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Multiple Gross Errors Detection in Surveying Measurements Using Statistical Quality Control

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Abstract: Most of the surveying tasks involve the acquisition and analysis of measurements. Such measurements are subject to random, systematic and gross errors. In practice, redundant measurements are made to provide quality control and errors check. In qualitative analysis and statistical evaluation, it is generally assumed that the measurements contain only random errors and are regarded as random variables. In reality, the measurements may contain gross and/or systematic errors. The effects of such errors are distributed over the residuals, after an adjustment and lead to questionable results and interpretation.

For high precision applications, gross and systematic errors need to be detected prior to the analysis. These errors should be tackled before the adjustment by means of screening. These few remaining gross errors in the measurements can be detected after the adjustment. These adjustment methods assume the presence of only one gross error. One of the most effective methods that can be used in detecting multiple gross errors is the statistical quality control method. Statistical quality control is a technique used to monitor a procedure with a goal of making it more efficient and ensures precise results.

Statistical control charts are used to provide an operational definition of a special cause for a given set of data. It is possible to construct multiples of sigma control limits. When all the points on a control chart are within a multiple of sigma control limits and there are no gross errors in the data, the process of measurements is said to be in a state of statistical control. Otherwise, the data indicate the presence of non-random gross errors. In this research work, different methods of statistical quality control were used. Results showed that statistical quality control method can be used successfully and efficiently in detecting multiple gross errors.

***Keywords**: Statistical Control Charts, Variables Control Charts, Attribute Control Charts, Probability, Statistical Quality Control.*

مستخلص:

كثير من المهام المساحية تتطلب الحصول على القياسات وتحليلها. تخضع تلك القياسات للأخطاء الجسيمة ، المنتظمة والعشوائية. عملياً هنالك أرصادات زائدة من أجل الوصول الى قيم مضبوطه والمساعدة فى اكتشاف الأخطاء. فى التحليل النوعى والتحليل الإحصائى من أجل الجودة ،عادةً مايفترض أن القياسات تحتوى على أخطاء عشوائية فقط واعتبارها متغيراتٍ عشوائية. فى الحقيقة تلك القياسات يمكن أن تحتوى على أخطاء جسيمة بالاضافه الى أخطاء منتظمة. تأثير هذه الأخطاء يتوزع على الأخطاء المتبقية بعد إجراء عملية الضبط مما تؤدى بدورها الى إثارة الاسئلة حول نتائجها وتفسيرها.

التطبيقات ذات الدقة العالية تتطلب كشف الأخطاء الجسيمة والمنتظمة قبل إجراء التحليل، وهذه الأخطاء يجب معالجتها قبل إجراء عمليتى الضبط و التقدير عن طريق دراسة البيانات. الأخطاء الجسيمة البسيطة المتبقية فى نتائج القياسات يمكن كشفها بعد عملية الضبط. الطرائق المختلفة التى تستخدم عادة ما تفترض وجود خطأ جسيم واحد فقط. احدى الطرق ذات الفاعلية والتى يمكن استخدامها فى كشف الأخطاء الجسيمة المزدوجة هى طريقة ضبط الجودة الإحصائى. ضبط الجودة الإحصائى هى تقنية لمتابعة اجراءٍ ما بهدف جعله إجراءً فعالاً ويضمن نتائج ذات دقةٍ عالية. تستخدم الرسومات البيانية الإحصائية لتوضيح المسببات الخاصة لمصفوفة بيانات محددة ومن الممكن إنشاء حدودٍ بعينها كمضروبات للانحراف المعيارى . عندما تقع كل النقاط فى الحدود المحددة ولاتوجد أخطاء جسيمة او نمط واضح للبيانات، يقال ان عملية القياس فى وضع الضبط الإحصائى والا فإن البيانات تحتوى على أخطاء غير عشوائية. أستخدم فى هذا البحث طرقاً مختلفة لضبط الجودة الإحصائى. الخلاصة الرئيسية من هذا البحث هى ان ضبط الجودة الإحصائى يمكن إستخدامه فى كشف الأخطاء الجسيمة المزدوجة.

Introduction

Generally speaking, gross errors can arise from an incorrect measuring, and recording procedure, from the observer or the computer software used [1, 2]. They are the most serious of all types of errors, simply because of their relatively large sizes. Therefore, care must be taken to avoid or eliminate them from observations, otherwise the results obtained from a process may be highly affected by their presence [3].

Within a certain interval, i.e. (-a to +a), the observations are considered to be normally distributed. Then the gross error may be defined as an error which does not belong within the interval (-a to +a). it is stated further that an assumption that the distribution of a gross error is unknown seems to be the only realistic alternative [4].

Also when carrying out a statistical test for gross errors, a so-called statistic is computed. The probability density function (pdf) of the statistic is known and its value is so high that it can be expected to be excluded (in say) 1% of cases; it is assumed that the observation must be generated by another process (i.e. it is centered about a different mean and is highlighted as a possible gross error (for probable rejection) [5].

The interpretation of gross errors given by above is the most widely used interpretation. It follows the so-called mean-shift model where it is assumed that the mean of an observation is shifted when a gross error is present but the shape of its distribution is not altered. i.e. it is still normally distributed with a given variance. The mean-shift model has advantage of allowing statistical tests to be applied. For this reason, that the mean-shift model has found good support in methods and techniques dealing with gross error detection [3].

There are two major types of gross errors. The first is related to the instrument performance and include measurement errors (bias), miscalibration and total instrument failure. The second is model related and include model inaccuracies due to inaccurate model parameters.

Various techniques have been designed for the detection and elimination of these two types of gross errors[6,7]. Any comprehensive gross error detection strategy should preferably have the following capabilities:

1) Ability to detect the presence of one or more gross errors in the data set (e.g., the detection problem).

2) Ability to identify the type and location of the gross error (e.g., the identification problem).

3) Ability to locate and identify multiple gross errors which may be present simultaneously in the data set (e.g., the

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multiple gross error identification problems).

4) Ability to estimate the magnitudes of the gross errors (e.g., the estimation problem).

Not all gross error detection strategies may fulfill all the above requirements. The last of the above requirements, although useful, is not absolutely necessary. A gross error detection strategy can be analyzed in terms of the component methods it uses to tackle the three main problems of detection, identification and multiple gross error identification, and the performance of the strategy is a strong function of these methods.

The objective of the preset work was to apply and evaluate the statistical quality control technique to detect multiple gross error in data collected through some surveying measurements. Data used include electronic distance measurements, angular measurements and coordinates determined from the Global Positioning System (GPS) measurements. Various techniques of Statistical Quality Control (SQC) are to be used.

The methodology used for data analyzing was the control charts generally applied in SQC technique.

DETECTION OF MULTIPLE GROSS ERRORS:

For a well maintained set of surveying observations, it should generally not expect more than one gross error to be present in the data. Therefore, a fundamental pre-requisite of any gross error strategy is that it should have good ability to detect and identify correctly a single gross error. However, if a large number of surveying measurements are to be carried out or if the instruments used are operated in a hostile environment, and/or procedures used in carrying out the measurements are inadequate, then it is possible for several gross errors to be simultaneously present in the data.

The most commonly used statistical techniques for detecting gross errors are based on hypothesis testing. In the gross error detection case, the null hypothesis H_0, is that no gross error is present, and the alternative hypothesis, H_A, is that one or more gross errors are present in the system. All statistical techniques for choosing between these two hypotheses make use of a statistical test which is a function of the measurements and model equations. The statistical test is compared with a pre-specified value and the null hypothesis is rejected or accepted, respectively, depending on whether the statistic exceeds the pre-specified value or not.

The outcome of the hypothesis testing is not always perfect. A statistical test may declare the presence of gross errors, when in fact there is no gross error. In other words, the null hypothesis is rejected when it is true and ought to have been accepted. In this case the test gives rise to a false alarm. On the other hand, the test may declare the measurements to be free from gross errors, when in fact one or more gross errors exist, i.e. the null hypothesis is accepted when it is wrong and to be rejected.

There are two major types of gross error detection. The first deals with the raw observations (e.g., pre-estimation gross error detection) and the second deals with the outcome from an estimation procedure (e.g., post-estimation gross error detection). In this paper only the first one is considered.

Pre-estimation gross error detection is meant to be the process of trying to detect, locate and eliminate or repair obvious large gross errors from the note-books or observation files with the help of simple set up conditions requiring little effort and without extensive and expensive computational work [3,8].

Statistical Quality Control (SQC) employs statistical methods to manage the quality of products and services. In 1924, Walter A. Shewart of the Bell Telephone Laboratories laid the foundation for statistical quality control [(9)]. Since then, the area of SQC has been enriched by the work of numerous statisticians, quality philosophers and researchers. No doubt, SQC is known in the quality literature. However, there is a lack of evidence that there is a chronological account of SQC to date in the literature.

The origination of statistical quality control was initiated and implemented in the Bell Laboratories in the mid 1920s [(10)]. It is stated:“as a young engineer at Western Electric's Haouthorne Works, I was drawn into a Bell Telephone Laboratories initiative to make use of the science of statistics for solving various problems facing Hauthorne's Inspection Branch. The end results of that initiative came to be known as Statistical Quality Control or SQC.”

It is evident from the above statement that the concept of SQC has begun in the Bell Laboratories in the mid-1920s.

A Control chart is one of the most important SQC methods in quality control and improvement. It is a statistical tool intended to monitor processes and signal (errors) when they go out of control. Nowadays the field of SQC can be broadly defined as those statistical and engineering models that are used in measuring, monitoring, controlling and improving quality [(11, 12, 13)].

Statistical Control Charts **(**SCCs**)** are used to provide an operational definition of a cause for a given set of process data. If the process is stable, then the distribution of any subgroup averages (means) will be approximately normal and it is possible to construct a 1-sigma, 2-sigma or even 3-sigma control limits, where sigma is generally taken to be the standard deviation of the mean. When all the points on a control chart are within the prescribed limits of 1, 2, and 3 sigma control limits and when there are no other non-random or systematic patterns in the data, the process is said to be in a state of statistical control or “in control”. Otherwise, the data indicate that the process is out of control.

The purpose of statistical quality control is to ensure, in a cost efficient manner, that the product delivered to customers meets their specifications. Inspecting every product is costly and inefficient, but the consequences of delivering nonconforming products can be significant in terms of customer dissatisfaction. Statistical quality control is the process of inspecting enough product from given lots to probabilistically ensure a specified quality level [(12)].

It is often required to compare the output of a stable process with the process specifications and make a statement about how well the process meets the specifications. To do this, it is necessary to compute the natural variability of a stable process with the process specification limits. A capable process is one where almost all the measurements fall inside the specification limits. This can be represented pictorially by the bell-shaped plot shown in Figure 1.

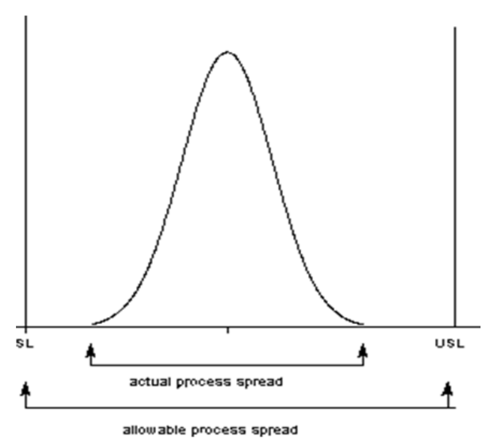

Figure 1: A capable Process

There is a close connection between control charts and hypothesis testing. Essentially, the control chart is a test of the hypothesis that the process is in a state of statistical control. A point fall within the control limits is means to accept the hypothesis of the statistical control, and vice versa.

The control chart is a device for describing exactly what is meant by statistical control. As such, it may be used in a variety of ways. If the sample values of the mean, say, fall within the control limits and do not exhibit any systematic pattern, the process is in control at the level indicated by the chart.

The control chart may also be used as an estimating device. That is, from a control chart that exhibits statistical control, we may estimate certain process parameters, such as the mean, standard deviation, and fraction nonconforming or fallout. These estimates may then be used to determine the capability of the process to produce acceptable products.

Control charts may be classified into two general types, Variables Control Charts (VCCs) and Attributes Control Charts (ACCs). Many quality characteristics can be measured and expressed as numbers in some continuous scale of measurement. In such cases, it is convenient to describe the quality characteristic with a measure of central tendency and a measure of variability. Control charts for central tendency and variability are collectively called Variables Control Charts (VCCs). The sample mean chart is the most widely used chart for monitoring central tendency; whereas charts based on either the sample range or the sample standard deviation are used to control process variability. Many quality characteristics are not measured on a continuous scale or even a quantitative scale. In these cases, each unit of product may be judged as either conforming or nonconforming on the basis of whether or not it possesses certain attributes; or the number of nonconformities (defects) appearing on a unit of product may be counted. Control charts for such quality characteristics are called Attributes Control Charts (ACCs).

Control charts have had a long history of use in industry. There are number of reasons for their popularity. These may include [9]:

1. Are proven techniques for improving productivity.
2. Are effective in defect prevention.
3. Prevent unnecessary process adjustments.
4. Provide diagnostic information.
5. Provide information about process capability.

The parameters required to design a control chart are the population mean μ, and population standard deviation, σ, (if they are known). Otherwise, these parameters can be replaced by their respective sample values. The average of the sample is then plotted on the chart. The Upper Control Limit (UCL) and Lower Control Limit (LCL) are then calculated and plotted on the chart. To do this, and because the control chart utilizes the sample mean to monitor the process mean (e.g., $\bar{X}$), it is usually called $\bar{X}$ control chart. If samples of size n are taken, then the standard deviation of the sample average, $\sigma_{\bar{X}}$, is given by;

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \qquad (1)$$

By using the central limit theorem to assume that $\bar{X}$ is approximately normally distributed, it would be expected that 100(1-α) % of the sample mean $\bar{X}$ fall between $\mu \pm k\sigma_{\bar{X}}$ where α is the level of significance. The constant k is customarily chosen to be a multiple of the standard deviation of $\bar{X}$. The "width" of the control limits UCL and LCL is inversely

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proportional to the sample size ***n*** for a given multiple of $\boldsymbol{\sigma}$.

Materials and methods:

To test the possibility of detecting multiple gross errors in surveying measurements, various tests were carried to find out whether multiple gross errors could be detected simultaneously using SQC methods. Various techniques and procedures were employed in order to achieve this objective. The data were provided using various surveying instruments including the Global Positioning System (GPS) and barometer.

Results:

In order to determine the height of a photo-control point, 30 measurements were taken using a barometer. Readings were taken every 10 minuets. The results of the measurements were as shown in Table (1). It is assumed that the measurements were taken under the same weather conditions. The formulae used to calculate the values given in Tables 1 and 2 were as follow:

Individuals Measurements:

$$\left.\begin{aligned} UCL &= \bar{x} + 3\frac{\bar{m}}{d_2} = \bar{x} + 3\frac{\bar{m}}{1.128} \\ CL &= \bar{x} \\ LCL &= \bar{x} - 3\frac{\bar{m}}{d_2} = \bar{x} - 3\frac{\bar{m}}{1.128} \end{aligned}\right\} \quad (2)$$

Moving ranges:

$$\left.\begin{aligned} UCL &= D_4\bar{m} = 3.267\bar{m} \\ CL &= \bar{m} \\ LCL &= D_3\bar{m} = 0 \end{aligned}\right\} \quad (3)$$

Ranges:

$$\left.\begin{aligned} UCL &= D_4\bar{r} \\ CL &= \bar{r} \\ LCL &= D_3\bar{r} \end{aligned}\right\} \quad (4)$$

3σ control charts:

$$\left.\begin{aligned} UCL &= \bar{x} + 3\sigma \\ CL &= \bar{x} \\ LCL &= \bar{x} - 3\sigma \end{aligned}\right\} \quad (5)$$

where: $\boldsymbol{D_3}$ and $\boldsymbol{D_4}$ are constants, tabulated for various sample sizes [9].

$D_4 = 2.115$, $D_3 = 0$, $d_2 = 1.128$

$\bar{r}$ is the sample average range.

$\bar{m}$ is the sample average moving range and individual measurements.

$\bar{x}$ is the sample average mean.

The test objective was to compare Shewart $\bar{X}$ control chart, individual control chart, moving range control chart, deviations control chart and cumulative sum control chart in detecting multiple gross errors.

In order to perform this comparison, ***3σ*** control charts for the above mentioned techniques were drawn. These are shown in Figure 2 (a, b, c, d, and e).

Because the control charts in Figure 2. a, b, c, d and e, can easily be constructed, the $\bar{X}$ and the R control charts were used to see whether multiple gross errors can be detected using these methods. Another test was carried out using 25 GPS measurement samples, to determine the height of a benchmark using GPS receiver. These measurements were carried out in five separate consecutive days. The results obtained are given in Table 2.

Again, the charts for the 25 samples, using the sample mean $\bar{X}$ is shown in Figure 3 and that for the sample ranges R, is shown in Figure 4.

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Table 1: Barometric Heights of a photo-Control Points

No	Observations (m)	CL	Deviation From The Mean	σ	$\bar{X}$ *Control Chart*		Individual Measurements		Moving range		Range	Cumulative Deviation
					UCL	LCL	UCL	LCL	UCL	LCL		
1	125.562	125.528	0.034	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0	0.034
2	125.501	125.528	-0.027	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.061	0.007
3	125.522	125.528	-0.006	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.021	0.001
4	125.56	125.528	0.032	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.038	0.033
5	125.498	125.528	-0.03	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.062	0.003
6	125.555	125.528	0.027	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.057	0.03
7	125.543	125.528	0.015	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.012	0.045
8	125.538	125.528	0.01	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.005	0.055
9	125.517	125.528	-0.011	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.021	0.044
10	125.499	125.528	-0.029	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.018	0.015
11	125.54	125.528	0.012	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.041	0.027
12	125.509	125.528	-0.019	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.031	0.008
13	125.538	125.528	0.01	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.029	0.018
14	125.544	125.528	0.016	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.006	0.034
15	125.547	125.528	0.019	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.003	0.053
16	125.565	125.528	0.037	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.018	0.09
17	125.497	125.528	-0.031	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.068	0.059
18	125.509	125.528	-0.019	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.012	0.04
19	125.514	125.528	-0.014	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.005	0.026
20	125.523	125.528	-0.005	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.009	0.021
21	125.548	125.528	0.02	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.025	0.041
22	125.533	125.528	0.005	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.015	0.046
23	125.571	125.528	0.043	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.038	0.089
24	125.507	125.528	-0.021	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.064	0.068
25	125.511	125.528	-0.017	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.004	0.051
26	125.519	125.528	-0.009	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.008	0.042
27	125.521	125.528	-0.007	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.002	0.035
28	125.5	125.528	-0.028	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.021	0.007
29	125.546	125.528	0.018	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.046	0.025
30	125.496	125.528	-0.032	0.023	125.597	125.459	125.6	125.456	0.088	0.027	0.05	-0.007
Total	3765.833											

Table 2: GPS height determination

Sample number	Observations (m)					$\bar{x}$	R	CL	σ	Three Sigma	
	1	2	3	4	5					UCL	LCL
1	629	636	640	635	640	636	11	629.3760	0.9033	634.707	624.045
2	630	631	622	625	627	627	9	629.3760	0.9033	634.707	624.045
3	628	631	633	633	630	631	5	629.3760	0.9033	634.707	624.045
4	634	630	631	632	633	632	4	629.3760	0.9033	634.707	624.045
5	619	628	630	619	625	624.8	11	629.3760	0.9033	634.707	624.045
6	613	629	634	625	628	625.8	21	629.3760	0.9033	634.707	624.045
7	630	639	625	629	627	630	14	629.3760	0.9033	634.707	624.045
8	628	627	622	625	627	625.8	6	629.3760	0.9033	634.707	624.045
9	623	626	633	630	624	627.2	10	629.3760	0.9033	634.707	624.045
10	631	631	633	631	630	631.2	3	629.3760	0.9033	634.707	624.045
11	635	630	638	633	635	634.2	8	629.3760	0.9033	634.707	624.045
12	623	630	630	627	629	627.8	7	629.3760	0.9033	634.707	624.045
13	635	631	630	630	630	631.2	5	629.3760	0.9033	634.707	624.045
14	645	640	631	640	642	639.6	14	629.3760	0.9033	634.707	624.045
15	619	644	632	622	635	630.4	25	629.3760	0.9033	634.707	624.045
16	631	627	630	628	629	629	4	629.3760	0.9033	634.707	624.045
17	616	623	631	620	625	623	15	629.3760	0.9033	634.707	624.045
18	630	630	626	629	628	628.6	4	629.3760	0.9033	634.707	624.045
19	636	631	629	635	634	633	7	629.3760	0.9033	634.707	624.045
20	640	635	629	635	634	634.2	11	629.3760	0.9033	634.707	624.045
21	628	625	616	620	623	622.4	12	629.3760	0.9033	634.707	624.045
22	615	625	619	619	622	620	10	629.3760	0.9033	634.707	624.045
23	630	632	630	631	630	630.6	2	629.3760	0.9033	634.707	624.045
24	635	629	635	631	633	632.6	6	629.3760	0.9033	634.707	624.045
25	623	629	630	626	628	627.2	7	629.3760	0.9033	634.707	624.045
total						15734.4	231	629.3760	0.9033	634.707	624.045
mean						629.376	9.24				

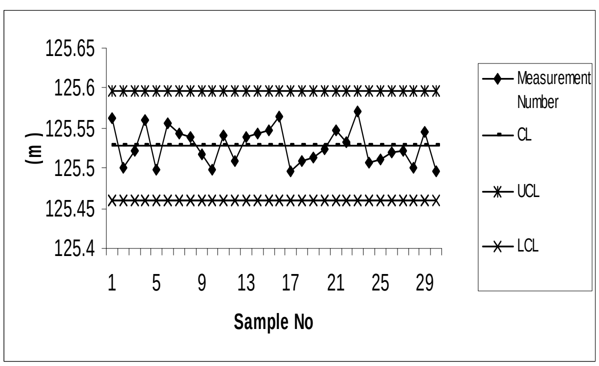


Figure 2a: 3σ $\bar{x}$ Control chart for Barometer readings

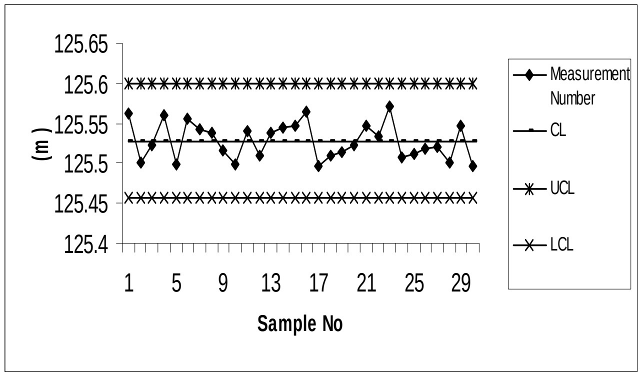
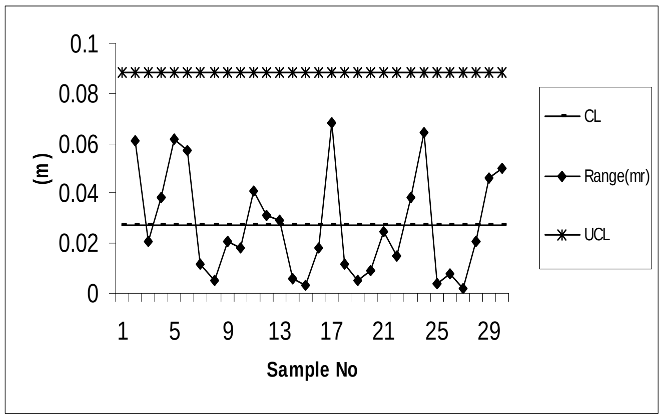


Figure 2b: 3σ Individual control chart for Barometer readings



(2c) 3σ Moving range control chart for Barometer readings

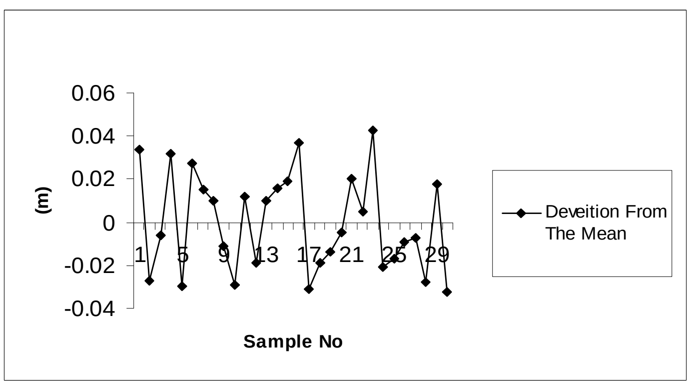


Figure 2d: 3σ Deviations from the mean control chart for Barometer readings

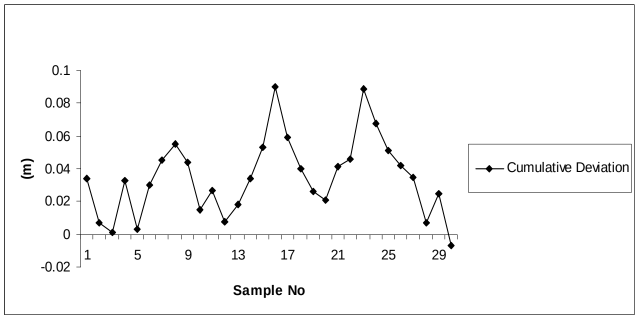


Figure 2e: 3σ Cumulative deviations control chart for the Barometer readings

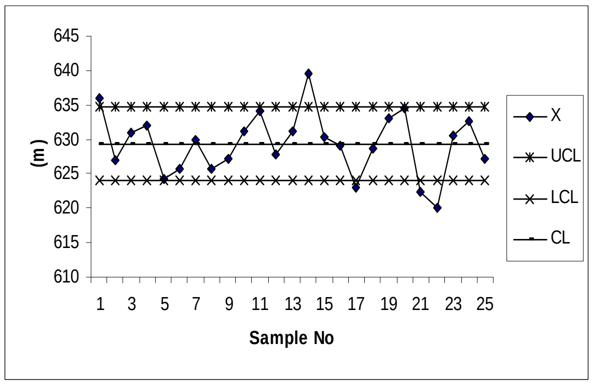


Figure 3: 3σ Control limits for a sample of 25 GPS heights (using $\bar{x}$)

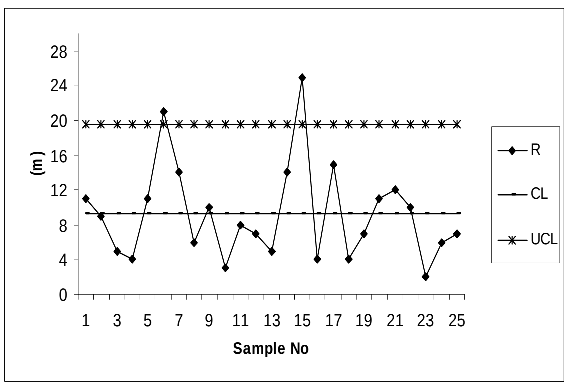


Figure 4: 3σ Control limits for a sample of 25 GPS heights (using $\bar{R}$)

DISCUSSION:

Referring to Figures 2 a , b, c, d and e it can be seen that the $\bar{X}$ control chart, the individual control chart and the deviations control chart, produce the same results. It is clearly that all measurement values fluctuate about the mean or around zero.

The use of the moving range gives results that were inconsistent. More observations fall on the lower side of the moving range than on the upper side. This means that the true mean of the measurements was underestimated. Also the shape of the chart using the moving range was identical to that when the cumulative deviations method was used. All values were positive except the measurement value number thirty in the cumulative deviations control chart. See Figure 2 e.

It should be noted that even when there was no gross errors in the measurements, there was out of control readings every 33 measurements for the 3σ control limits. In other words, the Average Run Length (ARL) was equal to 34 for the 3σ.

As it can be seen from the second test, using the mean ($\bar{X}$), the results obtained were more sensitive in detecting gross errors than using the range (R); five measurements were detected as gross errors (20%) compared to two measurements (8%) respectively. This is, because $\bar{X}$ uses all available information while R uses only the two extreme values i.e. the smallest and the largest values.

Conclusions

From the results obtained the following conclusions could be drawn:

- Using the mean was more sensitive in detecting multiple gross errors than the range .
- Both deviations from the mean method and the cumulative sum method give identical results in multiple gross error detection.
- The mean control chart, the individual control chart and the deviations control chart produce the same results when used in detecting gross errors.
- The use of the moving range in multiple gross error detection gives inconsistent results. Most of observations fall on one side of the base line than on the other.
- Moving range method and cumulative deviation method graphs are identical in shape but with different deviations from the true values.

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