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ELASTIC NEUTRINO-MUON AND ELECTRON SCATTERING IN THE STANDARD MODEL

التشتت المرن للنيوترينو ميون والإلكترون في النموذج العياري

A Graduate Project Submitted for the Degree of B.Sc. (honor) in Physics

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الآيـة

{ شَهِدَ اللَّهُ أَنَّهُ لَا إِلَهَ إِلَّا هُوَ وَالْمَلَائِكَةُ وَأُولُو الْعِلْمِ قَائِمًا بِالْقِسْطِ لَا إِلَهَ إِلَّا هُوَ الْعَزِيزُ الْحَكِيمُ }

[آل عمران :17]

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Abstract

In this research project we studied the theory of Fermi and the standard model of particle physics in some details. We calculated via a theoretically the cross-section of the elastic neutrino–muon and electron scattering in the Fermi theory and the standard model and found their values at $E = 4\ MeV$ to be

$$\boldsymbol{\sigma_{SM} = 5.28 \times 10^{-44} cm^2}$$

$$\boldsymbol{\sigma_{Fermi} = 8.61 \times 10^{-43} cm^2}$$

We have found that the cross-section in SM less than the cross-section in Fermi for a given energy. Furthermore, the Fermi theory breaks down at high energy but the standard model works very well at high energies.

ملخص البحث

في هذا البحث تم دراسة نظرية فيرمي للتفاعلات الضعيفة والنظرية العيارية لتوحيد القوي في الطبيعة. وتم حساب مساحه مقطع التشتت المرن للنيوترينو ميون في النموذجين حيث وُجد ان مساحة مقطع التشتت عند طاقة مقدارها 4 ميقا فولت تساوي

$$\sigma_{\text{العيارية}} = 5.2809 \times 10^{-44} cm^2$$

$$\sigma_{\text{فيرمي}} = 8.61184 \times 10^{-43} cm^2$$

وايضاً وُجد ان مساحة مقطع التشتت في النظرية العيارية اقل بكثير من مساحة مقطع التفاعل في نظرية فيرمي عند نفس الطاقة. إضافةً الي ذلك وُجد أن نظرية فيرمي تنهار عند الطاقات العالية بعكس النظرية العيارية التي تنجح بكفاءة عالية.

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Chapter One

INTRODUCTION

(1-1) Introduction

In this project we show and explore the importance of studying the neurino muon and electron scattering in Fermi theory of weak interaction and the standard model of particle physics. We will state the main objectives of this project and the outline of the project.

(1-2)The importance of the project

The cross section is a useful quantity to measure experimentally the probability of scattering or absorption event in particle physics, the concept of a cross section is used to express the interaction between elementary particles which is a measure of the probability of a specific scattering process under some given set of initial and final condition.

(1-3)The main objectives of the research

The aim of this project is to calculate the cross section for neutrino muon and electron in the standard model of particle physics and in Fermi theory of weak interaction and compare the two results.

(1-5)The outline of the research

This reserach project is structured as follow: In chapter one we give brief introduction and chapter two we introduce the Fermi theory of weak interaction and the standard model of particle physics, in chapter three we calculate the cross section for neutrino muon and electron scattering in both models, we present in chapter four our numerical results, discussions and conclude our results.

Chapter Two

INTRODUCTION TO THE STANDARD MODEL

(2-1) introduction

In this chapter we shall study the structure of the standard model and its beautiful mathematical structure, then we will introduce the spontanous symmetry breaking which refer also to the higgs mechanism to see how particles acquire their masses.

(2-2) what is the standard model (SM)

The Standard Model is theory that describes the interaction between subatomic particles. The SM is believed to describe all physical phenomena except gravity (Guigg, 1983).

We described the three fundamental interactions or force, the electromagnetic, weak and strong interactions. Electromagnetic force has infinite range but it is many times stronger than gravity. The weak and strong forces are effective only over a very short range and dominate only at the level of subatomic particles. The weak force is much stronger than gravity but it is indeed the weakest of the other three. The strong force, as the name suggests, is the strongest of all four fundamental interactions. Each force is mediated by a force-carrier particle called a gauge boson. Being a boson has integral spin. the gauge bosons for the electromagnetic, weak and strong forces are all spin-1. The standard model includes 12 elementary particles of spin (1/2) that make up matter (J.donoghue, 1994).

(2-2-1) Fermions

In the standard model the fermions are particles that make up matter and have spin (1/2) and can be divided in two groups, leptons and quarks. Each group comes in three different 'families' Lepton interact via the electromagnetic and weak interactions, but do not participate in the strong interaction. They can carry electrice charge, which we denote as -1 (the charge of the electron), or they carry electrically neutral (A.D.Martin, 1984). The leptons include the following particles:

The electron e carries charge -1 and has a mass of 0.511MeV/c^2.

The muon μ^- carries charge -1 and has a mass of 106M eV/c^2.

The tau τ^- carrries charge -1 and has a mass of 1777 MeV/c^2.

Each type of lepton described above defines one of the three families that make up the leptons. In short, the muon and tau are just heavy copies of the electron. Physicsts are not sure Why there are three familes of particles. The muon and tau are unstable and decay into elecrons and neutrinos (A.D.Martin, 1984).

Corresponding to each particle above, there is a neutrino. It was thought for a long time that neutrinos were massless, but the recent evidence indicates that this is not true, although experiment puts small bounds on their masses. Like the electron, muon, and tau, the three types of neutrinos come with masses that increase with each family. They are electrically neutral and are denoted by neutrino electron v_e, neurino muon v_μ and neutrion tau v_τ

Since they are electrically neutral, the neutrinos do not participate in the electromagnetic interaction. Since they are leptons, they do not participate in the strong intreaction either. They interact only via the weak interaction. For each lepton there is corresponding an anti lepton. The antiparticles corresponding to the electron, muon and tau all carry charge of +1, but they have the same masses. They are denoted as follows:

The positron e^+ carries charge +1 and has a mass of 0.511MeV/c^2 .

The antimuon μ^+ carries charge +1 and has a mass of 106M eV/c^2 .

The antitau τ^+ carries charge +1 and a mass of 1777 MeV/c^2.

Quarks that make up the neutron and proton. They are charged and thus participate in the electromagnetic ineraction. They also participate in the weak interaction . There are six types of quarks :

Up type quark ’u’ with charge +2/3 .

Down type quark ’d’ with charge -1/3 .

Strange type quark ’s’ with charge +2/3 .

Charmed type quark ’c’ with charge -1/3 .

Top type quark ’t’ with charge +2/3 .

Bottom type quark ’b’ with charge -1/3 .

Hadrons particles are classified into baryons and mesons. The baryons are fermions made of three quarks, qqq , as for instance the proton, $p \sim uud$, and the neutron, $n \sim ddu$, The mesons are bosons made of one quark and one antiquark as for example the poins , $\pi^+ \sim u\bar{d}$ and $\pi^- \sim d\bar{u}$. (Guigg, 1983)

(2-2-2) Bosons

Bosons are the particles that knwon as the force-carrier with spin (1) . the photon, γ, is the exchanged particle in the electromagnetic interaction, the eight gluons $g_\propto$;∝=1...8 mediate the strong interaction among quarks, and the three weak bosons, $w^\pm, Z$ are the corresponding mediater of the weak interaction (S.L.Glashow, 1961)

(2-3) The standard model lagrangian

Standard model of particle physics structured by using the rules of quantum field theory as we know the lagrangian defines the theory. It is written in terms of the elementary particles of the theory and contains the fundamental interaction of the theory. The SM lagrangian is based on gauge group theory.

The unitary group U(N) consist of all $N \times N$ unitary matrieces. Special unitary groups denoted by SU(N) are $N \times N$ unitary matrices with positive unit determinant, the dimension of SU(N) and hence the number of generators is given by $N^2 - 1$.

SU(2) has $2^2 - 1 = 3$ generators

SU(3) has $3^2 - 1 = 8$ generators

The simplest unitary group is the group U(1) . A ’1×1 matrix is just complex number written in polar representation .

When a lagrangian is invariant under a transformation there is symmetry , and in this case there is a U(1) symmetry. Force –mediating particle called gauge boson , will be associated with unitary symmetries. The guage boson associated with the U(1) symmetry of quantum electrodynamics is the photon (L.F.Li, 1991).

An particles appear to have second internal invarance under set of transformation that from SU(2) group, called the electroweak SU(2) invarince. The associated gauge boson necessary to maintain the invarance of the theory are called W_i^μ. There is one boson for each of the three generator of SU(2) transformation so $i = 1,2 \; or \; 3,$ these are called weak isospin transformation (A.J.G.hey, 1993)

All particles appear to have a third internal invariance, under a set or transformation that form an SU(3) group, the associated gauge bosons are labeled G_a^μ, where now $a = 1,2, \dots .8$ since ther is one spin-one boson for each of the eight generators of SU(3) they are called gluons.

The standad model is based on the $SU(3) \times SU(2) \times U(1)$ (J.donoghue, 1994)

So the lagrangian of the standard model can be written as:

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{higgs} + \mathcal{L}_{yukawa} \tag{2.1}$$

It defines the fundamental interaction of ferminos, gauge bosons and higgs boson. we will go through the details on next section, then we will write down the full standard model Lagrangian.

(2-3-1) The fermion sector

Quarks and leptons are organized in three families with identical peoperties except for mass. The particle content in each family is :

1^{st} family: $\binom{v_e}{e^-}_L$, e_R^-, $\binom{u}{d}_L$, $u_{R,}$ d_R (2.2)

2^{nd} family: $\binom{v_u}{\mu^-}_L$, μ_R^- , $\binom{c}{s}_L$, c_R, s_R (2.3)

3^{rd}family: $\binom{v_t}{\tau^-}_L$, τ_R^-, $\binom{t}{b}_L$, t_R, b_R (2.4)

And their corresponding antiparticles. The left-handed and left-handed fields are defined by means of the chirality operator γ_5 as usual

$$e_L^- = \frac{1}{2}(1-\gamma_5)e^- \, ; \, e_R^- = \frac{1}{2}(1+\gamma_5)e^- \quad (2.5)$$

They transform as doublets and singlets of $\mathrm{SU}(2)_L$ respectively.

We can write the fermions Lagrangian as (McMahon, 2008):

$$\mathcal{L}_{\text{fermions}} = \sum\{\overline{\psi}_L i\gamma^\mu D_\mu \psi_L + \overline{\psi}_R i\gamma^\mu D_\mu \psi_R \} \quad (2.6)$$

(2-3-2) Gauge boson sector

The gauge boson and the scalar Lagrangian give rise to the free Lagrangian for the photon, W , Z. and the higgs boson .The standard model gauge boson Lagrangian (gauge fields)is (Weinberg, 1996):

$$\mathcal{L}_G = -\frac{1}{4}G^a_{\mu v}\, G^{a\mu v} - \frac{1}{4}W^i_{\mu v}W^{i\mu v} - \frac{1}{4}B_{\mu v}B^{\mu v} \quad (2.7)$$

Where

$$G^a_{\mu v} = \partial_\mu G_v^a - \partial_v G_\mu^a + g_s f_{abc} G_\mu^b G_v^c \quad (2.8)$$

$$W^i_{\mu v} = \partial_\mu W_v^i - \partial_v W_\mu^i + g\epsilon_{ijk} W_\mu^i W_v^i \quad (2.9)$$

$$B_{\mu v}=\partial_{\mu}B_{v}-\partial_{v}B_{\mu} \quad (2.10)$$

$B_{\mu v}$ Is the hypercharge field strength, $W^{i}_{\mu v}$ contains the SU(2) field strength, so 'i' runs from one to three (over the three vector bosons of SU(2)), and $G^{a}_{\mu v}$ are the gluons kinetic term, so a=1.... 8.

(2-4) Higgs mechanism

In the particle physics the Higgs mechanism is needed to explain the generation mechanism of the mass property. An extra field called the Higgs field has to be added by hand to give the particles mass. The quantum of the Higgs field is spin-0 particle called the Higgs boson (Quigg, 2007)

Elementary particles acquire their masses through their interaction with the Higgs field, mathematically we introduce mass into a theory by adding interaction terms into the lagrangian that couple the field of the particle with the Higgs field . Normally, the lowest energy state of a field would have an expection value of zero. By summetry breaking, we introduce a non zero lowest energy state of the field. This procedure leads to the acquisition of mass by the particles in the theory. We can imagine the movement of elementary particles being resisted by the higgs field, with each particle interaction with the higgs field at a different strength. If the coupling between the higgs field and the particle is strong, then the masss of particle is large. If it is weak, then the particle has a smaller mass. A particle like the photon with zero rest mass dosen't interact with the higgs field at all. If the higgs field didn't exist at all, then all particle would be massless..

The higgs mechanism gives mass to the standard model particles and keeps the lagrangian invariant under guage symmetries. which will give the higgs a vacaum expectation value. This machanism added a new complex scalar Φ (Quigg, 2007). This scalar particle has been

discovered by the ATLAS (al, 2012) and CMS (al, 2012) experiments, which is compatible with the SM.
Higgs expectations with a mass of about 126 GeV

$$\boldsymbol{\mathcal{L}_{higgs}} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathrm{D}_{\mu}\,\Phi\right)^{\dagger}(\mathrm{D}^{\mu}\,\Phi\ ) - \mathrm{V}(\,\Phi\,) \tag{2.11}$$

$$\mathrm{V}(\,\emptyset\,) = \frac{\mu^2}{2}\,\Phi^{\,2} + \frac{\lambda}{4}\,\Phi^{\,4} \tag{2.12}$$

Therefore equation (2.11) becomes

$$\boldsymbol{\mathcal{L}_{higgs}} = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathrm{D}_{\mu}\,\Phi\right)^{\dagger}(\mathrm{D}^{\mu}\,\Phi) - \frac{\mu^2}{2}\,\Phi^{\,2} + \frac{\lambda}{4}\,\Phi^{\,4} \tag{2.13}$$

$\lambda \equiv$ Higgs self coupling.

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix} \tag{2.14}$$

By minimizeV(Φ):

$$\frac{\partial v}{\partial \Phi} = 0 \tag{2.15}$$

$$\frac{\partial v}{\partial \Phi} = -\mu^2\Phi + \lambda\,\Phi^3 \tag{2.16}$$

Solving this equation yeilds two solutions

$$\Phi(\mu^2 + \lambda\Phi^2\,) = 0 \tag{2.17}$$

$$\Phi = 0\ (\text{trivial solution})\ or\ (\mu^2 +\ \lambda\Phi^2) = 0 \tag{2.18}$$

$$\langle\Phi^2\rangle = \frac{-\mu^2}{\lambda} \tag{2.19}$$

$$\langle\Phi\rangle = \pm\sqrt{\frac{-\mu^2}{\lambda}} = v \tag{2.20}$$

Where v is known as the vacuum expecation value, $v = 246\ GeV$.

Equation (2.12) represents the Higgs potential, which involves two new real parameters μ and λ. We require that $\lambda > 0$ for the potential to be bounded; otherwise the potential is unbounded from below and there will be no stable vacuum state. μ takes the following two values:

1− $\mu^2 > 0$ in this case the vacuum corresponds to $\phi = 0$, the potential has a minimum at the origin (see Figure 2.1).

2− $\mu^2 < 0$ in this case the potential develops a non-zero Vacuum Expectation Value (VEV) and the minimum is along a circle of radius (see Figure 2.2).

$$(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = v^2 \tag{2.21}$$

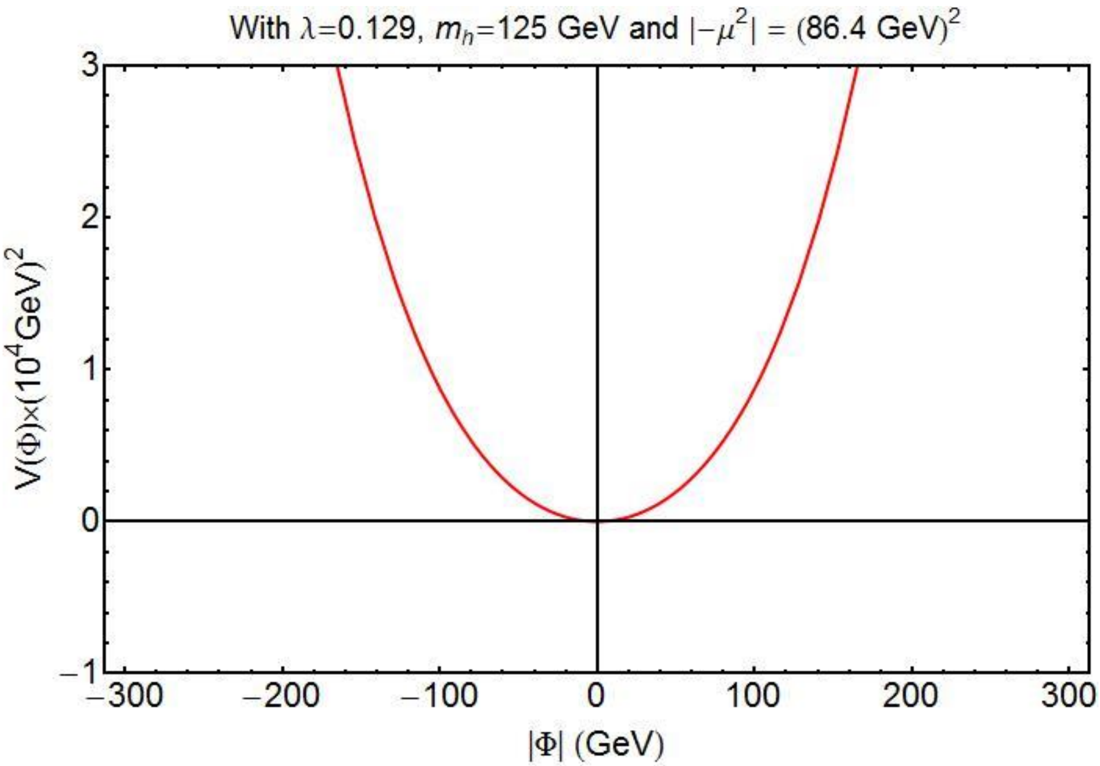

Figure 2.1. The Higgs potential with: the case $\mu^2 > 0$; as function of $|\Phi|$.

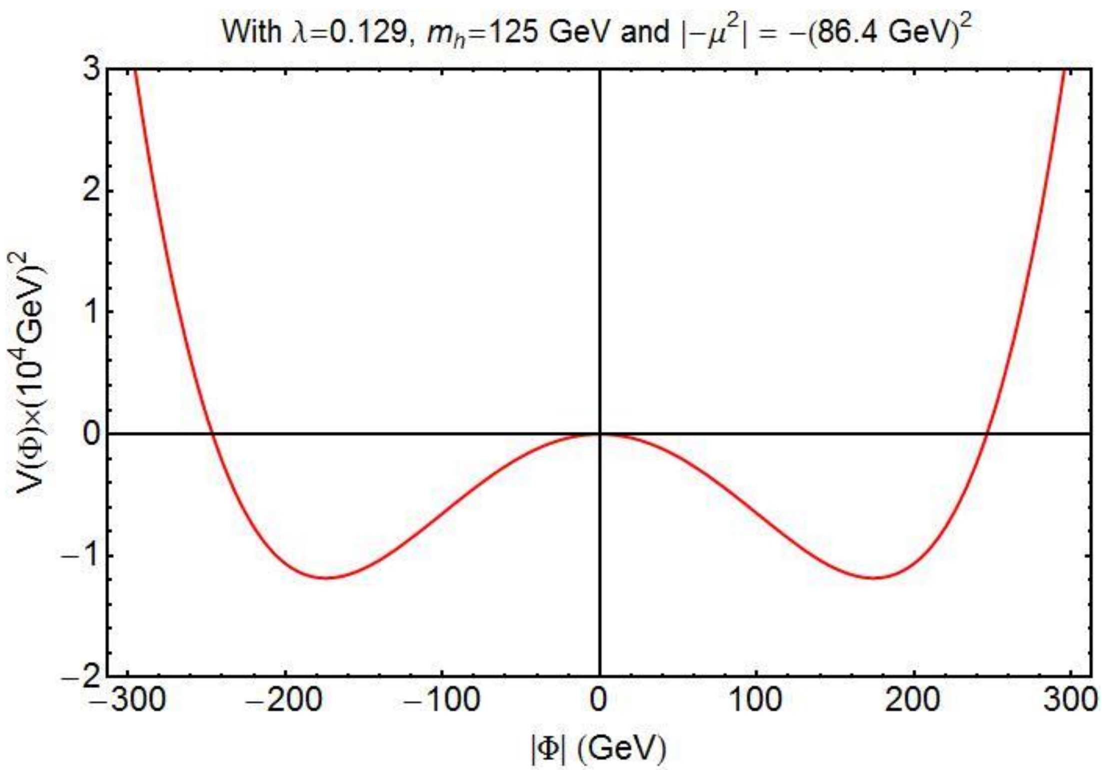

Figure 2.2. The Higgs potential with: the case $\mu^2 < 0$; as function of $|\Phi|$.

$$\Phi = \frac{1}{\sqrt{2}}\binom{0}{v} \tag{2.22}$$

Next we will see how to use this technique to give bosons and fermions a mass.

(2-4-1) Gauge bosons mass

To obtain the masses for the gauge bosons we will only need to study the scalar part of the Lagrangian (L.F.Li, 1991):

$$\mathcal{L} = \frac{1}{2}\left(D_\mu\phi\right)^\dagger (D^\mu\phi) - V(\phi) \tag{2.23}$$

Where $D_\mu$is the covariant derivative

$$D_\mu = (\partial_\mu + ig\tau^a W_\mu^a + i\acute{g}\frac{Y_\phi}{2}\boldsymbol{B}_\mu) \tag{2.24}$$

Now

$$D_\mu = \left[\partial_\mu + ig\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix} + i\acute{g}\frac{Y_\phi}{2}\boldsymbol{B}_\mu\right] \tag{2.25}$$

Therefore

$$D_\mu\phi = \left[\partial_\mu\phi + ig\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix}\phi + i\acute{g}\frac{Y_\phi}{2}\boldsymbol{B}_\mu\phi\right] \tag{2.26}$$

We have

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \tag{2.27}$$

Using (2.27) and (2.28) we get

$$D_\mu\phi = \frac{ig}{\sqrt{2}}\begin{pmatrix} W_3 & W^- \\ W^+ & -W_3 \end{pmatrix}\begin{pmatrix} 0 \\ \nu \end{pmatrix} + \frac{i\acute{g}Y_\phi\boldsymbol{B}_\mu}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix} \tag{2.28}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} igW_3 & igW^- \\ igW^+ & -igW_3 \end{pmatrix}\begin{pmatrix} 0 \\ \nu \end{pmatrix} + \frac{i\acute{g}}{\sqrt{2}}\begin{pmatrix} 0 \\ \boldsymbol{B}_\mu\nu \end{pmatrix} \tag{2.29}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{ig\nu}w^- \\ -\mathrm{ig\nu w_3} \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ i\acute{g}\boldsymbol{B}_\mu\nu \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{ig\nu}w^- \\ -\mathrm{ig\nu w_3} + i\acute{g}\nu\boldsymbol{B}_\mu \end{pmatrix} \tag{2.30}$$

$$\Rightarrow D_\mu\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{ig\nu}w^- \\ -\mathrm{ig\nu w_3} + i\acute{g}\nu\boldsymbol{B}_\mu \end{pmatrix} \tag{2.31}$$

Since $(D_\mu\phi)^\dagger$ is the complex conjugate of $D_\mu\phi$ then

$$(D_\mu\phi)^\dagger = \frac{1}{\sqrt{2}}(\mathrm{ig\nu}w^- \quad \mathrm{ig\nu w_3} - i\acute{g}\nu\boldsymbol{B}_\mu) \tag{2.32}$$

$$\left(D_\mu\phi\right)^\dagger(D^\mu\phi) = \frac{1}{\sqrt{2}}(\mathrm{igv}w^- \quad \mathrm{igvw}_3 - i\acute{g}v\boldsymbol{B}_\mu)\frac{1}{\sqrt{2}}\begin{pmatrix}\mathrm{igv}w^- \\ -\mathrm{igvw}_3 + i\acute{g}v\boldsymbol{B}_\mu\end{pmatrix} \tag{2.33}$$

$$= \frac{1}{2}\left[g^2v^2w^+w^- + v^2(gw_3 - \acute{g}B_\mu)^2\right] \tag{2.34}$$

$$\frac{1}{2}\left(D_\mu\phi\right)^\dagger(D^\mu\phi) = \frac{1}{4}g^2v^2w^+w^- + \frac{1}{4}v^2\left(gw_3 - \acute{g}B_\mu\right)^2 \tag{2.35}$$

From above equation (2.36) we obtain:

$$m_w^2 = \frac{1}{4}g^2v^2 \tag{2.36}$$

$$m_w = \frac{1}{2}vg \tag{2.37}$$

For Z boson, we have orthogonal combination as:

$$z_\mu = \frac{gw_3 - \acute{g}B_\mu}{\sqrt{g^2+\acute{g}^2}} = (\cos\theta_w\, w_3 - \sin\theta_w\ B_\mu) \tag{2.38}$$

And the photon field

$$A_\mu = \frac{1}{\sqrt{g^2+\acute{g}^2}}(\acute{g}w_3 + gB_\mu) \tag{2.39}$$

By using a rotation transformation:

$$\begin{pmatrix} z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix}\cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w\end{pmatrix}\begin{pmatrix} w_3 \\ B_\mu\end{pmatrix} \tag{2.40}$$

$$\cos\theta_w = \frac{g}{\sqrt{g^2+\acute{g}^2}} \quad \mathit{and} \quad \sin\theta_w = \frac{\acute{g}}{\sqrt{g^2+\acute{g}^2}} \tag{2.41}$$

Multiply the second part of equation (2.36) by $\frac{\sqrt{g^2+\acute{g}^2}}{\sqrt{g^2+\acute{g}^2}}$ we obtain

$$\frac{1}{4}\nu^2\left(gw_3 - \acute{g}B_\mu\right)^2 \cdot \frac{\sqrt{g^2+\acute{g}^2}}{\sqrt{g^2+\acute{g}^2}} = \frac{1}{4}\nu^2(\sqrt{g^2+\acute{g}^2})^2 z_\mu z^\mu \quad (2.42)$$

Thus

$$m_{z=}^2 \frac{1}{4}\nu^2(g^2+\acute{g}^2\,)^2 \quad (2.43)$$

$$m_z = \frac{1}{2}\nu\sqrt{g^2+\acute{g}^2} \quad (2.44)$$

Although since g and $\acute{g}$ are free parameters. The SM makes no absolute predictions for M_w and M_z, It was possible to set a lower limit before the W-and Z-boson were discovered. Their measured values are $\boldsymbol{M_w = 80.4\ GeV}$ and $\boldsymbol{M_z = 91.2\ GeV}$ (S.L.Glashow, 1961) (Salam, 1968)

(2-4-2) Fermions mass and Yukawa interaction

The Yukawa interaction is an interaction between a scalar field φ and a Dirac field ψ.

The Yukawa interaction can be used to describe the strong nuclear force between nucleons (which are fermions) mediated by pions (which are scalar mesons). The Yukawa interaction is also used in the standard model to describe the coupling between the higgs field and massless quark and electrons fields through spontaneous symmetry breaking, the fermions acquire a mass proportional to the vacuum expectation value of the higgs field (A.J.G.hey, 1993).

The Yukawa interaction is uniquely fixed by the dynamic of the system. It's given by

$$\mathcal{L}_{yukawa} = Y_d\bar{q}_L\phi d_R + Y_U\bar{q}_L\phi^* U_R \quad (2.45)$$

$$\mathcal{L}_{yukawa} = Y_d(\bar{U}_L \quad \bar{d}_L)\frac{1}{\sqrt{2}}\binom{0}{\nu} d_R + Y_u(\bar{U}_L \quad \bar{d}_L)\frac{1}{\sqrt{2}}\binom{\nu}{0} U_R + Y_e(\bar{\nu}_L \quad \bar{e}_l)\frac{1}{\sqrt{2}}\binom{0}{\nu} e_R \quad (2.46)$$

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}}(\bar{u}_L \quad \bar{d}_L)\begin{pmatrix}0\\ \nu\end{pmatrix} d_R + \frac{Y_u}{\sqrt{2}}(\bar{u}_L \quad \bar{d}_L)\begin{pmatrix}\nu\\ 0\end{pmatrix} u_R + \frac{Y_e}{\sqrt{2}}(\bar{\nu}_L \quad \bar{e}_l)\begin{pmatrix}0\\ \nu\end{pmatrix} e_R \tag{2.47}$$

$$\mathcal{L}_{yukawa} = \frac{Y_d}{\sqrt{2}}\, \nu \bar{d}_L\, d_R + \frac{Y_u}{\sqrt{2}}\, \nu \bar{u}_L u_R + \frac{Y_e}{\sqrt{2}}\, \nu \bar{e}_l e_R \tag{2.48}$$

From last equation and analog to previous section we find that (McMahon, 2008)

$$m_d = \frac{Y_d}{\sqrt{2}}\, \nu \tag{2.49}$$

$$m_u = \frac{Y_u}{\sqrt{2}}\, \nu \tag{2.50}$$

$$m_e = \frac{Y_e}{\sqrt{2}} \nu \tag{2.51}$$

Where **Y** is Yukawa coupling.

(2-5) Full SM Lagrangian

To summarize the standard model we collect together all the ingredients of the Lagrangian.

Therefore the complete (full) Lagrangian is:

$$\mathcal{L} = -\frac{1}{4}\mathrm{W}_{\mu\nu}\mathrm{W}^{\mu\nu} - \frac{1}{4}\mathrm{B}_{\mu\nu}\mathrm{B}^{\mu\nu}\{\mathrm{W}^{\pm}, \mathrm{Z}, \gamma \text{ Kinetic energies and self} - \text{interction.} + \bar{\mathrm{L}}\gamma^{\mu}\Big(\mathrm{i}\,\partial_{\mu} -$$

$$\mathrm{g}\frac{1}{4}\tau\mathrm{W}_{\mu} - \acute{\mathrm{g}}\frac{\mathrm{Y}}{2}\mathrm{B}_{\mu}\Big)\,\mathrm{L} + \bar{\mathrm{R}}\gamma^{\mu}(\mathrm{i}\,\partial_{\mu} - \acute{\mathrm{g}}\frac{\mathrm{Y}}{2}\mathrm{B}_{\mu})\mathrm{R}\begin{cases}\text{lepton and quark}\\ \text{kinetiv energies and their}\\ \text{interaction with } \mathrm{W}^{\pm}, \mathrm{Z}, \gamma\end{cases} + \Big|(\mathrm{i}\,\partial_{\mu} -$$

$$\mathrm{g}\frac{1}{4}\tau\mathrm{W}_{\mu} - \acute{\mathrm{g}}\frac{\mathrm{Y}}{2}\mathrm{B}_{\mu})\phi\Big|^2 - \mathrm{V}(\phi)\{\mathrm{W}^{\pm}, \mathrm{Z}, \gamma \text{ and higgs masses couplings} \quad (-\mathrm{G}_1\bar{\mathrm{L}}\phi\mathrm{R} + \mathrm{G}_2\bar{\mathrm{L}}\phi_c\mathrm{R} +$$

$$\text{hermition conjugate})$$

L denotes a left-handed fermion (lepton or quark) doublet, and R a right-handed fermion singlet.

Chapter Three

SCATTERING CROSS-SECTION

(3-1) Introduction

In this chapter we will derive the cross section formula for neutrino-muon and electron scattering, using two methods, that is Fermi effective theory and the standard model of particle physics.

(3-2)The cross section

The cross section is a hypothetical area measure around the target particles of the substance (using its atoms) that represent a surface, if a particle of the beam crosses this surface, there will be kind of interaction.

The cross section is a natural quantity to measure experimentally. The probability of some scattering or absorption event in nuclear and particle physics, the concept of a cross section is used to express the likelihood of interaction between particle which is a measure of the probability of a specific scattering process under some given set of initial and final condition. The term is derived from the purely classical picture of (a large number of) point like projectiles directed to an area that includes a solid target . Assuming that an interaction will occur (with 100% probability) if the projectile hits the solid, and not at all (0% probability) if it misses, the total interaction probability for the single projectile will be the ratio of the area of the section of the solid to the total targeted area (L.F.Li, 1991).

(3-3)Neutrino-Muon and electron scattering in the SM

In this section we will derive the cross-section formula that we used in our calculations in the two scenarios that is the SM and Fermi theory.

(3-4) Fermi approach

According to Pauli the Beta –decay ($\beta\ decay$) of

$$B_{i_{83}}^{216} \rightarrow po_{84}^{216} + e + \nu \tag{3.1}$$

The question was how ν can be kept in nuclei being neutral particle? the answer has been given by E. Fermi in 1934 who knew

1- Discovery of neutron.
2- Pauli hypothesis.

Fermi suggested that the β-decay of nuclei is the decay of one neutrons which is contained in nuclei as

$$n \rightarrow p + e^{-} + \overline{\nu_e} \qquad (\beta^{-} - decay) \tag{3.2}$$

Also inverse transition $p \rightarrow n$ is possible in nuclei

$$p \rightarrow n + e^{+} + \nu_e \qquad (\beta^{+} - decay) \tag{3.3}$$

Fermi used the analogy with electrodynamics , he established the Hamiltonian of weak interaction which described by the neutron decay

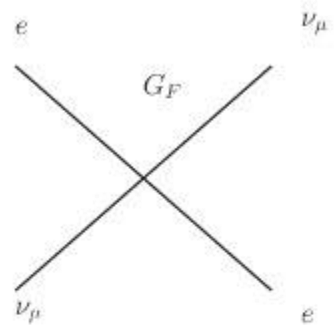

Fig 1.3 Feynman diagram of neutrino-electron scattering in Fermi theory

$$H = \frac{G_f}{\sqrt{2}} \bar{P}(x)\gamma^{\mu} n(x)\, \bar{e}(x)\gamma_{\nu}\ \nu_e(x) + \cdots \quad (3.4)$$

Here p(x) , n(x) , (x) ,andν_e (x) are the fields of proton ,neutron ,electron and neutrino

respectively.

G_f is the Fermi coupling constant .

All the fields are taken in the same space–time point therefore, the interaction occurs locally.

Let us estimate the probability" **p**" of neutrino interaction in a detector. It is given by the product

$$P = L\mathrm{d}\sigma \quad (3.5)$$

Where **L** is the length of the detector along the neutrino trajectory, **d** is the number density and $\boldsymbol{\sigma}$ is the cross-section – the characteristics of interaction that should have according to the above equation the dimension of $[\sigma] = cm^2 = GeV^{-1}$or inverse of mass (L.F.Li, 1991).

For very low energies; $E_\nu \ll m_P$.

The cross-section can be easily estimated on the basis of dimensional analysis, since

$$H \propto G_F \quad \Longrightarrow P \propto H^2 \tag{3.6}$$

Then the cross section is proportional to

$$\sigma \propto {G_F}^2 \tag{3.7}$$

If E<<m_P ,

Where m_P is mass of nucleon at rest , the only energy characteristics is neutrino energy E_V.

Therefore the correct dimension of $\boldsymbol{\sigma}$ can be represent as

$$\sigma = {G_F}^2 {E_\nu}^2 \tag{3.8}$$

(3-5)Elastic Neutrino-electron scattering in the SM:

$$\nu_\mu + e \longrightarrow \nu_\mu + e \tag{3.9}$$

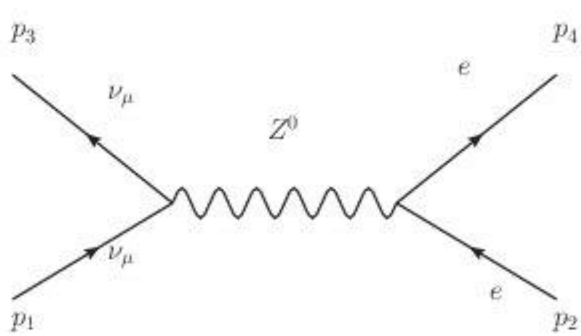

Fig. 3.2 Feynman diagram for neutrino-electron scattering in SM theory

The amplitude of this Feynman diagram can be written by using Feynman rules which can be found in Ref (L.F.Li, 1991)

$$M = \frac{g^2}{8M_Z^2}\ [\bar{u}(p_3)\gamma^{\mu}(1-\gamma^5)u(p_1)]\big[\bar{u}(p_4)\gamma_{\mu}(c_V - c_A\gamma^5)u(p_2)\big] \quad (3.10)$$

And hence

$$|M|^2 = 2\left(\frac{g}{4M_Z^2}\right)^4 Tr(\gamma^{\mu}(1-\gamma^5)p_1\gamma^{\nu}(1-\gamma^5)p_3) \times Tr\left(\gamma_{\mu}(c_V - c_A\gamma^5)(p_2+m_e)\gamma_{\nu}(c_V - c_A\gamma^5)(P_4+m_e)\right) \quad (3.11)$$

$$= \frac{1}{2}\left(\frac{g}{M_Z}\right)^4 ((c_V+c_A)^2(p_1.p_2)(p_3.p_4) + (c_V-c_A)^2(p_1.p_4)(p_2.p_3) - m_e^2(c_V^2-c_A^2)(p_1.p_3)$$

(3.12)

$c_A$and c_V are the neutral weak couplines for the electron .

In the center of mass:

$m_e \longrightarrow 0$

We find

$$|M|^2 = 2\left(\frac{gE}{M_Z}\right)^4 \left((c_V+c_A)^2 + (c_V-c_A)^2 \cos^4\frac{\theta}{2}\right) \quad (3.13)$$

E is the electron or neutrino energy and θ is the scattering angle.

$$\frac{d\sigma}{d\Omega} \propto |M|^2 \quad (3.14)$$

$$\rightarrow\ \frac{\partial\sigma}{\partial\Omega}=2\left(\frac{g}{4M_z}\right)^4 E^2\left[(c_V+c_A)^2 + \left((c_V-c_A)^2\cos^4\frac{\theta}{2}\right)\right] \quad (3.15)$$

Then total cross section now is simplified to

$$\rightarrow \sigma = \frac{2}{3\pi}\left(\frac{g}{2M_z}\right)^4 E^2(c_V^2 + c_A^2 + c_V c_A) \tag{3.16}$$

We have c_A and c_V for ν_μ=$\frac{1}{2}$

And for the electron

c_V = -$\frac{1}{2}$ + 2 sin$\theta\omega$, c_A= - $\frac{1}{2}$

Chapter Four

CALCULATIONS, RESULTS, DISCUSSIONS AND CONCLUSIONS

(4-1) Introducion

In this chapter we shall calaulate the scattering cross-section

$$\boldsymbol{\sigma(\nu_\mu + e \to \nu_\mu + e)}$$

In the two approaches Fermi theory and standard model theory.

(4-2) Calculation of $\boldsymbol{\sigma}$ using Fermi formula

$$\boldsymbol{\sigma(\nu_\mu + e \to \nu_\mu + e) = {G_F}^2 \times {E_\nu}^2}$$

Where

$${G_f}^2 \equiv \text{is coupling constant.}$$

$$E_\nu \quad \equiv \text{is the neutrino energy .}$$

$${G_f}^2 = 1.16\times 10^{-5} GeV^{-2}$$

$$E_\nu = 4\ MeV$$

$$1GeV^{-1} = 2 \times 10^{-14} cm$$

Then

$$\sigma = (1.16 \times 10^{-5})^2\ (4 \times 10^{-3})^2\ \mathrm{GeV}^{-2}$$

$$\sigma = 21.5296 \times 10^{-16} \mathrm{GeV}^{-2}$$

Thus

$$\sigma = 8.61184 \times 10^{-43} cm^2$$

(4-3) Calculation of σ using the SM formula:

$$\boldsymbol{\sigma(\nu_\mu + e \to \nu_\mu + e) = \frac{2}{3\pi} \left(\frac{g}{2M_z}\right)^4 E^2 (c_V^2 + c_A^2 + c_V c_A)}$$

Where

$$c_A = -\frac{1}{2}\ ,\quad c_V = 0.4591\ , g = 0.653\quad , M_z = 91\ GeV\quad , E = 4MeV$$

Then

$$\sigma = 1.32023 \times 10^{-16}\ GeV^{-2}$$

$$\sigma = 1.32023 \times 10^{-16} \times 4 \times 10^{-28}\ cm^2$$

Therefore

$$\sigma = 5.2809 \times 10^{-44} cm^2$$

(4-4) Plotting the results

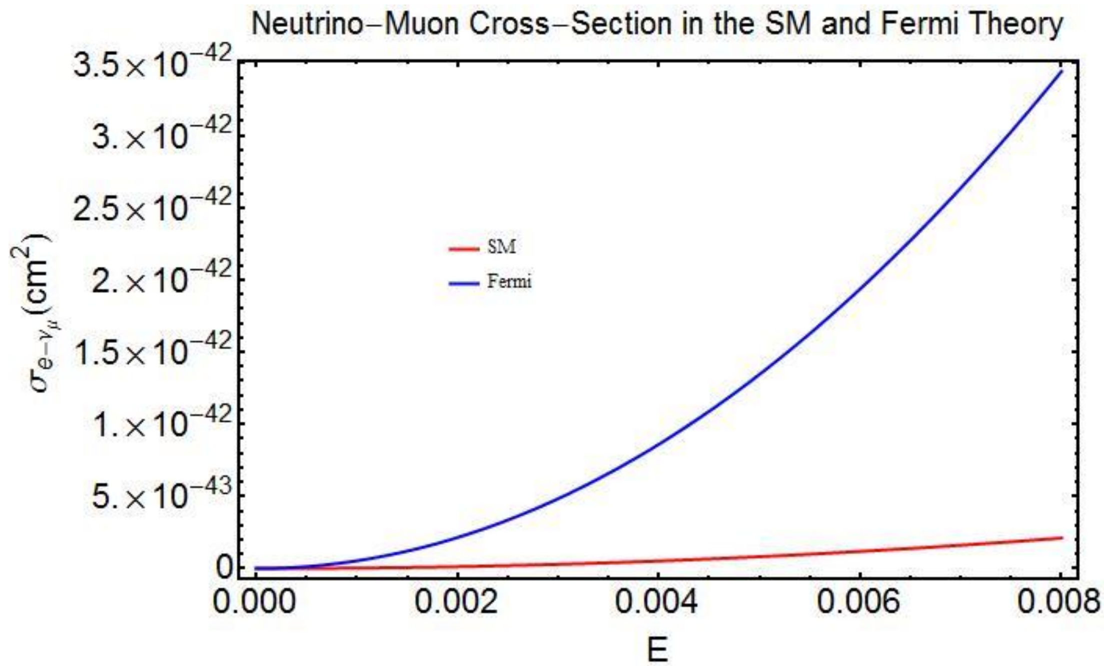

Figure (4-1) Neutrino-Muon Cross-Section in the SM and Fermi Theory, blue line represents the neutrino-muon cross-section in fermi theory and red line represents the neutrino-muon cross-section in SM.

(4-5) Discussions

As we see in figure (4-1) the cross-section increases as energy "increases" in both approaches Fermi theory and standard model. we found that the cross-section in SM less than the cross-section in Fermi theory. This deviation is due to the fact that Fermi theory work for low energies unlike the standard model works for high energies which mean the Fermi theory breaks down at high energies. Therefore we recommend using the standard model approach.

(4-6) Conclusions

In conclusion, we discussed the Fermi theory of weak interaction at length and the standard model of particle physics in detail. We had calculated the cross-section of the elastic neutrino–muon and electron scattering in Fermi theory and the standard model to be

$$\boldsymbol{\sigma_{SM} = 5.28 \times 10^{-44} cm^2}$$

$$\boldsymbol{\sigma_{Fermi} = 8.61 \times 10^{-43} cm^2}$$

We have found that the cross-section in SM is always less than the cross-section in Fermi for a given energy. Furthermore, the Fermi theory breaks down at high energy but the standard model works very well at high energies.

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