Sudan University of Science and Technology
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Running of Quark Mass and Flavor Mixing in Extra Dimension Model

نشأة كتله ومزج نكهه جسيم الكوارك في نموذج الابعاد الزائدة

A Dissertation submitted to the college of graduate studies, in partial fulfillment of the requirements for the degree of Master of Science in Physics.

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الآيـة

قال تعالي :

{اللَّهُ نُورُ السَّمَاوَاتِ وَالْأَرْضِ ۚ مَثَلُ نُورِهِ كَمِشْكَاةٍ فِيهَا مِصْبَاحٌ ۖ الْمِصْبَاحُ فِي زُجَاجَةٍ ۖ الزُّجَاجَةُ كَأَنَّهَا كَوْكَبٌ دُرِّيٌّ يُوقَدُ مِنْ شَجَرَةٍ مُبَارَكَةٍ زَيْتُونَةٍ لَا شَرْقِيَّةٍ وَلَا غَرْبِيَّةٍ يَكَادُ زَيْتُهَا يُضِيءُ وَلَوْ لَمْ تَمْسَسْهُ نَارٌ ۚ نُورٌ عَلَىٰ نُورٍ ۗ يَهْدِي اللَّهُ لِنُورِهِ مَنْ يَشَاءُ ۚ وَيَضْرِبُ اللَّهُ الْأَمْثَالَ لِلنَّاسِ ۗ وَاللَّهُ بِكُلِّ شَيْءٍ عَلِيمٌ}

سورة النور الآية 35

Abstract

In this research, the running of quarks masses and flavor mixing in five dimensional standard models has been studied. The evolution properties of quark mass and flavor mixing are performed for the one-loop renormalization group equations in the five dimensional standard model. It is found that the five dimension model has a significant effect on the running of the fermion masses, including both quark and lepton sectors. We quantitatively discussed these quantities for $\mathrm{R}^{-1} = 1\ \mathrm{TeV}, 5\ \mathrm{TeV}$ and $13\ \mathrm{TeV}$ resulting in similar behaviors for all values of the compactification radius.

ملخص البحث

في هذا البحث دُرس نشاة كتل ومزج ونكهه الكوراكات في نظرية الأبعاد الزائده (نظرية النموذج القياسي للجسيمات الاولية في خمسة أبعاد). تم دراسه وحساب نشاة كتله ومزج نكهه الكورك للرتبه الاولي باستخدام معادلات الحلقه الدولي للمجموعه المعايره في نظريه الابعاد الزائده. وجد ان نظريه الأبعاد الزائده تساهم بصوره كبيره علي نشاة كتله الفيرمونات كواركات وليبتونات. نوقش كتله ومزج نكهه الكورك عند طاقات $R^{-1} = 1\ \mathrm{TeV}, 5\ \mathrm{TeV}\ \mathrm{and}\ 13\ \mathrm{TeV}$ ووجد نتائج متشابه لهذه الكميات عند هذه الطاقات.

Dedication

All praise to Allah today we fold the day tiredness and the errand summing up between the cover of this humble work.

To the spring that never stops giving

To my mother.

To whom he strives to bless comfort and welfare and never-stints what he owns to push me in the success way who taught me to promote life stairs wisely and patiently

To my dearest father God mercy.

To whose love flows in my veins and my heart

To my brothers and sisters.

To those who taught us letters of gold and teach us their knowledge simply

To my teachers.

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I would like to express my appreciation to my supervisor ***Dr. Ammar Ibrahim Abdalgabar*** who *has cheerfully answered my queries, provided me with materials, checked my examples ,assisted me in a myriad ways with the writing and helpfully commented on earlier drafts of this project .*

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CHAPTER I

1-1 General Introduction

All known elementary particle physics phenomena are extremely well understood within the Standard Model (SM) of elementary particles and their fundamental interactions (Salam, 1968). The SM of particle physics is a theory that describes the interaction between elementary particles (Guigg, 1983). It combines the Glashow-Weinberg-Salam theory of electroweak interaction (that unified the weak and electromagnetic interactions), based on $SU(2)_L \times U(1)_Y$group (S.L.Glashow, 1961) and the strong interactions known as Quantum Chromo dynamics (QCD), based on $SU(3)_C$group (Guigg, 1983).

The matter fields are fermions, have spin s = 1/2 and are classified into leptons and quarks. The known leptons are: the electron, e^-, the muon μ^-, and the tau τ^-, with electric charge Q= -1; and the corresponding neutrinos ν_e,ν_μ,and $\nu_\tau$with electric charge Q = 0.

The known quarks are consist of six different flavors u, d, c, s, t and b with fractional electric charge Q = 2/3,−1/3, 2/3,−1/3, 2/3,−1/3 respectively (J.donoghue, 1994).

The SM gauge sector is composed of 12 gauge fields have spin s = 1, which mediate the interactions between the matter fields; the photon (mediate the electromagnetic interactions) being neutral, the three weak gauge bosons (charged $W^{\pm}$and neutral Z mediate the weak interactions) which are the four gauge bosons of $SU(2)_L \times U(1)_Y$and eight gluons (g$_\alpha$;α= 1, 2,...,8 mediate the strong interactions) which are the gauge bosons of $SU(3)_C$.

The Z boson in the Glashow-Salam-Weinberg model is a mixture of the neutral spin $SU(2)_L$ and the hypercharge $U(1)_Y$ gauge fields, with the mixing parameterized by $sin^2\theta_W$ The Z boson interacts with vector and axial-vector currents of matter (Veltman, 2003).

The Z -matter couplings, including the mixing angle, are affected by radiative corrections so that high-precision analyses allow both tests at the quantum level and extrapolations to new scales of virtual particles. Despite the success of the standard model. However, there are some issues with the standard model; these issues suggest the extension of the standard model to account for these unsolved problems (Falcone, 2002).

A theory of fermion masses and the associated mixing angles provide an interesting puzzle and a likely suggested a window to physics beyond the Standard Model (SM), where one of the main issues in particle physics is to understand the fermion mass hierarchy and mixings (E.~Poppitz, 2001).

There have been many attempts to understand the fermion mass hierarchies and their mixings by utilizing the technique of Renormalization Group Equations (RGEs) especially for Universal Extra Dimension (UED) models and their possible extensions (A.~Abdalgabar, 2013) (A.~S.~Cornell, 2010) (L.~-X.~Liu, 2010).

There are several versions of UED models, the simplest being the case of one flat extra dimension compactified on an S_1/Z_2 orbifold which has a size 1/R ≈ 1 TeV. This compactification will lead to a new particle states in the effective 4-dimensional (4D) theory. As such, in the 4D effective theory there appears an infinite tower of massive Kaluza-Klein (KK) states, with a mass contribution inversely proportional to the radius of the extra-dimension (N.~Maru, 2010).

1-2 The importance of the study

There are many reasons to consider such models, primarily as they provide a way to address the “hierarchy problem”, that is, the question of why there is huge gap between Planck scale $M_{Pl} = 10^{19}$ GeV and weak scale 246 GeV, but also to provide a means of breaking the electroweak symmetry, the generation of fermion mass hierarchies, and in studying the CKM matrix and new sources of CP violation. Furthermore, TeV scale grand unification and sources of dark matter are also possible in these theories. To date much of the interest in UED models has

been for its source of beyond the SM TeV-scale physics, largely arising from the tower of KK states approximately degenerate in mass at the scale set by the inverse of the compactification radius. KK parity and the 4D conservation of momentum imply that contributions to SM particle masses occur only for interactions at loop level, and that the lightest KK particle will be stable and a suitable dark matter candidate.

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1-3 The main objectives of the study

The main objective is to discuss and study the evolution of quark mass and flavor mixing in extra dimension models at one loop level by using the techniques of renormalization group equation.

1-4 Outline of the Research

This research is structured as follow: In chapter II we briefly introduced the standard model of particle physics, which describe the interaction among the elementary particles. Chapter III is devoted to calculation of the renormalization group equations in five dimensional models. We present our numerical result, discussions and conclusions in chapter IV.

CHAPTER II

Introduction to the Standard Model and Beyond

(2-1)The Standard Model

After three decades of great cumulative theoretical and experimental effort, today it seems to possess the theory of both strong and electroweak interactions. This is the so-called Standard Model (J.donoghue, 1994), based on the invariance under the symmetry group

$$G_{st} = SU(3)_C \times SU(2)_L \times U(1)_Y \qquad (1.1)$$

More correctly phrased, it is based on the partial spontaneous symmetry breaking of G_{st}.

The basic constituents of matter, the elementary fermions, i.e. the Quarks and leptons have the following transformation properties under G_{st} (for simplicity we concentrate only in the first generation of Fermions) see table 1.1.

The $SU(3)_C$ gauge group or colour group is the symmetry group of strong interactions.

The $SU(2)_L \otimes U(1)_Y$ is the gauge group of the unified weak and electromagnetic interactions ‘where $SU(2)_L$is the weak isospin group, acting on left-handed fermions, and $U(1)_Y$ is the hypercharge group (Guigg, 1983).

(2-2) Symmetries in the Standard Model

1-*Global symmetries:-*

The continuous parameters of the transformation *do not depend* on the space-time coordinates. Some examples are: SU(2) Isospin symmetry, SU(3) flavor symmetry, $\mathrm{U}(1)_\mathrm{B}$ baryon symmetry, $\mathrm{U}(1)_\mathrm{L}$ lepton symmetry (L.F.Li, 1991).

2-Local (Gauge) symmetries:-

The continuous parameters of the transformation *do depend* on the space-time coordinates. Some examples are: $\mathrm{U}(1)_\mathrm{em}$electromagnetic symmetry, $\mathrm{SU}(2)_\mathrm{L}$weak isospin symmetry, $\mathrm{U}(1)_\mathrm{Y}$weak hypercharge symmetry, $\mathrm{SU}(3)_\mathrm{C}$color symmetry (Guigg, 1983).

Table 1.1: The SM fields with their representations under $SU(3)_c$ and $\mathrm{SU}(2)_\mathrm{L}$ and their charges under $U(1)_Y$ and $U(1)_{EM}$, Q is the electric charge and *s* is the spin of the field.

Field	Notation	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{EM}$
Quarks (s=1/2)	$Q_L = \binom{u_L}{d_L}, \binom{c_L}{s_L}, \binom{t_L}{b_L}$	3	2	1/6	$\binom{2/3}{-1/3}$
	u_R, c_R, t_R	3	1	2/3	2/3
	d_R, s_R, b_R	3	1	-1/3	-1/3
Leptons (s=1/2)	$L_L = \binom{\nu_e}{e_L}, \binom{\nu_\mu}{\mu_L}, \binom{\nu_\tau}{\tau}$	1	2	-1/2	$\binom{0}{-1}$
	e_R, μ_R, τ_R	1	1	1	-1
Gauge (s=1)	g	8	1	0	0
	$W^3, W^\pm$	1	3	0	0,±1
	B	1	1	0	0
Higgs (s=0)	$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 = \frac{1}{\sqrt{2}}(\mathrm{v} + h + i\varphi_0) \end{pmatrix}$	1	2	1/2	$\binom{1}{0}$

(2-3) The Standard Model Lagrangian

The SM Lagrangian can be divided as:

$\mathcal{L}_{SM}= \mathcal{L}_{Gauge} + \mathcal{L}_{\text{Fermions}} + \mathcal{L}_{\text{Higgs}} +\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Gauge.fixing}} +\mathcal{L}_{\text{Ghost}}.$

(2.1)

We shall now briefly introduce each sector of this Lagrangian

$\mathcal{L}_{\text{Gauge}} =$

$$-\frac{1}{4} G^{A}_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^{a}_{\mu\nu} w^{a\mu\nu} - \frac{1}{4} B_{\mu} B^{\mu} \quad (2.2)$$

$$\mathcal{L}_{\text{Fermion}} = \sum_f \bar{f}\, i\gamma^{\mu} D_{\mu} f \quad (2.3)$$

$\mathcal{L}_{\text{Higgs}} =$

$$(\mathrm{D}_{\mu}\Phi)^{\dagger}(\mathrm{D}^{\mu}\Phi) - \mathrm{V}(\Phi) \quad (2.4)$$

$\mathcal{L}_{\text{Yukawa}} =$

$$\mathrm{Y}^{\mathrm{d}}_{\mathrm{ij}} \mathrm{q}^{\mathrm{i}}_{\mathrm{l}} \Phi \mathrm{d}^{\mathrm{j}}_{\mathrm{R}} + \mathrm{Y}^{\mathrm{u}}_{\mathrm{ij}} \mathrm{q}^{\mathrm{i}}_{\mathrm{l}} \Phi^{\sim} \mathrm{u}^{\mathrm{j}}_{\mathrm{R}} + \mathrm{Y}^{\mathrm{e}}_{\mathrm{ij}} \mathrm{L}^{\mathrm{i}}_{\mathrm{l}} \Phi \mathrm{e}^{\mathrm{j}}_{\mathrm{R}} + \mathrm{h.c} \quad (2.5)$$

$\mathcal{L}_{\text{Gauge fixing}}$

$$= -\frac{1}{2}\frac{\zeta}{2}(\partial_{\mu}\mathrm{A}^{\mu})^2 \quad (2.6)$$

$\mathcal{L}_{\text{Ghost}} =$

$$\overline{\mathrm{C_b}}\, \partial^{\mu} \mathrm{D}^{\mathrm{ab}}_{\mu} \mathrm{C_a} \quad (2.7)$$

(2-4)The Higgs Mechanism

If mass terms for gauge bosons and for left/right-chiral fermions are introduced by hand, they break the gauge invariance of the theory. So we need a mechanism to give masses to these particles. This issue has been solved by means of the Higgs mechanism in which masses are introduced into gauge theories in a consistent way. The solution of the problem is achieved at the expense of a new fundamental degree of freedom, the Higgs field, which is a scalar field.

This Scalar field is denoted by ϕ and has hypercharge$Y_{\phi} = 1$ and can interact with each other, the interaction between fermion fields and the Higgs field is of Yukawa type (P.~W.~Higgs, 1964).

The Higgs doublet Lagrangian should contain a **"**spontaneous symmetry breaking**"** potential, this will give the Higgs a vev and self-interactions, and kinetic terms which will generate the gauge boson masses and interactions between the Higgs and the gauge bosons. This scalar particle has been discovered by the ATLAS (al, 2012) and CMS (al, 2012) experiments, which is compatible with the SM. Higgs expectations with a mass 126 GeV.

The Higgs potential is given by

$$V(\Phi) = -\frac{1}{2}\mu^2\Phi^+\Phi - \frac{\lambda}{4}(\Phi^+\Phi)^2 \tag{2.9}$$

Which involves two new real parameters $\mu\ and\ \lambda$ we demand $\lambda > 0$ for the potential to be bounded; otherwise the potential is unbounded from below and there will be no stable vacuum
state.

μ Takes the following two values:

• $\mu^2 > 0$ Then the vacuum corresponds to$\Phi = 0$, the potential has a minimum at the origin (see figure 2.2).

• $\mu^2 < 0$ Then the potential develops a non-zero Vacuum Expectation Value (VEV) and the minimum is along a circle of radius $\frac{v}{\sqrt{2}} = \frac{246}{\sqrt{2}}$ (see figure 2.1).

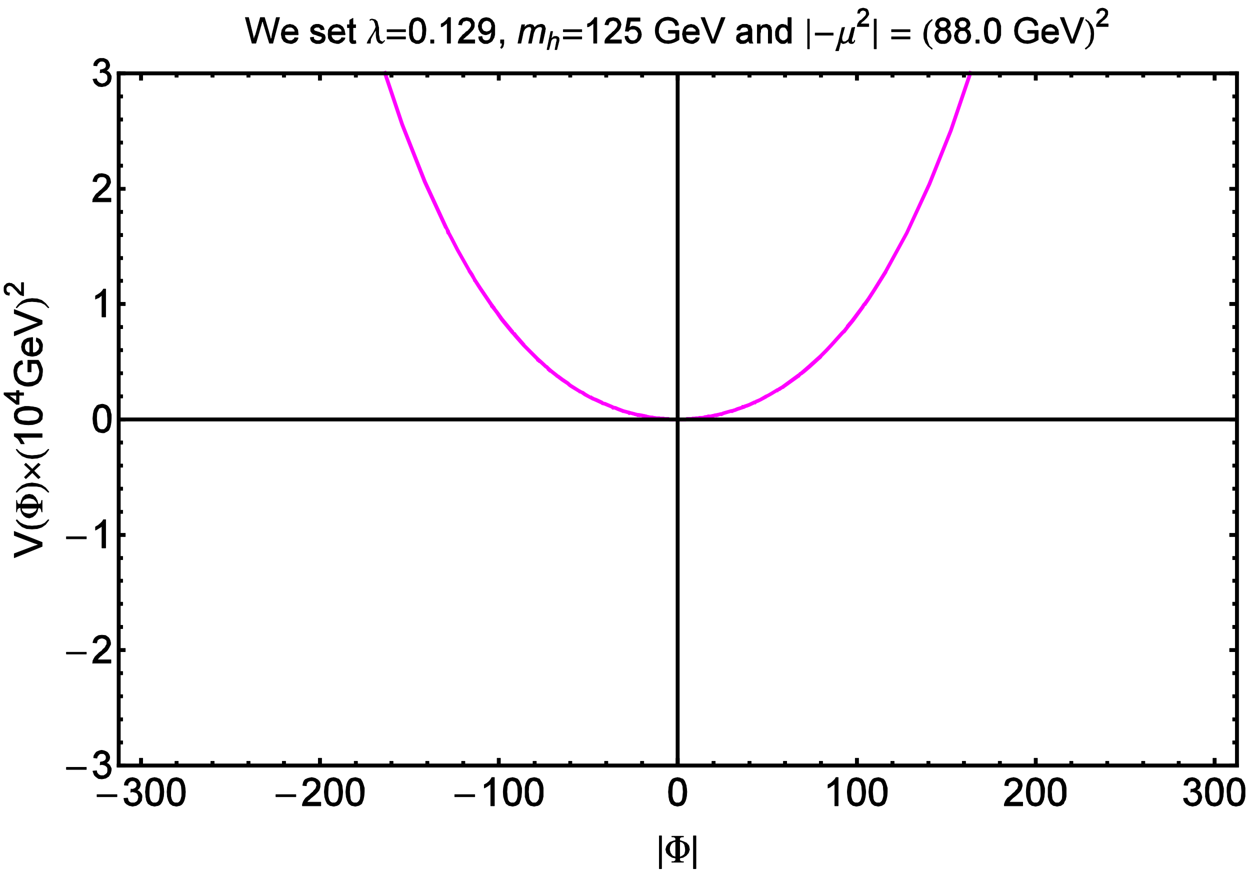

Figure (2.1): The Higgs potential $V(\Phi)$ with, the case $\mu^2 > 0$; as function of $|\Phi| = \sqrt{\Phi^\dagger \Phi}$

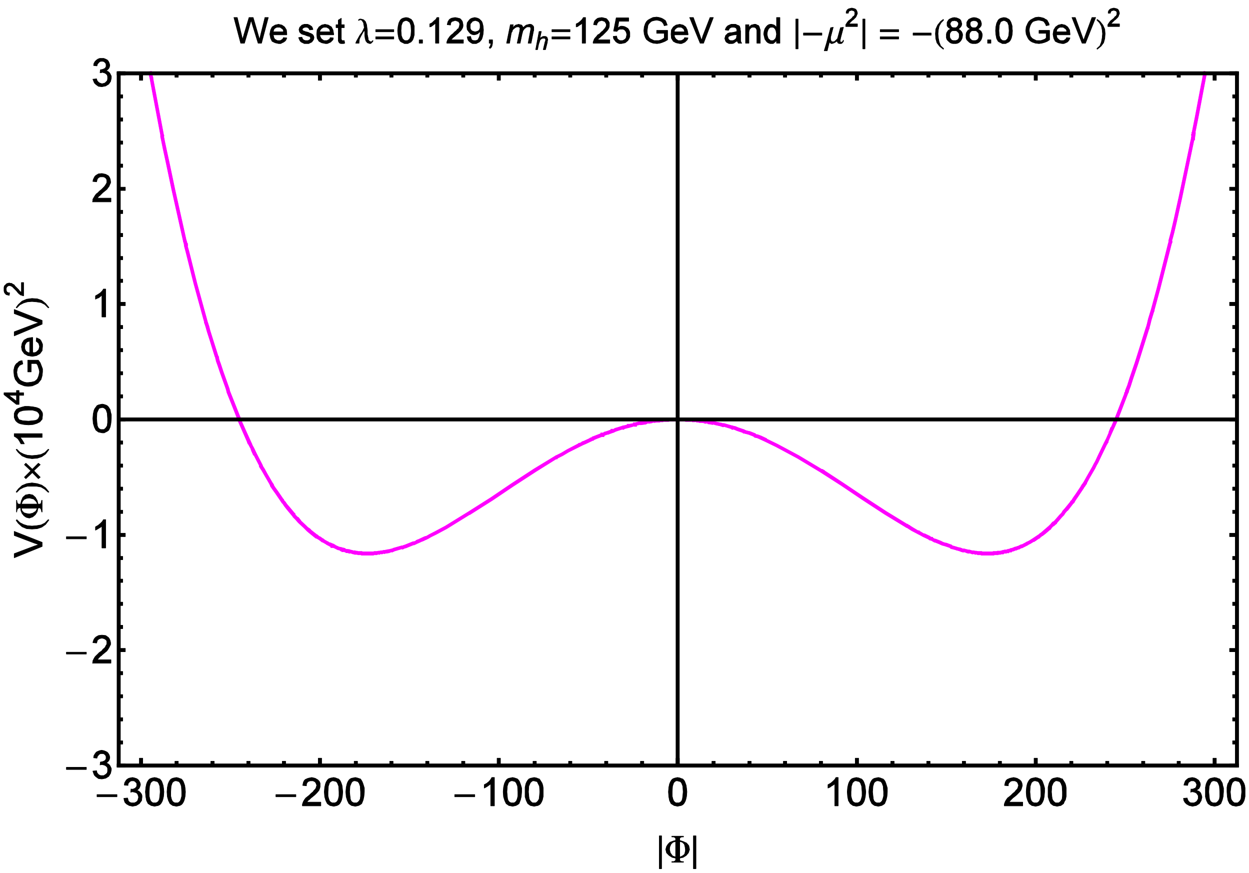

Figure (2.2): The Higgs potential $V(\Phi)$ with, the case $\mu^2 < 0$; as function of

$$|\Phi| = \sqrt{\Phi^{\dagger}\Phi}$$

(2-5) Problem of standard model

Despite the success of the standard model. Below we list some of unsolved problem in the standard model.

1- Cosmological consideration: The observed matter density of galaxies falls short of the measured matter as measured by the rotation curves. It is theorized that the baryon matter density

is∼ 4%. The rest of the universe is made up of ∼ 24% dark matter and ∼ 72% dark energy. In the last decade, the direct observation of gravitational lensing and observations in galactic

Collision (in the 'Bullet' cluster) events have provided hard evidence for the existence of *Dark Matter* (DM).The WMAP probe has measured the dark matter density to be between (0.087 < DMh2 <0.138) at 3σ range. SM neither provides any explanation for dark energy nor does it have a suitable dark matter candidate (Collaboration], 2011).

2- Gauge Hierarchy problem:

The gauge Hierarchy problem is the question of why there is such a huge difference between the electroweak scale $\boldsymbol{M_{EW} = \mathcal{O}(100)\ Gev}$ and the Planck scale $\boldsymbol{M_{PL} = \mathcal{O}(10^{18}) Gev}$. This is also known as the naturalness problem (L.F.Li, 1991).

3.-Gravity is not included: Gravity is not put on the same footing as other interactions in the

SM.

4-Fermion mass: In particle physics one of the major issues is to explain the fermion mass hierarchy and their mixings. The practical feature of the fermion mass spectrum gives us

$$m_u \ll m_c \ll m_t, \qquad m_d \ll m_s \ll m_b, \qquad m_e \ll m_\mu \ll m_\tau$$

where a completely satisfactory theory of fermion masses and the related problem of mixing angles is certainly lacking at present. However, there has been considerable effort to understand the hierarchies of these mixing angles and fermion masses in terms of the renormalization group equations (RGE) (K.~S.~Babu, 1987).

(2-6)Universal Extra Dimensions

Whilst our universe seems to consist of four space-time dimensions, the possibility of including extra spatial dimensions is an idea which dates back quite some time. In fact, as early as the 1920's Kaluza and Klein proposed the existence of an additional spatial dimension compactified in such a way as to make it too small to have as yet been observed (T.~Kaluza, 1921). There are several versions of this model, the simplest being the case of one flat extra dimension compactified on an S_1/Z_2 orbifold which has a size 1/R ~ 1 TeV. This compactification lead to a tower of new particle states in the effective four dimensional theory As such, in the four dimensional effective theory there appears an infinite tower of massive KK states, with a mass contribution inversely proportional to the radius of the extra-dimension (H.~-U.~Yee, 2003).

The Universal Extra Dimension (UED) model is an effective theory in four dimensions with a cutoff Λ, with the consequence that the tree level spectrum is highly degenerate and where loop corrections to masses become Important. The phenomenology of these UED models will arise when their flat extra dimensions allow all (or a subset) of the SM fields to propagate in the full space-time. There are many reasons to study such models (see section 2.5 for more detail). In this section we will discuss the model building of five extra dimensions in the universal extra dimensional model (N.~Maru, 2010)

(2-7) Decomposing the five dimensions Kaluza Klein fields

We study a generic model with one universal extra dimension called Universal Extra Dimension (UED) model, where all the SM fields propagate universally in 5D space-time. The space-time coordinate x_μ (μ= 1, 2, 3, 4) denotes the usual Minkowski space, and the fifth extra spatial dimension coordinates $x_5 = y$ is compactified on a circle (N.~Maru, 2010).

The Lagrangian for a scalar field Φ is

$$\mathcal{L}_{\text{Higgs}} = \int \text{dy}\, (\text{D}_\mu\Phi)^\dagger(\text{D}^\mu\Phi)$$
$$- \text{V}(\Phi) \qquad (2.7)$$

To get the effective 4 dimension we integrate the fifth coordinate y and after Fourier decomposition along the extra coordinates, the fields can be written as a sum of KK modes.as

$$\Phi(x,y) = \frac{1}{\sqrt{\pi R}}\left(\Phi(x\,,y) + \sqrt{2}\,\sum_{n=1}^{\infty}\Phi_n(x)\cos\frac{ny}{R}\right) \quad (scalar\ field\ or\ higgs\ field) \qquad (2.8)$$

The Lagrangian for an Abelian gauge field (also for non-Abelian gauge symmetries at quadratic level) is

$$\mathcal{L}_{\text{Gauge}} = \int dy\left(-\tfrac{1}{4}\,G^A_{MN}G^{AMN} - \tfrac{1}{4}\,W^a_{MN}w^{aMN} - \tfrac{1}{4}B_M B^M\right)$$

Therefore each of the gauge fields has five components and decomposes into towers of 4D spin-1 fields and one tower of real scalar belonging to the adjoint representation, and their Fourier decomposition are

$$B_\mu(x\,,y) = \frac{1}{\sqrt{\pi R}}\left(B_\mu^0(x)\right.$$

$$\left. + \sqrt{2}\sum_{n=1}^{\infty} B_\mu^n(x)\cos\frac{ny}{R}\right) \tag{2.9}$$

$$B_5(x\,,y)$$

$$= \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} B_5^n\,(x)\sin\frac{ny}{R} \tag{2.10}$$

$$W_\mu(x\,,y) = \frac{1}{\sqrt{\pi R}}\left(W_\mu^0(x)\right.$$

$$\left. + \sqrt{2}\sum_{n=1}^{\infty} W_\mu^n(x)\cos\frac{ny}{R}\right) \tag{2.11 }$$

$$W_5(x\,,y)$$

$$= \sqrt{\frac{2}{\pi R}}\,\sum_{n=1}^{\infty} W_5\,(x)\sin\frac{ny}{R} \tag{2.13}$$

$$G_\mu(x\,,y) = \frac{1}{\sqrt{\pi R}}\left(G_\mu^0(x)\right.$$

$$\left. + \sqrt{2}\sum_{n=1}^{\infty} G_\mu^n\;(\mathrm{x})\cos\frac{ny}{R}\right) \tag{2.14}$$

$$G_5(x\,,y)$$

$$= \sqrt{\frac{2}{\pi R}}\,\sum_{n=1}^{\infty} G_5^n(x)\sin\frac{ny}{R} \tag{2.15}$$

So the gauge field in 5D $\mathrm{B}_\mu = \left(\mathrm{B}_\mu\,,\mathrm{B}_5\right), \mathrm{W}_\mu = \left(\mathrm{W}_\mu\,,\mathrm{W}_5\right), \mathrm{G}_\mu$

$$= \left(\mathrm{G}_\mu, \mathrm{G}_5\right) \tag{2.16}$$

The Lagrangian for fermions read:

$$\mathcal{L}_{\text{Fermion}} = \int dy\,\sum_f \bar{\psi}\,i\gamma^M D_M\psi$$

The decomposition of fermions can be written as

$$Q(x\,,y) = \frac{1}{\sqrt{\pi R}}(q_L(x)$$

$$+ \sqrt{2}\sum_{n=1}^{\infty}\left(Q_L^n(x)\cos\frac{ny}{R} + Q_R^n\,(\mathrm{x})\sin\frac{ny}{R}\right) \tag{2.17}$$

SU(2)quark doublet , $Q_L^n(x)$is the kk left handed doublet Q_R^n is the kk the right handed doublet

$$U(x\,,y) = \frac{1}{\sqrt{\pi R}}\bigg(U_R(x) + \sqrt{2}\sum_{n=1}^{\infty}\left(U_R^n\ (x)\cos\frac{ny}{R} + U_L^n\ (\mathrm{x})\sin\frac{ny}{R}\right)\bigg) \qquad (2{,}18)$$

$SU(2)singlet\ ,up\ type\ quark$

$$d(x\,,y) = \frac{1}{\sqrt{\pi R}}\bigg(d_R(x) + \sqrt{2}\sum_{n=1}^{\infty}\left(d_R^n(x)\cos\frac{ny}{R} + d_L^n\sin\frac{ny}{R}\right)\bigg) \qquad (2.19)$$

SU(2) singlet down type quark

$$L(x\,,y) = \frac{1}{\sqrt{\pi R}}\bigg(L_L(x) = +\sqrt{2}\sum_{n=1}^{\infty}\left(L_L^n(x)\cos\frac{ny}{R} + L_R^n\sin\frac{ny}{R}\right)\bigg) \qquad (2.20)$$

Lepton SU(2) doublet

e (x ,y) =

$$\frac{1}{\sqrt{\pi R}}\left(e_R(x) + \sqrt{2}\sum_{n=1}^{\infty}\left(e_R^n(x)\cos\frac{ny}{R} + e_L^n\ (\mathrm{x})\sin\frac{ny}{R}\right)\right) \qquad (2.21)$$

electron right handed SU(2) singlet.

Chapter III

(3-1)Renormalization group equation

This chapter shall discuss the RGEs and dimensional regularization method. Basically the renormalization theory is implemented to remove all the divergences in loop integrals from the physical measurable quantities. These loop diagrams are supposed to give finite results to the physical quantities but they give infinities instead (L.F.Li, 1991). In order to understand and study some of the issues in the SM listed in chapter 2, such as the mixing angles and fermion masses hierarchies, a great deal of work has gone into analyzing the RGEs of UEDs and their possible extensions (see Refs. (A.~S.~Cornell, 2010) (A.~Abdalgabar, 2013)and references therein).

(3-1-1) RGEs for gauge couplings in SM

In the Standard Model (SM), the one-loop corrections to the gauge couplings are given by

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 \qquad (3.1)$$

Where $b_i^{SM} = \left(\frac{41}{10}, -\frac{19}{6}, -7\right)$, and can be calculated by summing all the contribution in figure 3.1, $t = \ln(\frac{E}{M_Z})$, and M_Z is the Z boson mass. These equations lead directly to the well-known gauge unification around 10^{14} GeV scale.

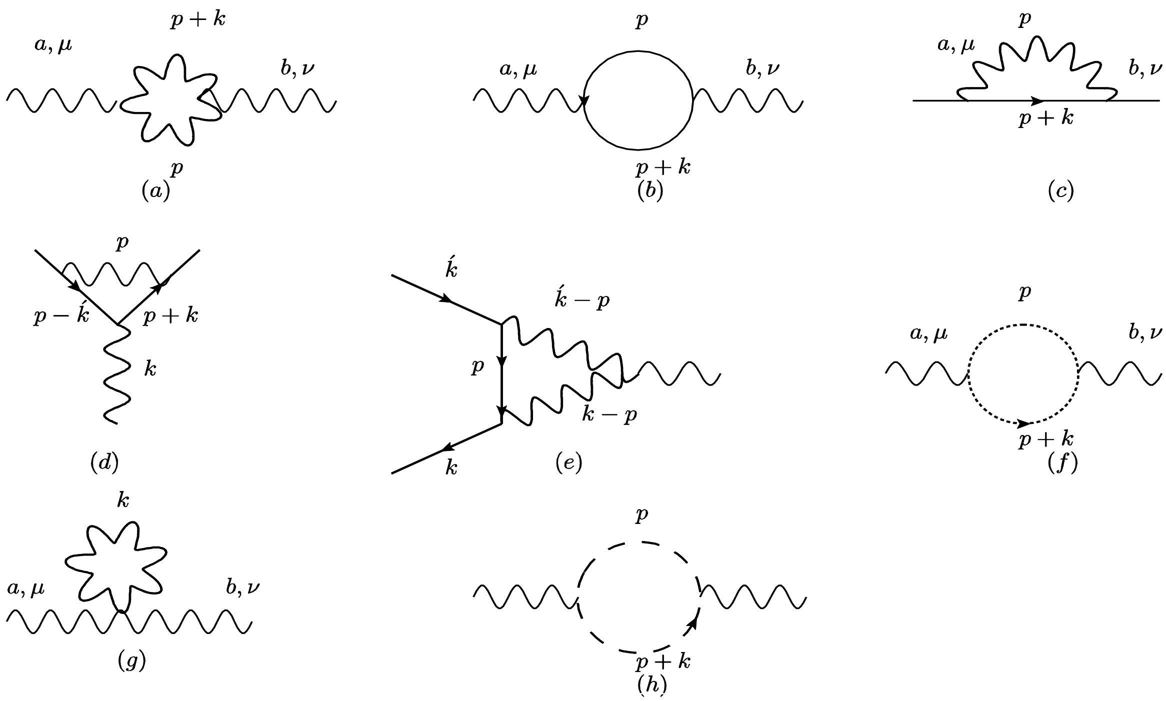

Figure 3.1 Feynman diagrams for self gauge couplings correction up to one-loop level in SM

(3-2-1) RGEs for Guage couplings in five dimensional model

At each excited KK level, the one-loop corrections to the gauge couplings arise from the diagrams exactly mirroring those of the SM ground states. Note that for the closed fermion loop diagrams one need to count the contributions from both the left-handed and right-handed KK modes of each chiral fermion to the self-energy of the gauge filed (A.~Abdalgabar, 2013).

The gauge coupling constants equation is given by

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 + b_i^{5D} g_i^3 \tag{3.2}$$

. Where $b_i^{5D} = \left(\frac{81}{10}, -\frac{7}{6}, \frac{5}{2}\right)$, and, $S(t) = m_Z R e^t = \mu R$, for $m_Z < \mu < \Lambda$ (Λ is the cut-off scale as shall be discussed in more detail latter).

(3-2) Renormalization group equation of Yukawa couplings

The evolution of a generic Yukawa coupling (which describes the fermion-scalar-boson interactions) is given by a beta function. The proper Yukawa vertex renormalization depends on the corresponding beta functions, and includes contributions from the anomalous dimensions of the field operators.

$$\mathrm{Y}^0\bar{\Psi}_\mathrm{L}^0\Psi_\mathrm{R}^0\Phi^0 = \mathrm{Y}^\mathrm{R}\mathrm{Z}_\mathrm{coupling}\bar{\Psi}_\mathrm{L}^\mathrm{R}\Psi_\mathrm{R}^\mathrm{R}\Phi^\mathrm{R} \quad (3.3)$$

We have the normalized fields defined by:-

$$\Psi_L^0 = Z_{\Psi_L}^{\frac{1}{2}}\Psi_L^R \quad (3.4)$$

$$\Psi_R^0 = Z_{\Psi_R}^{\frac{1}{2}}\Psi_R^R \quad (3.5)$$

$$\Phi^0 = Z_\Phi^{\frac{1}{2}}\,\Phi^R \quad (3.6)$$

So equation (3.1) becomes

$$Y^0\,\bar{\Psi}_L^0\,\Psi_R^0\,\Phi^0 = Y^0\left[Z_{\Psi_L}^{\frac{1}{2}}\bar{\Psi}_L^R\right]\left[Z_{\Psi_R}^{\frac{1}{2}}\,\Psi_R^R\right]\left[Z_\Phi^{\frac{1}{2}}\,\Phi^R\right] \quad (3.6)$$

$$= Z_{coupling}\,Y^R\,\bar{\Psi}_L^R\,\Psi_R^R\,\Phi^R \quad (3.6)$$

Therefore

$$Y^0 Z_{\Psi_L}^{\frac{1}{2}} Z_{\Psi_R}^{\frac{1}{2}} Z_{\Phi}^{\frac{1}{2}} =$$
$$Z_{coupling}\, Y^R \tag{3.7}$$

$$Y^0$$
$$= Z_{coupling}\, Z_{\Psi_L}^{\frac{-1}{2}} Z_{\Psi_R}^{-\frac{1}{2}} Z_{\Phi}^{-\frac{1}{2}} Y^R \tag{3.8}$$

We would like to track the arbitrariness in the mass scale μ to that end we take the bone parameter $Y^0$to be independent of μ and deterimind how the renormalized coupling so that λ^0 remain constant, we have therefore

$$\mu \frac{d}{d\mu} \lambda^0$$
$$= 0 \tag{3.9}$$

$$0 = [\mu \frac{d}{d\mu} Z_{coup}][Z_{\Psi_L}^{-\frac{1}{2}} Z_{\Psi_R}^{-\frac{1}{2}} Z_{\emptyset}^{-\frac{1}{2}} Y^R] - \frac{1}{2}\, Z_{coup}\, Z_{\Psi_L}^{\frac{3}{2}} \mu\, \frac{d}{d\mu}\, Z_{\Psi_L}. Z_{\Psi_R}^{\frac{1}{2}} Z_{\Phi}^{-\frac{1}{2}} Y^R -$$
$$\frac{1}{2} Z_{coup}\, Z_{\Psi_L}^{-\frac{1}{2}} Z_{\Psi_R}^{-\frac{3}{2}} \mu \frac{d}{d\mu} Z_{\Psi_R} Z_{\Phi}^{-\frac{1}{2}} Y^R - \frac{1}{2} Z_{coup} Z_{\Psi_L}^{-\frac{1}{2}} Z_{\Psi_R}^{-\frac{1}{2}} Z_{\Phi}^{\frac{3}{2}} \mu \frac{d}{d\mu} Z_{\Phi} Y^R +$$
$$Z_{coup} Z_{\Psi_L}^{-\frac{1}{2}} Z_{\Psi_R}^{-\frac{1}{2}} Z_{\Phi}^{-\frac{1}{2}} \mu \frac{d}{d\mu} Y^R$$

(3.10)

Dividing both side by $Z_{coup} Z_{\Psi_L}^{-\frac{1}{2}} Z_{\Psi_R}^{-\frac{1}{2}} Z_{\Phi}^{-\frac{1}{2}} Y^R$

We obtain:-

$$0 = \mu \frac{1}{Z_{coup}} \frac{d}{d\mu} Z_{coup} - \frac{1}{2}\, \mu\, \frac{d}{Z_{\Psi_L}} \frac{d}{d\mu} Z_{\Psi_L} - \frac{1}{2} \mu \frac{1}{Z_{\Psi_R}}\, \frac{d}{d\mu}\, Z_{\Psi_R} - \frac{1}{2}\, \mu\, \frac{1}{Z_{\Phi}}\, \frac{d}{d\mu}\, Z_{\Phi} +\, \mu\, \frac{1}{Y^R}\, \frac{d}{d\mu}\, Y^R$$

(3.11)

$$0 = \mu \frac{d}{d\mu} \ln Z_{coup} - \frac{1}{2}\, \mu\, \frac{d}{d\mu} \ln Z_{\Psi_L} - \frac{1}{2}\, \mu\, \frac{d}{d\mu} \ln Z_{\Psi_R} - \frac{1}{2}\, \mu\, \frac{d}{d\mu} \ln Z_{\Phi} - \frac{1}{2}\, \mu\, \frac{d}{d\mu}\, \ln Z_{\Phi} +$$
$$\mu\, \frac{d}{d\mu}\, \ln Y^R \tag{3.12}$$

Thus we obtain the renormalization group equation

$$\mu \frac{d}{d\mu} \ln Y^R = \frac{1}{2} \mu \frac{d}{d\mu} \ln Z_{\Psi_L} + \frac{1}{2} \mu \frac{d}{d\mu} \ln Z_{\Psi_R} + \frac{1}{2} \mu \frac{d}{d\mu} \ln Z_{\Phi} - \mu \frac{d}{d\mu} \ln Z_{coup} \qquad (3.13)$$

The gauge g_{i}, Yukawa couplings Y_{i}, RGEs at one-loop in the UED model will be calculated by utilizing the technique of dimensional regularization. Here we show one example figure 3.1b

$$I = \int \frac{d^d p}{(2\pi)^{\mathrm{d}}} \; (-\mathrm{ig}\gamma^{\mu} \mathrm{t}^{\mathrm{a}}) \left(\frac{\mathrm{ig}_{\mu\nu}}{\mathrm{p}^2} \right) (-\mathrm{ig}\gamma^{\nu} \mathrm{t}^{\mathrm{b}}) \frac{\mathrm{i}(\not{\mathrm{p}} + \not{\mathrm{k}})}{(\mathrm{p}+\mathrm{k})^2} \qquad (3.14)$$

$$\mathrm{I} = -\mathrm{g}^2 \, \mathrm{tr}(\mathrm{t}^{\mathrm{a}} \mathrm{t}^{\mathrm{b}}) \int \frac{\mathrm{d}^{\mathrm{d}} \mathrm{p}}{(2\pi)^{\mathrm{d}}} \frac{\gamma^{\mu} \mathrm{g}_{\mu\nu}}{\mathrm{p}^2} \frac{\gamma^{\nu}(\not{\mathrm{p}} + \not{\mathrm{k}})}{(\mathrm{p}+\mathrm{k})^2} \qquad (3.15)$$

Now

$$N(\text{numerator}) = \gamma^{\mu} g_{\mu\nu} \gamma^{\nu} (\not{p} + \not{k}) = \gamma^{\mu} \gamma_{\mu} \gamma^{\rho} (p_{\rho} + k_{\rho}) \qquad (3.16)$$

We have

$$\gamma^{\mu} \gamma^{\rho} \gamma_{\mu} = -(d-2)\gamma^{\rho} \qquad (3.17)$$

Therefore

$$\mathrm{N} = -(\mathrm{d}-2)\gamma^{\rho} (\mathrm{p}_{\rho} + \mathrm{k}_{\rho}) = (\mathrm{p}+\mathrm{k}) \qquad (3.18)$$

$$I = g^2\, T^a\, T^b (d - 2) \int \frac{d^d p}{(2\pi)^d}\, \frac{((\not{p}+\not{k})\,)}{p^2 (p+k)^2} \tag{3.19}$$

We have also the following relation

$$T^a\, T^b = c_2(r) \tag{3.20}$$

Then

$$I = g^2\, (d - 2) c(r)_2 \int \frac{d^d p}{(2\pi)^d}\, \frac{((\not{p}+\not{k}))}{p^2 (p+k)^2} \tag{3.21}$$

To evaluate the above integral we use Feynman parameterization integral:-

$$\frac{1}{ab} = \int dz\, \frac{1}{(b + (a-b)z)^2} \tag{3.22}$$

$$let\ b = p^2\ \ and\ \ a = (p+k)^2 \tag{3.23}$$

Then equation (3.21) becomes

$$I = g^2\, (d - 2) c_2\, (r) \int \frac{d^d p}{(2\pi)^d} \int_0^1 \frac{\left((\not{p} + \not{k})\right)\ dz}{(p^2 + (k^2 + 2pk)z)^2} \tag{3.24}$$

By introducing new variable q

$q = p + kz \Rightarrow dq =$
dp (3.25)

so that $(p^2 + (k^2 + 2pk)z) = (q^2 + k^2 z(1 - z))$.
(3.26)

And the numerator becomes

$$N = \left(\left(\not{p} + \not{k}\right)\right) = (\not{q} - \not{k}z + \not{k})$$
$$= \not{k}(1 - z) \quad (3.27)$$

drop all linear term in q as give zero, we get

$$\Rightarrow g^2(d - 2)c_2(r)\int \frac{d^d q}{(2\pi)^d} \int_0^1 \frac{\not{k}(1-z)dz}{(q^2 + k^2 z(1-z)} \quad (3.28)$$

$$= \int \frac{d^d q}{(2\pi)^d} \frac{\left(\not{k}(1-z)\right)}{\left(p^2 + k^2 z(1-z)\right)^2} \quad (3.29)$$

Comparing Eq (3.29) with the standard integral

$$\int d^d q \, \frac{1}{(q^2 + S + i\varepsilon)^2} = \frac{i\pi^{\frac{d}{2}}\,\Gamma(2 - \frac{d}{2})}{S^{2-\frac{d}{2}}} \quad (3.30)$$

we obtain

$$I = \int \frac{d^d q}{(2\pi)^d} \, \frac{\not{k}(1-z)}{(q^2 + k^2 z(1-z) + i\varepsilon)^2} = \frac{\not{k}(1-z)}{(2\pi)^d} \left(\frac{i\,\pi^{\frac{d}{2}}\,\Gamma\left(2 - \frac{d}{2}\right)}{\left(k^2 z(1-z)\right)^{2-\frac{d}{2}}} \right) \quad (3.31)$$

So

$$I = i\,\pi^{\frac{d}{2}} \, \frac{g^2(d-2)\,c_2\,(r)\,\not{k}\,\Gamma(2-\frac{d}{2})}{(2\pi)^d\,(k)^{2^{2-\frac{d}{2}}}} \int_0^1 \frac{(1-z)dz}{(z(1-z))^{2-\frac{d}{2}}} \quad (3.32)$$

$$I = \int_0^1 \frac{(1-z)dz}{(z(1-z))^{2-\frac{d}{2}}}$$

$$= \int_0^1 dz\, z^{\frac{d}{2}-2}\ (1-z)^{\frac{d}{2}-1} \qquad (3.33)$$

Compare Eq (2.33) by β(m ,n) function

$$\beta(m\,,n) = \int_0^1 x^{m-1}\,(1-x)^{n-1}\,dx \qquad (3.34)$$

$$\mathrm{I} = \int_0^1 \mathrm{z}^{\left(\frac{\mathrm{d}}{2}-1\right)-1}\,(1-\mathrm{z})^{\frac{\mathrm{d}}{2}-1}\mathrm{dz} \qquad (3.35)$$

$$\mathrm{m} = \frac{d}{2} - 1 \quad \text{and n} = \frac{\mathrm{d}}{2} \qquad (3.36)$$

$$\beta(\mathrm{m}\,,\mathrm{n}) = \frac{\Gamma(\mathrm{m})\Gamma(\mathrm{n})}{\Gamma(\mathrm{m}+\mathrm{n})} \qquad (3.37)$$

$$\beta\left(\frac{\mathrm{d}}{2}-1\,,\frac{\mathrm{d}}{2}\right) = \frac{\Gamma\left(\frac{\mathrm{d}}{2}-1\right)\Gamma\left(\frac{\mathrm{d}}{2}\right)}{\Gamma(\mathrm{d}-1)} \qquad (3.38)$$

Therefore

$$\mathrm{I} = \frac{\mathrm{i}\pi^{\frac{\mathrm{d}}{2}}\,\mathrm{g}^2\,(\mathrm{d}-2)(\,\mathrm{c}_2(\mathrm{r})\cancel{\mathrm{k}}\,\Gamma\left(2-\frac{\mathrm{d}}{2}\right)\Gamma\left(\frac{\mathrm{d}}{2}-1\right)\Gamma\left(\frac{\mathrm{d}}{2}\right)}{(2\pi)^{\mathrm{d}}\,\mathrm{k}^{2^{2-\frac{\mathrm{d}}{2}}}\,\Gamma(\mathrm{d}-1)} \qquad (3.39)$$

$$I = \frac{i\pi^{\frac{d}{2}} g^2(4-2)c_2\,(r)\, \not{k}\, \Gamma\left(2-\frac{d}{2}\right)\Gamma(2-1)\Gamma(2)}{(2\pi)^4\, k^{2^{2-\frac{d}{2}}}\Gamma 3} \tag{3.40}$$

$$I = \frac{i\, g^2\, c_{2(r)}\, \not{k}\, \Gamma(\, 2-\frac{d}{2})}{(4\pi)^2\, k^{2^{2-\frac{d}{2}}}} \tag{3.41}$$

(3-3)The beta function for the Yukawa couplings

The beta function for Yukawa couplings in five dimensions can be written as (A.~S.~Cornell, 2010)

$$16\pi^2 \frac{dY}{dt} = \beta_Y^{SM} = +\beta_Y^{UED} \tag{3.42}$$

And

$$\beta_{Yu}^{UED} = (s-1)\{-\left(\frac{28}{3}g_3^2 + \frac{15}{8}g_2^2 + \frac{101}{120}g_1^2\right) + \frac{3}{2})(y_u^2 - y_d^2)\}y_u + 2(s-1)[\, Y_l + 3Y_u + 3Y_d]y_u \tag{3.43}$$

$$\beta_{Yd}^{UED} = (s-1)[-\left(\frac{28}{3}g_3^2 + \frac{15}{8}g_2^2 + \frac{9}{40}g_1^2\right) + \frac{3}{2}\left(y_u^2 + y_d^2\right)]y_u + 2(s-1)[Y_l + 3Y_u + 3Y_d]y_u \tag{3.44}$$

The standard model REGs for the Yukawa coupling reads

$$\beta_{Yu}^{SM} = Y_u \left[\mathrm{Tr}\left(Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + 3Y_e^\dagger Y_e\right) - \left(\frac{17}{12} g_1^2 + \frac{9}{4} g_2^2 + 8g_3^2\right) + \frac{3}{2}\left(Y_u^2 - Y_d^2\right) \right] \qquad (3.45a)$$

$$\beta_{Yd}^{SM} = Y_d \left[\mathrm{Tr}\left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + 3Y_e^\dagger Y_e\right) - \left(\frac{5}{12} g_1^2 + \frac{9}{4} g_2^2 + 8g_3^2\right) + \frac{3}{2}\left(Y_d^2 - Y_u^2\right) \right] \qquad (3.45b)$$

We need to rescale $\frac{17}{12}\, g_1^2 \rightarrow \frac{17}{12} \times \frac{3}{5} g_1^2 = \frac{17}{20} g_1^2$.

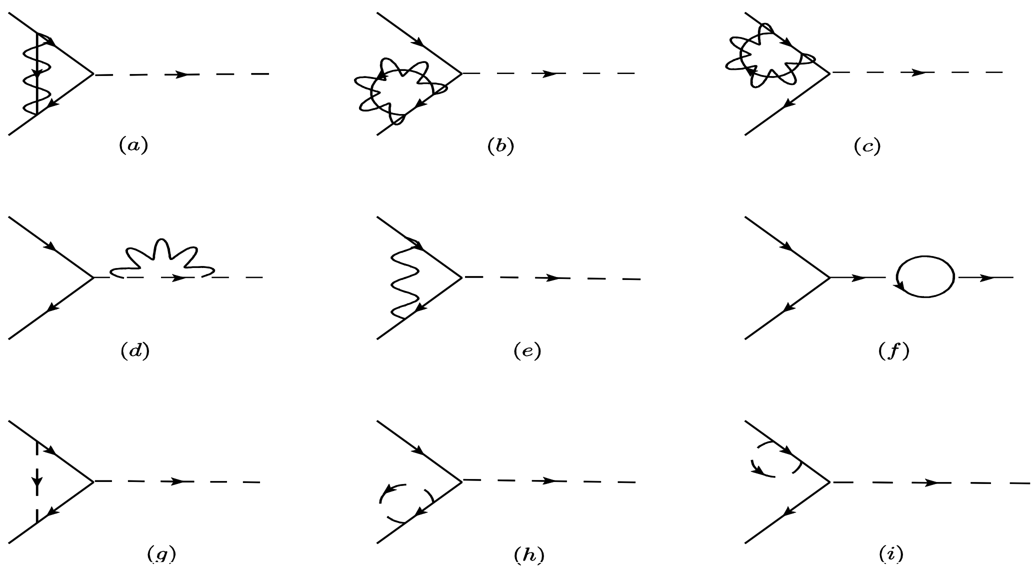

Figure 3.2: Diagrams contributing to Yukawa coupling RGEs in five dimensional models in the Landau gauge. Solid (broken) lines correspond to fermions (SM scalars), while wavy lines (wavy+solid lines) represent ordinary gauge bosons (fifth components of gauge bosons).

To calculate the factor g_3^2 in equation (3.43) we use dimensional regularization to calculate the contribution from figure 3.2(a), we get

$$Z_{coup} = 1 - g_3^2 \, \frac{8}{6} \, \frac{1}{16\pi^2} \, \frac{1}{\varepsilon} \, (\mu^2)^{-\varepsilon} \qquad (3.46)$$

Then

$$-\mu\,\frac{\partial}{\partial\mu}\ln Z_{coup}$$

$$= -g_3^2\,\frac{8}{3}\,\frac{1}{16\pi^2} \qquad (3.47)$$

Calculation of figure 3.2 (b) gives

$$Z_{u_R}$$

$$= 1 - g_3^2\,\frac{8}{6}\,\frac{1}{16\pi^2}\,\frac{1}{\varepsilon}\,(\mu^2)^{-\varepsilon}\,\frac{1}{2} \qquad (3.48)$$

then

$$\frac{1}{2}\,\mu\,\frac{\partial}{\partial\mu}\ln Z_{u_R}$$

$$= \frac{1}{2}\,g_3^2\,\frac{8}{6}\,\frac{1}{16\pi^2} \qquad (3.49)$$

Likewise figure 3.2 (c) give us

$$\frac{1}{2}\,\mu\,\frac{\partial}{\partial\mu}\ln Z_{q_l}$$

$$= \frac{1}{2}\,g_3^2\,\frac{8}{6}\,\frac{1}{16\pi^2} \qquad (3.50)$$

And figure 3.2(e) similar to the standard model as in equation (3.45a) give us

$$-\mu\,\frac{\partial}{\partial\mu}\ln Z_{coup}$$

$$= -8\,g_3^2\,\frac{1}{16\pi^2} \qquad (3.51)$$

Consider Equations (3.55), (3.56), (3.58), and (3.59) gives us

$$-g_3^2\,\frac{8}{3}\,\frac{1}{16\pi^2} + \frac{1}{2}\,g_3^2\,\frac{8}{6}\,\frac{1}{16\pi^2} + \frac{1}{2}g_3^2\,\frac{8}{6}\,\frac{1}{16\pi^2} - 8\,g_3^2\,\frac{1}{16\pi^2} =$$

$$-\frac{28}{3}\,g_3^2\,\frac{1}{16\pi^2} \qquad (3.52)$$

which exactly matches the factor of g_3^2 in equation (3.43)

Calculate the factor of the g_2^2 in equation (3.43)

Consider figure 3.2 (i)

$$\frac{1}{2}\,\mu\,\frac{\partial}{\partial\mu}\,\ln Z_{q_L}$$

$$= \frac{1}{2}\,g_2^2\,\frac{3}{4}\,\frac{1}{16\pi^2} \qquad (3.53)$$

And figure 3.2 (f) is identical to the standard model in equation (3.45 a) gives us

$$\frac{1}{2}\,\mu\,\frac{\partial}{\partial\mu}\,\ln Z_{\Phi}$$

$$= -\frac{9}{4}\,g_2^2\,\frac{1}{16\pi^2} \qquad (3.54)$$

And figure 3.2 (h) has no contribution to g_2^2 since the right handed fermion does not couple to W bosons

Then consider equation (3.61) + equation (3.62) gives us

$$\frac{1}{2}\,g_2^2\,\frac{3}{4}\,\frac{1}{16\pi^2} - \frac{9}{4}\,g_2^2\,\frac{1}{16\pi^2}$$

$$= -\frac{15}{8}\,g_2^2\,\frac{1}{16\pi^2} \qquad (3.55)$$

Which exactly matches the factor of the g_2^2 in Eq (3.45)

Calculate the factor of the g_1^2 in Eq (3.45)

Similar to figure 3.2 (i), the calculation is similar, only the vertex factor is different, so we take

off the $C_2(r) = \frac{3}{4}$ and replaced by $(\frac{Y_Q}{2})^2 = (\frac{1}{6})^2$

$$\frac{1}{2}\,\mu\,\frac{\partial}{\partial\mu}\,\ln Z_{q_L}$$

$$= \frac{1}{2}\,g_1^2\,(\frac{1}{6})^2\,\frac{1}{16\pi^2} \qquad (3.56)$$

Similar to Figure 3.2.(h) in equation (3.45), the calculation is similar, only the vertex factor is different, so we take off the $C_2(r) = \frac{8}{6}$ and replaced by $(\frac{Y_u}{2})^2 = (\frac{2}{3})^2$

$$\frac{1}{2}\ \mu\ \frac{\partial}{\partial\mu}\ \ln Z_{u_R}$$

$$= \frac{1}{2}\ g_1^2\ (\frac{2}{3})^2\ \frac{1}{16\pi^2} \qquad (3.57)$$

Similar to Figure 3.2.(g) in Eq.(3.45), the calculation is similar, only the vertex is different, so we take off the $C_2(r) = \frac{8}{6}$ and replaced by $\frac{Y_Q}{2}\frac{Y_u}{2} = \frac{1}{6}\frac{2}{3}$

$$Z_{\mathrm{coup}}$$

$$= 1 - g_1^2\ \frac{1}{6}\ \frac{2}{3}\ \frac{1}{16\pi^2}\ \frac{1}{\varepsilon}\ (\mu^2)^{-\varepsilon} \qquad (3.58)$$

Then

$$-\mu\ \frac{\partial}{\partial\mu}\ \ln Z_{\mathrm{coup}} = -g_1^2\ 2.\frac{1}{6}\ .\frac{2}{3}\ \frac{1}{16\pi^2}$$

$$= -g_1^2\ \frac{2}{9}\ \frac{1}{16\pi^2} \qquad (3.59)$$

And figure 3.2 (e) + figure 3.2 (f) like standard model in equation (3.45a) gives us $-\frac{17}{12}\ g_1^2\ \frac{1}{16\pi^2}$

Then consider equation the above equation and equations (3.56), (3.57), (3.59) we get

$$-\frac{17}{12}\ g_1^2\ \frac{1}{16\pi^2} + \frac{1}{2}\ g_1^2\ (\frac{1}{6})^2\ \frac{1}{16\pi^2} + \frac{1}{2}\ g_1^2\ \left(\frac{2}{3}\right)^2\ \frac{1}{16\pi^2} - g_1^2\ \frac{2}{9}\ \frac{1}{16\pi^2} = \frac{1}{16\pi^2}\ g_1^2\ \left(-\frac{17}{12} + \frac{1}{72} + \frac{2}{9} - \frac{2}{9}\right) = -\frac{1}{16\pi^2}\ g_1^2\ \frac{101}{72}$$

$$(3.60)$$

Rescale it with SU(5) normalization we get

$$-\frac{1}{16\pi^2}\,g_1^2\,\frac{101}{72}\rightarrow-\frac{1}{16\pi^2}\,g_1^2\,\frac{101}{72}\times\frac{3}{5}$$

$$=-\frac{1}{16\pi^2}\,g_1^2\,\frac{101}{120} \tag{3.61}$$

Note the equations (3.52) (3.55) and (3.61) are terms appear in the beta function of the Yukawa coupling in equation (3.43).

Chapter IV

(4-1)Numerical anlysis and Discussions

For our numerical calculations we have chosen the compactification scale to be $R^{-1} = 1\ \text{TeV}, 5\ \text{TeV}$ and $13\ \text{TeV}$. Only some selected figures will be shown and we will comment on the other similar cases not explicitly presented here. We quantitatively analyses these quantities in UED model, though we observed similar behaviors for all values of R^{-1}. The initial value we shall adopt at the M_z scale is presented in table 3.1.

Table 3.1. show the initial values at M_z scale used in our numerical calculations. Data is taken from Ref. (Z.~-z.~Xing, 2008)

Parameter	**Value (90% CL)**
$\alpha_1(M_z)$	0 • 01696
$\alpha_2(M_z)$	0 • 03377
$\alpha_3(M_z)$	0 • 1184
$m_u(M_z)$	0.00127 • • •
$m_c(M_z)$	0 • 619 • • •
$m_t(M_z)$	171 • 7 • • •
$m_d(M_z)$	2 • 90 • • •
$m_s(M_z)$	55 • • •
$m_b(M_z)$	2 • 83 • • •
$m_e(M_z)$	0 • 48657 • • •
$m_\mu(M_z)$	0.102718 • • •
$m_\tau(M_z)$	1.74624 • • •

As depicted in figure. 4.1. and figure 4.2. and equation (3.1) and (3.2), the one-loop evolution of the gauge couplings varies with energy scale drastically and brings the unification scale to lower value considerably. For instant, for the compactification scale $R^{-1} = 1$ TeV, 5 TeV and 13 TeV, we found that the gauge couplings approximately meet at around $10^{4.30}$ GeV = 20 TeV, $10^{4.97}$ GeV = 93.3 TeV and $10^{5.36}$ GeV = 229TeV respectively.

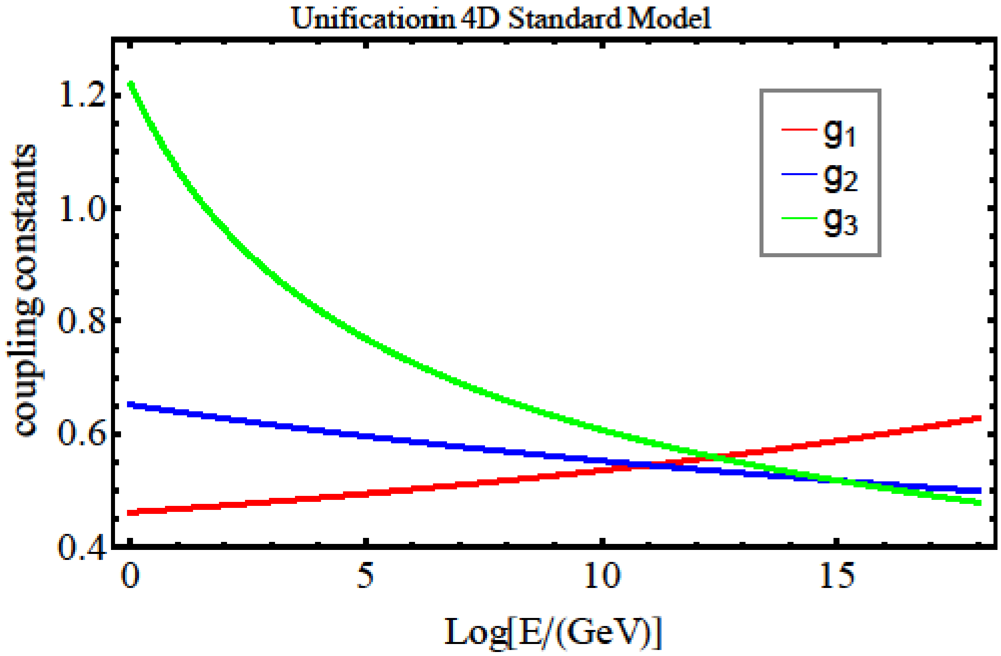

Figure 4.1. Running of Gauge couplings as a function of the energy in the Standard model

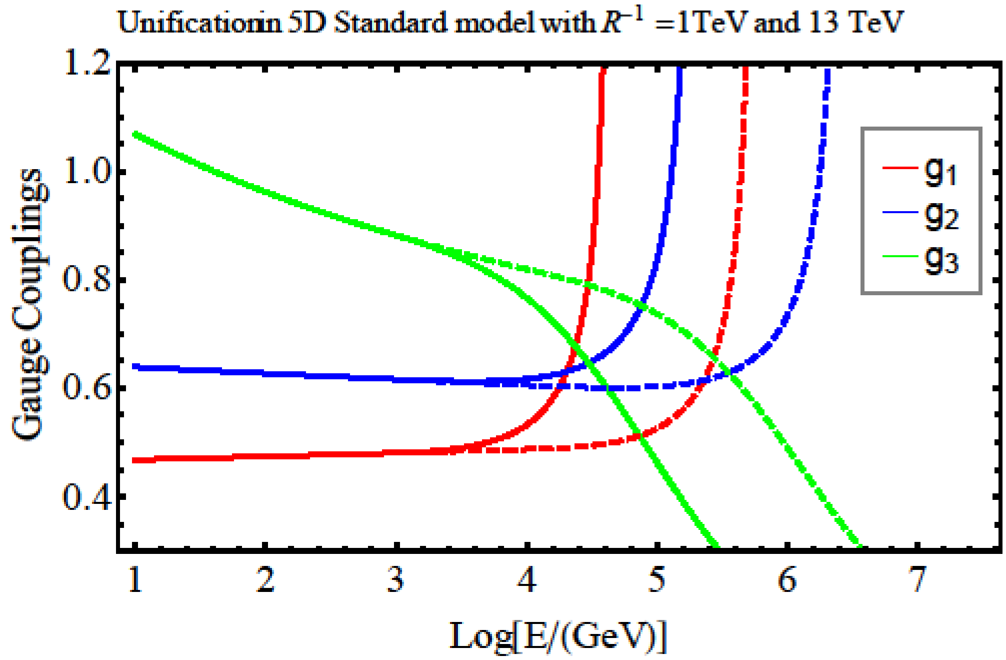

Figure 4.2. Running of gauge couplings as a function of the energy scale in UED model for three different compactification scales.

The Yukawa couplings also receive finite corrections at each KK level whose magnitudes depend on the cutoff energy scale, but we know that the mass of fermions is proportional to Yukawa couplings. As such the hierarchy between the first two light generations, in the leading order approximation, we found that the running behaviors of the mass ratios are controlled by the combination of the third family Yukawa couplings and the CKM matrix elements. This indicates that the mass ratios of the first two light generations have a slowed running well before the unification scale (where the gauge couplings meet). After that point new physics would come into life and should be accounted (for example, see figure 4.3).

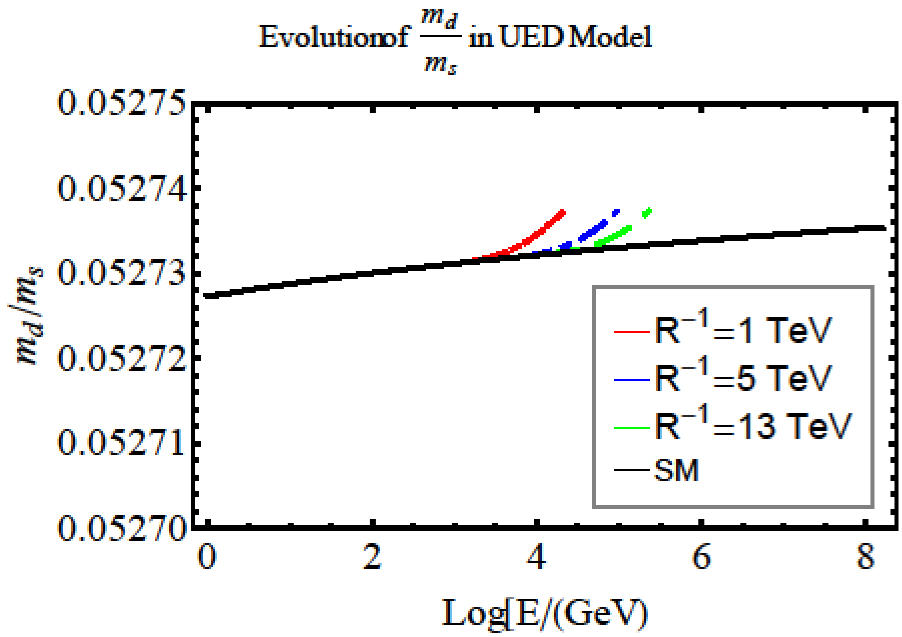

Figure 4.3. Evolution of m_d/m_s for three different compactification scales as function of energy scale.

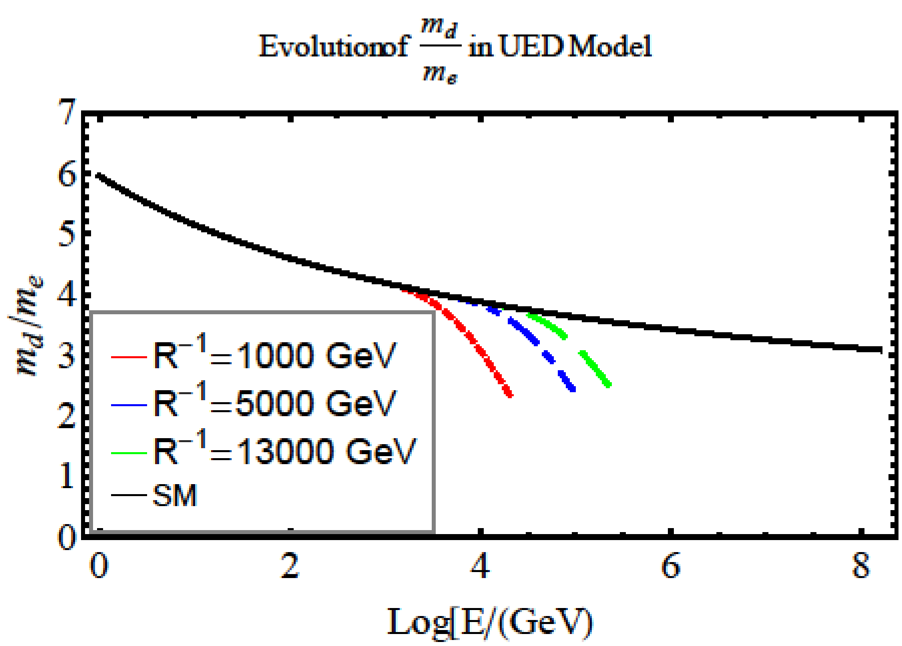

Figure 4.4. Evolution of m_d/m_e for three different compactification scales as function of energy scale.

Quantitatively, similar to the conclusions found in the SM, here we found the scaling dependence of m_u/m_c and m_e/m_μ is also have very slow running.

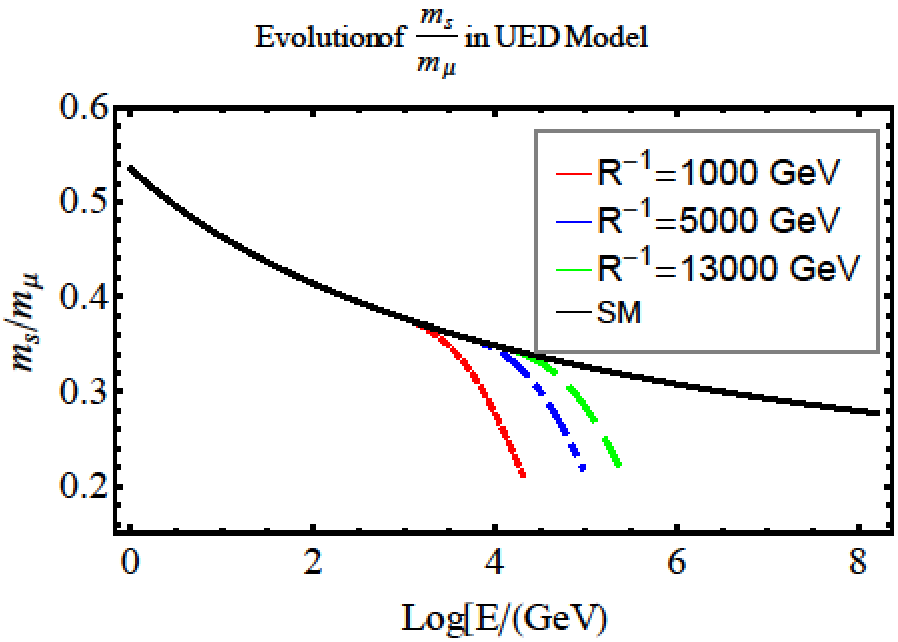

Figure 4.5. Evolution of m_s/m_μ for three different compactification scales as function of energy scale.

On the other hand, in Grand Unification Theories, such as the SU(5) and SO(10) theory, the quark and lepton fields are on the same footing when we fill out the field multiplet for the group representation. From the mass matrix relation we have $m_d = m_e$, $m_s = m_\mu$ and $m_b = m_\tau$ at the unification scale. These relations hold such that the differences of their mass values at the electro-weak scale are understood as a running effect. In the UED model, due to the power law enhancement of the Yukawa couplings, the RGEs effect on these relations can be large. In figure 4.4 , figure 4.5 and figure 4.6. we highlighted the numerical analysis of the one-loop calculation of the mass ratiosm_d/m_e, m_s/m_μ and m_b/m_τ respectively.

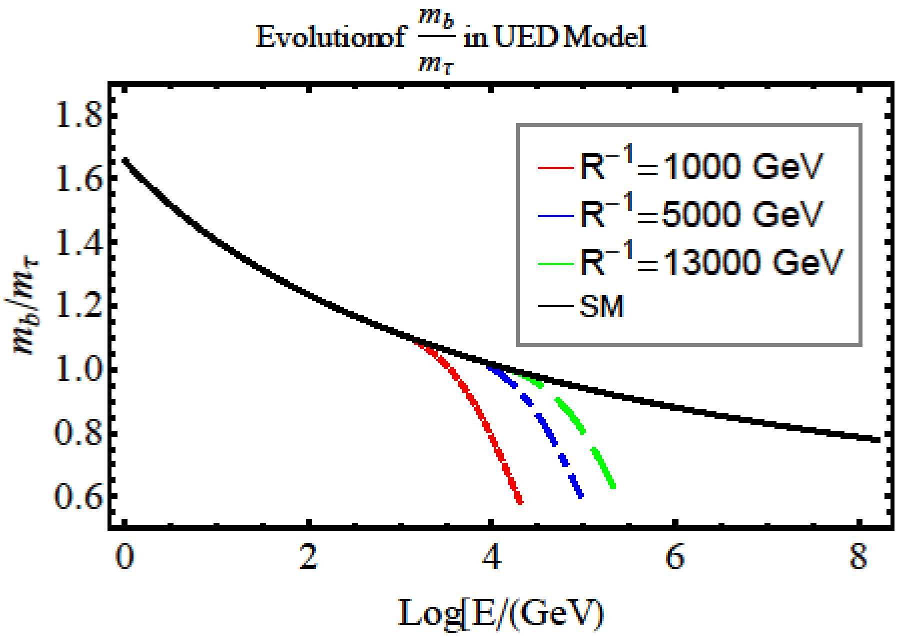

Figure 4.6. Evolution of m_b/m_τ for three different compactification scales as function of energy scale.

As illustrated, the mass ratios run in the usual logarithmic fashion when the energy is below 1TeV, 5TeV, and 13TeV for the three different compactification cases. However, once the first KK threshold is reached, the contributions from the KK states become more and more significantly important, at which point their evolution deviates from their normal trajectory and begin to run rapidly. As observed, the mass ratios decreasing with increasing energy, which agrees with what is observed in the SM, however, the mass ratios decrease at a much faster rate.

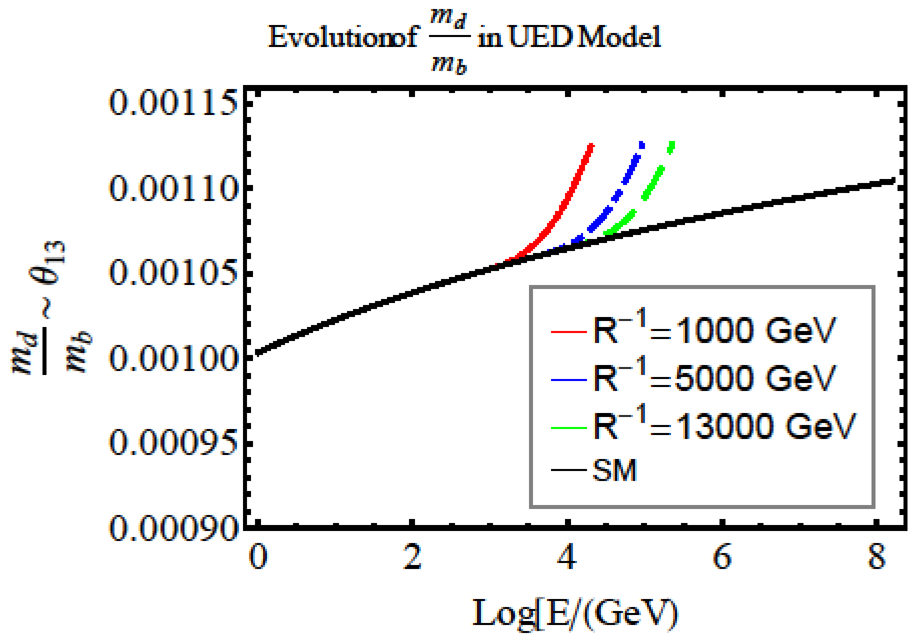

Figure 4.7. Evolution of m_d/m_s for three different compactification scales as function of energy scale.

In the UED model the mass ratios for the three families have a sizable variation, which is more than 60% across the whole range, and this is almost twice as great as that of the SM. This is an interesting feature that distinguishes these two models. Therefore, due to the fast power law running the unification of the Yukawa couplings is very desirable, where this feature has the potential to address the problem of fermion mass hierarchy.

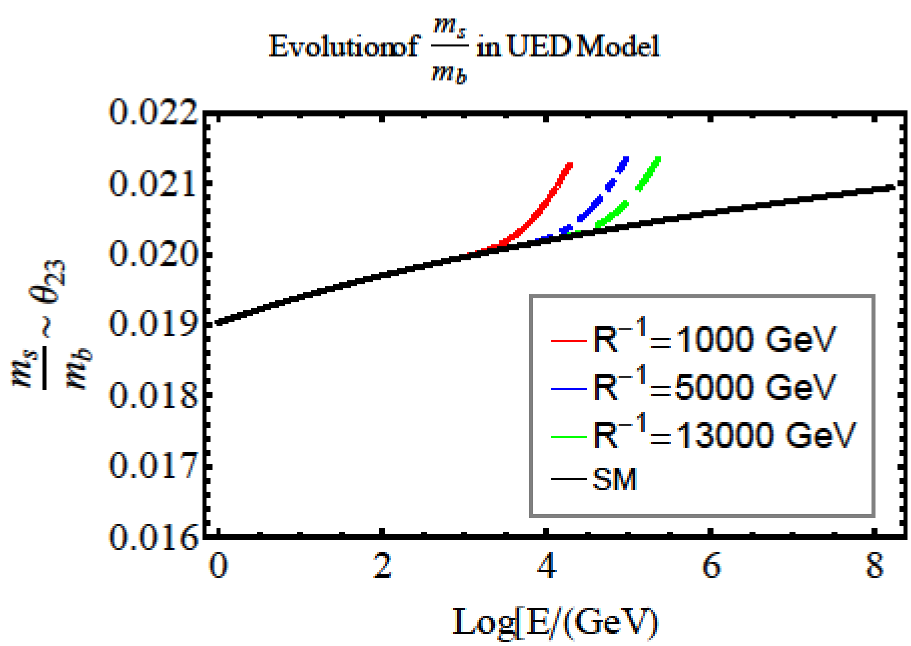

Figure 4.8. Evolution of m_d/m_s as function of energy scale

On the other hand, in the quark sector, both the mass ratios and mixing parameters exhibit rather large hierarchies. At the electroweak scale the observed pattern of fermion masses and mixings does not look accidental see figure 4.7, figure 4.8 and figure 4.9.

$$\theta_{13} \sim \frac{m_d}{m_b}$$

and

$$\theta_{23} \sim \frac{m_s}{m_b}.$$

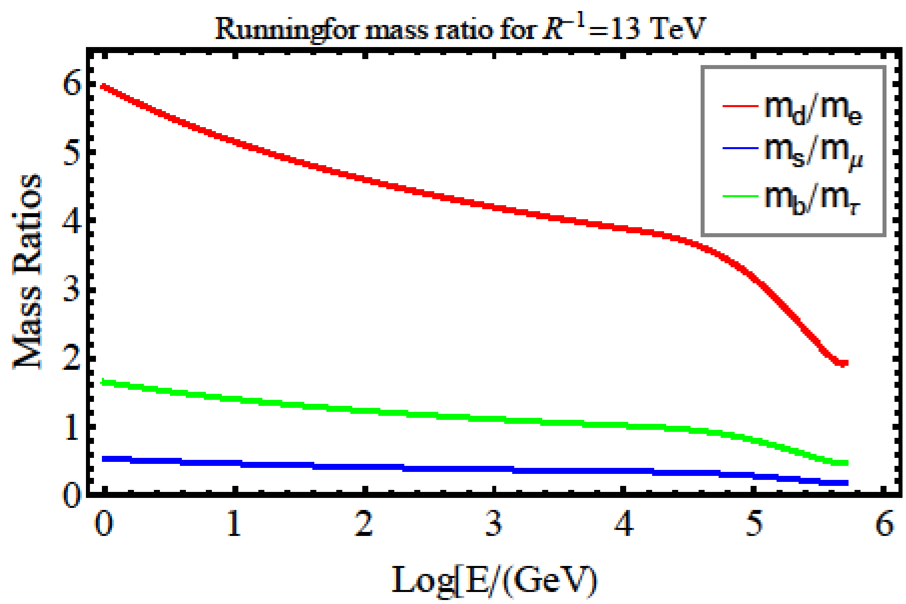

Figure 4.9. Evolution of m_q/m_l for three different compactification scales as function of energy scale

(4-2)Conclusions

To conclude, UED models with compactification radius near the TeV scale implies exciting phenomenology for collider physics. It is found that the running of the gauge couplings has a rapid variation in the presence of the KK modes and this leads to a much lower unification scale than the SM. The running of mass ratios for the three families has a sizable variation in UED model. We quantitatively discussed these quantities for $\mathrm{R}^{-1} =$

$1\ \mathrm{TeV}, 5\ \mathrm{TeV}$ and $13\ \mathrm{TeV}$ observing similar behaviors for all values of the compactification radius below these scale their evolution run in the usual SM logarithmic fashion. We have shown that the scale dependence is not logarithmic; it shows a power law behavior. The UED model has substantial effects on the hierarchy between the quark and lepton sectors and provides a very desirable scenario for grand unification.

(4-3)Recommendation

This work can be extended in a number of ways and we discuss just a few. In this work we considered only the bulk scenarios in which all SM field have access to full space. We leave other possibilities for future work in which the 1st and 2nd generation are in the bulk, with the 3rd generation either in the bulk or on a brane.

It is Also important to confirm these results and conclusions made at one loop that are sensitive to this scale are still consistent and under control at two (and higher) loops. For instance one might be concerned that one loop linear sensitivity to the cutoff behaving as ΛR do not result in terms of the form $(\Lambda R)^2$ at two-loop, which would then indicate a breakdown of perturbation theory at renormalization scales of the order of the compactification radius.

Bibliography

A.~Abdalgabar, A. A. a. A., 2013. Evolution of Yukawa couplings and quark flavour mixings in 2UED models. *Phys.\ Rev.\ D,* Volume 88, p. 056006.

A.~S.~Cornell, L.-X. a., 2010. Scaling of Yukawa Couplings and Quark Flavor Mixings in the UED Model. *PoS KRUGER,* Volume 2010, p. 045.

A.D.Martin, F. a., 1984. *Quarks and leptons:An introductory course in modern particle physices.* new york : John Wiley.

A.J.G.hey, I. a., 1993. *Gague theories in particlphysics.* second edition ed. s.l.:Bristol and philadelphia.

al, G. A. e., 2012. ATLAS collaboration. *Phys. Lett,* Volume 1, p. B716.

al, S. C. e., 2012. CMS collaboration. *Phys.Lett,* Volume 30, p. B716.

Collaboration], E. {. e. a. [., 2011. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. *Astrophys.\ J.\ Suppl.,* Volume 192, p. 18.

E.~Poppitz, B. a., 2001. Number of fermion generations derived from anomaly cancellation. *Phys.\ Rev.\ Lett.,* Volume 87, p. 031801.

Falcone, D., 2002. Fermion masses and mixings in gauge theories. *Int. J. Mod. Phys.,* Volume A17, p. 3981.

Guigg, c., 1983. *Gauge theories of the strong,weak and electromagnetic interaction.* s.l.: Benjiamin/cummings.

H.~-U.~Yee, T. a., 2003. Universal extra dimensions and the Higgs boson mass. *Phys.\ Rev.\ D,* Volume 67, p. 055002.

J.donoghue, E. a. B., 1994. *Dynamics of the standard model,Cambridge Monographs on particle ,Nuclear physics and Cosmology.* s.l.:s.n.

K.~S.~Babu, 1987. Renormalization Group Analysis Of The Kobayashi-maskawa Matrix. *Z.\ Phys. \ C,* Volume 35, p. 69.

L.~-X.~Liu, 2010. Renormalization Invariants and Quark Flavor Mixings. *Int.\ J.\ Mod.\ Phys.,* Volume A25, p. 4975.

L.~-X.~Liu, A. a., 2011. Evolution of the CKM Matrix in the Universal Extra Dimension Model. *Phys.\ Rev.\ D,* Volume 83, p. 033005.

L.F.Li, T. a., 1991. *Gauge theory of elementary particle physics.* s.l.:oxford univ.

McMahon, D., 2008. *Quantum Field theory Demystified.* s.l.:McGraw-Hill companies.

N.~Maru, T. J. a. M., 2010. The Universal Extra Dimensional Model with S**2/Z(2) extra-space. *Nucl.\ Phys.\ B,* Volume 830, p. 414.

P.~W.~Higgs, 1964. Broken symmetries, massless particles and gauge fields. *Phys.\ Lett.,* Volume 12, p. 132.

Quigg, C., 2007. *spontaneous symmetry Breaking as a Basic of particle mass,* s.l.: arXIV:0704.2232.

S.L.Glashow, 1961. *Nucl. Phys,* Volume 22, p. 579.

Salam, A., 1968. Elementary Partcile Physics (Nobel Symp. N.8). p. 367.

T.~Appelquist, H.-C. a. B., 2001. Bounds on universal extra dimensions. *Phys.\ Rev.\ D,* Volume 64, p. 035002.

T.~Kaluza, 1921. On the Problem of Unity in Physics. *Sitzungsber.\ Preuss.\ Akad.\ Wiss.\ Berlin (Math.\ Phys.\ ),* Volume 1921, p. 966.

Veltman, M., 2003. *Facts and mysteries in elementary particls physics,* s.l.: Co.Pte.Ltd.

Weinberg, S., 1967. *Phys. ReV. Lett,* Volume 19, p. 1264.

Weinberg, S., 1996. *the Quantum theory of field.* s.l.:Cambridge University.

Z.~-z.~Xing, H. a., 2000. Mass and flavor mixing schemes of quarks and leptons. *Prog.\ Part.\ Nucl.\ Phys.,* Volume 45, p. 1.

Z.~-z.~Xing, H. a. S., 2008. Updated Values of Running Quark and Lepton Masses. *Phys.\ Rev.\ D,* Volume 77, p. 113016.