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Solution of A fractional Differential Equation Based on Hilfer's Derivative Operator

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ABSTRACT

In this paper, the solution of the fractional differential equation $x(D_{0^+}^{u,v}y)(x) = {}^{\mathfrak{a}}(\pounds_{\sim,\ ,pw,0^+}^{¥,¢,q})(x)$ with the initial condition $(I_{0^+}^{(1-v)(1-u)}y)(0_+) = c_1$ was investigated, based on the Hilfer's fractional derivative.

المستخلص

في هذة الورقة تمت مناقشة حل المعادلة التفاضلية الكسرية $x(D_{0^+}^{u,v}y)(x) = {}^{\mathfrak{a}}(\pounds_{\sim,\ ,pw,0^+}^{¥,¢,q})(x)$ طبقا للشروط الابتدائية $(I_{0^+}^{(1-v)(1-u)}y)(0_+) = c_1$ المعتمدة علي المشتقة الكسرية لهيلفر.

KEYWORDS: Fractional differential equation, Hilfer derivative operator, Mittag-Leffler function, Fractional calculus, Laplace convolution theorem.

INTRODUCTION

Mittag-Leffler function has been studied in the early 1900s[1-3], its importance is realized during the last two decades due to its involvement in problems of physics, chemistry and applied science. Further properties of generalization of Mittag-Leffler function associated with fractional calculus operators have been studied [4-6].

Kilbas made relevant references to analytical solution of initial and boundary value problems associated with fractional differential equations[7], one of them is Cauchy type problem

$$(D_{a^+}^{\mathfrak{a}}y)(x) = {}^{\mathfrak{a}}(\pounds_{\sim,\ .w,a^+}^{\mathfrak{a},1})(x) + f(x) \qquad (1)$$

with the initial condition $(D_{a^+}^{\mathfrak{a}-k}y)(0_+) = b_k$.

The homogeneous differential equation corresponding to (1) when $f(x) = 0$ is a generalization of a certain first-order Volterra-type integral differential equation govering the unsaturated behavior of free electron laser [3,8].

Hence by using Laplace transform method [9], an explicit solution on $L(0,\infty)$ of a more complicated fractional differential equation than (1) can be given which contains the generalized Riemann-Liouville fractional derivative operator, [10-13]

$$D^{«}f(x) = D^{m}\left[D^{-v}f(x)\right] \quad «,t>0, \quad v = m-« > 0$$

The following definitions were used in this study,

Definition (1):-

(**i**)- Hilfer's fractional derivative [3] is defined as:

$$(D_{a^+}^{u,v}\,\cdot\cdot)(x)=\left[I_{a^+}^{v(1-u)}\frac{d}{dx}\left(I_{a^+}^{(1-u)(1-v)}\,\cdot\cdot\right)\right](x)$$

and its Laplace transformation [9] is given by

$$L(D_{0^+}^{u,v}\,\cdot\cdot)(s)=s^{u}L[\cdot\cdot(x)](s)-s^{v(1-u)}\left(I_{0^+}^{(1-v)(1-u)}\,\cdot\cdot(x)\right)(0_+)$$

(**ii**)- An integral operator [13,14] is defined as

$$(\pounds_{\sim,\ ,w,a^+}^{¥,¢,q}\,\cdot\cdot)(x)=\int_0^x(x-t)^{\ -1}E_{\sim,\ ,p}^{¥,¢,q}\left[w(x-t)^{\sim}\right]\cdot\cdot(t)dt$$

(**iii**)- First order differential equation [15- 17]
$y'+p(x)y=Q(x)$ has the solution

$$y(x)=e^{-\int p(x)dx}\left[c_2+\int Q(x)e^{\int p(x)dx}dx\right]$$

(**iv**)- Laplace transformation is given by :[9]

$$L\left\{z^{a-1}E_{\sim,\ ,p}^{¥,¢,q}(xz^{\pm})\ ;s\right\}=\frac{\sqrt{¢}s^{-a}}{\sqrt{¥}}\,{}_3\cdot{}_2\left[\begin{matrix}(¥,q),(a\pm),(1,1)\\(\ ,\sim),(¢,\pm)\end{matrix}\ ;\frac{x}{s^{\pm}}\right]$$

(v)- Also another Laplace transformation see[7,18-20] is given by

$$L\left\{\frac{1}{\sqrt{u}}\int_0^x t^{\mu-1}(x-t)^{\ -1}E_{\sim,\ ,p}^{¥,¢,q}\left[w(x-t)^{\sim}\right]dt;s\right\}=\frac{1}{s^{\mu}}\sum_{n=0}^{\infty}\frac{(¥)_{qn}w^{n}}{\sqrt{(¢)_{pn}(\sim n+\ )}}\frac{1}{s^{\sim n+}}$$

(vi)- The two –parameter function of Mittag-Leffler [1-3] is defined by

$$E_{\sim,\ }(z)=\sum_{k=0}^{\infty}\frac{z^{k}}{\sqrt{\sim k+}}\qquad \sim>0\qquad >0$$

(vii)- Generalized Mittag-leffler function [1-3] defined as

$$E_{\sim,\ ,p}^{¥,¢,q}(z)=\sum_{n=0}^{\infty}\frac{(¥)_{qn}z^{n}}{\sqrt{(\sim n+\ )}(¢)_{pn}}$$

We need the theorem proved by [18]

Theorem (2):-
In the space $L(0,\infty)$ if the fractional differential equation is considered

$(D_{0^+}^{u,v}y)(x)=\text{a}(\pounds_{\sim,\ ,pw,0^+}^{¥,¢,q})(x)+f(x)$

$0<u<1\ ,\ 0\le v\le 1\ ,\ ¥,\sim,\ \ ,w\in C,$

with $\min\{\mathrm{Re}(\sim),\mathrm{Re}(\ ),\mathrm{Re}(¥),\mathrm{Re}(¢)\}>0$

and $p,q>0$, and with initial condition

$(I_{0^+}^{(1-v)(1-u)}\,\cdot\cdot)(0_+)=c$, then we have the solution

$$y(x)=c\frac{x^{u-v(1-u)-1}}{\sqrt{u-v+uv}}+\text{a}x^{\ +u}E_{\sim,\ \ +u+1,p}^{¥,¢,q}(wx^{\sim})$$

$$+\frac{1}{\sqrt{u}}\int_0^x(x-t)^{u-1}\cdot\cdot(t)dt \qquad (3)$$

Proof:-
Using Definition (1) (i) and (ii), also making use of (iv) we can get

$$L(D_{0^+}^{u,v}(y))=L\left[\text{a}(\pounds_{\sim,\ \ ,p,w,0^+}^{¥,\ \ ,q})\right](x)+L(f(x)).$$

Hence

$$s^{u}L(y)-s^{v(1-u)}\left(I_{0^+}^{(1-v)(1-u)}(y)\right)(0_+)=$$

$$\text{a}L\left[\int_0^x(x-t)^{\ -1}E_{\sim,\ ,p}^{¥,¢,q}(x-t)^{\sim}dt\right]+L(f(x))$$

According to Laplace convolution theorem and by applying the initial condition we have

$$s^{u}Y(s)-cs^{v(1-u)}=\text{a}L\left[x^{\ -1}E_{\sim,\ ,p}^{¥,¢,q}(wx^{\sim}\right](s)+L(1)(s)+F(s)$$

$$=\frac{\text{a}s^{-}\sqrt{¢}}{\sqrt{¥}}\,{}_3\cdot{}_2\left[\begin{matrix}(¥,q),(\ ,\sim),(1,1)\\(\ ,\sim),(¢,p)\end{matrix}\ ;\frac{w}{s^{\sim}}\right]\frac{1}{s}+F(s)$$

$$=\frac{\text{a}s^{-\ -1}\sqrt{¢}}{\sqrt{¥}}\,{}_2\cdot{}_1\left[\begin{matrix}(¥,q),(1,1)\\(¢,p)\end{matrix}\ ;\frac{w}{s^{\sim}}\right]+F(s).$$

Divided the above equation by s^{u} we get

$$Y(s)=cs^{\gamma(1-u)-u}+\frac{a\Gamma(\phi)}{\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{\Gamma(\gamma+qn)\Gamma(1+n)}{n!\Gamma(\phi+pn)}\left(\frac{w}{s^{\sim}}\right)+s^{-u}F(s).$$

Taking Laplace inverse of both sides of the last equation, we get

$$y(x)=cL^{-1}\left[s^{\gamma(1-u)-u}\right](x)+\frac{a\Gamma(\phi)}{\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{\Gamma(\gamma+qn)\Gamma(1+n)}{n!\Gamma(\phi+pn)}w^{n}L^{-1}\left[s^{-n-\mu-u-1}\right]$$

$$+L^{-1}L\left(\frac{x^{\mu-1}}{\Gamma(u)}f(x)\right),$$

where

$$s^{-u}F(s)=L\left(\frac{x^{u-1}}{\Gamma(u)}f(x)\right).$$

Again applying Laplace convolution theorem, we get

$$y(x)=c\frac{x^{u-v(1-u)}}{\Gamma(u-v+uv)}+a x^{\mu+u}E^{\gamma,\phi,q}_{\sim,\mu+u+1,p}(wx^{\sim})$$

$$+\frac{1}{\Gamma(u)}\int_0^x(x-t)^{u-1}f(t)dt,$$

which gives the solution.

Remark (3):

If we put $f(t)=t^{\mu}E^{\gamma,\phi,q}_{\sim,\mu+1,p}(wt^{\sim})$ in (2) we get the following particular case of (3) which is stated in the next lemma.

Lemma (4):

Under the same conditions of theorem (2), the following fractional differential equation

$$(D^{u,v}_{0^+}y)(x)=a(\pounds^{\gamma,\phi,q}_{\sim,\mu,pw,0^+})(x)+x^{\mu}E^{\gamma,\phi,q}_{\sim,\mu+1,p}(wx^{\sim}),$$

with the initial condition $(I^{(1-v)(1-u)}_{0^+}\cdots)=c$ has the solution

$$y(x)=c\frac{x^{u-v(1-u)}}{\Gamma(u-v(1-u))}+(a+1)E^{\gamma,\phi,q}_{\sim,\mu+u+1,p}(wx^{\sim}).$$

Proof:-

Putting $f(x)=x^{\mu}E^{\gamma,\phi,q}_{\sim,\mu+u+1,p}(wx^{\sim})$ in (3) we get

$$y(x)=c\frac{x^{u-v(1-u)-1}}{\Gamma(u-v+uv)}+a x^{\mu+u}E^{\gamma,\phi,q}_{\sim,\mu+u+1,p}(wx^{\sim})$$

$$+\frac{1}{\Gamma(u)}\int_0^x(x-t)^{u-1}t^{\mu}E^{\gamma,\phi,q}_{\sim,\mu+1,p}(wt^{\sim})dt.$$

But

$$\frac{1}{\Gamma(u)}\int_0^x(x-t)^{u-1}t^{\mu}E^{\gamma,\phi,q}_{\sim,\mu+1,p}(wt^{\sim})dt=$$

$$L^{-1}L\left[\frac{1}{\Gamma(u)}\int_0^x(x-t)^{u-1}t^{\mu}E^{\gamma,\phi,q}_{\sim,\mu+1,p}(wt^{\sim})dt\right]$$

$$=L^{-1}\left\{L\left(\frac{x^{u-1}}{\Gamma(u)}\right)(s)L(x^{\mu+1-1}E^{\gamma,\phi,q}_{\sim,\mu+1,p}(wx^{\sim}))(s)\right\}.$$

From Definition (1) (iv) we get

$$\frac{1}{\Gamma(u)}\int_0^x(x-t)^{u-1}t^{\mu}E^{\gamma,\phi,q}_{\sim,\mu+1,p}(wt^{\sim})dt$$

$$=L^{-1}\left\{\frac{1}{s^u}\frac{\Gamma(\phi)}{\Gamma(\gamma)}s^{-\mu-1}\cdot{}_2\left[\begin{matrix}(\gamma,q),(\mu+1,\sim),(1,1)\\(\mu+1),(\phi,p)\end{matrix};\frac{w}{s^{\sim}}\right]\right\}(s)$$

$$=L^{-1}\left\{\frac{\Gamma(\phi)}{\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{\Gamma(\gamma+qn)\Gamma(1+n)}{n!\Gamma(\phi+pn)}\left(\frac{w}{s^{\sim}}\right)^{n}s^{(4)-1}\right\}(x)(4)$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{(\phi)_{pn}}w^{n}L^{-1}\left\{s^{-\sim n-\mu-u-1}\right\}(x)$$

$$= \sum_{n=0}^{\infty} \frac{(¥)_{qn}}{(¢)_{pn}} w^n \frac{s^{(\sim n+ \ +u+1)}}{\Gamma(\sim n+ \ +u+1)}$$

$$= x^{\ +u} E^{¥,¢,q}_{\sim,\ +u+1,p}(wx^{\sim}),$$

by applying Laplace's convolution theorem. Hence the solution is given by

$$y(x) = c \frac{x^{u-v(1-u)-1}}{\Gamma(u-v(1-u))} +$$

$$(a+1) E^{¥,¢,q}_{\sim,\ +u+1,p}(wx^{\sim}). \quad (5)$$

RESULTS

Now we give our main results

Theorem (5):

Under the same conditions of Theorem (2), the following fractional differential equation

$$x(D^{u,v}_{0^+} y)(x) =$$

$$a(£^{¥,¢,q}_{\sim,\ ,pw,0^+})(x) \quad (6)$$

with the initial condition $(I^{(1-v)(1-u)}_{0^+} y)(0_+) = c_1$ has the following solution

$$y(x) = c_2 \frac{x^{u-1}}{\Gamma(u)} + c_1 \frac{x^{u-v(1-u)-1}}{\Gamma(u-v(1-u))} +$$

$$\frac{a}{\Gamma(u)} \int_0^x t^{u-1}(x-t)^{\ -1} E^{¥,¢,q}_{\sim,\ +1,p} w(w-t)^{\sim} dt \quad (7)$$

where c_1, c_2 are constants.

Proof:

We will use the same procedures as in Theorem (2) in addition, we can use

$$\frac{\partial^n}{\partial s^n}[Ly(x)](s) = (-1)^n L[x^n y(x)](s) \quad (8)$$

Now take the Laplace transform for both sides of eqn. (6)

$$L[x(D^{u,v}_{0^+} y)](s) = aL[£^{¥,¢,q}_{\sim,\ ,pw,0^+}](s).$$

Now using (8) with $n=1$ we get

$$\frac{\partial}{\partial s}[L(D^{u,v}_{0^+} y)](s) = (-1)L[xD^{u,v}_{0^+} y](s)$$

From definition (1) (i) and theorem (2), we have

$$\frac{\partial}{\partial s}\left[s^u Y(s) - c_1 s^{v(1-u)}\right] = -a \frac{s^{\ -\ -1}\Gamma(¢)}{\Gamma(¥)} {}_3\cdot{}_2 \left[\begin{matrix}(¥,q),(\ \ ,\sim),(1,1)\\(¢,p),(\ \ ,\sim)\end{matrix}; \frac{w}{s^{\sim}}\right]$$

$$s^u Y'(s) - us^{u-1}Y(s) - c_1 v(1-u)s^{v(1-u)-1} +$$

$$a \frac{s^{\ -\ -1}\Gamma(¢)}{\Gamma(¥)} {}_2\cdot{}_1 \left[\begin{matrix}(¥,q),(1,1)\\(¢,p)\end{matrix}; \frac{w}{s^{\sim}}\right] = 0$$

Dividing the above equation by s^u, yields

$$Y'(s) - \frac{u}{s}Y(s) - c_1 v(1-u)s^{v(1-u)-u-1} +$$

$$a \frac{s^{\ -\ -u-1}\Gamma(¢)}{\Gamma(¥)} {}_2\cdot{}_1 \left[\begin{matrix}(¥,q),(1,1)\\(¢,p)\end{matrix}; \frac{w}{s^{\sim}}\right] = 0 \quad (9)$$

Which is a first order ordinary differential equation then, the solution of (9) is

$$Y(s) = \exp\left(-\int \frac{u}{s} ds\right) \times$$

$$\left[c_2 + \left(\int \begin{matrix} c_1 v(1-u)s^{v(1-u)-u-1} - \\ a \frac{s^{\ -\ -u-1}\Gamma(¢)}{\Gamma(¥)} {}_2\cdot{}_1 \left[\begin{matrix}(¥,q),(1,1)\\(¢,p)\end{matrix}; \frac{w}{s^{\sim}}\right] \end{matrix}\right) ds.\right.$$

$$\left.\exp\left(\int \frac{u}{s} ds\right)\right]$$

So that

$$Y(s) = s^{-u}\left[c_2 + \left(c_1 \int v(1-u)s^{v(1-u)-u-1} - a \frac{s^{\ -\ -u-1}\Gamma(¢)}{\Gamma(¥)} {}_2\cdot{}_1 \left[\begin{matrix}(¥,q),(1,1)\\(¢,p)\end{matrix}; \frac{w}{s^{\sim}}\right]\right) s^u ds\right],$$

$$= s^{-u}\left[c_2 + \left(c_1\int v(1-u)s^{v(1-u)-1} - \text{a}\frac{s^{-\;-1})¢}{)¥}\,{}_2\cdot{}_1\left[\begin{matrix}(¥,q),(1,1)\\(¢,p)\end{matrix};\frac{w}{s^{\sim}}\right]\right)ds\right],$$

$$= s^{=u}\left[\begin{matrix} c_2 + c_1\dfrac{v(1-u)s^{v(1-u)}}{v(1-u)} - \text{a}\dfrac{s^{-\;-1})¢}{)¥} \\ \displaystyle\int\sum_{n=0}^{\infty}\frac{)(¥+qn))(1+n)}{n!)(¢+pn)}\left(\frac{w}{s^{\sim}}\right)^n ds \end{matrix}\right]$$

$$= s^{-u}\left[c_2 + c_1 s^{v(1-u)} - \text{a}\sum_{n=0}^{\infty}\frac{(¥)_{qn}}{(¢)_{pn}}w^n\int s^{-\sim n-\;-1}ds\right]$$

$$= s^{-u}\left[c_2 + c_1 s^{v(1-u)} - \text{a}\sum_{n=0}^{\infty}\frac{(¥)_{qn}w^n}{(¢)_{pn}}\frac{s^{-\sim n-}}{-(\sim n+\;)}\right]$$

$$= c_2 s^{-u} + c_1 s^{v(1-u)-u} + \frac{\text{a}}{s^u}\sum_{n=0}^{\infty}\frac{(¥)_{qn}w^n}{(¢)_{pn}(\sim n+\;)}\frac{1}{s^{\sim n+}}$$

Taking the Laplace inverse of the last equation, gives

$$y(x) = c_2\frac{x^{u-1}}{)u} + c_1\frac{x^{u-v(1-u)-1}}{)(u-v(1-u))} +$$

$$L^{-1}\left[\frac{\text{a}}{s^u}\sum_{n=0}^{\infty}\frac{(¥)_{qn}w^n}{(¢)_{pn}(\sim n+\;)s^{\sim n+}}\right]$$

Now using Definition (1) (v), we can get the required solution

$$y(x) = c_2\frac{x^{u-1}}{)u} + c_1\frac{x^{u-v(1-u)}-1}{)(u-v(1-u))} +$$

$$\frac{\text{a}}{)u}\int_0^x t^{u-1}(x-t)^{\;-1}E_{\sim,\;\;+1,p}^{¥,¢,q}\,w(x-t)^{\sim}\,dt$$

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Evaluation of Normal Anatomy of the Mandible Using Three Dimensions Computed Tomography and Orthopantomography

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ABSTRACT
The three-dimensional (3D) multi planar reformatted images from conventional cross-sectional computed tomography data have been increasingly used to better demonstrate the anatomy and pathologic conditions of various organ systems. This study was conducted to assess the 3D CT and OPG techniques in demonstrating the anatomy of the mandible including body, ramus, symphysis menti and TMJ, lamina Dura, pulp canal, root of the teeth, mandibular canal, mandibular angle. Samples of twenty Sudanese adult male patients of ages between (20-60years) who were clinically diagnosed as normal, without any fractures, infection, caries, periodontal disease, per apical Pathology, or affected by any bone diseases, were included in this study. 3D facial bone was done for ten of the sample and OPG was done for the other ten. This study was done in Modern Medical Center and Mursi Medical Dental Center in the period from May to August 2010. The machines used were general electric CT/with dual slice (Germany) with: Voltage: 120KV, 140KV, Current four options 60 mA, 80 mA, 100 mA and 160 mA. The orthopantomograph, model OP 5 (Siemens Bens-him, Germany), with the setting were selected typically in the range 70-100 KV and 4-12 mA. The results of this study showed that the OPG technique was the best in demonstrating the root of the teeth (45%), TMJ(10%), Ramus (15%), Lamina Dura (5%), where the 3D CT was the best in demonstrating mandibular angle, and body of the mandible, ramus, smphysis menti for (95%), respectively, and TMJ for (70%). Both techniques failed to demonstrate pulp canal, and mandibular canal, (0%).

المستخلص

تعتبر الصور ثلاثية الابعاد التي اعيد تركيبها من بيانات المقاطع العرضية العادية بالتصوير الطبقي واسعة الانتشار لتوضيح التشريح و الحالات المرضية المختلفة لمختلف اجهزة الجسم. أجريت هده الدراسة بغرض تقويم تقنية الاشعة المقطعية ثلاثية الابعاد و البانوراما في توضيح الأجزاء التشريحية الطبيعية للفك الأسفل متضمناً الصفحيه الحافيه المعتمه للأشعه، قناة اللب، الحاجز الأنفي، قناة الفك الأسفل، زاوية الفك الأسفل، المفصل الصدغي الفكي، جسم الفك الأسفل، امتداد الفك الأسفل، الإرتفاق الذقني. تم إختيار عينة مكونة من عشرين ذكر سوداني بالغ في مدي العمر بين 20–60 عام تم تشخيصهم على أن الفك الأسفل طبيعي دون كسور او التهابات او امراض باللثة او امراض بالاسنان او عظم الفك لأغراض هذا البحث. اجري لعشره مرضى من العينه فحص بواسطة الأشعه المقطعيه ثلاثية الأبعاد والعشره الآخرون أجريت لهم فحوصات عن طريق البانوراما.أجريت الدراسه في المركز الطبي الحديث ومركز مرسي الطبي للأسنان في الفتره ما بين مايو الي أغسطس

2010باستخدام مقطعية جنرال الكتريك متعدد المقاطع باستخدام 120–140 كيلو فولت و تيار 160،100،80،60 ملي امبير و جهاز اشعة مقطعية فكية كلي باستخدام 70–100 كيلو فولت و 4–12 ملي امبير .
أوضحت نتائج الدراسه أن تقنية البانوراما هي التقنيه الأفضل لتوضيح جذور الأسنان (45%) و المفصل الصدغي الفكي (10%) و امتداد الفك (15%) و الصفحيه الحافيه المعتمه للأشعه (5%) بينما اوضحت النتائج بان تقنية الأشعه المقطعيه ثلاثية الأبعاد هي الأفضل في توضيح زاوية وجسم وامتداد الفك الأسفل، الإرتفاق الذقني بنسبة (95%) بالتتابع والمفصل الصدغي الفكي بنسبة (70%)، فشلت التقنيتان في توضيح قناة اللب وقناة الفك الأسفل (0%).

KEYWORDS: *OPG, mandible, Three Dimensions CT.*

INTRODUCTION

It has been shown that cross-sectional imaging (e.g. conventional spiral tomography or spiral CT) offers a better alternative for the precise visualization of anatomical structures in the oral region [1- 7].

Nowadays, panoramic radiography is often used in dental practice, because it provides visibility of anatomical structures in pathological changes of the teeth, jaws and temporo-mandibular joints. However, a panoramic radiograph is a two-dimensional (2D) image, lacking information in the bucco-lingual direction and magnifying in both vertical and horizontal directions.

The mandible has a curved body, the halves of which join in the midline at the symphysis menti, and two posterior ascending rami, each with a superior coronoid process (anteriorly) and condylar process (posteriorly, ending in the condyle of the temporo mandibular joint (TMJ)). The body and ramus meet at the angle. The inferior alveolar canal extends from the mandibular foramen on the medial aspect of the ramus to the mental foramen on the buccal surface of the body, opposite the second premolar. The tooth-bearing body of the mandible, and the maxilla, consist of alveolar bone, which supports the dentition, and basal bone. Bone has sparse trabeculae and together these factors can result in artefactual lucency in the distal body, which should not be mistaken for a lytic lesion[1].

Dentine, which makes up the bulk of the tooth, is covered by an enamel cap on the crown and by a layer of cementum on the root. The enamel is very dense and is readily distinguishable from dentine whereas cementum has similar density to dentine and is not radiographically visible separate from the dentine. The radiolucent pulp chamber in the crown is continuous with the root canal. A thin radiolucent periodontal ligament space separates the root from the dense bone making up the lamina dura [1].

Nevertheless, the fact that panoramic imaging is widely used for evaluation of the jaws, justifies the interest in determining the visibility of anatomical structures on these films. Number of radiologic technique had been applied to demonstrate different anatomical structures, the, Oblique lateral demonstrates large areas of mandible, is the projection that produces an image of both

jaws and their respective dentitions on a single extra oral film [8].

Computed tomography provides sectional images of facial bone, orbits, mandible and TMJ in axial, sagittal and coronal planes. The information obtained from the original axial scan can be manipulated by the computer to reconstruct the three dimensions CT (3D) [9].

MATERIAL and METHODS

Patients:

Twenty Sudanese adult male patients with ages between (20-60years) were selected in this study, ten patients referred to the dental clinic for OPG examination, to detect the impacted lower wisdom teeth. The radiographs that were clinically diagnosed as normal mandible; without any fractures, infection, caries, periodontal disease, per apical pathology, or affected by any bone diseases were selected.

Another ten patients were selected from CT department examined for 3D facial bone diagnosed as normal patients.

Equipments:

The machines used were (general electric GE CT/with dual slice - Germany) with the following features, Voltage: two options 120KV, 140KV. Current: four options 60mA, 80mA, 100 mA and160mA.

The machine of Pan Tomography consists of tube, cassette-carriage and patient support. The generator is a single-pulse generator provides a sufficient output. The x.ray tube was of small effective focal area. Panoramic radiographs were obtained with an orthopantomograph model OP 5 (Siemens. Bens-him, Germany) with setting selected, typically in the range 70-100 KV and 4-12 mA.

CT technique The patients were positioned on the CT examination table, lying flat on their back and head rested on the head holder. Straps and pillow were used to help in maintaining the correct position [10]. The 3D images were obtained from the CT axial projection using analytical software When performing the CT examination, a large amount of data is produced, the volume data set is transmitted to the computer where all CT examinations are analyzed using three-dimensional rendering software, the three-dimensional data set analysis uses several types of reconstruction algorithms and is considered part of the CT examination. Rapid scrolling of the axial image is carried out as the first step of the reviewing process; different planes were selected of examination and apply the selected reconstruction algorithm.

OPG Technique

Panoramic equipment is based upon simultaneous rotation movement of the tube head and film cassette carriage in equal but opposite directions around the patient head which remains stationary. The patients were in erect position and were placed accurately within the machines using the various head positioning devices and light beam marker positioning guides. Patients were instructed not to move throughout the exposure [11].

RESULTS

Table (1) lists the results which were obtained from evaluating the radiographs by two radiologists. The evaluation was for lamina dura, pulp canal, root, mandible canal, mandibular angle, TMJ, body of mandible, ramus of mandible, and symphysis mentioned in OPG and 3D CT techniques.

The evaluation was taken from the films diagnosed by two radiologists, the total number was twenty, and the excellent appearance was given according to normal

density (radio-frequency or radio-opacity in the OPG radiographs and the clarity of the anatomical structures in the rotated views in the 3D CT images.

Figures (1) and (2) show examples of radiographs for some patients.

Table (1): the results of the excellent appearance of different anatomical structure done by the radiologists in percentages (%).

The Anatomical structure	OPG Excellent appearance (%)	Frequency of OPG Excellent appearance	3D Excellent appearance (%)	Frequency of 3D Excellent appearance
1- lamina Dura	5	1	0	0
2-Pulp canal	0	0	0	0
3-Root of the teeth	45	9	0	0
4-Mandibular canal	0	0	0	0
5-Mandibular angle	25	5	90	9
6-TMJ	10	2	70	7
7-Body 0f mandible	0	0	90	9
8-Ramus of mandible	15	3	90	9
9-Symphysis menti	0	0	90	9

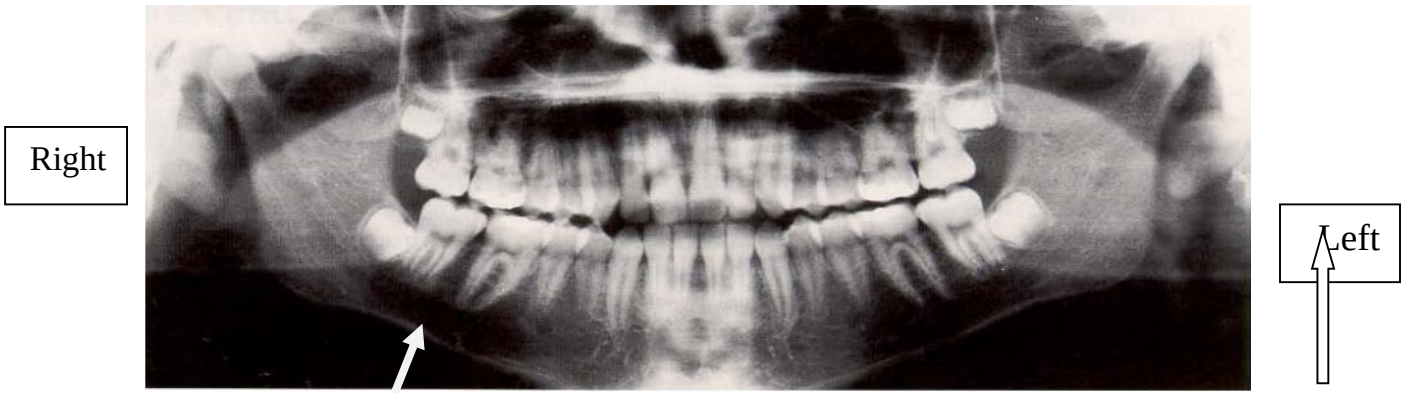

*Figure 1: the anatomical structures for a 20 years old male with permanent dentition showing the roots of the teeth (long arrow) and Lamina Dura (short arrow) as demonstrated In OPG technique.(90 KV and 12mA**).***

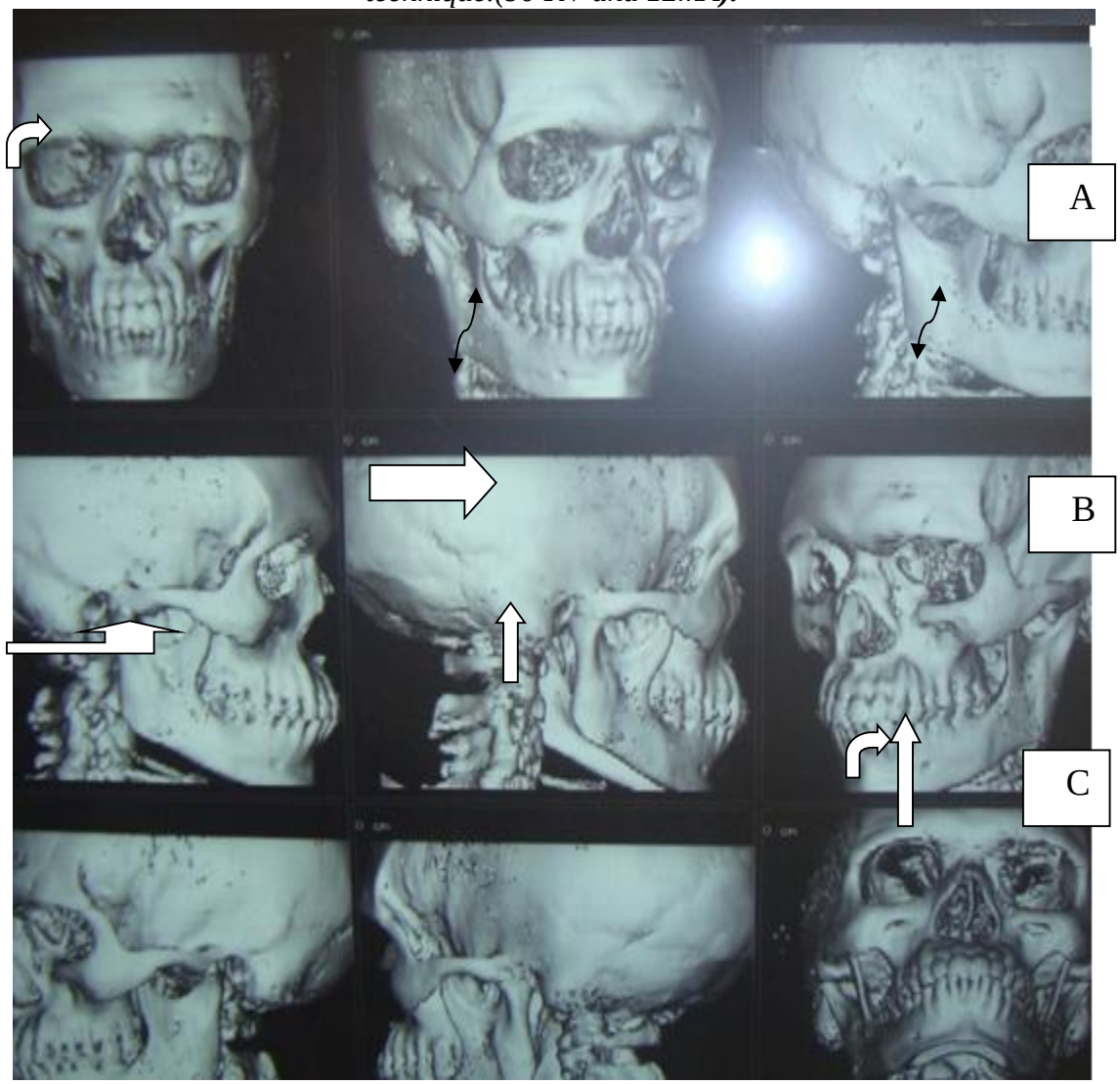

Figure 2: the anatomical structures for a 47 years old male with permanent dentition showing: Nasal septum curved arrow at (A) and (C), and body of the mandible thin arrow at (A), Ramus of the mandible curved arrow, angle of the mandible thin arrow and TMJ thick arrow at (B) the Symphesis menti Straight arrow at(C).140 KV and160 mA.

DISCUSSION

This study conducted to assess the 3D CT and OPG techniques in demonstration the anatomy of the mandible including body, ramus, symphysis menti and TMJ, lamina dura, pulp canal, nasal septum, hard palate, root, mandibular canal, mandibular angle.

The evaluation was from all films diagnosed by two radiologists, the total number was twenty, and the excellent appearance was given according to normal density (radio-lucency or radio-opacity in the OPG radiographs and the clearity of the anatomical structures in the rotated views in the 3D CT images.

Curved structure of the jaws requires a special nomenclature for anatomical relationship: Boeddinghaus, and because in the OPG ,the X-ray tube and film rotate around the head of the patient, at different velocities, resulting in a flat representation of the curved surfaces of the jaws[11]. Only objects in the image layer remain in focus, other structures appearing blurred and distorted. So the spatial resolution is therefore lower and subtle carious or normal radiolucency or radioopacity and periapical lesions may not be visible and this agree with Lurie [12], as well as the panoramic radiograph is a two-dimensional (2D) image, lacking information in the bucco-lingual direction and enhance in both vertical and horizontal directions. Therefore the findings of this study, in the OPG technique, showed that the appearance of root of the teeth which is radiolucent and of low attenuated value was found to be excellent in (45%) of the sample and TMJ (10%), Ramus (15%) Lamina Dura (5%) consigned with the above mentioned justifications.

CT studies have the advantage of allowing fairly detailed assessment of the bony structures; where Special software gathers information from the transaxial scan data to display 3D images on a 2D screen (Volume Rendering).

The results of this study showed that the 3D CT is best in demonstrating mandibular angle, and body of the mandible, ramus, smphysis menti for (95%) respectively, and TMJ for (70%) and Nasal Septum (50%). This is because of its ability to present the anatomy in a rotated form as well as it is a result from data collected after the images reconstruction.

Both techniques failed in demonstration of pulp canal, and mandibular canal, (0%), because the pulp canal bounded by high intensity material, so radiation was absorbed by the outer structure (no details), and this can be solved by using perpendicular plain to get the pulp canal image.

CONCLUSIONS

According to the results the main findings of the study were:

The 3DCT has value in demonstrating the angle of mandible, body and ramus of mandible, TMJ, and symphysis menti and nasal septum.

The OPG has value in demonstrating the root of the teeth and lamina Dura.

In all cases both techniques failed in demonstrating the pulp canal and mandibular canal.

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CR-Sub Manifolds of the Six Dimensional Sphere

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ABSTRACT
In this paper a proper CR-submanifolds of the 6-dimensional sphere was considered, S^6. some characterization theorems of their submanifolds were provided. In particular, 4- dimensional CR-submanifolds were introduced and their theorems were illustrated with examples.

المستخلص

فى هذه الورقة اعتبرنا عديدة الطيات الجزئية الفعلية لكوشى ريمان للكرة ذات الابعاد الست . استطعنا ان نقدم بعض نظريات التصنيف لهذه الفضاءات. على وجة الخصوص قدمنا عديدات الطيات ذات الاربعة ابعاد لكوشى ريمان وبينا النظريات التى توصلنا اليها من خلال بعض الامثلة.

***KEYWORDS*:** CR-submanifolds, submanifolds Kahlerian structure , holomorphic distribution, real submanifolds.

INTRODUCTION
The six-dimensional sphere S^6 is the most typical example of nearly Kaehlerian manifolds. The existence of such a nearly Kaehlerian structure for the 6-sphere was proved by Fukami and Ishihara(1) by making use of the properties of the Cayley division algebra. The almost complex submanifolds of the 6-dimensional sphere were studied by Gray and Sekigawa. A. Gray proved that with respect to the Canonical nearly Kaehlerian structure (2). S^6 has no 4-dimensional almost complex submanifolds. On the other hand Sekigawa studied the 2-dimensional almost complex submanifolds of S^6 (3). He proved that, among other things, a 2- dimensional almost complex sub manifold of S^6 with Gaussian curvature $K < 1$ is either diffeomorphic to a 2-dimensional torus or a 2-dimensional sphere. Bashir M. A also have many results in this article (4 - 7).

The six–dimensional space is any space that has six dimensions, that is six degrees of freedom, and that needs six pieces of data, or coordinates.
Formally six–dimensional Euclidean space S^6, is generated by considering all real 6-tuples as 6–vectors in this space. As such it has the properties of all Euclidean spaces, so it is linear, has a metric and a full set of vector operations. In particular, the dot product between two 6-vectors is readily defined, and can be used to calculate the metric. 6x6 metrics can be used to describe transformations such as rotations that keep the origin fixed.

More generally, any space that can be described with six coordinates not necessarily Euclidean ones, is six dimensional. One example is surface of 6-sphere S^6. This is the set of all points in seven dimensional Euclidean space R^7 that is equidistant from origin. This constraint reduces the number of coordinates need to describe a point on 6-sphere by one, so it has six dimensions they have for more applications.

The 6–sphere, or hypersphere is in seven dimensions, is the six dimensional surface equidistant from a point, e.g. the origin. It has symbol S^6, with formal definition for 6-sphere with radius r is:

$$S^6 = \{x \in R^7 : \|x\| = r\}$$

Let C be the set of all purely imaginary Cayley numbers. C can be viewed as a 7-dimensional linear subspace R^7 of R^8. Consider the unit hypersurface which is centered at the origin

$$S^6(1) = \{x \in C :< x, x >= 1\}$$

The tangent space $T_x S^6$ of S^6 at a point X may be identified with the affine subspace of C which is orthogonal to X. $A(1,1)$ tensor field J on S^6 is defined by: $J_x U = X \times U$

where the above product is defined for $x \in S^6$ and $U \in T_x S^6$. The tensor field J determines an almost complex structure (i.e. $J^2 = -id$) on S^6 If $\overline{\nabla}$ is the Riemannian connection on S^6, then $(\overline{\nabla}_x J)X = 0$ for any $X \in \chi(S^6)$, i.e. S^6 is nearly Kaehler. J is orthogonal with respect to indusced metric g and they are related by the form

$$\forall X, Y \in TM, g(X,Y) = g(JX,JY)$$

A 2p + q –dimensional submanifold M on S^6 is called a CR-Submanifold if there exists a pair of orthogonal complementary distributions D and $D^\perp$ such that JD = D and $JD^\perp \in \nu$ where ν is the normal bundle of M. The distributions D and $D^\perp$ are called the holomorphic distribution and the totally real distribution respectively with dim D = 2p and dim $D^\perp$ p. The normal bundle ν splits as $\nu = JD^\perp \oplus \ll$ where $\ll$ is invariant sub-bundle of ν under J. The CR-submanifold is said to be proper if neither $D = \{0\}$ nor $D^\perp = \{0\}$. A proper CR-submanifold M of S^6 is said to be a CR-product submanifold if it is locally the Riemannian product of a holomorphic submanifold and a totally real submanifold of S^6. It is known that there does not exist any CR-product submanifolds in S^6 (8).

In the area of number theory, the Euler numbers are a sequence E_n of integers defined by the following Taylor series expansion:

$$\frac{1}{\cosh t} = \frac{2}{e^t + e^{-t}} = \sum_{n=0}^{\infty} \frac{E_n}{n!} \cdot t^n$$

The Euler numbers appear as a special value of the Euler polynomials.

Given a real vector bundle E over M, its k-th Pontryagin class $p_k(E)$ is defined as

$$p_k(E) = p_k(E, \mathbb{C}) = (-1)^k c_{2k}(E \otimes \mathbb{C}) \in H^{4k}(M, \mathbb{C})$$

Here $c_{2k}(E \otimes \mathbb{C})$ denotes the 2k-th Chern class of the complexification $E \otimes \mathbb{C} = E \oplus iE$ of E and $H^{4k}(M, \mathbb{C})$, the Z^{4k}- homology group of M with integer coefficients.

The rational Pontryagin class $p_k(E, \mathbb{Q})$ is defined to be the image of $p_k(E)$ in $H^{4k}(M, \mathbb{Q})$, the 4k-cohomology group of M with rational coefficients.

The Pontryagin classes of a smooth manifold are defined to be the Pontryagin classes of its tangent bundle.

Novikov proved in (1966) that if manifolds are homeomorphic then their rational Pontryagin classes: $p_k(M,¤) \hat{I} H^{4k}(M,¤)$ are the same.

If the dimensions are at least five, there are at most finitely many different smooth manifolds with given homotopy type and Pontryagin classes.

G_2 manifold is a seven-dimensional Riemannian manifold with holonomy group G_2. The group G_2 is one of the five exceptional simple Lie groups.

A G_2-structure is an important type of M-structure that can be defined on a smooth manifold. If M is a smooth manifold of dimension seven, then a G_2-structure is a reduction of structure group of the frame bundle of M to the compact, exceptional Lie group G_2.

We denote by D the Levi-Civita connection of S^6. Then we have

$$(D_x J)Y = -X \times Y + < X \times Y, x > x \quad (1)$$

for $X, Y \in T_x S^6, x \in S^6$. Thus we see that the almost Hermitian structure J on S^6 is a nearly Kaehler structure $((D_x J)X = 0)$ which is not Kaehler one.

Now, we prepare fundamental formula for Riemannian submanifolds of S^6. Let $(M,¤)$ be a submanifolds of S^6 with the isometric immersion $¤ : M \to S^6$. We set $x = § \circ ¤$ and consider x as the corresponding position vector to the image of $¤$ in $\mathrm{Im}C$, where $§$ denote the inclusion map from S^6 to $\mathrm{Im}C$. We denote by ∇ and $\nabla^{\perp}$ the Riemannian connections on M and the normal bundle $T^{\perp}M$ induced by the Riemannian connection D on S^6, respectively.

Then, the Gauss and Weingarten formulas are given respectively by

$$D_X Y = \nabla_X Y + \pm(X,Y) \quad (2)$$

$$D_X ¶ = A_¶ X + \nabla^{\perp}_X ¶ \quad \ldots\ldots(3)$$

where $\pm$ and $A_¶$ are the second fundamental form and the shape operator (with respect to the normal vector field $¶$), respectively, and $X, Y \in \chi(M)$ where $\chi(M)$ denotes the Lie algebra of all smooth tangent vector fields on M. The second fundamental form $\pm$ and the shape operator $A_¶$ are related by

$$< \pm(X,Y), ¶ > = < A_¶ X, Y >.$$

The Gauss, Codazzi and Ricci equations are given respectively by

$$< R(X,Y)Z,W > = < X,W >< Y,Z > - < X,Z >< Y,W > + < \pm(X,W), \pm(Y,Z) > - < \pm(X,Z), \pm(Y,W) > \quad (4)$$

$$(\nabla'_X \pm)(Y,Z) = (\nabla'_Y \pm)(X,Z) \quad (5)$$

$$< R^{\perp}(X,Y)¶, ¦ > = < [A_¶, A_¦]X, Y > \quad (6)$$

where

$$(\nabla'_X \pm)(Y,Z) = \nabla^{\perp}_X \pm (Y,Z) - \pm(\nabla_X Y, Z) - \pm(Y, \nabla_X Z)$$

and

$$R^{\perp}(X,Y)¶ = \left[\nabla_X, \nabla_Y\right]¶ - \nabla^{\perp}_{[X,Y]}¶$$

for $X, Y, Z, W \in \chi(M)$ and $¶, ¦$ are vector fields normal to M.

CR-submanifolds of the six dimensional sphere:

There are many theorems of CR-submanifolds of S^6 have been studied by several mathematician.

For instance Sekigawa [8] proved that S^6 does not contain any CR-product submanifold. Gray has shown that S^6 does not admit a 4-dimensional complex manifold[2]. Ejiri N. proved the following[9]**:**

Theorem [1]: A 3-dimensional totally real submanifold of S^6 is orientable and minimal.

Proof: Let (M,g) be a 3-dimensional totally real submanifold of $(\overset{6}{S},J,g)$. First of all, we shall prove the following.

Lemma 4.3.1. G(X,Y) is normal to M for X,Y tangent to M.

Proof: From the second fundamental form of the immersion given by

$$(X,Y)=\bar{\nabla}_X Y-\nabla_X Y \tag{7}$$

For vector fields X,Y on M. For a normal vector fields $\P$, we denote by $-A_\P X$ and $\nabla^\perp_X\P$ the tangential and normal components of $\bar{\nabla}_X\P$ respectively so that

$$\bar{\nabla}\P_X Y=-A_\P X+\nabla^\perp_X\P \tag{8}$$

We have

$$g((\bar{\nabla}_x J)Y,Z)=g(J\pm(X,Z),Y)-g(J\pm(X,Y),Z),$$
$$g((\bar{\nabla}_z J)X,Y)=g(J\pm(Z,Y),X)-g(J\pm(Z,X),Y),$$
$$g((\bar{\nabla}_y J)Z,X)=g(J\pm(Y,X),Z)-g(J\pm(Y,Z),X),$$

for X,Y,Z tangent to M. Since $\bar{g}$ is Hermitian with respect to $J,\bar{\nabla}_x J$ is skew-symmetric with respect to $\bar{g}$. This, together with the fact $(\bar{\nabla}_X J)X=0$, holds for all vector fields X and S^6 implies that the left-hand sides of the above three equations are equal to each other. Therefore we have

$$g((\bar{\nabla}_X J)Y,Z)=0,$$

which means $G(X,Y)$ is orthogonal to M. By

$$G(X,JY)=-JG(X,Y) \tag{9}$$

we obtain

$$(\bar{\nabla}_x G)(JY,JZ)=\bar{\nabla}_x G(JY,JZ)-G(\bar{\nabla}_x JY,JZ)-G(JY,\bar{\nabla}_x JZ)$$
$$=-\bar{\nabla}_x G(Y,Z)-G((\bar{\nabla}_x J)Y,JZ)-G(J\bar{\nabla}_x Y,JZ)$$
$$-G(JY,(\bar{\nabla}_x J)Z)-G(JY,J\bar{\nabla}_x Z)$$
$$-\bar{\nabla}_x G(Y,Z)+JG(G(X,Y),Z)$$

$$+G((\bar{\nabla}_x Y,Z)+JG(Y,G(X,Z))+G(Y,\bar{\nabla}_x Z)$$

$$=-(\bar{\nabla}_x G)(Y,Z)+JG(G(X,Y),Z)+JG(Y,G(X,Z)$$

for X Y Z tangent to M.

This, combined with the fact

$$(\bar{\nabla}_x G)(Y,Z)=\bar{g}(Y,JZ)X+\bar{g}(X,Z)JY-\bar{g}(X,Y)JZ \tag{10}$$

holds for all vector fields X, Y, Z on S^6, implies

$$G(Y,G(Z,X)+G(Z,G(X,Y))=g(X,Y)Z-g(X,Z)Y$$

and hence

$$G(X,G(Y,Z))=g(X,Z)Y-g(X,Y)Z$$

or equivalently

$$JG(X,JG(Y,Z)=g(X,Z)Y-g(X,Y)Z \tag{11}$$

for X, Y, Z tangent to M. Since $JG(X,Y)$ is tangent to M by Lemma [1] we see from (11) that

$$g(JG(X,Y),Y)X-g(JG(X,Y),X)Y=$$
$$JG(JG(X,Y),JG(X,Y))=0$$

Thus $JG(X,Y)$ is orthogonal to X and Y if X and Y are linearly independent. This property, together with (11), implies that M is orientable, because the orientation can be defined by regarding JG(X, Y) as the vector product of X and

Y at each point of M. Next, we shall prove that M is minimal. It follows immediately from eqns. (7), (8) and Lemma [1] that

$$\nabla^{\perp}_X JY = G(X,Y) + J\nabla_x Y) \tag{12}$$

and

$$A_{JX} = -J\pm(X,Y) \tag{13}$$

hold for X, Y tangent to M. By (7), (8), (12), (13) and (9), we obtain

$$(\bar{\nabla}_x G)(Y,Z) = \bar{\nabla}_x G(Y,Z) - G(\bar{\nabla}_x Y, Z) - G(Y, \bar{\nabla}_x Z)$$
$$= -A_{G(Y,Z)}X + \nabla^{\perp}_X G(Y,Z) - G(\bar{\nabla}_x Y, Z) - G(Y, \bar{\nabla}_x Z)$$
$$= J\pm(JG(Y,Z),X) + JG(X,G(Y,Z)) - J(\nabla_x JG(Y,Z)$$
$$-G(\pm(X,Y),Z) - G(Y,\pm(X,Z))$$

for X, Y, Z tangent to M. This, combined with (10), implies

$$(\bar{\nabla}_x JG)(Y,Z) = g(X,Y)Z - g(X,Z) + G(X,GY,z)$$
$$+ \sigma(X,JGY,Z) + JG\sigma(X,Y),Z) + JG(Y,\sigma(Z,X))$$

Taking the normal component, we have

$$\pm(X,JG(Y,Z)) + JG(\pm(X,Y),Z) + JG(Y,\pm(Z,X)) = 0 \tag{14}$$

for X, Y, Z tangent to M. Let e_1, e_2, e_3 be a local field of orthonormal frames on M. Then we may assume without loss of generality that $JG(e_1, e_2) = e_3$, $JG(e_2, e_3) = e_1$ and Hence we have from eqn. (14) that the trace of $\sigma = 0$, which implies that M is minimal.

Theorem [2]: Let M be a 3–dimensional totally real submanifold of constant curvature C in S^6, then either C = 1 (i.e. M is totally geodesic) or $c = 1/16$.

Proof: Let M be a 3-dimensional totally real submanifold of constant curvature c in S^6. Then the equation of Gauss reduces to

$$(1-c)\{g(X,Z)g(Y,W) - g(X,W)g(Y,Z)\}$$
$$+\bar{g}(\pm(X,Z),\pm(Y,W) - \bar{g}(\pm(X,W),\pm(Y,Z)) = 0 \tag{15}$$

If C = 1, then M is totally geodesic. Therefore it is sufficient to consider the case C< 1.

Consider a cubic function $f(x) = \bar{g}(\pm(X,X),JX)$ defined on $\{X \in T_x; \|X\| = 1\}$. If f attains its maximum at x, then $\bar{g}(\pm(X,X),JX) = 0$ for Y orthogonal to x and hence $\pm(X,X)$ is proportional to JX. Therefore, if f is constant, $\pm(X,X) = 0$ for all X, since M is minimal. Thus f is not constant, since we are considering the case where M is not totally geodesic.

Choose e_1 to be the maximum point of f at each point $x \in M$. By the similar argument to the above, we see that f restricted to $\{X \in T_x M; \|X\| = 1\}$ and $g(X,e_1) = 0\}$ is not constant. Choose e_2 to be the maximum point of f restricted to $\{X \in T_x M; \|X\| = 1$ and $g(X,e_1) = 0\}$ and choose e_3 so that e_1, e_2, e_3 form an orthonormal frame field. Then we easily see that

$$\bar{g}(\pm(e_2,e_2),Je_3) = 0 \tag{16}$$

Put $a_i = \bar{g}(\pm(e_i,e_i),Je_1) = 0$. Then we have $a_1 + a_2 +, a_3 = 0$, since M is minimal. We see that $a_1 > 0$, because a_1 is the maximum value for the cubic function f and M is not totally geodesic. Moreover, from (15) we have

$$1 - c + a_1 a_2 - a_3^2 = 0$$

and

$$1 - c + a_1 a_3 - a_3^2 = 0,$$

since (13) implies that $\bar{g}(\pm(X,Y),JZ)$ is symmetric in X, Y, Z. Therefore we get

$$(a_1,a_2,a_3) = (2\sqrt{(1-c)/3}, -\sqrt{(1-c)/3}, -\sqrt{(1-c)/3}),$$

which implies that

$$\pm(e_1, e_1) = 2\sqrt{(1-c)/3}\,Je_1 \qquad (17)$$

and

$$\bar{g}(\pm(X,X), Je_1) = -\sqrt{(1-c)/3} \qquad (18)$$

for a unit vector X orthogonal to e_1. In particular, putting $X = (e_2 + e_3)/\sqrt{2}$, we obtain

$$\bar{g}(\pm(e_2, e_3), Je_1) = 0. \qquad (19)$$

In consideration of eqns. (16), (17), (18), (19) and minimality of M, we may put

$$\pm(e_2,e_2) = -\sqrt{(1-c)/3}\,Je_1 + \mathrm{a}\,Je_2,$$
$$\pm(e_3,e_3) = -\sqrt{(1-c)/3}\,Je_1 - \mathrm{a}\,e_2,$$
$$\pm(e_2,e_3) = -\mathrm{a}\,Je_3,$$

. Putting $X = W = e_2$ and $Y = Z = e_3$ in (15), we obtain $\mathrm{a} = \sqrt{2(1-c)/3}$. Therefore we have

$$\pm(e_2,e_2) = -\sqrt{(1-c)/3}\,Je_1 + \sqrt{2(1-c)/3}\,Je_2,$$
$$\pm(e_3,e_3) = -\sqrt{(1-c)/3}\,Je_1 - \sqrt{2(1-c)/3}\,Je_2,$$
$$\pm(e_2,e_3) = -\sqrt{2(1-c)/3}\,Je_3, \qquad (20)$$

which, together with (17), (18) and (19), implies

$$\pm(e_1,e_2) = -\sqrt{(1-c)/3}\,Je_2,$$
$$\pm(e_1,e_3) = -\sqrt{(1-c)/3}\,Je_3 \qquad (21)$$

Applying the Codazzi equation to (17), (20) and (21), we obtain $\nabla_{e_i} e_i = 0$, $\nabla_{e_1} e_2 = -\nabla_{e_2} e_1 = -\frac{1}{4}e_3$, $\nabla_{e_1} e_3 = -\nabla_{e_3} e_1 = -\frac{1}{4}e_2$, $\nabla_{e_2} e_3 = -\nabla_{e_3} e_2 = -\frac{1}{4}e_1$.

Therefore we have $R(e_1,e_2)e_1 = 1/16\,e_2$ and hence $c = 1/16$.

Bashir M.A. proved the following:

Theorem [3]: S^6 does not admit any compact proper CR-submanifold with non-negative sectional curvature and integrable holomorphic distribution(5).

Proof: Since D is integrable, then the integral submanifold of the distribution Dis a Kahler manifold. Since M is proper then $\dim D = 4$ is ruled out by a result of Gray(2), namely S^6 does not contain a 4-dimensional complex submanifold. Therefore dim D = 2. Since $\nu = J\overset{\perp}{D}\oplus$ « and M is a proper CR-submanifold of S6 we have $\dim \overset{\perp}{D} = 1$, i.e., M is 3-dimensional. Now let μ be a 2-form on the integral submanifold of D and let h be its dual. Since the integral submanifold of D is Kaehler, μ is harmonic. Using Poincare duality theorem, its dual h is also harmonic, i.e., $d\mu = \delta\mu = 0$.

Now from the hypothesis of the theorem, we get $Ric(z,z) \geq 0$. Using the integral formula

$$\int_M \{Ric(x,x) + \|\nabla X\|^2 - \frac{1}{2}\|d\mu\|^2 - (divX)^2\}dv = 0$$

and $Z \in \overset{\perp}{D}$ we have

$$\int_M \{Ric(z,z) - \frac{1}{2}\|d\mu\|^2 + \|\nabla Z\|^2 - (\delta\mu)^2\}dv = 0$$

from which we get $\nabla_X Z = 0$ for all $X \in \chi(M)$ and $Z \in \overset{\perp}{D}$, i.e., the distribution $\overset{\perp}{D}$ is parallel. Also $g(Y,Z) = 0$ for all $Y \in D$ gives $\nabla_X Y = 0$ for all $X \in \chi(M)$ and $Y \in D$ This means that

D is also parallel. D and $\overset{\perp}{D}$ being parallel implies that M is a CR-product(11), which is a contradiction to the fact that S^6 does not have any CR-product submanifold. Therefore our theorem is proven.

Theorem [4]: Let M be a complex totally real 2-dimensional submanifold of S^6. Then M is flat and minimal (see ref. (7)).

Chen B.Y. proved the following theorem for the case dimension $M \leq 3$, and Bashir M.A. proved it here for general.

Theorem [5]: Let M be a simply connected compact mixed foliate CR-submanifold of a hyperbolic complex space form $\overline{M}(-4)$. Then M is either a complex submanifold or a totally real submanifold of $\overline{M}$ (4).

4-Dimensional CR-Submanifold(10).

Concerning 4-dimensional CR-submanifolds of S^6, it is only known that there does not exist a 4-dimensional CR-product submanifold of S^6 (10,11)

First, we recall the following characterization for a 4-dimensional oriented submanifold of S^6 to be a CR-submanifolds (12,13).

Proposition [1]: Let $\ddot{} : M^4 \to S^6$ be an orientable 4-dimensional submanifold of S^6. Then $(M^4, \ddot{})$ is CR-submanifold of S^6 if and only if it satisfies one of the following conditions

(1) $\mu(T^{\perp}M^4) = 0$,

(2) $^*\mu(TM^4) = 0$,

where μ denote the Kaehler form of S^6.

Remark: Let $h \in SO(7)/G_2$ be a 4-dimensional CR-submanifold. If $g \in G_2$, then $g \circ f$ is also. However, if $h \in SO(7)/G_2$, then $h \circ ¤$ is not a CR-submanifold, in general (where G_2 is the compact Lie group of all automorphisms of the octonions (known also as the Cayley division algebra).

Let $\ddot{} : M^4 \to S^6$ be a 4-dimensional submanifold of S^6 and discuss some fundamental properties concerning $(M^4, \ddot{})$. Especially we discuss a (local) orthonormal CR-adapted frame field along $(M^4, \ddot{})$. Let $¶_1, ¶_2$ be a local orthonormal frame fields $¶_1, ¶_2$ of $H^{\perp}$. Then we have $span_R(J¶_1, J¶_2) = T^{\perp}M^4$. Then the exterior product $¶_1 \times ¶_2$ depends only on the given orientation of $H^{\perp}$ and is independent on the choice of the orthonormal frame fields. Also we have

$¶_1 \times ¶_2, J(¶_1 \times ¶_2) \in H$.

Therefore, the vector field $¶_1 \times ¶_2$ is well defined whole on M^4. Hence H has an absolute parallelizability. We see that $\{¶_1 \times ¶_2, J(¶_1 \times ¶_2), ¶_1 \times ¶_2\}$ is a local orthonormal frame field of M^4. We obtain

Proposition [2]: Let $\ddot{} : M^4 \to S^6$ be an orientable compact 4-dimensional CR-submanifold of S^6, then the Euler number of M^4 vanishes.

By Proposition (2), we may immediately see that 4-dimensional sphere, product of two 2-dimensional spheres and complex 2-dimensional projective space cannot be realized as a CR-submanifold of S^6. On the other hand, since $\dim H^{\perp} = 2$, and $H^{\perp}$ is orientable, we can define two kinds of almost complex structures J_1, J_2 on M^4 such that

$J_1 = J_H \oplus J', J_2 = J_H \oplus (-J')$

where J_H is the restriction of the almost complex structure of S^6 to the holomorphic distribution H. hence we have the following decomposition

$$TS^6\big|_{\varphi(M^4)} = H \oplus H^{\perp} \oplus T^{\perp}M^4$$

If we set $V = H^{\perp} \oplus T^{\perp}M^4$, then V is a C^2- vector bundle over M^4. Concerning the characteristic classes of these vector bundles, we have the following

Theorem [6]: Let $\varphi : M^4 \to S^6$ be a 4-dimensional CR-submanifold of S^6. The first Pontryagin class of the tangent bundle vanishes i.e., $p_1(TM^4) = 0$.

By taking account of the G_2- structure equation on S^6, we can also show that 2-dimensional totally real distribution $H^{\perp}$ is not integrable.

Examples

The above arguments assert that there exist many obstructions for the existence of 4-dimensional CR-submanifolds of S^6. However, contrary to this circumstances, we may construct several examples of such submanifolds. We herewith introduce two typical examples of 4-dimensional CR-submanifolds of S^6.

Example [1]: Let $\gamma : 1 \to S^2 \subset \mathrm{Im}\,H$ be any curve in the 2-dimensional sphere $S^2 \subset \mathrm{Im}\,H \simeq R^3$, and $(q \in)S^3 \subset H$ be the 3-dimensional sphere of the quaternion H. Then the product immersion $\cdot : 1 \times S^3 \to S^6$ which is defined by

$$\cdot(t,q) = a\gamma(t) + bq$$

Gives a 4-dimensional submanifold of S^6, for any $a,b>0$ with $a^2+b^2=1$. Here, t denotes the arc length parameter of γ.

In fact, let $(1\times S^3, \cdot)$ be the submanifold of S^6 gives in the above example 1. Then, we may choose the orthonormal frame field $\{v_1, v_2\}$ of the normal bundle in such a way that

$$v_1 = \gamma(t) \times \gamma(t), v_2 = b\gamma(t) - aq$$

Thus we have

$$J(v_2) = (b\gamma(t) - aq) \times (a\gamma(t) + bq) = \gamma(t) \times q = (q\gamma(t)) \in H$$

therefore, $< v_1, J(v_2) >= 0$. Thus (1) of Proposition [1] we get the desired result. Further, we may obtain the corresponding CR-frame along $(1\times S^3, \cdot)$ in the following way. A local orthonormal frame field of $H^{\perp}$ is given by

$$\cdot_*(\partial_1) = J(v_1) = v_1 \times \cdot = -a\gamma(t) + b(\gamma(t) \times \gamma(t)) \cdot q$$

$$\cdot_*(\partial_2) = J(v_2) = \gamma(t) \times q .$$

On the other hand, an orthonormal frame field of H is given by

$$\cdot_*(e_1) = J(v_1) \times J(v_2) = b\gamma(t) + b(\gamma(t) \times \gamma(t)) \cdot q,$$

$$\cdot_*(J(e_1)) = (\gamma(t)) \cdot q$$

Example [2]: The following immersion $\varphi : S^1 \times S^3 \to S^6$ is a 4-dimensional CR-submanifold of S^6;

$$\varphi(\theta, q) = a(qiq) + b(\gamma(\theta))q,$$

For $a,b>0$ with $a^2+b^2=1$, where

$$\gamma(\theta) = t\{-\sin(\theta) + \cos(\theta)i\} + s\{\cos(\theta)j + \sin(\theta)k\}$$

is a great circle of $S^3 \subset H$ for $t,s>0$ with $t^2 + s^2 = 1$.

Proposition [3]: Let $\varphi : S^1 \times S^3 \to S^6$ be a 4-dimensional CR-submanifold of S^6 in Example (2).

Then the map φ is not an imbedding. In fact, we have

$$\varphi(\theta + \pi, -q) = \varphi(\theta, q).$$

The immersion ¤ is full.

The immersion $¤ : S^1 \times S^3 \to S^6$ is minimal if and only if $a = \sqrt{(3+\sqrt{57})/24}, t = 1/\sqrt{2}$. For the other (a,t) in example (2), the length of the mean curvature vector field is constant, but the mean curvature vector field is not parallel with respect to the normal connection. In particular, the second fundamental is not parallel for any immersion of this type.

The normal curvature of the immersion ¤ is not flat.

The Ricci eigenvalues of the induced metric of the immersion ˙ 1 are constant, but the metric is not Einstein.

If $a = 1/\sqrt{3}$ and $t = 1/\sqrt{2}$, the holomorphic distribution H is integrable.

For more details see references (11, 14, 15).

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Effects of Hydroxyurea Hemoglobin F Level in Pediatric Patients with Sickle Cell Disease Attaining Jafaar Ibnouf Hospital - Khartoum

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ABSTRACT

Fetal hemoglobin (HbF) is the main hemoglobin throughout the fetal life and at birth, accounting for approximately 80% of total hemoglobin in newborn. HbF, when elevated during sickle cell disease (SCD), cause reduction of different crises associated with SCD. Elevation of HbF is achieved by hydroxyurea (HU) therapy as a tool to control SCD. This study was conducted in the period between May 2009 to December 2009 to determine the levels of HbF in SCD pediatric patients treated with hydroxyurea (HU) therapy and evaluate its effects on elevation of HbF which lead to the reduction of SCD crises. Ninety one Sudanese SCD pediatric patients with mean age of (6.0 years ± 2.9), who were referred to Jafaar Ibnouf hospital in Khartoum city, were included in this study. Sixty one of the patients, designated group T, had been treated with 22 mg /Kg/ day of HU for seven months; and thirty patients had been treated with conventional treatment were used as control designated group C. Two and half ml blood sample was collected from each participant in Ethylene Diamine Tetra Acetic Acid (EDTA) container. HbF level was determined by using the denaturation method. The results showed that the mean of HbF levels in group T was 8.00 ± 02.59 while the mean of HbF level in the control group (C) which was 0.8 ± 0.2.

المستخلص:

الهيموقلوبين من النوع (F) هو الهيموقلوبين الرئيس في المرحلة الجنينية وحتى الولادة حيث يمثل 80% من الهيموقلوبين عند الأطفال حديثي الولادة. يرتفع تركيز هيموقلوبين (F) في حالة الإصابة بمرض الأنيميا المنجلية مسبباً المضاعفات المرتبطة بمرض الأنيميا المنجلية. يستعمل علاج يسمى هيدروكسى يوريا للتحكم في مرض الأنيميا المنجلية. أجريت هذه الدراسة في الفترة ما بين مايو 2009م حتى ديسمبر 2009 لتحديد تركيز هيموقلوبين (F) عند الأطفال المصابين بمرض الأنيميا المنجلية تحت العلاج بالهيدروكسى يوريا ولتقييم إرتفاع هيموقلوبين (F) والذي يقلل من المضاعفات الناتجة من هذا المرض. واحد وتسعون مصاباً بالأنيميا المنجلية متوسط أعمارهم = 6 ± 2.9عاماً والمترددون على مستشفى جعفر بن عوف بولاية الخرطوم تم إختيارهم لهذه الدراسة. واحد وستون مريضاً تم علاجهم بالهيدروكسي يوريا بتركيز 22mg/kg يومياً ولمدة 7 أشهر. هذه المجموعة سميت المجموعة (T)، ثلاثون مريضاً تم علاجهم بالطريقة التقليدية أخذوا كمجموعة ضابطة وسميت المجموعة (C). جمع 2.5 مل من دم كل مشارك في إناء يحتوى على مادة مانعة للتجلط (EDTA) لتحديد مستوى هيموقلوبين (F) بطريقة التكسير. أظهرت نتائج الدراسة أن متوسط هيموقلوبين (F) في المجموعة (T) = 8.00 ± 2.59 جرام/ديسلتر، حيث إرتفع مقارنة بمتوسط هيموقلوبين (F) في المجموعة (C) = 0.8 ± 0.2 جرام/ ديسلتر.

***KEYWORDS*:** Fetal hemoglobin (HbF); sickle cell disease; hydroxyurea therapy.

INTRODUCTION

Sickle cell anemia (SCA) is a common genetic disorder that causes considerable morbidity and mortality throughout the world. SCA results from an amino acid substitution of valine for glutamic acid at position 6 of the β-globin chain, which results in the polymerization of hemoglobin upon deoxygenation, leading to deformed dense red blood cells the predominant pathophysiological feature of SCA is vaso-occlusion, which leads to acute and chronic complications such as painful crises, acute chest syndrome and strokes. Patients with SCA have a markedly decreased life expectancy and their quality of life is greatly compromised by their disease [(1)]. In 1910, Dr James Herrick working in Chicago, USA, reported 'Peculiar elongated and sickle shaped red blood corpuscles in a case of severe anemia[(2)]. The inherited disease was subsequently called SCA, and has continued to attract the attention of medical scientists to the present day. SCA includs homozygous (Hb SS) sickle cell disease and compound heterozygous states such as sickle cell haemoglobin C (Hb SC) disease, sickle cell thalassaemia (Hb S thal), HbS/Hb D Punjab (Los Angeles), HbS/HbO-Arab, HbS/HbE, and HbS/Hb [(3)]. Hemoglobin S becomes polymerized and becomes poorly soluble when the oxygen tension is lowered and red cells that contain this hemoglobin become distorted and rigid. SCD occurs when an individual is homozygous for the sickle cell mutation or is a compound heterozygote for sickle haemoglobin and β-thalassemia, haemoglobin C, and some less common β–globin mutations. Diagnosis depends upon demonstrating the presence of the abnormal haemoglobin (S) in the red cells. The disease is characterized by haemolytic and by three types of crises, painful (vasoocclusive), sequestration, and aplastic crisis. Complications include splenic infarction and autosplenectomy, stroke, bone infarcts and aseptic necrosis of the femoral head, leg ulcers, priapism, pulmonary hypertension, and renal failure [(4)]. Hemoglobin S occurs with greatest prevalence in tropical Africa. HbF is the main hemoglobin component throughout fetal life and at birth, accounting for approximately 80% of total hemoglobin in newborns. HbF is produced from the sixth week of gestation and during the rest of fetal life, replacing the embryonic hemoglobin. After birth, HbF synthesis rapidly declines and HbF is gradually substituted by HbA in the peripheral blood, so that within the first two years of life, the characteristic hemoglobin phenotype of the adult with very low levels of HbF (less than 1%) is found [(1,2)]. Functionally, HbF differs mostly from HbA because it has a slightly higher oxygen affinity, explained by the low interaction with 2,3-DPG. This characteristic makes the delivery of oxygen through placenta easer, giving the fetus better access to oxygen from the mother's bloodstream[(5)]. Moreover, HbF is known to inhibit the polymerization of HbS and different agents have been introduced to increase HbF production for therapeutic aim [(6)]. The levels of HbF in erythrocytes account for a large part of the clinical heterogeneity observed in patients with SCD and β - thalassemia[(6)]. The Cooperative study of SCD identified HbF as a major prognostic factor for several clinical complications including painful events and acute chest syndrome. These clinical and epidemiological observations provided important clues about the beneficial role of HbF in ameliorating the clinical complications of SCD which is a major public health concern that has great impact on both individuals and society. HU treatment allows γ-globin genes to be more actively expressed. By killing cycling cells, HU changes the kinetics of erythroid proliferation, forcing more F cells to be produced from primitive progenitors. HU also produces nitric oxide and directly stimulates fetal hemoglobin production. Because F cells are less likely in red cells with little Hb F to occlude vessels and cause membrane damage, HU treatment results in fewer symptoms, less severe hemolytic

anemia, and lower mortality[7]. Some patients also display increases in their anaerobic muscular performance and aerobic cardiovascular fitness[8]. The hemoglobin S–containing erythrocytes became less dense, and hemolysis was reduced. These changes and the reduction in painful episodes preceded the increase in the hemoglobin F concentration[9]. HU should be reserved for patients with sickle cell anemia who have complications that are sufficiently severe to justify the burdens of treatment and who can comply with the treatment regimen.

This study aimed at determining the levels of HbF in SGD pediatric patients treated with hydroxyl urea therapy in Jafaar Ibnouf hospital in Khartoum state.

MATERIALS and METHODS

Ninety one SCD patients, who were referred to Jafaar Ibnouf hospital in Khartoum city, were informed about the study objectives and agreements for their participation were obtained. Sixty one of the patients had been treated with HU called T group, and thirty patients had been treated with conventional treatment called C group. Patient's samples were selected randomly to be matched in age, sex with control. Venous blood (2.5 ml) was collected from each patient into Ethylene Diamine Tetra Acetic acid (EDTA) containers.

Principle of denaturation method

This method based on the resistance to denaturation by alkali of HbF compared to HbA, the denaturation being activated by the ionization of buried, weakly acidic side chains (one tyrosine and two cysteines) present in HbA and not in HbF [10]. This is only a relative difference, and the conditions have been optimized over time in order that during the time of exposure to alkali most of the HbA is denaturized while the HbF is largely unaffected. Before the exposure to alkali, all the hemoglobin forms are transformed in the more stable Cyanomehtemoglobin form by treatment with Drabkin's reagent. An optimized version of the method was proposed by Pembrey [11].

So HbF was estimated by denaturation method to measure the percentage of HbF in a mixture of hemoglobins [12]. Sodium hydroxide was added to a lysate and, after a set time, denaturation was stopped by adding saturated ammonium sulphate. The ammonium sulphate lowers the pH and precipitates the denatured hemoglobin. After filtration, the quantity of undenatured (unprecipitated) hemoglobin was measured. The proportion of alkali-resistant HbF was subsequently calculated as a percentage of the total amount of hemoglobin present [13].

Procedure

Lysate (0.25 ml) was added to 4.75 ml of cyanide to make a hemoglobin cyanide (HiCN), after that 2.8 ml of HiCN was transferred to a new glass tube and allowed to equilibrate at 20°C.To the same test tube 0.2 ml of 1.2 mol/l of NaOH was added and mixed well with HiCN on a vortex mixer for 2 to 3 seconds. After 2 minutes, 2 ml of saturated ammonium sulphate was added to the same test tube and also mixed well on a vortex mixer. Tube left to stand for 5 to 10 minutes at 20°C. Solution was filtered twice through a Whatman filter paper No.42. The filtrate containing alkaline- resistant hemoglobin (HbF). Total Hb was measured by transferring 0.4 ml of HiCN into another test tube and 13.9 ml of water was added to the same test tube.The absorbance of alkali- resistant and total Hb were detected using spectrophotometer set at 420nm against water blank. The percentage of alkali-resistant was calculated as follows:

$$HbF\ \% = \frac{\text{a 420 alkali} - \text{resistant Hb}}{\text{a 420 total Hb}} \times 100\%$$

RESULTS

The measured level of HbF in group T was (8.00 ± 2.59) which was found to be raised significantly compared with group C (0.81 ± 0.21), (P=0.000), (Table 1):

Table 1: HbF Levels and P values in the study groups

Group	(Mean ± SD)	P value
T	8.00 ± 2.59	0.000
C	0.81 ± 0.21	

It was noticed that most of the patients belonged to tribes living in the west of Sudan where the gene of sickle cell anemia is most prevalent. The highest frequency of participators was found to be from Meseria tribe (n=51) 35%. The distribution of participant's tribe is shown in figure 1 below.

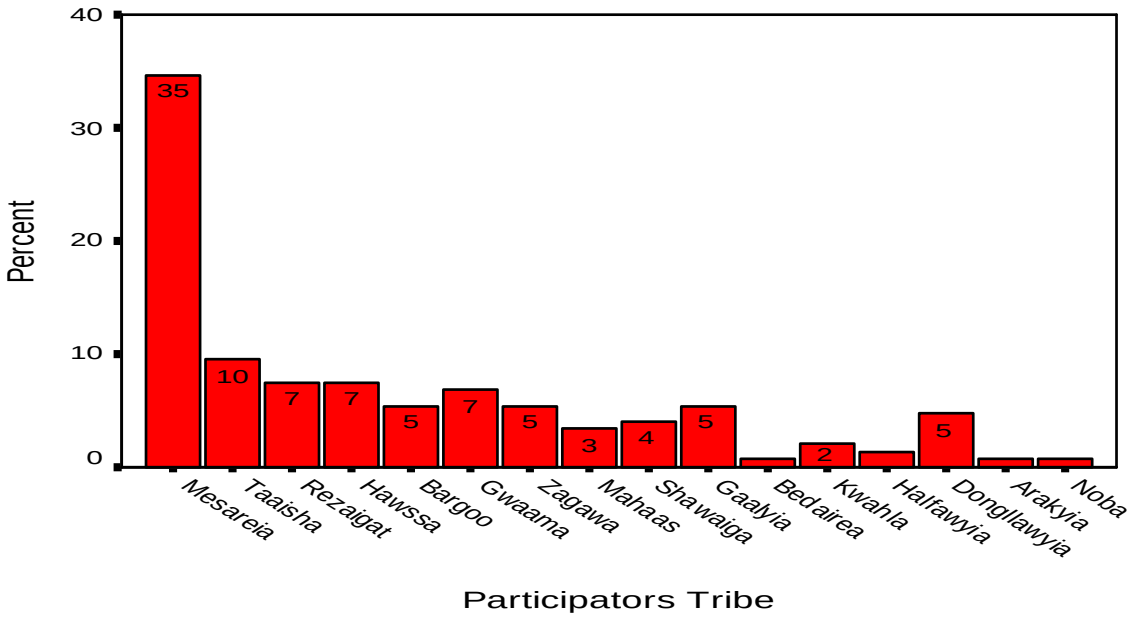

Figure 1: Frequency of participant's tribes in this study

DISCUSSION

SCA is a well-known haemoglobin-opathies considered as endemic disease in certain areas of the world. It has been recognized now that it may have a wide geographic distribution, whose clinical manifestations arise from the tendency of the haemoglobin HbS (or sickle haemoglobin) to polymerize and deform red blood cells into the characteristic sickle shape leading to various types of crises. In Sudan sickle cell disease considered as serious problem either in Khartoum state or in rural areas. The results of this study showed significant differences in HbF levels (Mean ± SD) when group T compared with group C, (P = 0.00), as shown in table 1. This finding agreed with several studies such as Multicenter Study of Hydroxyurea (MSH) and Griffin P. Rodgers *et al.* [(14)], Past Adragna N. C. *et al.* [(15)] , Steinberg MH and his colleagues, and recent such researches by Mary Catherine Beach and others [(16)]. Also this result is similar to the results of study done in India by Hraminder Singh *et al*. which reported that the level of HbF was increased in SCD patients treated with HU [(17)]. HU is now used as drug of choice in reduction of SCD crises.

CONCLUSIONS

HbF levels increased significantly in SCD patients treated with HU compared to the control group and this study proved that HU still had high efficiency to reduce the suffering of SCD patients.

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The Geometric Formulation of Electromagnetic Field

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ABSTRACT

This paper gives a geometrical interpretation of Maxwell's equations describing the electromagnetic field. The geometrical interpretation is not only based on 4 -dimensional Minkowski space, but extended to U 1 fiber bundle, describing the electromagnetic field. The gauge potential is described by the connection of this fiber bundle. A differential form of electromagnetic theory was also provided.

المستخلص

في هذه الورقة تم إعطاء دلالات هندسية لمعادلات ماكسويل لوصف الحقل المغنطيسي الكهربي. الدلالات الهندسية ليس لها أساس فقط على فضاء منكو سكاي ذو البعد −4. ولكن يمدد إلى طرفة الليفية (1) u الواصفة للحقل المغنطيسي الكهربي. الجهد القياسي يوصف بواسطة ربط هذه الحزمة الليفية. أيضاً تم إشتراط صيغة تفاضلية للنظرية المغنطيسية الكهربية.

KEYWORDS: Geometric Formulation, Minkowski space-time, continuity equation.

INTRODUCTION

Principal fiber bundles (PFBs) will be defined and some nontrivial examples will be given (e.i., double covering of the circle and the frame bundle of a manifold).The connection of PFBs with group $U(1)$ over space-time will physically be identified as the four-dimensional vector potential of electromagnetism.

The geometrization of electrical forces requires another dimension (the charge dimension) [(1)]. The resulting five dimensional spaces are actually PFBs with group $U(1)$. A connection (or gauge potential) en-dows the five component of a geodestic in five –space. If the charge component of a certain geodestices$q$, then the projection of this geodestic onto space-time is the non-geodestic path of an object charge q of subject to the force of gauge potential [(2,3)]. The invariance of the gauge potential under $U(1)$ action on the PFBs implies the conservation of charge. There are many other types of charge that respond to various other kind of force (e.g., isospin geometrization, of the force requires PFBs with larger non–abilian groups) and so five dimensions are not nearly enough. At the end of this paper we have provided a form of Maxwell's equations using the language of exterior derivative [(3)].

Principal Fiber bundle (PFB) consists of a manifold P (called the total space) [(4)], a Lie group G, a base manifold M, and a projection map $\pi : p \to M$ such that the following hold. For each $x \in M$ there is an open set U with $x \in U$ and diffeomorphism $T_u : \pi^{-1}\ U \to U \times G$ of the form $T_u\ P = \pi\ P\ , s_u\ p$ where $s_u : \pi^{-1}\ U \to G$ has property $s_u\ pg = s_u\ p\ g$ for all $g \in G, p \in \pi^{-1}\ U$. The map T_u is called a local trivialization (LT), or (in physical language) a choice of gauge.

Definition: Let $T_u : \pi^{-1}\ U\ \rightarrow U \times G$ and $T_v : \pi^{-1}\ V\ \rightarrow V \times G$ be two LTs of a PFB with a group. The transitions function from T_u to T_v be the map $g_{uv} : U \cap V \rightarrow G$ defined for $x = \pi\ P\ \in U \cap V$, by $s_u\ P\ \ s_v\ P\ ^{-1}$. Note that $g_{uv}\ x$ is independent of the choice of $P \in\ \pi^{-1}\ x$.

Because $s_u\ Pg\ \ s_v\ P\ g\ ^{-1} =$

$s_u\ P\ g\ s_v\ \ P\ g\ \ ^{-1} =$

$s_u\ P\ gg^{-1}\ s_v\ P\ ^{-1} =\ \ s_u\ P\ \ \ s_v\ P\ ^{-1}$

We have

(i) $g_{uu}\ y\ = e$ for all $y \in U$;

(ii) $g_{vu}\ y\ = g_{uv}\ y\ ^{-1}$ for all $y \in U \cap V$;

(iii) $g_{vu}\ y\ \ g_{vw}\ y\ \ g_{wu}\ y\ =$ e for all $y \in U \cap V \cap W$.

The transition functions describe how various product $U \times G, V \times G, \ldots..$ glue together to form the total space P [4]. Indeed P may be considered as the space obtained from disjoint union $U \times G\ \cup\ V \times G\ \cup \ldots..$ by identifying the point $x, g\ \in U \times G$ with $x, \acute{g}\ \in V \times G$ if $g = g_{uv}\ x\ \acute{g}$. Because of (i), (ii) and (iii), this identification is equivalence relation.

Definition: define a local section of a PFB $\pi :\ P \rightarrow M$ with group G to be a map $\sigma : U \rightarrow\ P\ \ U \subset M, U\ open$ such that $\pi\ ^{\circ}\sigma = I_u \equiv$ *the identity function on* $U\ \ x \rightarrow x$.

Theorem: There is a natural correspondence between local section and local trivialization, thus[5]: $\mathcal{G}\sigma_{g_{uv}}\ x\ ^{-1}\omega_u\ Y_x\ =$ $\sigma_{g_{uv}}\ x\ ^{-1}\omega_u\ Y_x\ \ g_{uv}\ x$.

Consequently the transform rule from ω_u to ω_v can be expressed as:

$\omega_v = g_{uv}^{-1}\ d\ g_{uv} + g_{uv}^{-1}\ \omega_v g_{uv}$.

Given a connection on1-form ω on a PFB $\pi: P \rightarrow M$ with group G, we can write any$X \in T_P\ P$ as $X = X^V + X^H$ where X^V is vertical $i.e, \pi_*\ X^V\ = 0$ and X^H is horizontal $i.e, \omega\ X^H\ = 0$.

Definition: The exterior derivative of $\varphi \in \Lambda^k\ P, \mathcal{G}$ is $D^{\omega}\varphi \equiv\ d\varphi\ ^{H} \in \Lambda^{k+1}\ P, \mathcal{G}$ where $d\varphi$ is the usual exterior derivative of φ , although the operator D^{ω} depends on ω. Because we consider functional on the space of connections and other situations where more than one connection is involved, we will usually not observe this custom.

Definition: The curvature of the $\varphi \in \Lambda^1\ P, \mathcal{G}$ is $\omega \in \Lambda^1\ P, \mathcal{G}$. That is $\Omega^{\omega} \equiv D^{\omega}\omega \in \Lambda^2\ P, \mathcal{G}$. When ω is regarded as a potential, Ω^{ω} is the field strength of ω [6].

Theorem (The Structural Equation): The curvature form is given by $\Omega^{\omega} = d\ \omega +$ $\frac{1}{2}\ \omega, \omega$ (i.e, $D^{\omega} = d\ \omega + \frac{1}{2}\ \omega, \omega$).

Maxwell's equations in Minkowski space-time:

Let M = R4 with coordinate $X^{\circ}, X^1, X^2, X^3\ = t, X, Y, Z$ and metric g such that $g\ \partial_{\circ}, \partial_{\circ}\ = 1$, $g\ \partial_i, \partial_i\ = -1$ for $i =$ 1,2,3 and $g\ \partial_i, \partial_j\ = 0$ for $i \neq j$.

then M, g is called Minkowski space.

Hodge star operator and wedge product:

On an oriented n-dimensional Riemannian manifold, the Hodge star is a linear function which converts alternating differential k – forms to alternating $n - k$ -forms. If ω is an alternating k – form, its Hodge star is given by [7]:

$\omega\ v_{k+1}\ ,\ldots\ldots\ldots, v_n\ = *_{\omega}\ \ v_1\ ,\ldots\ldots\ldots, v_n$ where $v_1\ ,\ldots\ldots\ldots, v_n$ is an oriented ortho-normal basis. Then the wedge product is the product in an exterior algebra. If α and β are differential k – forms of degrees p and q respectively, then

$$\alpha \wedge \beta = \ -1\ ^{pq}\beta \wedge \alpha \qquad (1)$$

It is not (in general) commutative, but it is associative,

$$\alpha \wedge \beta\ \wedge u = \alpha \wedge\ \beta \wedge u \qquad (2)$$

Exterior differential form of Maxwell's equations:

Consider the 2-form:

$$F = E_1 dx\Lambda dt + E_2 dy\Lambda dt + E_3 dz\Lambda dt + B_1 dy\Lambda dz + B_2 dz\Lambda dx + B_3 dx\Lambda dy.$$

For $dr = dx, dy, dz$ and $d\sigma = dy\Lambda dz, dz\Lambda dx, dx\Lambda dy$ [8], we employ the shorthand $F = E.dr\Lambda dt + B.d\sigma$. By simple computation, we obtain

$$dF = \nabla \times E + \frac{\partial B}{\partial t}.d\sigma\Lambda dt + \nabla.B\, d\tau$$

where $d\tau = dx \Lambda dy\Lambda dz$. Thus $dF = 0$ if $\nabla \times E + \frac{\partial B}{\partial t} = 0$ and $\nabla.B = 0$, which are two Maxwell's four equations (where E is the electric field and B is the magnetic field). Now $*F = E.d\sigma - B.dr\Lambda dt$ and so

$$d*F = \nabla.E\, d\tau - \nabla.B + \frac{\partial E}{\partial t}.d\sigma\Lambda dt$$

Now $\delta = -1^{g-1^{4k+1}} * d* = *d*$ on $\Lambda^k R^4$. Thus

$$\delta F = *d*F = \nabla.E\, dt - \nabla \times B + \frac{\partial E}{\partial t}.dr$$

$j \in \Lambda^1 R^4$ is defined by $j = \rho dt - j.dr$ Then

$$\delta F = j$$

is equivalent to the other. Let the maps $\rho: R^4 \to R$ and $j: R^4 \to R^3$ be the charge density and the current density [9]. The source 1- form two (unhomogeneous) Maxwell's equation $\nabla.E = \rho$ and $\nabla \times B - \frac{\partial E}{\partial t} = j$. Thus, the four Maxwell equations are summarized by $dF = 0$ and $\delta F = j$, and we can obtain:

$$0 = \delta^2 F = = \delta * j = *d\ \rho d\tau - j.d\sigma\Lambda dt - * \frac{\partial \rho}{\partial t} dt\Lambda d\tau - \nabla.j\, d\tau\Lambda dt = -\frac{\partial \rho}{\partial t} - \nabla.j\ .$$

Thus, we obtain the so-called *continunity equation*

$$\frac{\partial \rho}{\partial t} - \nabla.j = 0\ .$$

which says that charge is conserved.

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Molecular Characterization and Sensitivity Pattern of *Pseudomona aeruginosa* Among Patients With Different Diseases in Khartoum, Sudan

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ABSTRACT

The aim of this study was to determine the presence of the genes implicated in resistance to agents used for chemotherapy of infectious diseases caused by *Pseudomonas aeruginosa*. Seventy four *P. aeruginosa* strains were isolated from patients hospitalised in the Khartoum Teaching Hospital, Sudan. The strains were isolated in the period from February to June 2011 and serotyped studied at the research laboratory in Sudan University of Science and Theology. The isolates were recovered from patients with multiple types of infections, mostly respiratory tract, urinary tract and postoperative wound infection. Direct PCR technique was used to identify the genes implicated in antimicrobial resistance mechanisms. DNA extraction was skipped and the bacterial cell wall was denaturized in the first step of the reaction. The presence of IMP family genes, in the tested isolates of *P. aeruginosa* was identified, namely IMP-7 and IMP-10. This was the first report of the presence of IMP-7 and IMP-10 type genes in *P. aeruginosa* isolates in Khartoum State, Sudan.

المستخلص

هدفت هذه الدراسة إلى دراسة الزائفة الزنجارية من بين المرضى مع مختلف الأمراض في ولاية الخرطوم، السودان وتم كشف الزائفة الزنجارية بواسطة الكشف الجزئي ونمط الحساسية. وقد أجريت هذه الدراسة خلال الفترة من فبراير إلي يونيو 2011م، وشملت (74) عينة تحمل مختلف الأمراض من الزائفة الزنجارية تم جمعها من الجنسين ومختلف الأعمار والغالبية كانت من عينات مسحات التهاب الجروح والبول. هذه العينات تم جمعها واختبارها في مستشفي الخرطوم التعليمي، وأجريت دراستها في معمل البحوث بجامعة السودان للعلوم والتكنولوجيا، واستخدمت الاختبارات البيوكيميائية للتعرف علي البكتيريا المعزولة بواسطة استخدام اختبار (بي سي آر) باستخلاص الـ (دي ان. آى). وقد أجري تفاعل البلمرة المتسلسل لعائلة أي أم بي للجين (آي أم بي – سفن) والجين (آي أم بي – تن) للزائفة الزنجارية المعزولة. خلصت الدراسة إلى أن هذا يعد أول تقرير لوجود أنواع الجينين (أي أم بي – سفن) و(أي أم بي – تن) بالنسبة للزائفة الزنجارية في ولاية الخرطوم – السودان.

KEYWORDS: *Pseudomonas aeruginosa*; molecular techniques; PCR.

INTRODUCTION

Nosocomial infections are an important source of morbidity and mortality in many hospitals affecting millions of patients each year [(1)]. *P. aeruginosa* is one of the most important nosocomial pathogens, being responsible for various types of infections with more and more limited therapeutic options [(2)]. Infection due to *P. aeruginosa* continues to be a major cause of mortality among critically ill and immuno-compromised patients despite the development of newer and more powerful antibiotics. *P. aeruginosa* is characterized by

inherent resistances to a wide variety of antimicrobials. Its intrinsic resistance to many antimicrobial agents and its ability to develop multidrug resistance impose a serious therapeutic problem [3]. Multi-drug-resistant (MDR) strains of *P. aeruginosa*, defined as resistant to at least three of the following antibiotics: ceftazidime, imipenem, gentamicin or ciprofloxacin, are often isolated from patients exposed to prolonged intensive care-type therapies [4]. Its resistance to anti-pseudomonal -lactams, advanced generation of cephalosporins, monobactams and carbapenems is also an increasing clinical problem. Carbapenems, mainly imipenem and meropenem, are potent agents for the treatment of infections due to MDR *P. aeruginosa*. The immunoevasive nature of *P. aeruginosa*, as well as its acquisition of multi-drug resistance makes elimination of this organism a particular challenge. Yet antibiotic resistance itself does not confer enhanced virulence [5], and therefore the ability to discriminate between virulent versus non- virulent phenotypes among MDR isolates would be a major step in predicting the particular threat of a colonizing strain of *P. aeruginosa*.

Historically, the analysis of nosocomial pathogens has relied on a comparison of phenotypic characteristics such as biotypes, serotypes and antimicrobial susceptibility profiles [6]. This approach has begun to change over the past 2 decades, with the development and implementation of new technologies based on DNA or molecular analysis. Studies of microbial pathogenicity at the molecular level have made substantial contributions to understanding of the epidemiology, clinical manifestations, diagnosis, treatment, and immunoprophylaxis of infectious diseases. One of the most exciting and profound technical advances in the past years has been the development of nucleic acid amplification techniques and their application to the study of microbial pathogenesis and the diagnosis of infectious diseases. Comparing with traditional methods, molecular analysis has a higher accuracy, and results are obtained much faster and much cheaper.

The goal of this study was to identify the presence of bacterial genes involved in multiple resistances to antimicrobials in *P. aeruginosa* using the PCR method.

MATERIALS and METHODS

Clinical isolates: Seventy four clinical isolates of *P. aeruginosa* were obtained from the Clinical Hospital for Infectious Diseases in Khartoum Teaching Hospital, Sudan medical center. The bacterial isolates were collected between February to June 2011from patients with multiple types of infections, mostly respiratory tract, urinary tract, postoperative wound (Septicaemia, Skin and soft tissue infection, Ecthyma gangrenosum), Eye, External ear, Respiratory tract (pulmonary infection), Gastrointestinal (Diarrheal infection), Chronic otitis media and external and Melioidosis infections. The isolates were identified with Compact for Gram-Negative Identification, card 2GN (bioMerieux-Vitek, Inc., Hazelwood, Mo.).

Antimicrobial susceptibility testing

Antimicrobial susceptibility testing was performed by Kirby-Bauer disk diffusion method. The minimum inhibitory concentration (MIC) was done with Vitek 2 Compact system for Gram-Negative bacteria, according to the recommendations of the Clinical and Laboratory Standards Institute (CLSI).

The MICs of: ciproflox- acin, pefloxacin, gentamicin, amikacin, tobramycin, ticarcillin, piperacillin, carboxipenicillin and ureidopenicillin with betalactamases, piperacillin, ticarcillin, ceftazidime, cefepime, cefpirome, aztreo- nam and colisin were determined. *P. aeruginosa* ATCC 27853, and *Escherichia*

coli ATCC 25922 were used as reference strains in susceptibility testing.

Preparation of samples and DNA amplification

Direct PCR technique was used, DNA extraction was skipped and the bacterial cell wall was denaturized in the first step of the reaction. One or two bacterial colonies from a plate that was incubated overnight were suspended in 100 ml of sterile water and then diluted to a concentration of approximately 10^6 CFU/ml. an eliquot of 3 « l of the suspension was used as the template for amplification by PCR. Using this technique skips the expensive DNA extraction and minimizes self contamination of workers [7].

PCR protocol

A typical 20 « 1 PCR mixture with 4 « 1 master mix which contained 0.5 « 1 5 x PCR reaction buffer, 0.2 « l of each primer, 200 « M concentrations of each dNTPs, 2.5 « 1 $MgC1_2$, 12.5 mM (2 mM final concentration), 0.75 U of *Taq* polymerase, blue dye (Migration equivalent to 3.5 - 4.5 kb DNA fragment), yellow dye (Migration rate in excess of primers in 1% agarose gel < 35 -45 bp and 3 « l bacterial suspension. PCR was performed in a Thermocycler, (Gradient Palm-Cycler™, Corbett Life Science). The parameters for amplification were as follows: initial denaturation at 94°C for 4 min, 30, cycles of: 1 min each at 94°C, 1 min. at 57-62°C (depending on the primer), 1 min. at 72°C and a final extension step at 72°C for 10 min. Amplicons have been separated on 1% agarose gel, stained with ethidium bromide. The optimization of PCR was made after McPherson and Møller [8], and Roux [9]. The primers (Table 1) were designed with "Pick Primers" programe according to the seque1nces found at NCBI data base.

Table (1) Primer used for PCR amplification of Pseudomonas aeruginosa ATCC 27853

Target gene	Function	Sequence ($\overline{5}-\overline{3}$)	Amplification size (pb)	Accession number
IMP- 7	-lactamase	AAGGCAGTATCTCCTCTCATTTC/ ACTCTATGTTCAGGTAGCCAAACC	243	EF606914
IMP-10	-lactamase	AATGCTGAGGCTTACCTAATTGAC/ CCAAGCTTCTATATTTGCGTCAC	388	DQ288156

IMP- 7:

IMP-7 Outer - F: AAGGCAGTATCTCCTCTCATTTC/

IMP-8 Outer - R: ACTCTATGTTCAGGTAGCCAAACC

Outer - R1: GGTTTGGCTACCTGAACATAGAGT

IMP-10 :

IMP-10 Outer-F: AATGCTGAGGCTTACCTAATTGAC/

IMP-11 Outer – R: CCAAGCTTCTATATTTGCGTCAC

Outer- R1: GTGACGCAAATATAGAAGCTTGG

RESULTS

Phenotypic traits: Out of one hundred and fifty samples, seventy four isolates were *P. aeruginosa*. Most of these tested strains were highly sensitive to antibiotic drugs. Seventy strains were susceptible to imipenem, sixty to ceftazidim and sixty of them were sensitive to ciprofloxacin and highly resistant to others antibiotics, fifty strains were resistant to carbenicillin and fourty strains were resistant to piperacillin. Fifty seven isolates were recognized to have IMP-7 and IMP-10, only twenty six isolates produce IMP-7 and the remainder thirty one isolates had IMP-10. Five isolates of them were giving same reactives in tow genes IMP-7 & IMP-10.

PCR used to detect IMP family tests (IMP-7 and IMP-10) genes yielded PCR products of the expected sizes. Amplicons of the expected sizes were obtained for IMP-7 and IMP-10 genes in the isolates that were resistant to antimicrobials (Fig. 1 to 3). At the sensitive isolates amplicons were not obtained

DISCUSSION

Pseudomonas aeruginosa is one of the most important nosocomial pathogens, being responsible for various types of infections with more limited therapeutic options. One of the most alarming characteristic of *P. aeruginosa* is its low antibiotic susceptibility. Since the first report of acquired metallo- -lactamases (MBL) in Japan in 1994 [10] genes encoding IMP-types (IMP-7 and IMP-10) enzymes have spread rapidly among *Pseudomonas spp*. [5,9,11]. Acquired metallo- -lactamases (MBLs) are mostly encoded by integron-borne genes and confer resistance against all -lactams except for the monobactams. IMP-types MBLs were reported from several countries [12]. The prevalence of MBL- producing Gram-negative bacilli has increased in some hospitals, particularly among clinical isolates of *P. aeruginosa* [13]. Since MBL production may confer phenotypic resistance to virtually all clinically available -lactams, the continued spread of MBL is a major clinical concern [14].

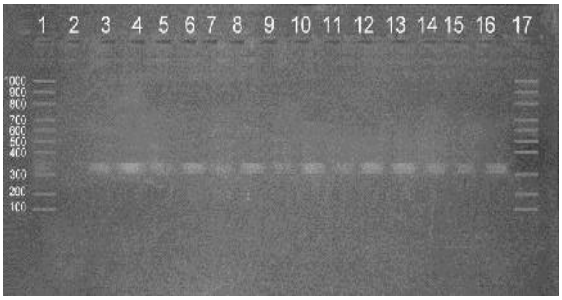

Figure (1) Agrose gel electrophoresis of PCR products after amplification of (IMP-7) gene. Lanes: 1 and 17 – molecular weight marker (O'Range Ruler 100bp DNA ladder); 2 – Negative control; 3 – 16

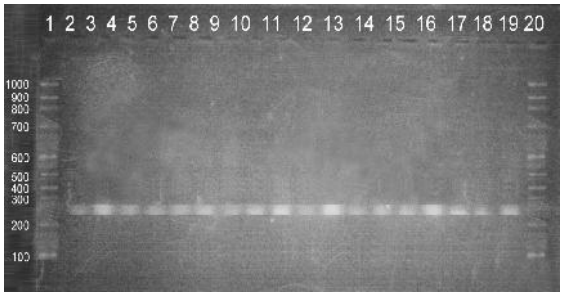

Figure (2) Agrose gel electrophoresis of PCR products after amplification of (IMP-10) gene. Lanes: 1 and 20 – molecular weight marker (O'Range Ruler 100bp DNA ladder); 2 – Positive control; 3 – 19 different strains of P. aeruginosa (IMP-10 gene

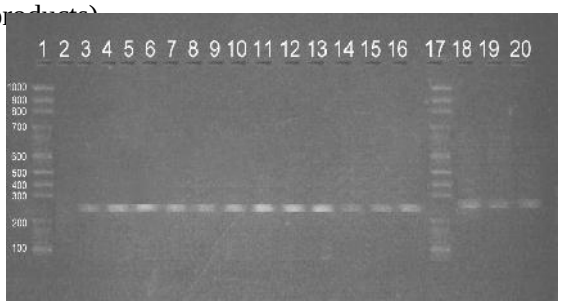

Figure (3) Agrose gel electrophoresis of PCR products after amplification of (IMP-7 and IMP-10) genes. Lanes: 1 and 17 – molecular weight marker (O'Range Ruler 100bp DNA ladder); 2 Negative control; 3 and 16 different strains of P. aeruginosa IMP-7 gene products;

With the recent detection of IMP-7 and IMP-10 producing strains in several Eastern-European countries [1], the appearance of MBL-producing clinical isolates of *P. aeruginosa* can be anticipated in Romania. Our results suggest the lack of these IMP-7 and IMP-10 genes in the tested isolates. Further studies are necessary to conclude that this genes family is not present in the *P. aeruginosa* isolates circulating in this area of the country.

Extended-spectrum -lactamases (ESBLs) that confer resistance to oxyimino- -lactams are frequently plasmid encoded. *Pseudomonas* extended resistant (PER). -lactamases are one of the rarer ESBL families; however, their prevalence may be increasing [2]. Since 1995, PER-producing organisms were disseminating in Italy [15, 16] and, more recently, in Belgium [17], France [18], Spain [19], Romania [20], Hungary and Serbia [21], Korea [22], Japan [5], and China [23].

The present study revealed that IMP-7 and IMP-10 genes are implicated in antimicrobial resistance mechanisms in MDR tested isolates of *P.aeruginosa*. Further studies are necessary to conclude if other genes contribute to the antimicrobial resistance in *P.aeruginosa* strains in Sudan.

This study is the first report of presence of IMP type genes in *P. aeruginosa* isolates, in Khartoum State, Sudan. A regular screening and monitoring system should be set up to prevent the spread of the resistance determinants in the country.

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Phenotypic and Genotypic Detection of *Moraxella Catarrhalis* Among Patients With Respiratory Tract and Otitis Media Infections

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ABSTRACT

Respiratory tract infections are considerably prevalent worldwide and identifying their aetiological agents is of great medical and therapeutic value. The aims of the present study are to determine the prevalence of *Moraxella catarrhalis* among Sudanese patients infected with respiratory tract infections (upper and lower) and to determine the antibiotic sensitivity pattern of *Moraxella catarrhalis* isolates as well as the risk factors. Samples, which were collected from four hundred patients with upper and lower respiratory tract infections, were cultured. Then suspected *Moraxella catarrhalis* colonies were biochemically tested. Next, positive isolates were confirmed using polymerase chain reaction technique. Finally, antibiotic sensitivity tests were carried out and beta-lactmase production was inspected for each isolate using nitrocefin disks. After tests, 19 (4.7%) from the collected samples were positive for *Moraxella catarrhalis*. Of these, 15 (78.9%) isolates showed typical bands of *M. catarrhalis* while 4 (21.0%) isolates were negative. This study shows that *Moraxella catarrhalis* is an important respiratory tract pathogen in Sudan. The emergence of antibiotic resistance in *Moraxella catarrhalis* suggests that the incidence of these infections may continue to rise.

المستخلص

تعتبر امراض الجهاز التنفسي من الامراض الشائعة والمنتشرة علي مستوي العالم , وتحديد المسبب يعتبر ذو فائدة تشخصية وعلاجية يساهم في تعجيل شفاء المريض ويقلل من نسبة المضاعفات.اجريت هذه الدراسة لتحديد وجود باكتيريا موراكسيلا النزلية في شريحة مرضي الجهاز التنفسي العلوي والسفلي وايضا هدفت الدراسة لتحديد حساسية المضادات الحيوية لهذه البكتريا وايضا العوامل التي يمكن ان تودي الي الاصابة بالبكتريا.جمعت العينات من 400 مريض وشملت البلغم ومسحه من الاذن وتم زراعة العينات في اوساط غذائية تساعد البكتريا علي النمو وتم عمل الاختبارات الكيميائية لتأكيد نوع البكتريا المستهدفة .ثم تم اجراء اختبار البلمرة التسلسلي للبكتريا المعزولة التي تم التعرف علي انها باكتريا موراكسيلا النزلية وبعد ذلك تم عمل اختبار الحساسية للمضادات الحيوية واختبار انزيم البيتا لاكتميز. تم عزل 19 ميكروب باكتيريا موراكسيلا النزلية بنسبة 4,7% وتم تحديد نسبة 78,9% من البكتريا المعزولة موجبة لتفاعل البلمرة التسلسلي ونسبة21,1 % سالبة لهذا الاختبار. وخلصت الدراسة الي وجوب التعاطي مع هذه البكتريا علي الوجه السليم خاصة وان كل البكتريا المعزولة افرزت انزيم البيتا لاكتميز.

KEYWORDS: Beta-lactamase, tributyrin, polymerase chain reaction, nitrocefin disks.

INTRODUCTION

Moraxella catarrhalis, formerly called Neisseria catarrhalis and Branhamella catarrhalis, is a Gram negative aerobic diplococcus frequently found as a commensal of upper respiratory track [1, 2]. Over the last 20 to 30 years, the bacterium has emerged as a genuine pathogen and now is considered as an important cause of upper respiratory tract infections in otherwise healthy children and elderly people [3-5]. Moreover, Moraxella catarrhalis is an important cause of lower respiratory tract infections, particularly in adults with chronic obstructive pulmonary disease (COPD) [6].

Additionally, it has been reported as one of the main pathogens of community-acquired pneumonia (CAP) [7-11]. In immunocompromised hosts, the bacterium can cause a variety of severe infections including pneumonia, endocarditis, septicemia and meningitis [3,12]. *M. catarrhalis* is now accepted as the third most respiratory tract pathogen after *Streptococcus pneumonia* and *Hemophilus influenza* [3, 8].

Reports of hospital outbreaks of respiratory diseases caused by *M. catarrhalis* have established the bacterium as a nosocomial pathogen [13, 14].

A recent study recognized *M. catarrhalis* as a pathogen in cleft palate repairs [15].

M. catarrhalis population may be subdivided into two distinct genetic lineages phenotypically characterized by their ability to resist the destructive effect of human serum (i.e., complement resistant versus complement sensitive) and difference in their ability to adhere to human epithelial cells [16]. Recent report indicates that a population expansion (including the acquisition of virulent genes) probably occurred within seroresistant lineage of *Moraxella catarrhalis* around the time of hominid expansion 5 million years ago [17].

There has been rapid acquisition and spread of Beta–lactmase resistance of *Moraxella catarrhalis* in the last 20 to 30 years to the extent that approximately 95% to 99% of clinical isolates now appear to resist one or more beta lactamase [18-20].

MATERIALS and METHODS

Control strains of *Moraxella catarrhalis* (American type culture collection) were used as the reference strains. These included: ATCC2324, ATCC 25238, ATCC 25240, and ATCC 49143.

Clinical Samples

A total of 110 specimens were collected from children suffering from middle ear infections with ear discharge from Khartoum center of ENT, head and neck surgery and a total of 290 sputum samples were collected from patients attending Alshaab Teaching Hospital and Soba university hospital with signs and symptoms of lower respiratory tract infections. Regarding otitis media, the diagnosis was made by a pediatrician, a family physician and an otolaryngologist. Concerning sputum, samples were collected in clean, wide mouth, and leak proof specimen containers. Data were collected using standardized questionnaire eliciting information such as date, name, gender, smoking, underlying disease, and history of antibiotic treatment.

Bacterial Culture

The samples were inoculated on sheep blood agar chocolate blood agar , Columbia blood agar supplemented with5% sheep blood, vancomycin, amphotricin B, and acetozolamide [21], and incubated over night aerobically at 37^{o}C with 5% CO_2 as well as at room temperature. *Moraxella catarrhalis* bacteria were identified according to colonial appearance, Gram stain, catalase reaction, oxidase reaction, reduction of nitrate, ability to grow on nutrient agar at

room temperature, DNAs production and tributyrin test.

All the isolates were tested for beta – lactmase production using nitrocefin disks (Sigma- Aldrich, Germany).

DNA Extraction

Isolates were incubated over night at 37ºC on blood agar plates using GF-1 bacterial DNA extraction kit from (Vivantis Company, Germany). This kit uses a specially treated glass filter membrane fixed into a column to efficiently bind DNA in the presence of high salt. This kit applies the principle of mini column spin technology and the use of optimized buffers ensures only DNA is isolated while cellular proteins, metabolites, salts and other low molecular weight impurities are subsequently removed during the washing steps

Polymerase Chain Reaction

Moraxella catarrhalis isolates were confirmed using GenePack DNA PCR tests provided by (GeneON, Germany). The latter utilize a technique based on polymerase chain reaction. Thus, the specific DNA was detected in the suspected colonies.

All the reagents required for PCR (Tag DNA polymerase, dNTPs, specific primers, salt and stabilizers) were lyophilized in PCR tubes (0.2 ml).

The PCR thermocycler parameters used were initial denaturation step at 95ºC for 3 minutes, followed by repeated 37 cycles composed of three stages; 95ºC for 30 seconds, 56ºC for 60 second and 72ºC for 2 minutes, and then a final extension step at 72ºC for 10 minutes. After thermocycling, the samples were loaded on agarose gel stained by ethidium bromide. Typical band positive for *Moraxella catarrhalis* was 550 bp.

Susceptibility of Moraxella *catarrhalis* to antimicrobial agents was determined by Kirby Beauer disc diffusion method on Muller Hinton agar. Antibiotics tested included amoxyclav, ampicillin, azithromycin, ceftazidime, ceftriaxon, cephalexin, cephotaxime, chloramphenicol, ciprofloxacin, co- trimoxazole, gentamycin, and erythromycin.

RESULTS

Moraxella catarrhalis was isolated from 19 patients; 12 males (63.2%) and 7 females (36.8%). 14 of the patients were adult (73.7%) with the mean of age being 48.5 (range: 22-72) and 5 were children (26.3%) with the mean of age being 23month (range: 5-40 month).

14 (4.8%) out of 290 sputum samples, showed *Moraxella catarrhalis* isolates. 5 (4.5%) out of 110 specimens collected from children with ear discharge, confirmed *Moraxella catarrhalis* isolates (table 1).

Table 1: Clinical data of patients with Moraxella catarrhalis isolates

Clinical datum	Children	Adult	
		Present (%)	Absent (%)
Otitis media	5	-	-
Liver disease	-	1 (5.3)	18 (94.7)
Diabetes mellitus		2 (10.5)	17 (89.5)

Chronic bronchitis	-	1 (5.3)	18 (94.7)
Pneumonia	-	2(10.5)	17 (89.5)
Chronic cough	-	8 (42.1)	11 (57.9)
Smoking	-	3 (15.8)	16 (84.2)
Age more than 50 years	-	8 (42.1)	11(57.9)
History of pulmonary TB		2 (10.5)	17 (89.5)

All the isolates of *Moraxella catarrhalis* were Gram-negative diplococci, hockey puck sign positive, catalase positive, oxidase positive, DNAse producing, nitrate reducing, tributyrin positive and able to grow on nutrient agar at room temperature. All isolates showed beta lactmase production. Characteristics of Moraxella catarrhalis used in its identification are listed in table 2.

Table 2: Characteristics of Moraxella catarrhalis used in its identification

Colonial morphology on blood agar	round, opaque colonies
Colonial morphology on chocolate blood agar	pinkish-Brown hockey puck
Colonial morphology on semi selective media	round, opaque colonies
Gram stain	Gram-negative diplococcic
Oxidase test	Positive
Catalase test	Positive
Deoxyribonuclease (DANas) test	Positive
Reduction of nitrate	Positive
Growth on nutrient agar at room temperature	Positive
Tributyrin test	Positive

When PCR was performed to confirmed *Moraxella catarrhalis* isolates 15 (78.9%) isolates showed typical band of 550 bp size as indicated by the standard DNA marker while 5 samples were PCR negative (see figure 1).

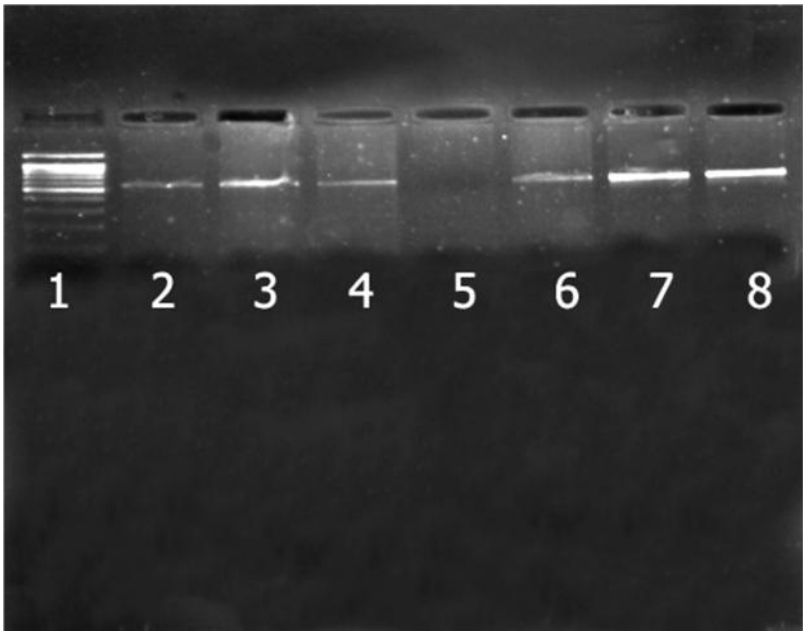

Figure1: The amplicon of Moraxella catarrhalis after PCR on 2% agarose gel: lane1= marker; lane 2= positive control; lane 5= negative control; lanes 3-4 and lanes 6-8 isolates positive for Moraxella catarrhalis (550 bp).

All isolates were noted to be sensitive to amoxyclav, azithromycin, ceftazidime, ceftriaxon, cephalexin, cephotaxime, chloramphenicol, ciprofloxacin and cotrimoxazole and 90% were sensitive to erythromycin, while they were wholly resistant to ampicillin.

DISCUSSION

Moraxella catarrhalis is a Gram negative aerobic diplococcus which is frequently found as a commensal of upper respiratory tract and has been recovered exclusively from human over the last few decades [22].

It has emerged as a genuine pathogen, it is now considered as an important cause of upper respiratory tract infections in otherwise healthy children and elderly adult [22,23]

The *Moraxella catarrhalis* rate in the present study is 4.7%. This is semi similar to that observed by Nishioka *et al.* who had carried out a study in Japan and found the isolation rate of *Moraxella catarrhalis* to be 4.26% [24]. A higher rate (6.9%) was detected among elderly population by Tamang and his fellows [25].

Pollard and colleagues in 1986 found the isolation rate of *Moraxella catarrhalis* was 5.3% which is more or less similar to that of present study [26].

Unlike the aforementioned findings, Ahmed *et al.* in 1994 and Sarubbi *et al.* in 1990 found the rate of isolation of *Moraxella catarrhalis* to be much lower (0.89 and 2.5%, respectively) [27,28]. On the other hand, Hager et al in 1987 found the rate of isolation of *Moraxella catarrhalis* in pure culture was 61% [6], and this figure is much higher from that of current study.

In the present study, 5 (4.5%) *M. catarrhalis* was isolated from children aged from 1 month to 5 years with a percentage of (4.7%). The bacteria were not isolated from those whose ages ranged between 6 and 15 years old**.** This result disagrees with the result obtained by Constantinescu et al who isolated *M. catarrhalis* in culture in 9% of children younger than 5 years and 33% of children aged 6-10 years in his study [29]. Nearly similar to the results obtained in this research, a study by Broides *et al.* in 2009 had reported that *M. catarrhalis* occur in 4.8% of children less than 5 years [30]. Another result reported by Vergison in 2008 shows that the *M. catarrhalis* proportion in children with otitis media was (3–20%) [31].

In some reports, an increasing rate of *M. catarrhalis* isolation from the middle ear fluid (MEF) in AOM has been shown. Kilpi et al have reported an increase from 10% to 23% within 15 years, and a similar pattern has also been reported in the United States [32]. In Costa Rica, the prevalence of *M. catarrhalis* isolated from the MEF of children with AOM aged 3–144 months increased from 2.5% of all pathogens during 1992–1997 to 7% during 1999–2004 and was most commonly found in children aged <24 months during the dry season [33]. However, in the present study, *M. catarrhalis* has been consistently found as a single pathogen in only~1% of MEF isolates. The reasons for the higher relative importance of *M. catarrhalis* as a pathogen in AOM in certain geographical areas and the different rates of *M catarrhalis* AOM in other areas are unknown.

The underlying factors which are associated with *Moraxella catarrhalis* were studied previously. Age was a critical determinant of the pathogenic significance of the isolates of *Moraxella catarrhalis*: the more the advanced age is, the greater the pathological significance of the isolates [25, 34].

In the present study, being of advanced age (42.1), having chronic cough (productive cough) (42.1) and smoking (15.8) are the common predisposing factors. This agrees with the study done by Tamang *et al.* which found the most important predisposing factors were old age and history of smoking [25].

Studies have shown that the elderly are at greater risk of respiratory tract infections which

are caused by *Moraxella catarrhalis* when compared to young adults. Most of these patients had underlying lung disease and other conditions like diabetes mellitus, corticosteroid therapy and malignancy [8, 26, 35, 36].

In the present study rate of *M. catarrhalis* isolated from sputum of elderly is (42.1%). The significance of this finding is strengthened by the fact that after the age of 50 years there may be a reduction of immunoglobulin G and M titers along with damage of the respiratory tract by viral infections that may promote invasion by *M. catarrhalis* [3].

This study noted that there has been no strong correlation between chronic obstructive pulmonary disease (COPD) and *Moraxella catarrhalis* in Sudan. This observation disagrees with that of Timothy and his fellows who concluded that *M.* catarrhalis likely causes approximately 10% of COPD [37].

Antibiotic sensitivity pattern showed high level resistance to penicillin, this is due to rapid increase in prevalence of beta lactamase producing strains of *Moraxella catarrhalis*. In the present study, 100% of isolates are B-lactamase producers. This is semi similar to the observation of Anita et al in 2011 and Hanan in 2000 who found that the proportion of beta lactamase producing isolates was 84 % [36, 38]. Proportions higher than 90% were found in several studies. In 2007, Esel and his fellow found that the percentage of b-lactamase producing strain of *Moraxella catarrhalis* was 94% [39]. Another report in turkey found that 96.9% of clinical isolates and 90.6% of carrier strains produced B-lactamase [40]. A recent study conducted in Taiwan showed the rate of beta lactamase production was 97.8% [41]. The dramatic increase in frequency of beta lactamase producing *M. catarrhalis* could be regarded as the fastest dissemination of any known beta-lactamase producing bacteria within all bacterial species [42].

The beta lactamase activity of *M. catarrhalis* is inhibited by beta- lactam inhibitors such as clavulanic acid and sulbactam[8]. This fact can be reflected by the finding that all isolates in this study were sensitive to amoxycilin- clavulanic acid combination. However, this sensitivity pattern is in discrepancy with that reported by Tamang et al which found that only 4% of *Moraxella catarrhalis* isolates were resistant to both amoxicillin-clavulanate and ceftriaxone and 8% were resistant to ciprofloxacin [25].

The present study shows that all isolates were sensitive to amoxicillin-clavulanate and ceftriaxone. This observation is similar to that of Tamang *et al.* [25].

It has been demonstrated in vitro that *Moraxella catarrhalis BRO* enzyme can confer protection against B-lactam to other co-existing respiratory pathogens residing in the host. This phenomenon, referred to as indirect pathogenicity, of *Moraxella catarrhalis* may lead to antibiotic failure when treating a mixed infection containing both susceptible bacteria and resistant *Moraxella catarrhalis* strains [8, 23].

This, along with the great increase in the percentage of strains producing B-lactamase, makes it necessary to report the isolation of *Moraxella catarrhalis* when seen both in isolation and when co-existing with other pathogens. This will help in the selection of appropriate antibiotics and also in combating infections by other pathogens which may be other- wise protected by B-lactamase produced by *Moraxella catarrhalis.*

CONCLUSIONS

This study shows that *Moraxella catarrhalis* is an important respiratortract pathogen in adults and children in Sudan. The emergence of antibiotic resistance in *Moraxella catarrhalis* suggests that the

incidence of these infections may continue to rise.

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Group Invariant Solutions of Partial Differential Equations

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ABSTRACT

In this paper we utilize the symmetry group of differential equations in reducing the number of variables, thus reducing the order of differential equations. The reduction procedure is discussed thorouly. We also illustrate the procedure with some examples.

المستخلص

استخدمنا في هذه الورقة تماثل المعادلات التفاضلية الجزئية في تخفيض عدد المتغيرات، وبذا تخفيض رتبة المعادلة التفاضلية. في هذا البحث ناقشنا بإتقان وتفصيل هذه المسألة ووضحناها ببعض الأمثلة.

***KEYWORDS*:** symmetry group, invariant solutions, partial differential equations.

INTRODUCTION

The methods used to find group-invariant solutions generalizing the well known techniques for finding similarity solutions, provide a systematic computational method for determining large classes or special solutions. These group-invariant solutions are characterized by their invariance under some symmetry group of the system can all be found by solving a system of differential equations involving r fewer independent variables than the original system [1,2].

Consider a system of partial differential equations Δ defined over an open subset $M \subset X \times U \approx R^p \times R^q$ of the space of independent and dependent variables. Let G be a local group of transformations acting on M. A solution u = f (x) of the system is said to be G -invariant if it remains unchanged by all the group transformations in G, i.e. for each g ∈ G [3].

The functions f and g . f agree on their common domains of definition. For example, the fundamental solution u = log $(x^2 + y^2)$ for the tow-dimensional Laplace equation $u_{xx} + u_{yy} = 0$ is invariant under the one-parameter rotation group:

SO(2) : (x,y,u) → (xcosΘ - ysinΘ, xsinΘ + ycosΘ,u) acting on the independent variables x,y. We can define a G -invariant solution of a system of partial differential equations as solution u = f (x) whose graph $S \equiv (x, f\ x\ ) \subset M$ is a locally G – invariant subset of M.

If G is a symmetry group of a system of partial differential equations Δ then, under some additional regularity assumptions on the action of G, we can find all the G – invariant solutions to Δ by solving a reduced system of differential equations, denoted by $\Delta_{G,}$ which will involve fewer independent

variables than the original system Δ [4].

Method of reducing the variables in partial differential equations

i) Find all the infinitesimal generators υ of symmetry group of the system using the basic prolongation methods.

ii) Decide on the "degree of symmetry" s of the invariant solutions. Here1 ≤ s ≤ p will correspond to the dimension of the orbits of some subgroup of the full symmetry group. The reduced systems of differential equations for the invariant solutions will depend on p – s independent variables [5]. Thus to reduce the system of partial differential equations to a system of ordinary differential equation, we need to choose: s = p -1.

iii) Find all s –dimensional subgroups G the full symmetry group found in part (i). This is equivalent to finding all s –dimenional subalgebra of the full Lie algebra of infinitesimal symmetries υ. To each subgroup or sub algebra there will correspond a set of group-invariant solutions reflecting the symmetries inherent in G itself.

iv) Fixing the symmetry group G we construct a complete set functionally independent invariants, which we divide into two classes

$$y^1 = \eta^1(x,u), \ldots\ldots\ldots, y^{p-s} = \eta^{p-s}(x,u) \qquad \upsilon^1 = \xi^1(x,u), \ldots\ldots\ldots, \upsilon^q = \xi^q(x,u) \quad \ldots. (1)$$

corresponding to the new independent and dependent variables, respectively. If G acts projectably, the choice of independent and dependent variables is prescribed by requiring the $\eta^{i,s}$ to be independent of u; in the more general ease, there is quite a bit of freedom in this choice [6], and different choices lead to seemingly different reduced systems, all of which are related by some form of "hodograph" transformation.

v) Provided G acts transversally, we can solve eqn. (1) for p – s of the x' s, which we denote by $\bar{x}$, and all of the u's in term of y, υ and the remaining s- parametric variables $\bar{x}$.

$$\bar{x} = \gamma(\bar{x}, y, \upsilon), \quad u = \delta(\bar{x}, y, \upsilon) \quad \ldots..(2)$$

Furthermore, considering υ as a function of y , we can use (1) , (2) and chain rule to differentiate and thereby find expressions for the x- derivatives of any G-invariant u in term of y, υ, y –derivatives of υ and the parametric variables $\bar{x}$,

$$u^n = \delta^n(\bar{x}, y, \upsilon^n) \quad \ldots\ldots 3$$

vi) Substitute the expressions (2) and (3) into the system $\Delta(x,u^{(n)}) = 0$, the resulting system of equations will always be equivalent to a system of differential equations for υ = h(y) independent of the parametric variables $\bar{x}$.

$$\Delta_G(y, \upsilon^n) = 0 \quad \ldots..(4)$$

vii) Solve the reduced system (4), for each solution υ = h(y) of Δ_G there corresponds a G - invariant solution u = f(x) of the original system, which is given implicitly by the relation:

$$\xi(x,u) = \eta(x,u) \quad \ldots\ldots(5)$$

Repeating (iv) through (vii) for each symmetry group G determined in step (iii) will yield a complete set of group – invariant solutions for our systems.

Applications of the method of reducing the variants in partial differential equations:

Example (1)

Consider the one-parameter scaling group

$$x, t, u \rightarrow \lambda x, \lambda t, u, \lambda \in R^+,$$

acting on $X \times U \approx R^3$. On the upper half space M ≡ {t > 0}, the action is regular, with global independent invariants y = x/t and υ = u. If we treat υ as a function of y, we can compute formulae for the derivatives of

u with respect to x and t in terms of y, x and the derivatives of υ with respect to y, along with a single parametric variable which we designate to be t, so that x will be the corresponding principal variable [7], by using the chain rule, that if $u = \upsilon = \square\, y = \square\, \frac{x}{t}$ then:

$u_x = t^{-1} \upsilon_y$, $u_t = - t^{-2} x\upsilon_y = - t^{-1} y\upsilon_y$

Further differentiations yield the second order formula:

$u_{xx} = t^{-2} \upsilon_{yy}$, $u_{xt} = - t^{-2} (y\upsilon_{yy} + \upsilon_y)$

$u_u = - t^{-2} (y^2\upsilon_{yy} + 2y\upsilon_y)$ ……(6)

One the relevant formulae relating derivatives of u with respect to x to those of υ with respect to y, have been determined, the reduced system of differential equations for the G –invariant solutions to the system Δ is found by substituting these expressions into the system wherever they occur. In general this leads to a system of equations of the form:

$$\Delta_\upsilon\ \bar{x}, y, u^{\,n} = 0, \upsilon = 1, \dots\dots I,$$

still involving parametric variables $\bar{x}$. If G is actually a symmetry group for Δ the resulting system is equivalent to a system of equations, denoted:

$$\left(\frac{\Delta}{G}\right)\ y, u^{\,n} = 0, \qquad \upsilon = 1, \dots\dots I$$

which are independent of the parametric variables, and thus constitute a genuine system of differential equations for Δ as a function of y.

Example (2):

The one-dimensional wave equation $u_{tt} – u_{xx} = 0$ is invariant under the scaling group. To construct the corresponding scale-invariant solutions, we need only substitute the derivative formula (6) into the wave equation, and solve the resulting ordinary differential equation, we find:

$t^{-2} (y^2\upsilon_{yy} + 2y\upsilon_y - \upsilon_{yy}) = 0$

This equation is equivalent to an equation

$(y^2 – 1) \upsilon_{yy} + 2y\upsilon_y = 0$

in which the parametric variable t no longer appears. This latter ordinary differential equation is the reduced equation for the scale-invariant solutions to the wave equation. It is easily integrated, with general solution:

$$\upsilon = c\, log\ \frac{y - 1}{y + 1} + \acute{c},$$

where c, $\acute{c}$ are arbitrary constants. Replacing the variables y and υ in the solution by their expressions in terms of x, t, u we deduce the general scale-invariant solution to the wave equation to be:

$$u = c\, log\ \frac{x-1}{x+1} + \acute{c}.$$

Example (3): (The heat equation)

The symmetry group of the heat equation consists of a six-parameter group of symmetries particular to the equation itself plus an infinite-dimensional subgroup stemming from the linearity of the equation. For each one-parameter subgroup-invariant solutions, which will be determined from a reduced ordinary differential equation, whose from will in general depend on the particular subgroup under investigation [8].

a) **Travelling Wave Solutions:** in general, travelling wave solutions to a partial differential equation arise as special group-invariant solutions in which the group under consideration is a translation group on the space of independent variables. Consider the translation group:

$$x, t, u \ \rightarrow\ x + c\varepsilon, t + \varepsilon, u\ , \varepsilon \in R$$

generated by $\partial_t + c\partial_x$ in which c is a fixed constant, which will determine the speed of the waves. Global invariants of this group are:

$y = x - ct$, $\upsilon = u$ …….(7)

so that a group-invariant solution $\upsilon = h(y)$ takes the familiar form $u = h(x-ct)$ determining a wave of unchanging profile moving at the constant velocity c. Solving

for the derivatives of u with respect to x and in terms of those of υ with respect to y we find:

$$u_t = - c\upsilon_y,\ u_x = \upsilon_y,\quad u_{xx} = \upsilon_{yy}$$

Substituting these expressions into the heat equation, we find the reduced ordinary differential equation for the travelling wave solutions to be:

$$\upsilon_{yy} + c\upsilon_y = 0$$

The general solution of this linear, constant coefficient equation is

$$\upsilon\ (y)\ = k\ e^{-cy} + I,$$

for k,I arbitrary constants. Substituting back according to (7), we find the most general travelling wave solution to the heat equation to be an exponential of the form:

$$u\ (x,t)\ = k\ e^{-c(x-ct)} + I,$$

b) **Scale-invariant solutions:** There are two one-parameter groups of scaling symmetries of the heat equation, and we consider a linear combination [9]:

$$x\ \partial_x + 2\ t\ \partial_t + 2\ au\ \partial_u,\ a \in R,$$

of their infinitesimal generators, which corresponds to the group:

$$x, t, u\ \rightarrow\ \lambda x + \lambda^2 t + \lambda^{2a} u\ ,\lambda \in R^+$$

On the half space {(x,t,u) : t > 0}, global invariants of this one-parameter group are provided by the functions:

$$y = \frac{x}{\sqrt{t}},\qquad \upsilon\ = t^{-a}\ u$$

Solving for the derivatives of u in terms of υ, we find:

$$u\ = t^{\ a}\ u\ \upsilon,\ u_x = t^{a-1/2}\upsilon_y$$
$$u_{xx} = t^{a-1}\upsilon_y$$
$$u_t = -\frac{1}{2}xt^{a-3/2}\upsilon_y + at^{a-1}\upsilon =$$
$$t^{a-1}(-\frac{1}{2}y\upsilon_y + av).$$

Substituting these expressions into the heat equation, we find:

$$t^{a-1}\upsilon_{yy} = t^{a-1} - \frac{1}{2}y\upsilon_y + av\ ,$$

This equation is equivalent to one in which the parametric variable t does not occur, namely:

$$\upsilon_{yy} + \frac{1}{2}y\upsilon_y + av = 0,$$

which forms the reduced equation for the scale-invariant solutions. If we set $w = \upsilon exp.(\frac{1}{8}y^2)$ then w satisfies a scaled from of Webers differential equation,

$$w_{yy} = (a + \frac{1}{4}) + \frac{1}{16}y^2\ w$$

The general solution of this equation is

$$w\ y\ = KU\ 2a + \frac{1}{2}, \frac{y}{\sqrt{2}}\ + \bar{K}V(2a + \frac{1}{2}, \frac{y}{\sqrt{2}})$$

Thus the general scale-invariant solution to the heat equation takes the form:

$$u\ x,t\ = t^a e^{x^2/8t}\{KU\ 2a + \frac{1}{2}, \frac{x}{\sqrt{2t}} + \bar{K}V\ 2a + \frac{1}{2}, \frac{x}{\sqrt{2t}}\ \}$$

If a = 0, we obtain the probability solution:

$$u\ x,t\ = k^*\ \mathrm{erf}\ \frac{x}{\sqrt{2t}}\ + \bar{k}^*$$

where erf is the error function.

c) **Galilean-Invariant Solutions:** The one-parameter group of Galilean boosts, generated by $\upsilon_s = 2t\partial_x - xu\partial_u$ has global invariants y = t, υ = u exp (x^2/4t) on the upper half space {t > 0} [9], we find

$$u_t = (\upsilon_y + \frac{x^2}{4t^2}\ \upsilon)e^{-x^2\ 4t}$$

$u_{xx} = (\frac{x^2}{4t^2} - \frac{1}{2t})\upsilon\, e^{-x^2\ 4t}$

Therefore, for the heat equation the reduced equation for Galilean- invariant solutions is a first order ordinary differential equation $2y\upsilon_y + \upsilon = 0$. The solution is $\upsilon(y) = \frac{k}{\sqrt{y}}$.

Hence the most general Galilean- invariant solutions is scalar multiple of the source solution,

$$u(x,t) = (\frac{k}{\sqrt{t}})\, e^{-x^2\ 4t}$$

which we earlier found as a scale- invariant solution.

CONCLUSIONS

There is a one-to-one correspondence between G -invariant functions u = f(x) on M and arbitrary functions υ = h(y) involving the new variables. To explain this correspondence, we begin by invoking the implicit function theorem to solve the system y = η (x) for p – s of the independent variables, say $\bar{x} = (x^{i_1}, \ldots\ldots x^{i_{p-1}})$, in terms of the new variables, $y^1, \ldots\ldots y^{p-s}$ and the remaining s old independent variables, denoted as $\bar{x} = x^{j_1}, \ldots\ldots x^{ij_s}$. Thus we have the solution $\bar{x} = \gamma(\bar{x}, y)$, for some well –defined function y. Then we solve the reduced system $\Delta_G\ y, \upsilon^{(n)} = 0$. For each solution υ = h(y) of Δ_G, there corresponds a G -invariant solution u = f(x) of the original system, which is given implicitly by the relation:

$\xi\ x,u = \{\eta\ x,u\ \}$.

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تطبيقات البرمجة الخطية في نماذج النقل

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المستخلص

يتناول هذا البحث تطبيقات البرمجة الخطية في نماذج النقل وخصوصا عند وجود مراحل نقل متعددة (Multi Stages) حيث تم التنبؤ بالطلب على المنتج (الحليب الجاف) ومن ثم بناء نموذج رياضي لمشكلة النقل حيث تعجز طرق النقل الاعتيادية عن حل نموذج نقل لعدة مراحل. وقد تم استخدام تحليل الحساسية (Sensitivity Analysis) لايجاد الكميات المثلى المنقوله وباقل كلفة ممكنة. وقد تم التطبيق في شركة المها التجارية المحدوده في جمهورية العراق – مدينة بغداد.

الكلمات المفتاحية: تحليل الحساسية – طريقة التمهيد الاسي الموسمي – اسعار الظل

ABSTRACT

This work aimed to apply the linear programming in transport models, especially when there are stages of the transfer of multiple (Multi Stages), where the demand for the product (dry milk) was predicted and then a mathematical model was built to solve the problem of transport where the normal transport routes are failed in solving the model of several stages. The sensitivity analysis was used to find the optimal quantities pushers at the lowest possible cost. The model was implemented in the Maha Trading Co. Ltd. in republic of Iraq – Baghdad.

المقدمة

ان البرمجة الخطية هي عبارة عن اسلوب رياضي يستخدم في ايجاد الحل الامثل لكيفية استخدام المشروع لموارده.

وتشير كلمة خطية الى ان العلاقات بين المتغيرات هي علاقة خطية اما كلمة برمجة فتشير الى التكنيك الرياضي المستخدم في ايجاد الحل.

ان اسلوب البرمجة الخطية (Linear programming) يستخدم في حل المشاكل المتعلقة بتخصيص الموارد النادرة من الاستخدامات البديلة المتاحة في افضل تخصيص بهدف تعظيم دالة منفعة متخذ القرار وذلك بتخصيص الموارد المتاحة بصورة تحقق اقصى ارباح ممكنة اذا كان الهدف تعظيم الربح (profit maximization) او تدنية الكلفة اذا كان الهدف هو تقليل الكلفة (cost minimization) واغلب مشاكل النقل يتم صياغتها بوساطة نموذج برمجة خطيه على اساس تقليل كلفة النقل. ان نموذج النقل هنا هو من النوع تقليل الكلفة (cost minimization)[1].

ان الهدف من هذا البحث هو التنبؤ بكمية الطلب ومن ثم تحويل انموذج النقل الى انموذج برمجة خطية حيث يتم ايجاد الحل الامثل باستخدام البرنامج الجاهز WinQSB ومن ثم يتم استخدام تحليل مابعد الامثلية او ما يسمى بتحليل الحساسية (sensitivity analysis) للوصول الى الكميات المثلى المنقولة بأقل التكاليف.

ان اهمية هذا البحث تكمن في انه يعالج مشكلة النقل ذات المرحلتين مع التنبؤ بحجم الطلب في حين ان الدراسات السابقة قد عالجت مشكلة النقل بمرحله واحدة وبطلب ثابت.

اما بالنسبة للدراسات السابقة فلم اجد أي دراسة عالجت موضوع النقل على عدة محطات باستخدام البرمجة الخطية حيث ان هذه الدراسة تعتبر من اولى الدراسات في هذا المجال .

لقد تم تطبيق الدراسة في شركة المها التجارية المحدودة لاستيراد المواد الغذائية في العراق حيث تم اخذ منتج واحد من منتجاتها وهو الحليب المجفف كونه يمثل اكثر انواع المواد الغذائية التي تستوردها الشركة طلبا في السوق المحلية حيث ان الهدف الرئيس من هذا البحث هو التنبؤ بحجم الطلب على الحليب المجفف ومن ثم ايجاد الكميات المثلى المنقوله من المجهز الى المسوق وباقل كلفة كلية ممكنة .

مقدمة عن البرمجة الخطية:

ان اسلوب البرمجة الخطية يعالج المشاكل المختلفة ببناء نموذج حيث يقوم بايجاد قيم (x1…x2…x3……xn) المثلى والتي تحقق اكبر منفعة ممكنة لمتخذ القرار سواء كانت دالة الهدف من نوع تعظيم الارباح (maximization profit) او تقليل التكاليف[2] (minimization of cost).

شروط البرمجة الخطية:

1.القدرة على تحديد المشكلة موضوع البحث تحديدا رياضيا دقيقا .

2.محدودية الموارد البشرية والمادية الخاضعة للبرمجة مثل محدودية راس المال .. عدد العمال ...البضاعة المستورده.... الطاقة الانتاجية وغيرها .

3.امكانية التعبير عن الفعاليات او المتغيرات موضوع البرمجة بصورة رقمية

4.ان تكون العلاقة بين المتغيرات هي علاقة خطية .

5.توفر استخدامات تنافسية للموارد البشرية والمادية موضوع البرمجة الخطيه مثلا انتاج منشأة سلعتين [3]. ان الصيغة الرياضية للبرمجة الخطية هي كالاتي: [4]

$$\text{Max (Min) } Z = C_1X_1+C_2X_2+\ldots\ldots\ldots\ldots +C_iX_i+\ldots\ldots\ldots\ldots C_nX_n$$

حيث ان X_i هي عدد الوحدات المنتجة من المنتج i وان C_i هي معامل رقمي يمثل ربح (كلفة) الوحدة الواحدة من المنتج X_i حيث ان $i=1\ldots\ldots\ldots\ldots\ldots n$.

اما القيود الخطية فهي تعرف كما ياتي: [4]

a_{11} | $X_1+a_{12}X_2+\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{1n}X_n(<=,>=,=)\ b_1$

a_{21} | $X_1+a_{22}X_2+\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{2n}X_n(<=,>=,=)\ b_2$

a_{m1} | $X_1+a_{m2}X_2+\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_{mn}X_n(<=,>=,=)\ b_m$

شرط عدم السلبية $X_i >= 0\ (i=1,2,\ldots\ldots,n)$

حيث ان n هي عدد المتغيرات و m تمثل عدد القيود b_j قيمة المتاح من الموارد, a_{ij} تمثل احتياجات المنتج i من المورد j حيث ان :

j =1, 2,…….. n
I = 1, 2,…….. m

هناك ثلاثة طرق رئيسية تستخدمها بحوث العمليات لحل مشاكل البرمجة الخطية وكما ياتي: [4]

1 – طريقة الرسم البياني(Graphic Method) : وهي اداة بيانية بسيطة جدا تستخدم رغم بساطتها في معالجة مشاكل متعددة في مجال التسويق والانتاج والافراد وغيرها من المجالات الادارية حيث تشترط هذه الطريقة وجود ثلاثة متغيرات على الاكثر بسبب تعذر رسم اكثر من ثلاثة ابعاد هندسية على الورق حيث يتم رسم دالة الهدف والقيود ومن ثم ايجاد منطقة الحل الامثل.

2 – طريقة الحل الجبري (The Algebra Method): وهي تمثل اسلوبا اخر من اساليب البرمجة الخطية وهذه الطريقة تتميز باتساع استخدامها في حالة زيادة عدد المتغيرات عن اثنين.

3 – الطريقة الثنائية (Simplex Method) : لقد استخدمت هذه الطريقة لاول مرة عام 1947 من قبل العالم الامريكي دانتز وهي اوسع نطاقا من الطريقتين السابقتين وهذه الطريقة تتميز بكونها تتكون من عمليات ومراحل متكرره حيث تمثل كل مرحلة حلا قائما بذاته مع ملاحظة ان كل حل افضل من سابقه وهكذا حتى الوصول للحل الامثل كما ان لكل حل من هذه الحلول دالة الهدف الخاصة به . وعلى العموم تتميز هذه الطريقة بدرجة عالية من الدقة والكفاءه في معالجة مشكلات البرمجة الخطيه بغض النظر عن عدد المتغيرات وسوف نستخدم هذه الطريقة في حل المشكلة قيد البحث.

نموذج النقل (transportation model) [5]:

تقوم فكرة نماذج النقل على اساس النقل الاقتصادي للوحدات الانتاجية المتجانسة من مصادر الانتاج او التسويق الى مواقع الطلب او الاستهلاك او بعبارة اخرى فأن نموذج النقل هو خطة النقل لعدد من المنتجات (سلع او خدمات) من عدد من مصادر الانتاج او التجهيز الى عدد من مواقع الطلب او الاستهلاك باقل كلفة نقل ممكنة.

ان نموذج النقل يعتمد على الافتراضات الاساسية الاتية:

. ان جميع المواد المنقولة بين المصادر ومناطق الطلب متجانسة (Homogeneous) .

. عدم وجود عوائق للنقل بين أي مصدر للتجهيز واي موقع للطلب .

. ان مجموع كمية الطلب المتوفرة لدى المصدر يساوي مجموع كمية الطلب في المواقع.

. ان تكاليف نقل المواد بين أي مصدر واي موقع للطلب معروفة ولن تتغير في الامد القريب.

. ان كلفة النقل بين أي مصدر واي موقع لا تتغير بتغير كمية المواد المنقولة .

. ان الهدف الرئيس لمشكلة النقل هو تخفيض تكاليف النقل الكلية بين مصادر التجهيز ومناطق الطلب والاستهلاك

الانموذج الرياضي لمشكلة النقل [6]:

1 –_نفترض ان عدد المصادر هو m ونفترض عدد مناطق الطلب هي n

2 – نفترض ان تكلفة نقل الوحدة الواحده من المواد المنقولة من المصدر (i) الى منطقة الطلب (j) , حيث ان (i) عبارة عن رقم من 1 الى m و (j) عبارة عن رقم من 1 الى n وان هذه الكلفة هي C_{ij}.

3 – ان كل مصدر يحتوي على كمية من البضاعة تصل الى حد معين ولنفترض ان المصدر (i) يحتوي على a_i وان احتياجات كل منطقة طلب (j) هي b_j.

4 – نفترض ان الكمية المنقولة هي X_{ij}.

5 – ولتسهيل دراسة المشكلة ومن ثم ايجاد الحلول لها نقوم بوضع مشكلة النقل على شكل جدول وهذا الجدول يسمى بجدول النقل حيث تنقسم جداول النقل الى قسمين هما جدول التكاليف وجدول التوزيع حيث ان جدول التوزيع هو عبارة عن الكميات المنقولة من المصدر الى منطقة الطلب اما جدول الكلفة فهو عبارة عن كلفة النقل من المصدر الى منطقة الطلب.

طرائق حل انموذج النقل [6]:

ان هناك ثلاث طرق رئيسه لايجاد الحل الاساسي الاولي لمشكلة النقل وهي :

1 – طريقة الركن الشمالي الغربي (North West Corner): تعتبر هذه الطريقة من اسهل الطرق لحل مشكلة النقل حيث تبدأ عملية ايجاد الحل الاساسي الاولي من الزاوية الشمالية الغربية ولذلك سميت هذه الطريقة بهذا الاسم.

2 – طريقة اقل كلفة (The Least –Cost Method) : يتم العمل بهذه الطريقة على اساس اقل الكلف حيث يتم مشاهدة جدول التكاليف وايجاد اقل الكلف ومن ثم تخصيص الكمية المطلوبة على اساس اقل الكلف.

3 – طريقة فوجل (Vogels Approximation Method) : تعتبر هذه الطريقة من افضل الطرق وادقها لما تتميز به هذه الطريقة من القدرة للوصول للحل الامثل او الحل القريب من الحل الامثل ونقصد بالافضلية هو الوصول للحل الامثل باسرع وقت ممكن.

تحويل انموذج النقل الى انموذج برمجة خطية[7]

ان فكرة تحويل مشكلة النقل(تدنية تكاليف النقل) الى انموذج برمجة خطيه هي بالاساس تتم بتحويل مشكلة النقل بجملتها الى دالة هدف (OBJECTIVE FUNCTION) من نوع تصغير (minimization)وقيود(CONSTRAINTS) ان الانموذج الرياضي العام لتحويل مشكلة النقل الى مشكلة برمجة خطية هو بالشكل الاتي:

$$\text{الهدف} \begin{cases} \text{Min } Z=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \quad \text{دالة} \\ \text{قيود} \quad \sum_{j=1}^{n} X_{ij}=S_i \quad i=1,2,\ldots..,n \\ \text{قيود} \quad \sum_{i=1}^{m} X_{ij}=d_j \quad j=1,2,\ldots,m \\ \text{الطلب} \quad X_{ij}\geq 0 \end{cases} \quad \text{s. t} \quad (1)$$

حيث ان X_{ij} هي الكمية المنقولة من المصدر I الى المنطقة j .

C_{ij} هي كلفة النقل من المصدر I الى المنطقة j .

S_i هي كمية التجهيز المتوفرة في المصدر I .

D_i هي الكمية المطلوبة للمنطقة j .

طرائق التنبؤ بالطلب [8]:

وبما ان سلوك الطلب موسمي ، تم استخدام طرائق التنبؤ الموسية،وهذه الطرائق هي كالاتي :

- نماذج بوكس – جينكيز.
- طريقة التمهيد الآسي الموسمي "طريقة ونترز".

تـم اعتمـاد طريقـة التمهيـد الآسـي الموسـمي فـي الجانـب التطبيقـي ، لـذلك سـيتم عـرض الجانـب النظـري لطريقـة التمهيد الآسي الموسمي فقط.

طريقة التمهيد الآسي الموسمي "طريقة ونترز":

تعد أساليب التمهيد الآسي من الأساليب الشائعة الاستخدام فـي عمليـة التنبـؤ لمعالجـة بيانـات السلسـلة الزمنيـة، وذلـك

$$\hat{X}_{T+t}(T)=\{\hat{a}(T)+\hat{b}(T)t\}\hat{C}_{T+t}(T+t-L)$$

$$\hat{a}(T)=\sim\frac{X_t}{\hat{C}_T(T-L)}+(1-\sim)\{\hat{a}(T-1)+\hat{b}(T-1)\}$$

$$\hat{b}(T)= \{\hat{a}(T)-\hat{a}(T-1)\}+(1-)\hat{b}(T-1)$$

$$\hat{C}_T(T)=¥\frac{X_T}{\hat{a}(T)}+(1-¥)\hat{C}_T(T-1)$$

Where $0< \alpha,\beta,\gamma<1$

إذا أن α, β, γ هي معلمات التمهيد.

ولبيان كفاءة طرائق التنبؤ يتم استخدم المعايير الإحصائية الآتية:

أولا : معدل القيم المطلقة للأخطاء (Mean absolute error)

$$MAE=1/M\ \Sigma e_t(L) \quad \ldots\ldots\ldots(6)$$

ثانيا: معدل مربعات الخطأ (Mean square error) :

$$MSE=1/M\ \Sigma e^2_t(L) \quad \ldots\ldots\ldots\ (7)$$

ثالثا: معدل القيم المطلقة لنسب الأخطاء (Mean absolute percentage) :

$$MAPE=(1/M\Sigma e_t(L)/X_{t+L}\ )100\% \quad \ldots\ldots\ldots\ldots(8)$$

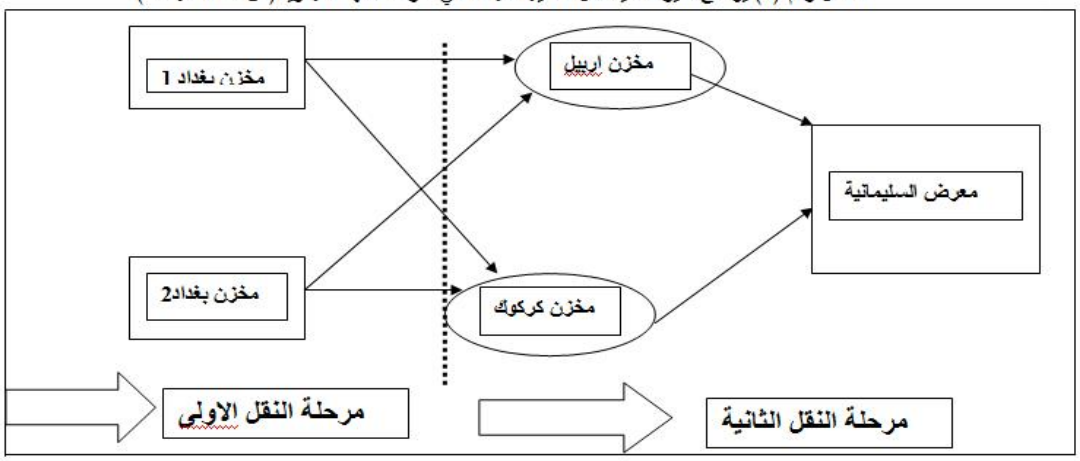
بسبب كفاءتها وبساطتها وتكيفها للتغيرات المستقبلية فضلاً عـن عـدم حاجتهـا الاحتفـاظ بعـدد كبيـر مـن البيانـات. وتسـتخدم طريقـة التمهيـد الآسـي الموسـمي "طريقـة ونتـرز لمعالجـة البيانـات الموسمية.تسـتخدم طريقـة ونتـرز عنـدما تكـون السلسـلة الزمنيـة موسـمية وتسـتند هـذه الطريقـة علـى المعادلات الاتية:

مشكلة البحث:

شركة المها من الشركات الكبرى الخاصة الموجودة في العراق والتي تعمل في استيراد الحليب المجفف بمختلف انواعه من دولة الامارات العربية المتحده عن طريق السفن

الصغيرة حيث تتحمل الشركة المصدرة للحليب تكلفة النقل من دولة الامارات العربيه المتحدة الى مخزني شركة المها الموجوده في بغداد مع التامينات وتاسست هذه الشركة سنة 2003 وهذه الشركة لها مخزنين رئيسيين في بغداد والتي تستقبل استيرادت الشركة من الحليب المجفف. ولدى الشركة ايضا مخزنين انتقاليين احدهما في محافظة اربيل والثاني في محافظة كركوك . كما ان للشركة ايضا معرض رئيس في محافظة السليمانية(والذي نعتبره مصدر الطلب او المحطة النهائية للحليب المجفف) حيث يستلم هذا المعرض الحليب المجفف من المخزنين الانتقاليين في اربيل وكركوك . من خلال ملاحظتنا لمسير عملية النقل في الشركة وجدنا انهم يطبقون اسلوب التخمينات والخبره السابقة على تجهيزهم للمواد كما ان تقدير الكمية المطلوبة يتم في الشركة عن طريق التخمينات لذلك ابدى السيد مدير الشركة رغبته الشديده في التنبؤ بكمية الطلب ومن ثم بناء نموذج برمجة خطية لعملية نقل الحليب المجفف وتحقيق اقل الكلف وامثل الكميات ضمن عملية النقل اي اننا امام مشكلة نقل وعلى مرحلتين ان الشكل رقم(1) يوضح سير نقل الحليب الجاف في شركة المها التجارية المحدودة.

الشكل رقم (1) يوضح سير عملية نقل الحليب الجاف في شركة المها التجارية(من اعداد الباحث)



التنبؤ بالطلب:

تم التنبؤ بالطلبات الموسمية للفصل الاول (كانون الثاني , شباط ,اذار) من سنة 2011 باستخدام طريقة ونترز الموسمية وقد استخدمنا طريقة ونترز الموسمية لان الطلب على الحليب الجاف هوموسمي بسبب زيادة الطلب عليه في فترة الصيف كونه مطلوبا وبشدة في صناعة المرطبات، و بسبب محدوية وقصر البيانات المتاحة لان الشركة حديثة العهد، لذلك تم الاعتماد على أسلوب التمهيد الآسي، وكانت النتائج لمعلمات طريقة ونترز للطلب على الحليب المجفف للمدة 2006–2010 والمعايير الإحصائية بالاعتماد على المعادلات (2–8) كالأتي:

قيم معلمات طريقة ونترز المثالية / المعايير الاحصائية	α=0.2	β=0.05	γ=0.1
MAE	16.36		
MSE	11.007		
MAPE	0.53		

حيث تم تقدير الكمية المطلوبة من الحليب المجفف من قبل المعرض الرئيس في السليمانية ب1000 طن خلال الفصل الاول (شهر 1 ، 2 و 3) من سنة 2012. يبين الجدول رقم (1) والجدول رقم (2) تكاليف النقل (مقاسة بالدينار للطن الواحد) والكميات المنقولة بين المخازن والمعرض (حسب بيانات الشركة).

الجدول رقم (1): تكاليف النقل (مقاسة بالدينار للطن الواحد) بين المخازن والمعرض

من / الى	مخزن اربيل	مخزن كركوك	معرض السليمانية
مخزن بغداد(1)	200000	170000	.....
مخزن بغداد(2)	160000	110000	.....
مخزن اربيل	.....	.....	40000
مخزن كركوك	.....	.....	80000

الجدول رقم (2): الكميات المنقولة بين المخازن والمعرض الرئيس (مقاسة بالاطنان)

من / الى	مخزن اربيل	مخزن كركوك	معرض السليمانية
مخزن بغداد (1)	300	300	.....
مخزن بغداد(2)	500	500	.....
مخزن اربيل	.....	.....	700
مخزن كركوك	.....	.....	700

بناء انموذج برمجة خطية لمشكلة النقل:

لصياغة الانموذج الرياضي (انموذج البرمجة الخطية) يجب اولا تعريف معالم الانموذج وهذه المعالم هي كالاتي :

نفرض ان عدد الاطنان المنقولة من مخزن بغداد (1) الى مخزن اربيل = XB1A

نفرض ان عدد الاطنان المنقولة من مخزن بغداد (1) الى مخزن كركوك= XB1K

نفرض ان عدد الاطنان المنقولة من مخزن بغداد (2) الى مخزن اربيل = XB2A

نفرض ان عدد الاطنان المنقولة من مخزن بغداد (2) الى مخزن كركوك= XB2K

نفرض ان عدد الاطنان المنقولة من مخزن كركوك الى السليمانية = XKS

نفرض ان عدد الاطنان المنقولة من مخزن اربيل الى السليمانية = XAS

ان الانموذج الرياضي للمشكلة قيد البحث يكون بالشكل الاتي :

MINIMIZATION Z= 200000XB1A+170000XB1K+160000XB2A +
110000XB2K +40000XAS+ 80000XKS

CONSTRAINT:

XB1A+XB1K=400..……..…….……(1)

XB2A+XB2K=600.……..…….……(2)

XB1A+XB2A-XAS=……..…….……...(3)

XB1K+XB2K-XKS=0 ……..……(4)

XAS+XKS=1000 …….…… (5)

XB1A<=300 ……..(6)

XB1K<=300 …….…..……...(7)

XB2A<=500 ……..…….(8)

XB2K<=500 ……..……..(9)

XAS<=700 …….……(10)
XKS<=700 …….……(11)

NONNEGATIVE INTEGAR:

XB1A,XB1K,XB2A,XB2K,XAS,XKS>=0 AND MUST BE INTEGAR

حل الانموذج الرياضي (انموذج البرمجة الخطية) [2]:

رغبة منا باستخدام احدى البرامج المخصصة لحل مشاكل البرمجة الخطية حيث ان استخدام البرامج يحقق سرعه ودقة في حل الانموذج كما انه يستوعب اي تغير مستقبلي في المتغيرات وببساطة شديدة جدا دون الحاجة الى تغير فرضيات الانموذج ومن هذه البرامج برنامج اكسل وبرنامج WINQSB وسنعتمد في حل هذا الانموذج على برنامج WINQSB باعتباره من اكثر البرامج تخصصا في حل مشاكل البرمجة الخطية. ان جدول الحل الامثل موضح بالجدول رقم(4) حيث يوضح هذا الجدول الحل الامثل لنموذج البرمجة الخطية.

تفسير النتائج: أن جدول الحل النهائي رقم (4) يعطينا تصورا واضحا عن الاسلوب الواجب اتباعه في عملية النقل

حيث يبين لنا عمود قيمة الحل (SOLUTION VALUE) الكمية التي يجب الالتزام بها من الحليب المجفف الذي يجب نقله بين المخازن فيما بينها وبين المعرض الرئيس فمثلا الكمية التي يجب نقلها من مخزن بغداد(1) الى مخزن اربيل هي 300 طن وبكلفة كلية هي 60 مليون دينار وهكذا ..... كما ان قيمة دالة الهدف والتي تمثل ادنى كلفة نقل هي 212 مليون دينار حيث يوضح الجدول رقم (3) الكميات التي يجب الالتزام بها للنقل بين المخازن والمعارض في حين ان الجدول رقم (4) يوضح الكميات المثلى التي يجب الالتزام بها للنقل بين المخازن وبين المعرض الرئيس.

الجدول رقم (3): الكميات التي يجب الالتزام بها للنقل بين المخازن والمعرض

من / الى	مخزن اربيل	مخزن كركوك	مخزن السليمانية
مخزن بغداد(1)	300 طن	100 طن	.....
مخزن بغداد(2)	100 طن	500 طن	.....
مخزن اربيل	.....	.....	400 طن
مخزن كركوك	.....	.....	600 طن

الجدول رقم (4): الحل لنموذج البرمجة الخطية

	21:23:43		Saturday	February	03	2007		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	XB1A	300.0000	200.000.0000	60.000.000.0000	0	basic	-M	210.000.0000
2	XB1K	100.0000	170.000.0000	17.000.000.0000	0	basic	160.000.0000	M
3	XB2A	100.0000	160.000.0000	16.000.000.0000	0	basic	150.000.0000	210.000.0000
4	XB2K	600.0000	110.000.0000	66.000.000.0000	0	basic	-M	120.000.0000
5	XAS	400.0000	40.000.0000	16.000.000.0000	0	basic	30.000.0000	50.000.0000
6	XKS	600.0000	80.000.0000	48.000.000.0000	0	basic	70.000.0000	90.000.0000
	Objective	Function	(Min.) –	212.000.000.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	400.0000	=	400.0000	0	0	400.0000	M
2	C2	600.0000	=	600.0000	0	50,000.0000	600.0000	700.0000
3	C3	0	=	0	0	210.000.0000	-100.0000	0
4	C4	0	=	0	0	170.000.0000	100.0000	0
5	C5	1.000.0000	=	1.000.0000	0	250.000.0000	900.0000	1.000.0000
6	C6	300.0000	<=	300.0000	0	10.000.0000	200.0000	400.0000
7	C7	100.0000	<=	300.0000	200.0000	0	100.0000	M
8	C8	100.0000	<=	500.0000	400.0000	0	100.0000	M
9	C9	500.0000	<=	500.0000	0	10.000.0000	200.0000	600.0000
10	C10	400.0000	<=	700.0000	300.0000	0	400.0000	M
11	C11	600.0000	<=	700.0000	100.0000	0	600.0000	M

استخدام اسعار الظل (shadow price) في تقليل تكاليف النقل الكلية[6]:

ان مصطلح اسعار الظل (shadow price) هو التغير الحدي في دالة الهدف عند زيادة الطرف الايمن من القيود وحدة واحدة. ان استخدام اسعار الظل يتطلب وبالدرجة الاساس دراسة ندرة الموارد في الطرف الايمن من القيود

وامكانية تغيرها زيادة او نقصانا وهذه الدراسة يجب ان تتم بحضور الادارة العليا لغرض الوقوف على الامكانية الفعلية للتغير.

ولقد تم اجراء الدراسة اللازمة للطرف الايمن من القيود وامكانية استخدام اسعار الظل في تقليل تكاليف الانموذج وابدى السيد مدير ادارة الشركة تعاونا كبيرا في اجراء التغير رغبة منه في تطبيق الجانب العلمي وبالتالي تقليل تكاليف النقل باقل ما يمكن فلو لاحظنا الجدول رقم (4) والذي يمثل جدول الحل النهائي سوف نجد ما ياتي:

1. ان قيمة اسعار الظل والمقابلة للقيد الثاني هو – 50 الف دينار وهذا يعني اننا لو غيرنا الطرف الايمن من القيد الثاني من 600 طن الى 601 طن سوف تقل الكلفة الكلية بمقدار 50 الف دينار ولكن وبعد التباحث مع الإدارة وجدنا عدم امكان استخدام هذا القيد لان استيعاب المخزن بغداد (2) هو 600 طن ولا يمكن زيادة الخزين.

2. ان قيمة اسعار الظل والمقابلة للقيد السادس وهي – 10 الف دينار وقد استخدمنا هذا القيد بعد ان عرفنا من الادارة انه من الممكن التغير في هذا القيد والذي يمثل اعلى حد مسموح به وهو 300 طن من المخزن بغداد(1) الى مخزن كركوك حيث تم رفع هذا الحد من 300 طن الى 400 طن وتم ايضا تخفيض الطرف الايمن للقيد السابع والذي يمثل اعلى حد مسموح به وهو 300 طن من مخزن بغداد(1) الى كركوك من 300 طن الى 200 طن وذلك لتعويض الزيادة في القيد السادس بمقدار 100 طن لانها تصدر من نفس المخزن وهو بغداد (1).

3. ان قيمة اسعار الظل والمقابلة الى القيد التاسع هي – 10 الف دينار وقد استخدمنا هذا القيد بعد ان عرفنا من الادارة انه من الممكن التغير في هذا القيد والذي يمثل اعلى حد مسموح به وهو 500 طن من المخزن بغداد(2) الى مخزن كركوك حيث تم رفع هذا الحد من 500 طن الى 600 طن وتم ايضا تخفيض الطرف الايمن للقيد الثامن والذي يمثل اعلى حد مسموح به وهو 500 طن من مخزن بغداد(2) الى مخزن اربيل من 500 طن الى 400 طن وذلك لتعويض الزيادة في القيد التاسع 100 طن لانها تصدر من نفس المخزن بغداد (2). ان جدول الحل الامثل بعد اجراء التغيرات على الطرف الايمن موضح بالجدول رقم (5).

تفسير االنتائج:

1 – لو لاحظنا عمود قيمة الحل (solution value) في الجدول رقم (5) لوجدنا ان كمية الحليب المثلى التي يجب نقلها من مخزن بغداد (1) الى مخزن اربيل هي 400 طن والغاء نقل الحليب من بغداد (1) الى مخزن كركوك حيث ان القيمة الموجودة في جدول الحل الامثل هي قيمة صفرية.

2 – لو لاحظنا عمود قيمة الحل (solution value) في الجدول (5) لوجدنا ان كمية الحليب المثلى التي يجب نقلها من مخزن بغداد(2) الى مخزن كركوك هي 600 طن والغاء نقل الحليب من مخزن بغداد(2) الى مخزن اربيل حيث ان القيمة الموجوده في جدول الحل الامثل هي قيمة صفرية.

3 – ان كلفة النقل الكلية المثلى هي 210 مليون دينار في الجدول رقم (5) أي انها اقل من الكلفة الكلية في الجدول النهائي (الجدول رقم 4) بمقدار 2 مليون دينار.

الجدول رقم (5): الحل الامثل بعد اجراء تغيرات في الطرف الايمن

	18:58:57		Wednesday	March	21	2007		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	XB1A	400.0000	200,000.0000	80,000,000.0000	0	basic	150,000.0000	210,000.0000
2	XB1K	0	170,000.0000	0	10,000.0000	at bound	160,000.0000	M
3	XB2A	0	160,000.0000	0	10,000.0000	at bound	150,000.0000	M
4	XB2K	600.0000	110,000.0000	66,000,000.0000	0	basic	-M	120,000.0000
5	XAS	400.0000	40,000.0000	16,000,000.0000	0	basic	30,000.0000	50,000.0000
6	XKS	600.0000	80,000.0000	48,000,000.0000	0	basic	70,000.0000	90,000.0000
	Objective	Function	(Min.) =	210,000,000.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	400.0000	=	400.0000	0	0	400.0000	M
2	C2	600.0000	=	600.0000	0	50,000.0000	600.0000	600.0000
3	C3	0	=	0	0	200,000.0000	-400.0000	0
4	C4	0	=	0	0	160,000.0000	-100.0000	0
5	C5	1,000.0000	=	1,000.0000	0	240,000.0000	600.0000	1,000.0000
6	C6	400.0000	<=	400.0000	0	0	400.0000	M
7	C7	0	<=	200.0000	200.0000	0	0	M
8	C8	0	<=	400.0000	400.0000	0	0	M
9	C9	600.0000	<=	600.0000	0	0	600.0000	M
10	C10	400.0000	<=	700.0000	300.0000	0	400.0000	M
11	C11	600.0000	<=	700.0000	100.0000	0	600.0000	M

(RHS)

الاستنتاجات:

ان اهم الاستنتاجات التي توصلنا اليها من خلال هذا البحث هي كالاتي :

1. ان القيمة التنبؤية للطلب على الحليب الجاف في معرض السليمانية والتي جرى تقديرها باستخدام طريقة ونترز الموسمية هي 1000 طن.

2 . اظهرت نتائج البحث ان اقل كلفة نقل كلية ممكن الحصول عليها هي 210 مليون دينار ككلفة نقل كلية مثلى حصلنا عليها بعد استخدام تحليل ما بعد الامثلية (تحليل الحساسية).

3 . ان استخدام تحليل ما بعد الامثلية (تحليل الحساسية) قد ساهم وبشكل كبير في تقليل كلفة النقل الكلية من 212 مليون دينار الى 210 مليون دينار حيث ان الكلفة الكلية للنقل قد انخفضت بمقدار 2 مليون دينار.

4 . لو نظرنا الى الجدول رقم (1) لوجدنا ان هناك فروقات كبيرة بين كلف النقل بين مخزني بغداد (1) وبغداد (2) الى بقية المخازن مع ان المخزنين يقعان في بغداد واتضح ان السيارات المستخدمة في النقل في مخزن بغداد (1) هي سيارات مؤجرة اما السيارات المستخدمة في مخزن بغداد(2) فهي ملك للشركة (كلفة النقل هنا محسوبة على اساس مجموع تكاليف صيانة السيارات شهريا + تكلفة الوقود) فمن الطبيعي ان تكون كلف النقل من مخزن بغداد (1) الى بقية المخازن اعلى من كلفة النقل من مخزن بغداد (2) الى بقية المخازن.

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