Dedications

To my great mother, father, husband, brothers, sisters and friends.

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ABSTRACT

We classify the functional model for dissipative and calculus for a pair of almost commuting selfadjoint operators. We find the estimates in operator class for a difference of function from the pick class of accretive operators and of operator moduli of continuity .We discuss the Hankel operators in the perturbation theory for unitary, selfadjoint operators, functions of perturbed and normal operators .We show an extension of the koplienko - Neidhardt trace formule and the differential properties of some dense subalgebras of C*-algebras with multiple operator integrals and higher operator derivatives. We determine the fully operator Lipschitz and Hölder–Zygmund functions .

الخلاصة

صنفنا النموذج الدالي المبدد والحسبان لأجل زوج مؤثرات المرافق الذاتي التبديلي تقريبا. أوجدنا التقديرات في عائلة المؤثر لأجل الفرق للدالة من عائلة الاختيار للمؤثرات التراكمية ولمقياس مؤثر الاستمرارية . ناقشنا مؤثرات هانكل في نظرية الاضطراب لأجل مؤثرات الواحدية والمرافقة الذاتية والدوال لمؤثرات الاضطراب والناظمية .أوضحنا التمديد لصيغة أثر كوبلينكو- نيد هاردت والخواص التفاضلية لبعض الجبريات الجزئية الكثيفة لجبريات -C* مع تكاملات المؤثر المضاعفة ومشتقات المؤثر العليا . حددنا مؤثر ليبشتيز الكامل ودوال هولدر – زيجموند.

introduction

The research has, basically, a methodical character. Dissipative operator: are investigated with the aid of the Cayley contraction-transformation on the basis of the "coordinate free model" of V. I. Vasyunin and N. K.Nikol'skii. it is shown that the known form of the characteristic junction and of the selfadjoint dilation can be obtained as various realization of a general scheme.

We make an attempt to introduce a new technique in the considered sphere of questions. The fundamental circumstance consists in the reduction of the mentioned problems to the investigation of the metric properties of Hankel operators.

Koplienko found a trace formula for perturbations of self-adjoint operators by operators of Hilbert Schmidt class $\mathbf{S_2}$. A similar formula in the case of unitary operators was obtained by Neidhardt.

We studie some classes of dense *-subalgebras B of C*-algebras A whose properties are close to the properties of the algebras of differentiable functions. In terms of a set of norms $\{\| \quad \|_i\}_{i=1}^{p}$ on B it defines (D_p^*)-subalgebras of A and establishes that they are locally normal Q*-subalgebras. If $x = x^* \in B$ and $f(t)$ is a function on *Sp(x)*, some sufficient conditions are given for *f(x)* to belong to *B*.

Let $A(\mathbb{D})$ be the disc algebra of all continuous complex-valued functions on the unit disc $\mathbb{D}$ holomorphic in its interior. Functions from $A(\mathbb{D})$ act on the set of all sequences of contraction operators $(\|A\| \leq 1)$ on Hilbert spaces.

We show that if $0 < \alpha < 1$ and f is in the Hölder class $\Lambda_\alpha(\mathbb{R})$, then for arbitrary selfadjoint operators A and B with bounded $A - B$, the operator $f(A) - f(B)$ is bounded and $\|f(A) - f(B)\| \leq \text{const} \|A - B\|^\alpha$. We show a similar result for functions f of the Zygmund class $\Lambda_1(\mathbb{R})$: $\|f(A + K) - 2f(A) + F(A - K)\| \leq \text{const} \|K\|$, where A and K are selfadjoint operators. Similar results also hold for all Hölder–Zygmund classes $\Lambda_\alpha(\mathbb{R})$, $\alpha > 0$.

In Alesksandrov and Peller sharp estimates for $f(A) - f(B)$ were obtained for self-adjoint operators A and B and for various classes of functions f on the real line $\mathbb{R}$. We extend those results to the case of functions of normal operators. We show that if a function f belongs to the H¨older

class $\Lambda_\alpha(\mathbb{R}^2)$,$0< \alpha < 1$, of functions of two variables, and N_1 and N_2 are normal operators, then

$\|f\ (N_1) - f\ \ (N_2)\| \leq \text{const}\ \|f\|_{\Lambda_\alpha}\ \|\ N_1 - N_2\ \|^\alpha$.

We consider the problem of the existence of higher derivatives of the function $t \to \varphi(A + tK)$, where φ is a function on the real line , A is a self-adjoint operator, and K is a bounded self-adjoint operator.

It is well known that a Lipschitz function on the real line does not have to be operator Lipschitz. We show that the situation changes dramatically if we pass to Hölder classes. Namely, we show that if f belongs to the Hölder class $\Lambda_\alpha(\mathbb{R})$with $0 < \alpha < 1$, then $\|\mathrm{f}(\mathrm{A}) - \mathrm{f}(\mathrm{B})\| \leq \text{const}\ \|\mathrm{A} - \mathrm{B}\|^\alpha$ for arbitrary self-adjoint operators A and B. We show a similar result for functions f in the Zygmund class $\Lambda_\alpha(\mathbb{R})$: for arbitrary self-adjoint operators A and K we have $\|f\ (A - \mathrm{k}) - 2f\ (A) + F(A + K)\| \leq \text{const}\ \|K\|$. We also obtain analogs of this result for all Hölder–Zygmund classes $\Lambda_\alpha(\mathbb{R})$, $\alpha > 0$. Then we find a sharp estimate for $\|f\ (A) - f\ (B)\|$ for functions f of class $\Lambda_\omega \stackrel{\text{def}}{=} \{f: \omega_f(\delta) \leq \text{const}\ \omega(\delta)\}$ for an arbitrary modulus of continuity ω.

Let (A. B) be a pair of almost commuting selfadjoint operators on Hilbert space. The aim is to construct a functional calculus defined on a class of functions two variables and satisfying natural properties (linearity and multiplicativity modulo the trace class).The problem is to make this class as big as possible. It is proved that it is impossible to construct such a functional calculus on the class of all continuously differentiable functions.

In Aleksandrov and Peller we obtained general estimates of the operator moduli of continuity of functions on the real line. We improve the estimates obtained in Aleksandrov and Peller for certain special classes of functions. In particular, we improve estimates of Kato and show that

$$\||S| - |T|\| \leq \mathrm{C}\|S - T\| \log(2 + \log\frac{\|S\| + \|T\|}{\|S - T\|})$$

for all bounded operators S and T on Hilbert space. Here $|S| \stackrel{\text{def}}{=} (S^*S)^{\frac{1}{2}}$. Moreover, we show that this inequality is sharp.

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