Sudan University of Science and Technology
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NEUTRINO MASS IN THE STANDARD MODEL AND SEE SAW MECHANISM

كتلة النيوترينو في النموذج العياري والآلية المتأرجحة

A Graduate Project Submitted for the Degree of B.Sc. (honor) in Physics

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الآيـة

(يا بُنـيَّ إنَّـهَا إِنْ تَكُ مِثْقَالَ حَبَّةٍ مِنْ خَرْدَلٍ فَتَكُنْ فِي صَخْرَةٍ أَوْ فِي السَّمَاوَاتِ أَوْ فِي الأَرْضِ يَأتِ بِهَا اللهُ إِنَّ اللهَ لَطِيْفٌ خَبِيْر).

[لقمان:16]

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Abstract

The standard model of particle physics is a remarkable work and has been examined to high level of accuracy. However, the SM predicts vanishing neutrino mass, but the recent evidence from neutrino oscillation suggested that the neutrino has finite mass. We discuss here a possible theory of neutrino mass in the minimal change of the standard model and the implementation of the three types of see saw mechanism in a way that leads to neutrino Majorana mass and to explain the smallness of neutrino mass.

ملخص البحث

تعتبر النظرية العيارية للجسيمات الاولية من النظريات الناجحة جدا وتمّ التاكد منها عمليا. لكن هذه النظرية لديها قصور في كتلة النيوترينو والتي تتعارض مع المشاهادات المرصودة في تذبذب جسيم النيوترينو والتي أكدت ان للنيوترينو كتلة صغيرة جدا. في هذا البحث دُرس ثلاثة انواع للآلية المتأرجحة والتي تؤدي الي كتلة من نوع نيوترينو مايورانا. وكذلك لتفسير صغر هذه كتلة النيوترينو.

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Chapter One

INTRODUCTION

(1-1) Introduction

The standard model (SM) of particle physics has been an extremely remarkable theory describing the interactions between elementary particles. Its predictions have been experimentally tested to a high level of accuracy. However, the minimal SM predicts vanishing neutrino mass, but the recent observations of neutrino oscillations strongly suggested that neutrinos have finite masses providing a likely window to new physics.

A major cornerstone for the theory research in this field has been the see-saw mechanism introduced in the late of seventies to understand why neutrino masses are so much smaller than the masses of other fermions of the standard model.

(1-2)The importance of the study

The problem of neutrino mass has attracted both theorists and experimentalists for a long time. Today we know for sure that neutrinos have finite mass, but these masses are tiny, order of few eV. The smallness of neutrino mass provides a window to new physics beyond standard model, since in the minimal standard model neutrino mass vanish.

(1-3)The main objectives of the study

In this project, we will discuss possible theories of neutrino mass, from the minimal changes of the standard model (by inserting right hand neutrino into the theory). We shall discuss at length the see-saw mechanism which leads to neutrino Majorana mass.

(1-5)The outline of the study

This reserach project is structured as follow: In chapter one we give brief introduction and chapter two we discuss the standard model of particle physics in details, in chapter three we discussed the three types of see saw mechanism, then we present in chapter four discussions and conclusions.

Chapter Two

INTRODUCTION TO THE STANDARD MODEL

(2-1) Introduction

In this chapter we will study the structure of the standard model of elementry particles and its mathematical foundation, then we shall discuss the higgs mechanism to see how particles obtian their masses.

(2-2) What is the standard model (SM)

The standard model describe the weak, strong and electromagnetic interactions in terms of "gauge theories". It was dated back to the latter half of the 20th century, as a collaborative effort of scientists around the world. The current formulation was finalized in the mid-1970s upon experimental confirmation of the existence of quarks. Since then, discoveries of the top quark (1995), the tau neutrino (2000), and more recently the Higgs boson (2013), have given further credence to the Standard Model. Because of its success in explaining many experimental results, the Standard Model is sometimes regarded as a "theory of almost everything". Mathematically, the standard model is a quantized Yang–Mills theory. In 1950's Yang and Mills considered (as purely mathematical exercise) extending gauge invariance to include non-abelian (i.e. non-commuting) transformations such as SU(2). In this case one needs a set of massless vector fields (three in the case of SU(2)), which were formally called "Yang-Mills" fields, but are now known as "gauge fields" (L.F.Li, 1991)

An important feature of the standard model is that "it works", it is consistent with, or varified by, all available data. Secondly, it is a unified picture, in terms of guage theories of all interactions of known particles except gravity (A.J.G.hey, 1993).

(2-3)The standard model Lagrangian

Quantum field theory provides the mathematical framework for the standard model in which a lagrangian controls the dynamics and kinematics of the theory. Each kind of particle is discribed in terms of dynamical field that pervades space-time. The construction of the standard model based on the modren method of construction of field theories by first postulating a set of symmetries of the system and then by writing down the most general renormlizabel lagrangian from it's particle (L.F.Li, 1991).

The standard model is a gauge theory representing the fundamental interactions as changes in a Lagrangian function of quantum fields. It contain spinless, spin-(1/2) and spin -1 fields interacting with one another in a way governed by the Lagrangian which is invariant by Lorentz transformations (Weinberg, 1996).

The Lagrangian of the standard model contains kinetic terms, coupling and interaction terms related to the gauge symmetries of the force carriers , mass terms and the Higgs mechanism term.

(3-1)The fermion sector

The fermionic sector consist of quarks and leptons come in three families with identical properties except for mass. The particle content in each family is:

$$1^{st}\ \text{family: lepton; l} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, e_R^- \quad (2.1)$$

$$\text{Quark; } q = \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R \quad (2.2)$$

$$2^{nd}\ \text{family: lepton; l} = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \mu_R^- \quad (2.3)$$

$$\text{Quark;} q = \begin{pmatrix} s \\ c \end{pmatrix}_L, c_R, s_R \quad (2.4)$$

$$3^{rd}\ \text{family: lepton; l} = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \tau_R^- \quad (2.5)$$

$$\text{Quark; } q = \begin{pmatrix} t \\ b \end{pmatrix}_L, t_R, b_R \quad (2.6)$$

(2-3-2) Gauge boson sector

The gauge boson and the scalar lagrangians give rise to the free lagrangian for the photon, W, Z. and the higgs boson .The standard model gauge boson lagrangian (gauge fields) is given by

$$\mathcal{L}_G = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} tr(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} tr(G_{\mu\nu} G^{\mu\nu}) \quad (2.7)$$

$\mathbf{G_{\mu\nu}}$ is the gauge field strength of the strong SU(3) gauge field.

$\mathbf{W_{\mu\nu}}$is the gauge field strength of the weak isospin SU(2) gauge field.

$\mathbf{B_{\mu\nu}}$is the gauge field strength of the weak hypercharge U(1) gauge field.

These fields are defined as

$$G_{\mu\upsilon}^{a} = \partial_{\mu} G_{\upsilon}^{a} - \partial_{\upsilon} G_{\mu}^{a} + g_s f_{abc} G_{\mu}^{b} G_{\upsilon}^{c} \tag{2.8}$$

$$W_{\mu\upsilon}^{i} = \partial_{\mu} W_{\upsilon}^{i} - \partial_{\upsilon} W_{\mu}^{i} + g\epsilon_{ijk} W_{\mu}^{i} W_{\nu}^{i} \tag{2.9}$$

$$B_{\mu\upsilon} = \partial_{\mu} B_{\upsilon} - \partial_{\upsilon} B_{\mu} \tag{2.10}$$

(2-4) Higgs mechanism

The masses of elementary particle can not be included in the lagrangian because they will break the gauge symmetry. Therefor we need a mechanism some how to give masses to these particle. An extra field called the Higgs field has to be added by hand to give the particles mass. The Higgs field has a spin-0 particle called Higgs boson. The higgs boson is electrically neutral. The extra field if it exists, is believed to fill all of empty space throughout the entire universe. Elementary particles acquire their mass through their interaction with the Higgs field. Mathematically we introduce mass into a theory by adding interaction terms into the Lagrangian that couple the field of the particle to the Higgs field. Basically, the lowest energy state of a field would have an expectation value of zero. By symmetry breaking we introduce a nonzero lowest energy state of the field. This procedure leads to the acquisition of mass by the particles in the theory (Quigg, 2007).

We can imagine the movement of elementary particles being resisted by the Higgs field, with each particle interact with the Higgs field at a different stregth. If the coupling between the Higgs field and the particle is strong then the mass of the particle is large. If it's weak then the particle has a smaller mass. A particle like the photon with zero rest mass doesn't interact with the Higgs field at all only through a loop. If the Higgs field didn't exist at all the all particles would be massless. This scalar particle has been discovered by the ATLAS (al, 2012) and CMS (al, 2012) experiments, which is compatible with the SM. Higgs expectations with a mass 126 GeV.

The addition of this new particle will add new terms into the lagrangian:

$$\mathcal{L}_{Higgs} = \frac{1}{2}\left(D_\mu\phi\right)\left(D_\mu\phi\right) - V(\phi) \tag{2.11}$$

$$V(\phi) = \frac{\mu^2}{2}\phi^*\phi - \frac{\lambda}{4}\phi^4 \tag{2.12}$$

Therefore equation (2.11) becomes:

$$\mathcal{L}_{Higgs}=\frac{1}{2}\left(D_\mu\phi\right)\left(D_\mu\phi\right) - \frac{\mu^2}{2}\phi^*\phi - \frac{\lambda}{4}\phi^4 \tag{2.13}$$

Where $\lambda \equiv$ Higgs self coupling.

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix} \tag{2.14}$$

By minimize V(ϕ):

$$\frac{\partial V}{\partial\phi} = 0 \tag{2.15}$$

We get

$$\frac{\partial V}{\partial\phi} = -\mu^2\phi + \lambda\phi^3 \tag{2.16}$$

This equation has two solutions

$$\phi(-\mu^2 + \lambda\phi^2) = 0 \tag{2.17}$$

$$\phi = 0\ (trivial\ solution)\quad or\quad (-\mu^2 + \lambda\phi^2) = 0 \tag{2.18}$$

Therefore

$$\langle\phi^2\rangle = \frac{\mu^2}{\lambda} \qquad (2.19)$$

$$\langle\phi\rangle = \sqrt{\frac{\mu^2}{\lambda}} = \nu \qquad (2.20)$$

Where ν is known as the vacuum expectation value (VEV), $\nu = 246\ GeV$.

$$\left(\phi_1{}^2 + \phi_2{}^2 + \phi_3{}^2 + \phi_4{}^2\right) = \nu^2 \qquad (2.21)$$

Equation (2.12) represents the Higgs potential, which involves two new real parameters μ and λ. We require that $\lambda > 0$ for the potential to be bounded; otherwise the potential is unbounded from below and there will be no stable vacuum state. μ takes the following two values:

1− $\mu^2 > 0$ in this case the vacuum corresponds to $\phi = 0$, the potential has a minimum at the origin (see Figure 2.1).

2− $\mu^2 < 0$ in this case the potential develops a non-zero Vacuum Expectation Value (VEV) and the minimum is along a circle of radius (see Figure 2.2).

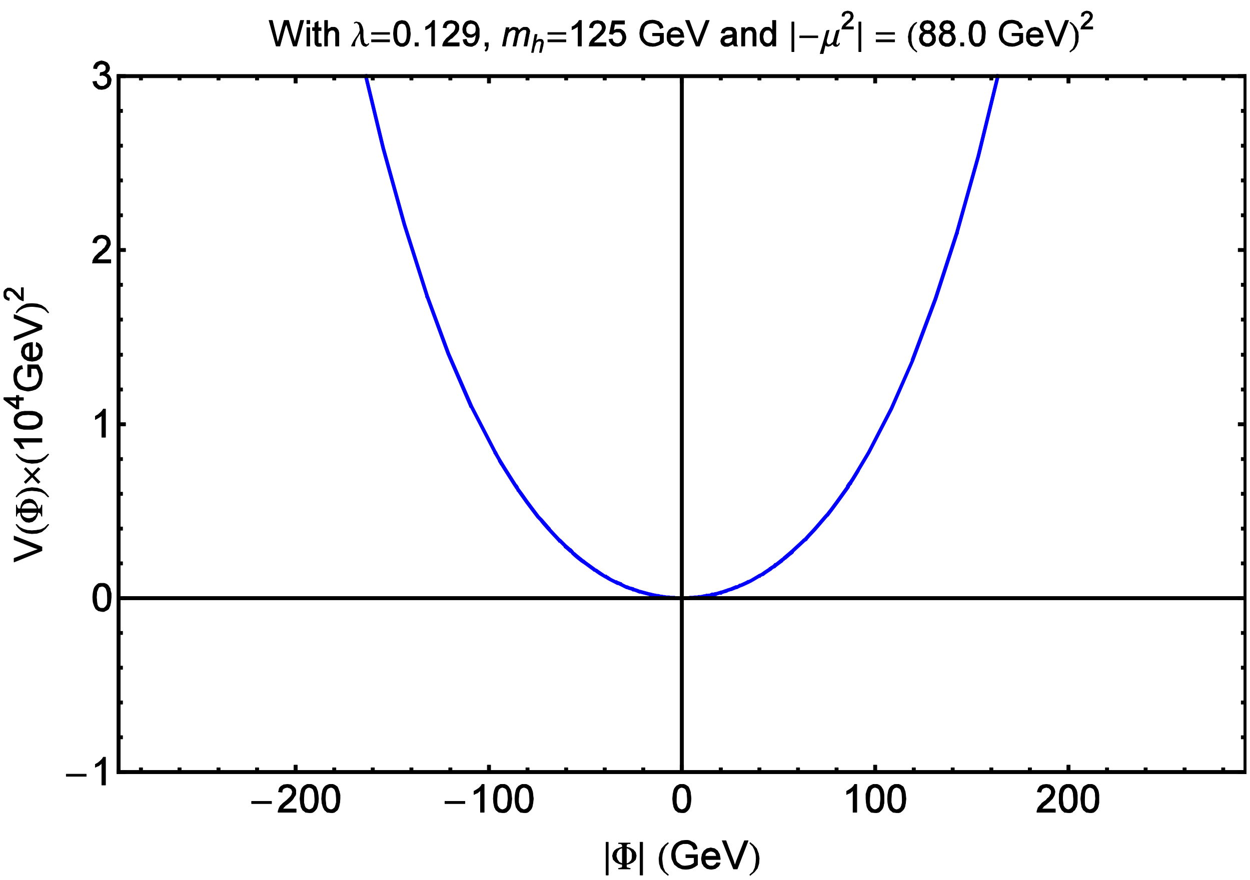

Figure 2.1. The Higgs potential with: the case $\mu^2 > 0$; as function of $|\Phi|$.

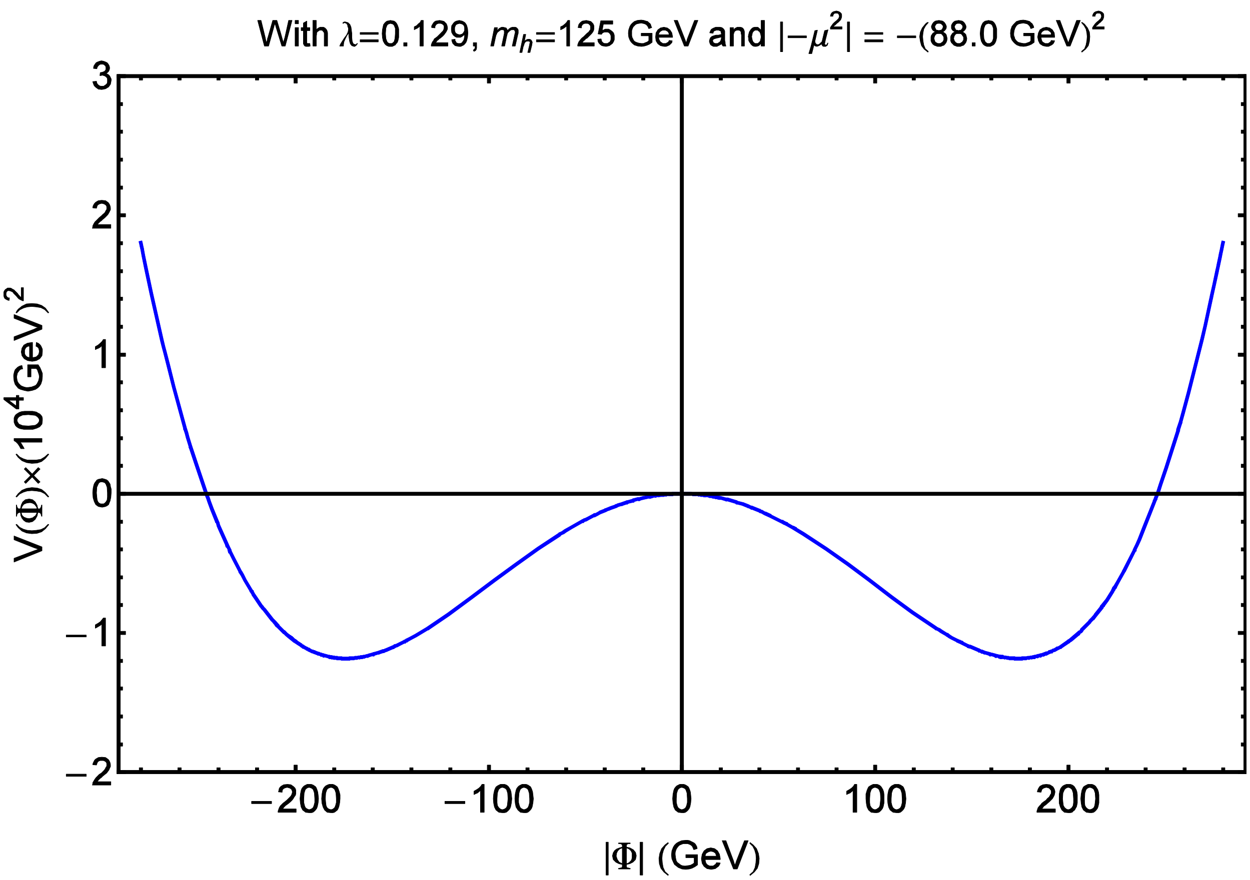

Figure 2.2. The Higgs potential with: the case $\mu^2 < 0$; as function of $|\Phi|$.

$$\phi = \frac{1}{\sqrt{2}}\binom{0}{\nu} \qquad (2.22)$$

Next we will see how to use this technique to give bosons and fermions a mass.

(2-4-1) Gauge bosons mass

To obtain the masses for the gauge bosons we will only need to study the scalar part of the lagrangian

$$\mathcal{L} = \frac{1}{2}\left(D_\mu \phi\right)^\dagger (D^\mu \phi) - V(\phi) \qquad (2.23)$$

Where D_μ is the covariant derivative.

$$D_\mu = \left(\partial_\mu + ig\tau^a W_\mu^a + i\acute{g}\frac{Y_\phi}{2}B_\mu\right) \tag{2.24}$$

$$D_\mu = \left[\partial_\mu + ig\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix} + i\acute{g}\frac{Y_\phi}{2}B_\mu\right] \tag{2.25}$$

Then

$$D_\mu\phi = \left[\partial_\mu\phi + ig\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix}\phi + i\acute{g}\frac{Y_\phi}{2}B_\mu\phi\right] \tag{2.26}$$

We have

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \tag{2.27}$$

Therefore, after a little algebra we get

$$D_\mu\phi = \frac{ig}{\sqrt{2}}\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix}\begin{pmatrix} 0 \\ \nu \end{pmatrix} + \frac{i\acute{g}Y_\phi B_\mu}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \tag{2.28}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} igW_3 & igW^- \\ igW^+ & -igW_3 \end{pmatrix}\begin{pmatrix} 0 \\ \nu \end{pmatrix} + \frac{i\acute{g}}{\sqrt{2}}\begin{pmatrix} 0 \\ B_\mu\nu \end{pmatrix} \tag{2.29}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{ig}\nu w^- \\ -\mathrm{ig}\nu \mathrm{w}_3 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i\acute{g}\boldsymbol{B}_\mu\nu \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{ig}\nu w^- \\ -\mathrm{ig}\nu\mathrm{w}_3 + i\acute{g}\nu B_\mu \end{pmatrix} \tag{2.30}$$

$$\Rightarrow D_\mu\phi = \tfrac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{ig}\nu w^- \\ -\mathrm{ig}\nu\mathrm{w}_3 + i\acute{g}\nu B_\mu \end{pmatrix} \tag{2.31}$$

Since $(D_\mu\phi)^\dagger$ is the complex conjugate of $D_\mu\phi$ then

$$\left(D_{\mu}\phi\right)^{\dagger} = \frac{1}{\sqrt{2}}(\mathrm{igv}w^{-} \quad \mathrm{igvw}_3 - i\acute{g}v\mathrm{B}_{\mu}) \tag{2.32}$$

$$\left(D_{\mu}\phi\right)^{\dagger}(D^{\mu}\phi) = \frac{1}{\sqrt{2}}(\mathrm{igv}w^{-} \quad \mathrm{igvw}_3 - i\acute{g}v\boldsymbol{B}_{\mu})\frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{igv}w^{-} \\ -\mathrm{igvw}_3 + i\acute{g}vB_{\mu} \end{pmatrix}$$

$$= \frac{1}{2}\left[g^2v^2w^+w^- + v^2(gw_3 - \acute{g}B_{\mu})^2\right] \tag{2.33}$$

So

$$\frac{1}{2}\left(D_{\mu}\phi\right)^{\dagger}(D^{\mu}\phi) = \frac{1}{4}g^2v^2w^+w^- + \frac{1}{4}v^2\left(gw_3 - \acute{g}B_{\mu}\right)^2 \tag{2.34}$$

From the above equation we obtain

$$m_w^2 = \frac{1}{4}g^2v^2$$

$$m_w = \frac{1}{2}vg$$

For Z boson, we use the orthogonal combination as

$$z_{\mu} = \frac{gw_3 - \acute{g}B_{\mu}}{\sqrt{g^2 + \acute{g}^2}} = \left(\cos\theta_w\, w_3 - \sin\theta_w\, B_{\mu}\right) \tag{2.35}$$

And the photon:

$$A_{\mu} = \frac{1}{\sqrt{g^2 + \acute{g}^2}}\left(\acute{g}w_3 + gB_{\mu}\right) \tag{2.36}$$

By using a rotation transformation

$$\begin{pmatrix} z_{\mu} \\ A_{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix}\begin{pmatrix} w_3 \\ B_{\mu} \end{pmatrix} \tag{2.37}$$

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + \acute{g}^2}} \quad and \quad \sin\theta_w = \frac{\acute{g}}{\sqrt{g^2 + \acute{g}^2}} \tag{2.38}$$

Multiply the second part of equation by $\frac{\sqrt{g^2+\acute{g}^2}}{\sqrt{g^2+\acute{g}^2}}$ we obtain

$$\frac{1}{4}\nu^2\left(gw_3 - \acute{g}B_\mu\right)^2 \cdot \frac{\sqrt{g^2 + \acute{g}^2}}{\sqrt{g^2 + \acute{g}^2}} = \frac{1}{4}\nu^2(\sqrt{g^2 + \acute{g}^2})^2 z_\mu z^\mu \tag{2.39}$$

Thus

$$m_{z=}^2 \frac{1}{4}\nu^2(g^2 + \acute{g}^2\,)^2$$

$$m_z = \frac{1}{2}\nu\sqrt{g^2 + \acute{g}^2}$$

Although since g and $\acute{g}$ are free parameters. The SM makes no absolute predictions for M_w and M_z, it has been possible to set a lower limit before the W-and Z-boson were discovered. Their measured values are $M_w = 80.4GeV$ and $M_z = 91.2GeV$ (Weinberg ،1967).

(2-4-2) Fermions mass and Yukawa interaction

In particle physics, Yukawa's interaction, named after Hideki Yukawa is an interaction between a scalar field and a Dirac field, the Yukawa interaction can be used to describe ;the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudo scalar mesons).

The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, as result these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field (J.donoghue, 1994).

The Yukawa interaction is uniquely fixed by the dynamic of the system. Its given by

$$\mathcal{L}_{yukawa} = Y_d \bar{q}_L \phi d_R + Y_U \bar{q}_L \phi^* U_R \tag{2.40}$$

Using equation (2.22) we get

$$\mathcal{L}_{yukawa} = Y_d (\bar{U}_L \quad \bar{d}_L) \frac{1}{\sqrt{2}} \binom{0}{\nu} d_R + Y_u (\bar{U}_L \quad \bar{d}_L) \frac{1}{\sqrt{2}} \binom{\nu}{0} U_R + Y_e (\bar{\nu}_L \quad \bar{e}_l) \frac{1}{\sqrt{2}} \binom{0}{\nu} e_R \tag{2.41}$$

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}} (\bar{U}_L \quad \bar{d}_L) \binom{0}{\nu} d_R + \frac{Y_u}{\sqrt{2}} (\bar{U}_L \quad \bar{d}_L) \binom{\nu}{0} U_R + \frac{Y_e}{\sqrt{2}} (\bar{\nu}_L \quad \bar{e}_l) \binom{0}{\nu} e_R \tag{2.42}$$

Then

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}} \nu \bar{d}_L d_R + \frac{Y_u}{\sqrt{2}} \nu \bar{U}_L U_R + \frac{Y_e}{\sqrt{2}} \nu \bar{e}_l e_R \tag{2.43}$$

From the last equation and analog to previous section we find that

$$m_d = \frac{Y_d}{\sqrt{2}} \nu$$

$$m_u = \frac{Y_u}{\sqrt{2}} \nu$$

$$m_e = \frac{Y_e}{\sqrt{2}} \nu$$

Where **Y** is Yukawa coupling.

(2-5)Full SM lagrangian:

To summarize the full standard model we gather together all the ingredients of the lagrangian.

Thus the complete (full) lagrangian is:

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{L}\gamma^{\mu}\left(i\partial_{\mu} - g\frac{1}{4}\tau W_{\mu} - \acute{g}\frac{Y}{2}B_{\mu}\right)L + \bar{R}\gamma^{\mu}(i\partial_{\mu} - \acute{g}\frac{Y}{2}B_{\mu})R + \left|(i\partial_{\mu} - g\frac{1}{4}\tau W_{\mu} - \acute{g}\frac{Y}{2}B_{\mu})\phi\right|^{2} - V(\phi) + (Y_{d}\bar{q}_{L}\phi d_{R} + Y_{U}\bar{q}_{L}\phi^{*}U_{R} + Y_{e}\bar{l}_{L}\phi e_{R} + \text{h.c.}) \quad (2.44)$$

L denotes a left-handed fermion (lepton or quark) doublet, and R a right-handed fermion singlet (J.donoghue, 1994).

Chapter Three

SEE SAW MECHANISMS

(3-1) Introduction

In this chapter we will the study the types of the see-saw mechanism which leads to Majoran neutrino at length.

(3-2) The see-saw mechanism

The see-saw mechanism is a generic model used to understand the relative sizes of observed neutrino masses, of the order of few eV, compared to those of quarks and charged leptons, which are millions times heavier (Pal, 1998).

The see-saw mechanism is the most likely way to explain how neutrinos got their mass, and why they are so small (W. Marciano, 1982).

(3-3) Types of the see-saw mechanism

There are several types of the see-saw mechanism each extending the standard model.

(3-3-1) Type I see-saw

Right-handed neutrino:

Introduction of right-handed neutrinoν_R, allows to insert additional term into Yukawa interaction

.

$$\mathcal{L}_Y^\nu = Y_D \bar{\ell}_L \sigma_2 \Phi^* \nu_R + \frac{M_R}{2} \nu_R^T C \nu_R + h.c. \tag{3.1}$$

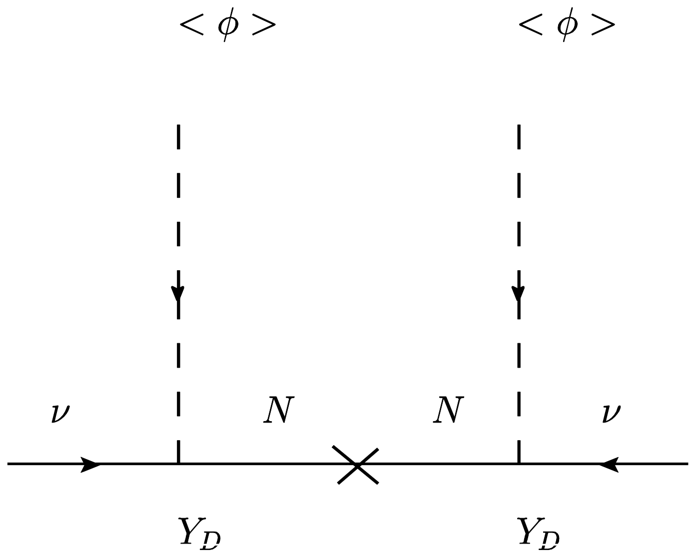

Figure 3.1. Diagrammatic representation of the Type I see-saw

The Yukawa interaction defines the quantum numbers of the right-handed neutrino , it carries the lepton number.

Majorana mass term is allowed for the right-handed neutrinos, consistent with the gauge symmetries of the theory (Majorana, 1937).

The Yukawa interaction, which couples left-handed and right-handed neutrinos yields after spontaneous symmetry breaking the Dirac neutrino mass matrix $m_D = Y_D v$, so that the complete mass terms are given by

$$\mathcal{L}_M^\nu = m_D \bar{\nu}_L \nu_R + \frac{1}{2} M_R \nu_R^T C \nu_R + h.c. \tag{3.2}$$

We can rewrite equation (3.2) in terms of two components spinners.

$$\nu \equiv \nu_L + C\bar{\nu}_L^T \tag{3.3}$$

$$N \equiv \nu_R + C\bar{\nu}_R^T \tag{3.4}$$

Using the properties of charge-conjugation matrix

$$C^T\gamma^\mu C = -\gamma_\mu^T\,, \qquad C^T = -C \tag{3.5}$$

We obtain the following relation

$$\bar{\nu}N = \bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L = \bar{N}\nu \tag{3.6}$$

$$\bar{N}N = \nu_R^T C\nu_R + h.c. \tag{3.7}$$

$$\nu\bar{\nu} = \nu_L^T C\nu_L + h.c. \tag{3.8}$$

So that the lagrangian can be written as

$$\mathcal{L} = \frac{1}{2}\,[i\bar{\nu}\gamma^\mu\partial_\mu\nu + i\bar{N}\gamma^\mu\partial_\mu N - M_L\bar{\nu}\nu - M_R\bar{N}N - m_D\bar{\nu}N - m_D\bar{N}\nu] \tag{3.9}$$

We now summarize the above equation with the following form of mass matrix

$$\mathcal{L}_m^{(\nu)} = \frac{1}{2}(\bar{\nu},\bar{N})\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu \\ N \end{pmatrix} + h.c. \tag{3.10}$$

The mass matrix for the fields ν and N

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \tag{3.11}$$

Where $m_D = Y_D\nu$. One can diagonalize this matrix by a similarity transformation using the orthogonal matrix

$$\begin{pmatrix} 1 & \epsilon \\ -\epsilon^T & 1 \end{pmatrix} \tag{3.12}$$

Where $\epsilon = \frac{m_D}{M_R}$. This diagonalization is correct up to terms smaller than of order ϵ^2 , one obtains the mass matrix for the light neutrino to be

$$m_\nu^{light} = -m_D \frac{1}{M_R} m_D^T \tag{3.13}$$

This is the original see-saw formula called type 1.

If $M_R << m_D$, neutrino would be predominantly Dirac particles. For $M_R \approx M_D$, we have a messy combination of Majorana and Dirac, where as for $m_D << M_R$ we would have a Majorana case. The diagrammatic representation of the see-saw in Figure 3.1.of the type I

show that the heavy neutrino propagator gives the see-saw result.

(3-3-2) Type II see-saw

Instead of right-handed neutrino ν_R we can choose Y=2, triplet scalar Δ_L this will lead to new term in Yukawa interaction

$$\mathcal{L}_\Delta = Y_\Delta^{-1} \ell^T C \sigma_2 \Delta_L \ell + h.c \tag{3.14}$$

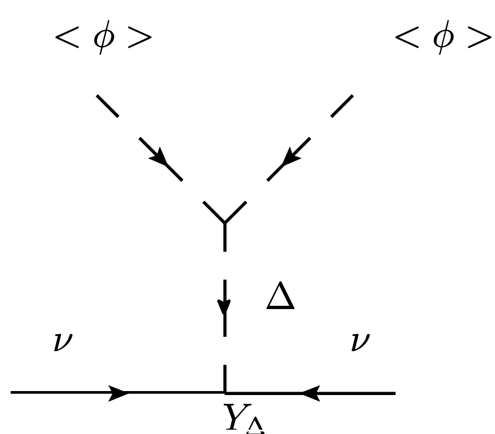

Figure 3.2. Diagrammatic representation of the Type II see-saw

Neutrino gets mass when Δ_L gets a vev

$$M_\nu = Y_\Delta \langle \Delta \rangle \tag{3.15}$$

The vev $\langle \Delta \rangle$ results from the cubic scalar interaction

$$\Delta V = \mu \Phi^t \sigma_2 \Delta_L^* \Phi + M_\Delta^2 Tr \Delta_L^* \Delta_L + \cdots \tag{3.16}$$

With

$$\langle \Delta \rangle \equiv \frac{\mu \upsilon^2}{M_\Delta^2} \tag{3.17}$$

$$m_\nu = Y_\Delta \frac{\mu \upsilon^2}{M_\Delta^2} \tag{3.18}$$

Where one expect μ of order M_Δ. If $M_\Delta >> \upsilon$, neutrinos are naturally light.

(3-3-3) Type III see-saw

In this type we introduce triplet fermions $\vec{T}_F$ in Majorana notation, (where for simplicity the generation index is suppressed and also an index counting the number of triplet).

$$\Delta \mathcal{L}(T_F) = Y_T \ell^T C \sigma_2 \vec{\sigma}.\vec{T}_F \Phi + M_T \vec{T}_F^T C \vec{T}_F \tag{3.19}$$

Exactly the same manner as before in type I, one gets a type III see-saw for $M_T >> \upsilon$

$$M_\nu = -Y_T^T \frac{1}{M_T} Y_T \nu^2 \tag{3.20}$$

Chapter Four

DISCUSSIONS AND CONCLUSIONS

(4-5) Discussions

When considering the SM family plus an additional SM gauge singlet, the right-handed neutrino, such theories allow type I and type II seesaw mechanisms for generating light neutrino masses.

In summary, the main message of this project should be that the Majorana neutrino mass is rather suggestive from the theoretical point of view. As such, it provides a window to new physics at scale M. The crucial prediction of this picture is the L = 2 lepton number violation in processes such as neutrino less double beta decay $\beta\beta0\nu$. However, $\beta\beta0\nu$ depends in general on the new physics at scale M, and it is desirable to have a direct probe of lepton number violation.

What happens if the neutrino has a pure Dirac mass In this case, and the smallness of Dirac mass simply requires the smallness of Yukawa. The smallness of Dirac mass remains a puzzle controlled by small Yukawa, as much as the smallness of electron mass is controlled by a small electron Yukawa coupling.

(4-6) Conclusions

In conclusion, we have studied the theory of standard model in some details and its extension to accommodate the neutrino mass by implementing the three types of see saw mechanisms. By now we have evidence that neutrino are massive.

From the standard model point of view masses could vanish if no right-handed neutrinos existed (no Dirac mass) and lepton number was conserved (no Majorana mass)

The see-saw mechanism explains the smallness of neutrino masses in term of the large scale where B-L violated. Thus neutrino masses are important to a probe into the physics at GUT scale.

Recommendation: In the left-right symmetric and SO(10) family unified both these assumptions are violated. The right-handed neutrinos are required in all unifying groups larger than SU (5). In SO (10) the 16 fermion fields in each family, including the right handed neutrino, exactly fit into 16-dimensional representation of this group.

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