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Optimally Tuned Proportional Integral Derivatives (PID) Controllers for Set-point

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Abstract: The proportional-integral-derivative (PID) controller is tuned to find its parameters values. Generally most of the tuning methods depend mainly on the experimental approach of open-loop unit step response. The controller parameters can be found if the system truly can be approximated by First Order Plus-Dead Time (FOPDT). The problem with such type of controllers is that: the performance of most of them deteriorates as the ratio (L/T) of approximated equivalent delay L to the overall time constant T changes. The optimum tuning always checks this ratio and considers it in its formulae. The performances of different PID tuning techniques are simulated for different systems and analyzed based on the transient responses. MATLAB simulation results are presented and compared for different higher order systems. For the same characterization procedure, optimally tuned PID controller shows better performances over Ziegler-Nichols (Z-N) and Cohn-Coon tuned. Superiority of the optimal PID tuning techniques sustained for variety of higher order systems.

Keywords: *Optimum Controller; PID Controller; Set-point Changes; Controller Tuning*

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مستخلص:

يتم تنغيم أو ضبط الحاكمة النسبية–التكاملية–التفاضلية لإيجاد قيم عناصرها. عموماً أغلب طرق التنغيمِ تَعتمدُ بشكل رئيسي على النهج التجريبي لإستجابة نظام الحلقةِ المفتوحةِ لوحدةِ الدالة الدرجية. ويمكن إيجاد عناصر هذه الحاكمة لأى نظام يمكن تقريبه بنظام من الدرجة الأولى زائداً زمن تأخُر. المشكلة مع هذا النوع من الحواكم هى أكثرها يتدهور أداءه كلما تغيرت نسبة زمن التأخُر التقريبى إلى الثابت الزمنى الكلى للنظام. التنغيم الأمثل لهذه الحاكمة دائماً يفحص هذه النسبةِ وتَعتبرُها في معادلاته الأساسية. إنّ أداء تقنياتِ تنغيم مختلفة تمت محاكاتها وتحليلها للأنظمةِ المختلفةِ مستندة على الإستجابة العابرةِ. نتائج المحاكاة بالماتلاب لأنظمة ذات رتب عليا مختلفة تمت مقارنتها. من المحاكاة إتضح ان حاكمة التنغيم الأمثل ذات أداء أفضل من الحاكمة المنغمة بطريقة زيقلر–نيكولس وطريقة كوهين–كوون ودت خصائص أداء ثابتة لمدى واسع من أنظمة الرتب العليا.

Introduction:

The PID controller has several important functions; it provides feedback, has the ability to eliminate steady state offsets through integral action, and it can anticipate the future through derivative action. PID controllers are the largest number of controllers found in industries sufficient for solving many control problems.

The determination of the controller parameters is called the controller tuning or design. Many approaches have been developed for tuning PID controller and getting its parameters for single input single output (SISO) systems. Among the well-known approaches are the Ziegler-Nichols (Z-N) method (1), the Cohen-Coon (C-C) method (2), integral of squared time weighted error rule (ISTE) (3), integral of absolute error criteria (IAE) (4), internal-

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model-control (IMC) based method [5], and gain-phase margin method [6]. This paper focuses on studying the optimum tuning method and comparing it with Z-N which has been explored since 1942 and is still used in industry and C-C.

The PID control law is the sum of three types of control actions: a proportional, an integral and a derivative control actions. Mathematically PID controller in the time-domain is given by the following equation**:**

$$u(t) = K_P[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d \frac{de(t)}{dt}] \qquad (1)$$

Where *u (t)* is the controller output (input signal to the plant model), the error signal *e(t)* is defined as *e(t) =r (t) − y (t),* and *r (t)* is the reference input signal while *y(t)* is the output. The controller parameters are proportional gain K_p, integral time T_i, and derivative time T_d [7].

If a mathematical model of the PID-controlled plant can be derived, then various design techniques for determining the controller parameters can be applied. However, if the plant is so complex that its mathematical model cannot easily be obtained, then analytical approach to design PID controller is not possible[8]. Then we must resort to experimental approaches for tuning of PID controllers.

In this work the open-loop step response of the given system is obtained and the three characterizing parameters (K, L, and T) are determined from this response. Then according to the tuning method, the controller parameters (K_p, T_i, and T_d) can be obtained. The transient step responses of the simulated closed-loop are then compared for different tuning rules.

OPTIMUM PID CONTROLLER DESIGN:

Optimum setting algorithms for a PID controller were proposed by Zhuang and Atherton [9] for various criteria. The methods involve searching for minimum of the cost function $J_n(\varphi)$ in its general form:

$$j_n(\varphi) = \int_0^\infty [t^n e(\varphi, t)]^2 dt \qquad (2)$$

Where e (φ, t) is the error signal, with φ as PID controller parameters. The optimum controller parameters are found when the partial derivative of $J_n(\varphi)$ with respect to φ equals zero. The error signal used for optimization can be a result set-point or of load disturbance. Therefore, it is possible to obtain two sets of parameters: one for the set-point input and the other for the disturbance signal. In particular, three values of n (n = 0, 1, 2) are discussed. These three cases correspond, respectively, to three different optimum criteria: the integral squared error (ISE) criterion, integral squared time weighted error (ISTE) criterion, and the integral squared time-squared weighted error (IST^2E) criterion. The expressions given were obtained by fitting curves to the optimum theoretical results [9,10].

A large number of industrial plants can approximately be modeled by the first order plus dead time (FOPDT) model with transfer function as follows [11]:

$$G(s) = \frac{K}{Ts+1}e^{-Ls} \qquad (3)$$

Sometimes one may want to design a controller having good rejection performances on the disturbance signal. The parameters equations to design controllers for disturbance rejection using the optimal method are different than the set point used here.

PID controller equation suggested by Z-N is:

$$G_{ZN}(s) = 1.2\frac{T}{L}(1 + \frac{1}{2Ls} + 0.5Ls) \qquad (4)$$

And C-C suggested gains setting as:

$$\left.\begin{aligned} K_p &= \frac{1.25}{a}(1 + \frac{0.18\tau}{1-\tau}) \\ T_i &= \frac{2.5 - 2\tau}{1 - 0.39\tau}L \\ T_d &= \frac{0.37 - 0.37\tau}{1 - 0.81\tau}L \end{aligned}\right\} \qquad (5)$$

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Where; $a = k_p L/T$ & $\tau = L/(L+T)$.

While the gains of optimal PID controller can be set as follows [9]:

$$\left.\begin{aligned} K_p &= \frac{a_1}{k}\left(\frac{L}{T}\right)^{b1} \\ T_i &= \frac{T}{a_2 + b_2(L/T)} \\ T_d &= a_3 T\left(\frac{L}{T}\right)^{b3} \end{aligned}\right\} \quad (6)$$

Where the parameters (a, b) should be determined according to Table A1 in appendix A. the selection of (a, b) depends mainly on the value of (L/T).

Tuning Rules of PID Controllers for Set-Point Changes:

The key feature in the optimum methods for PID controller tuning is to obtain the response of the plant to a unit step input. If it involves neither integrator nor dominant complex-conjugate poles, then such an open-loop unit step response curve may be characterized by three constants, gain K delay time L and time constant T [7].

The following is an example of PID-controlled systems and their responses for different ratios when tuned using Z-N and C-C methods.

These constants are either to be found experimentally or instead of experimental approaches, a simulation may be used to get these parameters. In the following are different systems examined to illustrate the method for tuning the controllers.

$$System1; \frac{120}{(s^2 + 6s + 3)} \quad (7)$$

From the step response we obtained the parameters (K, L, and T) as (K = 40, L = 0.174, T =1.826). The range of (L/T) from the given transfer function is equal to

$$\frac{0.174}{1.826} = 0.095$$

Table 1: The Controller Parameters of System1

	PID Controller Parameters		
Criterion	K_p	T_i	T_d
Z-N	0.3143	0.3485	0.0871
C-C	0.3330	0.507	0.0633
ISE	0..2155	1.574	0.111
IST^2E	0.2024	1.92	0.071

$$System\ 2; G_2(s) = \frac{200}{(s+1)(s+4)(s+6)} \quad (8)$$

From the step response we obtained the parameters (K, L, and T) as (K = 8.333, L = 0.3725, T = 1.0442). The range of (L/T) from the given transfer function is equal to

$$\frac{0.3725}{1.0442} = 0.3567$$

The parameters of the controller are obtained as in Table 2:

Table 2. The Controller Parameters of System2

	PID Controller Parameters		
Criterion	K_p	T_i	T_d
Z-N	0.4036	0.7450	0.1863
C-C	0.4475	0.8600	0.1291
ISE	0.3170	0.9816	0.2044
IST^2E	0.2949	1.1775	0.1316

$$System3; G_3(s) = 4/(s+1)^4 \quad (9)$$

From the step response we obtained the parameters (K, L, and T) as (K=2, L=2, T=2). The range of (L/T) from the given transfer function is equal to 1. Table 3 shows the parameters of the different controllers.

Table 3: The Controller Parameters of System 3

	PID Controller Parameters		
Criterion	K_p	T_i	T_d
Z-N	0.3	4	1
C-C	0.3687	3.500	0.622
ISE	0.262	2.418	0.978
IST^2E	0.2420	2.762	0.632

$$System4; G_4(s) = \frac{3}{(s+2)(s+1)^6} \quad (10)$$

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From the step response we obtained the parameters (K, L, and T) as (K =1.5, L =4, T=2.5), $\frac{L}{T}=1.6$; Therefore the parameters of the controller are obtained as in Table 4.

Table 4: The Controller Parameters of System 4

	PID Controller Parameters		
Criterion	K_p	T_i	T_d
Z-N	0.500	8	2
C-C	0.671	6.177	1.135
ISE	0.590	3.600	1.710
IST^2E	0.5378	3.981	1.164

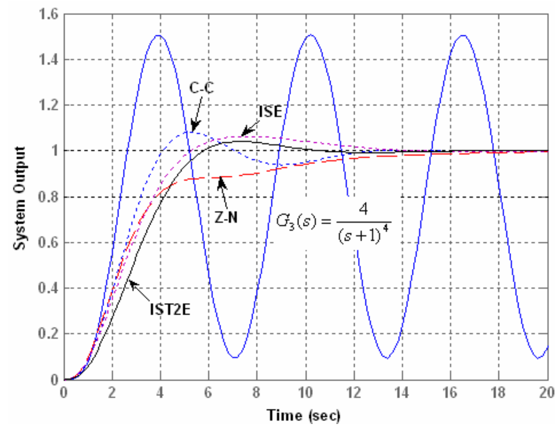

Figure 2: System 2 Step Response

Simulation Results and Discussions:

In this section, a simulation for the four different systems is carried out to obtain their transient responses. The comparison is based on the rise time (t_r), the settling time (t_s), and the peak over shoot (M_P) of the closed-loop step response for each method as shown in Figures 1 - 4 and the results are tabulated in Table 5.

For the different tested systems, the optimally tuned controller gives accepted rise time, generally the best settling time and they perform very much better than others in the overshoot behavior. For some systems the responses of both Z-N and C-C result in longer settling time and approaching critically damped systems which considered as their shortcoming.

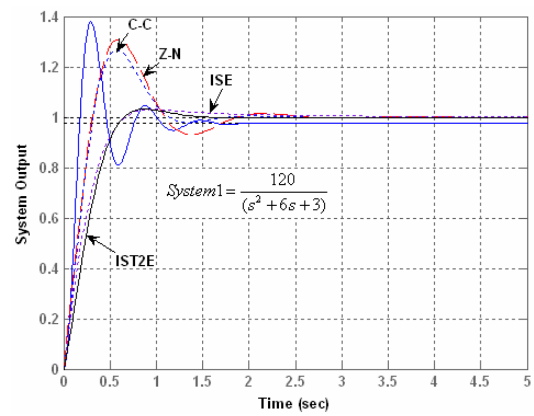

Figure 3: System 3 Step Response

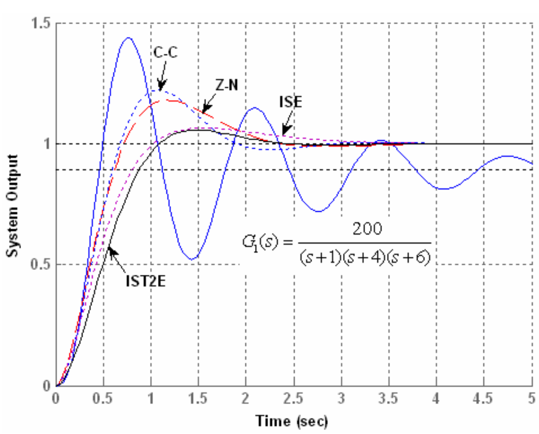

Figure 1: System1 Step Response

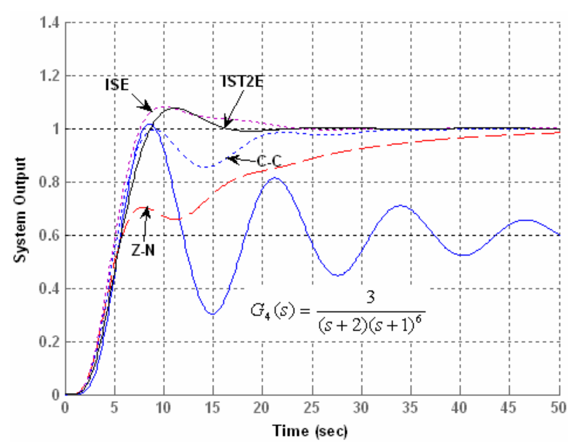

Figure 4: System 4 Step Response

Table 5: Comparison of Transient Responses

System	Criteria	t_r (Sec)	t_s (Sec)	% O.s
$G_1(s)$	Z-N	0.24	1.74	30.8
	C-C	0.24	1.44	26.5
	ISE	0.435	1.39	3.07
	IST^2E	0.437	1.11	3.2
$G_2(s)$	Z-N	0.496	2.15	18.0
	C-C	0.465	2.48	22.0
	ISE	0.675	2.65	6.52
	IST^2E	0.715	2.05	5.62
$G_3(s)$	Z-N	6.86	14.2	≅ Critical
	C-C	2.51	11.5	8.08
	ISE	3.25	11.0	6.33
	IST^2E	3.51	09.1	4.05
$G_4(s)$	Z-N	23.8	47.7	≅ Critical
	C-C	4.38	28.6	≅ Critical
	ISE	4.10	19.8	8.2
	IST^2E	4.72	14.8	7.6

Conclusions:

From the results obtained, it could be conclude that PID control is still of great interest, and is a promising control strategy that deserves further research and investigation. These tuning methods are only valid for open loop and those can be described by the first order plus dead-time model and for 'ideal' PID control structure case. Optimally tuned PID controllers show better results than Z-N and C-C. The responses of both the later deteriorate as the approximated equivalent delay L to the overall time constant T changes. Optimally tuned controller sustain for wide range of systems due to their consideration to L/T. However, among the optimum PID tuning methods, IST^2E was shown to be the best for the transient response specifications.

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APPENDIX A:

Table A1: Set-point PID controller parameters

L/T	0.1-1			1.1-2		
	ISE	*ISTE*	IST^2E	*ISE*	*ISTE*	IST^2E
a1	1.048	1.042	0.968	1.154	1.142	1.061
b1	−.897	−.897	−.904	−.567	−.579	−.583
a2	1.195	0.987	0.977	1.047	0.919	0.892
b2	−.368	−.238	−.253	−.220	−.172	−.165
a3	0.489	0.385	0.316	0.490	0.384	0.315
b3	0.888	0.906	0.892	0.708	0.839	0.832

APPENDIX B:

M-File:

```
G=tf(--, [-----])
[K,L,T]=getfod(G,1)
a=K*L/T; GP=feedback(G,1);
step(GP,'b-',50)
for n=1:4
switch n
case 1
  kp=1.2/a
  Ti=2*L
  Td=0.5*L
gc=tf(kp*[Ti*Td Ti 1],[Ti 0])
GZ=series(gc,G); Zeig=feedback(GZ,1);
step(Zeig,'r--')
   case 2
  tao=L/(L+T);   kp=(1.25/a)*(1+((0.18*tao)/(1-tao)))
 Ti=((3.3-tao)/(1+1.2*tao))*L
  Td=((0.37-0.37*tao)/(1-0.81*tao))*L
 gc=tf(kp*[Ti*Td Ti 1],[Ti 0])
GCC=series(gc,G); coh=feedback(GCC,1);
hold on
step(coh,'b:')
   case 3
  % kp=(1.048/K)*((L/T)^(-0.897)) %L/T<1
  % Ti=T/(1.195-0.368*(L/T))
  % Td=0.489*T*((L/T)^0.888)
 kp=(1.154/K)*((L/T)^(-0.567))     %L/T>1
 Ti=T/(1.047-0.22*(L/T))
 Td=0.49*T*((L/T)^0.708)
  gc=tf(kp*[Ti*Td Ti 1],[Ti 0])
GO1=series(gc,G); ISE=feedback(GO1,1);
step(ISE,'m:')
   case 4
 % kp=(0.968/K)*((L/T)^(-0.904))   %L/T<1
 % Ti=T/(0.977-0.253*(L/T))
 % Td=0.316*T*((L/T)^0.892)
     kp=(1.061/K)*((L/T)^(-0.583))      %L/T>1
     Ti=T/(0.892-0.165*(L/T))
     Td=0.315*T*((L/T)^0.832)
    gc=tf(kp*[Ti*Td Ti 1],[Ti 0])
GO2=series(gc,G); IST2E=feedback(GO2,1);
hold on
step(IST2E,'g-.')
end
```