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Effect of Magnetic Field Energy Gap and Resistance in Superconducting and Resonance

تأثير المجال المغنطيسي على فجوة الطاقة والمقاومة في التوصيل
الفائق والرنين

A Dissertation Submitted in fulfillment of the Requirements for the Degree of PhD in Physics

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Dedication

To the spirit of my parents, dear father and loved mother

Acknowledgement

Abstract

Magnetic properties of matter play an important role in human life. This requires developing the physical theories that describe magnetic phenomena. One of these important fields is the magnetic properties of superconductors. Till now there is no satisfactory theory that fully explains these magnetic properties of superconductors.

This set back encourages constructing a model based on temperature dependent Schrodinger equation to explain superconductivity destroy by external magnetic field. By using quantum resistance relation and treating resistance as consisting of real superconducting and imaginary part in one approach and positive superconducting beside negative part, it was shown that superconductivity is destroyed when the external magnetic field exceeds a certain critical value. This result agrees with observations. The magnetic field is shown also to be associated with the energy gap which depends on critical temperature. This relation also agrees with the empirical relation.

المستخلص

تلعب خواص المادة المغنطيسية دوراً مهماً في حياة الإنسان. وهذا يتطلب تطوير النظريات الفيزيائية التي تصف الظواهر المغنطيسية. أحد أهم المجالات المهمة مجال التوصيل الفائق. وحتى الآن لاتوجد نظرية تصف السلوك المغنطيسي للموصلات الفائقة بصورة كاملة.

هذا العيب شجع لعمل نموذج يعتمد علي معادلة شوردينجر الحرارية لتفسير تلاش التوصيل الفائق عبر تسليط مجال مغنطيسي خارجي. بإستخدام علاقة المقاومة الكمية وفصلها لجزء حقيقي فائق وجزء تخيلي في نمودج وجزء موجب فائق وآخر سالب في نمودخ آخر ثبت أن التوصيل الفائق يتلاشى عند تجاوز المجال الغنطيسي الخارجي لقيمة حرجة، وهذه النتيجة تتفق مع التجربة. وتم التوصل لوجود فجوة طاقة مصاحبة للمجال المغنطيسي بصيغة تعتمد علي درجة الحرارة الحرجة مماثلة للصيغة المتحصل عليها تجريبياً.

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CHAPTER ONE

Introduction

1.1 Magnetic properties of Matter

The history of magnetic field dates from the discovery of the peculiar of the peculiar property of some stones in attracting iron pieces.

These stones are called later as magnetic. The development of physical theories later on proposed that magnetic fields are generated by electrically moving like electrons [1, 2, and 3]. The electric by charges can also generate magnetic β current. The electric charges are also surrounded by an electric field. This means that the electric field and magnetic field are related to each other these relations are formulated mathematically by Maxwell, who propose the so called Maxwell's equations [4, 5, and 6]. It is well know that this relation is used widely in generating electric field by allowing wire coils to revolve in a magnetic field. The magnetic properties of matter are used widely in storing information in computers beside fabrication of the so called super conductors which are used in medical diagnosis by observing magnetic activities of organs like heart and brain [7, 8, 9].

One of these medical techniques known, use the so called SQUID which is abbreviation of the word (super conductor Quantum inter faience device). This device is very sensitive to very work magnetic field variation. Like that produced by brain and some vital organs [10]. This device is a super conductor device where its working principle is based on the physical properties, namely the magnetic properties of Super-conductors [11, 12, 13, and 14]. The other technique is based on magnetic resonance property, by using the so called magnetic resonance imaging (MRI) technique which is better than other physical imaging techniques especially for soft tissues [15, 16, and 17].

These important applications of magnetic properties of matter motivates researchers to construct theoretical models so as to know the physical parameter that control super-conducting properties [18, 19, 20,21] and the nature of magnetic resonance process [22.23,24,25].

1.2 Research Problem

The increasing need to use magnetic properties of matter requires developing the theoretical models that are concerned with the magnetic properties of matter.

1.3 Literature Review

Different attempts were made to study magnetic properties of matter [26, 27, 28, 29]. Like magnetic properties of super-conducting [30, 31, 32, 33]. In some of these attempts the magnetic flux and current in super-conducting ring is shown to be quantized [A51] [34]. The magnetic flux produced by cold atoms is also shown to be quantized when it is cooled [35]. These attempts [36, 37, 38, 39], however dose not study the reason why super-conducting is destroyed when an external magnetic field exceeding certain critical value is applied.

1.4 Aim of the Work

The aim of work is to construct a useful theoretical model that can explain some magnetic properties of super-conducting like the relation between super-conducting existence and magnetic field beside magnetic resonance phenomena.

1.5 Thesis Layout

The thesis consists of five chapters. Chapter one and two are devoted for introduction and theoretical back ground on magnetic properties of super-conducting. Chapters three, four and five are concerned with literature review and contribution.

CHAPTER TWO

Physical and Magnetic Properties of Conductors and Super-Conductors

2.1 Introduction

The physical properties of matter play an important role in physics and technology. This chapter is devoted for magnetic properties of matter in general and for super conductors.

2.2 Magnetic Moment of Atoms

The magnetic moment P_0 is defined, in terms of the current i and the area A enclosed by it, to be in the form [40, 41, 42].

$$P_0 = iA \qquad (2.2.1)$$

But the current generated by the electron of charge $-e$ moving around a nucleus in a circular orbit of radius is given by

$$i = -ef = \frac{e\omega}{2\pi} \qquad (2.2.2)$$

$f \equiv frequency$

Where the area is given by

$$A = \pi r^2 \qquad (2.2.3)$$

On the other hand the orbital angular L is given by [25]

$$Lmvr = m\omega r^2 \qquad (2.2.4)$$

Inserting (2.2.2), (2.2.3) and (2.2.4) in (2.2.1) yields the magnetic moment in the form

$$P_0 = iA = \frac{e\omega}{2\pi}(\pi r^2) = -\frac{e\omega r^2}{2}$$

$$P_0 = \frac{-em\omega r^2}{2m} = -\frac{e}{2m}\vec{L} \qquad (2.2.5)$$

Similarly the spin magnetic moment P_s is related to the spin angular momentum according to the relation

$$\vec{P_s} = \frac{-e}{m}\vec{S}$$
(2.2.6)

The total magnetic moment P_m resulting from spin and orbital motion is given by [25]

$$\vec{P_m} = \frac{e\hbar}{2m}g_j\vec{J}$$

$$\vec{P_m} = -\mu_g g_J\vec{J}$$
(2.2.7)

$\vec{J}$ is the quantum number, g_j is the g factor and μ_g is the susceptibility hence

$$\vec{P_m} = -\mu_\beta\langle\vec{L} + 2\vec{S}\rangle$$
(2.2.8)

Where

$$g_J\vec{J} = \langle\vec{L} + 2\vec{S}\rangle$$
(2.2.9)

$$\mu_\beta = \frac{e\hbar}{2m}$$
(2.2.10)

The parameter g_J can simply be given by

$$g_J = \frac{\frac{3}{2}\vec{J}^2 + \left(\frac{1}{2}\right)\vec{S}^2 - \frac{1}{2}\vec{L}^2}{\vec{J}^2}$$

$$= \frac{3}{2} + \frac{S(S + 1) - L(L + 1)}{2J(J + 1)}$$
(2.2.11)

$$\vec{J} = \hbar\sqrt{J(J + 1)}$$
(2.2.12)

This relation can be found by setting

$$\vec{J} = \vec{L} + \vec{S}$$
(2.2.13)

$$J^2 = L^2 + S^2 + 2\vec{L}.\vec{S}$$
(2.2.14)

$$\langle\vec{L} + 2\vec{S}\rangle = \vec{L} + 2\vec{S}$$
(2.2.15)

To get

$$g_J\vec{J}.\vec{J} = (\vec{L} + 2\vec{S}).(\vec{L} + \vec{S}) = L^2 + 2S^2 + 3\vec{L}.\vec{S}$$
(2.2.16)

Then

$$g_JJ^2 = L^2 + 2S^2 + 3L.S$$
(22.17)

4

From (2.2.14)

$$L.S = \frac{J^2 - L^2 - S^2}{2}$$

$$= \frac{1}{2}J^2 - \frac{1}{2}L^2 - \frac{1}{2}S^2$$

$$g_J J^2 = L^2 + 2S^2 + \frac{3}{2}J^2 + \frac{3}{2}L^2 = \frac{3}{2}S^2$$

$$= \frac{3}{2}J^2 + \frac{1}{2}L^2 = \frac{1}{2}S^2$$

$$g_J = \frac{\frac{3}{2}J^2 + \frac{1}{2}S^2 - \frac{1}{2}L^2}{J^2} \qquad (2.2.18)$$

If n atoms per unit volume align themselves along the x-axis thus the component of x is given by

$$M_x = nP_m = -n\mu_\beta g_J \vec{J} \qquad (2.2.19)$$

Where M_x changes from 0 to max value during a time$T$.

The electron revolving around nucleus can produce a magnetic field of flux density$B$. If the electron revolves with frequency f in circular orbit of radius$r$, then according to Bio-savart law it produces a magnetic field of flux density [26].

$$B_e = \frac{\mu_0 i}{2r} \qquad (2.2.20)$$

$$B_e = \frac{\mu_0 f e}{2r} \qquad (2.2.21)$$

Hence

$$i = fe$$

The magnetic moment produced by such an electron is given by

$$P_m = iA = i(\pi r^2) = \pi i r^2 \qquad (2.2.22)$$

Where A is the area enclosed by the current i

$$A = \pi r^2 \qquad (2.2.23)$$

For z electrons with mean radius r, the magnetic flux density of the atom B_a is given by

$$B_a = \frac{\mu_0 z f e}{2r} \tag{2.2.24}$$

Thus the internal field generated by one electron is $B_i = \frac{\mu_0 f e}{2r}$, and is related to magnetic moment through the relation

$$B_a = \frac{\mu_0 P_m}{2\pi r^3} \tag{2.2.25}$$

Since the current for the whole atom [27] is

$$i = z f e$$

Hence the field of the atoms is related to the magnetic moment also as follows

$$B_a = \frac{\mu_0 P_m}{2\pi r^3} \tag{2.2.26}$$

But the magnetic moment M is defined in terms of the number of dipoles N divided by the volume V to be

$$M = \frac{N P_m}{V} \tag{2.2.27}$$

IF the atomic radius r, thus one atom exists in a volume V_a is given by

$$\frac{N}{V} = \frac{1}{V_a} = \frac{1}{\frac{4}{3}\pi r^3} = \frac{3}{4\pi r^3} \tag{2.2.28}$$

This:

$$M = \frac{3 P_m}{\pi r^3} \tag{2.2.29}$$

Using (2.2.25), (2.2.27) and (2.2.28) in (2.2.29) yields:

$$M = \frac{3}{\pi r^3} \frac{B_a}{\mu_0} (2\pi r^3) = \frac{3 P_m}{2\mu_0} \tag{2.2.30}$$

2.3 Energy Splitting and Zeeman Effect

The Zeeman Effect is the name for the splitting of atomic energy levels or spectral lines due to the action of an external magnetic field. The effect was first

predicted by $H.A.$ Lorenz in 1895 as part of his classic theory of the electron, and experimentally confirmed some years later by $P.$ Zeeman.

Zeeman observed [27].

A line triplet instead of a single spectral line at right angles to a magnetic field, and a line doublet parallel to the magnetic field. Later more complex splitting of spectral lines were observed, which became known as the anomalous Zeeman Effect. To explain this phenomenon, Goudsmit and Uhlenbeck first introduced the hypothesis of electron spin in 1925. Ultimately, it became apparent that the anomalous Zeeman Effect was actually the rule and the normal Zeeman Effect the exception [28].

The normal Zeeman Effect only occurs at the transitions between atomic states with the total spin $S = 0$. The total angular momentum $J = L + S$ of a state is then a pure orbital angular momentum $J = L$. For the corresponding magnetic moment, we can simply say that

$$\mu = \frac{\mu_B}{\hbar} j \qquad (2.3.1)$$

Where

$$\mu_B = \frac{\hbar e}{-2m_e} \qquad (2.3.2)$$

$$\mu_B = Boher's\, magnton$$
$$m_e = mass\ of\ electron$$
$$e = elementary\ charage$$
$$\hbar = h/2\pi$$
$$h = Plank's\, constant$$

In an external magnetic field B, the magnetic moment has the energy

$$E = -\mu.B \qquad (2.3.3)$$

The angular-momentum component in the direction of the magnetic field can have the values $J_z = M_J.\hbar\ with\ M_J = J, J-1, \dots\dots, -J.$

Therefore, the term with the angular momentum J is split in to $2J + 1$ equidistant Zeeman components which differ by the value of M_J. The energy interval of the adjacent components M_J, M_{J+1} is

$$\Delta E = \mu_B . B \qquad (2.3.4)$$

Where an electron travelling in a circular orbital perpendicular to the Z axis has magnetic moment μ_{LZ} to the orbital angular momentum L by relation [29].

$$\mu_{LZ} = \frac{e}{2m_e} L_z \qquad (2.3.5)$$

On the other hand for electron spin it is experimentally observed that

$$\mu_{sZ} = g \frac{e}{2m_e} S_z \qquad (2.3.6)$$

Where g is lande factor

For an electron has both spin angular momentum, orbital angular momentum and total angular momentum we can write

$$\mu_{JZ} = g_L \frac{e}{2m_e} J_z \qquad (2.3.7)$$

Where Z component of the total angular momentum if the magnetic field is in the Z direction

$$\Delta E = -g_L \frac{e}{2m_e} J_z B \qquad (2.3.8)$$

Thus

$$\Delta E = -g_L \frac{eh}{4\pi m_e} m_j B$$
$$= -g_L \mu_B m_j B \qquad (2.3.9)$$

$\mu_B = 9.274 \times 10^{-24} J/T$ is called Bohr magneton.

Then the change in photon energy is

$$\Delta E = g_{eff} \mu_B B \qquad (2.3.10)$$

g_{eff} is the effective g factor for the transition.

We can observe the normal Zeeman Effect e.g. in the red spectral line of cadmium($\lambda_0 = 643.8nm, f_0 = 465.7THz$). It corresponds to the transition 1D2 ($J = 2, s = 0$) →1P1 ($J = 1, S = 0$) of an electron of the fifth shell. In the magnetic field, the 1D2 level splits into five Zeeman components having the spacing calculated using equating (2.3.4).

Optical transitions between these levels are only possible in the form of electrical dipole radiation. The following selection rules apply for the magnetic quantum numbers M_J of the ststes involved:

$\Delta M_J = \pm 1$ For σ components

$$\Delta M_J = \pm 0 \text{ for } \pi \text{ components} \tag{2.3.11}$$

Thus, we observe a total three spectral lines; the π component is not shifted and the two σ components are shifted by [46, 47].

$$\Delta f = \pm \frac{\Delta E}{h} \tag{2.3.12}$$

$$\Delta E = V_m \equiv magnetic\ energy$$

With respect to the original frequency. In this equation, ΔE is the equidistant energy split calculated in (2.3.4).

Depending on the angular momentum component ΔM_J in the direction of the magnetic field, the emitted photons exhibit different angular distributions. The angular distributions in the form of two-dimensional polar diagrams. The can be observed experimentally, as the magnetic field is characterized by a common axis for all cadmium atoms [68].

In classical terms, the case $\Delta M_J = 0$ corresponds to an infinitesimal dipole oscillating parallel to the magnetic field. No quanta are emitted in the direction of the magnetic field, i.e.

The π component cannot be observed parallel to the magnetic field. The light emitted perpendicular to the magnetic field is linearly polarized, where by the

$E - vector$ oscillates in the direction of the dipole and parallel to the magnetic field.

Conversely, in the case $\Delta M_J = \pm 1$ most of the quanta travel in the direction of the magnetic field. In classical terms, this case corresponds to two parallel dipoles oscillating with a phase difference of$90°$. The superposition of the two dipoles produces a circulating current. Thus in the direction of the magnetic field, circularly polarized light is emitted in the positive direction it is clockwise for $\Delta M_J = +1$ and anticlockwise-circular for$\Delta M_J = -1$.

The Zeeman Effect enables spectroscopic separation of the differently polarized components. To demonstrate the shift, however we require a spectral apparatus with extremely high resolution, as the two π components of the red cadmium line are shifted e.g. at magnetic flux density $B = 1\,T$ by only $f = 14GHz$, respectively $\Delta \lambda = 0.02nm$.

2.4 Super-Conductors

A super-conductor is the material that has zero resistance beyond critical temperature. A super conductor has also very interesting [30, 31].

Magnetic properties. For instance super-conductor can expel weak magnetic field out of it completely. Thus super-conductor is a perfect diamagnetic.

These properties are exhibited in details in the sections below.

2.5 Electrical Properties of Super-conductor

Every super-conductor has zero resistivity i.e., infinite conductivity for a small amplitude DC current any temperature belowT_c. This property of the super-conducting state was demonstrated by inducing a small-amplitude DC current around a closed ring of a conventional super-conductor. The experiment continued over two and a half years there was no measurable decay of the current [32].
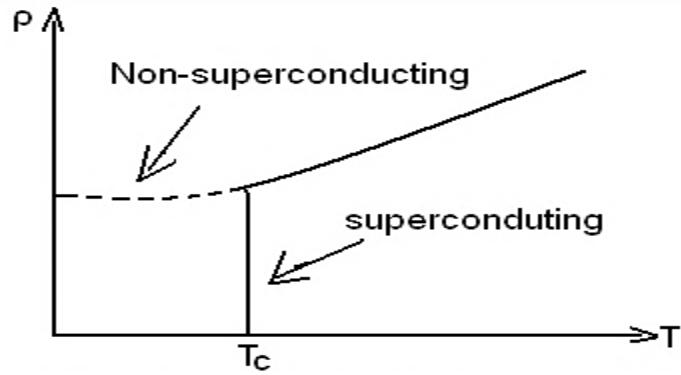
Figure (2.1) Resistivity of typical metal as a function of temperature

Temperature while for a super-conductor all sings of resistance disappears suddenly below a certain temperature T_c[33].

2.6 The Meissner-Ochsenfeld Effect

Super-conductivity can flourish only if the external magnetic field is smaller than critical value critical magnetic field H_c, a quantity which varies from a maximum value H_0 at $T = 0$ to zero field at the critical temperature. For many Super-conductors the temperature dependence of the form [34, 35, 36].
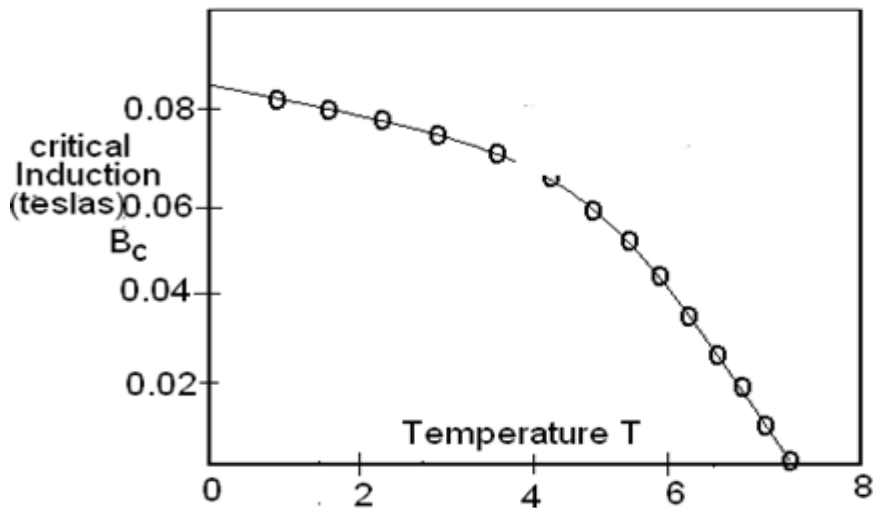
Figure (2.2) Temperature dependence of the magnetic field necessary to quench super-conductivity in lead.

$$H_c = H_0 \left[1 - \left(\frac{T}{T_C} \right)^2 \right]$$ (2.6.1)

Super-conductivity in lead ($T_C = 7.19k$, $B_0 = 0.0803$ tesla)[37]

Nowadays the fact that the resistivity zero, $\rho = 0$, is not taken as true definition of super-conductivity. The fundamental proof that super-conductivity occurs in a given material is demonstration of the Meissner-ochsenfeld effect [38].

In 1933 Mesissner and ochsenfeld discovered another distinct property of the super-conductivity state perfect diamagnetism. They noticed that the magnetic flux is expelled in weak external magnetic field in the figure (2.3).
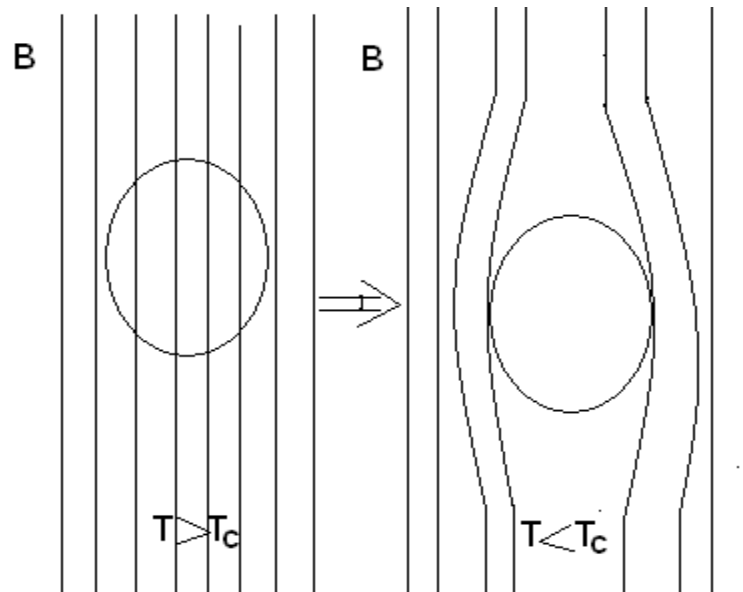
Figure (2.3) Meissner Effect

The explosion of weak external magnetic field from the interior of a super-conductor [39].

We noticed in the normal state, at temperatures above T_C the field lines pass through the metallic specimen. Upon cooling below T_C, a phase transition into the super-conducting state takes place and the magnetic flux gets expelled out of the interior of the metallic sample [40]. Due to electric currents known as screening currents flowing on the surface of the super-conductor in such a way as to generate a field equal and opposite to the applied field [41]. Meissner-ochsenfeld cannot be deduced from the infinite conductivity of a super-conductor.

From ohm's law

$$E = \rho J \qquad\qquad (2.6.2)$$

Where E represented the electric field, ρ the resistivity and J the electric current density in the sample. Zero resistivity implies zero electric field. So, if we take the Maxwell equation

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad\qquad (2.6.3)$$

We have

$$\frac{\partial B}{\partial t} = 0 \qquad\qquad (2.3.4)$$

We see that the magnetic induction in the interior of the sample has to be constant as a function of time. Then the super-conducting metal always expels the field from its interior, and has $B = 0$ in its interior. So the expulsion of the magnetic field ensures that super-conducting state is a true thermodynamic state [42].

Perfect Diamagnetism

In order to maintain $B = 0$ inside the sample whatever (small) external fields are imposed as required by the Meissner-ochsenfeld effect there obviously must be screening currents flowing around the edges of the sample. These produce a magnetic field which is equal and opposite the applied external field, leaving zero field in total [43, 44].

The total current is separated into the external applied currents (e.g. in the coils producing the external field), J_{ext}. And the internal screening currents, J_{ext}.

$$J = J_{ext} + J_{int} \qquad\qquad (2.6.5)$$

The screening currents produce a magnetization in the sample. M per unit volume defined by

$$\nabla \times H = J_{ext} \qquad\qquad (2.6.6)$$

The three vectors M, H and B are related by

$$B = \mu_0(H + M) \qquad\qquad (2.6.7)$$

Maxwell's equation also tell us that

$$\nabla . B = 0 \qquad (2.6.8)$$

Equation (2.6.8) shows that the component of B perpendicular to the surface must remain constant; while from condition equation (2.6.6) one can prove that components of H parallel to the surface remain constant. The two boundary conditions are therefore.

$$\Delta B_\perp = 0 \qquad (2.6.9)$$

$$\Delta B_\parallel = 0 \qquad (2.6.10)$$

Imposing the Meisser condition $B = 0$ in equation (2.6.7) we find the magnetization is

$$M = -H \qquad (2.6.11)$$

The magnetic susceptibility is defined by

$$\chi = \frac{dM}{dH}\bigg|_{H=0} \qquad (2.6.12)$$

And so we find that for Super-conductors

$$\chi = -1 \qquad (2.6.13)$$

Solids with negative value of χ are called diamagnetism (in contrast positive χ are paramagnets). Diamagents screen out part of the external magnetic field, and they become magnetized oppositely to the external field. In Super-conductors the external field is completely screened out, therefore we can say that Super-conductors are perfect diamagnetism [45].

2.7 Types of Super-conductors

Super-conductors divide into two classes according to behavior in a magnetic field. All pure samples of super-conducting elements, exceptNb, exhibit type − 1 behavior and their super-conductivity destroyed by a modest applied magnetic field B_c, known as the critical field [46,47].

2.7.1 Type − 1 Super-conductor

The behavior of type − 1 and super-conductor, at a given temperature T and in a uniform external magnetic field H, can be described as follows. If H is

smaller than a critical value $H_c(T)$, the super-conductor completely expels the magnetic flux from its interior (Meissner effect); as the external field is increased above the critical value $H_c(T)$, the entire specimen reverts from the super-conducting to the normal state.

A plot of the magnetization M versus the applied magnetic His shown in fig (2.3) for $H < H_c(T)$ we have $B = H + 4\pi m = 0$ and thus $-4\pi m = H$
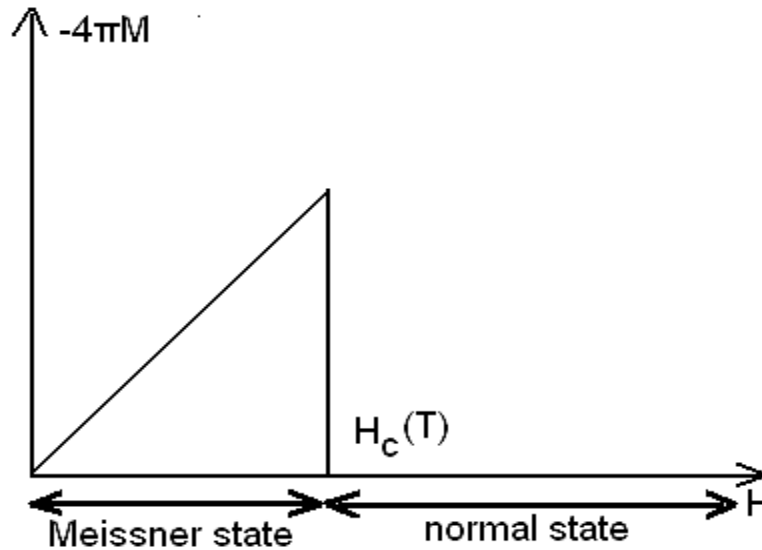
Figure (2.4) Magnetization versus applied field for type-1 super-conductor

2.7.2 Type $-$ 2 Super-conductor

Although Nb is the only element that is Type $-$ 2 is pure state, other elements generally become Type $-$ 2 when the electron mean free path is reduced sufficiently by alloying [48]. In Type $-$ 2 Super-conductors, the transition a normal state is quite gradual. This is shown in fig (2.3) where super-conductivity is only partially destroyed for $H_{c1} \leq H \leq H_{c2}$. The region between H_{c1} and H_{c2} is known as intermediate state as it contains partially both normal and the super-conducting states. H_{c1} is called the lower critical field, whereas H_{c2} is known as upper critical field. At H_{c1} the field begins to penetrate the sample, and the penetration increases until H_{c2}, the magnetization vanishes and the sample reaches the normal. Type $-$ 2 Super-conductors exhibit imperfect diamagnetism

[49]. Type $-$ 2 Super-conductors are known in which H_{c2} is large as $2.8 \times 10^7 A/m$ at absolute zero. Such materials are used now for practical super-conducting magnet coils, and are anticipated for use in the generation and distribution of electrical power.
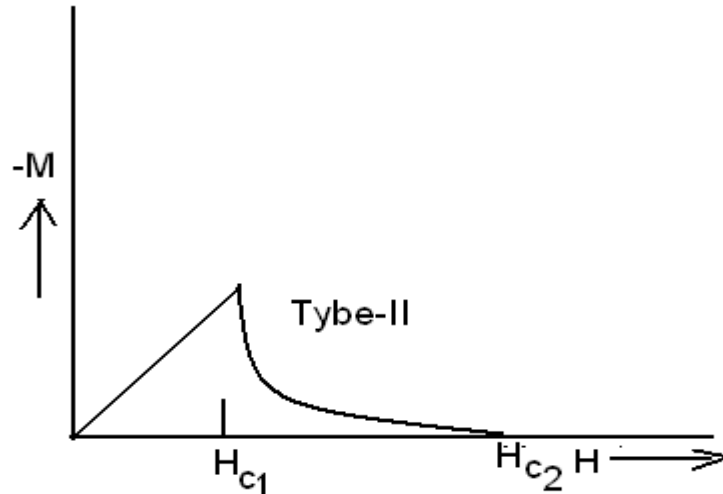
Figure (2.5) Magnetization curve for type-2 super-conductor

2.8 Energy Gap in the Excitation Spectrum

At $T = 0$, the elementary excitation spectrum of a super-conductor has an energy gap. In conventional Super-conductors, however, however at some special conditions there may exist gapless super-conductivity, since conventional Super-conductors have only one energy gap the pairing one. The energy gap in a super-conductor is carried by the Fermi surface and occurs on either side of the Fermi level E_F, as shown in Fig (2.6). Theory and experiment both concluded that energy gap (Δ) in a super-conductor is of the order of $k_B T_c$. Nevertheless it is found to be a function of temperature such as [50].
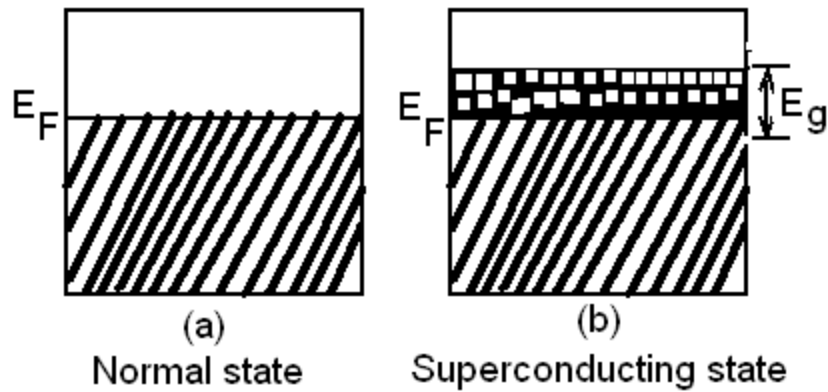
Figure (2.6) Fermi level in normal state (a) and super-conductor state (b)

$$E_g = 2\Delta(T) = 2bk_BT_c$$

$$2 bis\ about\ 2.3$$

The variation of energy gap with temperature is shown in Fig (2.7). The gap is found to decrease with the increase of temperature and vanishes entirely at $T = T_c$.
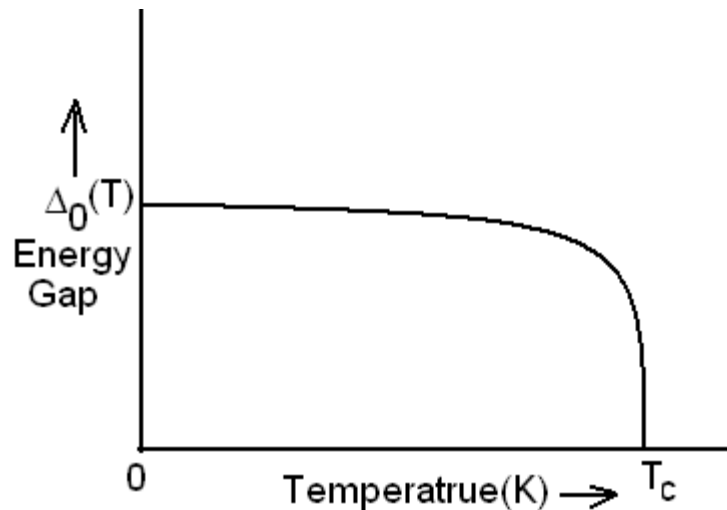
Figure (2.7) Variation of energy gap with tempreature

2.9 Isotope Effect

It has been confirmed that the critical temperature for different isotopes of super-conductor metal is different other. Maxwell and Reynolds they observed that for mercury isotope the critical temperature, T_c varies from 4.185K to 4.146K as the isotopic mass M varies from 199.5amu to 203.4amu.The

experimental results on lead, tin and isotopes of other metal suggest isotope effect may be well fitted by relation of the form

$$M^{\alpha}T_c = \text{Constant} \tag{2.9.1}$$

Where α is the isotope effect coefficient.

In the early years of development of BCS theory it was observed that for most the metals

$$T_c \alpha M^{-\frac{1}{2}} \tag{2.9.2}$$

Science the phone processes depend on $M^{-\frac{1}{2}}$, the isotope effect suggests that the super-conductivity include a large electron-phonon interaction, and hence T_c depends on isotopic mass [51].

2.10 Thermodynamic Properties

The transition from the normal state to the super-conducting is the second-order phase transition. At a second-order phase transition, the first derivatives of the Gibbs discontinuities. The Gibbs free energy G for a system in thermal equilibrium is defined (in CGS units) as [52].

$$G \equiv U - TS - \frac{BH}{4\pi} + pV \equiv -B.\frac{H}{4\pi} + pV \tag{2.10.1}$$

Where U is the total internal energy or the system; T is the temperature of the system; S is the entropy per unit volume; p in the pressure in the system; V is the volume of the system, H and B are the applied magnetic field and flux, respectively. The function $F \equiv U - TS$ is the Helmholtz free energy. The Helmholtz free energy of super-conducting state F_s is lower than that of the normal state F_n by the value called the condensation energy.

$$F_n - F_s = \frac{H_c^2}{8\pi} \tag{2.10.2}$$
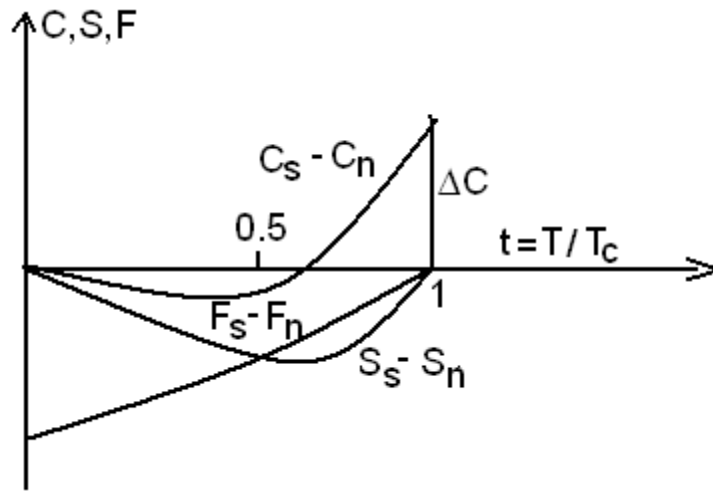
Figure (2.8) Temperature dependences of the specific heat C_s

The free energy F_s of a super-conductor in $H = 0$ with respect to their values in the normal state, C_n, S_n and F_n.

In the normal state thus above T_c, the specific C_n linearly decreases as the temperature decreases, $C_n = \gamma T$ as shown in Fig (2.9) such a linear dependence of specific heat is typical for normal metals and represents the electronic specific heat. In the super-conducting state thus below T_c, the specific heat falls exponentially, as the temperature decreases as schematically shown in Fig (2.9).

$$C_s \alpha e^{\left(-\frac{\Delta(T)}{k_B T}\right)}$$
(2.10.3)
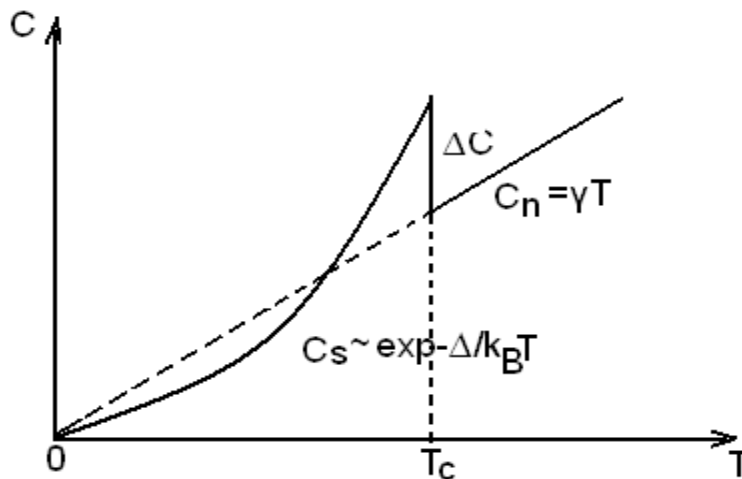
Figure (2.9) Temperature dependences of the specific heat of a super-conductor

2.11 London Equations

The London brothers proposed a simple theory to explain the Meissner effect. The London equations provided an early simple model for describing experimental results.

The 1935 theory of London brothers provides the first and second London equations, which relate the electric and magnetic fieldsE and B, respectively inside a super-conductor to the current density.

2.11.1 Derivation of First London Equation

A potential difference applied along a conducing wire produces an electric filed E on any electron is given by [53].

$$F = -eE = m\frac{dv}{dt}$$

Where e is the electron charge, m represents its mass, while v stands for its velocity. Electrons undergo successive periods of acceleration interrupted by collision, and during the average time [relaxation time (scattering on defects)] τ between collisions. The velocity is given by

$$v = \frac{eE}{m}\tau \tag{2.11.1}$$

Which called the drift velocity the negative sing means that the electrons move in direction opposite to that of the electric field?

When the electron is assumed to move in a resistive medium which have frictional force proportional to the velocity the electron equation of motion is given by

$$m\frac{dv}{dt} = eE - m\frac{v}{\tau} \tag{2.11.2}$$

Where the frictional force is given by:

$$F = ma, v = v_0 + a\tau = 0 + a\tau \Rightarrow F = \frac{mv}{\tau}$$

For steady state in normal metal, no acceleration exists. i.e.

$$\frac{dv}{dt} = 0$$

Therefore

$$v = \frac{eE}{m}\tau \qquad (2.11.3)$$

Hence the current density given by

$$J = nev = \frac{ne\tau}{m}E = \sigma E \qquad (2.11.4)$$

Where n the density of electrons is σ is electrical conductivity.

In the two fluid models we have the temperature dependent expression for the super n_n electrons densities respectively

$$n_s(T) + n_n(T) = n \qquad (2.11.5)$$

Where the total electron density n is independent of temperature and at $T = 0$ we have$n_n(0) = 0$ and $n_s(0) = n$, and the simple theory predicts the following temperature dependences:

$$n_s(T) = n_s\left(\frac{T}{T_c}\right)^4 \qquad (2.11.6)$$

Where T_c is the critical temperature.
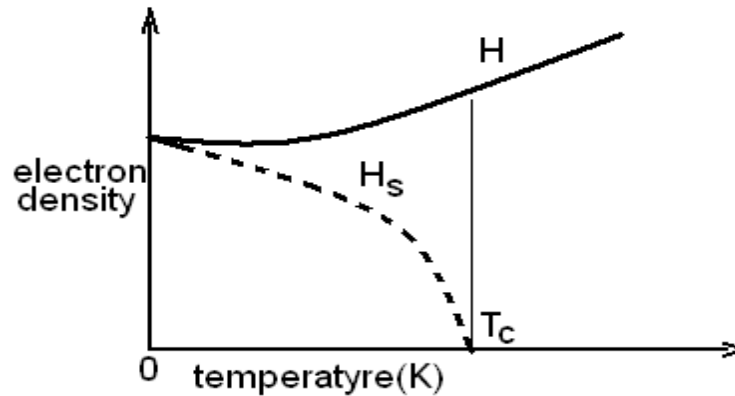
Figure (2.10) Temperature dependence of density of super-conducting

For super-conductor below. T_c The resistivity is zero we obtain equation (2.11.2) become:

$$\frac{dv}{dt} = \frac{eE}{m} \tag{2.11.7}$$

Taking the derivative of J in equation (2.11.7) with respect to time

$$\frac{dJ}{dt} = n_s e \frac{dv}{dt} = \frac{n_s e^2}{m} E \tag{2.11.8}$$

The term $\frac{m}{n_s e^2} = A$ is a phenomenological parameter

Equation (2.11.8) can be rewrite as

$$E = \frac{d}{dt}(AJ) = A\frac{dJ}{dt} \tag{2.11.9}$$

This equation is known as the first London equation

2.11.2 Second London Equation

This equation relates to time dependent fields and important for Meissner effect.

The electric current density is given quite by [54].

$$J = nqv \tag{2.11.10}$$

Where n is concentration of carriers of chargeq. In the presence of a magnetic field described by the vector potentialA, the velocity v is related to the total the momentum p by

$$p = mv + \frac{q}{c}A \quad v = \frac{1}{m}\left(p - \frac{q}{c}A\right) \tag{2.11.11}$$

Where m is the mass, c the speed of light in vacuum.

Thus equation (2.11.10) can write as

$$J = \frac{nq}{m}p - \frac{nq^2}{mc}A \tag{2.11.12}$$

In the super-conducting state, the total momentum p is zero although it not equal to zero in normal state. i.e.

p = 0, and equation (2.11.12) reduces to

$$J = \frac{nq^2}{mc}A \tag{2.11.13}$$

For electrons $q = e, n = n_s$

$$J - \frac{n_s e^2}{mc} A \qquad (2.11.14)$$

The vector potential is related to the magnetic field by

$$B = \nabla \times A \qquad (2.11.15)$$

Equation (2.11.14) can be rewritten as

$$J - \frac{c}{4\pi\lambda_L^2} A \qquad (2.11.16)$$

This equation is known as the second London equation where

$\lambda_2^2 = \frac{mc^2}{4\pi n_s e^2}$ (Where λ_s is known as London penetration depth) equation (2.11.16) can be expressed in another way by taking the curl of both sides and using equation (2.11.15) to obtains

$$\nabla \times J = -\frac{c}{4\pi\lambda_L^2} (\nabla \times A) = -\frac{c}{4\pi\lambda_L^2} B \qquad (2.11.17)$$

$$B = -cA\nabla \times J \qquad (2.11.18)$$

Where $A = \frac{m}{n_s e^2} = \frac{4\pi\lambda_L^2}{c^2}$ is a phenomenological parameter. Equation (2.11.18) is other form of the second equation of London.

2.12 London Penetration Depth in Super-conductors
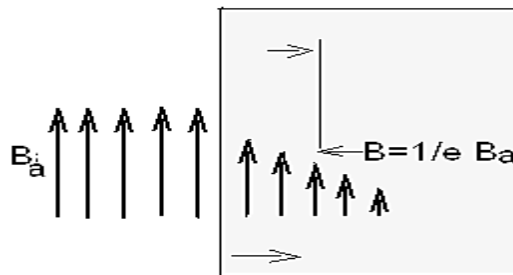
Figure (2.11) Exponentially damped magnetic field in semi-infinite super-conductor

One of the theoretical approaches for the description of the super-conducting state is the London equation. It relates the curl of the current density J to the magnetic field according to equation (2.11.17) as [55].

$$\nabla \times J = -\frac{c}{4\pi\lambda_L^2}(\nabla \times A) = \frac{c}{4\pi\lambda_L^2}B \qquad (2.12.1)$$

However form Maxwell equation under static conditions

$$\nabla \times B = \frac{4\pi J}{c} = \mu_0 J \qquad (2.12.2)$$

Where

$$\mu_0 = \frac{4\pi}{c}$$

This equation can be expressed in terms of the magnetic flux density by taking the curl of both sides of equation (2.12.2) one obtains

$$\nabla \times \nabla \times B = \nabla(\nabla.B) - \nabla^2 B = \frac{4\pi}{c}(\nabla \times J)$$

Where $\nabla.B = 0 \Rightarrow \nabla(\nabla.B) = 0$ according to Maxwell equation.

$$\nabla^2 B = -\frac{4\pi}{c}(\nabla \times J)$$

With equation (2.12.1) one gets

$$\nabla^2 B = +\frac{4\pi}{c}B \qquad (2.12.3)$$

If $B = B_a = $ constant. Then $\nabla^2 B_a$ is always zero, but $\frac{B_a}{\lambda_L^2}$ is not zero unless B_a is zero. This result shows that if the magnetic flux density is constant inside a super-conductor in must be identically zero everywhere inside a super-conductor.

In the pure super-conducting state, the only allowed solution is the damped exponentially one. This can be shown from the above equation. To solve this equation let a semi-infinite super-conductor occupy the space on the positive side of the X-axis in figure (2.11.2) at the top. If B_a is the field outside the super-conductor and at the plane boundary, then the field inside the super-conductor is given according to equation (2.12.3) as

$$B(\times) = B(0)\exp\left(-\frac{X}{\lambda_L}\right) \qquad (2.12.4)$$

WhereB($\times = 0$) $= B_a$, i.e B_a is the field outside the super-conductor and at the boundary therefore substituting in equation (2.12.4) yields:

$$B(\times) = B_a \exp\left(-\frac{x}{\lambda_L}\right) \qquad (2.12.5)$$

Where λ_L is known as London penetration depth

$$\lambda_L = \left(\frac{m}{\mu_0 n_s e^2}\right)^{\frac{1}{2}} \qquad (2.12.6)$$

$$\lambda_L = \left(\frac{mc^{\frac{1}{2}}\varepsilon_0}{n_s e^2}\right)^{\frac{1}{2}} \qquad (2.12.7)$$

Where μ_0 permeability of vacuum ε_0 permittivity (Dielectric constant of vacuum the above equations show that the nature of decay depends upon the super-conducting electron density n_s.

2.13 Coherence Length

The coherence length is a measure of the distance with in which the gap parameter cannot change drastically varying magnetic field. The London equation is a local equation: it relates the current density at a point r to the vector potential at the same point. As long as J(r) is given as a constant timesA(r), the current is required to follow exactly any variation in vector potential. However the coherence length is a measure of the range over which we should average A to obtainJ.

Any spatial variation in the state of an electronic system requires extra kinetic energy. It is reasonable to restrict the spatial variation of J(r) in such a way that the extra energy is less than the stabilization energy of the super-conducting state. A suggestive argument (based on the uncertainty principle) for coherence length at absolute zero follows [56].

The electron motion can be described by the plane wave

$$\psi(x) = e^{ikx}$$

Considering that the electron have two state one characterized by k, and the other is characterized by $k + q$. Then the wave function of the electron in the superposition of the two states is as follows

$$\psi(x) = 2^{-\frac{1}{2}}\left(e^{i(k+q)x} + e^{ikx}\right) \tag{2.13.1}$$

The probability density as associated with single plane wave is uniform in space. Where

$$\psi^*\psi = e^{-ikx}e^{ikx} = 1 \tag{2.13.2}$$

Where the probability $\varphi^*\varphi$ is modulated by the wave vector q.

Probability $|\varphi|^2$ of two states $= \varphi^*\varphi$

$$= \frac{1}{2}\left(e^{-i(k+q)x} + e^{-ikx}\right)\left(e^{i(k+q)x} + e^{ikx}\right)$$

$$= \frac{1}{2}\left(2 + e^{-iqx} + e^{-iqx}\right) = 1 + \cos(qx) \tag{2.13.3}$$

The kinetic energy of the wave $\psi(x)$ at a single state k is

$$H = \frac{P^2}{2m} + v \tag{2.13.4}$$

For free electrons

$$v = 0, p = \frac{\hbar}{i}\frac{d}{dx}, H\psi = E\psi \ \psi(x) = e^{ikx} \tag{2.13.5}$$

$$E = \frac{\hbar^2}{2m}k^2 \tag{2.13.6}$$

The kinetic energy of the wave function φ is

$$\langle E \rangle = \int \varphi^* H\varphi \, dx \tag{2.13.7}$$

For

$$v = 0 \quad H = \frac{P^2}{2m} = \frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

Then

$$\langle E \rangle = \int \varphi^* \left(\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \right) \varphi \, dx$$

$$\frac{1}{2} \frac{\hbar^2}{2m} [(k+q)^2 + +k^2]$$

$$= \frac{1}{2} \frac{\hbar^2}{2m} (k^2 + 2kq + k^2) \int \phi^* \phi \, dx$$

$$\frac{\hbar^2}{2m} (k^2 + kq) \tag{2.13.8}$$

Where we neglect q^2 on the assumption that $q \ll k$ comparing (2.13.6) and (2.13.7)

The increase of energy required for modulation is $\frac{\hbar^2 q}{2m}$. If this increase exceeds the energy gap E_g, super-conductivity will destroyed. The critical value q_0 of modulation wave vector is defined by

$$\frac{\hbar^2 q_0}{2m} k_F = E_g \tag{2.13.9}$$

We define an intrinsic coherence length λ_0 related to the critical modulation by.

$$\lambda_0 = \frac{2\pi}{q_0}$$

Since

$$\hbar k = p = mv$$

Then from (2.13.9) one obtains

$$\lambda_0 = \frac{2\pi\hbar^2}{2mE_g} k_F$$

$$= \frac{\pi\hbar}{E_g} v_F \tag{2.13.10}$$

Where

$$v_F = \frac{\hbar k_F}{m}$$

The electron velocity at the Fermi surface.

On the BCS theory a similar result can be obtained. i.e.

$$\lambda_0 = \frac{2\hbar v_F}{\pi E_g} \qquad (2.13.11)$$

The intrinsic coherence length λ_0 is characteristic of a pure super-conductor. In impure materials and in allays the coherence length λ_c is shorter thanλ_0. This maybe understood qualitatively in impure material the electron Eigen functions already have wiggles in them. We can construct a given localized variation of current density with less energy from functions with wiggles than from smooth wave function.

Arising in the theoretical and experimental investigations of super-conductivity are two characteristic lengths, the London penetration depth and the coherence length.

The London penetration depth refers to the exponentially decaying magnetic field at the surface of a super-conductor. It is related to the density of super-conducting electrons in the material. The fact of exclusion of magnetic field from the interior of the super-conductor is called the Meissner effect. An independent characteristic length is called the coherence length. It is related to the Fermi velocity for the material and the energy gap associated with the super-conducting electron density cannot change quickly there is a minimum length over which a given charge can be made, lest it destroy the super-conducting state. For example a transition layer of finite thickness which is related to the coherence length. Experimental studies of various Super-conductors have led to the following calculated values for these two types of characteristic lengths.

2.14 The Meissner Effect

The super-conductor has the important property of having zero resistance accordingly; the electric field in its interior must be zero. According to Faraday's law of induction of the line integral of the electric fieldEaround any close loop is equal to the negative rate of change in the magnetic flux through the loop. Since Eis zero everywhere inside the super-conductor the integral over any closed path

in the super-conductor is zero. Hence the rate of change of magnetic flux in the super-conductor is constant. Therefore, if we transport a super-conducting cylinder, e.g. in to a magnetic field it will push the magnetic lines a side so that none of these penetrate the cylinder. As the super-conductor touches the magnetic field of these currents are induced on the surface and the magnetic field, the current are induced on the surface and the magnetic field of these currents produces just the rich deformation of the magnetic field lines to prevent their penetration in to the cylinder. The super-conductor also expels any magnetic field lines that are initially inside the material before it becomes super-conducting. When a material makes the transition from the normal to super-conducting state, it actively excludes magnetic field form its interior; this phenomena is called the Meissner effect [57].

This constraint of zero magnetic fields inside a super-conductor is distinct from the perfect diamagnetism, which would arise from its zero electrical resistance. Zero resistance would imply that if you tried to magnetize a super-conductor, current loops would be generated to exactly cancel the imposed field (Lenz's Law). However if the material already had a steady magnetic field through it when it was cooled trough the super-conducting transition the magnetic field would be expected to remain. If there were no change in the applied would be expected to remain. If there were no change in the applied magnetic field, there would be no generated voltage (Faraday's Law) to drive current even in a perfect conductor. Hence the active exclusion of magnetic field must be considered an effect distinct from just zero resistance.

One of the theoretical explanations of the Meissner effect comes from the London equation. It shows that the magnetic field decays exponentially inside the super-conductor over a distance $20-40$nm. It is described in terms of a parameter called the London penetration depth [58, 59].

In type-II super-conductors the magnetic field is not excluded completely, but is constrained in filaments within the material. These filaments are in the normal state, surrounded by super currents in what is called a vortex state. Such materials can be subjected to much higher external magnetic fields and remain super-conducting [60].

2.15 Flux Quantization

When a type $-$ II super-conductor is immersed in an intermediate magnetic field to transfer it in to a mixed state the bulk of the material is super-conducting, but it is thread by thin filaments of normal material. The vortex lines are oriented parallel to the external magnetic field and they serve as paths for the magnetic flux lines of the external field.

A current circulates around the perimeter of each vortex line. This current shields the bulk of the super-conductor from the magnetic field in the filaments. The flow of this current has the character of a vortex and that is why that the filaments were calling as vortex lines.

Increasing the magnetic field would not cause an increase of the flux associated with each vortex line instead it will cause an increase in the number of vortex line threading the super-conductor. The stronger the external field he more densely will pack the vortex line. The ends of the vortex lines at surface of a super-conducting (Type $-$ II) metrical in the mixed state have been made visible by dusting the surface with powdered iron. The vortex line is packed in the form of heaps having regular pattern on the surface. Knowing the magnetic field intensity and the number of vortex lines per square cm, it was found that the a mount of flux associated with each vortex line has a fixed value related to Plank's constant h, and the electric charge of the electron e.

The quantum of flux [61].

$$\phi_0 = \frac{h}{2e} = 2.o7 \times 10^{-7} \text{Tm}^2 \qquad (2.15.1)$$

In general the

$$\phi = n\phi_0$$

Where n is integer $= 1,2,3, ...$

This result confirms the significance of electron pairs in the composition of the super-conducting state. Flux quantization is a beautiful example of a long-range quantum effect; in the instance of a ring the coherence of super-conducting state extends over the ring [62, 63].

The electromagnetic field is an example of a boson field. The electric field intensity E(r) acts qualitatively as field amplitude. The energy density may be written as, in a semi classical approximation.

$$\frac{E^*(r)E(r)}{4\pi} \approx n(r)\hbar\omega \qquad (2.15.2)$$

Where $n(r)$ is the number of photons of frequency ω per unit volume.

We assume that the total number of photons in the volume is large in comparison with unity.

Then:

$$E(r) \approx (4\pi\hbar\omega)^{\frac{1}{2}}(n)r^{\frac{1}{2}}e^{i\theta(r)} \qquad (2.15.3)$$

$$E^*(r) \approx (4\pi\hbar\omega)^{\frac{1}{2}}(n)r^{\frac{1}{2}}e^{-i\theta(r)} \qquad (2.15.4)$$

Where is $\theta(r)$ the phase of the field.

We now introduce similar particle probability amplitudes in to the description of particle bosons, where a particle is an electron pair (the analogy with photons is not exact but it is helpful).

The ground state of cooer pairs. An electron pair will act as a boson although a single electron is fermions. The arguments that follow apply specifically to boson gas with a very large number of bosons in the same orbital. We then can treat the boson probability amplitude as a classical quantity, just as the electromagnetic field is used for photons. The arguments do not apply to a metal in the normal state because an electron in normal state acts as single unpaired fermions.

We first show that a charged boson gas obeys the London equation in the form:

$$\nabla \times J = -\frac{c^2}{4\pi\lambda_L^2} B = -\frac{1}{\mu_0\lambda_L^2} B \qquad (2.15.5)$$

Let $\psi(r)$ be the particle probability amplitude. We suppose that the concentration.

$$n = \psi^*(r)\psi(r) = \text{const} \qquad (2.15.6)$$

At absolute zero n is one-half of the concentration of electrons in the conduction band for n refers to pairs. Then we can write:

$$\psi(r) = n^{\frac{1}{2}}e^{i\theta(r)}, \psi^*(r) = n^{\frac{1}{2}}e^{-i\theta(r)} \qquad (2.15.7)$$

The phase $\theta(r)$ is important for what follows

We make the good approximation that $\psi(r)$ is classical amplitude rather than a quantum field operator. The velocity of a particle is

$$v = \frac{1}{m}\left(p - \frac{q}{c}A\right) = \frac{1}{m}\left(-i\hbar\nabla - \frac{q}{c}A\right) \qquad (2.15.8)$$

The particle flux is given by:

$$nv = \psi^*(r)\psi(r)v = \psi^*(r)v\psi(r) = \psi^*(r)\left(-i\hbar\nabla - \frac{q}{c}A\right)\frac{\psi(r)}{m}$$

$$= \psi^*(r)\left(-i\hbar\nabla - \frac{q}{c}A\right)\frac{n^{\frac{1}{2}}e^{i\theta(r)}}{m}$$

$$\frac{\psi^*(r)}{m}\left(-i\hbar n^{\frac{1}{2}}e^{i\theta(r)}\nabla\theta(r) - \frac{q}{c}An^{\frac{1}{2}}e^{i\theta(r)}\right)$$

$$\frac{n^{\frac{1}{2}}e^{-i\theta(r)}}{m}\left(-i\hbar n^{\frac{1}{2}}e^{i\theta(r)}\nabla\theta(r) - \frac{q}{c}An^{\frac{1}{2}}e^{i\theta(r)}\right)$$

$$\frac{n}{m}\left(\hbar\nabla\theta - \frac{q}{c}A\right) \qquad\qquad (2.15.9)$$

So that the electric density in the ring (which is a multiply- connected region).

Is

$$J = nqv = \psi^*(r)\psi(r)qv = nq\left(\hbar\nabla\theta - \frac{q}{c}A\right)\frac{1}{m} \qquad (2.15.10)$$

Taking the curl of both sides one obtains

$$\nabla \times J = \frac{nq}{m}\left(\hbar\nabla \times \nabla\theta(r) - \frac{q}{c} \times \nabla A\right)$$

For equation $(2.11.15)\,B = \nabla \times A$, with use of the fact that the curl of the gradient of a scalar is identically zero, i.e.

$$\nabla \times \nabla\theta = 0$$

The above equation is one form of the London equation.

$$\nabla \times J = -\frac{nq^2}{m}B \qquad\qquad (2.15.11)$$

The quantization of the magnetic flux through a ring is a dramatic consequence of the equation of the electric current density j above. Let use take a close path C through the interior of the super-conducting material well away from the surface [64].
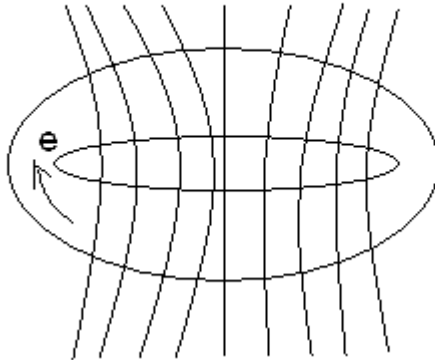
Figure (2.12) Path of integration C through the interior of super-conducting

The Meissner effect tells us that B and J are zero in the interior. Now from equation (2.15.10)

$$j = 0$$

If

$$\hbar \nabla \theta c = qA \qquad (2.15.12)$$

However, we have

$$\oint_C \nabla \theta \cdot dl = \theta_2 - \theta_1 \qquad (2.15.13)$$

Hence

$$c\hbar \oint_c \nabla\theta \cdot dl = q \oint_c A \cdot dl \qquad (2.15.14)$$

For the change of phase on going once around the ring, the boson probability amplitude is measurable in the classical approximation, so that ψ must be single valued and we must have [65].

$$\theta_2 - \theta_1 = 2\pi s \qquad (2.15.15)$$

Where s is an integer

$$\psi(\theta) = \sqrt{n}\, e^{i\theta}$$

$$\psi(\theta_1) = \sqrt{n}\, e^{i\theta_1} \quad \psi(\theta_2) = \sqrt{n}\, e^{i\theta_2}$$

$$\psi(\theta_1) = \psi(\theta_2)$$

$$= e^{i(\theta_2 - \theta_1)} = 1$$

$$\cos\theta + i\sin\theta = 1$$

$$\cos\theta = 1 \quad \sin\theta = 0$$

$$\theta = \theta_2 - \theta_1 = 2\pi s$$

$$s = 1,2,3,\dots$$

We also have by the stocks theorem and the fact that $\text{curl}A = B$

$$\oint_c A \cdot dl = \oint_c (\text{curl}A) \cdot d\sigma = \oint_c B \cdot d\sigma \qquad (2.15.16)$$

Where dσ is an element of area on a surface bounded by the curve C, and ϕ is the magnetic flux through C. From equation (2.15.12), (2.15.15) and (2.15.16). We have

$$2\pi\hbar sc = q\phi \qquad (2.15.17)$$

Alternatively

$$\phi = \left(\frac{2\pi\hbar c}{q}\right)s \qquad (2.15.18)$$

Thus, the flux through the ring is quantized as an integral multiples of $\frac{2\pi\hbar c}{q}$. By experiment $q = -2e$ as appropriate for electron pairs, so that the quantum of flux in a super-conductor is [66].

$$\frac{2\pi\hbar c}{q} \approx 2.07 \times 10^{-7}\,\text{Gauss/cm}^2$$

This unit of flux is calling a fluoride. It is important to note that the simple result (2.15.18) does not hold if the flux penetrates the ring itself as the material of the ring is thin.

The flux through the ring is the sum of the flux ϕ_{Ext} from external sources and the ϕ_{Sc} from the super-conducting currents, which flow in the surface of the ring:

$$\phi = \phi_{Ext} + \phi_{Sc}$$

The flux ϕ is quantized [1, 2, 8, and 13].

CHAPTER THREE

Literature Review

3.1 Introduction

Different attempts were made to account for the magnetic properties of matter these include magnetic resonance and magnetic properties of super-conductor [70, 71, 72 and73]. There one tries to exhibit some of these attempts.

3.2 Quantum Explanation of Conductivity at Resonance

The work is done by Asma. M to explain the conductivity frequency relation theoretically [74].

3.2.1 Experimental Change of Conductivity with Frequency

In the experiment a transmitter coil emits electromagnetic waves. This electromagnetic wave is allowed to incident on certain materials. The re emitted electromagnetic wave are receipted by a receiver.

Apparatus

10Resistors(10Ω,2.2GΩ,39kΩ),12capacitor(0.1μF,0.01μF,220μF), 6 Transistors (NPN), 2 transmitter and receptor coils (400,500,600,700,1000 turns), wire connection, Speakers, cathode Ray oscillator, Board connection, battery (9V), signal generator.

Samples:

Pieces of metal(Cu, Al, Fe, Au, Ag, Sn).

Method:

The transmitter coil current is varied by using signal generator. The emitted photons are allowed to incident on the sample. The sample absorbs photons and reemits them. The metal detector design is the circuit which connected as shown in fig (3.1). The signals appearing at oscilloscope were taken before mounting the sample, and after photon emission. The frequency and the corresponding conductivity of sample are recorded and determined from signal generator, current voltage, the length and cross sectional area of samples. The current and voltage gives resistance, which allows conductivity determination from the dimensions of the sample.

Table (3.1) Relation between frequency (f) and Conductivity (σ) without applied magnetic field for Cu, AI, Fe, Au, Ag, Sn

Frequency(Hz)	Conductivity($10^6 cm.\,\Omega$)
24	0.425
27	0.596
29	0.0993
34	0.0917
50	0.143
56	0.337

Table (3.2) Relation between frequency (f) and Conductivity (σ) for different magnetic flux densities for gold

Frequency(Hz)	Conductivity $(10^6 cm.\Omega)$ In97.3μT	Conductivity $(10^6 cm.\Omega)$ In77μT	Conductivity $(10^6 cm.\Omega)$ In116.7μT	Conductivity $(10^6 cm.\Omega)$ In136.2μT	Conductivity $(10^7 cm.\Omega)$ In194.53μT
55.25746	0.50026	0.12637	0.04739	0.15165	0.06824
47.48594	0.26365	0.03724	0.01396	0.04468	0.02011
40.49329	0.16368	0.03439	0.0129	0.04126	0.01857
35.4219	0.24726	0.03562	0.02011	0.06435	0.02896
26.8109	0.39107	0.1695	0.06356	0.2034	0.09153
24.81177	0.58753	0.2235	0.08381	0.2682	0.12069

3.2.2 Quantum Theoretical Model

Klein-Gordon equation reads

$$\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -c^2.\hbar^2 \nabla^2 + m_0^2 c^4 \psi \qquad (3.1.1)$$

Using separation of variable method

$$\psi = u(r)f(t)$$

In (3.2.1) yields

$$-u\hbar^2 \frac{\partial^2 f}{\partial t^2} = -fc^2.\hbar^2\nabla^2 u + m_0^2 c^4 fu \qquad (3.1.2)$$

$$\frac{1}{f}\hbar^2 \frac{\partial^2 f}{\partial t^2} = -\frac{1}{u}c^2.\hbar^2\nabla^2 u + m_0^2 c^4 = E^2 \qquad (3.1.3)$$

Where

$$-\frac{1}{f}\hbar^2\frac{\partial^2 f}{\partial t^2} = E^2 \qquad (3.1.4)$$

$$\therefore -\hbar^2\frac{\partial^2 f}{\partial t^2} = E^2 f \qquad (3.1.5)$$

Where

$$\hbar\omega_0 = electron\ energy\ in\ bounded\ state.$$

$$\hbar\omega = energy\ given\ to\ the\ electron.$$

$$E = \hbar\omega - \hbar\omega_0 = exciation\ energy.$$

Consider solution

$$f = \sin\alpha t \qquad (3.1.6)$$

$$\hbar^2\alpha^2 f = E^2 f \qquad (3.1.7)$$

$$\hbar\alpha = E$$

$$\alpha = \frac{E}{\hbar} = \omega - \omega_0 \qquad (3.1.8)$$

$$\therefore \sigma = \frac{ne^2\tau}{m} = \frac{|\psi^2|e^2\tau}{m} = \frac{e^2\tau}{m}|\sin(\omega - \omega_0)t|^2 \qquad (3.1.9)$$

3.2.3 Classical Absorption Conductivity Resonance Curve

Consider an electron of mass m oscillate with natural frequency ω_0. If and electric field of strength

$$E = E_0 e^{i\omega t} \qquad (3.2.1)$$

Was applied, then the equation of motion of the electron, in a frictional medium of friction coefficientγ, is given by

$$m\ddot{x} = eE - m\omega_0^2 x - \gamma\dot{x} \qquad (3.2.2)$$

Consider the solution

$$x = x_0 e^{i\omega t} \qquad (3.2.3)$$

Thus

$$v = \dot{x} = i\omega x \qquad \ddot{x} = -\omega^2 x \qquad (3.2.4)$$

Interesting (14). (15) And (12) in (13) yields

$$-\omega^2 x = e \frac{E_0}{x_0} x - m\omega_0^2 x - \gamma v$$

Thus

$$v = -m \frac{(\omega - \omega_0)}{\gamma} x + e \frac{E_0}{\gamma x_0} x \qquad (3.2.5)$$

For simplicity consider large displacement amplitudex_0 compared to the electrical one E_0.

Thus the last term in (4.5) can be neglected to get

$$v = -m \frac{(\omega - \omega_0)}{\gamma} x \qquad (3.2.6)$$

But the conductivity is given by

$$\sigma = \frac{e\tau}{m} n = \frac{e\tau}{m} n_0 e^{\frac{-\beta m v_e^2}{2}} \qquad (3.2.7)$$

Where the effective value v_e is related to the maximum value through the relation

$$v_e = \frac{v_0}{\sqrt{2}} \qquad (3.2.8)$$

For small value of the power of$e$, one can expand exponential term to be

$$e^{-x} = 1 - x \qquad (3.2.9)$$

Therefore equation (3.2.9)

$$\sigma = \frac{e\tau}{m} n_0 \left[1 + \frac{\beta m v_e^2}{4} \right] \qquad (3.2.10)$$

Inserting (3.2.5) in (3.2.4) yields

$$\sigma = \frac{e\tau}{m} n_0 \left[1 + \frac{\beta m^2 x_0^2 (\omega - \omega_0)^2 (\omega + \omega_0)^2}{4\gamma} \right]$$

$$\sigma = \frac{e\tau}{m} n_0 \left[1 + \frac{\beta m^2 \omega_0^2 x_0^2 (\omega - \omega_0)^2}{4\gamma} \right] \qquad (3.2.11)$$

Where near resonance

$$\omega \approx \omega_0 \qquad \omega + \omega_0 \approx 2\omega_0 \qquad (3.2.12)$$

The relation between conductivity and frequency resembles that of (2.2.1) in its dependence onω. This relation is displayed graphically in fig (3.2.12)

3.2.4 Discussion

The experimental work which was done shows variation of gold according to figs (2.4.2.1) and (2.4.2.2). The conductivity decreases then attains a minimum value in the range of (40-05Hz), then increases a gain.

The theoretical expression (3.1.11) which is displayed graphically in fig (3.1.1) is based on the ordinary expression for the conductivity. The electrons density n is found by solving Klein-Gordon equation for free particle. This is obvious as far as conduction electrons are free. The electron density is found from the

square of the wave function, which is a sin function. Since at resonance ω is very near to ω_0, thus one can replace $\sin x$ by x. The theoretical relation for f and σ obtained by this model resembles the experimental one in fig (3.1.1).

Another classical approach based on Maxwell-Boltzmann distribution in section (3) shows a relation between σ and f in fig (3.2.1) similar to experimental relations between σ and f resembles that of resonance, with minimum conductivity.

It is very interesting to note that each element has its own resonance conductivity at which conductivity is minimum.

In this model the ordinary expression for σ in equation (3.1.19) is used. But n here is found from Maxwell statistical distribution.

3.3 Relation between Matter Density, Atomic number, Magnetic field and resonance Frequency on the basis of Non Equilibrium Statistical Law and Zeeman Effect

Asma A.M. also tries to relate matter density and atomic number to the magnetic flux at resonance [75].

3.3.1 Experimental Relation of Matter Density and Magnetic Field with Resonance Frequency

In this experiment a transmitter coil emits electromagnetic waves. These electromagnetic waves are allowed to incident on certain materials. The re emitted electromagnetic waves are receipted by a receiver.

-Apparatus

10 Resistors (10Ω, 2.2GΩ, 39kΩ), 12 capacitors 0.1μF, 0.01μF, 220μF), 6Transistors (NPN). 2Coils400, 500.600, 700 100 turns), wire connection, Speakers, cathode Ray oscillator, Board connection, battery (9V), signal generator.

Samples:

Pieces of metal(Cu, Al, Fe, Au, Ag, Sn).

Method:

The transmitter coil current is varied by using signal generator. The emitted photons are allowed to incident on the sample. The sample absorbs photons and re emits them. The metal detector design is the circuit which connected as shown in fig (3.1). The signals appearing at oscilloscope were taken before mounting the sample, and after photon emission. The frequency and the corresponding conductivity of sample are recorded and determined from signal generator, current voltage, the length and cross sectional area of samples. The current and voltage gives resistance, which allows conductivity determination from the dimensions of the sample. The current and voltage gives resistance, which allows conductivity determination from the dimensions of the sample.

Table (3.3) Relation between frequency (f) and magnetic field in different voltages

Frequency(Hz)	Magnetic field (μT)In 400mV	Magnetic field (μT)In 200mV	Magnetic field (μT)In 180mV
55.25746	194.53	96.73	90.94
47.48594	136.2	68.06	63.66
40.49329	116.7	58.33	54.6
35.4219	97.3	48.6	45
26.8109	77.8	38.9	36.4

Table (3.4) Relation between frequency (f) and Electron Affinity Atomic number and Density

elements	Frequency (Hz)	Electron Affinity (KJ/mole)	Electron number (KJ/mole)	Density (kg/m^2)
AI	56	44.2	13	2700
Fe	29	16	26	7870
Sn	34	116	50	7300
Ag	50	112	47	8900
Au	24	223	79	19300
Cu	27	118	29	8960

3.3.2 Zeeman Effect and Statistical Theoretical Model

The Zeeman Effect is the name given to the splitting of the energy levels of an atom when it is placed in an externally applied magnetic field. The occurs because of the interaction of the magnetic moment μ of the atom with magnetic field B slightly shifts the energy of the atomic levels an amount.

$$\Delta E = -\mu B \qquad (3.3.1)$$

This energy shift depends on the relative orientation of the magnetic moment and the magnetic field. Nuclear magnetic resonance (NMR) and electron spin resonance (ESR) both depend on the Zeeman Splitting of a single energy level within the atom.

The first order perturbation theory gives a corresponding energy shift by

$$\Delta E = \mu_B g_s M_J H \qquad (3.3.2)$$

Where

$$M_J \equiv is\ orbital\ angular\ momentum$$

$$g_s \equiv is\ g - factors\ (Lande)$$

In the optical Zeeman Effect atoms are excited to level above the ground state by collisions with electrons in an electrical discharge. When they return to the ground state, they emit by extra energy as a visible photon whose energy corresponds to the difference in energy between the excited and ground state. According to Maxwell distribution the density of is given by:

$$n = n_0 e^{\frac{E}{\bar{\bar{E}}}} \qquad (3.3.3)$$

Where

$\bar{E} \equiv Stands\ for\ uniform\ energy.$

Assuming the oscillating frequency is uniform, then

$$\bar{E} = hf$$

Therefore equation (4.3) becomes

$$n = n_0 e^{\frac{-E}{hf}} \qquad (3.3.4)$$

It is quite too natural to assume that the density of photons is proportional to the exited atoms or electrons density i.e.

$$n_P = C_0 n = C_0 n_0 e^{\frac{-E}{hf}} \qquad (3.3.5)$$

By neglecting kinetic term, when the potential is very high in this case

$$E = -V_0 \qquad (3.3.6)$$

Therefore equation becomes (3.3.2)

$$n_P = C_0 n_0 e^{\frac{V_0}{hf}} \qquad (3.3.7)$$

$$\frac{V_0}{h} \sim \frac{10^{-5}}{r_0}$$

For

$$r_0 \sim 10^{-5}m \frac{V_0}{h} \sim 1 \tag{3.3.8}$$

The light intensity of emitted photons is given by

$$I = Cn_P = CC_0 n_0 e^{\frac{-V_0}{hf}} = I_0 e^{\frac{V_0}{hf}} \tag{3.3.9}$$

By a suitable choice of (3.3.8) and using (3.3.9) parameters one can choose

$$I = I_0 e^{\frac{1}{f}}$$

$$I_0 = 10 \tag{3.3.10}$$

3.3.3 Discussion

The metrical relation between applied magnetic field and resonance frequency resembles the theoretical one which shows exponential decay in fig (2.4.1.1). This confirms the readability of work done.

The empirical relation between resonance frequency on one hand with matter density, atomic number and electron affinity on the other hand which shows exponential decay. Can be explained also on the basis of the statistical equation for non-equilibrium state as which shows relation between density and frequency theoretically.

The relation for atomic number Z is related to the fact that the number of free electrons n_f is proportional to the atomic number Z where

$$n_f \propto Z - n_b$$

Where

n_b are the number of bounded electrons.

3.4 Using the Tight Binding Approximation in Deriving the Quantum Critical Temperature Super-conductivity Equation

R.A.ELhai was plasma equation and quantum Laws to derive condition for critical temperature resistance [76].

3.4.1 Plasma Equation

According to plasma equation, a fluid of particles of mass m, number density n, velocity v, force F and pressure P is given by

$$mn\left[\frac{\partial v}{\partial t} + v.\nabla v\right] = F - \nabla P \tag{3.4.1}$$

If F is a field force then

$$F = -n\nabla V$$

Where V is the potential of one particle in one dimension.

$$mn\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = n\nabla V - \nabla P = -\frac{dV}{dx} - \frac{dp}{dx} \tag{3.4.1}$$

$$dv = \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial x}dx$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\frac{dx}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} \tag{3.4.2}$$

Thus according to equation (3.4.1) in one dimension

$$mn\frac{dv}{dt} = -n\frac{dv}{dx} - \frac{dp}{dx}$$

3.4.2 Schrodinger Temperature Dependent Equation

Schrodinger equation can be derived by using new expression of energy obtained from the plasma equation to do this one can use (3.4.2) to get

$$mn\frac{dv}{dx}\frac{dx}{dt} = -n\frac{dV}{dx} - \frac{dP}{dx}$$

Multiplying both sides by dx and integrating yields

$$mn\int vdv = -n\int dV - \int dP$$

Considering the pressure to be $p = \gamma nkT$ in general thus

$$mn\frac{v^2}{2} = -nv - P = -nV - \gamma nkT$$

Hence

$$mn\frac{v^2}{2} + V + \gamma kT = const$$

This constant conserved quantity looks the ordinary energy beside the ordinary thermal energy term γkT.

$$E = \frac{p^2}{2m} + \gamma kT \qquad (3.4.3)$$

To find Schrodinger equation for it, consider ordinary wave function

$$\psi = Ae^{i/\hbar(px - Et)}$$

Differentiating both sides by t and x yields

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}E\psi \implies i\hbar\frac{\partial \psi}{\partial t} = E\psi$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{p^2}{\hbar^2}\psi \implies -\hbar^2\nabla^2\psi = p^2\psi \qquad (3.4.4)$$

Multiplying both sides of equation (3.4.3) by ψ yields

$$E\psi = \frac{p^2}{2m}\psi + V\psi + \gamma kT\psi$$

Substituting equation (3.4.4) one gets

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + \gamma kT\psi$$

This equation represents Schrödinger equation when thermal motion is considered. The solution for time free potential can be

$$\psi = Ae^{i/\hbar(Et)}u \implies \frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + \gamma kT\psi$$

The time independent Schrodinger equation thus takes the form

$$Eu = -\frac{\hbar^2}{2m}\nabla^2 u + Vu + \gamma kTu \tag{3.4.5}$$

For constant potential, the solution can be

$$u = e^{ikx} \qquad V = V_0$$

Inserting this solution in equation (3.4.5) yields

$$Eu = \frac{\hbar^2 k^2}{2m}u + V_0 u + \gamma kTu$$

$$E = \frac{\hbar^2 k^2}{2m} + V_0 + \gamma kT$$

If one set the kinetic term to be $E_0 = \frac{\hbar^2 k^2}{2m}$, one can thus writ the energy in the form

$$E = E_0 + V_0 + \gamma kT \tag{3.4.6}$$

This quantum energy expression involves a thermal term beside kinetic and potential term.

3.4.3 Quantum Resistance

The resistance, z per unit length $(L = 1)$ per unit area $(A = 1)$can be found from the ordinary definition of z. The resistance z is defined to be the ratio of the potential$u$, to the current per unit area, J,i.e.

$$z = \frac{u}{I} = \frac{u}{JA} = \frac{u}{J} = \frac{u}{nev} = \frac{mu}{nep} \tag{3.4.7}$$

With $n$and e standing for the free bole or electron density and charge respectively, while p represents the momentum of electron of mass m, where

$$P = mv$$

This resistance (it actually stands for resistivity) can be found by using the laws of quantum mechanics for a free charge which are responsible for generating the electric current, where the wave function takes the form.

$$\psi = Ae^{ikx} \tag{3.4.8}$$

This selection of ψ comes from the fact that the resistance property comes from the motion of the free charges the potential u is related to the Hamiltonian H through the relation

$$H = eu$$

Thus for freely moving charge one gets

$$\hat{H} = eu = \frac{1}{2}mv^2 = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

In view of equation (3.4.8) and according to the correspondence principle V takes the form

$$u = \frac{\langle \hat{H} \rangle}{e} = \frac{\int \bar{\psi}\hat{H}\psi dx}{e} = \frac{\int \bar{\psi}\hat{p}^2\psi dx}{2me} \tag{3.4.9}$$

$$= \frac{\hbar^2 k^2}{2me} \int \bar{\psi}\psi dx = \frac{\hbar^2 k^2}{2me}$$

While P becomes

$$p = \langle \hat{p} \rangle = \int \bar{\psi}\hat{p}^2\psi dx = \hbar k \int \bar{\psi}\psi dr = \hbar k \tag{3.4.10}$$

Thus inserting equation (3.4.9), (3.4.10) one obtains

$$Z = \frac{m\hbar^2 k^2}{2me^2\hbar kn} = \frac{\hbar k}{2e^2 n} = \left(\frac{h}{2\pi}\right)\left(\frac{2\pi}{\lambda}\right)\frac{1}{2e^2 n}$$

$$z = \frac{h}{2\lambda e^2 n} = \frac{hf}{2f\lambda e^2 n} = \frac{hf}{2e^2 nv} = \frac{hf\sqrt{\mu\varepsilon}}{2e^2 n} = \frac{\hbar\omega\sqrt{\mu\varepsilon}}{2e^2 n} \tag{3.4.11}$$

Where the expression $f\lambda$ for velocity is found by assuming charges to be waves, then following the electromagnetic theory (EMT), the speed of the waves is affected by electric permittivity ε and magnetic permeability through the relation

$$v = f\lambda = \frac{1}{\sqrt{\mu\varepsilon}} \tag{3.4.12}$$

Where the effect of medium changes the wave length λ, while the frequency f, is unchanged. Thus assuming the charge density n, to be constant the only change of z can be caused by μ and ε.

It is also important to note that in Super-conductors, the current can flow without the aid of deriving potential u. The role of u is confined only in enabling electrons to gain kinetic energy through the relations.

$$eu = \frac{1}{2}mv^2 = k \qquad (3.4.13)$$

Where this potential can be applied between any two arbitrary points in the super-conductor then remove it. The role of resistive force is neglected here as done as done in deriving London equations.

The expression for Z can also be found by inserting equation (3.4.13) in to get

$$z = \frac{u}{J} = \frac{u}{nev} = \frac{mv^2}{2ne^2v} = \frac{mv}{2ne^2} = \frac{p}{2ne^2} = \frac{h}{2\lambda ne^2}$$

$$z = \frac{hf}{2\lambda f e^2 n} = \frac{hf}{2e^2 nv} = \frac{hf}{2e^2 n} = \frac{\hbar\omega\sqrt{\mu\varepsilon_0(1+x)}}{2e^2 n} \qquad (3.4.14)$$

It is important to note that this quantum resistance expression resembles the ones found by Tsui[3] where one uses De Broglie hypothesis [4], i.e.

$$p = \frac{h}{\lambda}$$

3.4.4 Calculation HTSC by Electric Susceptibility

Consider holes in a conductor having resistive force F_r, magnetic force F_m and pressure force F_p, beside the electric force F_e, the equation motion then becomes [3]

$$F = F_r + F_m + F_e + F_p$$

Where

$$F_p = -\nabla P \quad F_r = -\frac{mv}{\tau} \quad F_m = Bev \quad F_e = eE = eE_0 e^{iax}$$

P, x, m, v, τ, B, e and E stands for the pressure, displacement, mass, velocity, relaxation time, magnetic flux density, electron charge and electric field intensity respectively. Thus the equation of motion takes the form

$$m\ddot{x} = -\frac{mv}{\tau} + Bev + eE - \nabla P \qquad (3.4.15)$$

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The solution of this equation can be suggested to be

$$x = x_0 e^{iax}$$

$$v = v_0 e^{iax}$$

$$E = E_0 e^{iax} \tag{3.4.16}$$

Inserting (16) in (15) yields

$$-m\omega^2 x = \left[-\frac{mv_0}{E_0\tau} + \frac{Bev_0}{E_0} - \frac{kT\nabla n}{E_0} + e \right] E \tag{3.4.17}$$

$$x = \frac{\left[\frac{mv_0}{E_0\tau} + \frac{Bev_0}{E_0} - \frac{kT\nabla n}{E_0} + e \right] E}{m\omega^2}$$

This expression of x can be utilized in the formula which relates the electric polarization vector P to the susceptibility χ on one hand and to the number of atoms N via the following relation

$$P = \varepsilon_0 \chi E = +eNx \tag{3.4.18}$$

Motivated by the important role of holes in HTSC, displacement can be assumed to result from the motion of holes or positive nuclear charges, thus inserting equation (3.4.17) in (3.4.18) yields

$$\varepsilon_0 \chi E = eN \frac{\left[\frac{mv_0}{E_0\tau} + \frac{Bev_0}{E_0} - \frac{kT\nabla n}{E_0} + e \right] E}{m\omega^2}$$

$$\chi = \frac{eN}{m\omega^2 \varepsilon_0 E_0} \left[\frac{mv_0}{E_0\tau} - Bev_0 + kT\nabla n - eE_0 \right] \tag{3.4.19}$$

The electric flux density assumes the following relation

$$D = \varepsilon E = \varepsilon_0 E + \varepsilon_0 \chi E = \varepsilon_0 (1 + \chi) E = P + \varepsilon_0 E$$

The electric permittivity is given by

$$\varepsilon = \varepsilon_0 (1 + \chi) \tag{3.4.20}$$

The electric permittivity is thus given according to equation (3.4.20) to be

$$\varepsilon = \varepsilon_0(1 + \chi)$$

$$\left[1 + \frac{eN}{m\omega^2 E_0}\left(\frac{mv_0}{E_0\tau} - Bev_0 + kT\nabla n - eE_0\right)\right] \tag{3.4.21}$$

The resistance Z can be found by inserting (3.4.21) in (3.4.14)

$$m\omega^2\varepsilon_0 E_0 + eN\left[kT\nabla n + \frac{mv_0}{\tau} - Bev_0 - eE_0\right] < 0$$

$$kT\nabla n < Bev_0 + eE_0 - \frac{mv_0}{\tau}$$

$$T < +\frac{Bev_0}{k\nabla n} + \frac{(e - m\omega^2\varepsilon_0)E_0}{eNk\nabla n} - \frac{mv_0}{\tau k\nabla n}$$

The resistance Z can be found by inserting (3.4.21) in (3.4.14) to get

$$z = \frac{\hbar\omega}{2ne^2}\sqrt{\mu\varepsilon_0\sqrt{1 + \frac{eN}{m\omega^2\varepsilon_0 E_0}\left(\frac{mv_0}{E_0\tau} - Bev_0 + kT\nabla n - eE_0\right)}} \tag{3.4.22}$$

$$z = \frac{\hbar\omega}{2ne^2}\sqrt{\mu\varepsilon_0\sqrt{\frac{m\omega^2\varepsilon_0 E_0 + eN\left(kT\nabla n + \frac{mv_0}{\tau} - Bev_0 - eE_0\right)}{m\omega^2\varepsilon_0 E_0}}}$$

$$T_c < +\frac{Bev_0}{k\nabla n} + \frac{(e - m\omega^2\varepsilon_0)E_0}{eNk\nabla n} \tag{3.4.23}$$

If the internal field B results from N_0 atoms each having a verge flux density μB then [5].

$$B = \mu_B N_0 \tag{3.4.24}$$

Therefore T_c can be take the form

$$T_c = \frac{(\mu_B N_0 e\tau - m)v_0}{\tau k \nabla n} + \frac{(e - m\omega^2 \varepsilon_0)E_0}{eNk\nabla n} \tag{3.4.25}$$

3.4.5 Tight Binding Critical Temperature and Energy Gap

In tight binding model [5] the energy of electrons in the crystal is given by

$$\varepsilon = \varepsilon_0 + \alpha_1 + 2\gamma coska \tag{3.4.26}$$

Where ε_0 is the energy in the absence of energy field. While the other terms describe the effect of the crystal field. The energy ε_0 can split in to two terms the kinetic

Part which can describe the thermal motion in the form $\frac{f_0}{2}kT$ beside the potential term $-V_0$ for attractive farce or bounded particle.

Thus or bounded particle. Thus one write

$$\varepsilon_0 = \frac{\hbar^2 k_0^2}{2m} + \frac{f_0}{2}kT - V_0 \tag{3.4.27}$$

$$E = \frac{\hbar^2 k_0^2}{2m} + \gamma kT + V$$

$$\varepsilon_0 = \frac{f_0}{2}kT - V_0 - \alpha_0$$

$$\alpha_0 = \frac{\hbar^2 k^2}{2m}$$

f_0 Represents the degrees of freedom. The terms describing the effect of the crystal force are

$$\alpha_1 = \langle \phi_m | \hat{H}_{cry} | \phi_m \rangle$$

$$\gamma = \langle \phi_J | \hat{H}_{cry} | \phi_m \rangle \qquad (3.4.28)$$

$$\alpha = \alpha_0 + \alpha_1$$

In view of equation (3.4.26) and (3.4.27)

$$\varepsilon = \frac{f_0}{2} kT - V_0 + \alpha + 2\gamma \cos ka \qquad (3.4.29)$$

Here H_{cry} stands for the crystal force Hamiltonian part, while ϕ_m and ϕ_J are the states of particles located at the site m and J respectively. The super-conductor is characterized by the existence of energy gap. This gap can be under stood here in two ways. If the electrons or holes are not free. This requires E to negative. Thus equation (3.4.27) and (3.4.26) needs

$$\varepsilon = \frac{f_0}{2} kT - V_0 + \alpha + 2\gamma \cos ka < 0 \qquad (3.4.30)$$

Or the max value of ε where $\cos ka = -1$ is less than zero, i.e.

$$\varepsilon_{max} = \frac{f_0}{2} kT - V_0 + \alpha + 2\gamma \cos ka < 0 (3.4.31)$$

For constant attractive crystal force

$$H_{cry} = -V_{cry}$$

$$\alpha_1 = \langle \phi_m | H_{cry} | \phi_m \rangle = -\langle \phi_m | V_{cry} | \phi_m \rangle = -V_{cry} \delta_{mm}$$

$$\gamma = \langle \phi_J | -V_{cry} | \phi_m \rangle = -V_{cry} \langle \phi_J | \phi_m \rangle = -V_{cry} \delta_{Jm=0} \qquad (3.4.32)$$

Thus

$$\frac{f_0}{2} kT \leq V_0 - \alpha$$

Thus the critical temperature is given by

$$\frac{f_0}{2} kT_c = V_0 - \alpha \tag{3.4.33}$$

Substituted equation (3.4.33) beside equation (3.4.32) in equation (3.4.30) one gets

$$\varepsilon = \frac{f_0}{2} kT - \frac{f_0}{2} kT_c \tag{3.4.34}$$

The energy gap Δ equal to the difference between zero energy in conduction band and the negative energy in the valence band. Thus

$$\Delta = 0 - \varepsilon = \frac{f_0}{2} kT_c - \frac{f_0}{2} kT$$

Since this relation holds for $T < T_c$ one can neglect T since it is small to get

$$\Delta = \frac{f_0}{2} kT_c$$

Equation (3.4.30) can also be utilized to get the forbidden energy states which characterizes Super-conductors, where

$$coska = \frac{\varepsilon - \frac{f_0}{2} kT + V_0 - \alpha}{2\gamma}$$

The energy is forbidden when $coska \geq 1$

$$coska = \frac{\varepsilon - \frac{f_0}{2} kT + V_0 - \alpha}{2\gamma} \geq 1$$

$$\varepsilon - \frac{f_0}{2}kT + V_0 - \alpha \geq 2\gamma$$

$$\frac{f_0}{2}kT + \alpha - \varepsilon - V_0 \leq -2\gamma$$

$$\frac{f_0}{2}kT \leq +V_0 - 2\gamma - \alpha$$

Thus the critical temperature

$$\frac{f_0}{2}kT_c = \varepsilon + V_0 - 2\gamma - \alpha \qquad (3.4.35)$$

The forbidden energy is thus related to the critical temperature through the relation

$$\varepsilon = \frac{f_0}{2}kT_c - V_0 + 2\gamma + \alpha \qquad (3.4.36)$$

3.5 Complex Quantum Resistance Model

M. Dirar uses quantum Laws based on plasma equation to explain the SC resistance vanishing [77].

Plasma equation describes ionized particles in a gaseous or liquid form. This equation can thus describe the electron motion easily. This is since the electrons be behaves as ionized particles in side matter for pressure exerted by the gas plasma equation becomes:

$$mn\frac{dv}{dt} = -\nabla P + F \qquad (3.5.1)$$

But for pressure exerted by the medium on the electron gas. The equation become

$$mn\frac{dv}{dt} = -\nabla P + F = \nabla P - \nabla V \qquad (3.5.2)$$

In one dimensions, the equation becomes

$$mn\frac{dv}{dt}\frac{dx}{dt} = \frac{d(nkT)}{dX} - \frac{dnv}{dx}$$

$$mn\frac{vdv}{dx} = \frac{d}{dx}[nkT - nv]$$

Where V is the potential for one particle

$$mn\frac{d1/2\,v^2}{dx} = \frac{d}{dx}[nkT - nV]$$

Thus in integrating both sides by assuming n to be constant, or in-dependent of K, yields

$$\frac{n}{2}mv^2 = nkT - nv + c$$

$$\frac{1}{2}mv^2 + v - kT = \frac{c}{n} = constant = E$$

This constant of motion stands for energy, thus

$$E = \frac{P^2}{2m} + v - kT \qquad (3.5.3)$$

Multiplying by ψ, yields

$$E\psi = \frac{P^2}{2m}\psi + v\psi - kT\psi \qquad (3.5.4)$$

According to the wave nature of particles

$$\psi = Ae^{\frac{i}{\hbar}(px-Et)}$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$\hbar^2\nabla^2\psi = P^2\psi \tag{3.5.5}$$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + v\psi - kT\psi \tag{3.5.6}$$

The time in dependent equation becomes

$$\frac{-\hbar^2}{2m}\nabla^2\psi + v\psi - kT\psi = E\psi \tag{3.5.7}$$

Consider the case when these electrons wave subjected to constant crystal field. This assumption is quite natural as far as particles are distributed homogenously. Thus equation (3.5.7) becomes

$$\frac{-\hbar^2}{2m}\nabla^2\psi + v_0\psi - kT\psi = E\psi \tag{3.5.8}$$

One can suggest the solution to be

$$\psi = Ae^{kx} \tag{3.5.9}$$

A direct substitution yields

$$\left(\frac{\hbar^2}{2m}k^2 + v_0 - kT\right)\psi = E\psi$$

Therefore

$$k = \frac{\sqrt{2m(E + kT - V_0)}}{\hbar} \qquad (3.5.10)$$

This wave number K, is related to the momentum according to the relation

$$P = mv = k\hbar\sqrt{2m(E + kT - V_0)} \qquad (3.5.11)$$

This relation can be used to find the quantum resistance R of a certain material. According to classical laws

$$R = \frac{V}{I} \qquad (3.5.12)$$

For electrons accelerated by the potential. The wave done is related to the potential V and kinetic energy K according to the relation

$$W = V = \frac{12}{mv^2} \qquad (3.5.13)$$

But since the current I is given by

$$I = nevA \qquad (3.5.14)$$

$$R = \frac{mv^2}{2nevA} = \frac{mv}{2nevA} = \frac{P}{2nveA} (3.5.15)$$

From (3.5.12) and (3.5.13):

$$R = \frac{\sqrt{2m(E + kT - V_0)}}{2nev} \qquad (3.5.16)$$

Splitting R to real part R, and imaginary, when

$$R = R_s + R_i \tag{3.5.17}$$

According to equation (3.5.16) R becomes pure imaginary, when

$$E + kT - V_0 < 0$$

$$kT < V_0 - E$$

$$T < \frac{(V_0 - E)}{K} \tag{3.5.18}$$

Thus the critical temperature is given by

$$T_c = \frac{V_0 - E}{K}$$

This requires

$$v_0 > E \tag{3.5.19}$$

In this case (see equation (3.5.17))

$$R = jR_i$$

$$R_s = 0 \tag{3.5.20}$$

Thus the super-conductivity resistance R_s becomes zero beyond a certain critical temperature given by equation (3.5.17). Which requires binding energy to dominate.

Another direct approach can also be found by considering the pressure exerted by the electrons. In this case [6] the Hamiltonian becomes:

$$\hat{H} = \frac{\hat{p}^2}{2m} + kT + V \tag{3.5.21}$$

For spin repulsive force

$$V = -V_0$$

Thus

$$\hat{H} = \frac{\hat{P}^2}{2m} + kT - V_0 \qquad (3.5.22)$$

Thus the average energy which is equal to the classical energy is given by

$$\langle \hat{H} \rangle = \langle \frac{\hat{P}^2}{2m} \rangle + kT - V_0 = E_0 + kT - V_0 \qquad (3.5.23)$$

Using the quantum definition of K[6]

$$R = \frac{\langle \hat{H} \rangle}{I} = \frac{E_0 + kT - V_0}{I}$$

$$R = R_+ + R_- \qquad (3.5.24)$$

Where one splits R to positive and negative one. When

$$E_0 + kT - V_0 < 0 \qquad (3.5.24)$$

$$R_- = \frac{E_0 + kT - V_0}{I} \quad R_+ = 0 \qquad (3.5.26)$$

From equations (3.5.25) and (3.5.26) the super conductivity resistance R_S vanishes i.e.

$$R_+ + R_S = 0$$

When

$$kT < V_0 - E_0$$

$$T < \frac{V_0 - E_0}{k} \qquad (3.5.27)$$

Thus the critical temperature is given by

$$T_c = \frac{V_0 - E_0}{k} \qquad (3.5.28)$$

Again for T_c to be positive $V_0 > E_0$

Thus for:

$$T < T_c$$

$$R_{sc} = R_+ = 0$$

Using plasma equation (3.5.1) a useful energy expression containing thermal energy is found in equation (3.5.3). Assuming electrons are free. Free wave solution is given by equation (3.5.9). This gives quantum momentum relation in equation (3.5.11). This relation is used in quantum resistance R. expression (3.5.15) which splits R to real and imaginary part. Thus one gets condition for zero resistance in equation (3.5.18) and (3.5.19). This happens beyond a critical temperature given by equation (3.5.19). This critical temperature T_c requires binding energy domination, which means that condition takes place by hopping.

Anther quantum resistance expression, which splits R to positive and negative terms, is also proposed in equation (3.5.24). The super-conductivity positive resistance vanishes beyond critical temperature given by equation (3.5.27). Again this T_c requires binding energy domination and hopping mechanism.

3.6 New Derivation of Simple Josephson Effect Relation using New Quantum Mechanical Equation

Rashida.I uses modified Schrodinger equation to derive Josephson Effect [78].

3.6.1 New Quantum Equation

The Newtonian energy E is a sum of kinetic and potential energy v, i.e.

$$E = \frac{1}{2}mv^2 + V = \frac{P^2}{2m} + V \qquad (3.6.1)$$

Where m, v, p are the mass, velocity and momentum respectively. According to a theorem of Bloch's [7], in such Super-conductors the momentum p is zero.

$$p = \frac{mv^2}{2} + \frac{qA}{c}$$

Thus (3.6.1) becomes

$$E = V \qquad (3.6.2)$$

This is related to the fact that in Josephson Effect the tunneling potential is considered to be larger then kinetic term squaring both sides' yields

$$E^2 = V^2 \qquad (3.6.3)$$

Multiplying both sides by ψ, one gets

$$E^2\psi = V^2\psi \qquad (3.6.4)$$

The wave function of a free particle is given by

$$\psi = Ae^{\frac{i}{\hbar}(Px - Et)} \qquad (3.6.5)$$

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Differentiating both sides with respect x and t twee

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \psi$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{i}{\hbar} E \frac{\partial \psi}{\partial t} = \frac{i^2}{\hbar^2} E^2 \psi = \frac{E^2}{\hbar^2} \psi$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = E^2 \psi \tag{3.6.6}$$

Similarly

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar} P \psi$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} = \frac{iP}{\hbar} \frac{\partial \psi}{\partial x} = \frac{iP}{\hbar} \left[\frac{iP}{\hbar}\right] \psi = \frac{i^2 P^2}{\hbar^2}$$

$$-\hbar^2 \nabla^2 \psi = P^2 \psi \tag{3.6.7}$$

Substitute (3.6.6) in (3.6.4) to get

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = V^2 \psi \tag{3.6.8}$$

3.6.2 Josephson Effect Equation

In Josephson Effect are considered as having small kinetic energy compared to the potential. Thus Schrodinger equation (3.6.8), in which kinetic term is neglected is suitable for describing the Josephson Effect. To derive Josephson Effect equation, consider the solution

$$\psi = D \sin(\alpha t + \phi) \tag{3.6.9}$$

The tunneling potential is constant inside a super-conductor, thus

$$V = V_0 \qquad (3.6.10)$$

From (3.6.9) one can differentiate ψ with respect to time twice to get

$$\frac{\partial \psi}{\partial t} = -\alpha D \cos(\alpha t + \phi)$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\alpha^2 D \sin(\alpha t + \phi) = -\alpha^2 \psi \qquad (3.6.11)$$

Substitute (3.6.10) and (3.6.11) in (3.6.8) to obtain

$$+\hbar^2 \alpha^2 \psi = V_0^2 \psi$$

$$\alpha^2 = \frac{V_0^2}{\hbar^2}$$

$$\alpha = \pm \frac{V_0}{\hbar} \qquad (3.6.12)$$

By substituting (3.6.12) in (3.6.9) and choosing a negative sing, that is in dealing with the change in potential energy one gets

$$\psi = D \sin\left[-\frac{eV_0}{\hbar} t + \phi\right] \qquad (3.6.13)$$

But the energy density J is given by:

$$J = e\frac{\partial n}{\partial t} = e\frac{\partial |\psi|^2}{\partial t} = 2e|\psi|\frac{d|\psi|}{dt}$$

$$= 2eD\sin(\alpha t + \phi)\left[-\frac{e}{\hbar}V_0\right]\cos(\alpha t + \phi) \qquad (3.6.14)$$

$$= 2\frac{e^2 DV_0}{\hbar}\sin\theta\cos\theta$$

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$$\theta = \phi - \frac{eV_0 t}{\hbar}$$

Bu using mathematical identity

$$sin2\theta = 2sin\theta cos\theta$$

One can rewrite equation (3.7.14) to be

$$J = -\frac{e^2 DV_0}{\hbar} \sin\left[2\phi - \frac{2eV_0}{\hbar}t\right] = Asin\left[2\phi - \frac{2eV_0}{\hbar}t\right] \qquad (3.6.15)$$

Setting:

$$2\phi = \delta$$

The current density is given by:

$$J = J_0 \sin\left[\delta(0) - \frac{2eV_0}{\hbar}t\right] \qquad (3.6.16)$$

3.6.3 Discussion

Equation (3.6.2) shows a new energy equation based on Newtonian mechanics, with the neglected kinetic term. This equation is used a new quantum equation in (3.6.8). This new equation is based on Newtonian energy with no kinetic term to beside the wave equation of a free particle. His derivation resembles simple derivations of Schrodinger equation except the fact that the kinetic term is neglected.

This equation is used to derive simple Josephson current density equation. This (3.6.16) is the same as the old one, but derived using simple arguments.

3.7 Generalized Special Relativity Quantum Theory and Josephson Super-conducting Effect

M.Dirar utilizes GSR quantum equation to derive Josephson Effect.

3.7.1 New Version of GSR Energy Formula and GSR Quantum Theory:

The mass expression given according to energy momentum conservation is given by [79].

$$m = \gamma m_0 = \frac{m_0}{\sqrt{g_{00} - \dfrac{v^2}{c^2}}} \tag{3.7.1}$$

This appears to be in direct conflict with the expression derived by [7]

$$m = g_{00}\gamma m_0 \tag{3.7.2}$$

But this conflict can remove by re deriving the expression of energy [8].

Where

$$T^{00} = \gamma m_0 c^2 \tag{3.7.3}$$

$$E = T^{00} = g_{00}T^{00} = g_{00}\gamma m_0 c^2 \tag{3.7.3}$$

This conflict can be removed by lowering the insides in flat space by taking

$$E = T^{00} = \mu_{00}T^{00} = 1X\gamma m_0 c^2 = \frac{m_0}{\sqrt{g_{00} - \dfrac{v^2}{c^2}}} \tag{3.7.5}$$

Therefore expression (3.7.1) and (3.7.5) are the same. Thus the energy is given by

$$E = m_0 c^2 \left(g_{00} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \left(1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} m_0 c^2$$

$$= \left(\frac{m^2c^4 + 2(mc^2)(m\phi)}{m^2c^4} - \frac{m^2v^2c^2}{m^2c^4} \right)^{-\frac{1}{2}} m_0c^2$$

$$= (E^{-2})^{-\frac{1}{2}}(E^2 + 2VE - P^2c^2)^{-\frac{1}{2}}m_0c^2$$

Therefore

$$(E^2 + 2VE - P^2c^2)^{-\frac{1}{2}} = m_0c^2$$

$$E^2 + 2VE - P^2c^2 + m_0c^2 \quad (3.7.6)$$

It is very interesting to note that when the potential vanishes, i.e.

$$v = 0$$

Equation (3.7.6) reduces to

$$E^2 = P^2c^2 + m_0c^2 \tag{3.7.7}$$

This is the ordinary Einstein energy-momentum relation. The quantum new GSR equation can by using the free particle wave equation

$$\psi = e^{\frac{i}{\hbar}(px+Et)}$$

Where

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t} = E^2\psi \qquad i\hbar \frac{\partial \psi}{\partial t} = E\psi$$

$$-\hbar^2 \nabla^2 \psi = P^2\psi \tag{3.7.8}$$

Then by multiplying (3.7.6) by ψ and substituting (3.7.8)

$$E^2\psi + 2VE\psi = P^2c^2\psi + m_0^2c^4\psi \tag{3.7.9}$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t} + 2i\hbar V \frac{\partial \psi}{\partial t} = -c^2\hbar^2\nabla^2\psi + m_0^2 c^4\psi \tag{3.7.10}$$

This is the new quantum GSR equation.

3.7.2 Josephson Effect Current Expression According to New GSR

Consider solution of (5.10.10) in the form

$$\psi(r,t) = f_0(t)C = f_0 e^{ikr} \tag{3.7.11}$$

A direct substitution of (3.9.11) in (3.9.10) yields

$$\left[\hbar^2 \frac{\partial^2 f_0}{\partial t^2} + 2i\hbar V \frac{\partial f_0}{\partial t}\right] e^{ikr} = c^2\hbar^2 k^2 f_0 e^{ikr} + m_0^2 c^4 f_0 e^{ikr}$$

Cancelling exponential terms on both sides yields

$$\hbar^2 \frac{\partial^2 f_0}{\partial t^2} + 2i\hbar V \frac{\partial f_0}{\partial t} = P^2 c^2 f_0 + m_0^2 c^4 f_0 \tag{3.7.12}$$

Consider very small mass and momentum such that

$$P^2 c^2 \to 0 \quad m_0^2 c^4 \to 0$$

In this case equation (3. 7.12) reads

$$\hbar^2 \frac{\partial^2 f_0}{\partial t^2} + 2i\hbar V \frac{\partial f_0}{\partial t} = 0 \tag{3.7.13}$$

Where v_e is the potential affecting one electron consider the solution

$$f_0 = De^{\pm(\alpha t + c)i} \frac{\partial f_0}{\partial t} = \pm i\alpha f_0$$

$$\frac{\partial^2 f_0}{\partial t^2} = -\alpha^2 f \tag{3.7.14}$$

Substituting (3.7.14) in (3.7.13) yields

$$[\hbar^2\alpha^2 \pm 2\alpha V_e]f_0 = 0 \hbar^2\alpha^2 = \pm 2\alpha V_e$$

$$\alpha = \pm\frac{2V_e}{\hbar}$$

Where

$$\theta = \phi - \alpha t \qquad (3.7.15)$$

According to equation (3.7.14) the general solution is in the form

$$f = D_1 e^{i\theta} + D_2 e^{-i\theta} \qquad (3.7.16)$$

The current is given by

$$I = \frac{\partial Q}{\partial t} = e\frac{\partial |\psi|^2}{\partial t} = e\frac{\partial \psi\bar{\psi}}{\partial t}$$

$$I = e\frac{\partial f\bar{f}}{\partial t} \qquad (3.7.17)$$

When no potential applied and when no separation is made by insulator between the super conductor sides.

$$I = 0 \qquad V_0 = 0 \qquad \alpha = 0 \qquad (3.7.18)$$

One of the possible solutions is to set

$$f = 0$$

In view of (3.7.18) and (3.7.14) one gets

$$0 = (D_1 + D_2)e^{i\theta}$$

Thus

$$D_1 + D_2 = D \tag{3.7.19}$$

Hence the solution will be according to equation (3.7.14), (3.7.16) and (3.7.19) in the form

$$f = D\left[e^{+i\theta} - e^{-i\theta}\right]$$

$$D[\cos\theta + iD\sin\theta - \cos\theta + i\sin\theta]$$

In view of equation (3.7.15) one gets

$$f = 2iD\sin\theta = 2iD\cos(\phi - \alpha t) \tag{3.7.20}$$

According to cooper theory on have two electron pairs, if the total electric potential on both pairs is V_0. Thus the potential for each is $\frac{1}{2}V_0$. Thus the total potential energy on the pair is

$$V = 2e\left(\frac{1}{2}V_0\right) = eV_0 \tag{3.7.21}$$

The total potential of the cooper electron pair is also double that of a single electron. Thus

$$V = 2V_e \tag{3.7.22}$$

Thus α in equation (5.11.5) is given by

$$\alpha = \frac{\forall}{\hbar} = \frac{eV_0}{\hbar} \tag{3.7.23}$$

The electric current can be obtained by inserting (3.7.19) in (3.7.17) to get

$$I = e\frac{\partial}{\partial t}[4(i)(-i)D^2 \sin^2(\phi - \alpha t)]$$

$$8eD^2 sin(\phi - \alpha t)[-\alpha cos(\phi - \alpha t)]$$

$$I = -8e^2 \alpha D^2 sin2(\phi - \alpha t)cos(\phi - \alpha t)$$

$$= -4e^2 \alpha D^2 sin2(\phi - \alpha t)$$

$$I = D_0 sin(\phi_0 - 2\alpha t) \qquad (3.7.24)$$

Where

$$D_0 = 4e^2 \alpha D^2 \phi_0 = 2\phi \qquad (3.7.25)$$

According to equation (3.7.14), (3.7.24) and (3.7.25) the super current is given by

$$I = D_0 sin\left(\phi_0 - \frac{2eV_0 t}{\hbar}\right) \qquad (3.7.26)$$

The periodicity of current requires

$$I(t + T) = I(t) \qquad (3.7.27)$$

According to equation (3.7.26)

$$sin[\phi_0 - 2\alpha t(t + T)] = sin[\phi_0 - 2\alpha t]$$

$$sin[\phi_0 - 2\alpha t] cos2\alpha T - cos[\phi_0 - 2\alpha t]sin2\alpha T = 0$$

This requires

$$cos2\alpha T = 1 \qquad sin2\alpha T = 0$$

$$2\alpha T = 2n\pi$$

$$\alpha = \frac{n\pi}{T} = n\pi f \qquad (3.7.28)$$

If one choose n to by unity then

$$n = 1 \quad \alpha = \pi f \tag{3.9.29}$$

Thus equation (3.7.24) becomes

$$I = D_0 \sin(\phi_0 - 2\pi f t) \tag{3.7.30}$$

Comparing equation (3.7.26) and (3.7.30) yields

$$2\pi f = \frac{2eV_0}{\hbar}$$

Hence

$$f = \frac{2eV_0}{\hbar} \tag{3.7.31}$$

It is interesting to one that expression (3.7.26), (3.7.30) and (3.7.31) for super current and frequency are completely consistent with Josephson super current formula or expression.

3.7.3 Discussion

Generalized special relativity energy relation (2.7) beside the wave equation (2.8) for free particle is used to derive new relativistic equation as shown by equation (2.10). t is very interesting to observe that this equation reduces to Klein-Gordon equation in the absence of potential. This new equation is more advanced than Klein-Gordon one since it contains an expression of potential energy for any field. In Klein-Gordon the potential energy for each field requires deriving the quantum equation for each case which is very complex and time consuming.

3.8 Summery and Critique

The exhibited models describe successfully some of the important physical properties of SC like zero resistance beyond T_c, Josephson effect and energy gap.

Many papers are also proposed to solve some of these problems [80, 81, 82], but none of them take care of the magnetic properties take care of the magnetic properties [83, 84, 85].

CHAPTER FOUR

Effect of Magnetic Field on Energy Gap and Super-conductor Resistance

4.1 Introduction

Super conductors have very interesting magnetic properties. These properties include effect of magnetic field on SC resistance and the energy gap. Anew simple theoretical explanation based on quantum plasma equation is exhibited here.

4.2 Complex Quantum Resistance Model

Plasma equation describes ionized particles in a gaseous or liquid form. This equation can thus describe the electron motion easily. This is since the electrons be behaves as ionized particle in side matter. For pressure exerted by the gas plasma equation becomes:

$$mn\frac{dv}{dt} = -\nabla P + F \tag{4.2.1}$$

But for pressure exerted by the medium on the electron gas, the equation become

$$mn\frac{dv}{dt} = \nabla P + F = \nabla P - \nabla V \tag{4.2.2}$$

In one dimensions, the equation become

$$mn\frac{dv}{dx}\frac{dx}{dt} = \frac{d(nkT)}{dx} - \frac{dnv}{dx} - \frac{dnv}{dx}$$

$$mn\frac{vdv}{dx} = \frac{d}{dx}[nkT - nv]$$

Where V is the potential for one particle

$$mn\frac{d1/2v^2}{dx} = \frac{d}{dx}[nkT - nV]$$

Thus in integrating both sides by assuming n to be constant, or in-dependent of k, yields

$$\frac{n}{2}mv^2 = nkT - nV + c$$

$$\frac{n}{2}mv^2 + V - kT = \frac{c}{n} = constant = E$$

This constant of motion stands for energy, thus

$$E = \frac{P^2}{2m} + V - kT \qquad (4.2.3)$$

Multiplying by ψ, yields

$$E\psi = \frac{P^2\psi}{2m} + V\psi - kT\psi \qquad (4.2.4)$$

According to the wave nature of particles

$$\psi = Ae^{\frac{i}{\hbar}(px-Ex)}$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$-\hbar^2\nabla^2\psi = P^2\psi \qquad (4.2.5)$$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi - kT\psi \qquad (4.2.6)$$

The time in dependent equation becomes

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi - kT\psi = E\psi \qquad (4.2.7)$$

Consider the case when these electrons wave subjected to constant crystal filed v_0. This assumption is quite natural as for as particles are distributed homogenously. Thus equation (4.2.7) becomes

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V_0\psi - kT\psi = E\psi \qquad (4.2.8)$$

One can suggest the solution to be

$$\psi = e^{ikx} \qquad (4.2.9)$$

A direct substitution yields

$$\left(\frac{\hbar^2}{2m}k^2 + V_0 - kT\right)\psi = E\psi$$

Therefore

$$k = \frac{\sqrt{2m(E + kT - V_0)}}{\hbar} \qquad (4.2.10)$$

This wave number k is related to the momentum according to the relation

$$P = mv = \hbar k = \sqrt{2m(E + kT - V_0)} \qquad (4.2.11)$$

This relation can be used to find the quantum resistance$R$ of a certain. According classical laws

$$R = \frac{V}{I} \qquad (4.2.12)$$

For electrons accelerated by the potential. The wave done is related to the potential V and kinetic energy K according to the relation

$$w = V = \frac{1}{2}mv^2 \tag{4.2.13}$$

But since the current I is gives by

$$I = nevA \tag{4.2.14}$$

$$R = \frac{mv^2}{2nevA} = \frac{mv}{2neA} = \frac{P}{2neA} \tag{4.2.15}$$

From (4.2.12) and (4.2.13)

$$R = \frac{\sqrt{2m(E + kT - V_0)}}{2neA} \tag{4.2.16}$$

Splitting R real part R_s and imaginary part R_i

$$R = R_s + R_i \tag{4.2.17}$$

According to equation (4.2.16) R becomes pure imaginary, when

$$E = KT - V_0 < 0$$

$$KT < V_0 - E$$

$$T < \frac{(V_0 - E)}{K} \tag{4.2.18}$$

Thus the critical temperature is given by

$$T_c = \frac{(V_0 - E)}{K} \tag{4.2.19}$$

This requires

$$V_0 > E$$

In this case (see equation (4.2.17))

$$R = jR_i$$

$$R_s = 0 \qquad (4.2.20)$$

Thus the super-conductivity resistance R_s becomes zero beyond a certain critical temperature given by equation (4.2.17). Which requires binding energy to dominate.

4.3 Energy Gap and Photon Absorption

It is known that in some Super-conductors, the materials behave as an anti-ferromagnetic this means that it is possible to consider electrons in the atoms as having spin up and down atoms with number N_u and N_d respectively, in the ground lower and excited states respectively, such that the magnetic flux density inside the medium is given by

$$B_m = B_e(N_u - N_d) \qquad (4.3.1)$$

Where B_e is the magnetic flux density of one electron. If a photon beams was absorbed this will change B_m by the transition of electrons from ground state to the excited state. If the number of incident photons is N_p the new internal flux density is given by

$$B_m = B_e\left(N_u - N_d + 2N_p\right) \qquad (4.3.2)$$

This changes the potential of electrons to V_m and split the energy levels to be

$$V_m = m_L \left(\frac{e\hbar}{2m}\right) B_m \qquad (4.3.3)$$

Thus the energy gap is given by

$$E_g = \frac{e\hbar}{2m} B_m \qquad (4.3.4)$$

Here one assumes that any electron is affected by the magnetic field of the spinning electron gap. When electrons are affected by internal magnetic field the

resistance in equation (4.2.16) and by the definition of T_c in equation (4.2.19) is given by

$$R = \frac{\sqrt{2mk(T + T_m - T_c)}}{2ne\lambda} = R_s + jR_i \qquad (4.3.5)$$

Where

$$kT_m = V_m \qquad (4.3.6)$$

The super-conductivity is destroyed when

$$T_m \geq T_c \qquad (4.3.7)$$

Thus

$$V_m \geq kT_c \qquad (4.3.8)$$

Since V_m is proportional to B_m according to equation (4.3.4) the energy gap corresponds to the minimum voltage that destroy super-conductivity. Thus

$$E_g = c_m V_{mg} \qquad (4.3.9)$$

But according to equation (4.3.8) the minimum magnetic energy that can destroy super-conductivity is

$$V_{mg} = kT_c \qquad (4.3.10)$$

Thus equation (4.3.9) indicates that the energy gap takes the form

$$E_g = c_m kT_c \qquad (4.3.11)$$

It is very interesting to note that this expression for E_g conforms to the well-known ordinary relation. In this model the photon plays a double role. When it is incident and absorbed by the super-conductivity it increases the internal field by causing more electrons with spin down to be in an excited state. This increase in the internal field B_m causes splitting of energy levels by the amount.

$$\Delta E = gm_s \mu_B H_m \qquad (4.3.12)$$

Thus the super-conductivity resistance vanishes for all T less than the critical value.

When an external magnetic field of flux density B is applied, the total the medium filed is given by:

$$B_m = B - B_i \qquad (4.4.23)$$

Where B_i is the internal flux density. The corresponding potential applied on electrons or charges is given by V_m thus the total potential in equation (4.4.7) becomes

$$V = V_0 \pm V_m \qquad (4.4.24)$$

$$V_m = V_m \left(\frac{e\hbar}{2m}\right) B_m = C_0^{-1} B_m$$

When the net magnetic potential opposes the crystal field

$$V = V_0 - V_m = V_0 - C_0^{-1} B_m \qquad (4.4.25)$$

In this case one can rewrite the expression of R in equation (4.4.16) to be

$$R = \frac{\sqrt{2mk(E + kT - V_0 + V_m)}}{2enA} = \frac{\sqrt{2mk(T - T_c + T_m)}}{2enA}$$

Where

$$2mT_m = V_m$$

$$R = \frac{\sqrt{2mk(T - T_c + T_m)}}{2enA} \qquad (4.4.26)$$

Consider now the case when T_m is greater T_c, i.e. when

$$T_m \geq T_c \qquad (4.4.27)$$

According to equation (4.4.23) and (4.4.25) and (4.4.27) this critical value is given by

$$B_c = C_0(2mKT_c) + B_{ic} \qquad (4.4.28)$$

In this case the term under the square root is positive always. This means that, it

$$R = R_s + jR_i \qquad (4.4.29)$$

$$R_i = 0 \qquad R_s \neq 0 \qquad (4.4.30)$$

This means that the super-conductivity is destroyed when applying an external magnetic filed having strength exceeding the critical valve (4.4.28).

4.4 Quantum Plasma Based on Negative Resistance Model and String Mode

Another direct approach can also be found by considering the pressure exerted by the electrons. In this case [9]. The Hamiltonian becomes:

$$\hat{H} = \frac{\hat{P}^2}{2m} + KT - V_0 \tag{4.5.1}$$

For spin repulsive force

$$V = -V_0$$

Thus

$$\hat{H} = \frac{\hat{P}^2}{2m} + KT - V_0 \tag{4.5.2}$$

Thus the average energy which is equal to the classical energy is given by:

$$H < \frac{\hat{P}^2}{2m} > +KT - V_0 = E_0 + KT - V_0 \tag{4.5.3}$$

Resistance for harmonic oscillator where

$$x = x_0 e^{iwt} \quad v = i\omega t \quad T = \frac{1}{2}m|v|^2 = 1/2m\omega^2 x^2$$

$$V = \frac{1}{2}Kx^2 = \frac{1}{2m\omega^2 x^2} = T \quad H = T + V = 2V \quad eV_e = V = \frac{H}{2}$$

Where V_e is the potential, thus

$$V_e = \frac{H}{2e}$$

Using the quantum definition of [10]:

$$R = \frac{V_e}{I}$$

$$R = \frac{\langle \hat{H} \rangle}{2eI} = \frac{E_0 + KT - V_0}{2eI}$$

$$R = R_+ + R_- \tag{4.5.4}$$

Where one splits R to positive and negative one.

When

$$E_0 + KT - V_0 < 0 \tag{4.5.5}$$

$$R_- = \frac{E_0 + KT - V_0}{2eI} \qquad R_+ = 0 \tag{4.5.6}$$

From equation (4.5.5) and (4.5.6) the super conductivity resistance R_s vanishes i.e.:

$$R_+ = R_s = 0 \tag{4.5.7}$$

When

$$KT < V_0 - E_0$$

$$T < \frac{V_0 - E_0}{k} \tag{4.5.8}$$

Thus the critical temperature is given by

$$T_C < \frac{V_0 - E_0}{k} \tag{4.5.9}$$

Again for T_C to be positive $V_0 > E_0$

Thus for

$$T < T_C \tag{4.5.10}$$

$$R_{sc} = R_+ = 0 \tag{4.5.11}$$

In the case when external magnetic field of flux density B is applied on the super-conductivity, the total magnetic field B_m and potential V_m resulting from both external and internal to magnetic fields are given by:

$$B_m = B - B_i \tag{4.5.12}$$

$$V_m = V - V_i \tag{4.5.13}$$

Where B_i and V_i stands for the internal magnetic density and potential respectively. When the magnetic field attracts electrons, the Hamiltonian and the average energy in equations (4.5.1), (4.5.2) and (4.5.3) are given

$$\hat{H} = \frac{\hat{p}^2}{2m} + KT + V_m - V_0$$

$$\langle \hat{H} \rangle = E_0 + KT + V_m - V_0 \tag{4.5.14}$$

Thus according to equation (4.5.4) the quantum resistance is given by

$$R = \frac{KT + E_0 - V_0 + V_m}{2eI} = R_+ + R_- \tag{4.5.15}$$

In view of equation (4.5.9) and by denoting V_m to be

$$V_m = KT_m = m_l \left(\frac{e\hbar}{2m} \right) B_m = C_m^{-1} B_m \tag{4.5.16}$$

Equation (4.5.15) reads

$$R = \frac{K(T + T_m - T_c)}{2eI} \tag{4.5.17}$$

When

$$T < T_C \tag{4.5.18}$$

Thus the critical V_m and B are given by (4.5.18), (4.5.12) to be

$$B_c = B_{mc} + B_{ic} = C_m V_{mc} + B_{ic} = C_m KT_c + B_{ic} \tag{4.5.19}$$

In this case R is positive always, no matter what the value of T is therefore

$$R = R_+ + R_- = \frac{K(T + T_m - T_c)}{2eI} \tag{4.5.20}$$

Thus

$$R_+ \neq 0$$

Always, when condition (4.5.18) is satisfied, i.e. when the external field B exceeds the critical valve B_c given by (4.5.19).

4.5 Discussion

In view of plasma equation (4.2.1) a useful time dependent energy equation (4.2.3) and corresponding Schrodinger equations (4.2.6), (4.2.7) were found.

Using equation (4.2.11) and (4.2.12) a useful temperature quantum resistance expression (4.2.16) was found. By splitting resistance to real super conducting part and imaginary part [see (4.2.17)], equation (4.2.18) and (4.2.19) shows that R is imaginary, thus the SC real part vanishes when T is Less than a critical value given by equation (4.2.19), which requires very Large binding energy.

This suggests conducting by hopping.

When a magnetic field was induced by photon absorption this induces internal magnetic flux density given by (4.3.2).

This leads to energy splitting and existence of energy gap given by (4.3.4). This Leads to destroy of SC, since R, given by (4.3.5) become real when the magnetic potential exceeds certain critical value given by (4.3.8). This critical value corresponds to critical existence of energy gap given by (4.3.11).

Surprisingly this expression of energy gap is typical to that well known relation.

The effect of external magnetic field is discussed section (4.4). The total medium field is assumed to result from external one B and internal local field B_i [see (4.4.23)]. According to equation (4.4.28) the SC is destroyed when the external field exceeds this critical value. This result agrees with experimental observations.

Using quantum plasma Schrodinger equation, but treating charge carriers as strings a useful quantum resistance relation was found [see 4.5.4], where we split R into positive SC part and negative part. The Sc positive part is destroyed when R is negative [see 4.5.5, 6] The SC takes place when T is less than T_c [see (4.5.9.10.11)]. In this case R_s vanishes

One can discuss what happens when external magnetic field B is applied, by considering medium field to result from local internal and external field [see (4.5.12)]. Again the SC is destroyed when B exceeds a critical value given by (4.5.19), in agreement with experiments.

CHAPTER FIVE

Hyper Fine Rubidium

5.1 Introduction

Hyper fine interaction Leads to energy splitting due to spin interaction with magnetic field. This experiment was designed to examine the hyperfine structure of atomic states.

5.2 Theoretical Background

(Zeeman Splitting of Hyperfine Structure)

The relation between Zeeman energy splitting and magnetic fields, measure the Landed g-factor, also known as the gyro magnetic ratio. The g-factor relates the magnetic dipole moment to the angular momentum of a quantum state. With angular momentum in units of ħ, one has the equation relating magnetic dipole moment to angular momentum in the form

$$\mu_S = -g_s\mu_B S \tag{5.2.1}$$
$$\mu_L = -g_L\mu_B L \tag{5.2.2}$$
$$\mu_J = -g_J\mu_B J \tag{5.2.3}$$
$$\mu_I = -g_I\mu_N I \tag{5.2.4}$$
$$\mu_F = -g_F\mu_B F \tag{5.2.5}$$

Where μ_B and μ_N are the Bohr and nuclear magneton, respectively. Given the relations (5.2.1), (5.2.3) and $J{\sim} = L{\sim} + S{\sim}$, and using the properties of the dot product, one obtains

$$g_J J = g_L L\cos\theta_{JL} + g_s S\cos\theta_{JS} \tag{5.2.6}$$

With θ AB representing the angle between the vectors A and B. Similarly, one can use the equation for the total angular momentum of the electron J to get

$$L{\sim}2 = S{\sim}2 + J{\sim}2 - 2SJ\cos JS \tag{52.7}$$
$$S{\sim}2 = L{\sim}2 + J{\sim}2 - 2LJ\cos JL \tag{52.8}$$

Quantum mechanically, the expectation value for the angular momentum of A 2 is $A(A+1)$. In the end, solving equation (5.2.7) and equation (5.2.8) for the cosine of the angles and substituting the result into (5.2.6) yields

$$gJ = gs\,\frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$
$$+ gL\frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)} \tag{5.2.9}$$

For this experiment, we are studying the Zeeman splitting of the ground state. Here $L = 0$ and $J = S = \frac{1}{2}$.

Therefore, $g_J = g_S$. Similarly, one can use the exact same procedure for the total angular momentum of the atom to find that it has a g-factor

$$gf = gs\,\frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}$$
$$- gi\,\frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} \tag{5.2.10}$$

We are interested in the case where $F = 2 \ and \ J = 1/2$. Substituting these values and g$g_J = g_S$ into equation (5.2.10) for85Rb $(I = \frac{5}{2})$, we find

$$g_F = -\frac{1}{6}(g_S + 7g_t) \tag{5.2.11}$$

And for 87Rb $(I = \frac{3}{2})$, we find

$$g_F = -\frac{1}{4}(g_S - 3g_t) \tag{5.2.12}$$

The negative sign on equation (5.2.11) indicates that decreasing values of m$_F$ yield higher energy levels. Using the method of optical pumping, we will be determining the absolute value of g_F. Thus, we will only be quoting positive values of g_F.

Experimentally, the gyro magnetic ratio of the electron has been measured, yielding a value $g_S = 2.002319304$ [5]. In addition, the nuclear to electronic g-factor ratio, gI/gJ, for [85]Rb has been determined to be 1.46649093 equation

(5.2.11) $\times 10^{-4}$, independent of magnetic field [6]. Assuming that ^{87}Rb has a similarly small g-factor ratio, then equation (5.2.11) and equation (5.12) provide theoretical predictions (to 3 significant figures) of $g_F = 0.334$ and $g_F = 0.500$ for ^{85}Rb and ^{87} Rb, respectively. The number of significant figures was chosen to correspond with the precision to which this experiment was performed and to emphasize the fact that calculations are valid only to first order.

Finally, the Zeeman energy splitting of m_F sublevels in a weak magnetic field is (to first order) given by [1]

$$E = g_F \frac{c\hbar}{2m_e} B m_F = g_F \mu_B B m_F \tag{5.2.13}$$

5.3 Establishing Steady State Polarization

After the rubidium atoms have been subjected to a radio frequency $(r.f.)$ field of the right energy to induce a transition, the intensity of the D_1 radiation passing through the rubidium vapor will drop sharply. Once this $r.f.$ field is removed, the atoms will be pumped on again until a steady state polarization is established. There is a characteristic time associated with this phase. Following Benumb of [4], we can determine the change in population of the $mF = 3$ ground state of 87Rb relative to the other sublevels. This can be written as:

$$\frac{dn}{dt} = -nW_d + NW_u \tag{5.3.1}$$

Where n is the population of the $m_F = 3$ sublevel and N is the population of every other sublevel. The rate of transitioning down out of the pumped state is given by W_d, and W_u is the rate of transitioning into $m_F = 3$. The characteristic time to establish steady state polarization is found from equation (5.3.1).

Do we expect this time to depend on the intensity of the light Clearly, the relaxation processes causing the transition out of the pumped state, W_d, should not depend on the light intensity. W_d is a result of the atoms colliding with the glass walls. Next, we can break up W_u into two different steps. This rate depends

both on the rate of transition into the $2P\,1/2$ excited state, $\Gamma\uparrow$, and the rate of transition back down into the $m_F = 3$ ground state, $\Gamma\downarrow$. The longer rate, $\Gamma\uparrow$ or$\Gamma\downarrow$, will determine W_u. The intensity of the light is a measure of rate that optical photons are incident on the sample. Since these photons are responsible for excitation of the ground state, $\Gamma\uparrow$ will depend directly on the intensity. However, the transition back to the ground state is caused by the spontaneous emission of the optical photon. From the Fermi's Golden Rule [2], the spontaneous emission should depend on the frequency of the emitted photon and not the intensity of the light. We expect timescales of spontaneous emission to be much longer than those of photon absorption from the light. Therefore, the characteristic time should not depend on intensity.

5.4 Dynamic Response of Spins to Time Varying Magnetic Fields

We wish to first discuss passage through zero field (applied field canceling the Earth's Field) in the absence of any $r.f.$ field. In a static coordinate system, the nuclear spins obey the classical equation of motion [7]

$$\frac{d\vec{I}}{dt} = \gamma\left(\vec{I} \times \vec{B}\right) \tag{5.4.1}$$

Due to the variation of B~ with time, this is a difficult problem to solve. For instructive purposes, we examine limiting cases. The magnetic field vector can be separated into components perpendicular and parallel to the axis of the primary Helmholtz coils. The two cases [8] are adiabatic passage through zero field given by

$$\frac{I}{B_\perp}\frac{dB_\parallel}{dt} \ll \gamma B_\perp \tag{5.4.2}$$

And sudden passage given by

$$\frac{I}{B_\perp}\frac{dB_\parallel}{dt} \gg \gamma B_\perp \tag{5.4.3}$$

If the magnetic field is varying slowly, then equation (5.4.2) is satisfied, and I~ will stay nearly parallel with B~. As B~ is swept through zero, so is I~, and the

z −component of I~ will reverse sign. This will be as if the pumping light was reversed Light that was right-circularly polarized and resulted in raising m_F by one unit will now remove one unit of angular momentum. The pumping process will begin again, pumping atoms in the opposite way as before, and absorption of D_1 light will increase dramatically. When this happens, there will be a large optical signal.

On the other hand, it might be the case that the sweep rate of the magnetic field is too fast for the magnetization to follow. In this situation, equation (5.4.3) is satisfied. Now, magnetization remains always pointed in the same direction. So, when the field changes sign, it did so quickly enough to not affect the spins. Therefore, right-circularly polarized light will still increase mF by one unit. The net result is that pumping remains unchanged, so the optical signal is relatively steady.

When considering a resonance in the presence of a $r.f.$ magnetic field, it is useful to transform into a rotating coordinate system. Here, we wish to transform into a coordinate frame that rotates at the frequency of the $r.f.$ field. The rate of change for any vector in a fixed reference frame is given by

$$\left(\frac{d\vec{I}}{dt}\right)_{fixed} = \vec{\omega} \times \vec{I} + \left(\frac{d\vec{I}}{dt}\right)_{rotating} \tag{5.4.4}$$

Using this translation along with equation (5.4.1), it can be shown [8] that $B_\perp$ become replaced by $\text{B}r.f.$. If we define the quantity

$$X \equiv \frac{\mu B}{\Upsilon} \frac{1}{B} \frac{dB}{dt} \tag{5.19}$$

Then the adiabatic and sudden conditions can be written as $X \ll 1 \ and \ X \gg 1$, respectively. Although the argument above was based on classical mechanics, the result holds quantum mechanically.

5.5 Experimental Setup

The layout for this experiment is shown in Figure (5.1). Two sets of Helmholtz coils were used to provide the weak external magnetic field. The primary coils (shown in the figure) were aligned so that they would produce a magnetic field either parallel or anti-parallel to the Earth's magnetic field. Secondary coils were placed at right angles to these fields in an effort to minimize the transverse field and it's in homogeneities. Current through the primary coils was controlled with a trapezoidal sweep by using a function generator and control circuit.

Production of circularly polarized waves was achieved by using a linear polarizer and a quarter-wave plate. The two were aligned to obtain maximum circular polarization of optical light from a lamp.

The D_1 radiation was removed from the light for the previously mentioned reasons. A converging lens focused the light going through a resonance bulb full of rubidium vapor. Due to the low melting point of rubidium$(38.5 C^{\circ})$, the resonance bulb was heated by blowing hot air onto it to achieve optimum vapor pressures.

A signal generator supplied the radio frequency field necessary to induce transitions out of the pumped state. The magnitude of the $r.f.$ field was determined by measuring the emf induced in a small pickup coil. For each frequency, the magnetic field from the primary Helmholtz coils was swept through a range of values. When the magnetic field passed through resonance for a given isotope satisfying equation (5.2.12), absorption of the D_1 radiation increased sharply. Since the sweep of the primary coils was trapezoidal, there were two peaks for each isotope during one period of the sweep. Figure (5.2) shows both the sweep and the resonant peaks for each isotope at a fixed frequency. For all measurements, the oscilloscope was run in Average 256 mode to reduce the amount of noise in the signal.

In order to determine the effect of light intensity on the characteristic time to establish steady state polarization, it was necessary to vary the amount of incident light. This was achieved by placing a second linear polarizer between the lamp and the first linear polarizer. By varying the angle between the two polarizer's, it was possible to control the intensity of light shining on the sample.
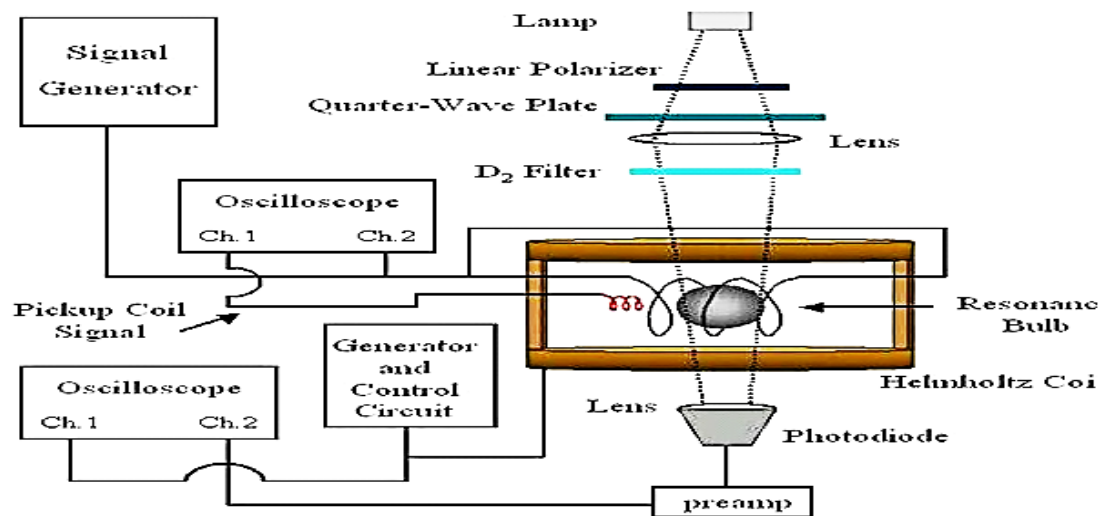
Figure (5.1) Diagram of optical pumping setup. Only the primary Helmholtz coils are shown in this diagram. The secondary coils used to trim the transverse fields to zero are not shown

5.6 Results
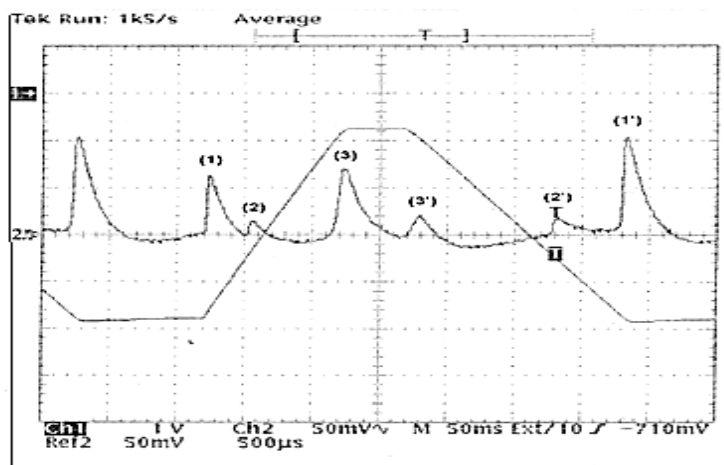
Figure (5.2) Oscilloscope trace of resonant peaks at a frequency of 1.005 MHz Ch. 1- Trapezoidal sweep for the primary Helmholtz coils. CH

2 - Signal from the photodiode. Peaks (1) and (1') correspond to ^{85}Rb, and peaks (2) and (2) correspond to ^{87}Rb. Note the appearance of an extra pair of peaks (3) and (3) corresponding to zero applied field

Table (5.1) G-factor measurements for two rubidium isotopes. Also included is the magnitude of the Earth's magnetic field determined using each isotope

R biotopes	g_F	B $_{Earth}$(G)a
85	0.335 ± 0.008	0.488 ± 0.046
87	0.505 ± 0.006	0.501 ± 0.050

Table (5.2) Characteristic time to establish steady state polarization at 0.900 *MHz*.

Δθ (deg)	I/I$_o$	τ(ms)
0	1.00	12.6 ± 1.0
25	0.82	12.3 ± 1.1
43	0.53	12.9 ± 1.4
60	0.25	13.4 ± 2.8

Table (5.3) Characteristic time and its variation per degree separation between linear polarizer's for three frequencies

frequency (MHz)	τ (ms)a	(ms/deg)
1.005	11.5 ± 0.4	-0.002 ± 0.026
0.900	12.6 ± 0.6	0.016 ± 0.034
0.800	12.8 ± 0.6	0.018 ± 0.033

TABLE (5.4) Measurements of the peak voltage for different r.f. field strengths. Included is the defined variable X which provides a measure of sudden passage ($X \gg 1$) and adiabatic passage ($X \ll 1$)

Peak (mV)	a B $r.f.\times\ 10^{-7}(T)$	x^B
12	0.12	216
25	0.26	54
42	0.37	27.5
59	0.48	16.7
76	0.7	7.98
91	0.97	4.16
93	1.19	2.79
101	1.47	1.85
101	1.69	1.4
105	1.96	1.04
107	2.46	0.67
108	3.01	0.45
108	3.45	0.34
108	3.84	0.28
110	4.39	0.21
111	4.94	0.17
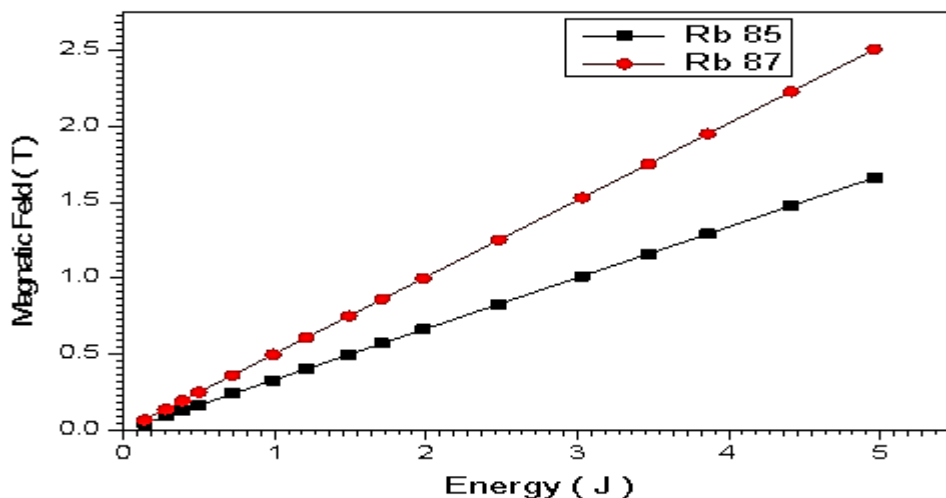
Figure (5.3) Finding the gF values for 85Rb and 87Rb.

The slope of these lines represent the (gF μB) $^{-1}$. The y-intercept of each line (not shown) was used to infer the magnitude of the Earth's magnetic field. Resonance measurements for twenty different frequencies were used.
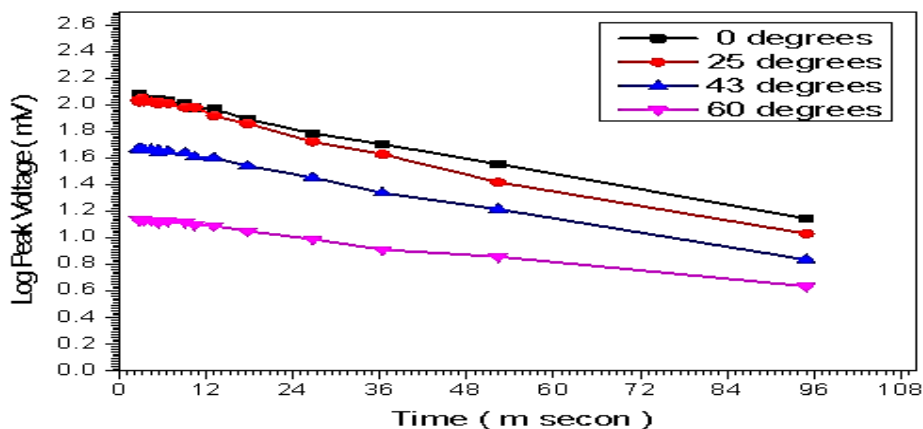
Figure (5.4) Dependence of $_T$ on light intensity at 0.900MHz. The different lines indicate different intensities

It was assumed that the decay of the voltage reading from the photodiode was exponential. Therefore, the slope of each line is τ^{-1}. The fact that these lines are roughly parallel suggests that τ is insensitive to changes in intensity.
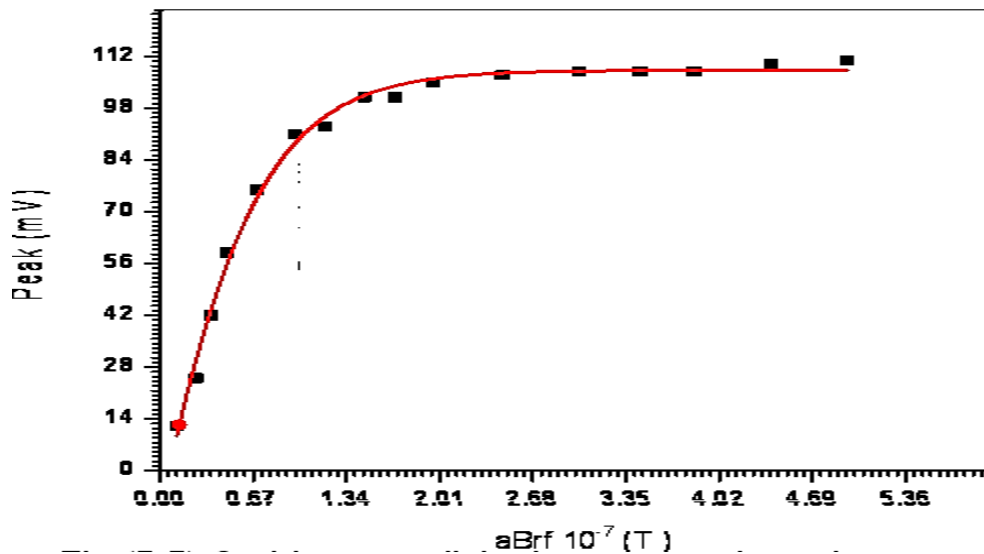
Figure (5.5) Sudden vs. adiabatic passage through resonance

Above a certain $r.f.$ field strength, the peak voltages were roughly equal (adiabatic passage). However, decreasing the $r.f.$ field eventually led to attenuation of the peak voltages (sudden passage).

5.7 Discussion

The magnetic field due to the primary Helmholtz coils was determined from the geometry of the coils and the amount of current flowing through them. These coils were designed to provide as uniform of a field as possible. To that end, the separation between the coils was measured to be the same (up to uncertainty) as the radius of each coil. Therefore, the magnetic field (in Tesla) in between and on the axis the coils is given by [10].

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 NI}{r} \tag{5.7.1}$$

Where N the number of turns in the coil and r is is the radius of the coils. The energy level splitting was determined by measuring the frequency of the $r.f.$ field and using $E = h\nu$. The relationship between the Zeeman energy splitting and applied magnetic field for both isotopes of rubidium is shown in Figure (5.3). The two lines were found using a least-squares weighted fit [11]. The magnetic field B was graphed vs. the energy E, instead of the reverse, due to the

larger error in measurements of B. The slope of each line determined the value of $(g_F\mu_B)^{-1}$, from which the g-factor could be obtained. In addition, the y-intercept of each line is a measure of the Earth's magnetic field. According to equation (13), one would expect the zero of energy splitting to occur in the presence of zero applied magnetic fields. The fact that the y-intercept is not zero indicates that there is an additional magnetic field other than the one produced by the Helmholtz coils. This extra field is attributed solely to field of the Earth. From the intercepts of both lines, we obtain a weighted average for the Earth's magnetic field of 0.496 ± 0.029 G. Table (5.I) summarizes the results. The characteristic time, τ, to establish steady state polarization was measured for different frequencies and intensities. Assuming an exponential decay of photodiode signal, the logarithm of the voltage was graphed versus time so that the points would fit a straight line with slope τ^{-1}. Again, each line was created using a least-squares weighted fit. Figure 5 displays the results at a fixed frequency of $0.900\ MHz$. Notice that each line has approximately the same slope. Table (5.2) provides the characteristic time for different polarizations. These characteristic times are all in good agreement, indicating that τ does not depend upon the light intensity, as hypothesized. The measured values of τ and their uncertainties for each intensity and for three different frequencies are detailed in Table (5.3). Note that each of the variations of τ over the range of intensities agree with zero, suggesting that there is no dependence of τ on light intensity. In addition to the peaks corresponding to the two rubidium isotopes, it was observed that there was another apparent resonance when the field was swept through zero (applied field canceling the earth's field) in the absence of an $r.f.$ field. See the *Theory* section for explanation. Figure (5.4) shows the dependence of the peak voltage on the magnitude of the $r.f.$.

5.8 Conclusion

The hyperfine structure of Rubidium shows the g factors for ^{85}Rb and ^{87}Rb to be 0.335 $\pm$ 0.008 and 0.505 $\pm$ 0.006, respectively, in agreement with theoretical predictions of 0.334 and 0.500. The Earth's magnetic field is found to be 0.496 $\pm$ 0.029G.

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