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Scheduling Problem under Constrained Resources: A Historical Review of Solution Methods and Computer Application

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Abstract- In construction projects, project execution time is a major concern of the involved stakeholders (client, contractors and consultants). Optimization of project scheduling through time control is considered as a critical factor in project management. Many studies were carried out and many models and software packages were developed since the fifties and till now, but no clear cut methods, to optimize resources while satisfying different constraints were found. The importance of the subject stemmed from the fact that project time completion affects the overall project cost. Considering the most two widely applied scheduling methods: Critical Path method (CPM) and Program Evaluation and Review Technique (PERT), it is found that negligence of handling limitation of resources is evident in most cases. On the other hand, a resource leveling technique which is used to reduce the sharp variations in the resource demand histogram cannot handle the issue of[1] minimizing project duration since it is used when there are enough resources. So the leveling process is accomplished by shifting only the non-critical activities within their floats. This paper show a number of heuristics and models to solve scheduling problem of projects subjected to limited resources. Different heuristic methods applied in past studies were examined in order to be tested and applied in a simple example, as a pilot study, so as to be used in real complex projects.

Keywords: *Project time control/optimization, constrained resources, heuristics.*

مستخلص

ظـــل اطـــــراف مثلث التشييد (المالك والمقاول والاستشاري) في السودان يعانون وباستمرار من مشكلة التحكم في زمن المشروع بصورة عامة وفي حالة الموارد المحدودة او المقيدة بصورة خاصه ،وعليه فان امثلية الجدولة للمشروع يعتبر هو العامل الحرج والموثر في ادارته. ان عدة دراسات وتطبيقات ونماذج وبرامج كمبيوتر قد تم اصدارها منذ الخمسينات وحتي الان ولكن ليست هناك طرقا قاطعه او نهائيه لامثلية استخدام الموارد عند تعرضها لقيود المحدوديه. ان هذا الموضوع يستند علي حقيقة ان اي تاخير في زمن اكمال المشروع يودي بالضرورة الي الارتفاع تكلفتة الكلية. وبالنظر الي افضل طريقتين للجدوله واكثرها استخداما واوسعها انتشارا وهما طريقتي المسار الحرج {Critical Path Method (CPM)} وبيـــــــرت {Program Evaluation and Review Technique (PERT)}. وجد ان هانين الطريقتين تتجاهلان حالة محدودية الموارد في أغلب الحالات. كما ان اسلوب اعادة توزيع الموارد (Resource leveling) والذي يستخدم عادة لتقليل التفاوتات الحاده في مخطط احتياجات الموارد (Resource demand histogram) لايمكن الاعتماد عليه في

حل مشكلة محدودية الموارد او تقليل زمن المشروع لانه يستخدم عادة في حالة توفيرهذه الموارد وبالتالي فهو اسلوب يعني فقط بتبادل الانشطه غير الحرجه وفق ازمان العوم (Floats) الخاصة بها. إن هذه الورقه تسلط الضؤ علي عدد من الفرضيات التجريبيه (Heuristics) والنماذج وبرامج الكمبيوترالمستخدمه في حل مشكلة الجدوله (Scheduling Problem) للمشاريع في حالة المواردالمقيده (Constrained resources). تم اختبار عدد من الفرضيات التجريبيه (Heuristics) والنماذج وبرامج الكمبيوتر المشار اليها اعلاه والتي تم دراستها سابقا من خلال مثال مبسط كدراسة استرشاديه توطئة لالستخدامها في مشاريع حقيقيه كبيره ومعقده.

Introduction:

Scheduling problem of simple and complex projects have been proposed, implemented, and evaluated for over fifty years. Optimization of project scheduling through time control is considered as the most important factor in project management. Many studies were carried out and many models and software packages were developed since II, and till now, to treat optimization of project scheduling through project time control. Heuristic methods used to optimize scheduling of construction projects. And analyze activities and schedule only one at a time. A proposed heuristic algorithm may rank possible heuristics' combinations every time and simultaneously schedules all activities in a selected combination. They compared the performance of the created heuristics with optimal solutions founded by a bounded enumeration method [(1)].

Eight standard heuristics were compared on a set of single-mode resource-constrained project scheduling problems, ranged from simple priority rules to very complex dispatch rules. The results showed that the minimum slack heuristic performed best; however it did not perform well when the resources were tightly capabilities of mainframe computer systems during the 1990s eventually made it possible to overcome many deficiencies in the scheduling techniques being used in project management in the 1970s and 1980s [(2)].

Development of a wide variety of affordable project management software packages (Harvard, Time line, Super project, PERT master, Microsoft and Primavera Project Planner etc.) make problems handling easier. These packages allow the project manager and team to plan and control projects in a completely interactive mode. Microsoft project and Primavera Project Planner are two popular software programs used in construction industry. However, both programs cannot guarantee a successful project plan but Primavera project planner contains more features to facilitate solving resource scheduling problems as compared to Microsoft project. In project scheduling problems, a single project consists of a set of tasks, or activities. The tasks have precedence relationships, i.e. some tasks cannot be started until their predecessors have been completed.

The tasks also have estimated durations and may include various other measures such as cost, but the most common objective in the project scheduling problem is to minimize the time to complete the entire project. In multimodal project scheduling problems, each task may be executed in more than one mode, and each mode may have different resource requirements and more than one project may be scheduled. In many scheduling problems an implicit assumption mode is that sufficient resources are available and only the technological constraints (precedence relationships) are important for setting schedules.

In most cases however, resources constraints cannot be ignored, i.e. manpower, raw materials and equipments. Critical Path

Method (CPM) and Program Evaluation and Review Technique (PERT) were the most popular network techniques for scheduling.

However, the two types of methods do not consider the limited resources availability in many circumstances. However both methods are considered as feasible procedures for producing non-feasible schedule. On the other hand, resource leveling which is used to reduce the sharp variations in the resource demand, histograms, cannot handle the issue of minimizing project duration. So, because it is used when there are enough resources, the leveling process is accomplished by shifting only the non-critical activities within their floats [1,3]. Ultimately, alternative methods for project scheduling problems with limited multi-modes resources associated with different durations were developed: Carruthers and Battersby et al. (1966-1976), applied exact methods; Stinson, et al. (1976-1998), presented optimization methods [4]. Davis and Heidorn [5] and Patterson and Davis et al. applied heuristic methods [6].

Problem Statement

When none repetitive construction projects are subjected to limited resources, the clients, contractors and consultants (stakeholders) generally suffer from the elongation of project time completion specifically in the case of limited resources. This problem illustrates the conflict between the stakeholders during the execution of any project. Consequently, this problem leads, frequently, to hanging of project, then the project time completion will be delayed and, hence, influences the overall project cost.

Objectives of the Paper

The objective of this paper was to show the solution methods and computer applications which were developed and led to a number of heuristics, models and computer programs in order to solve, in a second stage, scheduling problems of none repetitive construction projects which are subjected to limited resources.

Scope of the Work

A simple problem is taken, as a pilot example, to test several solution techniques and heuristics, from simple priority rule to complex ones, used throughout the last sixty years period of time. The scope includes traditional linear programming, deterministic and stochastic approaches.

Previous Studies

Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT) were applied since 1959. PERT is used in scheduling the development and the manufacture of the Fleet Ballistic Missile (FBM) weapon system for the Polaris submarine and CPM in scheduling the construction and the maintenance of a chemical plant facility [3,1]. In spite of their popularity as network techniques for scheduling, however, they do not consider resources availability. Therefore, CPM and PERT could not deal with the project scheduling problem under limited resources. Verhines advocated general use of the "minimum late-finish-time" (LFT) priority rule, apparently on the basis of its ability to produce shorter schedules than other rules tested for a few selected problems. Patterson et al. [6] reported nine heuristic rules for constrained resource project scheduling in chronological order and indicates the type of problems examined and the sequencing rule found generally most effective in terms of project duration measure (time slippage) for single-and-multi-project studies.

Lawlerand Wood, Johnson and Stinson applied branch and bound approach [7]. Jerome D. Wiest [8] in his “heuristic model for scheduling large projects with limited resources” presented PERT- type scheduling models. Edward developed a study that compared the performance of the heuristics with optimal solutions founded by a bounded enumeration method; then George E. Heidorm programmed the study for computation [3,5]. Davis and Patterson, [7] compared eight standard heuristics on a set of single-mode resource-constrained project scheduling problems; they compared the performance of the heuristics with optimal solutions found by a bounded enumeration method by Davis and Heidorn. The selected heuristics are described below:

a. Minimum Job Slack (MINSLK),
b. Resource Scheduling Method (RSM),
c. Minimum Late Finish Time (LFT),
d. Greatest Resource Demand (GRD),
e. Greatest Resource Utilization (GRU),
f. Shortest Imminent Operation (SIO),
g. Most Jobs Possible (M JP),
h. Select Jobs Randomly (RAN).

The Min. slack (MINSLK) rule produced an optimal schedule span, most of the times. Continuing comparing the other rules (heuristics) for a single-project, multi-resource scheduling, researchers found that either the late finish time (LFT) or late start time (LST) rules are the most effective ones; thus the three rules, (MINSLK/LFT/and LST). Taken as a group produces; better results than the others. (See Appendix 1).

Davis et al. surveyed a range of heuristics from simple priority rules to very complex dispatch rules, confirmed previous studies regarding LFT and LST as the most effective rules, and hence their results supported the previous findings of Davis and Patterson. Joel P. Stinson developed a branch and bound (skip tracking) procedure to solve the multiple constrained resource project scheduling problem. F. Brain Talbot developed an implicit enumeration procedure (back tracking) for solving the resource constrained, project scheduling problem, the procedure consists of a systematic evaluation (enumeration) of all possible job finish times for the activities of a project [6]. Davis, Khumawala and Patterson noted that exact approaches based on bounded enumeration, implicit enumeration and branch and bound are able to solve smaller problems and they are guaranteed to find the optimal solution.

However, they become impractical when faced with problems of significant sizes or large sets of constraints, and, hence, some indication of no solution can be found. In addition, the linear programming formulations typically do not scale well, so they can be used only for specific instances or small problem [4]. In genetic algorithm, throughout a period of 18 years (1985-2003), Alcaraz and Maroto [2] developed a genetic algorithm based on the activity list representation and the serial SGS.

Patterson presented an overview of optimal solution methods for project scheduling. He noted that the linear programming can be used only for specific instances or small problems. He presented a comparison of exact procedures for solving the multiple constrained resources for single project scheduling problem. In his conclusion he noted that the implicit enumeration procedure of Talbot required far less computer storage than do the other two approaches. Branch and bound solution procedures of Stinson need minimum amount of computation time, and likely would be the preferred solution approach in those instances in which computer memory is not limited. Finally, the bounded enumeration procedure of Davis is likely to produce the optimal solution in the minimum amount of computation time [5].

MilosSeda [9] proposed a new implementation of the computing for the resource-constrained project scheduling (RCPSP). The activities-shifting was replaced by prolonging their durations and dividing them into active and sleeping parts. Francisco Ballestin [10] performed heuristic algorithms work with stochastic durations instead of deterministic ones. J. J. M. Mendesa et al. [11] presented a new genetic algorithm for finding cost-effective solutions for the Resource

Constrained Project Scheduling Problem (RCPSP).

Francisco Ballestin and Rosa Blanco [(12)], presented a study deal with multi-objective optimization in resource-constrained project scheduling problems (MORCPSPs), in which they described the project scheduling is an inherently multi objective problem. GuoqiangLi et al. [(13)] presented "Development and investigation of efficient artificial bee colony algorithm for numerical function optimization" study in which they noted that it is more effective than genetic algorithm (GA).

Khattab noted that the most efficient way to develop a construction schedule is to use computer programs. Fortunately, the advancements in the memory capabilities of mainframe computer systems during the 1990s eventually made it possible to overcome many deficiencies in the scheduling techniques being used in project management in the 1970s and 1980s. A wide variety of affordable project management software packages are available for purchase (Harvard, Time line, Super project, Pert master, Microsoft project, Primavera Project Planner etc). These packages allow the project manager and project team to plan and control projects in a completely interactive mode. The computer programs give adequate solutions but when resource requirements exceed the resources available, the computer programs do not provide the optimum scheduling solutions.

Speed and accuracy of mathematical scheduling computations and analysis of the information produced make computerized scheduling indispensible tool for construction project controls. Various computer programs have been established so far with regard to preparing schedules (Harvard, Time line, Super project, Pert master, etc) out of which Microsoft project and Primavera Project Planner are two popular software programs used in construction industry [(14)].

Summary of the Solution Methods:

Critical Path Method (CPM) & Program Evaluation and Review Technique (PERT)

CPM & PERT give the shortest possible make span assuming infinite resources, but they cannot solve problems that include restrictions on the number of resources that are available. They he not consider temporal or resource constraints and do not consider the limited resources availability.

Other solution methods come to existence: exact and optimization methods, heuristic methods. These classes may be categorized further into stochastic and deterministic approaches.

Exact and Optimization Solution Methods

Exact approaches based on implicit enumeration with branch and bound and optimal solution methods for project scheduling under constrained resources. The formulation may be an integer program requiring some types of bounded enumeration or branch and bound procedures.

Linear and Integer Programming

Formulated in traditional linear or integer programming form with significant simplifications, these methods depend on characteristics of the objective function and specific constraint formulations [(6)].

Bounded Enumeration

These solution methods search a decision tree generated from the precedence relations in the project plan, where the root of the tree corresponds to the first task. The second level of the tree is the set of tasks that can be scheduled once the first task has been scheduled, and so on; the final tree represents a precedence-feasible set of task sequences. Enumerative methods are typically bounded using heuristics in order to reduce the size of the tree [(6)].

Branch and Bound Solution Method

The method is proposed by Lawler and Wood et al. (1966) and Stinson (1978). The method generates a tree by scheduling activities starting with the first task then adding a node to the tree for each task that could be scheduled based upon precedence and resource constraints. Bounds, based on partial schedules, were used to prune the search tree [(6)].

Implicit Enumeration Method

This method is developed by F. Brain Talbot [(6)] for solving the resource constrained project scheduling problem. The procedure consists of a systematic evaluation (enumeration) of all possible job finish times for the activities of a project. Most noteworthy in this regard is the concept of a "cut" introduced in his procedure to eliminate possible inferior completion times for activities earlier in the enumeration phase of the algorithm.

Heuristics Solutions Methods

Heuristics methods are used to optimize scheduling of construction projects. They analyze activities and schedule only one at a time. The proposed heuristic algorithm ranks possible combinations heuristics every time and simultaneously schedules all activities in the selected combination [(1)].

Scheduling Heuristic Method

Scheduling heuristic methods, typically, follow three steps: planning, sequencing, then scheduling. Some methods use heuristics to search the combinatorial space of permutations in task sequences; others use heuristics to determine feasible time/task/resource assignments during schedule generation, and others use heuristics to combine sequencing and scheduling. A few include planning in the generation of schedules by permitting more than one plan and allowing the search to choose between plans as it schedules. Precedence constraints typically dominate the search in the sequencer, whereas resource constraints dominate in the scheduler.

Scheduling Heuristics

Scheduling heuristics operate on a set of tasks and determine when each task should be executed. If a task may be executed in more than one execution mode or on any one of a set of resources, the heuristic must also determine which resources and/or execution mode to use. The scheduler enforces constraint satisfaction by assigning a task to a resource (or a resource to a task) at a time when the resource is available and the task can be executed. The following are some of the most commonly-used scheduling heuristics:

- MIN SLK: choose the task with the minimum total slack.
- MIN LFT: choose the task with the minimum latest finish time.
- SFD: choose the execution mode with the shortest feasible duration.
- LRP: choose the execution mode with the least resource proportion.

Sequencing Heuristics

Whereas scheduling heuristics operate on tasks to decide when they should be executed, sequencing heuristics determine the order in which the tasks will be scheduled. These heuristics are often used in combination with decision trees to determine which part of the tree to search or to avoid.

Scheduling Approaches

Serial Schedule Generation Approach

Two types of schedule generation schemes (SGS) are used in resources scheduling problem, serial and parallel. In the serial scheduling scheme, a priority list of activities is determined at time zero. This list is based on some heuristic such as latest finish time (LFT). The ordering of activities in a given priority list must, of course, follow precedence constraints, but it is independent of the resource constraints. Given a priority list, activities are scheduled in the given order at the earliest possible clock time at which the precedence constraints are satisfied and the resources are available. In this approach, processing times of activities are parameterzed using a weight factor. The problem of optimally scheduling a given project is then posed as the problem of finding the optimal set of weights in the weight search space, similar to the way non-linear mapping functions are determined in neural networks. Reinforcement and backtracking techniques are applied as part of weight modification strategies.

Priority-Rule

Based heuristics combine one or more priority serial, parallel rules and schedule generation schemes (SGS) or both in order to construct one or more schedules. It is easy to implement and fast in terms of the computational effort and it has two methods: If only one schedule is generated, it is called a single pass method and if more than one schedule is generated is called X-pass (multi-pass) method. Some of the well known priority rules are: minimum late finish time (LFT), minimum start time (MST), minimum total float (MTF), greatest resource demand (GRD), and earliest start time (EST).

Single-Pass Methods

A variety of priority single-pass methods have been widely used to solve the project scheduling problems (Patterson and Davis, 1975; Cooper, 1976; Patterson, 1984; and Alvares-Valdes and Tammarit, 1989). These methods are easy to implement and fast in terms of the computational effort.

Multi-Pass Methods

Can be categorized as multi-priority rule methods and sampling methods. Multi-priority rule methods combine the schedule generation scheme (SGS) with a different priority rule at each iteration.

Parallel Schedule Generation Approach

In parallel schedule generation, the order in which the activities are scheduled is not decided at time zero. The scheduling decisions are made on a clock timer, i.e., at times when activities can start and resources are available.

Methaheuristic Methods

On the other hand, the methaheuristic approaches (genetic algorithms, GA; simulated annealing, SA; tabu search; TS; and ant colonies, AC) have been widely applied to solve the scheduling problem under limited resources. They are characterized by their outstanding performance consistency, and the ability to determine global optima [(2)].

Deterministic Methods

Operate the same way each time for a given problem; many hybrid methods exist that combine the characteristics of these classes. When resource-constrained scheduling solutions were first proposed, simple models were used with exact methods for solving the problem (neural network, NN; Aug NN approach; hierarchical and artificial intelligence approaches) [(4)].

Stochastic Methods: Stochastic models were added to account for the uncertainty of real schedules. They include probabilistic operations, i.e., probabilistic estimates of task duration when the durations of activities are given by a distribution of probability. So, they may never operate the same way twice on a given problem, but two different runs may result in the same solution [(4)].

Case Study

To explain the previous studies and applications the following simple example has been solved as a pilot case study Table 1, using different methods and computer programs so as to be applied, in a second future phase, to none repetitive projects.

Solution by Critical Path Method (CPM)

To find critical path (C.P.) and then the project completion time we computed four quantities for each activity (Figure 1 and 2):

1. Earliest starting time (ES) by forward pass from node (1) to final node.
2. Earliest finishing time (EF) by forward pass from node (1) to final node.

3. Latest finishing time (LF) by backward pass from final node to node (1).
4. Latest starting time (LS) by backward pass from final node to node (1).

The complete summary of the calculations for example is shown in Table 2.

The Critical path is represented by activities (A- C- E- G- I)

Project completion time is the sum of durations of critical activities.

Project completion time (P.C.T.) =3+ 2 + 5 + 7 + 8 = **25 weeks.**

Solution by Project Evaluation and Review Technique (PERT)

To provide estimates of the effect of uncertainly on project completion time, PERT defines the following quantities: Minimum activity time = a, Most likely activity time = m. Maximum activity time = b. In PERT the beta distribution assumption is used to justify the approximation of the mean (μ) and standard deviation (σ) of each activity time by using the following formulas:

$$\mu = \frac{a+4m+b}{6} \quad , \quad \sigma = \frac{b-a}{6} \qquad (1)$$

(σ) is the square root of σ^2 (variance) which is obtained by the formula: $\sigma^2 = \frac{(b-a)^2}{36}$

The complete summary of the calculations is shown in Table 3. PERT method computes the critical path (C.P.) based on the mean activities times. Using PERT it was found that the critical path remains the same as in CPM solution method (A-C-E-G-I) so, the expected project completion time (P.C.T.) is the sum of mean activities times along the critical path.

P.C.T = 3 + 2 + 5 + 6.83 + 9.17 = 26 weeks

Variance σ^2 = 0.11+ 0 + 1 + 1.36 +4.69 = 7.16

Standard deviation $= \sqrt{\sigma^2} = \sqrt{7.16} = 2.68$

Then, total project completion time is a normal random variable with mean (μ) = 26 weeks with standard deviation = 2.68.

Solution By Exact and Optimization Solution Methods

As an example of these methods we will solve the example through linear programming: From Figure 2- network “activity – on – arrow” a total of eight nodes was found. The formulation of the “Objective Function” of the problem that gives earliest start times is:

$$Min \sum_{i=1}^{8} x_i \qquad (2)$$

Subjected to the following:

$$X_i \geq 0 \text{ for } 1 \leq i \leq 8T$$

The activities are shown in parentheses, Shown in Table 4 and the relevant portion of the Excel program output is given in Table 5. It is worth to mention that the Lindo program gives the same results (output).

Solution by Primavera Program (P_3)

To solve the example by primavera program (P_3) (15,16,17), same nine activities were entered with their same durations, and then they were given the same calendar (calendar no. 1) with five nonworking days per month. Unlimited resources were then allocated to each activity, and then scheduling and leveling steps were carried out.

Table 1: Data for the pilot case study

No.	Task description	Task name	Duration	Predecessors tasks
1.	Planning and digging work	A	3	-
2.	Import bases filling materials	B	4	A
3.	Bases and short columns materials	C	2	A
4.	Base filling and import materials for plain concrete floor and columns	D	6	B, C
5.	Grade beam works	E	5	C
6.	Buildings around grade beam works	F	3	C, G
7.	Steel reinforcement for long columns works	G	7	E
8.	Grade beam filling works	H	5	E, F
9	Plain concrete on floor and L. cols con-works	I	8	D, G, F

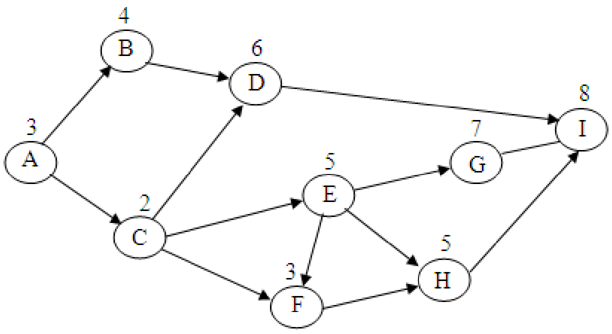

Figure 1: Activity-on- node method

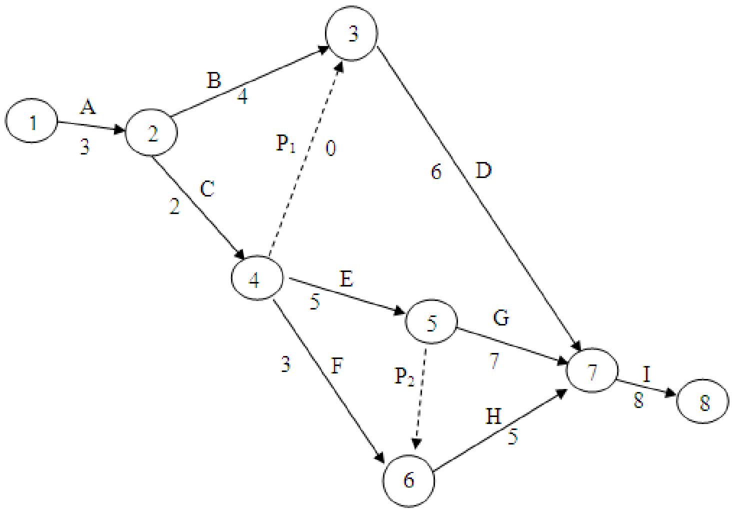

Figure 2: Activity-on-arrow method

Table 2: Summary of the CPM calculations

No.	Activity name	Duration	Predecessors activities	ES	EF	LS	LF	Total Float (T.F.)	Critical path C.P.
1	A	3	-	0	3	0	3	0	**A**
2	B	4	A	3	7	7	11	4	
3	C	2	A	3	5	3	5	0	**C**
4	D	6	B, C	7	13	11	17	4	
5	E	5	C	5	10	5	10	0	**E**
6	F	3	C,G	5	8	9	12	4	
7	G	7	E	10	17	10	17	0	**G**
8	H	5	E,F	10	15	12	17	2	
9	I	8	D,G,F	17	25	17	25	0	**I**

Table 3: Summary of the PERT calculations

Activity	Min time (a)	Most likely time (m)	Max time (b)	Mean: $\mu = \frac{a+4m+b}{6}$	Variance $\sigma^2 = \frac{(b-a)^2}{36}$	Critical path C.P.
A	2	3	4	3	0.11	A
B	2	4	10	4.67	1.78	
C	2	2	2	2	0	C
D	4	6	12	6.67	1.78	
E	2	5	8	5	1	E
F	2	3	8	3.57	1.21	
G	3	5	6	6.83	1.36	G
H	3	3	7	5.28	2.37	
I	5	2	8	9.17	4.69	I

The project is assumed to start in first day of October 2011 and project completion date is 23 of March 2012 which is equal to 25 weeks.

Table 4: The activities

$x_2 - x_1 \geq 3$	(A)
$X_3 - x_2 \geq 4$	(B)
$X_4 - x_2 \geq 2$	(C)
$X_3 - x_4 \geq 0$	(P1) Dummy activity
$X_5 - x_4 \geq 5$	(E)
$X_6 - x_4 \geq 3$	(F)
$X_6 - x_5 \geq 0$	(P2) Dummy activity
$X_7 - x_3 \geq 6$	(D)
$X_7 - x_6 \geq 5$	(H)
$X_7 - x_5 \geq 7$	(G)
$X_8 - x_7 \geq 8$	(I)

Solution By Heuristic Method (Priority Rule Using Primavera Program) When Project Is Subjected To Limited Resources

The same activities and were entered together with the calendar (calendar no. 1) to the project but normal resources were allocated to the activities. The scheduling was then performed followed by the leveling step, so the project completion date (time) is still the same as in the previous solution (23 of March 2012), but over allocation was evident in resources. To treat this over allocation of resources, the activities were rescheduled with different kinds of heuristics through single path approach and priority rule using primavera program (P_S) as a simulator tool. The heuristics used were:

a. Minimum Total Float- MTF (Min. slack).
b. Minimum Late start Time – MLS.
c. Minimum Late Finish Time – MLF.
d. Greatest Resource Demand – GRD.

In each heuristic we did scheduling and leveling steps using forward leveling and none time constraints choices. The summary of results is shown in the Table 6. From results it is clear that the project completion time was delayed every time a specific heuristic was used and each heuristic gave different performance and, hence, different completion time.

Conclusion

CPM and PERT are the most applicable methods, however, both do not consider the resources availability. So, the two methods are considered as feasible procedures for producing non-feasible schedules. On the other hand, resource leveling which is used to reduce the sharp variations in the resource demand histogram cannot handle the issue of minimizing project duration. The Min. slack (MINSLK) rule produces an optimal schedule span, most often. Considering many heuristic rules it was found that the MINSLK, LST, LFT rules are the most effective and efficient ones that produce better results than others.

Linear programming formulations can be used only for specific instances, i.e., small problems. The implicit enumeration procedure of Talbot requires far less computer storage. Branch and bound solution procedure of Stinson and bounded enumeration procedure of Davis are likely to produce the optimal solution while taking minimum amount of computation time.

Enumerative methods cannot solve large problems and the methaheuristic methods cost much more computational time than the heuristic methods which are simple to understand, easy to apply, and are able to rationalize the scheduling process and make it manageable for practical-size projects. Heuristics methods can produce schedules with lower average tardiness costs. The priority-rule based heuristics are easy to implement and fast in terms of the computational effort, so, a variety of priority single-pass methods have been widely used to solve the project scheduling problem under limited resources.

In genetic algorithm many real scheduling problems incorporate layers of ill-defined constraints that are often difficult, if not impossible, to represent using traditional math and thus susceptible to entrapment in sub-optimal regions of the search space. Genetic algorithms avoid sub-optimal solutions of the general resource- constrained scheduling problems. Computer programs give adequate solutions but, when resource requirements exceed the resources available, the computer programs do not provide the optimum scheduling solutions. Advancements in the memory capabilities of mainframe computer systems made it possible to overcome many deficiencies in the scheduling techniques being used in project management.

Microsoft project and Primavera Project Planner are two popular software programs used in construction industry. Both programs allow the user to prepare detailed schedule and allocate resources and level the type and quantity of resources required for each activity on the schedule, although this software cannot guarantee a successful project plan. Primavera project planner contains more features to facilitate solving resource scheduling problems as compared to Microsoft project.

Table 5: Excel program output

Adjustable Cell	Cells Name	Original value	Final value
B5	x_1	0	0
C5	x_2	0	3
D5	x_3	0	7
E5	x_4	0	5
F5	x_5	0	10
G5	x_6	0	10
H5	x_7	0	17
I5	x_8	0	25

The min. project completion time is the value of x_8, which is equal to 25 week.

Table 6: Scheduling and leveling steps using forward leveling and none time constraints choices

No.	Heuristic name	New project finish date	Old project finish date	Time delayed (days)	Time delayed (weeks)
1	Minimum total float (MTF)	13/4/2012	23/3/2012	21	3
2	Minimum late start time (MLS)	6/4/2012	23/3/2012	4	2
3	Minimum late finish time (MLF)	4/5/2012	23/3/2012	42	6
4	Greatest res demand (GRD)	11/5/2012	23/3/2012	49	7

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Appendix (1)

In order to show that the Min. LST and MINSLK rules are equivalent, it must be shown that if the float (slack) of activity *i (TFi)* is greater than or equal to the slack of activity j (TFj), then LSTi ≥ LSTj.

Proof: *Assume that* TFi ≥ TFj. *On a dynamic basis (i.e., parallel scheduling) the total slack of an activity is defined as:* TF = LST - Max {EST; TIME}, *where* TIME is *equal to the "current" schedule value of time when scheduling could occur.*

For example: if the EST *for an activity is 22 and the* LST *is 25, then slack "currently" equals 3 time units. But if the activity was not scheduled at* TIME = *22, then at* TIME = *23 only 2 units of slack would be present, etc. For scheduling consideration, technological constraints require that an activity's* EST *be less than or equal to the value of* TIME, *hence* TIME = MAX {EST; TIME} *And* TFi = LSTi –TIME *where:* TIME ≥ ESTi *Therefore, if* TFi>TFj , LSTi – TIME ≥ LSTj -TIME.

And since TIME ≥ 0, TIME *can be added to both sides of the last inequality without changing the direction of the inequality, then,*LSTi ≥ LSTj