Chapter I

1.1. General Introduction of Elementary Particles

1.1.1. What is Matter Made of?

Matter at the subatomic level consists of tiny particles, with vast empty spaces in between. Even more remarkable, these tiny particles come in a small number of different types (electrons, protons, neutrons, pi mesons, neutrinos, and so on). Particles that make up matter called atoms.

1.1.2. How do we Produce Elementary Particles?

Electrons and protons are no problem to produce; these are the stable constituents of ordinary matter. To produce electrons we simply heats up a piece of metal, and they come boiling off. If we want a beam of electrons, we then set up a positively charged plate nearby, to attract them over, and cuts a small hole in it; the electrons that make it through the hole constitute the beam. To obtain protons we ionize hydrogen (in other words, strip off the electron). In fact, if we're using the protons as a target, we don't even need to bother about the electrons; they're so light that an energetic particle coming in will knock them out of the way. Thus, a tank of hydrogen is basically a tank of protons. For more exotic particles there are three main sources:

Cosmic Rays: The earth has been constantly bombarded with high-energetic particles (mainly protons) coming from outer space. What is the source of these particles might be remains as something of mystery; at any rate, when they hit atoms in the upper atmosphere they produce showers of secondary particles (mostly muons, by the time they reach ground level), which rain down on us all the time. As a source of elementary particles, cosmic rays have two advantages: they are free, and their energies can be enormous-far greater than we could possibly produce in the laboratory. But they have two major disadvantages: The rate at which they strike any detector of reasonable size is very low, and they are completely uncontrollable. So cosmic ray experiments need patience and luck (J.donoghue, 1994).

Nuclear Reactors: When a radioactive nucleus disintegrates, it may emit a variety of particles-neutrons, neutrinos, what is used to be called alpha rays (actually, alpha particles, which are bound states of two neutrons plus two protons), beta rays (actually, electrons or positrons), and gamma rays (actually, higher energetic photons).

Particle Accelerators: We start with electrons or protons, accelerate them to high energy, and smash them into a target. By skillful arrangements of absorbers and magnets, we can separate out of the resulting debris the particle species we wish to study. Nowadays it is possible in this way to generate intense secondary beams of positrons, muons, pions, kaons, and antiprotons, which in turn can be fired at another target. The stable particles-electrons, protons, positrons, and antiprotons-can even by fed into giant storage rings in which they would be guided by powerful magnets, and circulate at high speed for hours at a time, to be extracted and used at the required moment, In general, the heavier of the particle we want to produce, the higher must be the energy of the collision. That's why, historically, lightweight particles tend to be discovered first, and as time goes on, and accelerators become more powerful, heavier and heavier particles are found, at present, the heaviest known particle is the Z, with nearly 100 times the mass of the proton. It turns out that the particle gains enormously energy if we collide two high-speed particles head-on, as opposed to firing one particle at a stationary target; there is another reason why particle physicists are always pushing towards higher energies. In general, the higher the energy of the collision, the closer the two particles come to one another. So if we want to study the interaction at very short range, we need very energetic particles (A.D.Martin, 1984).

1.1.3. How do we detect Elementary Particle?

There are many kinds of particle detectors-Geiger counters, cloud chambers, bubble chambers, spark chambers, photographic emulsions, Cerenkov counters, scintillates and photomultipliers. Actually, a typical modern detector has whole arrays of these devices, wired up to a computer that tracks the particles and displays their trajectories on a television screen, Most detection mechanisms rely on the fact that when high-energy charged particles pass through matter they ionize atoms along their path. The ions then act as "seeds" in the formation of droplets (cloud chamber) or bubbles (bubble chamber) or sparks (spark chamber), depending on of the detector the case. But electrically neutral particles do not cause ionization, and they leave no tracks (Veltman, 2003).

1.1.4. Particles Accelerator and High Energy Physics:

The study of particles physics began with the discovery of electron in 1897 by J.J.Thomson. Around 1930 and above, new particles were detected using Cosmic Rays as a source of energy, since it was the only high energy source known by then, starting with the discovery of the positron in 1931, and the muon in 1937 (A.D.Martin, 1984).

The Construction of High Energy Accelerators was improved, providing intense beams of known energy that lead us to discover the quark substructure of matter. One reason for why high energies became so important came from quantum mechanics, Which describes particles as waves, whose wavelength is established by the DE Broglie's Expression $\boldsymbol{\lambda} = \frac{\boldsymbol{h}}{\boldsymbol{p}}$ where $\boldsymbol{p}$ the beam momentum, and $\boldsymbol{h}$ the Planck's constant, which means that beams with higher momentums have shorter wavelengths, bringing higher resolutions, providing nicer detail in the structure of fundamental particles. To reach very high collision energies, many of the current accelerators are colliders in which two particles beams are accelerated in opposite directions for collision them , doing this almost all the particle energy can be employed for production of new particles, being able to obtain high collision energies to study the structure of matter . The Energy needed for particles discovery is increasing more and more with time. Some examples for this type of colliders are Tevatron at Fermi-lab, or the LHC at CERN. Since 1939, the accelerators development has grown so much that the energy has improved from the 80 KeV from the original cyclotron of 13cm of diameter to the 10 TeV from the LHC of 27 km of diameter (al, 2012).

1.2. Problem of the Study: There have been many proposed Unified Theories, but we need data to pick which, if any, of these theories describes nature. In many grand unified theory the gauge couplings constants (which define the electromagnetic, weak and strong interactions or forces, are combined into one single force.) are predicted to meet at some high energy unification scale. In the standard model the gauge couplings do not meet at single point. However, the unification works very well in Supersymmetry theory but at high scale approximately 10^{16}GeV , such a high energy scale is beyond the reach of any present or future experiments. Extra dimensions offer power law running, that brings down the unification scale to an explorable range.

There are several versions of extra dimension models, the simplest being the case of one flat extra dimension compactified on an S_1/Z_2 orbifold which has a size 1/R ≈ 1 TeV. This compactification will lead to a new particle states in the effective 4-dimensional (4D) theory. As such, in the 4D effective theory there appears an infinite tower of massive Kaluza-Klein (KK) states, with a mass contribution inversely proportional to the radius of the extra-dimension (N.~Maru, 2010).

1.3. Objectives:

There are many different ways to build a model with an extra dimensional space-time, the easiest one is the universal extra dimension (UED) model in which case all particles (bulk case) or some (brane case) do propagate in higher dimension space time. So we will study the unification of gauge coupling constants in various scenarios.

1.4. Outline of the thesis:

The outline of the thesis is as follow:
We introduce in Chapter I general introduction of particle physics. Chapter II will discuss the theory of the standard model, supersymmetry and extra dimension models. In Chapter III, we will apply the technique of renormalization group equations to the evolution of gauge couplings for different fields localization. Chapter IV devoted to our numerical results, discussions and conclusions.

Chapter II

The Standard model and Beyond

2.1. Introduction:

In this Chapter will present the Standard Model of elementary particle physics and discuss of its related issues. In addition to that, we also present alternative solution (beyond the standard model, supersymmetry and extra dimension models) to address some of these issues.

2.2. What is the Standard Model?

The Standard Model (SM) of particle physics is believed to be a remarkably successful theory describing elementary particles and their interactions. Its predictions have been tested well experimentally to a high level of accuracy, such as the structures of the neutral and charged currents, which agree with experiment. The standard model asserts that the material in the universe is made up of elementary fermions interacting through fields; the particles associated with the interacting fields are called bosons (mediator) (A.J.G.hey, 1993).

The assignments of elementary particles in the standard model are as follows:

$$\text{Quarks} \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad , \quad \begin{pmatrix} t \\ b \end{pmatrix} \tag{2.1}$$

$$\text{Leptons} \begin{pmatrix} v_e \\ e \end{pmatrix} \quad , \quad \begin{pmatrix} v_\mu \\ \mu \end{pmatrix} \quad , \quad \begin{pmatrix} v_\tau \\ \tau \end{pmatrix} \tag{2.2}$$

$$\text{Gauge bosons} \begin{cases} photon \gamma \\ weak(gauge) bosons W^{\mp}, Z^{\circ} \\ gluons g \end{cases} \tag{2.3}$$

$$\text{Higgs boson} \{ \text{H responsiple of mass} \tag{2.4}$$

Quarks and leptons are fundamental building blocks of matter; all of them are fermions and have spin$\left(\frac{1}{2}\right)$, They are classified as left-handed isospin doublets and right-handed isospin singlet's and will be described by the Dirac equation. Quarks interact through the electromagnetic (if they are charged quarks) and weak interaction and also through the strong interaction (quark comes with colors) (A.D.Martin, 1984).

Leptons interact only through the electromagnetic interaction (if they are charged) and the weak interaction.

Gauge bosons having spin (1) are mediators of interactions between quarks or leptons; there is a massless boson, the photon γ, and three massive ones, the $W^+, W^-$and the $Z^\circ boson$.

Electromagnetic, weak and strong interactions are mediated by photons γ, weak bosons $W\pm$, Z° and gluons g, respectively.

Interaction strength depends on which gauge bosons propagate between quarks or leptons.

The scalar Higgs boson has spin(0) is introduced for the Higgs mechanism to give mass to elementary particles, which is operative in the theories with spontaneous symmetry breaking of local gauge symmetries(Guigg, 1983).

Table 1.1: The SM fields with their representations under $SU(3)_c$ and $\mathrm{SU(2)_L}$ and their Charges under $U(1)_Y$ and $U(1)_{EM}$ Q is the electric charge and s is the spin of the field.

Field	Notation	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{EM}$
Quarks (s=1/2)	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	1/3	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
	u_R, c_R, t_R	3	1	4/3	2/3
	d_R, s_R, b_R	3	1	-2/3	-1/3
Leptons (s=1/2)	$L_L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	1	2	-1	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	e_R, μ_R, τ_R	1	1	-2	-1
Gauge (s=1)	g	8	1	0	0
	W^3,$W^\pm$	1	3	0	0,±1
	B	1	1	0	0
Higgs (s=0)	$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 = \frac{1}{\sqrt{2}}(\nu + h + i\varphi_0) \end{pmatrix}$	1	2	1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

2.3. Symmetries and Particle Content in Standard Model:

The Standard Model is a successful example of a minimal model based on the local gauge group (Salam, 1968).

The gauge group for the standard model is

$$SU(3) \otimes SU(2)_L \otimes U(1)_Y \tag{2.5}$$

Where:

-The SU (3) gauge group or color group is the symmetry group of strong interactions. This group acts on the quarks which are the elementary constituents of matter and the interaction force is mediated by the gluons which are the gauge bosons of the group. The quarks and the gluons are colored fields. The corresponding coupling is denoted by α, The SU (3) color symmetry is exact and consequently the gluons are massless.

- The theory of strong interactions based on color SU (3) is called Quantum Chromo dynamics (QCD).

- The $SU(2)_L \otimes U(1)_Y$ is the gauge group of the unified weak and electromagnetic interactions. Where $SU(2)_L$ is the weak isospin group, acting on left-handed fermions, and $U(1)_Y$ is the hypercharge group(Salam, 1968).

-The SU (2) group has three gauge bosons are denoted by $W_\mu^1,\ W_\mu^2, W_\mu^3$. None of these gauge bosons (and neither B_μ photon) are physical particles, linear combinations of these gauge bosons will make up the photon as well as the W± and the Z boson, as matter content for the first family, we have

$$q_L \equiv \binom{u_L}{d_L}\ ;\ u_R\ ;\ d_R \quad and\ \ l_L \equiv \binom{v_L}{e_L}\ ;\ e_R \tag{2.6}$$

The explicit values for the hypercharges of the particles above are as follows

$$Y(l_L) = -1\ \ , Y(e_R) = -2\ , Y(v) = 0\ , Y(q_L) = \frac{1}{3}\ , Y(u_R) = \frac{4}{3}\ , Y(d_R) = -\frac{2}{3}$$

Under $SU(3)$ the lepton fields are l_L, e_R, v_R singlets, i.e. they do not transform at all. This Means that they do not couple to the gluons. The quarks on the other hand form triplets Under SU (3). The strong interaction does not distinguish between left- and right-handed Particles.

However, since we ultimately want massive weak gauge bosons, we will have to break $U(1)_Y \times SU(2)$ gauge group spontaneously, by introducing some type of Higgs scalar(P.~W.~Higgs, 1964).

2.4. The Higgs Mechanism:

As was presented in the previous section, a Dirac mass term will violate the Gauge symmetry. As such we need a mechanism that gives mass to the SM Particles and keeps the Lagrangian invariant under gauge symmetries (P.~W.~Higgs, 1964). This can be done through the mechanism of spontaneous gauge symmetry breaking also known as the Higgs mechanism. This mechanism adds a new complex scalar field which is a doublet under the $\mathrm{SU}(2)_L$ group, a singlet with respect to SU(3) and has hypercharge $Y_\Phi = 1$**,** and

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{2.7}$$

Where ϕ_1, ϕ_2, $\phi_3$and $\phi_4$are real scalars. This new scalar Φ adds extra terms
To the SM Lagrangian:

$$\mathcal{L}_{\text{Higgs}} = \left(D_\mu \Phi\right)^\dagger (D^\mu \Phi) - \mathrm{V} \tag{2.8}$$

Where the covariant derivative D_μ is defined as

$$D_\mu = \partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\sigma^a}{2}W_\mu^a \tag{2.9}$$

The general gauge invariant renormalizable potential involving Φ is given by

$$V(\Phi) = -\frac{1}{2}\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 \tag{2.10}$$

Equation (2.10) describes the Higgs potential, which involves two new real parameters μ and λ. We demand $\lambda > 0$ for the potential to be bounded, otherwise the potential is unbounded from below and there will be no stable vacuum state. μ takes the following two values:

• $\mu^2 > 0$ then the vacuum corresponds to $\Phi = 0$, the potential has a minimum at the origin (See Fig (2.1)).

• $\mu^2 < 0$ Then the potential develops a non-zero Vacuum Expectation Value (VEV) and the minimum is along a circle of radius $\frac{\nu}{\sqrt{2}} = \frac{246}{\sqrt{2}}$ (See Fig (2.2)). Minimizing the potential we get

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = -\frac{\mu^2}{\lambda} = \nu^2 \tag{2.11}$$

As such, we need to choose one of these minima as the ground state ($\phi_3 = \nu, \phi_1 = 0$, $\phi_2 = 0$ and $\phi_4 = 0$) (A.J.G.hey, 1993). Thus the vacuum does not have the original symmetry of the Lagrangian, and therefore spontaneously breaks the symmetry. In other words, the Lagrangian is still invariant under the$SU(2)_L \times U(1)_Y$, while the ground state is not.

We choose the VEV in the neutral direction as the photon is neutral, so Φ becomes

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu\end{pmatrix} \tag{2.12}$$

With this particular choice of the ground state, the electroweak gauge group$SU(2)L \times U(1)Y$ is broken to electromagnetism, $U(1)em$,

$$SU(2)L \times U(1)Y \overset{\Phi}{\rightarrow} U(1)em \tag{2.13}$$

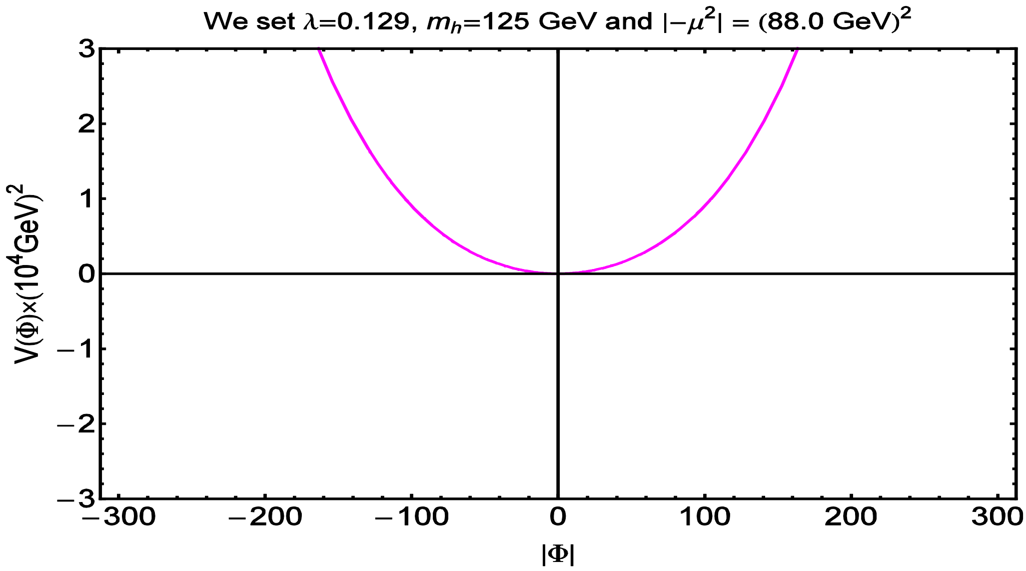

Figure (2.1): The Higgs potential $V(\Phi)$ with, the case $\mu^2 > 0$; as function of $|\Phi| = \sqrt{\Phi^{\dagger}\Phi}$

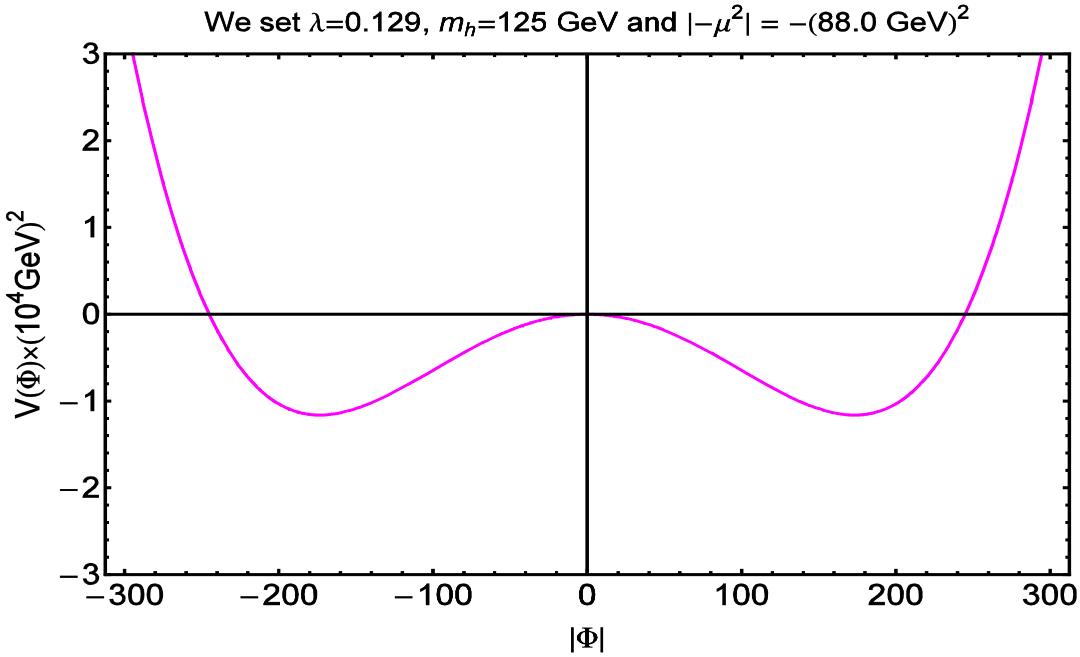

Figure (2.2): The Higgs potential $V(\Phi)$ with, the case $\mu^2 < 0$; as function of $|\Phi| = \sqrt{\Phi^{\dagger}\Phi}$

2.5. The Lagrangian of the Standard Model:

The Standard Model Lagrangian can be split into six parts the gauge sector$\mathcal{L}_G$, the Fermions sector$\mathcal{L}_F$, the Higgs sector$\mathcal{L}_H$, the Yukawa sector$\mathcal{L}_Y$, the Gauge,fixing sector $\mathcal{L}_{Gauge.fixing}$,and the Ghost sector $\mathcal{L}_{Ghost}$ (A.J.G.hey, 1993)

$$\mathcal{L}_{SM} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H + \mathcal{L}_Y + \mathcal{L}_{Gauge.fix} + \mathcal{L}_{Ghost} \qquad (2.14)$$

2.5.1. Gauge Sector

The gauge sector is composed of 12 gauge fields which mediate the interactions among the fermions fields

$$\mathcal{L}_G = -\frac{1}{4} W^i_{\mu v} W^i_{\mu v} - \frac{1}{4} B_{\mu v} B_{\mu v} - \frac{1}{4} G^a_{\mu v} G^a_{\mu v} \qquad (2.15)$$

Where $B_{\mu v}$,$W^i_{\mu v}$and $G^a_{\mu v}$is the field strength of the associated gauge fields given by:

$$W^i_{\mu v} = \partial_v W^i_\mu - \partial_\mu W^i_v - g_w \epsilon^{ijk} W^j_\mu W^k_v \qquad (2.16)$$

$$B_{\mu v} = \partial_\mu B_v - \partial_v B_\mu \qquad (2.17)$$

$$G^a_{\mu v} = \partial_v G^a_\mu - \partial_\mu G^a_v - g_s f^{abc} G^a_\mu G^b_v \qquad (2.18)$$

The tensors $\epsilon^{ijk}$and $f^{abc}$are the SU(2) and SU(3) structure constants, g_w and $g_s$are the weak-isospin and the strong coupling, respectively (L.F.Li, 1991).

2.5.2. Fermions Sector

The gauge interaction of fermions can be derived from the covariant derivative, once the various charges of the fields are known. The peculiarity of the SM is that the left-handed part of fermions has a different coupling compared to the right-handed one. For instance, only left-handed fields couple to W bosons (Falcone, 2002).

$$\mathcal{L}_F = \sum i\bar{\psi}\gamma_\mu D^\mu \psi \qquad (2.19)$$

Where ψ is the electron field, with the sum running over the left- and right-handed field components of the leptons and quarks.

Depending on the fermions species, the covariant derivative takes the form

$$D_\mu \binom{u}{d}_L = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig \frac{\sigma^a}{2} W_\mu^a - i\frac{1}{6} g' B_\mu\right) \binom{u}{d}_L \quad (2.20)$$

$$D_\mu u_R = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig \frac{\sigma^a}{2} W_\mu^a - ig' \frac{2}{3} B_\mu\right) u_R \quad (2.21)$$

$$D_\mu d_R = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig \frac{\sigma^a}{2} W_\mu^a + ig' \frac{1}{3} B_\mu\right) d_R \quad (2.22)$$

$$D_\mu \binom{\nu}{e}_L = \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a + i\frac{1}{2} g' B_\mu\right) \binom{\nu}{e}_L \quad (2.23)$$

And

$$D_\mu e_R = \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a + ig' B_\mu\right) e_R \quad (2.24)$$

Here $\gamma_\mu$are the usual Dirac matrices, g' is the coupling strength of the hypercharge interaction, W is the hypercharge, σ^a are the generators of $\mathrm{SU}(2)_L$ (simply the Pauli matrices), and $\lambda^a$are the generators of $\mathrm{SU}(3)_C$(the Gell-Mann matrices) (A.J.G.hey, 1993)

2.5.3. Higgs Sector

The Higgs-gauge boson interactions generated by the covariant derivative

$$\mathcal{L}_\mathrm{H} = \left(D_\mu \Phi\right)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad (2.25)$$

Where

$$D^\mu = \partial^\mu - ig_\mu W^{\mu,a} t^a - ig_1 Y_h B^\mu \quad (2.26)$$

This scalar particle has been discovered by the ATLAS (al, 2012) and CMS (al, 2012) experiments, which is compatible with the SM. Higgs expectations with a mass 126 GeV.

2.5.4. Yukawa sector

From symmetry considerations we are free to add gauge-invariant interactions between the Scalar fields and the fermions, these are called the Yukawa terms in the Lagrangian and they are responsible of generating fermions masses and the mixing between different families(Falcone, 2002).

$$\mathcal{L}_Y = Y_{ij}^d q_L^i \Phi d_R^j + Y_{ij}^u q_L^i \Phi^\sim u_R^j + Y_{ij}^e L_L^i \Phi e_R^j + h.c \quad (2.27)$$

2.5.5. Gauge Fixing and Ghosts

Gauge fixing is necessary when the gauge fields are quantized. Quantization means to develop a path integral formalism for the gauge theory. The path integral is diverging as one integrate over an infinite set of gauge-equivalent configuration, here the gauge fixing is used to pick up one arbitrary representative, therefore, giving meaning to the path integral. On other hands, the gauge invariance we look for in gauge theory, a naive path integral approach would spoil it. The solution is given by what is called the Faddev-Popov procedure, where they introduced an identity expression consisting of a functional integral over a gauge fixing condition times a functional determinant over anticommuting fields in the path integral. The latter gives rise to what is known as ghost fields, which keep the gauge freedom within the theory, but are not physical particles (because ghost violate the spin-statistics relation)(L.F.Li, 1991). As such, we need to add terms in the Lagrangian like

$$\mathcal{L}_{Gaugefixing} = -\frac{\zeta}{2}(\partial_\mu A^\mu)^2 \tag{2.28}$$

$$\mathcal{L}_{Ghost} = \bar{c_b}\partial^\mu D_\mu^{ab} \tag{2.29}$$

2.6. Problems with the Standard Model:

Although the Standard Model of particle physics is very successful, with no confirmed accelerator data that contradict it, there are many theoretical reasons to consider it unsatisfactory and to expect some physics beyond the Standard Model(Majee, March, 2008).

2.6.1. Dark Matter:

The SM does not have any dark matter candidates, as opposed to observational cosmology.

2.6.2. Gauge Hierarchy problem:

The symmetry lack of protecting the Higgs mass and the large hierarchy between the weak scale and the Planck scale makes it difficult to explain light Higgs mass within the SM. This is known as the gauge hierarchy problem which is basically a naturalness issue withThe SM. On the other hand, the other parameter of this theory, namely the Φ^4coupling λ is natural. This is so because, in the limit $\lambda \rightarrow 0$, we have a free scalar theory, which indeed has higher symmetry (L.F.Li, 1991).

2.6.3. Gravity is not included:

Though the unification of the electromagnetic and weak interactions was achieved in the SM and the strong interaction appears to be part of the unification, the SM does not include the effects of gravity. Note that the effects of gravity become important at energies of the order of the Planck scale, $M_{Pl} = 10^{18}$GeV. The SM is treated as an effective theory at a natural cut-off scale. The ultimate goal in particle physics is to unify all the fundamental forces in nature (Weinberg, 1996).

2.7. Beyond the Standard Model:

Most of the Beyond the Standard Model (BSM) physics have been constructed to solve the gauge hierarchy problem (Majee, March, 2008). The models that have been discussed in the literature may be categorized as follows:

2.7.1. Super Symmetry (SUSY):

Super symmetry (SUSY) is a space-time symmetry which relates the bosonic degrees of freedom to the fermionic degrees of freedom. The beautiful idea of supersymmetry helps to solve the gauge hierarchy problem, the one loop radiative correction for the Higgs mass due to scalar particles in the loop. In a supersymmetric the transformation changes a boson to a fermion and vice versa. Thus, if Qis the generator of this transformation then

$$\mathrm{Q}|\mathrm{boson}\rangle \equiv |\mathrm{fermion}\rangle, \qquad \mathrm{and} \mathrm{Q}|\mathrm{fermion}\rangle \equiv \mathrm{bosoni}\rangle$$

Therefore in Super symmetry there is" a superpartner" for each elementary particle**:** selectron, smuon,stau,squarks, photino, higgsino, etc see table 1.2.

Supersymmetry offer the unification of gauge couplings at single point 2×10^{16}GeV see figure (2.3). Henceforth,Supersymmetry has been studied intensively as a direct possible extension to the SM which we refer to as the Minimum Supersymmetry (MSSM).

For more details or further explanation on supersymmetry we refer the interested reader to Ref.(J. Louis, 1998) and references therein.

Table 1.2: Supersymmetric partners with the Standard Model members

Nature		spin 0	spin ½	$SU(3)_C, SU(2)_L, U(1)_Y$
Squarks , quarks	$\begin{cases} Q \\ \bar{u} \\ d \end{cases}$	$\tilde{u}_l, d_l^{\sim}$	u_l, d_l	$\left(3,2,\frac{1}{3}\right)$
		$\tilde{u}_R^*$	u_R^+	$(3,1,\frac{4}{3})$
		$d_R^{\sim *}$	d_R^+	$(\bar{3},1,\frac{-2}{3})$
sleptons, leptons	L	$v^{\sim}, \tilde{e}_l$	v, e_l	$(1,2,-1)$
	ē	$\tilde{e}_R^*$	e_R^+	$(1,1,-2)$
Higgs, higgsinos	H_1	H_1^+, H_1^0	$H_1^{\sim +}, H_1^{\sim 0}$	$(1,2,+1)$
	H_2	H_2^0, H_2^-	$H_2^{\sim 0}, H_2^{\sim -}$	$(1,2,-1)$
		spin ½	spin 1	
gluino, gluon		$g^{\sim}$	g	(8,1,10)
winos, W-bosons		$W^{\sim\pm}, W^{\sim 0}$	$W^{\pm}, W^0$	(1,3,0)
bino, B-boson		$B^{\sim 0}$	B^0	(1,1,0)

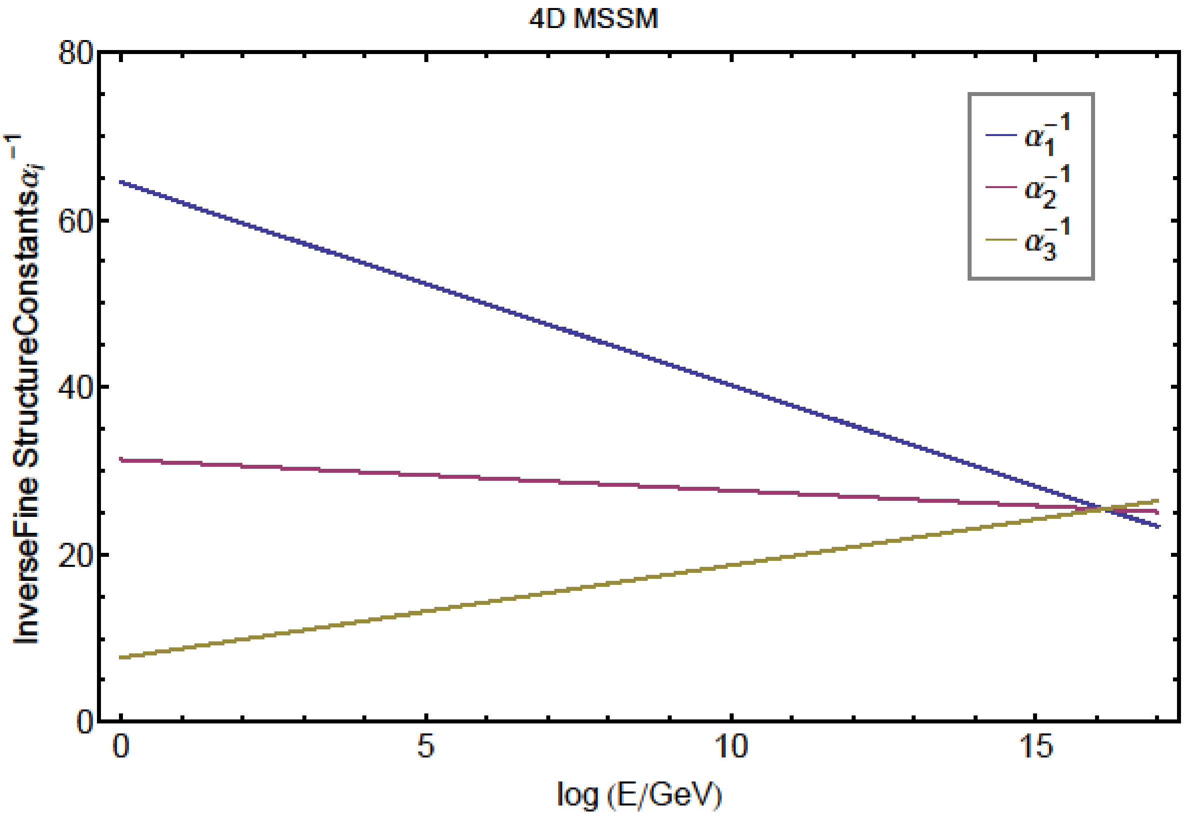

Figure (2.3) Unification of inverse fine structure constants in 4D MSSM, the unification takes place at 2×10^{16}GeV.

2.7.2. Extra Dimensions:

In the SM the hierarchy problem is arising due to the huge ratio of the Planck scale, M_{pl}, or the General unification theory(GUT) scale, M_G, to the electroweak scale. As discussed in the previous section, SUSY provides a natural way to solve this hierarchy problem (Majee, March, 2008). In that case, the supersymmetric particles are situated around the TeV scale. Actually to solve the hierarchy problem if we incorporate any new physics it should appear around that scale to address the huge ratio. More recently, a new kind of physics, Extra Dimension (ED), was introduced in particle physics. One might ask a question how do we distinguish supersymmetry particles from extra dimension particles? Practically we can distinguish a fermion from a bosonic particle by measuring the spin of the particle at the Large Hadron Collider (LHC) or the International Linear Collider (ILC), and then we can have a distinct signature of the physics of extra dimension from that of supersymmetry(Majee, March, 2008).

Historically, Extra dimension was first introduced by Kaluza and Klein in 1920, to unify the electromagnetic interaction with the gravitational one by generating the photon from the extra components of the five-dimensional metric (T.~Kaluza, 1921). Nowadays in a more popular and fundamental theory, namely, string theory, it is common to use more than one space dimension, as the theory is consistent only in the extra-dimensional scenario. There are many open questions about the extra dimension, *e.g*, what would be nature of the extra dimension, what is the size of it and many more. A huge number of phenomenological studies have been pursued in this subject in this decade (H.~-U.~Yee, 2003). Let us have a closer look on some of these.

2.7.2.1. Scalar Particle in ED:

In addition to the four space-time coordinates x= (x, t), let us denote the extra space-type coordinate y, compactified on a circle of radius R. Thus, the Lagrangian of a free complex Scalar$\Phi(x, y)$ with mass m will be a function of both x and y coordinates with a condition that the field at y = 2πR will match with that at y = 0, i.e. it has a periodicity of 2πRalong the y direction. So one can expand it in a Fourier series as**:**

$$\Phi(x, y) = \frac{1}{\sqrt{2\pi R}} \Phi^{(0)}(x) + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left[\Phi^{(n)}(x) \cos\left(\frac{ny}{R}\right) \right] \qquad (2.30)$$

The five dimensional Lagrangian is given by

$$\mathcal{L}_{scalar} = \int dx_5 \{ D_M \Phi^\dagger D^M \Phi - M^2 \Phi^\dagger \Phi - \lambda_5 (\Phi^\dagger \Phi)^2 \} \qquad (2.31)$$

With M= 0, 1,2,3,5

The equation of motion can be obtained by varying the above integral**:**

$$(\partial_5^2 + p^2 - M^2)\Phi = 0 \qquad (2.32)$$

Where the n-the KK mode mass is given by**:**

$$m_n^2 = M^2 + \frac{n^2}{R^2} \qquad (2.33)$$

Integrating the equation (2.31) and comparing with the effective 4D dimensional lagrangian we get

$$
\begin{aligned}
\mathcal{L}_{4D} = \quad & (D_\mu\Phi)^{(0)\dagger}(x)(D^\mu\Phi)^{(0)}(x) \\
& + (D_\mu\Phi)^{(n)\dagger}(x)(D^\mu\Phi)^{(n)}(x) \\
& + \ (D_5\Phi)^{(n)\dagger}(x)(D^5\Phi)^{(n)}(x)
\end{aligned} \tag{2.34}
$$

Now the covariant derivatives read

$$(D_\mu\Phi)^0 = D_\mu^{(0)}\Phi^{(0)} - \left(ig\frac{\sigma^i}{2}W_\mu^{(n)i} + ig'\frac{Y}{2}B_\mu^{(n)}\right)\Phi^{(n)} \tag{2.35}$$

$$(D_\mu\Phi)^n = D_\mu^{(ns)}\Phi^{(s)} - \left(ig\frac{\sigma^i}{2}W_\mu^{(n)i} + ig'\frac{Y}{2}B_\mu^{(n)}\right)\Phi^{(0)} \tag{2.36}$$

$$(D_5\Phi)^n = D_5^{(ns)}\Phi^{(s)} - \left(ig\frac{\sigma^i}{2}W_5^{(n)i} + ig'\frac{Y}{2}B_5^{(n)}\right)\Phi^{(n)} \tag{2.37}$$

2.7.2.2. Gauge Fields and Gauge Fixing:

The Lagrangian for an Abelian gauge field and gauge fixing is given by

$$\mathcal{L}_{Gauge+GF} = \int dy\ (-\frac{1}{4}F^{MN}F_{MN} - \frac{1}{2\xi}(\partial_\mu A^\mu - \xi(\partial_5 A_5)^2 \tag{2.38}$$

Where ξ is the gauge fixing parameter and $F_{MN} = \partial_M A_N - \partial_N A_M$. The gauge fixing term eliminates the mixing between $A_\mu$and the extra polarization A_5.

In the Feynman-’t Hooft gauge$\xi = 1$, the equations of motion for A_5can be obtained**:**

$$(\partial_5^2 - \partial_\mu^2)A_5 = 0 \tag{2.39}$$

Assigning even parity to the field A_μ^a and its Fourier expansion is

$$A_\mu^a(x,y) = \frac{1}{\sqrt{2\pi R}}A_\mu^{(0)a}(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\left[A_\mu^{(n)a}(x)\cos\left(\frac{ny}{R}\right)\right] \tag{2.40}$$

Similarly the effective gauge field lagrangian in 4D is given by

$$
\begin{aligned}
\mathcal{L}_{4D} \quad = \ & -\frac{1}{4}\left(G_{\mu\nu}^{(0)a}G^{(0)a\mu\nu} + G_{\mu\nu}^{(n)a}G^{(n)a\mu\nu} + 2G_{\mu5}^{(n)a}G^{(n)a\mu5}\right) \\
& -\frac{1}{4}\left(W_{\mu\nu}^{(0)a}W^{(0)a\mu\nu} + W_{\mu\nu}^{(n)a}W^{(n)a\mu\nu} + 2W_{\mu5}^{(n)a}W^{(n)a\mu5}\right) \\
& -\frac{1}{4}\left(B_{\mu\nu}^{(0)}B^{(0)\mu\nu} + B_{\mu\nu}^{(n)}B^{(n)\mu\nu} + 2B_{\mu5}^{(n)}B^{(n)\mu5}\right)
\end{aligned} \tag{2.41}
$$

2.7.2.3. Fermion Particle in ED

The Lagrangian for fermion is given by the Dirac lagrangian**:**

$$\mathcal{L}_{Fermion} = \overline{\Psi}(i\Gamma^M D_M - m)\Psi \tag{2.42}$$

$$\text{where } \Gamma^{\mathrm{M}} = (\gamma^{\mu}, \mathrm{i}\gamma^5) \text{and } \gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$$

Fermions are assigned to 5D representations of this group $\psi(x, y)$ to yield:

$$\psi(x,y) = \frac{1}{\sqrt{2\pi R}}\psi^{(0)}_{(L/R)}(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\left[\psi^{(n)}_{(L/R)}(x)\cos\left(\frac{ny}{R}\right) + \psi^{(n)}_{(L/R)}(x)\sin\left(\frac{ny}{R}\right)\right] \tag{2.43}$$

then the effective 4D dimension is

$$\mathcal{L}_{4D} = i\Psi_L^{(0)}\gamma^{\mu}D_{\mu}\Psi_L^{(0)} + i\Psi_L^{(n)}\gamma^{\mu}D_{\mu}\Psi_L^{(n)} + i\Psi_R^{(0)}\gamma^{\mu}D_{\mu}\Psi_R^{(0)} + i\Psi_R^{(n)}\gamma^{\mu}D_{\mu}\Psi_R^{(n)} + \Psi_L^{(n)}D_5\Psi_L^{(n)} + \Psi_R^{(n)}D_5\Psi_R^{(n)}\,. \tag{2.44}$$

Chapter III

Renormalization Group Equations

3.1. Introduction

This chapter we will discuss the Renormalization Group Equations (RGEs) in which that will be used in our numerical calculations and results.

3.2. Renormalization Group Equations:

The renormalization group, in quantum field theory (QFT), tells us how different couplings evolve with energy (Collins, 1984).

3.2.1. What is renormalization?

In Quantum Field Theory (QFT), Green function is a most important thing to be calculated. In perturbative QFT these quantities are divergent. The systematic way to remove these divergences is known as renormalization. The renormalization theory is implemented to remove all the divergences in loop integrals from the physical measurable quantities. These loop diagrams are supposed to give finite results to the physical quantities but they give infinities instead. This tells us that our theory has missed some information. One might ask a question where these infinities are come from. These infinities arise from the integration over all momentum. In other words, the infinities occur because we let our theory go to arbitrary high energy (UV). There are different ways to cancel these infinities. In order to renormalize the theory we need a reference point which is also arbitrary, different choices of this reference point lead to different sets of parameters for the theory, but physics should not depend on the arbitrary choice of the reference point and be invariant. This invariance leads to the renormalization group equation. In quantum field theory it is a useful method to examine the behavior of physics at a different scale knowing the same at some other scale. Thus, measuring the observables in a low energy experiment one can compare with the values predicted from a theory at a higher scale, *e.g*at the GUT scale and certify about the correctness of the theory. In the standard model, variations of the gauge coupling constants with energy are given by the following renormalization group equations (RGEs) (Collins, 1984).

$$16\pi^2 E \frac{dg_i}{dE} = b_i g_i^3 = \beta_{SM}(g_i) \qquad (3.1)$$

Where i stands for $\mathrm{U(1)_Y}$, $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$ and the right-hand-side is known as the β Function of the corresponding coupling constants.

In the above equations the co-efficient b_i can be calculated for any $SU(N)$group as

$$b_i = \left[\frac{11}{3}C_2(G) - \frac{4}{3}n_f C(R) - \frac{1}{3}n_s C_2 R\right] \qquad (3.2)$$

Where:

In the above equation $n_f$is the number of fermions and $n_s$is the number of higgs scalar and $i = 1{,}2{,}3$. For the representation the $C_2(G)$, $C(R)$, $C_2(R)$ refer to the gauge boson,fermions, and higgs scalar contribution respectively(A.~Abdalgabar, 2013).

3.3. Calculation of Beta Function for the Gauge Couplings Constant in the SM:

Equation (3.2) can be calculated from the Feynman diagrams presented in figure (3.1). We do not calculate here all the diagrams (only their results will be given). We give a detail calculation for one diagram figure (3.1 b).

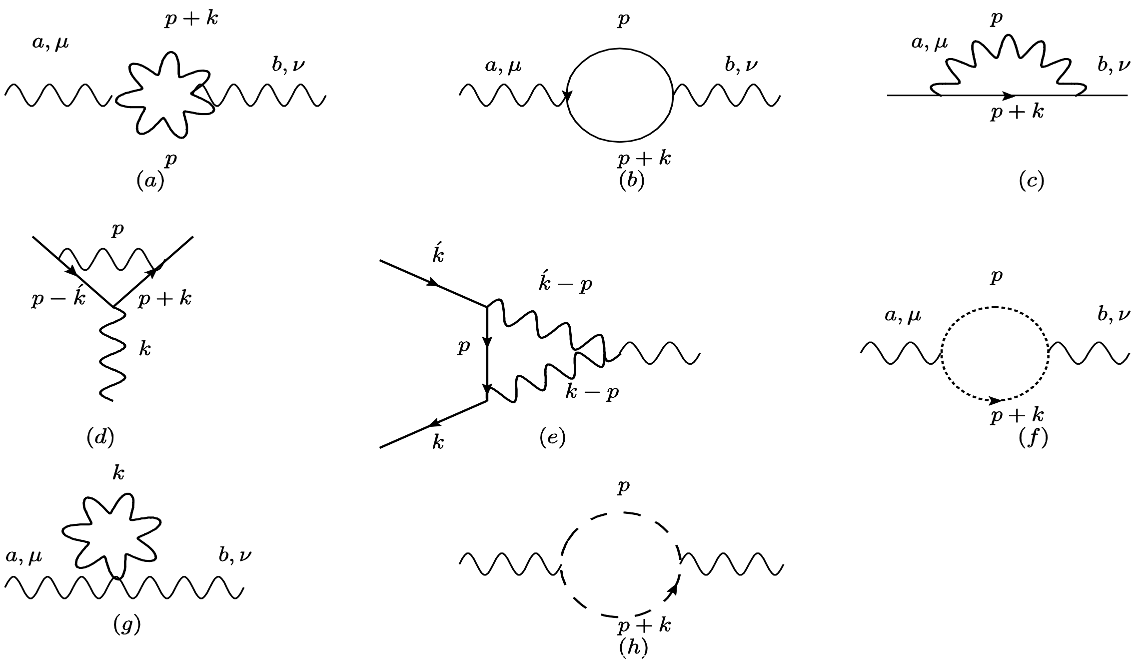

Figure (3.1) The one-loop gauge field self correction diagrams in the Standard model

$$figure(3.1.a) = \frac{ig^2 C_2(G)\delta^{ab}}{6(4\pi)^2}\Gamma\left(2-\frac{d}{2}\right)\left[\frac{19}{12}g^{\mu\nu}k^2 - \frac{11}{6}k^\mu k^\nu\right] \quad (3.3)$$

We will calculate the contribution of figure (3.1.b) in details:

$$figure(3.1.b) = \Pi^{\mu\nu}(p,k) = -n_f \int \frac{d^d p}{(2\pi)^d} Tr\left(-igt^a\gamma^\mu \frac{i(\not{p})}{p^2}\right)\left(-igt^b\gamma^\nu \frac{i(\not{p}+\not{k})}{(p+k)^2}\right) (3.4)$$

We have

$$Tr(t^a t^b) = C(r)\delta^{ab} \quad , \quad \not{p} = \gamma^\rho p^\rho \quad (3.5)$$

Then equation (3.4) becomes

$$\Pi^{\mu\nu}(p,k) = -n_f C(r)\delta^{ab} g^2 \int \frac{d^d p}{(2\pi)^d}\frac{Tr(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma(p^\rho(p^\sigma+k^\sigma))}{p^2(p+k)^2} \quad (3.6)$$

From Trace Approach we have

$$Tr(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma) = f(D)(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho}) \quad (3.7)$$

Using the above equation the numerator simplified to

$$N^{\mu\nu}(p,k) = Tr(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma\big(p^\rho(p^\sigma + k^\sigma)\big) \quad (3.8)$$

$$N^{\mu\nu}(p,k) = f(D)(g^{\mu\rho}g^{\nu\sigma}(p^\rho(p^\sigma+k^\sigma)) - g^{\mu\nu}g^{\rho\sigma}(p^\rho(p^\sigma+k^\sigma)) + g^{\mu\sigma}g^{\nu\rho}(p^\rho(p^\sigma+k^\sigma))$$

$$N^{\mu\nu}(p,k) = f(D)(p^\mu(p^\nu+k^\nu) - p(p+k)g^{\mu\nu} + p^\nu(p^\mu+k^\mu) \quad (3.9)$$

So

$$\Pi^{\mu\nu}(p,k) = -n_f C(r)\delta^{ab} g^2 \int \frac{d^d p}{(2\pi)^d}\frac{N^{\mu\nu}(p,k)}{p^2(p+k)^2} \quad (3.10)$$

To calculate the above integral we using Feynman integral parameterization

$$\frac{1}{ab} = \int_0^1 \frac{1}{(b+(a-b)z)^2}dz \quad (3.11)$$

Now Let

$$\mathrm{b} = \mathrm{p}^2, \qquad \text{and } \mathrm{a} = (\mathrm{p}+\mathrm{k})^2 \quad (3.12)$$

Insert equation (3.12) in equation (3.11) we obtain

$$\frac{1}{p^2(p+k)^2} = \int_0^1 \frac{1}{(p^2+(k^2+2pk)z)^2} dz \tag{3.13}$$

Inserting equation (3.13) in equation (3.10) yield

$$\Pi^{\mu\nu}(p,k) = -\frac{n_f C(r)\delta^{ab} g^2}{(2\pi)^d} \int d^d p \int_0^1 dz \frac{N^{\mu\nu}(p,k)}{(p^2+(k^2+2pk)z)^2} \tag{3.14}$$

Introduce new variable q

$$q = p + kz \implies p = q - kz \tag{3.15}$$

The numerator of equation (3.14) becomes

$$N^{\mu\nu}(q-kz,k) = f(D)((q-kz)^\mu\left(q^\nu + k^\nu(1-z)\right)$$

$$-(q-kz)(q+k(1-z))g^{\mu\nu} + (q-kz)^\nu\left(q^\mu + k^\mu(1-z)\right)$$

$$= f(D)(2q^\mu q^\nu - q^2 g^{\mu\nu} - 2k^\mu k^\nu z(1-z) + k^2 z(1-z) g^{\mu\nu})$$

$$= f(D)(2q^\mu q^\nu - q^2 g^{\mu\nu} + 2z(1-z)(k^2 g^{\mu\nu} - k^\mu k^\nu) - k^2 z(1-z) g^{\mu\nu}) \tag{3.16}$$

And the denominator in equation (3.14) becomes

$$(p^2 + (k^2 + 2pk)z)^2 = (q^2 + k^2 z(1-z))^2 \tag{3.17}$$

Plugging equation (3.16) and (3.17) in equation (3.14) yield

$$\Pi^{\mu\nu}(q-kz,k) = -n_f C(r)\delta^{ab} g^2 f(D) \begin{pmatrix} \int_0^1 dz \int \frac{d^d q}{(2\pi)^d} \frac{2q^\mu q^\nu - q^2 g^{\mu\nu}}{(q^2+k^2 z(1-z))^2} \\ + \int_0^1 dz \int \frac{d^d q}{(2\pi)^d} \frac{2z(1-z)\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)}{(q^2+k^2 z(1-z))^2} \\ - \int_0^1 dz \int \frac{d^d q}{(2\pi)^d} \frac{k^2 z(1-z) g^{\mu\nu}}{(q^2+k^2 z(1-z))^2} \end{pmatrix} \tag{3.18}$$

This equation can be written as

$$\Pi^{\mu\nu}(q-kz,k) = -n_f C(r)\delta^{ab} g^2 f(D)(I_1 + I_2 - I_3) \tag{3.19}$$

Using the following standard integrals

$$\int d^d q \frac{q^\mu q^\nu}{(q^2+k^2 z(1-z))^2} = \frac{i\pi^{\frac{d}{2}}\Gamma(1-\frac{d}{2})\; g^{\mu\nu}}{2\Gamma(2)(k^2 z(1-z))^{1-d/2}} \tag{3.20}$$

$$\int d^d q \frac{q^2}{(q^2+k^2 z(1-z))^2} = \frac{d i\pi^{\frac{d}{2}}\Gamma(1-\frac{d}{2})}{2\Gamma(2)(k^2 z(1-z))^{1-d/2}} \tag{3.21}$$

$$\int d^d q \frac{1}{(q^2+k^2 z(1-z))^2} = \frac{i\pi^{\frac{d}{2}}\Gamma\left(2-\frac{d}{2}\right)}{(k^2 z(1-z))^{2-\frac{d}{2}}} \tag{3.22}$$

We get

$$I_1 = \int_0^1 dz \int \frac{d^d q}{(2\pi)^d} \frac{2q^\mu q^\nu - q^2 g^{\mu\nu}}{(q^2 + k^2 z(1-z))^2}$$

$$= \int_0^1 dz \frac{i\pi^{\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right) 2\, g^{\mu\nu} - d i\pi^{\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right) g^{\mu\nu}}{(2\pi)^d\; 2\Gamma(2)(k^2 z(1-z))^{1-\frac{d}{2}}}$$

$$= i\frac{\pi^{\frac{d}{2}}}{(2\pi)^d}(1-\frac{d}{2})\Gamma(1-\frac{d}{2})k^{d-2} g^{\mu\nu}\int_0^1 z^{\frac{d}{2}-1}(1-z)^{\frac{d}{2}-1} dz \tag{3.23}$$

From the beta function

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx = \frac{\Gamma(\mathrm{m})\Gamma(\mathrm{n})}{\Gamma(\mathrm{m+n})} \tag{3.24}$$

$$\mathrm{n}\Gamma(n) = \Gamma(n+1) \tag{3.25}$$

From equations (3.24) and (3.25) equation (3.23) becomes

$$I_1 = i\pi^{\frac{d}{2}}\left(1-\frac{d}{2}\right)\Gamma\left(1-\frac{d}{2}\right)k^{d-2} g^{\mu\nu}\int_0^1 z^{\frac{d}{2}-1}(1-z)^{\frac{d}{2}-1} dz$$

$$= i\frac{\pi^{\frac{d}{2}}}{(2\pi)^d} k^{d-2} g^{\mu\nu}\left(\frac{\Gamma(d/2)\Gamma(d/2)}{\Gamma(d)}\right)\Gamma\left(2-\frac{d}{2}\right) \tag{3.26}$$

Similarly I_2 becomes

$$I_2 = \int_0^1 dz \int \frac{d^d q}{(2\pi)^d} \frac{2z(1-z)(k^2 g^{\mu\nu} - k^\mu k^\nu)}{(q^2 + k^2 z(1-z))^2}$$

$$= 2(k^2 g^{\mu\nu} - k^\mu k^\nu) i \frac{\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) \int_0^1 dz \frac{z(1-z)}{(k^2 z(1-z))^{2-\frac{d}{2}}}$$

$$= 2(k^2 g^{\mu\nu} - k^\mu k^\nu) i \frac{\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) k^{d-4} \int_0^1 z^{\frac{d}{2}-1} (1-z)^{\frac{d}{2}-1} dz$$

$$= 2(k^2 g^{\mu\nu} - k^\mu k^\nu) i \frac{\pi^{\frac{d}{2}}}{(2\pi)^d} k^{d-4} \left(\frac{\Gamma(\mathrm{d}/2)\Gamma(\mathrm{d}/2)}{\Gamma(\mathrm{d})}\right) \Gamma\left(2 - \frac{d}{2}\right) \qquad (3.27)$$

And

$$I_3 = \int_0^1 dz \int \frac{d^d q}{(2\pi)^d} \frac{k^2 z(1-z) g^{\mu\nu}}{(q^2 + k^2 z(1-z))^2} = k^2 g^{\mu\nu} i \frac{\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) \int_0^1 dz \frac{dzz(1-z)}{(k^2 z(1-z))^{2-\frac{d}{2}}}$$

$$= g^{\mu\nu} i \frac{\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) k^{d-2} \int_0^1 dz\, z^{\frac{d}{2}-1} (1-z)^{\frac{d}{2}-1}$$

$$= g^{\mu\nu} i \frac{\pi^{\frac{d}{2}}}{(2\pi)^d} k^{d-2} \left(\frac{\Gamma(\mathrm{d}/2)\Gamma(\mathrm{d}/2)}{\Gamma(\mathrm{d})}\right) \Gamma\left(2 - \frac{d}{2}\right) \qquad (3.28)$$

From equations (3.26) and (3.28) we can see that

$$I_1 = I_3 \implies I_1 - I_3 = 0 \qquad (3.29)$$

$$\therefore \Pi^{\mu\nu}(k) = -n_f C(r) \delta^{ab} g^2 f(D) I_2$$

$$= -2 n_f C(r) \delta^{ab} g^2 f(D) (k^2 g^{\mu\nu} - k^\mu k^\nu) i \frac{\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) k^{d-4} \left(\frac{\Gamma(\mathrm{d}/2)\Gamma(\mathrm{d}/2)}{\Gamma(\mathrm{d})}\right) \qquad (3.30)$$

$$\therefore figure(3.1.b) = i(k^2 g^{\mu\nu} - k^\mu k^\nu) \delta^{ab} \left[\frac{-g^2}{(4\pi)^2} \frac{4}{3} n_f C(r)\right] \left(\Gamma\left(2 - \frac{d}{2}\right)\right) \qquad (3.31)$$

$$figure(3.1.c) = i\frac{g^2}{(4\pi)^2}\left[C_2(R)\not{k}\right]\Gamma\left(2-\frac{d}{2}\right) \quad (3.32)$$

$$figure(3.1.d) = \frac{ig^3}{(4\pi)^2}t^a\gamma^\mu\left[C_2(r)-\frac{1}{2}C_2(G)\right]\Gamma\left(2-\frac{d}{2}\right) \quad (3.33)$$

$$figure(3.1.e) \quad = \frac{3}{2}\frac{ig^3}{(4\pi)^2}\left[C_2(G)t^a\gamma^\mu\right]\Gamma\left(2-\frac{d}{2}\right) \quad (3.34)$$

$$figure(3.1.f) \ = \frac{ig^2C_2(G)\delta^{ab}}{6(4\pi)^2}\left[\frac{g^{\mu\nu}k^2}{2}+k^\mu k^\nu\right]\Gamma\left(2-\frac{d}{2}\right) \quad (3.35)$$

$$figure(3.1.g) = 0 \ (3.36)$$

$$figure(3.1.h) = \frac{-ig^2C_2(R)\delta^{ab}}{6(4\pi)^2}\left[g^{\mu\nu}k^2-k^\mu k^\nu\right]\Gamma\left(2-\frac{d}{2}\right) \quad (3.37)$$

Therefore,

$$b_i = [figure(3.1.a)+figure(3.1.b)+figure(3.1.c)+figure(3.1.d) + figure(3.1.e)+figure(3.1.f)+figure(3.1.g)+figure(3.1.h)]$$

$$= \left[\frac{11}{3}C_2(G)-\frac{4}{3}n_fC(R)-\frac{1}{3}C_2(R)\right] \quad (3.38)$$

Now calculation of the coefficients $b_i$is as follow

For the strong interaction (b_3): $SU(3)_c$: We have

$$C_2(R) = 0 \ ,C_2(G) = C_2(SU(N)) = N = 3,\mathrm{C(R)} = 1 \quad (3.39)$$

Thus

$$b_3 = \frac{11}{3}\times(3)-\frac{4}{3}\times(1\times 3)-0 = 7 \quad (3.40)$$

For the weak interaction (b_2): $SU(2)_L$: We have

$$C_2(R) = \frac{1}{2}\ ,C_2(SU(N)) = C_2(G) = N = 2,\mathrm{C(R)} = 1 \quad (3.41)$$

Therefore,

$$b_2 = \frac{11}{3} \times (2) - \frac{4}{3} \times (1 \times 3) - \frac{1}{3} \times \frac{1}{2} = \frac{19}{6} \tag{3.42}$$

For the electromagnetism interaction (b_1): $U(1)_Y$:

$$C_2(G) = 0 \, , C_2(R) = \frac{1}{2} \, , C(R) = \frac{20}{12} \tag{3.43}$$

Then

$$b_1 = \left(0 - \frac{4}{3} \times 3 \times \frac{20}{12} - \frac{1}{6}\right) \times \frac{3}{5} = \frac{41}{10} \tag{3.44}$$

$$\therefore b_i = \left[\frac{41}{10}, -\frac{19}{6}, -7\right] \tag{3.45}$$

3.4 Beta Function for Gauge Coupling Constant in ED

We assume all the standard model particles can access the full space-time (bulk scenario).So all the fields have Kaluza-Klein (KK) expansions. The zero-mode will be identified as the standard model fields and the rest will be the excited KK states and will contribute at energy≥ R^{-1}.

The gauge coupling constants RGEs in ED are given by

$$16\pi^2 \frac{\mathrm{dg_i}}{\mathrm{dt}} = \mathrm{b_i^{SM} g_i^3} + (S(t) - 1)\mathrm{b_i^{5D} g_i^3} \tag{3.46}$$

The beta-function coefficients $\mathrm{b_i^{SM}}$ are those of the usual SM given in (3.45), which correspond to the zero-mode states, while the new beta-function coefficients $\mathrm{b_i^{5D}}$ are given by $\mathrm{b_i^{5D}} = \left(\frac{81}{10}, -\frac{7}{6}, \frac{5}{2}\right)$ comes from the exicted KK states, and, $S(t) = m_Z R e^t = \mu R$, is the sum of KK states for $m_Z < \mu < \Lambda$ (Λ is the cut-off scale in which the couplings constant do meet).These beta-function coefficients correspond to the contributions of the appropriate Kaluza-Klein states at each massive Kaluza-Klein excitation level.

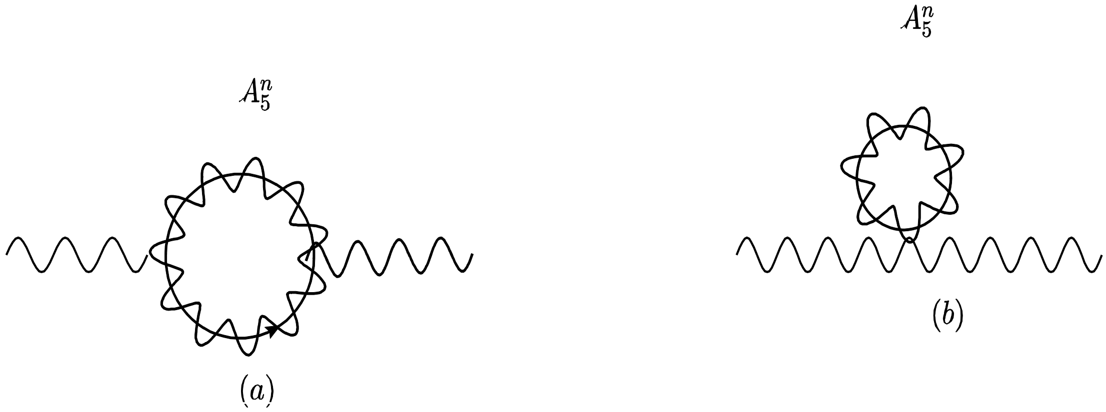

Figure (3.2) Contribution of extra dimension fields to gauge couplings unification

$$figure(3.2.a) = \Omega^{\mu\nu}(p,k) = \frac{1}{2}\int \frac{d^dp}{(2\pi)^d}\frac{i}{p^2}\frac{i}{(p+k)^2} g^2(2p+k)^\mu(-(2p+k))^\nu f^{acd}f^{bcd} \quad (3.47)$$

We have

$$f^{acd}f^{bcd} = C_2(G)\delta^{ab} \quad (3.48)$$

Insert equation (3.48) in equation (3.47) yield

$$\Omega^{\mu\nu}(p,k) = \frac{g^2C_2(G)\delta^{ab}}{2}\int \frac{d^dp}{(2\pi)^d}\frac{(2p+k)^\mu(2p+k)^\nu}{p^2(p+k)^2} \quad (3.49)$$

The nominator of equation (3.49) becomes

$$(2p+k)^\mu(-(2p+k))^\nu = 4p^\mu p^\nu + 2p^\mu k^\nu + 2k^\mu p^\nu + k^\mu k^\nu \quad (3.50)$$

By using Feynman integral in equation (3.11) yield

$$\Omega^{\mu\nu}(p,k) = \frac{g^2C_2(G)\delta^{ab}}{2}\int \frac{d^dp}{(2\pi)^d}\int_0^1 dz \frac{4p^\mu p^\nu+2p^\mu k^\nu+2k^\mu p^\nu+k^\mu k^\nu}{(p^2+(k^2+2pk)z)^2} \quad (3.51)$$

By introduce new variable q

$$q = p + kz, \quad p = q - kz \quad (3.52)$$

$$\Omega^{\mu\nu}(q-kz,k) = \frac{g^2C_2(G)\delta^{ab}}{2}\int \frac{d^dp}{(2\pi)^d}\int_0^1 dz \frac{4(q-kz)^\mu(q-kz)^\nu+2(q-kz)^\mu k^\nu+2k^\mu(q-kz)^\nu+k^\mu k^\nu}{(q^2+k^2z(1-z))^2}$$

(3.53)

$$\Omega^{\mu\nu}(q-kz,k) = \frac{g^2C_2(G)\delta^{ab}}{2}\int \frac{d^dp}{(2\pi)^d}\int_0^1 dz \frac{4(q^\mu q^\nu + k^\mu k^\nu z^2) - 4k^\mu k^\nu z + k^\mu k^\nu}{(q^2+k^2z(1-z))^2}$$

$$= \frac{g^2 C_2(G)\delta^{ab}}{2} \int \frac{d^d p}{(2\pi)^d} \int_0^1 dz \left(\frac{4q^\mu q^\nu + k^\mu k^\nu (4z^2 - 4z + 1)}{(q^2 + k^2 z(1-z))^2} \right) \qquad (3.54)$$

Let

$$I_1 = \int \frac{d^d p}{(2\pi)^d} \int_0^1 dz \frac{4q^\mu q^\nu}{(q^2 + k^2 z(1-z))^2} \qquad (3.55)$$

And

$$I_2 = \int \frac{d^d p}{(2\pi)^d} \int_0^1 dz \frac{k^\mu k^\nu (4z^2 - 4z + 1)}{(q^2 + k^2 z(1-z))^2} \qquad (3.56)$$

Then

$$\Omega^{\mu\nu}(p,k) = \frac{g^2 C_2(G)\delta^{ab}}{2} (I_1 + I_2) \qquad (3.57)$$

Comparing with the standard integrals we get

$$I_1 = \frac{i4\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2})\ g^{\mu\nu} k^2}{2(2\pi)^d} \int_0^1 dz\, z^{(\frac{d}{2}-1)} (1-z)^{\frac{d}{2}-1} \qquad (3.58)$$

$$I_2 = \frac{i\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) k^\mu k^\nu \int_0^1 dz \left(\frac{4z^2}{(k^2 z(1-z))^{2-\frac{d}{2}}} - \frac{4z}{(k^2 z(1-z))^{2-\frac{d}{2}}} + \frac{1}{(k^2 z(1-z))^{2-\frac{d}{2}}} \right)$$

$$= \frac{-i2\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) g^{\mu\nu} k^2 \left[\frac{\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}\right)}{\Gamma(d\,)} \right] = -\frac{1}{3(4\pi)^2} i g^{\mu\nu} k^2 \Gamma\left(2 - \frac{d}{2}\right)$$

$$= \frac{i\pi^{\frac{d}{2}}}{(2\pi)^d} \Gamma\left(2 - \frac{d}{2}\right) k^\mu k^\nu \int_0^1 dz \left(4z^{d/2}(1-z)^{\frac{d}{2}-2} - 4z^{\frac{d}{2}-1}(1-z)^{\frac{d}{2}-2} + z^{\frac{d}{2}-2}(1-z)^{\frac{d}{2}-2} \right)$$

$$= \frac{i\pi^{\frac{d}{2}}}{(2\pi)^d}\Gamma\left(2-\frac{d}{2}\right)k^\mu k^\nu\left[\frac{4\Gamma\left(\frac{d}{2}+1\right)\Gamma\left(\frac{d}{2}-1\right)}{\Gamma(d)} - \frac{4\Gamma\left(\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)}{\Gamma(d-1)} + \frac{\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}-1\right)}{\Gamma(d-2)}\right]$$

$$= \frac{i}{(4\pi)^2}\Gamma\left(2-\frac{d}{2}\right)k^\mu k^\nu\left[\frac{4}{3}-2+1\right] = \frac{i}{3(4\pi)^2}k^\mu k^\nu\Gamma\left(2-\frac{d}{2}\right) \tag{3.59}$$

Therefore,

$$\Omega^{\mu\nu}(k) = \frac{g^2 C_2(G)\delta^{ab}}{2}(I_1+I_2) = \frac{-ig^2C_2(G)\delta^{ab}}{6(4\pi)^2}(g^{\mu\nu}k^2 - k^\mu k^\nu)\Gamma\left(2-\frac{d}{2}\right) \tag{3.60}$$

And

$$figure(3.2.b) = 0 \tag{3.61}$$

Therefore between the scale R^{-1}, where the first KK states areexcited, and the cutoff scale, there are finite quantum corrections of the KK states to the gauge coupling. The one-loop evolution equation for the gauge coupling from these cumulative effects of the KK modes change equation (3.2) to

$$b_i = \left[\frac{11}{3}C_2(G) - \frac{8}{3}n_f C(R) - \frac{1}{3}n_s C_2(R) - \frac{1}{6}C_2(G)\right] \tag{3.62}$$

Now calculation of the coefficients b_i are modified in the presence of the KK particles as

for the strong interaction (b_3): $SU(3)_c$

$$C_2(R) = 0\,, C(R) = 1\,,\ C_2(G) = 3 \tag{3.63}$$

Therefore,

$$b_3 = -\left[\frac{11}{3}\times 3 - \frac{8}{3}\times 3 - \frac{1}{6}\times 3\right] = -\frac{5}{2} \tag{3.64}$$

For the week interaction (b_2): $SU(2)_L$

$$C_2(R) = \frac{1}{2}\,, C(R) = 1\,, C_2(G) = 2 \tag{3.65}$$

Then

$$b_2 = -\left[\frac{11}{3} \times 2 - \frac{8}{3} \times 3 - \frac{1}{6} \times 2 - \frac{1}{6}\right] = \frac{7}{6} \qquad (3.66)$$

Finally For the electromagnetic interaction (b_1): $U(1)_Y$ we have

$$C_2(R) = \frac{1}{2}, C(R) = \frac{20}{12}, C_2(G) = 0 \qquad (3.67)$$

So

$$b_1 = -\left[-\frac{8}{3} \times 3 \times \frac{20}{12} - \frac{1}{6}\right] \times \frac{3}{5} = \frac{81}{10} \qquad (3.68)$$

Thus

$$b_i = \left[\frac{81}{10}, \frac{7}{6}, -\frac{5}{2}\right] \qquad (3.69)$$

3.5. ED Beta Function for Gauge Coupling Constant in ED (Brane Case):

We shall now consider the case of Brane localized matter fields for coupling. In this case there are no contributions from fermions to the gauge couplings in ED

(Ammar Abdalgabar, 2016).

The general case in ED:

$$b_i = -\left[\frac{21}{6} C_2(G) - \frac{1}{3} C_2(R)\right] + \frac{8}{3} \times C(R) \times \eta \qquad (3.70)$$

And we calculate the gauge couplings coefficients in this scenario of the three forces (b_1, b_2 *and* b_3) without excited KK fermions we obtain.

$$b_i = \left[\frac{1}{10}, \frac{41}{6}, -\frac{21}{2}\right] + \left[\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right] \times \eta \qquad (3.71)$$

Where: η is number of fermions generations among interaction and takes the values

($\eta = 0,1,2,3$)

When $\eta = 0$ the fermions fields are localized in the Brane (Brane case) the equation (3.70) becomes

$$b_i = -\left[\frac{21}{6} C_2(G) - \frac{1}{3} C_2(R)\right] \qquad \textbf{(}3.72\textbf{)}$$

And equation (3.71) becomes

$$b_i = \left[\frac{1}{10}, \frac{41}{6}, -\frac{21}{2}\right] \qquad (3.73)$$

When $\eta = 3$ the fermions in the Bulk and we recover our early equation (3.69)

The equation (3.71) for $\eta = 1$ becomes

$$b_i = \left[\frac{1}{10}, \frac{41}{6}, -\frac{21}{2}\right] + \left[\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right] \times 1 \qquad (3.74)$$

And for $\eta = 2$

$$b_i = \left[\frac{1}{10}, \frac{41}{6}, -\frac{21}{2}\right] + \left[\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right] \times 2 \qquad (3.75)$$

Chapter IV

Numerical Results, Discussions and Conclusion

4.1. Numerical Results and Discussions:

This chapter shall present our numerical results and discussion for some selected results. We will utilize the technique of the renormalization group equations at one-loop level for gauge couplings in the standard model and extra dimensions; we will obtain a set of RGEs and solve them numerically by using Mathematica software. For our numerical calculations we assume the fundamental scale is not far from the range of the LHC run 2 and set the compactification radii to be $\mathrm{R}^{-1} = 1000\ \mathrm{GeV}, 5000\ \mathrm{GeV}$ and $13000\ \mathrm{GeV}$with the initial values adopted at the $\mathrm{M_Z}$ scale discussion as follows: for the gauge couplings $\mathrm{g}_1(\mathrm{M_Z}) = 0.462,\ \mathrm{g}_2(\mathrm{M_Z}) = 0.651$ and$\mathrm{g}_3(\mathrm{M_Z}) = 1.22$.

Here in this thesis we discuss different possibilities for the matter fields, such as the case of bulk propagating or brane localized fields. We will discuss in both scenarios the evolution of the inverse fine structure constant which is related to the gauge couplings by $\alpha^{-1} = 4\pi/\mathrm{g}^2$.In brane case the SM chiral fermions are located on a boundary and in the 5D picture do not have Kaluza-Klein (KK) modes so they will not contribute to our RGEs. The SM Higgs live in the bulk. The gauge fields also live in the bulk. We will also explore a model in which the third generation lives in the bulk, this too may unify. We compute the one-loop RGEs for the gauge couplings in different localization scenarios. We present some selected plots and comment on other similar cases.

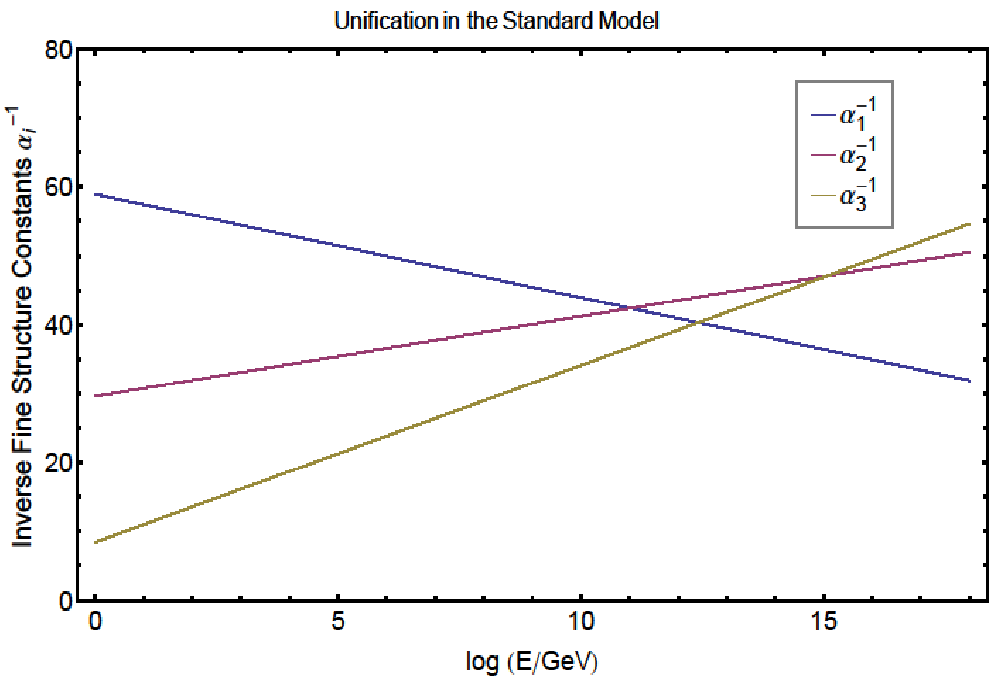

Figure (4.1) Evolution of the inverse fine structure constants in the standard model

Figure (4.1) shows that all three standard model inverse fine structure constants $\alpha_i = \frac{g_i^2}{4\pi}$are trying to unify themselves at some higher scale for one-loop level and in fact higher order loop correction does not change the result much as its effects is very small that means we need a new physics that have new particles to change the running of gauge coupling. This new physics could be either Supersymmetry or Extra Dimension models.

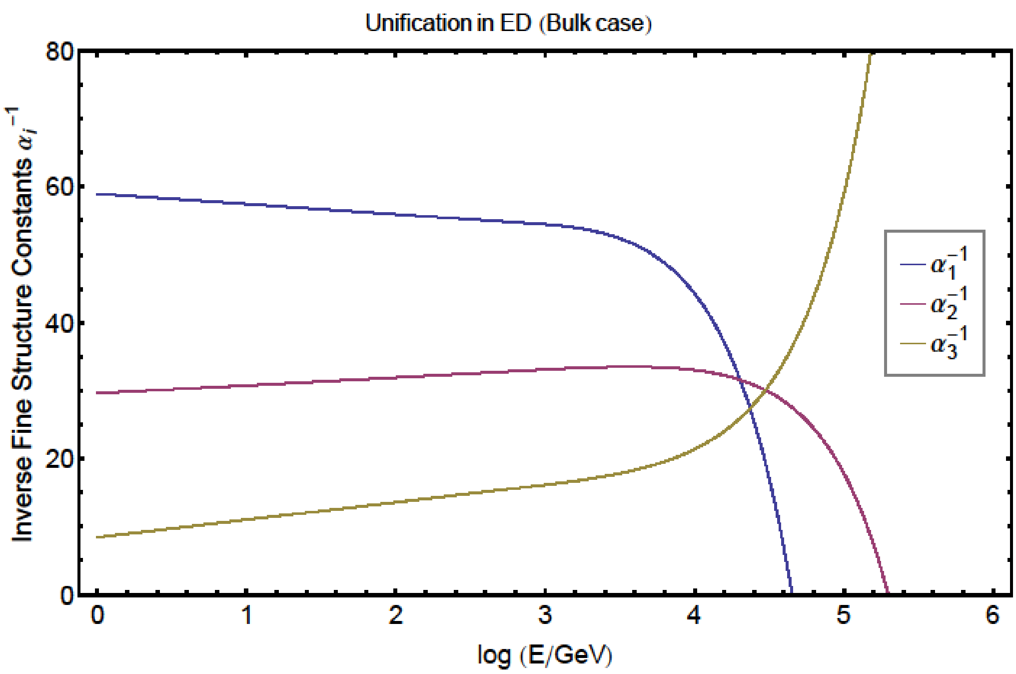

Figure (4.2) Evolution of the inverse fine structure constants in ED (Bulk Case)

It is clear from equ(3.45) that the presence of the extra dimensions has a substantial effect on the values of these gauge couplings. Remarkably, however, it turns out that there always exists a value of R^{-1} for which the gauge couplings may unify.

Figure (4.2) show the inverse fine structure constant α_i^{-1} in Bulk Case with $R^{-1} = 1$ TeV. As can be seen from the figure the ED particles give very good unification as expected and extra dimension may bring the unification down to a lower value, and the inverse fine structure constants nearly meet around $10^{4.3}$GeVin comparison with the standard model case. As can be noted from figure (4.2) the unification scale of the energy lowered from (10^{14}GeV) in the SM see figure (4.1) to ($10^{4.3}$) GeV in ED. This is due to the fact that the couplings constant run as power law rather than logarithmic fashion running.

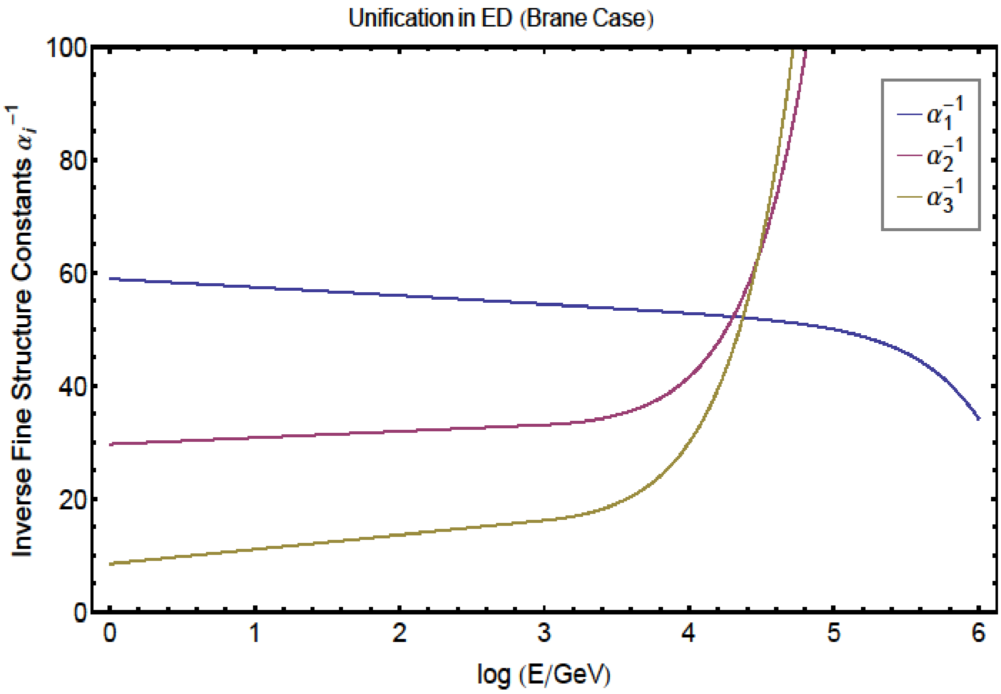

Figure (4.3) Evolution of the inverse fine structure constants in ED Brane Case

Figure (4.3) shows the running of coupling constants, the inverse fine structure constantsα_i^{-1} with $R^{-1} = 1$TeV. As one cross the threshold KK particle the contribution of KK particle becomes more and more significant and the unification scale also in this scenario is decreased as observed in the bulk scenario but the only difference is that the α_3^{-1} and $\alpha_2^{-1}$is increased and α_1^{-1} is decreased in Brane Case and all the inverse fine structure constants α_i^{-1} nearly meet around $10^{4.30}$, $10^{4.97}$and$10^{5.36}$GeV for $\mathrm{R}^{-1} = 1$ TeV, 5TeV and 13 TeV respectively.

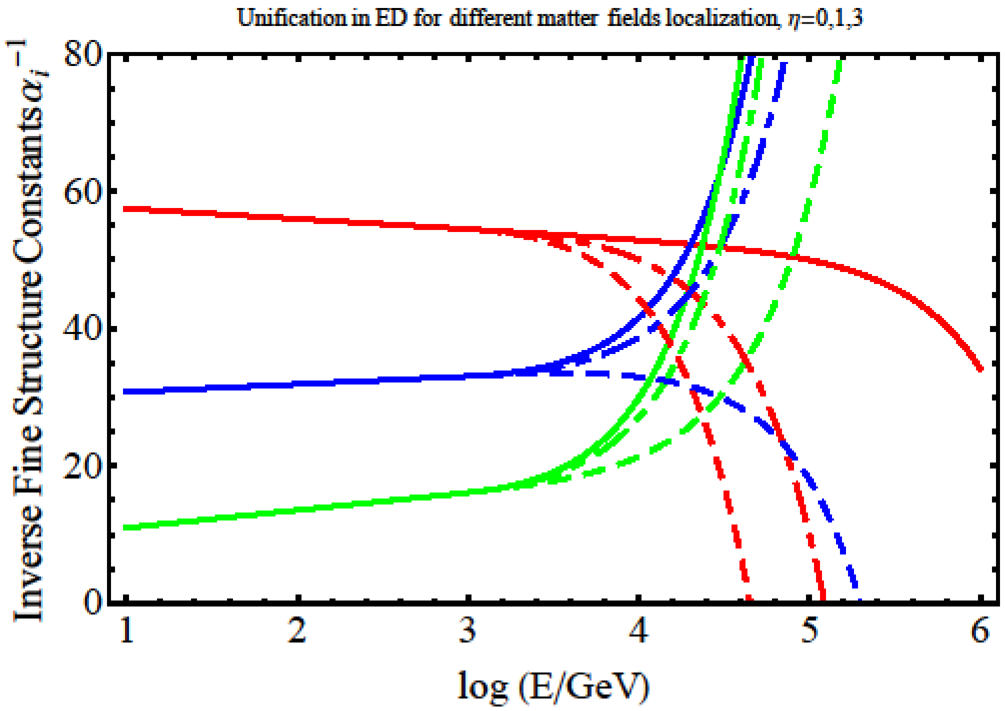

Figure (4.4) Evolution of the inverse fine structure constants in ED in general case

Figure (4.4) present the evolution of inverse fine structure constants in ED model for general case $\eta = 0, 1$ and 3where (solid line)is for $\eta = 0$ (brane case),(dot-dashed line) is for $\eta = 1$ and(dashed line) is for $\eta = 3$(bulk case),the evolution of the bulk field and brane localized cases for several choices of compactification scale for the extra-dimensions in the ED model were performed. We find that there is a difference in the α_2^{-1}evolution where it increases in the brane case and decreases in the bulk propagating case.

A sufficient condition for unification in a five dimensional model is that

$$R_{ij} = \frac{b_i^{5D} - b_j^{5D}}{b_i^{SM} - b_j^{SM}}$$

does not depend on (i, j), where $b_i^{5D}$are the five dimensional beta function coefficients, at one-loop.

It is easy to check that although these relations are not satisfied exactly in our case,
The y are nevertheless approximately satisfied:

$$\frac{R_{12}}{R_{13}} = \frac{0.954}{0.955} = 0.999, \qquad \frac{R_{13}}{R_{23}} = \frac{0.955}{0.956} = 0.997$$

This remains true independently of the value of η, which shifts allb_i^{5D} by a fixed Amount. Thus, we expect that gauge coupling unification will continue to hold to a good degree of accuracy.

In order to get precise unification additional fields are required, here a precise unification of the three gauge coupling is achieved by adding three real scalar fields in extra dimension. We assume these three additional fields transforming in the adjoint of SU(2) and have odd parity as result the contribution of these new fields modify the RGEs in equation (3.71) to

$$b_i = \left[\frac{1}{10}, \frac{35}{6}, -\frac{21}{2}\right] + \left[\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right] \times \eta$$

Which lead to unification of the three gauge couplings as shown in figure (4.6) and figure (4.7) Clearly these additional fields will appear at scale of R^{-1}.

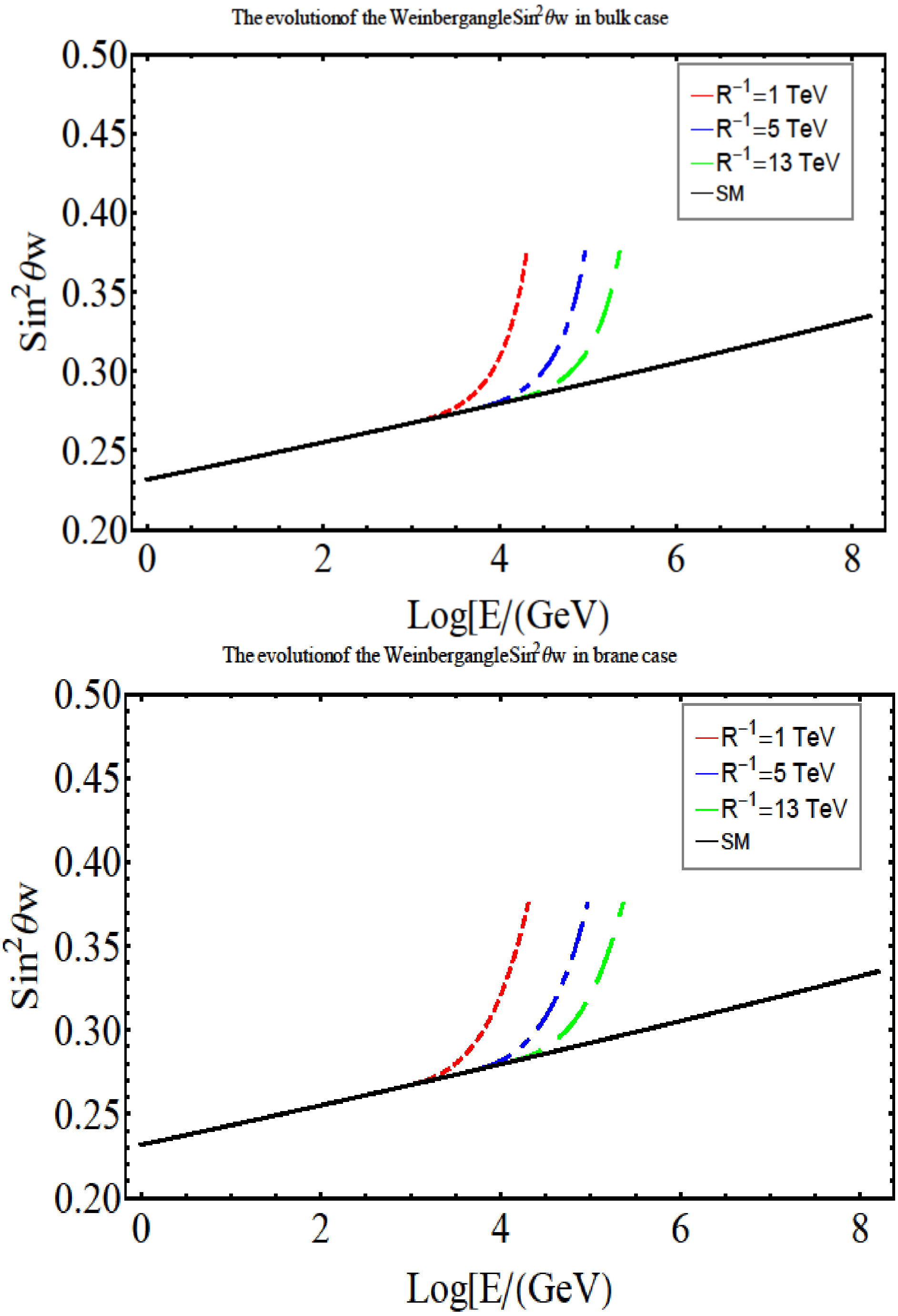

Figure (4.5) the evolution of the Weinberg angle for three different values of the

compactification where the solid line represents the SM case.

Furthermore we present the evolution of Weinberg mixing angle as function of energy scale. Note that evolution trajectories evolve until the unification scale of gauge couplings unify. Once the KK states begin to contribute the new contributions from the extra dimensions change the behavior; that is, it increases until we reach the cutoff scale. One can see that for $R^{-1} = 1$ TeV, $sin^2\Theta_W$ can rise from 0.23 to 0.4. This result may be useful, at least from a

model building perspective, as many extra-dimensional models such as gauge-Higgs unification models in ED predict for many choices of the gauge group large values of $sin^2\theta_W$ from a group theory point of view. However, this value is the one expected in the energy range of coupling unification, which once evolved back to the
electroweak scale and may indeed be close or compatible to the measured value.

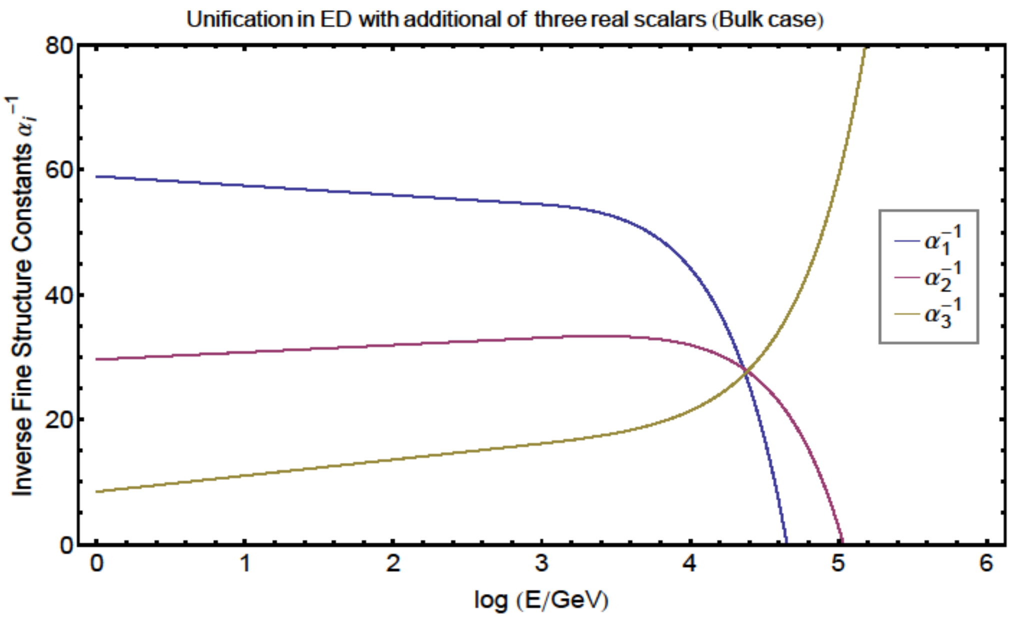

Figure (4.6) Unification of couplings with additional scalar fields in bulk case.

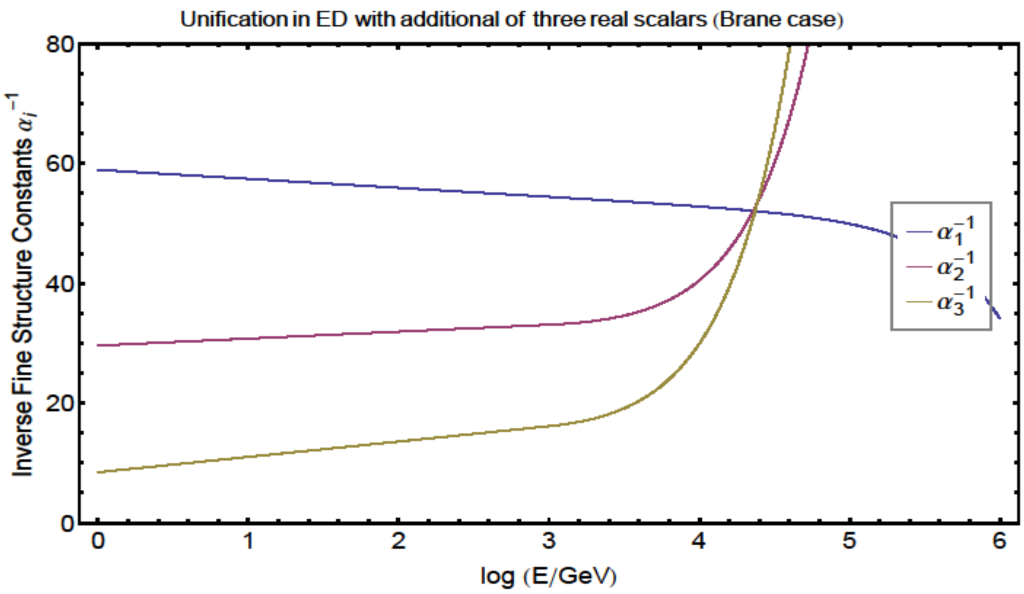

Figure (4.7) Unification of couplings with additional scalar fields in brane case.

4.2. Conclusion:

We derived the one loop renormalization group equation for the gauge couplings in the universal extra dimension for different matter field localization (brane case and bulk case) compactified in a circle. We discussed the evolution of inverse fine structure constants as function of energy scales in various scenarios in the five dimensional descriptions with a large enough extra dimensional scale as to make the extra dimensional features practically relevant to the phenomenology of the model. In other words we require a compactifications scale $\mathrm{R}^{-1} = 1\ \mathrm{TeV},\ 5\ \mathrm{TeV}$ and $13\ \mathrm{TeV}$ scale and not simply an (almost) GUT scale of extra dimension. Such a criteria is useful to rule out certain models, for instance by this criteria one can straightforwardly rule out at extra dimensional models in which the first and second generation are in the bulk, with the third generation either in the bulk or on a brane, as such a models can unify only with an extra dimensional scale of the order of the GUT scale. We have shown here in both scenarios (brane and bulk) the gauge couplings do not unify at single energy scale as observed in the SM only the unification scale is lowered to small energy scale for different compactification scale. We proposed a model in which the gauge couplings indeed unified at single point by adding three extra scalar fields in the extra dimension. Furthermore we studied the evolution of Weinberg angle in both scenarios.

4.3 Recommendation

We may go to the evolution up to two loop level even higher, the two loop RGEs for the gauge and Yukawa couplings are entangled, so we expect few percent change on the evolution of gauge couplings due to the appearance of Yukawa couplings this also might change α_3^{-1} because of the large size of Y_t. In models with extra dimension the one-loop running of Yukawa couplings is clearly insufficient, since higher order corrections can be just as important at scales few times above $\frac{1}{R}$. Although this type of large corrections, to claim unification one needs to make sure that Y_t stays perturbative up to the unification scale. As we highlighted in our discussion the coupling constants do not unify at single energy scale this will open the possibility to look for different model such as supersymmetry or supersymmetry in extra dimensions even with additional field to get a precise unification. We leave these two points to future works.

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