inserting equation (3.4.26) in(3.4.16) yields

$$\psi = Ae^{\frac{\sigma}{2c\varepsilon}x - i\beta t} \tag{3.4.27}$$

Thus the number of photons is given by

$$n = |\psi|^2 = \psi\bar{\psi} = A^2 e^{\frac{\sigma}{c\varepsilon}x} \tag{3.4.28}$$

This a gains means that lasing can take place.

Chapter Four

Results and Discussion

4-1: Discussion

The difference between matter and free space manifests itself through frictional term in the expression of momentum as shown by equation(3.2.3). Two terms recognize friction, according to equation(3.2.11). This equation reduces to ordinary Schrodinger equation in the absence of friction(see equation(3.2.15)).

Since string theory treats particles as harmonic oscillators it is thus quite obvious to try harmonic solutions. Suggesting solution equation(3.2.18) for the spatially oscillating system, the wave number is imaginary according to equation(3.2.22). This leads to spatially decaying wave function(3.2.26). This wave function (3.2.26). This wave function can describe inelastic scattering press in which the number of particles in a beam is reduced, where

$$n \sim |\psi|^2 = e^{-\frac{2x}{c\tau}}$$

It is very interesting to note that the energy E equation(3.2.25) shows no change of energy per particle. This means that energy loss due to friction changes the number of particles and does not change the energy of a single particle. This resembles inelastic scattering , which leads to atomic excitation that changes their energy by an amount ($\hbar\omega$). Here the increase or decrease due to friction and collision is caused by the change of the number of excited atoms.

Due to string theory electrons in any atom behaves as a harmonic oscillator. Thus it is natural to solve the Schrodinger equation for harmonic oscillator in the presence of friction Schrodinger equation becomes in the form(3.3.9). The wave number K is imaginary as shown by equation(3.3.10).

Thus the wave function and the number of particles increases with x as equations(3.3.12) and (3.3.13) indicates. This means that amplification can take place according to equation (3.3.14). This is since the number of emitted photons n_p is proportional to the number of excited electrons n given by equation(3.3.13). This means that resistive medium can induce laser. This is due to the fact that collision process takes place in a resistive medium. This collision leads to electrons excitation. The smaller collision time τ the larger the number of excited electrons, which causes a larger number of photons to be emitted. Thus amplification factor β should increase as τ decreases. This is strictly what is equation (3.3.15) state.

It is very interesting to note that the wave number as well as the momentum are quantized as shown by equation (3.3.31).

The Klein-Gordon equation for frictional medium shown in equation (3.4.14) . For free space the equation reduces to ordinary energy – momentum relativistic relation. However for conductor it predict that lasing can take place. This expression for lasing is similar to that obtained by some researchers.

4-2: Conclusion

The modified spatial Schrodinger equation and klein- Gordon that accounts for the effect of friction shows that friction affect on the wave function. This effect shows the possibility of lasing due to the effect of friction which causes atoms to gain energy by friction to emit coherent photons when one treats electrons and photons as vibrating strings.

4-3: Recommendations

1. This Quantum equations prediction needs to be used to see how physical fields affect physical quantities and material properties
2. The effect of Quantum friction on lasing process should also extend to include a wide variety of theoretical models
3. This equations can be applied to describe elementary particles interactions .

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Spatial Wave Function Diminishing Due to Friction Force

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Abstract: ***Schrodinger equation for resistive medium is obtained through momentum operator. This new equation reduces to ordinary Schrodinger equation in the absence of friction. This equation describes spatially decaying wave function when particles are treated as strings. This solution can describe the inelastic scattering process.***

Keywords: Resistive medium, friction, momentum, string theory, inelastic scattering

1. Introduction

Quantum mechanics is the body of scientific laws that describe the wacky behavior of photons, electrons and the other particles that make up the universe. It helps us understand the nature and behavior of matter and energy on the atomic and subatomic level [1].

Quantum mechanics include two independent formulations. The first formulation, called matrix mechanics, was developed by Heisenberg (1925) to describe atomic structure starting from the observed spectral lines. Inspired by Planck's quantization of waves and by Bohr's model of the hydrogen atom, Heisenberg founded his theory on the notion that the only allowed values of energy exchange between microphysical systems are those that are discrete: quanta. Expressing dynamical quantities such as energy, position, momentum and angular momentum in terms of matrices, he obtained an eigenvalue problem that describes the dynamics of microscopic systems; the diagonalization of the Hamiltonian matrix yields the energy spectrum and the state vectors of the system. Matrix mechanics was very successful in accounting for the discrete quanta of light emitted and absorbed by atoms.

The second formulation, called wave mechanics, was due to Schrödinger (1926); it is a generalization of the de Broglie postulate. This method, more intuitive than matrix mechanics, describes the dynamics of microscopic matter by means of a wave equation, called the Schrodinger equation; instead of the matrix eigenvalue problem of Heisenberg, Schrödinger obtained a differential equation. The solutions of this equation yield the energy spectrum and the wave function of the system under consideration. In 1927 Max Born proposed his probabilistic interpretation of wave mechanics: he took the square moduli of the wave functions that are solutions to the Schrodinger equation and he interpreted them as probability densities [2,3].

These two ostensibly different formulations Schrödinger's wave formulation and Heisenberg's matrix approach were shown to be equivalent. Dirac then suggested a more general formulation of quantum mechanics which deals with abstract objects such as kets (state vectors), bras, and operators. The representation of Dirac's formalism in a continuous basis the position or momentum representations gives back Schrödinger's wave mechanics. As for Heisenberg's matrix formulation, it can be obtained by representing Dirac's formalism in a discrete basis. In this context, the approaches of Schrödinger and Heisenberg represent, respectively, the wave formulation and the matrix formulation of the general theory of quantum mechanics.

Combining special relativity with quantum mechanics, Dirac derived in 1928 an equation which describes the motion of electrons. This equation, known as Dirac's equation, predicted the existence of an antiparticle, the positron, which has similar properties, but opposite charge, with the electron; the positron was discovered in 1932, four years after its prediction by quantum mechanics.

In summary, quantum mechanics is the theory that describes the dynamics of matter at the microscopic scale. Fine! But is it that important to learn? This is no less than an otiose question, for quantum mechanics is the only valid framework for describing the microphysical world.

It is vital for understanding the physics of solids, lasers, semiconductor and superconductor devices, plasmas, etc. In short, quantum mechanics is the founding basis of all modern physics: solid state, molecular, atomic, nuclear, and particle physics, optics, thermodynamics, statistical mechanics, and so on. Not only that, it is also considered to be the foundation of chemistry and biology.

Despite the remarkable successes of quantum equations, but they suffer from noticeable set backs. For example, the quantum equation can not differentiate between the behavior of two particles subjected to the same potential, but one moves in free space and the other moves inside matter. This is in direct conflict whit experimental observations. Thus one needs new quantum equation that differentiates between

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the two situations. This is done in section(2). Section (3) is concerned with harmonic oscillator solution, sections (4) and (5) are the discussion and conclusion.

2. Friction Effect on Momentum Term in Schrodinger Equation

The ordinary quantum mechanical laws no terms feeling the effect of friction [4]. Recently some attempts made by M.Dirar and others[5,6] recognize the effect of friction on energy. This effect shows how energy and wave function decays with time due to friction effect.

When particle move in frictional medium , the frictional energy of it is given.

$$E_f = \left(\frac{\hbar}{\tau}\right) i \quad (1)$$

Where τ is the relaxation time.

To find the corresponding frictional momentum one can use special relativistic energy – momentum relation. According to this (SR) relation the relativistic frictional energy gives by

$$E_f = CP_f \quad (2)$$

Thus using (1) and (2) yields:

$$P_f = \frac{E_f}{C} = \left(\frac{\hbar}{c\tau}\right) i \quad (3)$$

Which is the momentum loss by friction .

For any system moving with velocity v , the momentum is given by

$$P = mv \quad (4)$$

Thus the total momentum for frictional medium is given by

$$\tilde{P} = P - P_f \quad (5)$$

For the situation in which there is both a kinetic energy and a potential present, the total energy of the system in Newtonian mechanics (SR for law speed) is given by

$$E = \frac{(P-P_f)^2}{2m} + V$$

Thus

$$E = \frac{P^2}{2m} - \frac{PP_f}{m} + \frac{P_f{}^2}{2m} + V \quad (6)$$

Multiplying both sides of equation (6) by ψ , yields

$$E\psi = \frac{P^2}{2m}\psi - \frac{PP_f}{m}\psi + \frac{P_f{}^2}{2m}\psi + V\psi \quad (7)$$

Relation (7) can be used to find Schrodinger equation for particles moving in a resistive medium. This wave function for a free particle of momentum P and E. This wave function is given by

$$\psi = e^{\frac{i}{\hbar}(Px-Et)}$$

$$\frac{\partial\psi}{\partial x} = \frac{iP}{\hbar} e^{\frac{i}{\hbar}(Px-Et)}$$

Thus

$$\nabla\psi = \frac{\partial\psi}{\partial x} = \frac{iP}{\hbar}\psi$$

$$P\psi = \frac{\hbar}{i}\nabla\psi \quad (8)$$

Differentiating again ω.r.t x gives

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2} e^{\frac{i}{\hbar}(Px-Et)}$$

Hence

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2}\psi$$

$$P^2\psi = -\hbar^2\nabla^2\psi \quad (9)$$

The wave function can also be differentiated ω.r.t t to get

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar} E e^{\frac{i}{\hbar}(Px-Et)}$$

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar} E\psi$$

$$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \quad (10)$$

By Substituting equations (3) ,(8),(9) and (10) in equation (7) the modified Schrodinger equation , yields

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2}{mc\tau}\nabla\psi - \frac{\hbar^2}{2mC^2\tau^2}\psi + V\psi \quad (11)$$

The coefficient of friction γ is given by

$$\gamma = \frac{m}{\tau} \quad (12)$$

Thus Schrodinger equation is given by

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2\gamma}{m^2c}\nabla\psi - \frac{\hbar^2\gamma^2}{2m^3C^2}\psi + V\psi \quad (13)$$

In the absence of friction

$$\gamma = 0 \quad (14)$$

The equation (13) reduces to ordinary Schrodinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad (15)$$

The time independent Schrodinger equation (13) can be written in the form

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2\gamma}{m^2c}\nabla\psi - \frac{\hbar^2\gamma^2}{2m^3C^2}\psi + V\psi \quad (16)$$

3. Harmonic Oscillator Solution

Electrons moving in a circular orbit around the nucleus is a harmonic oscillator. This is since for such motion

$$F = ma = -m\omega^2 r$$

$$V = -\int F ds = m\omega^2 \int r(2\pi r d\theta) = m\omega^2 (2\pi)^2 r^2 \sim Kr^2 \quad (17)$$

To find solution for harmonic oscillator, consider the wave function is given by

$$\psi = e^{ikx} u(x) \quad (18)$$

Which can be differentiated ω.r.t to x to get

$$\nabla\psi = ike^{ikx}u + e^{ikx}\nabla u$$

$$\nabla^2\psi = -k^2e^{ikx}u + 2ike^{ikx}\nabla u + e^{ikx}\nabla^2 u \quad (19)$$

substituting equations (18) and (19) in equation (16) yields:

$$\frac{\hbar^2k^2}{2m}u - \frac{ik\hbar^2}{m}\nabla u - \frac{\hbar^2}{2m}\nabla^2 u - \frac{ik\hbar^2}{mc\tau}u - \frac{\hbar^2}{mc\tau}\nabla u - \frac{\hbar^2}{2mC^2\tau^2}u + Vu = Eu \quad (20)$$

For

$$\frac{ik\hbar^2}{m} = -\frac{\hbar^2}{mc\tau} \quad (21)$$

The wave number is given by

$$k = \frac{i}{c\tau} \quad (22)$$

Substituting equations (21) and (22) in equation (20) ,yields :

$$-\frac{\hbar^2}{2m}\nabla^2 u + \left(-\frac{ik\hbar^2}{m} + \frac{ik\hbar^2}{m}\right)\nabla u + \left(\frac{\hbar^2k^2}{2m} - \frac{\hbar^2k^2}{m} + \frac{\hbar^2k^2}{2m}\right)u + Vu = Eu$$

Thus

$$-\frac{\hbar^2}{2m}\nabla^2 u + Vu = Eu \quad (23)$$

Comparing this expression it is clear that this is typical to that of the ordinary harmonic oscillator. For Harmonic Oscillator the potential and energy are given by:

$$V = \frac{1}{2}kx^2 \quad (24)$$

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$$E = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (25)$$

Which is the ordinary energy for oscillator in the absence of friction. However the wave function in equation(18) for k given by equation (22) takes the form:

$$\psi = e^{-\frac{x}{c\tau}} u(x) \qquad (26)$$

which represents a decaying wave function this means that friction decreases the number of particles.

Consider now the solution

$$u = C_0 e^{\alpha x^2}$$

$$\nabla u = 2\alpha x C_0 e^{\alpha x^2} = 2\alpha x u$$

$$\nabla^2 u = (2\alpha + 4\alpha^2 x^2)u \qquad (27)$$

Substituting equations (24),(25) and (26) in equation (23) ,yields :

$$-\frac{\hbar^2}{2m}(2\alpha + 4\alpha^2 x^2)u + \frac{1}{2}kx^2 u = \left(n + \frac{1}{2}\right)\hbar\omega u$$

$$-\frac{\hbar^2}{2m}(2\alpha + 4\alpha^2 x^2) + \frac{1}{2}kx^2 = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (28)$$

Comparing the free terms and coefficients of x^2 on both sides yields:

$$-\frac{2\hbar^2\alpha^2}{m} + \frac{1}{2}k = 0$$

$$\alpha^2 = \frac{m}{4\hbar^2}k$$

$$\alpha = \frac{\sqrt{mk}}{2\hbar} \qquad (29)$$

Thus

$$-\frac{\hbar^2}{m}\alpha = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\alpha = -\frac{m}{\hbar}\left(n + \frac{1}{2}\right)\omega \qquad (30)$$

Thus from equation (29) and (30) :

$$\frac{\sqrt{mk}}{2\hbar} = -\frac{m}{\hbar}\left(n + \frac{1}{2}\right)\omega$$

$$k = 2m\left(n + \frac{1}{2}\right)^2 \omega^2$$

$$m = \frac{k}{2\left(n + \frac{1}{2}\right)^2 \omega^2} \qquad (31)$$

This means that The mass is quantized

4. Discussion

The difference between matter and free space manifests itself through frictional term in the expression of momentum as shown by equation (3). Two terms recognize friction, according to equation(11). This equation reduces to ordinary Schrodinger equation in the absence of friction(see equation(15)).

Since string theory treats particles as harmonic oscillators it is thus quite obvious to try harmonic solutions. Suggesting solution equation (18) for the spatially oscillating system, the wave number is imaginary according to equation(22). This leads to spatially decaying wave function (26). This wave function (26). This wave function can describe inelastic scattering press in which the number of particles in a beam is reduced, where

$$n \sim |\psi|^2 = e^{-\frac{2x}{c\tau}}$$

It is very interesting to note that the energy E equation(25) shows no change of energy per particle. This means that energy loss due to friction changes the number of particles and does not change the energy of a single particle. This resembles inelastic scattering , which leads to atomic excitation that changes their energy by an amount ($\hbar\omega$). Here the increase or decrease due to friction and collision is caused by the change of the number of excited atoms.

5. Conclusion

The effect of friction on Schrodinger equation leads to describing the inelastic scattering process. It shows that in the elastic scattering number of particles is not conserved references

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STRING QUANTUM MECHANICAL LASING DUE TO FRICTION

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ABSTRACT

Background: Lasers have numerous applications and can be considered as one of the main technological outputs of quantum physics. Due to string theory electrons in any atom behaves as a harmonic oscillator. Thus, it is natural to solve the Schrodinger equation for harmonic oscillator in the presence of friction Schrodinger equation. **Objectives**: The absence of a relation between frictional effects in quantum mechanics relativity and Laser amplification led to conduct a theoretical study so as to verify the Laser amplification due to friction. String theory treats particles as harmonic oscillators. **Methods** In this work Schrodinger equation for frictional medium is derived, and the Harmonic oscillator solved for resistive medium. **Results:** The results showed no change of energy per particle. This means that energy loss due to friction changes the number of particles and does not change the energy of a single particle. This resembles inelastic scattering, which leads to atomic excitation that changes their energy by an amount ($\hbar\omega$). Here the increase or decrease due to friction and collision is caused by the change of the number of excited atoms. **Conclusions:** The Schrodinger equation for frictional medium is derived; Harmonic oscillator solution for resistive medium shows the possibility of lasing within the framework of string theory.
Keywords: *Friction, lasing, String theory, Harmonic oscillator, Schrodinger equation.*

1. INTRODUCTION

In physics the discovery of the laser (light amplification by stimulated emission of radiation) was based on Einstein's theory of stimulated emission of radiation. However, it was only in the 1950's that this theoretical study led to the creation of lasers. Nowadays lasers found numerous applications and can be considered as one of the main technological outputs of quantum physics [1].

The most important features of quantum mechanics leading to laser theory were obtained already in the old quantum mechanics: i.e. the discrete structure of energy levels for atoms and the quantum structure of electromagnetic radiation; spontaneous and stimulated emission and absorption. Here the discrete structure of energy levels of atoms was simply postulated by Bohr to derive the stability of atoms. Then Einstein (motivated by Plank's study on black body radiation) postulated the quantum structure of radiation [2, 3].

By using the quantum structures for atoms and radiation and thermo dynamical considerations, he derived spontaneous and stimulated emissions and absorption which are fundamental in laser theory.
However, the "old fashioned considerations" in the spirit of Bohr and Einstein clarify the basic assumptions leading to the functioning of the laser in a more intuitive and less formal way [4, 5, 6].
Although quantum equations explain many subatomic phenomena; but it fails to account the effect of resistive medium. Some attempts were made by M. Dirar et al. (2015) and Lutfi et al. (2016), but they concentrate on the effects of friction on the time evolution of the physical system [7, 8]. Rare attempts to account for spatial effects. The principle objective of this study is to verify the Laser amplification due to friction.

2. LASER AMPLIFICATION

For any system moving with velocity *v*, the momentum is given by:

$$P = mv \qquad (1)$$

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Thus the total momentum for frictional medium is given by:

$$\widetilde{P} = P - P_f \tag{2}$$

For the situation in which there is both a kinetic energy and a potential present, the total energy of the system in Newtonian mechanics (SR for law speed) is given by:

$$E = \frac{(P - P_f)^2}{2m} + V$$

Thus:

$$E = \frac{P^2}{2m} - \frac{PP_f}{m} + \frac{P_f^{\,2}}{2m} + V \tag{3}$$

Multiplying both sides of Eq. (3) by ψ , yields:

$$E\psi = \frac{P^2}{2m}\psi - \frac{PP_f}{m}\psi + \frac{P_f^{\,2}}{2m}\psi + V\psi \tag{4}$$

Relation (4) can be used to find Schrodinger equation for particles moving in a resistive medium.

Consider:

$$P_f = -\left(\frac{\hbar}{c\tau}\right) i \tag{5}$$

$$P\psi = \frac{\hbar}{i}\nabla\psi \tag{6}$$

From Eq.s (4), (6):

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \frac{iP_f\hbar}{m}\nabla\psi + \frac{P_f^{\,2}}{2m}\psi + V\psi = E\psi \tag{7}$$

The wave function is given by:

$$\psi = e^{ikx}u(x) \tag{8}$$

$$\nabla\psi = ike^{ikx}u + e^{ikx}\nabla u \tag{9}$$

$$\nabla^2\psi = -k^2e^{ikx}u + 2ike^{ikx}\nabla u + e^{ikx}\nabla^2 u \tag{10}$$

Substituting Eq.s (5), (9) and (10) in Eq. (7), yields:

$$\frac{\hbar^2k^2}{2m}u - \frac{ik\hbar^2}{m}\nabla u - \frac{\hbar^2}{2m}\nabla^2 u + \frac{ik\hbar^2}{mc\tau}u + \frac{\hbar^2}{mc\tau}\nabla u + \frac{\hbar^2}{2mC^2\tau^2}u + Vu = Eu \tag{11}$$

This equation can be made reduced to Schrodinger equation by assuming:

$$\frac{ik\hbar^2}{m} = \frac{\hbar^2}{mc\tau}$$

Thus:

$$k = -\frac{i}{c\tau} \tag{12}$$

Which can be written:

$$-\frac{\hbar^2}{2m}\nabla^2 u + \left(\frac{\hbar^2k^2}{2m} - \frac{\hbar^2k^2}{m} - \frac{\hbar^2k^2}{2m}\right)u + Vu = Eu$$

$$-\frac{\hbar^2}{2m}\nabla^2 u + Vu = \left(E + \frac{\hbar^2k^2}{m}\right)u \tag{13}$$

From Eq.s (8) and (12):

$$\psi = e^{\frac{1}{c\tau}x}u(x) \quad (14)$$

Thus the numbers of electrons are given by:

$$n = |\psi|^2 = \psi\bar{\psi}$$

$$n = \left(e^{\frac{1}{c\tau}x}u\right)\left(e^{\frac{1}{c\tau}x}u\right) = e^{\frac{2}{c\tau}x}u^2 \quad (15)$$

We assume that n is the number of excited electrons, then the number of emitted photons n_p is proportional to it. Hence:

$$I = n_p c = cu^2 e^{\frac{2x}{c\tau}} \quad (16)$$

Comparing with laser amplification condition:

$$I = I_0 e^{\beta x} \quad (17)$$

$$\beta = \frac{2}{c\tau}$$

This lasting can take place.

The fact that:

$$\beta \sim \frac{1}{\tau}$$

Is related to the fact that smaller relaxation time τ means very small distances between adjacent atoms. Which means existence of more intensive excited states.

For harmonic oscillator:

$$-\frac{\hbar^2}{2m}\nabla^2 u + V = E_0\psi \quad (18)$$

Where:

$$\mathrm{V} = \frac{1}{2}k_0 x^2 \quad (19)$$

$$E_0 = (n + \tfrac{1}{2})\hbar\omega \quad (20)$$

Thus comparing Eq.(13) and(14) yields:

$$E_0 = \left(n + \frac{1}{2}\right)\hbar\omega = E + \frac{\hbar^2 k^2}{2m} \quad (21)$$

For

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \mathrm{V}\psi \quad (22)$$

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar^2\gamma}{\mathrm{m}^2\mathrm{c}}\nabla\psi - \frac{\hbar^2\gamma^2}{2\mathrm{m}^3\mathrm{C}^2}\psi + \mathrm{V}\psi \quad (23)$$ [9].

But comparing Eq.s (4), (6), (13), (21) and (23) yields:

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \quad (24)$$

Let:

$$\psi = g(x)u(x) \quad (25)$$

To get:

$$i\hbar\frac{\partial\psi}{\partial t} = E\mathcal{f} \tag{26}$$

The solution for $\mathcal{f}$ gives:

$$\mathcal{f} = \mathcal{f}_0 e^{-\frac{iEt}{\hbar}} = e^{-i\omega t} \tag{27}$$

The periodicity condition for harmonic oscillator yields:

$$\mathcal{f}(t+T) = \mathcal{f}(t) \tag{28}$$

$$e^{-i\omega t} = 1 \tag{29}$$

$\cos\omega T + i\sin\omega T = 1$

Thus:

$\cos\omega T = 1$

$\sin\omega T = 0$

Hence:

$$\omega T = 2n_0\pi \tag{30}$$

Thus:

$$\omega = \frac{2\pi n_0}{T} = 2\pi\mathcal{f}_0 n_0 = n_0\omega_0 \tag{31}$$

$n_0 = 1,2,3,\ldots$

From Eq.(27) and Eq. (31):

$$E = \hbar\omega = n_0\hbar\omega_0 \tag{32}$$

Thus from (21) and (32):

$$\frac{\hbar^2K^2}{2m} = \frac{P^2}{2m} = E_0 - E = \left(n+\frac{1}{2}\right)\hbar\omega - \hbar\omega = \left(n-\frac{1}{2}\right)\hbar\omega$$

$$= \left(n-\frac{1}{2}\right)\hbar_0\omega_0 \tag{33}$$

3. DISCUSSION

Due to string theory electrons in any atom behaves as a harmonic oscillator. Thus, it is natural to solve the Schrodinger equation for harmonic oscillator in the presence of friction Schrodinger equation becomes in the form Eq.(11). The effect of friction on Schrodinger equation leads to describing the inelastic scattering process. The wave number K is imaginary as shown by Eq.(12). Thus the wave function and the number of particles increases with x as Eq.s(14) and (15) indicates. This means that amplification can take place according to Eq. (16). This is since the number of emitted photons n_p is proportional to the number of excited electrons n given by Eq. (15). This means that resistive medium can induce laser. This is due to the fact that collision process takes place in a resistive medium. This collision leads to electrons excitation. The smaller collision time τ the larger the number of excited electrons, which causes a larger number of photons to be emitted. Thus amplification factor β should increase as τ decreases. This is strictly what is Eq. (17) stated. It is very interesting to note that the wave number as well as the momentum is quantized as shown by Eq. (33).

This a study agrees with some attempts, one of them is proposed by Lutfi Mohammed AbdAlgadir and others [8]. It is a Schroedinger quantum equation from classical and quantum harmonic oscillator, from this model shows Schrödinger equation and energy Eigen equations are affected by friction. Another attempt was also made by Mobarak Ibrahim and others to construct a quantum relativistic equation and string mass quantization, this expression includes mass energy beside potential energy, with energy conserved. The special relativistic energy in the presence of friction is found. Treating particles as vibrating string the mass is quantized [7]. But no one of them uses this equation to construct a quantum equation that accounts for lasing

4. CONCLUSION

The spatial quantum effect on particles and quantum systems manifests themselves via the momentum. The Schrodinger equation for frictional medium is derived; Harmonic oscillator solution for resistive medium shows possibility reference of lasing.

This study concludes that atomic excitation that changes their energy by an amount ($\hbar\omega$). Here the increase or decrease due to friction and collision is caused by the change of the number of excited atoms.

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