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Geometrically Nonlinear analysis using Plane Stress / strain Elements based on Alternative Strain Measures

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Abstract: The description of deformation and the measure of strain are essential parts of nonlinear continuum mechanics. In this paper, a new formulation for geometric nonlinear plane stress/strain based on Logarithmic strains (GNLGS) is presented. This is coupled with a formulation based on the well known Green's strains and coupled with modifying a formulation based on geometric strains (conventional strains). A geometric nonlinear total lagrangian formulation applied on two-dimensional elasticity using 4-node plane finite elements is used. The formulations were implemented into the finite element program (NUSAP), which is developed for the analysis of plane stress/strain problems subjected to static loading. The solution of nonlinear equations was obtained by the Newton-Raphson method. The program was applied to obtain displacements for the different strain measures. The accuracy of the results was demonstrated by using two numerical examples and the results are in good agreement with other available published solutions and those obtained using commercial finite element solvers such as ANSYS. It could be concluded that the geometrically nonlinear formulations converge to the correct solution with coarse meshes and are computationally efficient. In addition, the resulting displacements clearly showed the effect of the nonlinearity in the deflected shape. It is also observed that all results were approximately identical when applying a small value of load and when a large value of a load was applied there was a difference between the results of the three strain measures.

Keywords: *Geometric Nonlinear, Logarithmic, Geometric, strain measures*

مستخلص:

وصف التشوه وقياس الانفعال من المكونات الاساسية للميكانيكا اللاخطية للاجسام المتصلة. تعرض هذه الورقة تقنين جديد للاَّخطية الهندسية للاجهاد/الانفعال المستوى بُنى على الانفعالات اللوغرثمية . تُزاوج الورقة بين هذا التقنين وبين التقنين المعلوم لانفعالات قرين(Green) وبين تقنين مستحدث تم الحصول عليه بتعديل تقنين مبنى على الانفعالات الهندسية (المتعارف عليها) استخدام تقنين لاقرانج (Lagrange) الكلِّى للاَّخطية الهندسية تطبيقاً على المرونة ثنائية الابعاد لعنصر محدد مستوى ذى أربعة عقـد. تمت حوسبة التقنينات الثلاثة بوضعها فـى برنامـج العنصر المحدد (N U S A P) الذى طُوِّر لتحليل مسائل الاجهاد/ الانفعال المستوى المعرضة لأحمال ساكنة. تم الحصول على حل المعادلات اللاَّخطية باستخدام طريقة نيوتن – رابسون. طُبق البرنامج للحصول على الازاحات الناتجة عن قياسات الانفعال المختلفة. وبُينت دقة النتائج بناءً على مثالين عدديين ، وأظهرت النتائج توافقاً جيداً مع الحلول المنشورة ومع التى تم الحصول عليها باستخدام برامج العنصر المحدد التجارية مثل (A N S Y S). تخلص الورقة الى أن تقنينات اللاَّخطية الهندسية تتقارب الى الحل الصحيح باستخدام عناصر محددة قليلة وهى ذات كفاءة محوسبة مناسبة. وبالاضافة لهذا تُظهر الازاحات المتحصل عليها بوضوح أثر اللاَّخطية على الشكل المنحنى. ويلاحظ ، ايضاً ، أن كل النتائج متطابقة تقريباً عند تطبيق الاحمال الصغيرة ، وهنالك فرق واضح بين نتائج قياسات الانفعالات الثلاثة عند تطبيق الأحمال العاليه.

Introduction

Many engineering problems of interest are inherently nonlinear. In solid mechanics, generally, two sources of nonlinearity exist in the analysis of solid continua, namely, material nonlinearity and geometric nonlinearity. The former occurs when the stress-strain behavior given by the constitutive relation is nonlinear, whereas the latter is important when changes in geometry, whether large or small, have a significant effect on the load deformation behavior. The solution of geometrically non-linear problems is based on either the total lagrangian formulation where all variables are referred to initial configuration or the updated lagrangian formulation where all the variables are referred to the configuration at the beginning of the load step considered.

For genuine geometric non-linearity,' incremental' procedures were originally adopted by Argyris [(1)] using the 'geometric stiffness matrix' in conjunction with an up dating of coordinates and, possibly, an initial displacement matrix. Newton-Raphson iteration was used by Mallet and Marcal [(2)]. Zienkiewicz [(3)] and Oden [(4)] also recommended a modified Newton-Raphson procedure. A special form using the initial, elastic stiffness matrix was referred to as the 'initial stress' method [(5)]. Brebbia and Connor [(6)] introduced the concept of combining incremental and iterative methods. The plane stress problem is well suited for introducing continuum finite elements, from both the historical and technical standpoint. The first continuum structural finite elements were developed at Boeing in the early 1950s to model delta-wing skin panels [(7)]. A plane stress model was naturally chosen for the panels.

There are many applications of plane stress/strain in different fields of analysis, Pida, Yang and Soedel (1989) [(8)] used large strain 8-node plane stress isoparametric finite element for prediction of rubber fraction. The formulation is based on total Lagrangian description and incremental formulation.

Seki, and Atluri (1994) [(9)] used 2D plane stress/strain element in application of analysis of strain localization in strain-softening hyper elastic material, using assumed stress hybrid elements.

Fernando. Fores (2006) [(10)] used an assumed strain approach for a linear triangular element based on a total lagrangian formulation and its geometry is defined by three nodes with only translational degrees of freedom.

The nonlinear strain and stress measures in definition of stress-strain relation are one of the key concepts of several nonlinearities. There are alternative strain measures used to derive finite element equations, such as Green strain, which is associated with Piola-Kirchoff stress, Geometric strain, which is associated with Engineering stress, and Logarithmic strain, which is associated with true (Cauchy) stress. The Green strain is the most common definition applied to materials used in mechanical and structural engineering problems, which are subjected to small deformations. On the other hand, for some materials, subjected to large deformations, the engineering definition of strain is not applicable [(11)]. Thus other more complex definitions of strain are required, such as logarithmic strain and Almansi strain.

Turner, et. al, [(12)] reported the finite element procedure to geometrically nonlinear structure. Zienkiwicz [(13)]. introduced the geometric nonlinear analysis using the total lagragian formulations, with incremental procedure combined with Newton-Raphson (NR) iterative techniques. Mohamed (1983) [(14)] used both Green strain and geometric strain measures to solve the problem. He, also, proposed a total lagrangian modified incremental equations for a two-dimensional state of stress based on the

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geometric strains. This has been adopted as the base for the formulation based on the geometric strain In this Paper, geometrically nonlinear formulations based on two-dimensional 4-node plane stress and plane strain isoparametric finite elements are developed. The nonlinear formulations are based on the total Lagrangian formulation and using the Green's strains, Geometric strains and Logarithmic strains. The adopted formulations were implemented into a general-purpose nonlinear finite element program NUSAP and the displacements values obtained from the different strain measures were compared. Numerical examples were used to show the performance of the proposed formulations.

Geometrically Non-linear Formulations:

As stated above the geometrically non-linear finite element formulation based on Green's strains is well established. In the following sections the other two formulations are outlined.

Geometrically Non-linear Finite Element Formulation for Plane Stress/strain based on Geometric strain:

In this section, the formulation based on Geometric (conventional) strains is outlined.

In two dimensions the geometric strains ε_x' and ε_y' are defined by the change in length per unit initial length of line elements originally oriented parallel to the $\boldsymbol{x}$ and $\boldsymbol{y}$ axes respectively. The shear strain γ_{xy}' is the change in right angle. This shear strain is assumed to be small so that $\sin\gamma_{xy}'$ can be assumed to be equal to γ_{xy}'

The geometric strains as defined above by the change in length per unit initial length, are given by:

$$\varepsilon_x' = (g_x g_x)^{\frac{1}{2}} - 1 = (1 + 2\varepsilon_x)^{\frac{1}{2}} - 1 \quad \text{(1a)}$$

$$\varepsilon_y' = (g_y g_y)^{\frac{1}{2}} - 1 = (1 + 2\varepsilon_y)^{\frac{1}{2}} - 1$$

Assuming that the shear angle is small, we obtain the shear strain as

$$\gamma'_{xy} = \frac{\gamma_{xy}}{(1+2\varepsilon x)^{\frac{1}{2}}(1+2\varepsilon y)^{\frac{1}{2}}} \quad \text{(1b)}$$

where Ex, Ey and Yxy are the Green strains.

The variation in the geometric strains is given by equations 2 and 3.

$$\delta\varepsilon' = \begin{Bmatrix} \delta\varepsilon_x' \\ \delta\varepsilon_y' \\ \delta\gamma_{xy}' \end{Bmatrix} = \begin{bmatrix} \frac{1}{(1+2\varepsilon_x)^{\frac{1}{2}}} & 0 & 0 \\ 0 & \frac{1}{(1+2\varepsilon_y)^{\frac{1}{2}}} & 0 \\ \frac{-\gamma_{xy}}{(1+2\varepsilon_x)^{\frac{3}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}} & \frac{-\gamma_{xy}}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{3}{2}}} & \frac{1}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}} \end{bmatrix} \begin{Bmatrix} \delta\varepsilon_x \\ \delta\varepsilon_y \\ \delta\gamma_{xy} \end{Bmatrix} \quad (2)$$

or $\delta\varepsilon' = H\delta\varepsilon$ (3)

From equations (1a) & (1b), the variations in the geometric strains are given by:

$$\delta\varepsilon' = HB\delta a = B^*\delta a \quad B^{*T} = B^T H^T \quad (4)$$

In which B is the strain matrix, and H relates variation in geometric strain to variation in Green's strain.

The equilibrium equations in terms of engineering stresses are:

$$\psi = \int_v B^T H\sigma dv - f = 0 \quad (5)$$

By taking the variation of equation (5) we have:

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$$\delta\psi = \int_v B^T H^T \delta\sigma dv + \int_v \delta B^T H^T \sigma dv + \int_v B^T \delta H^T \sigma dv \qquad (6)$$

using equation (4) we have:

$$\int_v B^T H^T \delta\sigma dv = \left(\int_v B^T H^T DHB dv \right) \delta a$$
$$= \left(\int_v B^{*T} D B^* dv \right) \delta a = \left(K_o^* + K_L^* \right) \delta a \qquad (7)$$

Where

$$K_o^* = \int_v B_o^T H^T DHB_o dv \qquad (8)$$

and

$$K_L^* = \int_v B_o^T H^T DHB_L dv + \int_v B_L^T H^T DHB_o dv + \int_v B_L^T H^T DHB_L dv \qquad (9)$$

$$\int \delta B^T H^T \sigma dv = \int_v \delta B_L^T H^T \sigma dv = \int_v G^T \delta A^T H^T \sigma dv = K_\sigma^* \delta a \qquad (10)$$

where K_σ^* is the symmetric matrix dependent on the engineering stress, and can be written as:

$$\delta A^T H^T \sigma = \begin{bmatrix} \sigma_x^* [I] & \tau_{xy}^* [I] \\ \tau_{xy}^* [I] & \sigma_y^* [I] \end{bmatrix} \delta\theta \qquad (11)$$
$$= P^* G \delta a$$

where I is 2×2 unit matrix, and

$$\theta = \left\{ \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \right\}^T$$

is the vector containing displacement derivatives w.r.t cartesian coordinates, and is related to the nodal displacements by the form:

$$\theta = Ga$$

where G is a matrix containing shape function derivatives.

and σ^* is the engineering stress vector given by:

$$\sigma^* = \begin{Bmatrix} \sigma_x^* \\ \sigma_y^* \\ \tau_{xy}^* \end{Bmatrix} = H^T \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \qquad (12)$$

Substituting equation (11) in (10) then;

$$K_\sigma^* = \int_v G^T P^* G dv \qquad (13)$$

Using equations (12) we have

$$\delta H^T \sigma = \begin{bmatrix} \frac{-\sigma_x}{(1+2\varepsilon_x)^{\frac{3}{2}}} + \frac{3\gamma_{xy}\tau_{xy}}{(1+2\varepsilon_x)^{\frac{5}{3}}(1+2\varepsilon_y)^{\frac{1}{2}}} & \frac{\gamma_{xy}\tau_{xy}}{(1+2\varepsilon_x)^{\frac{3}{2}}(1+2\varepsilon_y)^{\frac{3}{2}}} & \frac{-\tau_{xy}}{(1+2\varepsilon_x)^{\frac{3}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}} \\ & \frac{-\sigma_y}{(1+2\varepsilon_y)^{\frac{3}{2}}} + \frac{3\gamma_{xy}\tau_{xy}}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+\varepsilon_y)^{\frac{5}{2}}} & \frac{-\tau_{xy}}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{3}{2}}} \\ \textit{symmetric} & & 0 \end{bmatrix} \begin{Bmatrix} \delta\varepsilon_x \\ \delta\varepsilon_y \\ \delta\gamma_{xy} \end{Bmatrix} \qquad (14a)$$

In simple form, equation (14a) can be re-written as;

$$\delta H^T \sigma = P \delta\varepsilon = PB\delta a$$

where P is given by;

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$$\begin{bmatrix} \frac{-\sigma_x}{(1+2\varepsilon_x)^{\frac{3}{2}}} + \frac{3\gamma_{xy}\tau_{xy}}{(1+2\varepsilon_x)^{\frac{5}{3}}(1+2\varepsilon_y)^{\frac{1}{2}}} & \frac{\gamma_{xy}\tau_{xy}}{(1+2\varepsilon_x)^{\frac{3}{2}}(1+2\varepsilon_y)^{\frac{3}{2}}} & \frac{-\tau_{xy}}{(1+2\varepsilon_x)^{\frac{3}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}} \\ & \frac{-\sigma_y}{(1+2\varepsilon_y)^{\frac{3}{2}}} + \frac{3\gamma_{xy}\tau_{xy}}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+\varepsilon_y)^{\frac{5}{2}}} & \frac{-\tau_{xy}}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{3}{2}}} \\ symmetric & & 0 \end{bmatrix} \quad (14b)$$

Then

$$\int_v B^T \delta H^T \sigma dv = \left(\int_v B^T PBdv\right)\delta a = K_\sigma^{**} \qquad (15)$$

where K_σ^{**} is the additional geometric stiffness matrix, which is clearly asymmetric matrix.
Therefore from equations (6), (10) & (15), equation (5) can be written as:

$$\delta\psi = \left(K_o^* + K_L^* + K_\sigma^* + K_\sigma^{**}\right)\delta a = K_{TG}^*\delta a \qquad (16)$$

in which
K_{TG}^* is the tangent stiffness matrix due to Geometric Strains

Geometrically Non-linear Finite Element Formulation for Plane Stress/strain based on Logarithmic strain:

In this section, the formulation based on Logarithmic strains is outlined.

From the principle of virtual work, the equilibrium equations can be written in terms of the true Cauchy stresses as:

$$\psi = \int \overline{B}^T \sigma dv - f = 0 \qquad (17)$$

where $\overline{B}^T = B^{*T} S^T = B^T H^T S^T$ and $\sigma = D\overline{B}a = DSB^*a = DSHBa$ where S relates variation in logarithmic strain to variation in Geometric strain
Then

$$\psi = \int_v B^T H^T S^T \sigma dv - f = 0 \qquad (18)$$

On taking the variation of Equation (18) the results are:

$$\delta\psi = \int_v B^T H^T S^T \delta\sigma d + \int_v \delta B^T H^T S^T \sigma dv + \int_v B^T \delta H^T S^T \sigma dv + \int_v B^T H^T \delta S^T \sigma dv \qquad (19)$$

in which:

$$\int_v B^T H^T S^T \delta\sigma dv = \left(\int_v B^T H^T S^T DSHBdv\right)\delta a = (K_o + K_L)\delta a \qquad (20)$$

Where

$$K_o = \int_v B_o^T H^T S^T DSHB_o dv \qquad (21)$$

$$K_L = \int_v B_o^T H^T S^T DSHB_L dv + \int_v B_L^T H^T S^T DSHB_o dv + \int_v B_L^T H^T S^T DSHB_L dv \qquad (22)$$

Since: $B^T = B_o^T + B_L^T$, then:

$\delta B^T = \delta B_L^T = G^T \delta A^T$, therefore

$$\int_v \delta B^T H^T S^T \sigma dv = \int_v G^T \delta A^T H^T S^T \sigma dv$$

and:

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$A^T H^T S^T \sigma' =$

$$\begin{bmatrix} \frac{\partial u}{\partial x}\left[\frac{\sigma_x'}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+\varepsilon_x')} - \frac{3\gamma_{xy}\tau_{xy}'}{(1+\varepsilon_x')(1+2\varepsilon_x)^{\frac{3}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}}\right] + \frac{\partial u}{\partial y}\left[\frac{\tau_{xy}'}{(1+\gamma_{xy}')(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}}\right] \\ \frac{\partial v}{\partial x}\left[\frac{\sigma_x'}{(1+2\varepsilon_x)^{\frac{1}{2}}(1+\varepsilon_x')} - \frac{3\gamma_{xy}\tau_{xy}'}{(1+\varepsilon_x')(1+2\varepsilon_x)^{\frac{3}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}}\right] + \frac{\partial v}{\partial y}\left[\frac{\tau_{xy}'}{(1+\gamma_{xy}')(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}}\right] \\ \frac{\partial u}{\partial y}\left[\frac{\sigma_y'}{(1+2\varepsilon_y)^{\frac{1}{2}}(1+\varepsilon_y')} - \frac{3\gamma_{xy}\tau_{xy}'}{(1+\varepsilon_y')(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{3}{2}}}\right] + \frac{\partial u}{\partial x}\left[\frac{\tau_{xy}'}{(1+\gamma_{xy}')(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}}\right] \\ \frac{\partial v}{\partial y}\left[\frac{\sigma_y'}{(1+2\varepsilon_y)^{\frac{1}{2}}(1+\varepsilon_y')} - \frac{3\gamma_{xy}\tau_{xy}'}{(1+\varepsilon_y')(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{3}{2}}}\right] + \frac{\partial v}{\partial x}\left[\frac{\tau_{xy}'}{(1+\gamma_{xy}')(1+2\varepsilon_x)^{\frac{1}{2}}(1+2\varepsilon_y)^{\frac{1}{2}}}\right] \end{bmatrix}$$

$$A^T H^T S^{T/} \sigma' = P^/ \theta \qquad (23)$$

Therefore, Equation (23) can be re-written as:

$$A^T H^T S^T \sigma = P^/ \theta = P^/ Ga$$

where $P^/$ is the initial stress matrix (symmetric matrix) Hence taking variation results in: $\delta A^T H^T S^T \sigma = P^/ G \delta a$

Therefore:

$$\int_v \delta B^T H^T S^T \sigma dv$$

$$= \int_v G^T \delta A^T H^T S^T \sigma dv$$

$$= \left(\int_v G^T P^/ G dv \right) \delta a = K_\sigma \delta a \qquad (24)$$

where $K_\sigma = \int_v G^T P^/ G dv$ is the initial stress stiffness matrix. Also:

$$\delta H^T \sigma' = L \begin{bmatrix} \delta\varepsilon_x \\ \delta\varepsilon_y \\ \delta\gamma_{xy} \end{bmatrix}$$

where $\quad \sigma' = S^T \sigma^* \qquad (25)$

In simple form, Equation (25) can be written as:

$$\delta H^T \sigma' = T' \delta\varepsilon = T' B \delta a \qquad (26)$$

where T' is second initial stress matrix.(symmetric matrix)

$$\int_v B^T \delta H^T \sigma' \sigma dv = \left(\int_v B^T T' B dv \right) \delta a = K_\sigma^/ \delta a$$

(27)

where

$K_\sigma^/ = \int_v B^T T' B dv$ is additional initial stress stiffness matrix. Similarly $H^T \delta S \sigma^* = M^/ B \delta a$

where $M^/$ is the a third initial stress matrix and is given in terms of the initial stresses and strains. Therefore;

$$\int_v B^T H^T \delta S^T \sigma^* dv = \left(\int_v B^T M^/ B dv \right) \delta a$$

$$= K_\sigma^{//} \delta a \qquad (28)$$

where $K_\sigma^{//}$ is the second additional initial stress stiffness matrix. From Equations, (20), (21), (22), (27) and (28) the tangent stiffness matrix due to logarithmic strains can be defined by:

$$\delta\psi = \left(K_o + K_L + K_\sigma + K_\sigma^{/} + K_\sigma^{//}\right)\delta a$$
$$= K_{TL}^{*}\delta a \qquad (29)$$

where

$K_{TL}^{*} = K_o + K_L + K_\sigma + K_\sigma^{/} + K_\sigma^{//}$

is the tangent stiffness matrix due to logarithmic strains.

Numerical Results and Discussion:

The finite element formulation described in the above section was implemented in the FORTRAN based NUSAP. The two numerical examples of large deformation problems were examined to demonstrate the degree of accuracy that can be obtained by using the geometrically non-linear formulations based on 4-node isoperimetric plane stress/strain element by using Green's strains, geometric strains and the new formulation, namely logarithmic strains. The results of displacements of the different strain measures are compared with those obtained from published finite element solutions and commercial finite element solvers such as ANSYS.

Cantilever under pure bending at free end: A cantilever subjected to pure moment was considered.

The cantilever was of dimensions L=3000 mm, D=300 mm and thickness t=60 mm as shown in Figure 1.

The numerical values of material property parameters are Young's modulus, E=210 GPa, and Poisson’s ratio, υ=0.3. The structure is modeled with a mesh of 40-isoparametric elements, and the integration order is 2x2. The mesh is of equal size elements of 150×150mm. The deformed shape when P=18000N, 21000N and 30000N is shown in Figure 3 and Table 2. The locus or path followed by the point A with load increments as computed by the present formulations is compared with that generated by finite element solver (ANSYS). Results are presented in Table 1 and Figure 2.

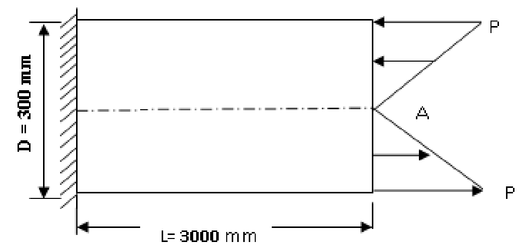

Figure 1**:** Cantilever under pure bending

It is observed that there was a slight difference between geometric and logarithmic values, whereas the logarithmic results showed reasonable agreement with finite element solver (ANSYS). It was also observed that under a lower load or initial values of load all results were identical. Figure 3 clearly shows the effect of the nonlinearity in the deflected shape. It is also observed that all results are approximately identical when applying a load of (18000N) whereas when the maximum load of (30000N) was applied there is a difference between the results of the three strain measures.

Clamped beam under point force: A beam with two-fixed end was considered. The beam length $\boldsymbol{L}$=200 mm, height $\boldsymbol{b}$=10 mm and thickness 1 mm as shown in Figure 5. The numerical values for material property parameters are Young's modulus, $\boldsymbol{E}$ =210GPa, Poisson's ratio, $\boldsymbol{\upsilon}$=0.3. The beam is modeled with mesh of 20-elementes, the integration order used was 2×2.

The computed results of the displacement at the centre point A obtained from the present formulations and finite element solver (ANSYS) are listed in Table 3; the responses of the normal deflection at point A, V_A, to the applied force F are presented in Figure 6.

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Table 1: Deflection at point A

LOAD	GREEN u	GEOM u	LOG u	GREEN v	GEOM v	LOG v	ANSYS v
0	0	0	0	0	0	0	0
3000	3.75703	3.73788	3.72309	99.9727	99.9841	99.9032	141.18
6000	15.7544	15.7539	15.7273	199.242	199.931	199.996	270.595
9000	35.7768	36.0832	36.1821	297.087	300.037	301.055	405.595
12000	63.4792	64.792	65.4045	392.848	400.459	403.837	517.66
15000	98.4102	101.964	103.85	485.956	501.306	509.061	623.54
18000	140.04	147.685	152.101	575.941	602.63	617.401	729.43
21000	187.788	202.027	210.849	662.442	704.421	729.472	823.55
24000	241.054	265.033	280.88	745.216	806.616	845.831	917.67
27000	299.237	336.712	363.054	824.102	909.104	966.961	974.73
30000	361.758	417.024	458.287	899.04	1011.74	1093.28	1047.08

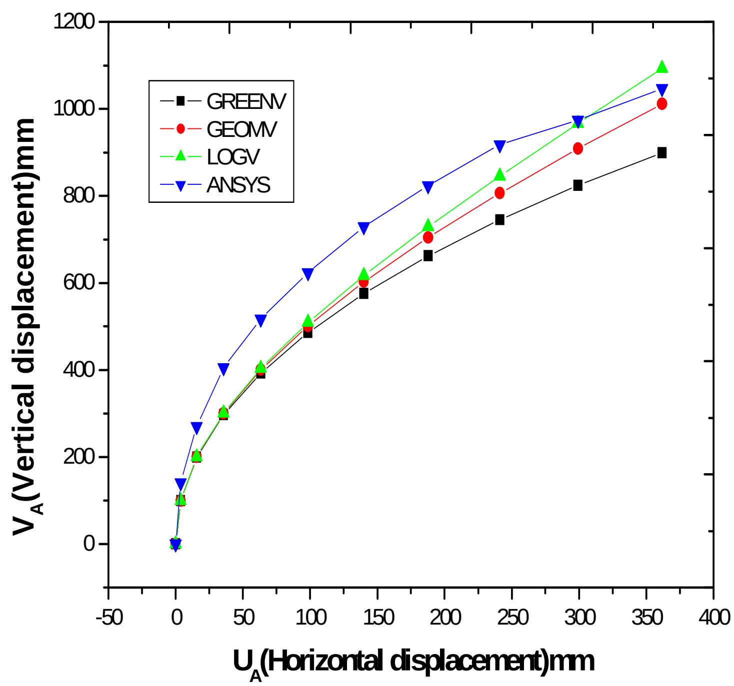

Figure 2: The path followed by the point A with load step increments for cantilever beam under pure bending

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Table 2: Deformed shape along center line

	LOAD 18000N			LOAD 21000N			LOAD 30000N		
NOED	GREEN	GEOM	LOG	GREEN	GEOM	LOG	GREEN	GEOM	LOG
23	0.0	0.0	0.0	0.0	0.0	0.00	0.00	0.00	0.00
24	-13.8	-13.3	-12.7	-16.1	-15.5	-14.7	-23.1	-21.9	-20.4
25	-37.9	-37.0	-35.9	-44.3	-43.1	-41.6	-63.2	-61.1	-58.4
26	-72.1	-71.0	-69.6	-84.0	-82.7	-80.8	-119.0	-118.0	-114.0
27	-116.0	-115.0	-114.0	-135.0	-135.0	-133.0	-191.0	-192.0	-189.0
28	-170.0	-170.0	-169.0	-198.0	-198.0	-197.0	-278.0	-283.0	-284.0
29	-233.0	-236.0	-235.0	-271.0	-275.0	-275.0	-378.0	-393.0	-399.0
30	-306.0	-311.0	-313.0	-354.0	-363.0	-367.0	-492.0	-520.0	-536.0
31	-387.0	-398.0	-402.0	-448.0	-464.0	-473.0	-617.0	-666.0	-696.0
32	-478.0	-495.0	-504.0	-551.0	-578.0	-593.0	-753.0	-830.0	-882.0
33	-576.0	-603.0	-617.0	-662.0	-704.0	-729.0	-899.0	-1010.0	-1090.

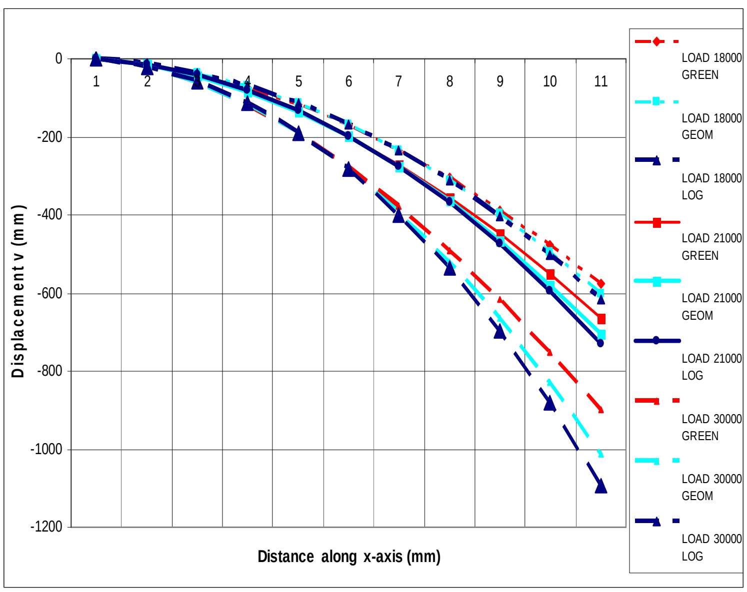

Figure 3: Deformed shape along center line

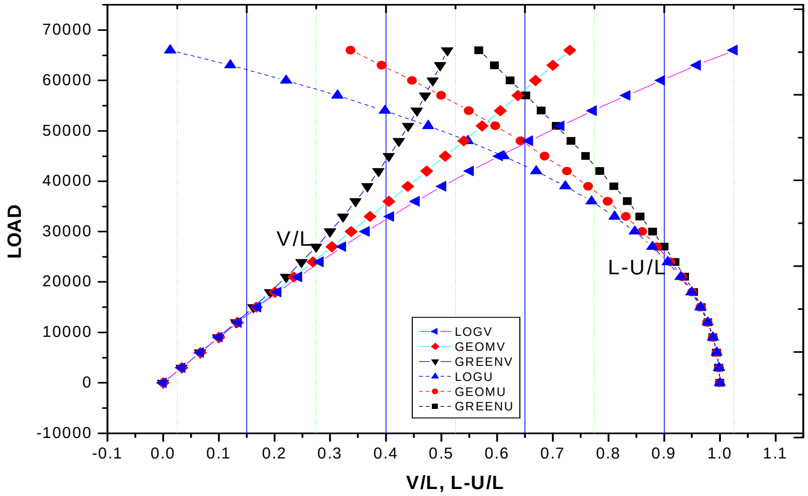

Figure 4: Displacements V/L, L-U/L at Point A

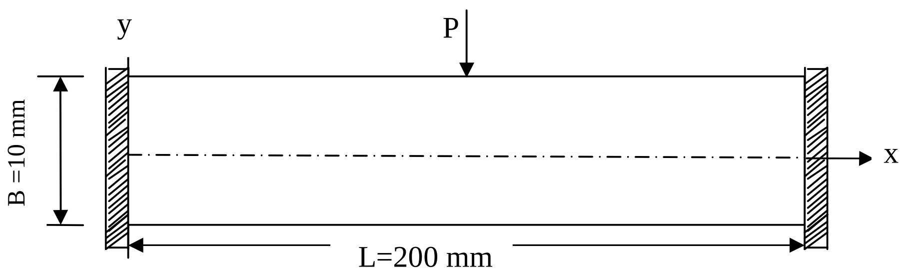

Figure 5**:** Clamped beam under point force

Table 3: Vertical displacement at point A

LOAD (N)	ANSYS VA	LUSAS VA	GEOM VA	LOG VA
25	0.062	0.062	0.062	0.062
50	0.120	0.126	0.126	0.125
100	0.240	0.253	0.253	0.253
175	0.419	0.449	0.449	0.449
287.5	0.687	0.752	0.751	0.751
456.25	1.104	1.223	1.122	1.122

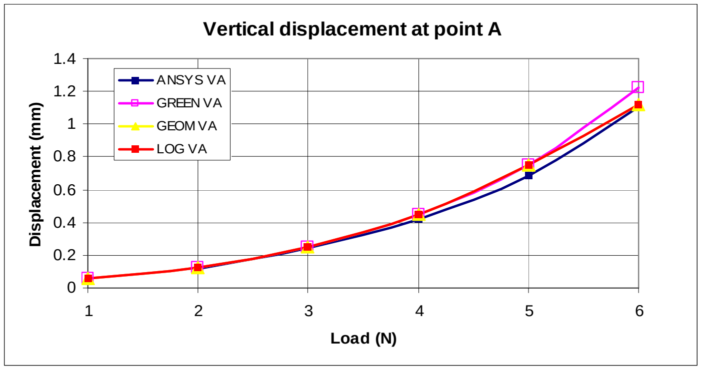

Figure 6: Vertical displacement at point A

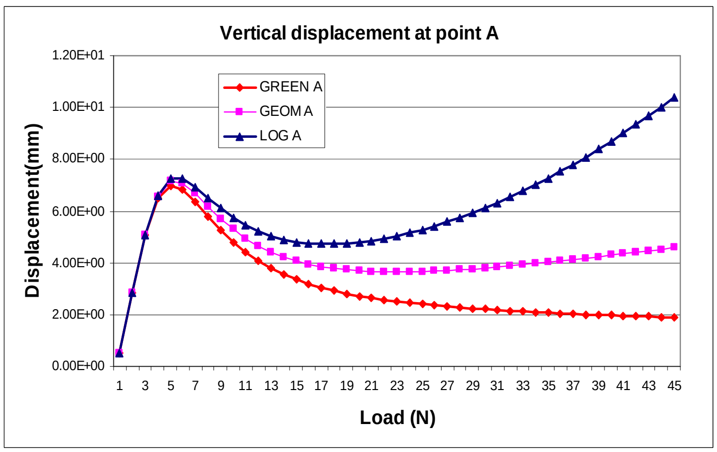

Figure 7: Vertical displacement at mid span (point A)

Six load increments were applied to the clamped beam and resulted in a close agreement of the displacements for the formulations especially the ANSYS and logarithmic strain solutions as shown in Figure 6.

The application on this example of 45 load increments resulted in the displacements shown in Figure 7. The results obtained showed that at the early stages of load, the three formulations curves are coincident. The application of large loads resulted in a variation between the results, and the log-curve tendsed in an infinite result as expected.

Conclusions:

Based on the results of the numerical examples, it can be concluded that:

- The resulting displacements showed good agreement with available solutions.
- For a clamped beam subjected to central load increments the resulting displacements are identical in the early stages of load for the three formulations. Variations occur as load increases and the GNLGS results tend to infinite values.
- The geometric strain solutions are suitable to be used with the elastic constants based on engineering stress.
- The logarithmic strain formulation can be used when the true stresses are required.

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