A Simplified P-Delta Second Order Numerical Analysis of High Rise

Buildings: Formulation and Validation

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ABSTRACT **-** In this paper, a simplified numerical method of global and local second order P-Delta 2D and 3D analysis of tall buildings subjected to vertical and horizontal loads is presented. The method was based on developing the moment transformation (MT) and the moment-force transformation (MFT) methods those are formulated using the moment distribution methods and have been successfully used in linear analysis of tall buildings neglecting and/or taking into account axial deformation in vertical members. The method was developed to include second order effects, by coupling the axial force and the bending moments in each of the vertical members with large lateral displacements at floor levels. Validity of the method was established by comparing the results of two 2D and 3D problems with those resulted from a reliable finite element approach. The comparisons show that, the results are in good agreement thus verifying the accuracy of the proposed method.

Keywords: Simplified Analysis Method, Tall buildings, nonlinear analysis, second order effects, P-Delta analysis.

المستخلص: تعرض هذه الورقة طريقة عددية مبسطة للتحليل اللاخطي للمباني العالية (المست*وية أو في الفراغ) والمعرضة* ألحمال رأسية و أفقية. الطريقة هي تطوير لطريقة نقل العزوم و طريقة نقل العزوم و القوى. الطريقتان مشتقتان من طرق توزيع العزوم و قد استخدمتا بنجاح في التحليل الخطي للمباني العالية مع تجاهل أو الأخذ في الإعتبار التشوهات المحورية في األعضاء الرأسية.أدمجت الالخطية في التحميل، بعدم تجاهل العالقة بين العزوم و القوى المحورية في األعضاء الرأسية في وجود إزاحات جانبية كبيرة. استخدمت الطريقة المقترحة لدراسة مسائل مختمفة، وقد قورنت النتائج المتحصل عميها بنتائج حصل عليها باستخدام طرق العناصر المحددة القياسية. وقد أثبتت المقارنة أن هنالك توافقاً جيداً بين النتائجيؤكد دقة الطريقة المقترحة.

INTRODUCTION

In the conventional linear analysis methods, the stiffness matrix for each element in the structure, and accordingly the global stiffness matrix, remainsunchanged throughout the analysis. If the building is very tall and slender and the axial forces are large or the individual columns are slender, then the lateral displacements become very large and affect the building geometry. This results in extra increase of the displacements and stresses, and second order or P-Delta analysis should be incorporated [1], [2].

In some of the available commercial analysis packages, the consideration of the nonlinearity in the static and the dynamic analysis of tall buildings is not exact and is subjected to several limitations. Examples of these are incorporation of the geometric stiffness while neglecting or approximately including the stress stiffening of the members due to the effects of the axial loads (e.g. assumption of cubic function deformed shape instead of trigonometric function for compression force or hyperbolic function for tension force) ^[3]. Sometimes in some commercial packages

there is no possibility to include the effects of geometric nonlinearity during the dynamic analysis mode. Also some packages use iterative methods of P-Delta analysis $[4]$, $[5]$. In the iterative methods of P-Delta analysis, the results tend to diverge when the vertical loads tend to reach the critical buckling load at any of the vertical members. Since the final forces are not known before performing the analysis, the convergence of the results to the correct answers will not be ensured.

Also in the design codes, the effects of the nonlinearity are incorporated approximately by modifying some of the design parameters, e.g. amplified moments $[6]$, $[7]$ and, extended effective lengths $[8]$, $[9]$. In methods of analysis of tall buildings and in order to incorporate the P-Delta effects, some authors suggest the introduction of an equivalent fictitious member of negative properties [1], [10]. Even this, is not acceptable in most of the analysis packages.

 It is well known that the analysis of tall buildings needs some simplifications especially in the preliminary analysis and design stage, in order to reduce the large amount of unknowns when using the conventional exact methods of analysis. This problem, if not solved, will affect the computer storage and increase the analysis running time. In addition to this, the nonlinear analysis also needs extra storage and extra time because most of the methods require several iterations for the results to converge to correct values.

METHODOLOGY

 The importance of performing the nonlinear analysis for tall buildings has been pointed out by various researchers $[11]$, $[12]$. In most of the simplified methods of analysis, there exist assumptions that lead to erroneous results in some of the practical cases. For example methods based on the continuum theory or the equivalent column theory should always be applied for buildings of equal floor heights, buildings with no set back, cases of contra flexure in the mid of the members, sometimes neglecting the flexural stiffness of the floors, or very regular structures where the geometric and stiffness characteristics of structural elements are constant throughout the building's height $^{[13], [14]}$.

 In this paper a simplified numerical method for second order analysis of tall buildings is presented. The method is based on the Moment Transformation (MT) $^{[15]$, $^{[16]}$ and the Moment-Force Transformation (MFT) methods [17], previously proposed and used for linear static analysis of tall buildings neglecting or including the axial deformations in the vertical members. Due to its simplicity, the proposed method greatly saves the effort faced from the difficulties of the data entry and the interpretation of the vast amount of the output results when using the conventional finite elements methods of analysis (FEM).

The algorithms of the moment transformation program (MTProg) and the moment-force transformation program (MFTProg) based on Visual Basic have been developed and implemented for the proposed method and used in the verification works.

The transformation methods are formulated from the moment distribution methods. Thus, they may be classified in the categories of the simplified displacement methods of analysis that treat the fixed-end moments produced from the applied loads and from the lateral translations of the members ends.

They are similar to the slope deflection method, successive sway correction method and substitute frame method $[18]$. In all the later methods, the moments are distributed between the end joints of each individual member. In the moment transformation method the distributions are carried out for a coupled group of moments at the same time from one level toward the next level.

Using this stream or bundle of distribution (or transformation), permits the axial deformation (shortening or elongation) of the vertical members to be incorporated in the analysis, as

manipulated in the Moment-Force transformation Method. By coupling of the moments and the axial forces in each of the vertical members in the floors levels during the transformation procedure, the second order P-Delta effect can be directly included in the analysis. Also using the proposed method, structural instability with reference to overall buckling or failure of columns subjected to axial load and bending, can be investigated.

 The transformation methods simplify the 2D and 3D analysis of tall buildings in three ways, summarized as follows:

- 1. The typical floors are analyzed only one time, by condensation of the floor degrees of freedom (DOFs) into only the supported DOFs with all the other remaining DOFs translating and rotating freely.
- 2. In 3D analyses, the considered DOFs in the vertical members are only two principal rotations in each floor level, as manipulated in the (MT) method, which can be reasonably used for moderate tall buildings or shear wall structures with negligible axial deformations in the vertical members. But for super tall buildings with the axial deformation in the vertical members dominant (e.g. tube and outrigger systems), (MFT) method can be used with one translational DOF added to each of the vertical members in each floor level, to represent their axial deformations. Hence, with some modifications in stiffness and carryover moment, the second order analysis can be incorporated with no extra cost.
- 3. The solution for the unknowns are carried out in each floor level separately by use of the calculated equivalent rotationaltranslational stiffness matrices and balancing the fixed and the transformed moments and forces in the concerned level.

To sum up, the overall objective of this research is to develop a simplified numerical method of analysis and a simple computer program able to perform the second order global and local P-Delta analysis of tall buildings easily and accurately.

FORMULATION OF METHOD Transformation of Moments and Forces:

Figure 1: Moment and Force Transformation

Referring to Figure 1 (a) and (b), and using the displacement method of analysis, the equivalent stiffness and the transformation factor $^{[15]}$, $^{[16]}$, $^{[17]}$, are given as follows:

$$
S_e = \left[S_2 - \frac{t_2^2}{(S_1 + S_2)} \right]
$$
 (1)

$$
TF = \frac{-t_2}{(S_1 + S_2)}
$$
 (2)

where:

Si: is the rotational or translational axial stiffness of member*i*, $(i = 1, 2)$.

t2: is the carryover moment or force for member 2.

 S_e : is the equivalent rotational or translational axial stiffness of the members 1 and 2, at joint 2.

TF: is the transformation factor used to transform the moment or force from joint 1 to joint 2.

2D and 3D Building Analysis:

 By combining the two transformation procedures, the generalized moment-force transformation procedure can be formulatedto calculate the rotational-translational equivalent stiffness matrices and themomentforce transformation factors matrices of the building, $^{[17]}$.

Condensed Stiffness and Carryover Matrices for Multiple Vertical Members, including P-Delta effects:

Considering a system of two vertical members, Figure 2, the stiffness matrix equation corresponding to the three degrees of freedom 1, 2 and 3, condensed into 1 and 2, is as follows:

Figure 2: Rotations and Translations DOFs of Two Vertical Members System

Figure 3: Translational Stiffness of a member including P-Delta effect

The translational stiffness *S³³* (Equation 3), is a summation of the translational stiffness (S_T) of each vertical member including its Global P-Delta effect (i.e. -P/L), as shown in Figure 3. The effect of the local p-delta in any member may be incorporated by using the rotational stiffness (S), and the carryover moment (t) of the member, which are trigonometric functions of axial compression forces (for positive P values), or hyperbolic functions of axial tension forces (for negative P values), $^{[18]}$.

Figure 4: Carryover moment including P-Delta effect

The lateral displacement, D, and the internal interaction force, *F*, Figure 4, are obtained from the different rotational stiffness configurations and hence the elements of the carryover moment matrix, including the P-Delta effects, are calculated from the following equation:

$$
t_{ji}^* = -S_{ji}^* + F.L + P.D \tag{4}
$$

NUMERICAL VALIDATION Results and Discussion:

 Using the computerized proposed method,two caseswere studied. A case of a 2D frame of 15 floors subjected to vertical and lateral loads, and a case of a 3D asymmetrical 25 floors building subjected to vertical and wind loads. The results obtained were compared with those obtained using StaadPro_2004 $^{[5]}$ and ETABS $^{[4]}$. In StaadPro_2004, the second order P-Delta results were obtained from 10 iterations, and in ETABS, the results were obtained from displacements relative tolerance of $1x10^{-3}$ and maximum 10 iterations.

The Fifteen Floors 2D Building Model:

The displacements and bending moments were obtained using the proposed method for a fifteenmulti-storey2D frame under the vertical

and horizontal loading shown in Figure 5.All building members are concrete of elasticity, E $= 29x10^{6}$ kN/m²

, and Poisson's ratio, $v = 0.2$

Linear and second-order (P-Delta) analyses have been carried out, and comparisons of the results with exact results are shown in Tables 1 to 4.

The displacements and the bending moments results obtained using the proposed method compared with results obtained using StaadPro 2004 ^[5], are shown in Tables 1 to 4. The comparison of the results shows very close agreement and sometimes the results are identical, both in the linear and second-order analysis.

As shown in Tables 1 to 4, the lateral displacements which are calculated including the P-Delta effects are greater than that calculated using ordinary linear analysis. As general, the second order analysis values may be increased with the increase of the vertical loads and or increase in the building height. Including the local p-delta effects in the analysis, results in extra increase in the lateral displacements.

The Twenty Five Floors 3D Building Model

 The building plan area, shown in Figure 6 is: 24 m x 12 m. The floor slab is of thickness = 0.2 m. The building is composed of 25 floors of floor height $= 3.5$ m for all floors except the lower floor which is of height $= 5.5$ m.

All building members are concrete of elasticity, $E = 29x10^6$ kN/m²

, and Poisson's ratio, $v = 0.2$

The section properties of the vertical elements (in meters) are:

All Columns: 0.60 m x 0.60 m for the 10 lower floors, 0.50 m x 0.50 m for the 10 middle floors, 0.40 m x 0.40 m for the 5 upper floors.

The Shear walls are of lengths 3.0 m (walls 1, 2 and 20), and 4.0 m (wall 3), and thicknesses are: 0.30 m for the 10 lower floors and 0.25 m for the 15 upper floors.

The building is subjected to vertical area load of 18 kN/ $m²$ at all floors, and to lateral loads (F, in Y-direction and in the location shown in Figure 6, at column 13), of 151.2 kN at the lower floor level, and 117.6 kN at all other floors levels.

 The slab was modeled by finite plate elements from Ghali et al. [18], of meshes size 0.5 m x 0.5 m. The columns and walls were modeled by frame members. The edge shear wall and the U-shaped core were connected at the floor levels with torsion released rigid beams represent the rigid parts of the walls [19] .

Linear and second-order (P-Delta) analyses have been carried out, and comparisons of the obtained results with exact results from different packages, $ETABS$ ^[4] and StaadPro 2004 [5], based on FEM, are shown in Tables 5 to 7 and Figures 7 to 11.

Comparisons using $ETABS$ ^[4], are performed for two options. The first option is based on thin-plate (Kirchhoff) formulation, which neglects the transverse shearing deformations, and the other option is thickplate (Mindline/Reissner) formulation which includes the effects of transverse shearing deformations^[3].

 Comparison of displacements in Y-direction and the twist rotation of the floors at the building center (Column 10),obtained using theproposed method and the different packages is shown in Table 5 and Figures 7 to 9. Comparisons of the bending moments of the U-shaped core (assembly of walls 1, 2 and 3) and the edge shear wall (wall 20) are shown in Tables 6 and 7, and Figures 10 and 11.

In all the comparisons of the displacements and the bending moments, for both linear and second order analysis, the differences are found to be very small.

The differences in the models displacements are proportional to the building height. ETABS (thick-plate) model has more rigid floor and less displacements and twist rotations than the other exact models, Figures

7 and 8. The assumption of the rigid diaphragm in the proposed method is extra resisting and reducing the twist rotations in the lower levels of the building compared with StaadPro 2004 and ETABS (thin-plate) models, Figure 8. This is due to the fact that, the torsion stiffness of the vertical members in the lower levels are very large compared with that in the upper levels, and the twist rotations in the vertical members are constrained to follow the rigid diaphragms twist rotations. This effect may be illustrated by comparing the results of the models with all the vertical members released for torsion, Figure 9. In this case the differences in the models twist rotations are almost proportional to the building height and with no such effects.

Additional Discussion of Results

 The differences in the results of the different programs models are due to the following factors:

- 1.The differences in the finite element formulation of the different programs, which are affecting the floor rotationaltranslational stiffness, and accordingly the building deformations andstresses.
- 2.The small deformations in the floor slab of the exact models due to the induced in-plane stresses, compared with the non-deformable rigid diaphragm of the proposed model. These deformations proofed to be negligible, as the differences in the twist rotations of the different models were not much affected by releasing the torsional stiffness of the vertical members, Figures 8 and 9.

In order to examine the effects of the finite element formulation on the results of the different models, a special subroutine has been created and implemented in the developedprogram. The subroutine is designed to calculate the floor rotational-translational stiffness from StaadPro one floor model. Therefore it permits the proposed method to use the Finite elements formulation of StaadPro program. By using this subroutine, the floor stiffness of StaadPro can beborrowed

and used in the proposed method instead of the embedded one.

 The subroutine has been created using the capabilities of OpenStaad, the Application Programming Interface (API), of StaadPro package, and executed by constructing a one floor StaadPro model supported by fully enforced supports in the locations of the columns and walls. A unit rotation or translation is exerted in each support in the directions of the different DOFs, and the corresponding reactions in all supports are retrieved and arranged systematically to construct the rotational-translational stiffness of the floor. Comparison of theresults of the proposed model including the borrowed floor, with the results obtained usingStaadPro_2004exact model, show zero or very small differences, as shown in Tables 8 to 10.

Comparison of Number of Unknowns:

In order to show the efficiency of the proposed method, the floor slab idealized by 48 x 24 finite elements with 20 vertical members (columns and walls) shown in Figure 6, wasused to compare the proposed method with the conventional matrix methods of analysis.The total number of unknowns for a building with same floor and of total N floors is:

(a) Conventional matrix methods (6 DOFs/joint):

 $S1 = [(49x25xN + 20)x6]$

(b) Proposed Method:

The unknowns in the proposed method are composed of two parts:

- 1.Coupled unknowns for one floor with 3 DOFs/joint, solved simultaneously and used to obtain the floors level stiffness.
- 2.Two Rotations plus one axial translation for each column/wall at all levels including the supports level. The unknowns solved separately, each (20x3) unknowns per each level.

 $S2 = [49x25x3] + [20x3]x(N+1)$

Note: coupled unknowns are in square brackets [].

For N= 150 floors:S1= 1,102,620 Coupled unknownsand, $S2 = 12,735$ unknowns (partially coupled),Ratio= 86 times.

Program Running Time

The floor, Figure 6,was used in a 150 floors building,with same materials and arbitrary properties of the vertical members and same loadings asbefore.All floors heights=3.5m The problem was solved for elastic linear analysis using theproposed program. The elapsed running time was 84 seconds.

Conclusion

 In this paper, a simplified numerical method of global and local second order P-Delta 2D and 3D analysis of tall buildings was presented. The method is suitable for the analysis of super-tall buildings with tubes and outrigger systems. The results obtained using the proposed method were close to the results obtained using the FEM. The saving in computer storage and computing time provided by the developed program, based on the proposed method, allows rapid re-analysis of the building to be accomplished in the preliminary analysis and design stages.

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TABLE 3. DISPLACEMENTS IN THE TOP FLOOR LEVEL (MM), (2D FRAME), P-DELTA ANALYSIS:

¹Including only Global P-Delta. ²Including Global and local P-Deltas.

TABLE 4. MAXIMUM BENDING MOMENT IN COLUMNS (KN.M), (2D FRAME), P-DELTA ANALYSIS:

TABLE 5. DISPLACEMENTS AND ROTATION IN THE TOP FLOOR LEVEL (MM, RAD), (3D FRAME):

Analysis	Linear	Second order
Proposed	43505.86	49183.91
StaadPro	44108.82	50071.03
$\Lambda^{0/2}$	-1.37	-1.77

TABLE 6. MAXIMUM BENDING MOMENT IN U-SHAPED CORE (KN.M), (3D FRAME):

 TABLE 7. MAXIMUM BENDING MOMENT IN EDGE SHEAR WALL (KN.M), (3D FRAME):

Analysis	Linear	Second order
Proposed	13130.94	15985.60
StaadPro	13431.23	16541.75
∧%	-2.24	-3.36

TABLE 8. DISPLACEMENTS AND ROTATION IN THE TOP FLOOR LEVEL (MM, RAD), (3D FRAME), (BORROWED

STAADPRO FLOOR):

TABLE 9. MAXIMUM BENDING MOMENT IN U-SHAPED CORE (KN.M), (3D FRAME), (BORROWED STAADPRO

FLOOR):

TABLE 10, MAXIMUM BENDING MOMENT IN EDGE SHEAR WALL (KN.M), (3D FRAME), (BORROWED STAADPRO

FLOOR):

Figure 5: Fifteen floors 2D Frame, properties and loading

Figure 6: 24 m x 12 m floor plan for 25 Storey Building

 Figure 7: P-Delta Analysis, Displacements in ydirection

Figure 8: P-Delta Analysis, Rotations in radians

Figure 9: P-Delta Analysis, Rotations in radians (torsion released)

Figure 11:P-Delta Analysis, B.M.D. for edge shear

wall