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The Possibility of Utilizing a Nano and Micro Rectangular Waveguide as a Solar Cell and a Lasing Cavity

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ABSTRACT: In this work the expression of the absorption coefficient of a rectangular waveguide, which is dependent on its dimensions, was utilized to show the conditions under which this guide act as a nano solar cell. It was shown that such guide can act as an optical amplifier for a solar cell on a nano scale. This expression can also considered as an expression of the energy for a particle in a box.

المستخلص:

في هذا البحث أستخدمت صيغة معامل الامتصاص للمرشد الموجي والتي تعتمد على أبعاده للتعرف على الشروط التي يعمل فيها هذا الدليل كخلية شمسية نانوية. تم إثبات أن هذا المرشد يمكن أن يعمل كمضخم ضوئي لخلية شمسية في الأبعاد النانوية. يمكن أن تعتمد هذه الصيغة لإيجاد طاقة الجسيم في صندوق.

KEYWORDS*: waveguide, solar cells, nano cells, lasing action.*

INTRODUCTION

The interaction of electromagnetic (e.m) waves with matter can be applied in a wide variety of fields and one of the most interesting applications is in the lasing process and in solar cells(1,2).

The recent application of laser in quantum computer needs a laser device on a nano scale. The ordinary laser devices are difficult to be build on this very small scale. Thus one needs a new lasing mechanism by which a device can be designed on a nano scale.

On the other hand the limitation of solar cell efficiency and its expensive fabrication needs a search for a new trend for solar cell mechanism of conversion of light to electric energy(3).

Rectangular Waveguides

Waveguides are used to transmit e.m wave of high frequency in the range of gega-hertz where ordinary transmission lasing give high energy losses in this range. The energy loss of transmission lines depends on the properties of the materials, μ , ε and σ via the absorption coefficient γ , according to the relation (4, 5).

$$\gamma = \sqrt{j\omega u\,(\sigma + j\omega\varepsilon)} \quad (1)$$

where ω stands for angular frequency of the e.m wave, while, μ , ε and σ represent the magnetic permeability, electric permittivity and conductivity, respectively .

In view of (1), it is clear that the control of the absorption coefficient, which can be done via the martial parameters μ , ε , σ is highly limited as far as the variation of μ , ε and σ is restricted to wave vector E which is given by Partha [4] and Kosa [6].

$$E = E_0 e^{-\gamma} e^{j\omega x} \qquad (2)$$

In view of equation (2) complete absorption takes place if γ is real, in this case the e.m wave become a standing wave. If γ is imaginary, the e.m wave travel without absorption. When γ a complex number (e.m) wave is transmitted with partial absorption. The form of relation (1) shows that γ is impossible to perform complete absorption of e.m wave.

There is a need thus to change the form of (2) by incorporating additional terms in the expression of γ by same means. Fortunately, this task can be performed by utilizing waveguides. In waveguides, the form γ depends on the dimensions of the waveguides. In some geometrical structures of the waveguides as of the case of the rectangular waveguide, the absorption coefficient of the rectangular waveguide with length b and height a is given by

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \mu\varepsilon\omega^2} \qquad (3)$$

This expression provides us with more freedom to adjust the dimensions of the waveguide to make complete absorption or to get rid of it completely.

To see the conditions under which this expression is obtained, one can recall Maxwell equations which describe the (e.m) waves propagating with frequency ω . Such equation takes the form: [4,8]

$$\nabla^2 \underline{E} = j\mu\omega\,(j\mu\varepsilon)\, E \qquad (4)$$

where

$$E = \underline{E_0} e^{j\omega t} \qquad (5)$$

Since the e.m wave propagate in the air which has non conductivity (σ = 0) then equation (4) takes the form :

$$E = j^2 \mu\omega^2 \varepsilon E = -\mu\varepsilon\omega^2 E \qquad (6)$$

Consider now the solution of the form

$$E = E_O(x, y) e^{\pm\gamma} \qquad (7)$$

It is important to note that for both $\pm\gamma$, inclusion of (7) in (6) leads to

$$\partial_x^2 E_0 + \partial_y^2 E_0 = (\gamma^2 + \mu\varepsilon\omega^2) E_0 \qquad (8)$$

This relation can be simplified by separation of variables in the form: [7]

$$E_0 = X(x) + Y(y) \qquad (9)$$

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Direct substitution leads to, $Y\partial_x^2 X + X\partial_x^2 Y = -(\gamma^2+\mu\varepsilon\omega^2)XY$, dividing both sides by XY yields

$$X^{-1}\partial_x^2 X + Y^{-1}\partial_x^2 Y = -(X^2+\mu\varepsilon\omega^2)XY \quad (10)$$

Thus the solution can be in the form

$$Y^{-1}\partial_x^2 X = -M^2 \qquad Y^{-1}\partial_x^2 Y = -N^2 \quad (11)$$

where M and N are two arbitrary constants. The solution of (11) takes the form

$$X = X_1\sin Mx + X_2\cos Mx, \qquad Y = Y_1\sin Ny + Y_2\cos Ny \quad (12)$$

Utilizing (11), in (10) yields

$$M^2+N^2 = \gamma^2+\mu\varepsilon\omega^2 \rightarrow \gamma = \sqrt{M^2+N^2-\mu\varepsilon\omega^2} \quad (13)$$

where e.m waves flow inside a hollow rectangular metallic waveguide with dimension x=a and y=b at interface between metal and air the boundary conditions lead to

$$E = 0 \; at \quad x = \overline{0},\ y=0,\ x=a,\ y=b \quad (14)$$

In view of relations (7), (9), (12) and the boundary conditions (14) are gets, E (x=0) = [X(x=0) ,Y(y)]$e^{\pm\gamma}$ = 0, $X_2 Y e^{-\gamma}$=0, Thus

$$X_2 = 0 \quad (2.15)$$

Similarly, E(y=0) =[X,Y (y=0)]$e^{\pm\gamma}$ = 0, $XY_2 e^{-\gamma}$ = 0, thus $Y_2 = 0$ (2.16)

As a result E can be written as $E = C\sin(Mx)\sin(Ny) = e^{\mp\gamma}$ (17)

where, C $= X_1 y_1$, according to equation (14) and (17) the boundary condition E=0 at x=a $\overline{\text{and}}$ y=b leads to, C sin(Ma) sin(Ny) $e^{\mp\gamma}$ = 0, hence

$$Ma = m\pi \rightarrow M = m\pi/a,\ m = 1,2,3 \quad (18)$$

C sin (Mx) sin (Nb) $e^{\mp\gamma}$ =0

Hence

$$Nb = m\pi \rightarrow N = \frac{n\pi}{b}, \qquad n = 1,2,3\ldots \quad (19)$$

The direct substitution of equations (18) and (19) in (14) leads to

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \mu\varepsilon\omega^2} \quad (20)$$

Lasing by a waveguide:

Amplification can be achieved and lasing is possible if the intensity I increases with z, this condition can be satisfied when one takes the positive sign in equation (17) where [7,9]

$$E = C\sin(Mx)\sin(Ny)e^{\gamma z}$$
$$I = \rho hf = \frac{hf}{4\pi}E^2 \quad (21)$$

For amplification to take place γ in (20) should be positive. For a = b, this require

$$2\left(\frac{n\pi}{b}\right)^2 > \mu\varepsilon\omega^2 \rightarrow \frac{b}{n\pi} < \frac{\sqrt{2}}{\sqrt{\mu\varepsilon}}\frac{1}{\omega} \quad (22)$$

For simplicity take n=2 to get

$$b < \frac{\sqrt{2}}{\sqrt{\mu\omega}}\frac{1}{f} \qquad (3.4)$$

Since, $v = \frac{1}{\sqrt{\mu\omega}} = \frac{c}{n_0}$, where n_0 is the refractive index it follows that :

$$b < \frac{\sqrt{2}}{n_0}\frac{c}{f} \rightarrow b < \frac{2}{n_0}\lambda \qquad (23)$$

But for visible light

λ=.05μm ≈ 50 x 10^{-9} ≈50 nm (24)

Thus for amplification to place the electric field intensity must be in the form (21). Thus form represents an e.m wave propagating in the –z direction. The dimension of the waveguide should satisfy relation (23),i.e

$b < (2/n_0)$ $50x10^{-9}$, $b < 66.6nm$ (25)

where n_0 = 1.5

To understand physically how amplification can be achieved, it is important to note that the e.m wave in (21) is stationary non traveling wave in the cavity. Thus the stream of e.m wave entering the waveguide does not leave it but tends to accumulate itself within the guide thus increasing its energy till the internal conducting field become larger to balance the external one such that E_z dose vanish.

Nano Solar Cells:

It is well known that solar cells are expensive, due to the silicon crystallization process. They have low efficiency which is highly limited by the optical as well as electrical and magnetic properties of matter, which are difficult to modify, so as to increase cell efficiency [4]. Thus one needs to find a parameter which has more freedom than the material properties so as to increase the efficiency. In view of (23) equation (20) it's apparent that the efficiency can be increased by increasing optical absorption through the terms γ. According to relations (21, 22, 23, 24, 25) this is possible when

b <66.6 nm (26)

Particle in a box:

It is well known in quantum mechanics that the energy of free electrons in the crystal resembles that of a particle in a box, which is obtained from Schrödinger equation:

$$E = \frac{\hbar^2 n^2}{8ma^2} \qquad (27)$$

if one considers electrons as waves, when the same result (27) can be obtained from e.m wave solution for waveguide. At equilibrium when no decaying exists, γ = 0 and in the case of one dimensional lattice, equation (20) reads

$$K = \frac{\omega}{v} = \sqrt{\mu\varepsilon\omega} = \frac{n\pi}{a} \qquad (28)$$

where (1/ v) is replaced by $\sqrt{\mu\varepsilon}$. For free electron the energy is given by

$$E = \frac{\hbar^2 k^2}{2m} = \frac{h^2 k^2}{8\pi^2 m} \quad (29)$$

Utilizing (28) in (29) yields

$$E = \frac{h^2 n^2 \pi^2}{8\pi^2 m a^2} = \frac{h^2}{8ma^2} n^2 \quad (30)$$

This is typical to the expression for a particle in a box obtained from Schrödinger equation with replacing *h* by $\hbar$.

RESULTS and DISCUSSION

Relations (20) and (21) showed that when γ is real and an e.m wave propagates in the negative z direction the light intensity increases and lasing can takes place. From equation (25), the visible light amplification is possible if the dimensions of the cavity are less than 66.6 *nm*.

A waveguide can act as micro or nano solar cell if the e.m wave propagates in the positive z direction as indicated by equation (7). In this case the wave is absorbed almost completely when γ is a real quantity. This requires the waveguide scale to be less than 66.6 *nm*. Finally, related to equation (30), a quantum expression for the electron obtained by considering non absorbed non traveling standing wave by setting γ =0 .

CONCLUSIONS

The waveguide can act as a nano or a micro lasing cavity or as a solar cell if the dimensions of the guide are small compared to the e.m wavelength. It is also interesting to note that the e.m wave is stationary non traveling wave in the cavity.

REFERENCES:

1. Silfvast, T. W., (1999). *Laser Fundamentals*, Cambridge University, Press printed in U.S.A.

2. Michael, J. H., (1996). *Modren Spectroscopy*, 3rd ed, Copyright by John Wiley and Sons 1td, England.

3. Martin, A. G., (1989). *Solar Cell*, chapter 5 and 6, Mawsel University, Iraq.

4. Partha, P. B., and Ting-Chung P, (1991). *Principle of Applied Optics*, Printed in USA, © Richard D. Irwin, Inc., and Aksen Associates, Inc.

5. Wintner E., (2001). *Ultra Short Pulse Laser Technology and Application*, Institute of Laser Engineering, Osaka University, Copyright.

6. Kosa, M., (2006). *Laser Physics and Applications*, Damascus University, Syria.

7. Shalom, E., (2002). *The Interaction of High Power Laser with Plasmas*, Institute of Phys. Publication, USA.

8. Yariv, A., (1987). *Quantum Electronics*, John Wiley and Sons, New York.

9. Matthews, P., T. and Frank, R. S., (1974). *Introduction to Quantum Mechanics*, McGraw, London.