

A CORRECTION FACTOR FOR PRESSURE DISTRIBUTION ANALYSIS OF AN AEROFOIL IN A SUBSONIC FLOW

By

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ABSTRACT

This study investigates the geometrical solution for a given pressure (i.e. velocity) distribution of subsonic flow around an aerofoil section using an analytical method. The results show that the analytical method is not precise on the leading edge, when it is used to solve the mean camber line of updated profile RAE (Royal Aircraft Establishment). Therefore, a correction parameter with changeable values ranging between 0.32 and 0.42 was created, and it gives appropriate results.

المخلص:

مسألة تحديد أبعاد الشكل الهندسي لقطع الجناح بمعرفة طبيعة توزيع الضغط (أو السرعة) تمت معالجتها باستخدام الطريقة التحليلية لجريان هوائي ثنائي البعد أقل من سرعة الصوت حول الجناح. أظهرت النتائج أن الطريقة التحليلية لا تعطي نتائج صحيحة لقذمة الجناح عند إجراؤها على نوع RAE (Royal Aircraft Establishment) لذا في هذا البحث تم استحداث معامل تصحيح له قيم متغيرة تتراوح بين 0.32- 0.42 يمكن من إيجاد نتائج دقيقة عند الإجراء التحليلي على هذا النوع من المقاطع.

INTRODUCTION

In wing section design, highly expensive working power is invested in order to find configurations that are stable and efficient in ample working ranges. The main goal is to pick out wing aerofoil profiles which achieved the desired working range stability and efficiency. This will automatically result in aircraft performance. At the same time, constraints arising from aerodynamic, aeroelasticity, mechanical and manufacturing considerations have to be satisfied.

Since to optimize aerofoil design manually is time consuming, we need to implement inverse design method (Berak, 1986)^[1], which can calculate the aerofoil shape to achieve desirable flow fields within short period of time.

Inverse Design Method:

The principle of inverse method considered by Carlson and Leland (1987)^[2] consists of the determination of a single profile and corresponding design angle α_D for the given distribution of the pressure coefficients $C_{PU}(x)$ and $C_{PL}(x)$ along the profile chord.

Assumptions of the Inverse Solution: There are two assumptions considered in Foundation of Aerodynamically Design (Kenthe and Chow, 1988)^[3] to apply the inverse design method to aerofoil design: The solution is linearizable from the physical point of view and the medium bypassing the profile is incompressible and non-viscous.

Physical Model: The solved profile shape is obtained by linear transformation of two basic geometric characteristics of the profile mentioned by Weber (1957)^[4] and illustrated in (Fig.1), namely: Mean camber line of profile $y_A(x)$ and thickness function $y_S(x)$.

The solution of the mean camber line of profile is actually a solution of bypassing infinitesimally thin profile for a given pressure distribution of differences of coefficients.

$$\Delta c_p(x) = c_{pL}(x) - c_{pU}(x) \dots\dots\dots (1)$$

where: C_p : pressure coefficient, L : lower surface of the profile, U : upper surface of the profile.

Theoretically, the infinitesimally thin profile is substituted by a continuous distribution of elemental potential vortices; density of distribution k of these elementary vortices is:

$$k(x) = \frac{1}{2} \Delta C_p(x) \dots\dots\dots (2)$$

Solution of the thickness function of the profile presents a solution of symmetrical bypassing the symmetrical profile of finite thickness for a given distribution of the mean pressure coefficient

$$C_{pmean}(x) = \frac{1}{2} [C_{pL}(x) + C_{pU}(x)] \dots\dots\dots (3)$$

Theoretically, the symmetrical profile of the finite thickness is substituted by a continuous distribution of source and sinks; the distribution density q of these sources and pancakes is:

$$q(x) = 2V_{ys}(x), \dots\dots\dots (4)$$

while

$$V_{xs}(x) = -\frac{1}{2}C_{PMEAN} \dots\dots\dots (5)$$

Mathematical Description of the Physical Model

Basic equations for solution of the profile mean camber line in Fundamentals of Aerodynamics (Anderson, 2001)^[5] are:

$$V_{IND.A}(x) = -\frac{1}{4\pi} \int_0^1 \frac{\Delta C_P(\xi)}{x - \xi} d\xi \dots\dots\dots (6)$$

$$\alpha_D = -\int_0^1 V_{IND.A}(x).dx \dots\dots\dots (7)$$

$$V_{YA}(x) = Y_A^1(x) = \alpha_D + V_{IND.A}(x) \dots\dots\dots (8)$$

$$y_A(x) = \int_0^x y_A^1(x).dx \dots\dots\dots (9)$$

where: $V_{IND.A}$: is induce velocity at the camber line, V : is the velocity, α_D : is design angle of attack, x : is aerofoil chord and y_A : is the coordinate of the mean camber line in Y-direction for unsymmetrical shape.

Basic equations for solution of the profile thickness function in Fundamentals of Aerodynamics (Anderson, 2001)^[5] (Fig.1) are

$$\int_0^1 V_{IND.S}(\xi) \frac{d\xi}{x - \xi} = -\frac{\pi}{2} C_{PMEAN}(x) \dots\dots\dots (10)$$

$$\int_0^1 V_{IND.S}(x).dx = \int_0^1 V_{ys}(x).dx = 0 \dots\dots\dots (11)$$

$$V_{ys}(x) = y_s^1(x) = V_{IND.S}(x) \dots\dots\dots (12)$$

$$y_s(x) = \int_0^1 y_s^1(x).dx \dots\dots\dots (13)$$

where: $V_{IND.S}$ is The induce velocity at the thickness line in Y-direction and y_s is the coordinate of the thickness line in Y-direction for unsymmetrical shape.

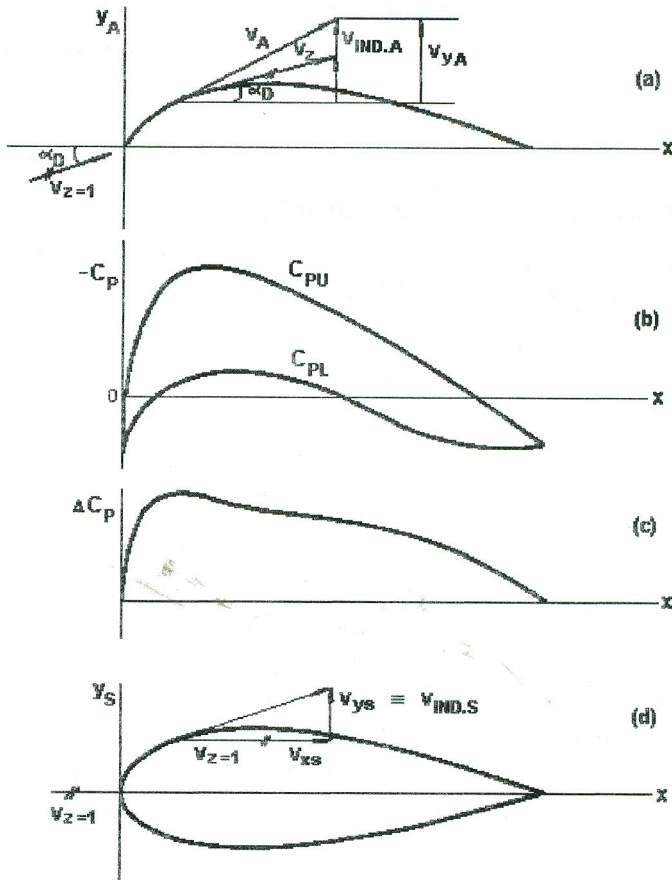


Fig. (1): Standard Aerofoil Model Configuration

(a) Mean Camber Line of Profile (b) Pressure Coefficients Distribution

(c) Differences of Coefficients Distribution (d) Thickness Function of Profile

Analytical Solution of the Problem:

The application of trigonometric series in Rational Design of an Airfoil for a High-Performance Jet Trainer (Powers and Sattler, 1981)^[6] requires an

introduction of new variable φ : (Fig.2) represents the following relations hold between variables x and φ :

$$x = \frac{1}{2}(1 + \cos \varphi) \dots\dots\dots (14)$$

$$\varphi = \arccos (2x - 1) \dots\dots\dots (15)$$

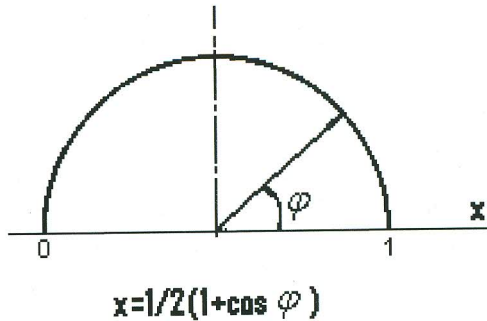


Fig. (2): Relation between the Profile Chord x and the Variable φ

Solution of the Mean Camber Line of the Profile

The input set of solution forms is a distribution of the pressure coefficient difference $\Delta C_p(x)$ indicated by equation (1).

This distribution $\Delta C_p(x) \equiv \Delta C_p(\varphi)$ is substituted by a trigonometric series in $(n+1)$ suitably selected discrete points:

$$A_0 \tan \frac{\varphi}{2} + \sum_{i=1}^n A_i \sin(i\varphi) = \frac{1}{4} \Delta C_p(\varphi) \dots\dots\dots (16)$$

Coefficients of this series $A_0, A_1, A_2 \dots A_i \dots A_n$, which determined by solution of a set of $(n+1)$ linear equations are a basic requirement for the following computing relations:

$$V_{IDN.A}(x) = V_{IDN.A}(\varphi) = - \left[A_0 + \sum_{i=1}^n A_i \cos(i\varphi) \right] \dots\dots\dots (17)$$

$$\alpha_D = A_0 + \frac{1}{2} \sum_{i=2}^n A_i \frac{(-1)^{i-1} - 1}{i^2 - 1} \dots\dots\dots (18)$$

Evaluation of Analytical Solutions:

The analytical solutions of the profile shape presented in the two preceding sections are based on application of two mathematical procedures; a theoretical qualitative comparison of this method of solution is virtually not feasible.

Thus, these different results of solution can be considered, using actual applications:

The solutions of mean camber line and thickness function for a selected standard in which the distribution of the pressure coefficient $C_p = C_p(x)$ and the shape of mean camber line $y_A = y_A(x)$ and the shape of thickness function $y_s = y_s(x)$ are available.

The standard selected model is update profile "ARE 101.14.30.70" considered by collective of authors (1972)^[8], where the results of camber line profile and thickness function are illustrated in (Fig. 3 a, b and c) represent the pressure and mean pressure distribution of the model respectively.

Comparison of the analytical solution results (which are carried out by application of program "INVSOL") with the selected standard model ARE 101.14.30.70 is represented by (Fig. 4 and 5) and result in: The analytical solution of the thickness function represents appropriate matching with the selected standard (etalon) result; the result is illustrated in (Fig.5) and The analytical solution to obtain the mean camber line shows inaccurate result in (Fig.4), since the surrounding region of the leading edge absolutely misfits. In the x-range from (0.1 to 1) it gives surely qualitative agreement according to the etalon, however, it sets up considerable different quantitative results. Thus, in the following article we carry out a solution using correction parameter for the influence of the leading edge, this parameter has optimization characteristic.

Principle of Numerical Solution Using Optimization Elements:

The originally numerical solution in Basic Optimization Methods (Bunday, 1992)^[9] has its main justification and importance in eliminating problematic accuracy of setting distributions of pressure coefficients $C_{PU}(x)$ and $C_{PL}(x)$ in a close vicinity of the of the leading edge.

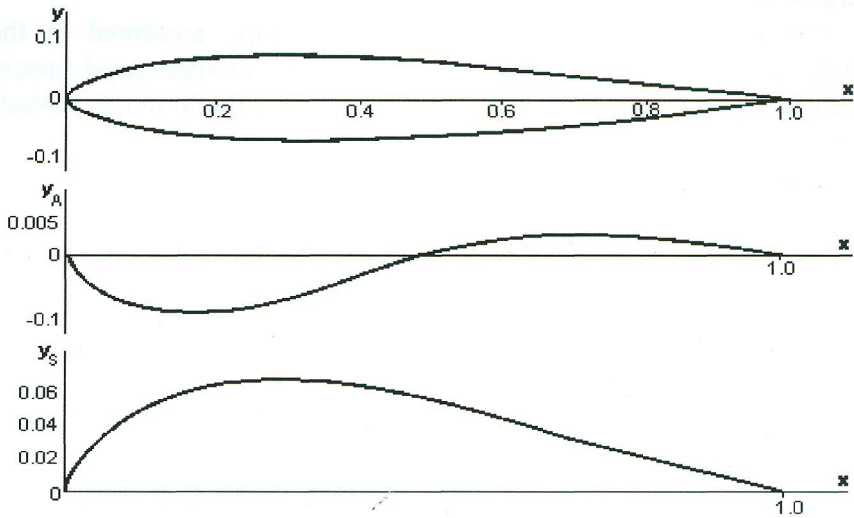


Fig. (3-a): Mean camber line and thickness function of the standard aerofoil RAE 101.14.30.70

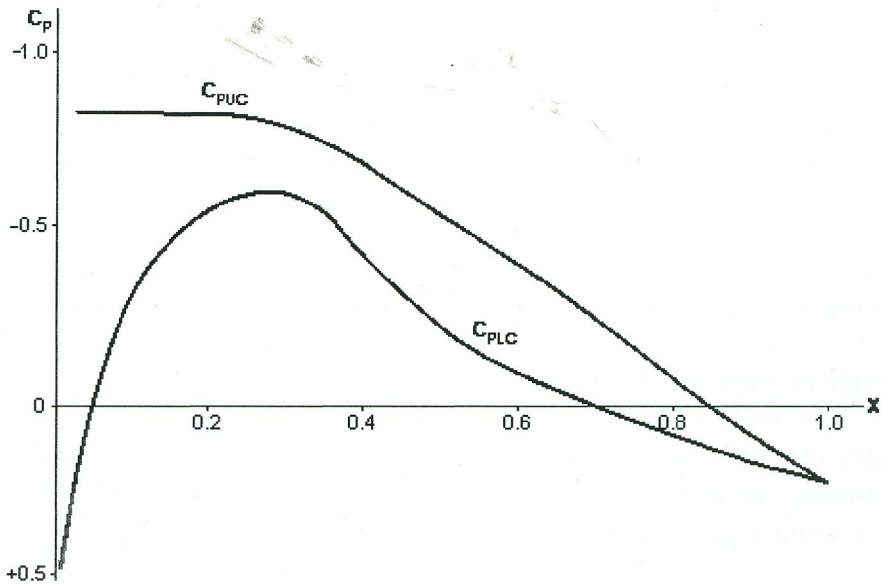


Fig. (3-b): Distribution of pressure coefficients of RAE 101.14.30.70

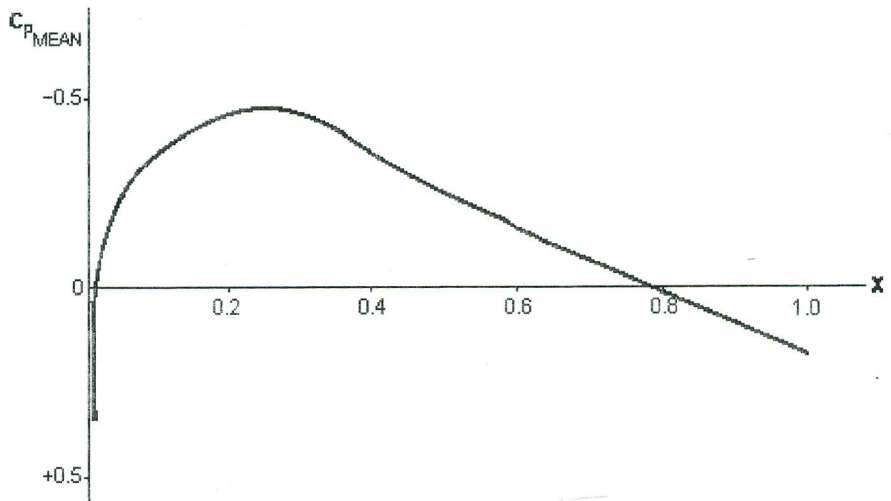


Fig. (3-c): Distribution of Mean pressure coefficient of RAE 101.14.30.70

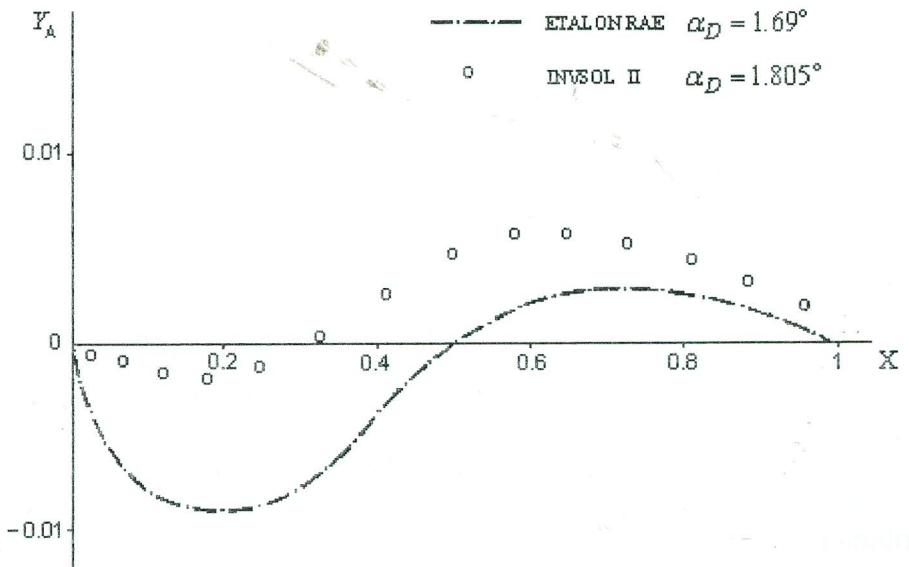


Fig. (4): Comparison of INVSOL II result with RAE 101.14.30.70 for mean camber line of the profile

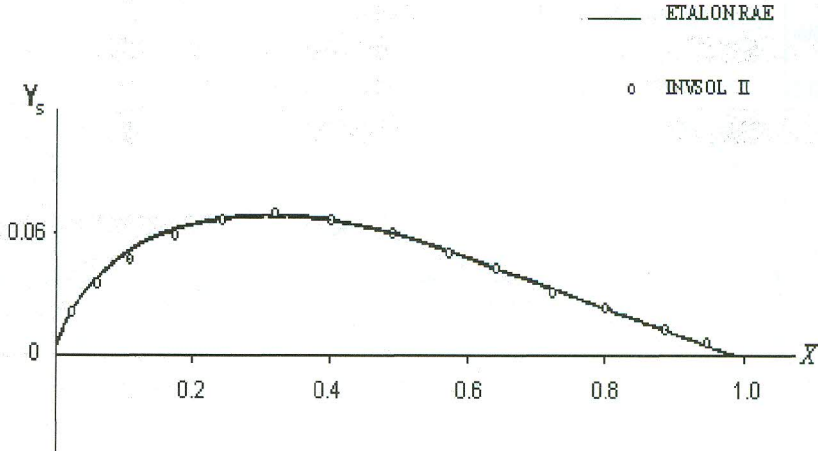


Fig. (5): Comparison of INVSOL II result with RAE 101.14.30.70 for thickness function of the profile

In this numerical processing of the problem, a numerical solution of two complex sets of basic relations is carried out without any additional modification (Cerna, Machlioky and Zlatnik, 1987)^[10]. The first set is composed of equations (6) till (9), while the second set contains equations (10) till (13).

The input quantities $\Delta C_P(x)$ and $C_{P\text{MEAN}}(x)$ in the close vicinity of the leading edge present optimization parameters: though their extent is not large, even a small change may influence substantially both shape of the profile mean camber line and shape of the profile thickness function.

The optimization criteria of the proposal for the mean camber line and thickness function mentioned by (Reneaux and Thibert, 1985)^[11] can be as follows: Minimum deflection of the mean camber line, Minimum profile thickness, highest critical Mach number of the proposed profile and Behavior of boundary layer on the proposed profile, and possibly some others.

The preceding examples have been chosen to demonstrate the proposal optimization criteria to solve the mean camber line of the profile 101.14.30.70: Distribution of differences of the pressure coefficients $\Delta C_P(\xi)$ is set as in (Fig. 6) and the determination of main values of the induced velocity $V_{IND.A.0}(x)$ is carried out only for a part of distribution of $\Delta C_P(\xi)$ within an interval of

$x \in (0.1; 1.0)$; i.e. provisionally without referring to the influence of a close vicinity at the leading edge.

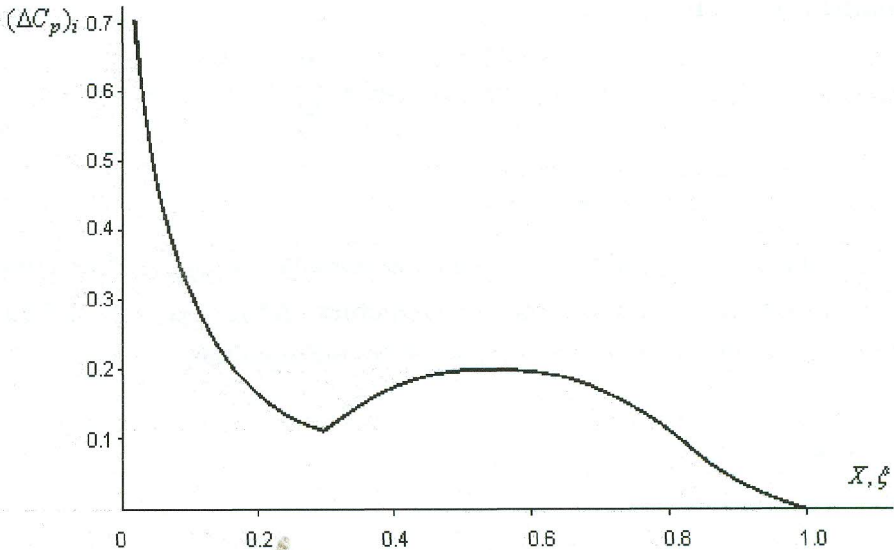


Fig. (6): Distribution of pressure differences of coefficients $(\Delta C_p)_i$

A modified equation (6) is:

$$V_{IND.A.O}(x) = \frac{1}{4\pi} \int \frac{\Delta C_p(\xi)}{x - \xi} .d\xi = \dots\dots\dots (24)$$

$$= K \left[\frac{\Delta C_p(0.125)}{x - 0.125} + \frac{\Delta C_p(0.175)}{x - 0.175} + \dots\dots\dots \frac{\Delta C_p(0.975)}{x - 0.975} \right]$$

The integration is carried out numerically, using steps $\Delta\xi = 0.05$. Thus, for a constant “K” given by

$$K = -\frac{1}{4\pi} \Delta\xi = -0.003976 \dots\dots\dots (25)$$

The resulting important values of the induced velocity $V_{INDAO}(x)$ are presented in (Fig.7): Determination of complementary value of induced velocity

$\Delta V_{IND.A}$ in the close vicinity of the leading edge is carried out for the value $\Delta C_P = 1.0$ within an interval of $\xi \in < 0.0; 0.1 >$.

A modified equation (6) is:

$$\Delta V_{IND.A}(x) = -\frac{1}{4\pi} \int_0^{0.1} \frac{\Delta C_P}{X - \xi} d\xi = \frac{1}{4\pi} \left[\int_0^{0.05} \frac{\Delta C_P}{X - \xi} d\xi + \frac{1}{4\pi} \int_{0.05}^{0.1} \frac{\Delta C_P}{X - \xi} d\xi \right] \dots\dots\dots (26)$$

$$= K \left[\frac{1}{X - 0.025} + \frac{1}{X - 0.075} \right]$$

Complementary values of the induced velocity $\Delta V_{IND.A}(x)$ are presented in (Fig.7). Determination of induced velocities $\Delta V_{IND.A}(x)$ is carried out, introducing the optimization coefficient λ from the equation:

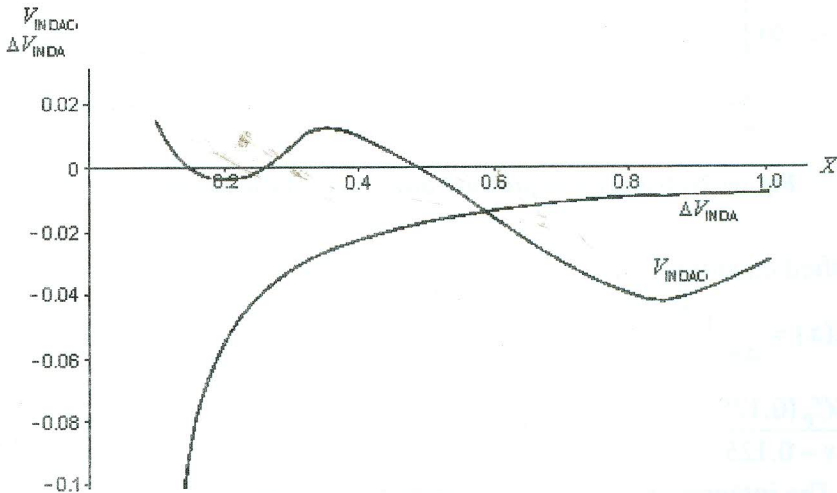


Fig. (7): Induced velocity profiles $V_{IND.AO}$ and $V_{IND.A}$

$$V_{IND.A}(x) = V_{IND.AO}(x) + \lambda \cdot \Delta V_{IND.A}(x) \dots\dots\dots (27)$$

The induced velocity $V_{IND.A}(x)$ in the close vicinity of the leading edge is determined from the equations:

$$V_{IND.A}(x = 0.05) = 2V_{IND.A}(x = 0.1) - V_{IND.A}(x = 0.15) \dots\dots\dots (28)$$

$$V_{IND.A}(x = 0) = 3V_{IND.A}(x = 0.1) - 2V_{IND.A}(x = 0.15) \dots\dots\dots (29)$$

Determination of design angle α_D obtained by application of modified equation (7).

$$\left. \begin{aligned} \alpha_D &= -\int_0^1 V_{IND.A}(x)dx = \\ &= -\frac{1}{2}[V_{IND.A}(x = 0) + V_{IND.A}(x = 1)]\Delta x - \\ &- [V_{IND.A}(x = 0.05) + V_{IND.A}(x = 0.1) + \dots V_{IND.A}(x = 0.95)]\Delta x \end{aligned} \right\} \dots\dots\dots (30)$$

Determination of additional velocity $V_{YA}(x)$ i.e. the first-order derivative of mean camber line $\bar{Y}_A(x)$, is carried out directly from equation (8).

$$V_{YA}(x) \equiv \bar{Y}'_A(x) = \alpha_D + V_{IND.A}(x) \dots\dots\dots (31)$$

Mean camber line $Y_A(x)$ is determined through the numerical integration of equation (9).

$$y_A = \int_0^x \bar{Y}'_A(x).dx, \dots\dots\dots (32)$$

using steps $\Delta x = 0.05$ as follows:

$$\left. \begin{aligned} y_A(x = 0) &= 0 \\ y_A(x = 0.05) &= y_A(x = 0) + y'_A(x = 0.025)0.05 \\ y_A(x = 0.1) &= y_A(x = 0.05) + y'_A(x = 0.075)0.05 \\ y_A(x = 0.95) &= y_A(x = 0.9) + y'_A(x = 0.925)0.05 \\ y_A(x = 1) &= 0 \end{aligned} \right\} \dots\dots\dots (33)$$

RESULTS AND DISCUSSION

The pervious procedure of the mean camber line solution as a whole represents a simple algorithm of the program “INVSOL II”.

At the practical application of program we appreciate the influence of the leading edge on the camber line by using the optimization parameter “ λ ” as input data basic value.

We carry out the calculation for range $(0.1 < \lambda < 0.4)$ and the results are illustrated in (Fig. 8).

For the selected standard etalon:

$$\frac{y_{AMIN}}{y_{AMAX}} = -2.127$$

Which permits the mean camber line to be close to the etalon, and this occurs at value $\lambda = 0.32$ and $\lambda = 0.42$.

The induced velocity result for the different rage of correction parameter is represented by (Fig. 9), whereas the validation of INVSOL II for the different rage of correction parameter with the model RAE 101.14.30.70 is illustrated by (Fig. 10).

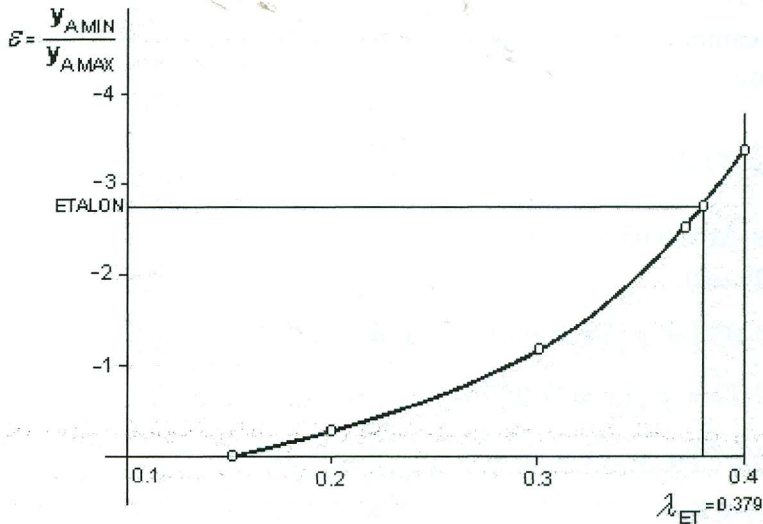


Fig. (8): Relation between ϵ and λ

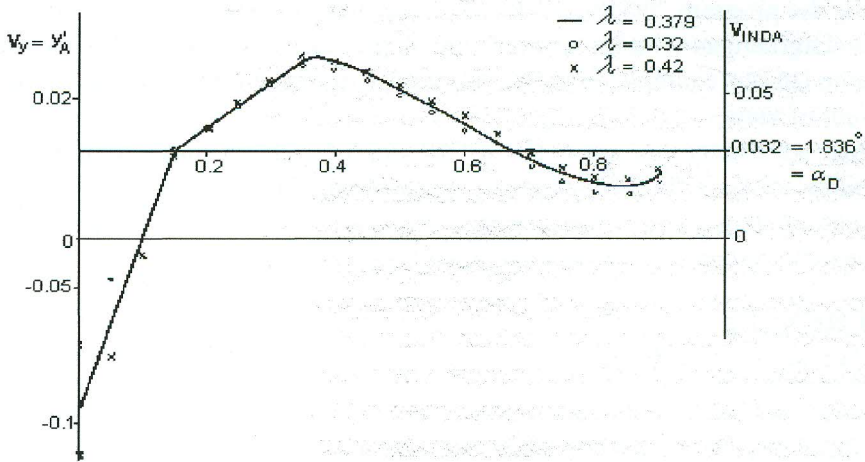


Fig. (9): Validation of INVSOL II for the Induced velocity V_{INDA} of the aerofoil

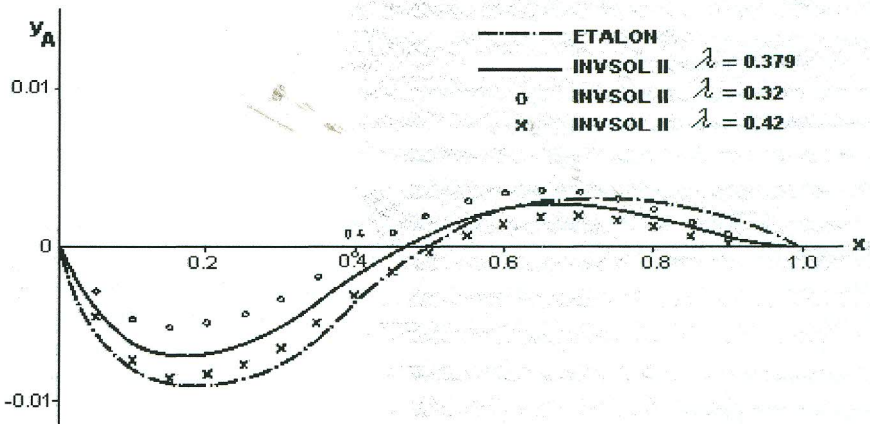


Fig. (10): Validation of INVSOL II for the leading edge of the aerofoil

CONCLUSION

From the previous analyses, we approach the consequent conclusion:

Using analytical solution to solve the mean camber line of an airfoil such as RAE does not give precise results on the leading edge, so we introduce a correction parameter and the results indicate that:

- i) Using the program "INVSOL II" to solve the mean camber line reaches a very good agreement with the etalon from the qualitative point of view, for certain value (in our example, $\lambda=0.375$) this a good agreement from the quantitative point of view.
- ii) When we change the value of (λ) we get a family of mean chamber line, which creates a set of input data to solve the profile's geometry.
- iii) From all sides of view the program "INVSOL II" (with optimized parameter) has better qualitative results than the program "INVSOL I".

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