# **Dedication**

- To My father spiritual God's mercy,
  - My dear mother,
    - And Loyal friends.

# Acknowledgements

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### **Abstract**

We show that Primitively of unital full free products of residually finitedimensional  $C^*$ -algebra and clear that the homeomorphisms into Z-Stable  $C^*$ -algebra. We explained that the exponential rank and exponential length to  $C^*$ -algebra and Z-Stable as well as we explained equivalence approximate unitary of tracial Rank one. We studied homology of unitaries in simple  $C^*$ -algebra and through that we present to the result of equivalence approximate unitary with tracial rank one. Finally we give the uniqueness and the theory of existence and classification theorms to obtain nuclear simple  $C^*$ -algebraresults of tracial topological rank one.

#### الخلاصة

اثبتنا بدائية نواتج الضرب الكامل الحر الاحادي للبعد المنتهي المتبقي في جبر و بينا -\* و C\* التشاكلات في جبر \* و C\* المستقر. وضحنا الرتبة الاسية والطول الاسي لجبر C\* و المستقر وكذلك وضحنا لتكافؤ الوحدوي التقريبي من تاثير الرتبة الاولى. درسنا علم التشاكل للاحاديات في جبر C\* البسيط ومن خلاله توصلنا إلي نتيجة التكافؤ الأحادي التقريبي مع تأثير الرتبة الأولى. وأخيراً قدمنا نظرية التفرد ونظرية الوجود والتنصيف للحصول علي نتائج بسيطة نووية جبر C\* من تأثير بنية الرتبة الأولى.

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