Chapter I

Introduction

1.1 General Introduction:

The Universe is made of elementary particles, ruled by a few fundamental forces of nature. Some of these particles are stable, but some of them just have a lifetime of a fraction of second. It's said that all these particles coexisted together during the Big Bang.

Particles Physics is the study of the basic nature of energy, of matter, of force, of time or space. It works on discovery the simplest constituents of matter (elementary particles), and to understand the fundamental forces interacting among them.

Elementary Particles are too small to see or study directly, so we examine them by colliding particles at high energies and analyzing the results.

1.2 Particles Accelerator and High Energy Physics:

The study of particles physics began with the discovery of electron in 1897 by J.J.Thomson. Around 1930, new particles were detected using Cosmic Rays as a source of energy, since it was the only high energy source known by that time, starting with the discovery of the positron in 1931, and the muon in 1937. The discovery of new particles was so much that the construction of High Energy Accelerators was impulse, providing intense beams of known energy that lead us to discovery of the quark substructure of matter.

One reason for why high energies became so important comes from quantum mechanics, which describes particles as waves, whose wavelengths are established by the de Broglie's expression $\boldsymbol{\lambda} = \frac{\boldsymbol{h}}{\boldsymbol{p}}$ where $\boldsymbol{p}$ the beam momentum, and $\boldsymbol{h}$ the Planck's constant, which means that beams with higher momentums have shorter wavelengths, bringing higher resolutions, providing finer detail in the structure of fundamental particles.

To reach very high collision energies, many of the current accelerators are colliders in which two particles beams are accelerated in opposite directions in order to collide them; doing this almost all particle energy can be employed for production of new particles, being able to obtain high collision energies to study the structure of matter.

The energy needed for particles discovery is increasing more and more with time.

Some examples of colliders are Tevatron at Fermilab, or the LHC at CERN.

Since 1939, the accelerators development has grown so much that the energy has increased from 80 KeV from the original cyclotron of 13cm diameter to the 10 TeV for the LHC of 27 km diameter.

1.3 Important of the study:

An important feature of extra-dimensional models is the impact of the large number of KK modes on the renormalization group (RG) running of physical parameters. The RG running in extra-dimensional models has been investigated and studied. It has been shown that the RG evolution changes from the typical logarithmic running in four-dimensional standard models to an effective power-law running at high energies in extra dimension. This means that sizable running could take place at relatively low energy scales. As such Extra space-time dimensions naturally lead to unification to gauge coupling constant at intermediate mass scales, and moreover it provides a natural mechanism for explaining the Yukawa fermion couplings hierarchy and this has the potential to address the fermion masses hierarchy. Furthermore in extra dimension Yukawa couplings too evolve with a power-law dependence on the mass scale.

1.4 Objectives:

The main objective is to derive the renormalization group equations for the Yukawa coupling constants at one loop level in extra dimension model, and to study the evolution of Yukawa couplings as function of energy scale in universal extra dimension model.

1.5 Outlines:

The outline of the dissertation is as follows:

We introduce in Chapter I a general introduction to topics. Chapter II will discuss the theory of the standard model and extra dimension model. Chapter III will concern with the technique of renormalization group equations. Chapter IV shall present our numerical results, discussions and conclusions.

Chapter II

The Standard model and new physics

2.1. Introduction:

This Chapter will present a general review of the Standard Model of particle physics (SM), extension of the SM such as supersymmetry and extra dimension.

2.2. What is the standard model (SM?):

The standard model of particle physics is the mathematical theory that describes the weak, electromagnetic and strong interactions between leptons and quarks; the theory of electroweak of the standard model was introduced by Glashow, Salam and Weinberg in the early 1970's (Salam, 1968) (Weinberg S. , 1967) (S.L.Glashow, 1961). The standard model asserts that the material in the universe is made up of elementary fermions interacting through fields; the particles associated with the interacting fields are called bosons.

The elementary particles assignments in the standard model are as follows:

Quarks $$\binom{u}{d}_L, u_R, d_R \qquad \binom{c}{s}_L, c_R, s_R \qquad \binom{t}{b}_L, t_R, b_R \tag{2.1}$$

Leptons $$\binom{\boldsymbol{\nu_e}}{\boldsymbol{e}}_{\boldsymbol{L}}, e_R \qquad \binom{\boldsymbol{\nu_\mu}}{\boldsymbol{\mu}}_{\boldsymbol{L}}, \mu_R \qquad \binom{\boldsymbol{\nu_\tau}}{\boldsymbol{\tau}}_{\boldsymbol{L}}, \tau_R \tag{2.2}$$

Gauge bosons $$\begin{cases} \boldsymbol{photon\ \ \gamma} \\ \boldsymbol{weak(gauge)bosons\ W^{\mp},\ Z^{\circ}} \\ \boldsymbol{gluons\ \ g} \end{cases} \tag{2.3}$$

Higgs boson $$\{\ \boldsymbol{H} \tag{2.4}$$

Table 2.1 shows characteristics of quarks in the Standard Model

Quark	$\boldsymbol{Q}^*$	$\boldsymbol{I_3^*}$	$(\frac{\boldsymbol{Y}}{\boldsymbol{2}})^* = \boldsymbol{Q} - \boldsymbol{I_3}$	**Mass**
u	**+2/3**	**1/2**	**1/6**	**1.5 ~ 5 MeV**
d	**-1/3**	**-1/2**	**1/6**	**3 ~ 9 MeV**
s	**-1/3**	**-1/2**	**1/6**	**60 ~ 170 MeV**
c	**+2/3**	**1/2**	**1/6**	**1.47 ~ 1.83 GeV**
b	**-1/3**	**- 1/2**	**1/6**	**4.6 ~ 5.1 GeV**
t	**+2/3**	**1/2**	**1/6**	**174.3 ± 3.2 ± 4.0 GeV**

(* Q is the charge, I_3 is the triplet isospin and $\frac{\boldsymbol{Y}}{\boldsymbol{2}}$ is the hyper charge**)**

Quarks and leptons are fundamental building blocks of matter; all of them are fermions and have spin$\left(\frac{\mathbf{1}}{\mathbf{2}}\right)$ **,** they are classified as left-handed isospin doublets and right-handed isospin singlet's; and will be described by the Dirac equation. Quarks interact through the electromagnetic and weak interactions and also through the strong interaction. Leptons interact only through the electromagnetic interaction (if they are charged) and the weak interaction (A.D.Martin, 1984).

Gauge bosons having spin **(1)** are the mediators of interactions between quarks or leptons; there are massless bosons, the photon$s$ γ, and three massive ones, the W+, W− and the Z°$bosons$ **.**

Electromagnetic, weak and strong interactions are mediated by photons γ, weak bosons $W\pm$, $Z°$ and gluons g, respectively. Interaction strength depends on which gauge bosons propagate between quarks or leptons (Guigg, 1983).

The Higgs boson with spin **(0)** is introduced for the Higgs mechanism to generate mass to elementary particles, which is operative in the theories with spontaneous symmetry breaking of local gauge symmetries (P.~W.~Higgs, 1964)**.**

Table 2.2 shows elementary particle interactions

Interaction	**Coupling strength**	**Mediator**	**Spin**
Electromagnetic	$\boldsymbol{\alpha = \frac{e^2}{4\pi} \cong \frac{1}{137}}$	**Photon**	**1**
Weak	$\boldsymbol{G_F \cong 1.16 \times 10^{-15} Gev^{-2}}$	**3 Weak bosons**	**1**
Strong	$\boldsymbol{\alpha_s = \frac{g_s^2}{4\pi} \cong 0.1}$	**8 Gluons**	**1**
Gravitational	$\boldsymbol{G_N \cong 6.71 \times 10^{-39} (Gev/c)^{-2}}$	**Graviton (Still to be discovered)**	**2**

Table 2.3 shows possible Quarks and Lepton interaction with all the forces

Elementary particles	$\boldsymbol{\gamma}$	$\boldsymbol{W^+, W^-, Z^0}$	**Gluons**
Quarks	√	√	√
Leptons	√	√	×

(√ means interact and × means don't interact**)**

2.3. Symmetries and Particle Content in standard model:

The Standard Model is believed to be a (highly successful theory) based on the local gauge groups. The gauge group for the standard model is**:**

$$\mathbf{SU(3)} \otimes \boldsymbol{SU(2)_L} \otimes \boldsymbol{U(1)_Y} \tag{2.5}$$

Where:

The **SU (3)** gauge group or color group is the symmetry group of strong interactions, This group acts on the quarks which are the elementary constituents of matter and the interaction force is mediated by the gluons which are the gauge bosons of the group (J.donoghue, 1994).

The quarks and the gluons are colored fields. The corresponding coupling is denoted by $\boldsymbol{\alpha_s}(\boldsymbol{the\ coupling\ strength\ constant})$, The **SU (3)** color symmetry is exact and consequently the gluons are massless.

The theory of strong interactions based on color SU (3) is called Quantum Chromo dynamics (**QCD**).

The $\mathbf{SU(2)}_L \otimes \mathbf{U(1)}_Y$ is the gauge group of the unified weak and electromagnetic interactions. Where $SU(2)_L$ is the weak isospin group, acting on left-handed fermions, and $\mathbf{U(1)}_Y$ is the hypercharge group.

The **SU (2)** group has three gauge bosons are denoted by $W_\mu^1,\ W_\mu^2\ , W_\mu^3$ None of these gauge bosons (and neither B_μ) are physical particles, linear combinations of these gauge bosons will make up the photon as well as the W± and the Z◦ boson.

As matter content for the first family, we have**:**

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}\ ;\ u_R\ ;\ d_R \qquad and \qquad l_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\ ;\ e_R\ ; \nu_R \tag{2.6}$$

The explicit values for the hypercharges of the particles above are as follows

$$Y(l_L)\ = -\tfrac{1}{2}\ , Y(e_R) = -1\ , Y(\nu_R) = 0\ , Y(q_L) = \tfrac{1}{6}\ , Y(u_R) = \tfrac{2}{3}\ , Y(d_R) = -\tfrac{1}{3} \tag{2.7}$$

Under SU (3) the lepton fields $l_L\ ;\ e_R\ ; \nu_R$ are singlets, i.e. they do not transform at all. This Means that they do not couple to the gluons. The quarks on the other hand form triplets Under

SU (3). The strong interaction does not distinguish between left- and right-handed Particles (Guigg, 1983).

However, since we ultimately want massive weak gauge bosons, we will have to break the $\mathbf{U(1)}_Y \times SU(2)$ gauge group spontaneously, by introducing some type of Higgs scalar (Quigg, 2007).

2.4. The Higgs mechanism:

A mechanism that gives mass to the SM particles and keeps the Lagrangian invariant under gauge symmetries (P.~W.~Higgs, 1964) (Quigg, 2007).

If mass terms for gauge bosons and for left/right-chiral fermions are introduced by hand into the theory, they destroy the gauge invariance of the theory. This problem has been solved by means of the Higgs mechanism in which masses are introduced into gauge theories in a consistent way. The solution of the problem is achieved at the expense of a new fundamental degree of freedom, the Higgs field, which is a scalar field.

This Scalar field is denoted by Φ and has hypercharge $Y_\phi = 1$ and can interaction with each other, the interaction between fermion fields and the Higgs field is of Yukawa type.

The Higgs doublet Lagrangian should contain a **"**spontaneous symmetry breaking**"** potential Which will give the Higgs a VEV (Vacuum Expectation Value) and self-interactions, and kinetic terms which will generate the gauge boson masses and interactions between the Higgs and the gauge bosons (L.F.Li, 1991).

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad (2.8)$$

And the potential is given by

$$V(\Phi) = -\frac{1}{2}\mu^2\Phi^+\Phi - \frac{\lambda}{4}(\Phi^+\Phi)^2 \qquad (2.9)$$

Which involves two new real parameters $\mu \; and \; \lambda$ we demand $\lambda > 0$ for the potential to be bounded; otherwise the potential is unbounded from below and there will be no stable vacuum state.

μ Takes the following two values:

• $\mu^2 > 0$ Then the vacuum corresponds to $\Phi = 0$, the potential has a minimum at the origin (see figure 2.1 right panel).

• $\mu^2 < 0$ Then the potential develops a non-zero Vacuum Expectation Value (VEV) and the minimum is along a circle of radius $\frac{v}{\sqrt{2}} = \frac{246}{\sqrt{2}}$ (see figure 2.1 left panel).

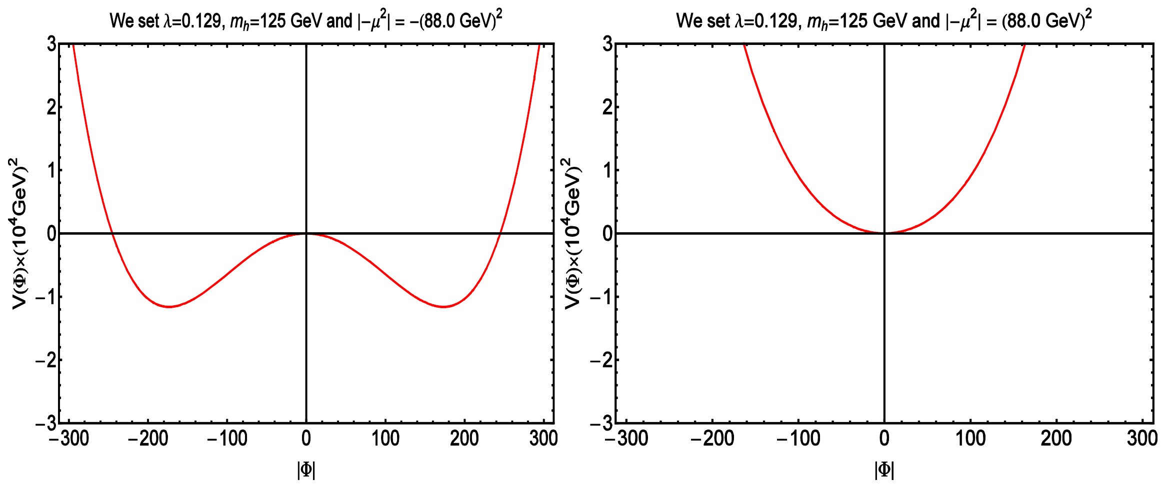

Fig 2.1 shows that The Higgs potential $V(\Phi)$ with: in the left panel, the case $\mu^2 < 0$ and the right panel for the case $\mu^2 > 0$ as a function of $|\Phi| = \sqrt{\Phi^+\Phi}$

2.5. The lagrangian of the standard model:

The Lagrangian of the standard model is the sum of the gauge, matter, Yukawa and Higgs interactions. It is given by**:**

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{Matter} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Gauge.fix} + \mathcal{L}_{Ghost} \tag{2.10}$$

The first term is built up by the gauge fields and their self- interactions**:**

$$\mathcal{L}_{Gauge} = -\frac{1}{4} W^i_{\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \tag{2.11}$$

With the field strengths defined as**:**

$$W^i_{\mu\nu} = \partial_\nu W^i_\mu - \partial_\mu W^i_\nu - g_w \epsilon^{ijk} W^j_\mu W^k_\nu \tag{2.12}$$

$$B_{\mu v} = \partial_\mu B_v - \partial_v B_\mu \tag{2.13}$$

$$G^a_{\mu v} = \partial_v G^a_\mu - \partial_\mu G^a_v - g_s f^{abc} G^a_\mu G^b_v \tag{2.14}$$

The tensors $\boldsymbol{\epsilon^{ijk}}$ and $\boldsymbol{f^{abc}}$ are the **SU (2)** and **SU (3)** structure constants, $\boldsymbol{g_w}$ and $\boldsymbol{g_s}$ are the weak-isospin and the strong coupling, respectively.

The second term summarizes the fermion-gauge boson couplings

$$\mathcal{L}_{Matter} = \sum \bar{\psi}\, i\, \gamma_\mu D_\mu\, \psi \tag{2.15}$$

Where $\boldsymbol{\psi}$ is the fermion field.

With the sum running over the left- and right-handed field components of the leptons and quarks. Depending on the fermion species, the covariant derivative takes the form**:**

$$iD_\mu = i\partial_\mu + g_W I^i W^i_\mu - g' \frac{Y}{2} B_\mu + g_S T^a G^a_\mu \tag{2.16}$$

g' is the hypercharge coupling.

The Higgs-gauge boson lagrangian generated by the covariant derivative:

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \tag{2.17}$$

Where the covariant derivative $\boldsymbol{D_\mu}$ is defined as:

$$D_\mu = \left(\partial_\mu - \frac{ig}{2} B_\mu - \frac{ig\sigma}{2} W^a_\mu\right) \tag{2.18}$$

Yukawa couplings are uniquely fixed by gauge invariance and the Lagrangian given by

$$\mathcal{L}_{Yukawa} = Y^d_{ij} q^i_l \Phi d^j_R + Y^u_{ij} q^i_l \Phi^\sim u^j_R + Y^e_{ij} L^i_l \Phi e^j_R + h.c \tag{2.19}$$

2.6. Gauge Fixing and Ghosts:

Gauge fixing is necessary when the gauge fields are quantized. Quantization means to develop a path integral formalism for the gauge theory. The path integral is diverging as one integrate over an infinite set of gauge-equivalent configuration, here the gauge fixing is used to pick up one arbitrary representative, therefore, giving meaning to the path integral. On other hands, the gauge

Invariance we look for in gauge theory, a naive path integral approach would spoiled it. The solution is given by what is called the Faddev-Popov procedure, where they introduced an identity expression consisting of a functional integral over a gauge fixing condition times a functional determinant over anti commuting fields in the path integral. The latter gives rise to what is known as ghost fields, which keep the gauge freedom within the theory, but are not physical particles (because ghost violate the spin-statistics relation) (McMahon, 2008). As such, we need to add terms in the Lagrangian like**:**

$$\mathcal{L}_{Gauge\ fixing} = -\frac{1}{2}\frac{\xi}{2}(\partial_\mu A^\mu)^2 \qquad (2.20)$$

$$\mathcal{L}_{Ghost} = \bar{c_b}\partial^\mu D_\mu^{ab} C_a \qquad (2.21)$$

2.7. Problems with the Standard Model:

Although the Standard Model of particle physics is very successful, with no confirmed accelerator data that contradict it, there are many theoretical reasons to consider some new physics beyond the Standard Model**.** The list below summarizes some issues with the standard model (A.~Abdalgabar, 2013).

2.7.1. Dark Matter:

The SM does not have any dark matter candidates, as opposed to observational cosmology**.**

2.7.2. Gauge Hierarchy problem:

The Hierarchy problem is the question of why there is such a huge difference between the electroweak scale $\boldsymbol{M_{EW} = \mathcal{O}(100)\ Gev}$ and the plank scale $\boldsymbol{M_{PL} = \mathcal{O}(10^{18})Gev}$. This is also known as the naturalness problem.

2.7.3. Gravity is not included:

Though the unification of the electromagnetic and weak interactions was achieved in the SM and the strong interaction appears to be part of the unification, the SM does not include the effects of gravity. Note that the effects of gravity become important at energies of the order of the Planck scale $\boldsymbol{M_{PL} = \mathcal{O}(10^{18}) Gev}$. Since The ultimate goal in particle physics is to unify all the fundamental forces in nature.

2.7.4 Gauge group:

The SM does not explain the choice of $\boldsymbol{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$ why are there three families and four interactions**?** Why **3+1** space time dimensions**?**

2.8. Beyond the Standard Model:

The problems mentioned above in the SM cannot represent a fundamental theory of the Universe but it can be an effective field theory at low energies. A physical theory may exist beyond the SM, Supersymmetry (**SUSY**) and universal extra dimensions (**UED**) have evolved into a new paradigm with many tools to solve the large number of outstanding issues that remains unanswered in the SM.

2.8.1. Super symmetry (SUSY):

Supersymmetry is a transformation which turns bosons into fermions, and fermions into bosons. If it is asymmetry of the Lagrangian, then every fermion must have a bosonic partner and the vice versa, and the interactions are restricted by the symmetry. When we supersymmetrize (exactly) the Standard Model, we will therefore double the number of particles but the number of coupling constants stays (almost) the same.

Alternatively, it is to say that Supersymmetry is the idea that there is" a superpartner" for each elementary particle**: selectron**, **smuon, stau**, **squarks**, **photino**, **higgsino, etc**. Supersymmetry provides an elegant way to solve the hierarchy problem, gauge couplings unification. We will not discuss the supersymmetry furthermore; we refer interested reader to go to Ref (J. Louis, 1998).

2.8.2. Universal Extra Dimensions (UED):

A new kind of physics, Extra Dimension (ED), was introduced in particle physics by Kaluza and Klein in 1920, to unify the electromagnetic interaction with the gravitational one by generating the photon from the extra components of the five-dimensional metric (T.~Kaluza, 1921). Extra dimension and supersymmetry introduced new particles so we need to distinguish these new particles. In particle physics if we can distinguish a fermion from a bosonic particle by measuring the spin of the particle at the Large Hadron Collider (LHC) or the International Linear Collider(ILC), then we can have a distinct signature of the physics of extra dimension from that of Supersymmetry. Since the major difference between SUSY and UED is that the new heavy particles have different spin (Majee, March, 2008).

2.8.2.1 Scalar particle in UED:

In addition to the four space-time co-ordinates **x (x, t),** let us denote the extra space-type coordinate by y, compactified on a circle or radius R (A.~Abdalgabar, 2013). Thus, the Lagrangian of a free complex scalar $\boldsymbol{\Phi(x,y)}$ with mass m will be a function of both x and y co-ordinates with a condition that the field at **y = πR** will match with that at **y = 0**, i.e. it has a periodicity of **πR** along the **y**

direction. So one can expand it in a Fourier series by assigning an even parity to the scalar field as**:**

$$\Phi(x,y) = \frac{1}{\sqrt{2\pi R}}\Phi_0(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\left[\Phi_n(x)\cos(\frac{ny}{R})\right] \qquad (2.22)$$

The five dimensional Lagrangian is given by

$$\boldsymbol{\mathcal{L}_{scalar}} = \int dy\{D_M\Phi^\dagger D^M\Phi - M^2\Phi^\dagger\Phi - \lambda_5(\Phi^\dagger\Phi)^2\} \qquad (2.23)$$

With M= 0, 1,2,3,5

The equation of motion can be derived by varying the above integral with respect to the field Φ**:**

$$(\partial_5^2 + p^2 - M^2)\Phi = 0 \qquad (2.24)$$

Where the n-th KK mode mass is given as**:**

$$m_n^2 = m^2 + \frac{n^2}{R^2} \tag{2.25}$$

Plugging equation (2.22) into equation (2.23) we obtain the effective four dimensional lagrangian

$$\begin{aligned}\boldsymbol{\mathcal{L}_{4D}} = \; & (D_\mu \Phi)^{(0)\dagger}(x)(D^\mu \Phi)^{(0)}(x) \\ & +(D_\mu \Phi)^{(n)\dagger}(x)(D^\mu \Phi)^{(n)}(x) \\ & + (D_5 \Phi)^{(n)\dagger}(x)(D^5 \Phi)^{(n)}(x)\end{aligned} \tag{2.26}$$

Where:

$$(D_\mu \Phi)^0 = D_\mu^{(0)} \Phi^{(0)} - \left(ig \frac{\sigma^i}{2} W_\mu^{(n)i} + ig' \frac{Y}{2} B_\mu^{(n)} \right) \Phi^{(n)} \tag{2.27}$$

$$(D_\mu \Phi)^n = D_\mu^{(ns)} \Phi^{(s)} - \left(ig \frac{\sigma^i}{2} W_\mu^{(n)i} + ig' \frac{Y}{2} B_\mu^{(n)} \right) \Phi^{(0)} \tag{2.28}$$

$$(D_5 \Phi)^n = D_5^{(ns)} \Phi^{(s)} - \left(ig \frac{\sigma^i}{2} W_5^{(n)i} + ig' \frac{Y}{2} B_5^{(n)} \right) \Phi^{(n)} \tag{2.29}$$

σ^i being the usual three Pauli matrices.

2.8.2.2 Gauge fields and Gauge fixing:

The Lagrangian for an Abelian gauge field and gauge fixing given by

$$\boldsymbol{\mathcal{L}_{Gauge+GF}} = \int dy \, (-\frac{1}{4} F^{MN} F_{MN} - \frac{1}{2\xi} (\partial_\mu A^\mu - \xi (\partial_5 A_5)^2) \tag{2.30}$$

Where $\boldsymbol{\xi}$ is the gauge fixing parameter and $\boldsymbol{F_{MN} = \partial_M A_N - \partial_N A_M}$. The gauge fixing term eliminates the mixing between A_μ and the extra polarization A_5.

In the Feynman-'t Hooft gauge $\boldsymbol{\xi = 1}$, the equations of motion for $\boldsymbol{A_5}$ can be obtained**:**

$$(\partial_5^2 - \partial_\mu^2) A_5 = 0 \tag{2.31}$$

And the effective four dimensional lagrangian is given by

$$\begin{aligned}\boldsymbol{\mathcal{L}_{4D}} \quad = \quad & -\frac{1}{4}\left(G_{\mu\nu}^{(0)a}G^{(0)a\mu\nu} + G_{\mu\nu}^{(n)a}G^{(n)a\mu\nu} + 2G_{\mu 5}^{(n)a}G^{(n)a\mu 5}\right) \\ & -\frac{1}{4}\left(W_{\mu\nu}^{(0)a}W^{(0)a\mu\nu} + W_{\mu\nu}^{(n)a}W^{(n)a\mu\nu} + 2W_{\mu 5}^{(n)a}W^{(n)a\mu 5}\right) \qquad (2.32) \\ & -\frac{1}{4}\left(B_{\mu\nu}^{(0)}B^{(0)\mu\nu} + B_{\mu\nu}^{(n)}B^{(n)\mu\nu} + 2B_{\mu 5}^{(n)}B^{(n)\mu 5}\right)\end{aligned}$$

2.8.2.3 Fermion particle in UED:

The Lagrangian for the Dirac field in 5D is given by**:**

$$\boldsymbol{\mathcal{L}_{Fermion}} = \int dy\, \overline{\Psi}(i\Gamma^M D_M - m)\Psi \qquad (2.33)$$

$\boldsymbol{where}$

$\Gamma^M = (\gamma^\mu, i\gamma^5)$ are Dirac matrices and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$$\begin{aligned}\boldsymbol{\mathcal{L}_{4D}} \quad = \quad & i\overline{\Psi}_L^{(0)}\gamma^\mu D_\mu \Psi_L^{(0)} + i\overline{\Psi}_L^{(n)}\gamma^\mu D_\mu \Psi_L^{(n)} \\ & + \overline{\Psi}_L^{(0)} D_5 \Psi_L^{(0)} - \overline{\Psi}_L^{(n)} D_5 \Psi_L^{(n)} + L \to R\,. \qquad (2.34)\end{aligned}$$

Chapter III

Renormalization Group Equations

3.1 Introduction:

This chapter shall discuss the Renormalization Group Equations (RGEs) method and calculation of beta function for gauge couplings and Yukawa couplings in Standard model and Extra dimension model.

3.2 Renormalization Group:

The renormalization group in quantum field theory (QFT) tells us how different couplings behave with energy (L.~-X.~Liu A. a., 2011).

3.2.1 What is renormalization?

In Quantum Field Theory (QFT), Green function is the most important thing to be calculated. In perturbative QFT these quantities are divergent. The systematic way to remove these divergences is known as **renormalization** (Collins, 1984).

The renormalization theory is implemented to remove all the divergences in loop integrals from the physical measurable quantities. These loop diagrams are supposed to give finite results to the physical quantities but they give infinities instead. This tells us that our theory has missed some information. One might ask where do these infinities come from**.** These infinities arise from the integration over all momentum. In other words, the infinities occur because we let our theory go to arbitrary high energy (UV) (Majee, March, 2008).

There are different ways to cancel these infinities. In order to renormalize the theory we need a reference point which is also arbitrary, Different choices of this reference point lead to different sets of parameters for the theory, but physics should not depend on the arbitrary choice of the reference point and be invariant. This invariance leads to the **renormalization group.**

In quantum field theory it is a useful method to examine the behavior of physics at a different scale knowing the same at some other scale. Thus, measuring the observables in a low energy experiment one can compare with the values predicted from a theory at a higher scale, *e.g.* at the GUT scale and certify about the correctness of the theory. In the standard model, variations of the gauge coupling constants with energy are given by the following renormalization group equations (**RGEs**) (A.~Abdalgabar, 2013).

$$16\pi^2 \frac{dg_i}{dt} = \quad \beta_{SM}(g_i\ ) + \beta_{ED}(g_i\ ) \qquad (3.1)$$

$$\beta(g_{i)} = b_i g_i^3 \qquad (3.2)$$

Where i stands for $U(1)_Y$, $SU(2)_L$ *and* $SU(3)_C$ and the right-hand-side of equation (3.1) is known as the β function of the corresponding coupling.

In the above equations the coefficient b_i can be calculated for any $SU(N)$ group as

$$b_i = -\frac{11}{3} C_2(G) + \frac{2}{3} n_f C(R) + \frac{1}{3} n_s C_2(R) \qquad (3.3)$$

Where:

$\boldsymbol{C_2(G), C(R)}$ ***and*** $\boldsymbol{C_2(R)}$ Refer to the gauge boson, Fermionic and Higgs scalar contribution respectively. n_f is number of fermion flavor and n_s is the number of scalar field.

3. 3 Calculation of Beta Function ($\boldsymbol{\beta}$) for the gauge couplings constant in the SM:

We now turn our attention to diagrams with loops. In quantum field theory some of these loops diagrams will diverge and we must take care to treat the divergent integrals correctly. We will derive the gauge coupling relation given in equation (3.3) and use this relation to discuss the qualitative features of the renormalization group follow in renormalizable field theories in next chapter. Here we will compute the gauge boson self-energy in detail at the one-loop level. The contribution for the gauge couplings RGEs is shown in figure (3.1).

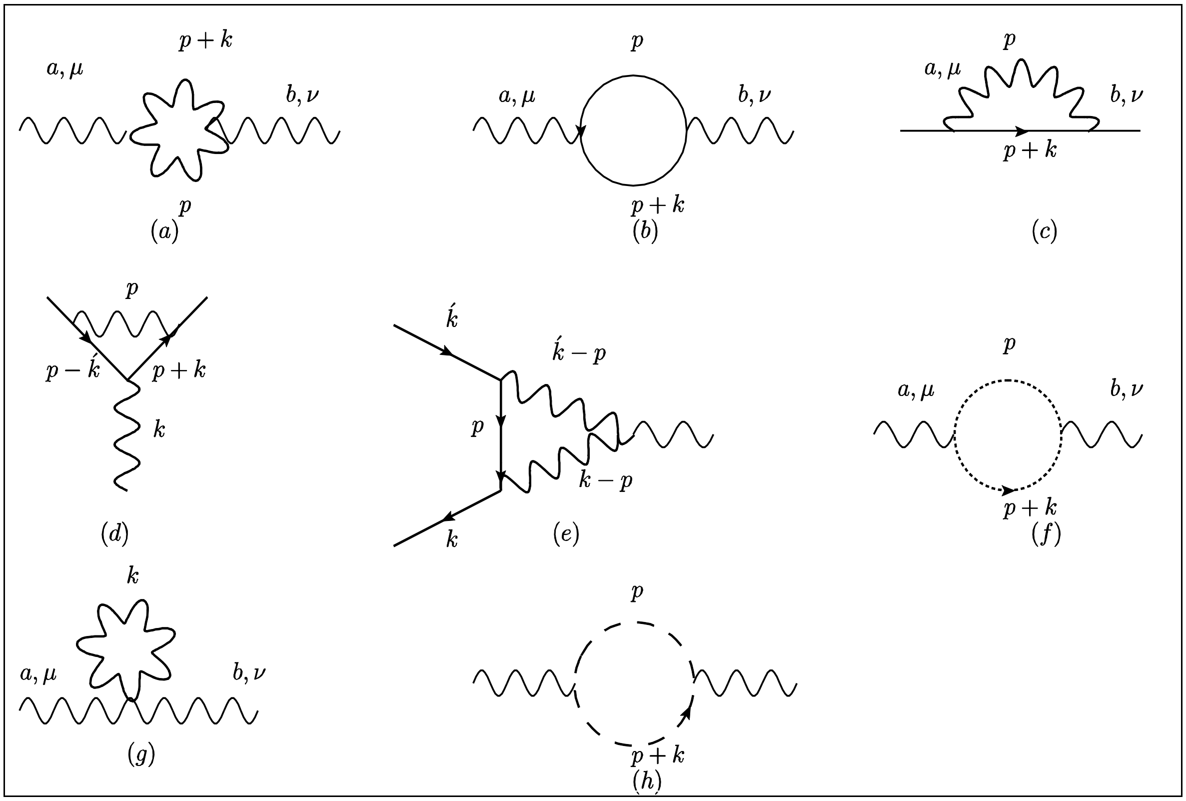

Figure 3.1 Feynman diagrams contributing to the gauge couplings in SM

Calculation of Fig (a) gives us:

$$\Pi(p,k) = \frac{-g^2 f^{abc} f^{acb}}{2} \int \frac{d^d p}{(2\pi)^d} \quad \frac{N(p,k)}{P^2(P+K)^2} \tag{3.4}$$

Where:

$$\begin{aligned} N(p,k) = {} & g^{\mu\nu} g_{\mu\nu} (2p+k)^{\rho} (2p+k)_{\rho} + g^{\nu\rho} g_{\nu\rho} (p-k)^{\mu} (p-k)_{\mu} + g^{\rho\mu} g_{\rho\mu} (p+2k)^{\nu} (p \\ & + 2k)_{\nu} - g_{\mu\nu}\, g^{\mu\rho} (2p+k)_{p} (p+2k)^{\nu} - g_{\mu\nu}\, g^{\rho\nu} (2p+k)_{\rho} (p-k)^{\mu} \\ & - g_{\nu\rho}\, g^{\nu\mu} (p-k)_{\mu} (2p+k)^{\rho} + g_{\nu\rho}\, g^{\mu\rho} (p-k)_{\mu} (p+2k)^{\nu} \\ & - g_{\rho\mu}\, g^{\nu\mu} (2k+p)_{\nu} (2p+k)^{\rho} + g_{\rho\mu}\, g^{\rho\nu} (2k+p)_{\nu} (p-k)^{\mu} \end{aligned} \tag{3.5}$$

Using the trace approach

$$g^{\mu\nu} g_{\mu\nu} = g^{\nu\rho} g_{\nu\rho} = g^{\rho\mu} g_{\rho\mu} = d$$

Hence

$$N = d((2p+k)^2 + (p-k)^2 + (p+2k)^2) - ((2p+k)(p+2k) - \big((2k+p)(2p+k)\big)$$
$$- ((2p+k)(p-k) - \big((p-k)(2p+k)\big) + \big((p-k)(p+2k)\big) \tag{3.6}$$
$$+ ((2k+p)(p-k))$$

Then

$$N = \quad d[((2p+k)^2 + (p-k)^2 + (p+2k)^2)] - 2\big((2p+k)(p+2k) +$$
$$(2p+k)(p-k) - (p+2k)(p-k)\big) \tag{3.7}$$

This lead to

$$N = d[4p^2 + 4pk + k^2 + p^2 - 2pk + k^2 + p^2 + 4pk + 4k^2] - 6(p^2 + k^2 + kp) \tag{3.8}$$

Then

$$N = d[6p^2 + 6pk + 6k^2] - 6(p^2 + pk + k^2) \tag{3.9}$$

After a little algebra we get

$$N = 6d(p^2 + pk + k^2) - 6(p^2 + pk + k^2) = 6(d-1)(p^2 + k^2 + pk) \tag{3.10}$$

So that

$$\Pi(p,k) = \frac{-g^2 f^{abc} f^{acb}}{2(2\pi)^d} d \int d^d p \;\; \frac{N(p,k)}{p^2(p+k)^2}$$
$$= \frac{-g^2 f^{abc} f^{acb}}{2} d \int \frac{d^d p (p^2 + pk + k^2) 6(d-1)}{p^2(p+k)^2} \tag{3.11}$$

We have

$$f^{abc} f^{acb} = C_2(G) \delta^{ab} \tag{3.12}$$

Then

$$\Pi(p,k) = \frac{-g^2 C_2(G) \delta^{ab}}{2(2\pi)^d} d \int \frac{d^d p (p^2 + pk + k^2) 6(d-1)}{p^2(p+k)^2} \tag{3.13}$$

By using Feynman parameterization

$$\frac{1}{ab} = \int_0^1 \frac{1}{(b+(a-b)z)^2} dz \tag{3.14}$$

And let

$$b = p^2 and\ a = (p+k)^2 \tag{3.15}$$

We obtain

$$\Pi(p,k) = \frac{-6g^2 C_2(G)\delta^{ab}}{2(2\pi)^d} d(d-1) \int d^d p \int_0^1 \frac{(p^2+pk+k^2)}{(p^2+(k^2+2pk)z)^2} dz \tag{3.16}$$

By introducing new variable

$$q = p + kz \tag{3.17}$$

the numerator $(p^2 + pk + K^2)$ becomes

$$N = ((q-kz)^2 + (q-kz)k + k^2) \tag{3.18}$$

$$N = (q^2 - 2qkz + k^2 z^2 + qk - k^2 z + k^2) \tag{3.19}$$

Then

$$N = \left(q^2 + k^2(z^2 - z + 1)\right) \tag{3.20}$$

And the denominator $(p^2 + (k^2 + 2pk)z)^2$ becomes

$$D = (q^2 + k^2 z(1-z))^2 \tag{3.21}$$

Thus

$$\Pi(q,k) = \frac{-6g^2 C_2(G)\delta^{ab}}{2(2\pi)^d} d(d-1) \int d^d q \int_0^1 dz \, \frac{\left(q^2 + k^2(z^2 - z + 1)\right)}{(q^2 + k^2 z(1-z))^2} \tag{3.22}$$

Using the standard integrals (Weinberg S. , 1996)**:**

$$\int d^d q \, \frac{q^2}{(q^2+\Delta)^n} = \frac{d\, i\pi^{\frac{d}{2}}\, \Gamma(n - \frac{d}{2} - 1)}{2\Gamma(n)(\Delta)^{n-\frac{d}{2}-1}} \tag{3.23}$$

$$\int d^d q \ \frac{1}{(q^2+\Delta)^n} \quad = \quad \frac{i\pi^{\frac{d}{2}}\,\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)(\Delta)^{n-\frac{d}{2}}} \tag{3.24}$$

Therefore Eq (3.22) becomes**:**

$$\Pi(q,k) = \frac{-6g^2C_2(G)\delta^{ab}}{2(2\pi)^d}d(d-1)\left(\frac{d\ i\pi^{\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right)}{2\Gamma(2)(k^2)^{1-\frac{d}{2}}}\int_0^1 dz\frac{1}{(z(1-z))^{1-\frac{d}{2}}}\right.$$
$$\left.+\ \frac{i\pi^{\frac{d}{2}}\,\Gamma\left(2-\frac{d}{2}\right)}{(k^2)^{2-\frac{d}{2}}}\int_0^1 dz\frac{(z^2-z+1)}{(z(1-z))^{2-\frac{d}{2}}}\right) \tag{3.25}$$

Where: $\Delta = k^2z(1-z)$

Comparing the beta function integral with Eq (3.25)

$$\mathrm{B}(m,n) = \int_0^1 (z)^{m-1}(1-z)^{n-1} \tag{3.26}$$

And the relation

$$\mathrm{B}(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \tag{3.27}$$

Yield

$$\int_0^1 dz\frac{1}{(z(1-z))^{1-\frac{d}{2}}} \ = \ \frac{\Gamma\left(\frac{\mathrm{d}}{2}\right)\Gamma\left(\frac{\mathrm{d}}{2}\right)}{\Gamma(\mathrm{d})} \tag{3.28}$$

And

$$\int_0^1 dz\frac{(z^2-z+1)}{(z(1-z))^{2-d/2}} = \frac{\Gamma\left(\frac{\mathrm{d}}{2}\right)\Gamma\left(\frac{\mathrm{d}}{2}\right)}{\Gamma(\mathrm{d})} + \frac{\Gamma\left(\frac{\mathrm{d}}{2}-1\right)\Gamma\left(\frac{\mathrm{d}}{2}-1\right)}{\Gamma(\mathrm{d}-2)} \tag{3.29}$$

So that Eq (3.25) becomes**:**

$$\Pi(q,k) = \frac{-6g^2C_2(G)\delta^{ab}}{2(2\pi)^d} d(d-1)\left(\frac{d\; i\pi^{\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right)}{2\Gamma(2)(k^2)^{1-\frac{d}{2}}}\frac{\Gamma\left(\frac{\mathrm{d}}{2}\right)\Gamma\left(\frac{\mathrm{d}}{2}\right)}{\Gamma(\mathrm{d})}\right.$$

$$\left. + \quad \frac{i\pi^{\frac{d}{2}}\,\Gamma\left(2-\frac{d}{2}\right)}{(k^2)^{2-\frac{d}{2}}}\left\{\frac{\Gamma\left(\frac{\mathrm{d}}{2}\right)\Gamma\left(\frac{\mathrm{d}}{2}\right)}{\Gamma(\mathrm{d})}+\frac{\Gamma\left(\frac{\mathrm{d}}{2}-1\right)\Gamma\left(\frac{\mathrm{d}}{2}-1\right)}{\Gamma(\mathrm{d}-2)}\right\}\right) \qquad (3.30)$$

We have

$$\Gamma(1+\mathrm{x}) = \mathrm{x}\Gamma(\mathrm{x}) \qquad (3.31)$$

Then

$$\Gamma\left(1-\frac{\mathrm{d}}{2}\right) = \frac{\Gamma(2-\frac{\mathrm{d}}{2})}{(1-\frac{d}{2})} \qquad (3.32)$$

So that we get

$$\Pi(\mathrm{k}) = \frac{\mathrm{ig}^2\mathrm{C}_2(\mathrm{G})\delta^{\mathrm{ab}}}{6(4\pi)^2}\Gamma\left(2-\frac{\mathrm{d}}{2}\right)\left[\frac{19}{12}\mathrm{g}^{\mu\nu}\mathrm{k}^2-\frac{11}{6}\mathrm{k}^{\mu}\mathrm{k}^{\nu}\right] \qquad (3.33)$$

Calculation of Fig (b) is similar to Fig (a) gives us

$$\Pi(p,k) = (-1)\int\frac{d^dp}{(2\pi)^d}Tr(\,(-ig\gamma^{\mu}T^a)\left(\frac{ip}{p^2}\right)(-ig\gamma^{\nu}T^b)\left(\frac{(\not{p}+\not{k})}{(p+k)^2}\right))n_f \qquad (3.34)$$

$$\Pi(p,k) = (-1)g^2Tr(T^aT^b)n_f\int\frac{d^dp}{(2\pi)^d}Tr\left(\;\gamma^{\mu}\;\;\frac{ip}{p^2}\;\gamma^{\nu}\,\frac{(\not{p}+\not{k})}{(p+k)^2}\;\right) \qquad (3.35)$$

We have

$$Tr(T^aT^b) = C(r)\delta^{ab} \qquad (3.36)$$

$$Tr(\gamma^{\mu}\gamma^{\rho}\gamma^{\nu}\gamma^{\sigma}) = f(D)[g^{\mu\rho}g^{\nu\sigma}-g^{\mu\nu}g^{\rho\sigma}+g^{\mu\sigma}g^{\rho\nu}] \qquad (3.37)$$

Likewise we get

$$\Pi(q,k) = -\frac{4}{3}\,\frac{\left(ig^2(k^2g^{\mu\nu}-k^{\mu}k^{\nu})\delta^{ab}n_fC(r)\Gamma\left(2-\frac{d}{2}\right)\right)}{(4\pi)^2} \qquad (3.38)$$

Similarly the calculation of Fig (c) gives:

$$\Pi(p,k) = \int \frac{d^d p}{(2\pi)^d}(-ig\gamma^\mu T^a)\left(-\frac{ig_{\mu v}}{p^2}\right)(-ig\gamma^v T^b)\frac{i(\not{p}+\not{k})}{(p+k)} \quad (3.39)$$

Then

$$\Pi(p,k) = -g^2 T^a T^b \int \frac{d^d p}{(2\pi)^d}\left(\frac{\gamma^\mu\, g_{\mu v}\gamma^v(\not{p}+\not{k})}{p^2(p+k)^2}\right) \quad (3.40)$$

We have

$$T^a T^b = C_2(r) \quad (3.41)$$

The numerator can be written as:

$$\gamma^\mu\, g_{\mu v}\gamma^v(\not{p}+\not{k}) = \gamma^\mu \gamma_\mu \gamma^\rho(p_\rho + k_\rho) \quad (3.42)$$

Hence

$$\gamma^\mu \gamma_\mu \gamma^\rho = -(d-2)\gamma^\rho \quad (3.43)$$

Therefore

$$\Pi(p,k) \quad = \frac{ig^2 C_2(r)\Gamma\left(2-\frac{d}{2}\right)}{(4\pi)^2 (k^2)^{2-\frac{d}{2}}} k \quad (3.44)$$

Calculations of Fig (d) give

$$\Pi(k) \;=\; \frac{ig^3}{(4\pi)^2} t^a \gamma^\mu \left[C_2(r) - \frac{1}{2}C_2(G)\right]\Gamma\left(2-\frac{d}{2}\right) \quad (3.45)$$

Calculations of Fig (e) give

$$\Pi(k) = \frac{3}{2}\;\frac{ig^3}{(4\pi)^2}[C_2(G) t^a \gamma^\mu]\Gamma\left(2-\frac{d}{2}\right) \quad (3.46)$$

Calculations of Fig (f) give

$$\Pi(k) = \frac{ig^2 C_2(G)\delta^{ab}}{6(4\pi)^2}\left[\frac{g^{\mu\nu}k^2}{2} + k^\mu k^\nu\right]\Gamma\left(2 - \frac{d}{2}\right) \qquad (3.47)$$

Calculations of Fig (g) give

$$\begin{aligned}\Pi(p,k) \quad &= \frac{1}{2}\int \frac{d^d p}{(2\pi)^d}(-ig^2 f^{eab} f^{ecd}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \\ &- ig^2 f^{eac} f^{ebd}\left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}\right) \qquad (3.48) \\ &- ig^2 f^{ead} f^{ebc}\left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}\right).\left(-\frac{ig^{\mu\nu}}{p^2}\right)\end{aligned}$$

$$\because f^{acd} f^{bcd} = C_2(G)\delta^{ab} \qquad (3.49)$$

Then

$$\begin{aligned}\Pi(p,k) \ &= -\frac{1}{2}g^2 C_2(G)\delta^{ab}\int \frac{d^d p}{(2\pi)^d}\ \left\{\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right) + \left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)\right. \\ &\left. + \left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}\right)\right\}\ \left(-\frac{ig^{\mu\nu}}{p^2}\right) \qquad (3.50)\end{aligned}$$

It is not difficult to show that Eq (3.50) give zero

$$\Pi(p,k) = 0 \qquad (3.51)$$

Calculations of Fig (h) give

$$\Pi(p,k) = \int \frac{d^d p}{(2\pi)^d}(-ig\gamma^\mu T^a)\frac{\left(i\left(\not{p} + \not{k}\right)\right)}{(p+k)^2}(-ig\gamma^\nu T^b)\frac{\left(i\left(\not{p} + \not{k}\right)\right)}{(p+k)^2}(-ig\gamma^p T^c)\frac{(-ig_{\mu p})}{p^2} \quad (3.52)$$

$$\Pi(p,k) = g^3 T^a T^b T^a \int \frac{d^d p}{(2\pi)^d}\ \ \frac{\left(\gamma^\mu(\not{p} + \not{k})\gamma^\nu(\not{p} + \not{k})\gamma^p g_{\mu p}\right)}{(p+k)^2(p+k)^2(p^2)} \qquad (3.53)$$

We have

$$T^a T^b T^a = \left(C_2(r) - \frac{1}{2}C_2(G)\right)T^b \qquad (3.54)$$

So we have

$$\Pi(k) = \frac{-ig^2 C_2(R)\delta^{ab}}{6(4\pi)^2}[g^{\mu\nu}k^2 - k^\mu k^\nu]\Gamma\left(2-\frac{d}{2}\right) \qquad (3.55)$$

Now summing all the results in equations

((3.33),(3.38),(3.44),(3.45),(3.46),(3.47),(3.51)and(3.55)) yield

$$\Pi(k) = i(k^2 g^{\mu\nu} - k^\mu k^\nu)\delta^{ab}\left[\frac{-g^2}{(4\pi)^2}\left(\frac{-5}{3}\right)c_2(\epsilon)\right]\Gamma\left(2-{}^{d}\!/_{2}\right) + {}^{4}\!/_{3}\, n_f \mathrm{c(r)}\Gamma\left(2-{}^{d}\!/_{2}\right)$$
$$+\left(\frac{-1}{6}\right)\frac{ig^2}{(4\pi)^2}c_2(\epsilon)\Gamma\left(2-{}^{d}\!/_{2}\right) \qquad (3.56)$$

Where

$$b_G = \left[\frac{11}{3}C_2(G) - \frac{4}{3}\, n_f C(R) - \frac{1}{3}\, n_h C_2(R)\right] \qquad (3.57)$$

3.4 Calculation of the coefficient $\boldsymbol{b_G}$ in the Standard Model (SM):

Firstly for the strong interaction $\boldsymbol{SU(3)}$

The gauge bosons (gluons) belong to the adjoint representation which imply$\mathrm{C}_2\big(\mathrm{SU}(3)\big) = 3$, and the fermion is belonging to the fundamental representation

For one generation of fermion only u_α and d_α contribute, therefore

$$T(1generation) = T_F(3) + T_F(3) = \frac{1}{2} + \frac{1}{2} = 1 \qquad (3.58)$$

If we work with Weyl fermions u_L , u_R *and* d_R , then we must include the factor $\frac{1}{2}$ for each helicity, which follows from $Tr\ L(R) = \frac{1}{2}$, $L(R) = \frac{1}{2}(1 \pm \gamma_5)$ *hence*

$$C(R) = 4 \times \frac{1}{2} \times \frac{1}{2} = 1 \qquad (3.59)$$

Since the Higgs is not colored under $SU(3)$ then $C_2(R) = 0$.
We thus finally

$$b_3^{SM} = \left(\frac{11}{3} \times 3 - \frac{4}{3} \times 1 \times 3\right) = 7 \tag{3.60}$$

Secondly for the weak interaction $SU(2)_L$

We have $C_2\big(SU(2)\big) = 2 \; and \; C(R) = 1 \; and \; C_2(R) = \frac{1}{2}$ thus we get

$$b_2^{SM} = \left(\frac{11}{3} \times 2 - \frac{4}{3} \times 1 \times 3 - \frac{1}{3} \times 1 \times \frac{1}{2}\right) = \frac{19}{6} \tag{3.61}$$

Finally for $U(1)_Y$

There will be no gauge boson contributions in b_Y since they do not carry hypercharge. For the fermions and Higgs scalar we take their hypercharges from Table 3.1, therefore

$$b_1^{SM} = \left(-3 \times \frac{4}{3}\left(\frac{2}{36} \times 3 + \frac{4}{9} \times 3 + \frac{1}{9} \times 3 + \frac{2}{4} + 1\right) \times \frac{1}{2} - \frac{1}{6}\right) = -\frac{41}{6} \tag{3.62}$$

But we always use the $SU(5)$ normalization, that is $g' = \sqrt{\frac{3}{5}}\, g_1$

Therefore

$$b_1^{SM} = -\frac{41}{6} \times \frac{3}{5} = -\frac{41}{10} \tag{3.63}$$

In general way

$$b_i^{SM} = \left(-\frac{41}{10}, \frac{19}{6}, 7\right) \tag{3.64}$$

3.4 The CKM Matrix:

When we consider all the generations of quarks, there are possibilities for their mixing. This mixing is described by the CKM Matrix, which has four observable parameters, including three mixing angles and one phase. It appears upon the diagonalization of Yukawa matrices by using two unitary matrices $U \; and \; V$ (K.~S.~Babu, 1987).

Where the CKM matrix is given by:

$$V_{CKM} = UV^{\dagger} \tag{3.65}$$

The form of the CKM matrix that describes the quark sector mixing is parameterized as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{3.66}$$

In other words, the CKM matrix arises from a consideration of the square of the quark Yukawa coupling matrices being diagonalized by using two unitary matrices U and V

$$\mathrm{diag}\,(f_u^2, f_c^2, f_t^2) = \mathrm{U}\mathrm{Y}_\mathrm{u}^\dagger \mathrm{Y}_\mathrm{u} \mathrm{U}^\dagger$$
$$\mathrm{diag}\,(f_d^2, f_s^2, f_b^2) = \mathrm{U}\mathrm{Y}_\mathrm{d}^\dagger \mathrm{Y}_\mathrm{d} \mathrm{U}^\dagger$$

Where f_u^2, f_c^2, f_t^2 and f_d^2, f_s^2, f_b^2 are the eigenvalues of the $\mathrm{Y}_\mathrm{u}^\dagger \mathrm{Y}_\mathrm{u}$ and $\mathrm{Y}_\mathrm{d}^\dagger \mathrm{Y}_\mathrm{d}$ respectively. More details about the variation of CKM matrix for example see (A.~S.~Cornell, 2010) (K.~S.~Babu, 1987).

3.5 One-loop Yukawa couplings

The beta function of the Yukawa couplings is given by:

$$16\pi^2 \frac{dy}{dt} = \beta_y^{SM} + \beta_y^{UED} \tag{3.67}$$

Where

$$\beta_{y_u}^{SM} = Y_u \left[Tr\left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + 3Y_e^\dagger Y_e\right) - \left(\frac{17}{12} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) + \frac{3}{2}(Y_u^2 - Y_d^2) \right] \tag{3.68}$$

$$\beta_{y_d}^{SM} = Y_d \left[Tr\left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + 3Y_e^\dagger Y_e\right) - \left(\frac{5}{12} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2\right) + \frac{3}{2}(Y_d^2 - Y_u^2) \right] \tag{3.69}$$

$$\begin{aligned} \beta_{y_u}^{UED} = (S-1)\left[-\left(\frac{28}{3} g_3^2 + \frac{15}{8} g_2^2 + \frac{101}{120} g_1^2\right) + \frac{3}{2}(y_u^2 - y_d^2) \right] y_u \\ + 2(S-1)[Y_l + 3Y_u + 3Y_d] y_u \end{aligned} \tag{3.70}$$

$$\begin{aligned} \beta_{y_d}^{UED} = (S-1)\left[-\left(\frac{28}{3} g_3^2 + \frac{15}{8} g_2^2 + \frac{17}{120} g_1^2\right) + \frac{3}{2}(y_u^2 - y_d^2) \right] y_u \\ + 2(S-1)[Y_l + 3Y_u + 3Y_d] y_u \end{aligned} \tag{3.71}$$

3.6 Calculation of the Yukawa couplings factor in the Landau gauge:

At each excited KK level, the one-loop corrections to the Yukawa couplings arise from the diagrams exactly mirroring those of the SM ground states. Note that for the closed fermion loop diagrams one need to count the contributions from both the left-handed and right-handed KK modes of each chiral fermion to the self-energy of the gauge filed. Additional contributions to the Yukawa couplings come from the fifth component of the 5D gauge field at each KK excited level as shown in figure (3.2).

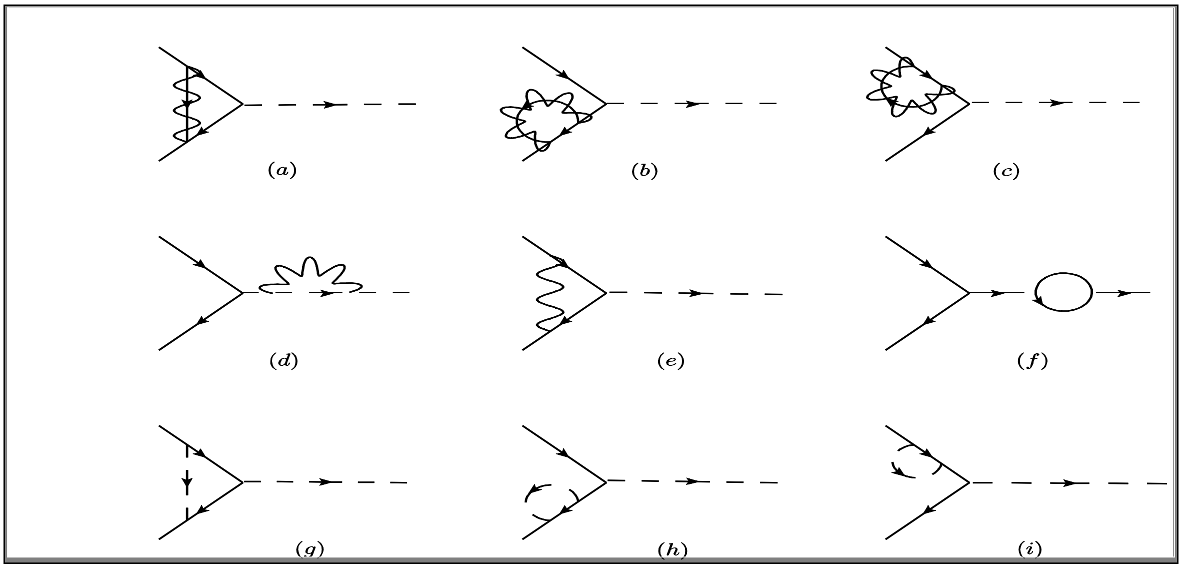

Figure 3.2 show the Feynman diagrams contributing to Yukawa coupling in the Landau gauge

Let us calculate the numerical coefficients appeared in equation (3.70).

Calculation of the factor g_3^2:

Calculation of Fig (a) gives

$$I = \int \frac{d^d p}{(2\pi)^d} \left(\frac{g_3^5}{\sqrt{\pi R}} \gamma^5 \frac{\lambda^A}{2}\right) \left(\frac{i}{\not{p} - \not{k}}\right) \left(\frac{g_3^5}{\sqrt{\pi R}} \gamma^5 \frac{\lambda^A}{2}\right) \left(\frac{i}{\not{p} - \not{k}}\right) \left(-\frac{iY_u^5}{\sqrt{2}\sqrt{\pi R}}\right) \left(\frac{i}{p^2}\right) \tag{3.72}$$

Use

$$(\gamma^5)^2 = 1 \quad and \quad \{\gamma^5, \gamma^\mu\} = 0 \quad and \quad \frac{Y_u^5}{\sqrt{\pi R}} = Y_u \quad and \quad \frac{g_3^5}{\sqrt{\pi R}} = g_3$$

Where

$$\frac{\lambda^A}{2}\frac{\lambda^A}{2} = C_2(r) \tag{3.73}$$

Then

$$I = -\frac{i}{\sqrt{2}}\, Y_u g_3^2 \int \frac{d^d p}{(2\pi)^d}\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{p^2}\right) C_2(r) \tag{3.74}$$

$$\because\, C_2(r) = \frac{N^2-1}{2N} \tag{3.75}$$

So we get

$$I = -\frac{i}{\sqrt{2}}\, Y_u g_3^2 \int \frac{d^d p}{(2\pi)^d}\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{p^2}\right)\frac{N^2-1}{2N} \tag{3.76}$$

For $SU(3)\quad N = 3$

$$I = -\frac{i}{\sqrt{2}}\, Y_u g_3^2 \int \frac{d^d p}{(2\pi)^d}\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{p^2}\right)\frac{8}{6} \tag{3.77}$$

After integration the above integral we get

$$I = -\frac{i}{\sqrt{2}}\, Y_u g_3^2 \frac{8}{6}\frac{1}{(16\pi^2)}\frac{1}{\epsilon}(\mu^2)^{-\epsilon} i \tag{3.78}$$

Then

$$Z_{coupling} = 1 - g_3^2 \frac{8}{6}\frac{1}{(16\pi^2)}\frac{1}{\epsilon}(\mu^2)^{-\epsilon} i \tag{3.79}$$

Or

$$-\mu\frac{\partial}{\partial\mu}\ln Z_{coupling} = g_3^2\frac{8}{3}\frac{1}{(16\pi^2)} \tag{3.80}$$

Calculation of Fig (b) gives

$$I = \int \frac{d^d p}{(2\pi)^d}\left(\frac{g_3^5}{\sqrt{\pi R}}\gamma^5\frac{\lambda^A}{2}\right)\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{g_3^5}{\sqrt{\pi R}}\gamma^5\frac{\lambda^A}{2}\right)\left(\frac{i}{p^2}\right) \tag{3.81}$$

By similar way we get

$$I = g_3^2\frac{8}{6}\frac{1}{(16\pi^2)}\frac{1}{\epsilon}(\mu^2)^{-\epsilon}i\ \frac{1}{2} \tag{3.82}$$

Then

$$Z_{u_R} = 1 - g_3^2\frac{8}{6}\frac{1}{(16\pi^2)}\frac{1}{\epsilon}(\mu^2)^{-\epsilon}i\frac{1}{2} \tag{3.83}$$

Or

$$\frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_{u_R} = g_3^2\frac{8}{3}\frac{1}{(16\pi^2)}\frac{1}{2} \tag{3.84}$$

Calculation of Fig (c) likewise gives

$$\frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_{q_L} = g_3^2\frac{8}{3}\frac{1}{(16\pi^2)}\frac{1}{2} \tag{3.85}$$

Calculation of Fig (d) likewise gives

$$-\mu\frac{\partial}{\partial\mu}\ln Z_{coupling} = -8g_3^2\frac{1}{(16\pi^2)} \tag{3.86}$$

Then adding Eqs ((3.80), (3.84), (3.85), (3.86)) together gives

$$g_3^2\frac{8}{3}\frac{1}{(16\pi^2)}+g_3^2\frac{8}{3}\frac{1}{(16\pi^2)}\frac{1}{2}+g_3^2\frac{8}{3}\frac{1}{(16\pi^2)}\frac{1}{2}-8g_3^2\frac{1}{(16\pi^2)}$$
$$= -\frac{28}{3}g_3^2\frac{1}{(16\pi^2)} \tag{3.87}$$

Which match exactly the number in equation (3.70).

Calculation of the factor g_2^2

Calculation of fig(c) gives

$$I = \int \frac{d^d p}{(2\pi)^d}\left(\frac{g_2^5}{\sqrt{\pi R}}\gamma^5\frac{\tau^a}{2}\right)\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{g_2^5}{\sqrt{\pi R}}\gamma^5\frac{\tau^a}{2}\right)\left(\frac{i}{p^2}\right) \qquad (3.88)$$

Use

$$(\gamma^5)^2 = 1 \quad and \ \{\gamma^5,\gamma^\mu\} = 0 \quad and \quad \frac{Y_u^5}{\sqrt{\pi R}} = Y_u \quad and \ \frac{g_2^5}{\sqrt{\pi R}} = g_2$$

Then

$$\mathrm{I} = -g_2^2 \int \frac{d^d p}{(2\pi)^d}\left(\gamma^5\frac{\tau^a}{2}\right)\left(\frac{i}{\not{p}-\not{k}}\right)\left(\gamma^5\frac{\tau^a}{2}\right)\left(\frac{i}{p^2}\right) \qquad (3.89)$$

But we have

$$\frac{\tau^a}{2}\frac{\tau^a}{2} = C_2(r) \qquad (3.90)$$

Equation (3.89) becomes

$$I = -g_2^2 \int \frac{d^d p}{(2\pi)^d}\ C_2(r)\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{p^2}\right) \qquad (3.91)$$

We have also

$$C_2(r) = \frac{N^2-1}{2N} \qquad (3.92)$$

Then

$$I = -g_2^2 \int \frac{d^d p}{(2\pi)^d}\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{p^2}\right)\frac{N^2-1}{2N} \qquad (3.93)$$

For $SU(2)\ \ N = 2$

$$I = -\frac{3}{4}g_2^2 \int \frac{d^d p}{(2\pi)^d}\left(\frac{i}{\not{p}-\not{k}}\right)\left(\frac{i}{p^2}\right) \qquad (3.94)$$

After integration we get

$$I = -\frac{3}{4} g_2^2 \frac{1}{(16\pi^2)} \frac{1}{\epsilon} (\mu^2)^{-\epsilon} i \frac{1}{2} \qquad (3.95)$$

Or

$$\frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_{q_L} = g_2^2 \frac{3}{4} \frac{1}{(16\pi^2)} \frac{1}{2} \qquad (3.96)$$

Likewise calculation of Fig (f) gives

$$\frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_{\emptyset} = -g_2^2 \frac{9}{4} \frac{1}{(16\pi^2)} \qquad (3.97)$$

And Fig (b) has no contribution to g_2^2 since the right handed fermion does not couple to W bosons.

So adding Eq (3.96) to Eq (3.97) gives

$$g_2^2 \frac{3}{4} \frac{1}{(16\pi^2)} \frac{1}{2} - g_2^2 \frac{9}{4} \frac{1}{(16\pi^2)} = -\frac{15}{8} \; g_2^2 \frac{1}{(16\pi^2)} \qquad (3.98)$$

This confirms the coefficient of g_2^2 in equation (3.70).

Calculation of the factor g_1^2 :

Calculation of fig (c) gives

$$I = -g_1^2 \int \frac{d^d p}{(2\pi)^d} \left(\gamma^5 \frac{Y_Q}{2}\right) \left(\frac{i}{\not{k}}\right) \left(\gamma^5 \frac{Y_Q}{2}\right) \left(\frac{i}{p^2}\right) \qquad (3.99)$$

Use

$$(\gamma^5)^2 = 1 \quad and \; \{\gamma^5, \gamma^\mu\} = 0 \quad and \quad \frac{Y_Q^5}{\sqrt{\pi R}} = Y_Q \qquad and \; \frac{g_1^5}{\sqrt{\pi R}} = g_1$$

Then

$$I = -g_1^2 \int \frac{d^d p}{(2\pi)^d} \; (\frac{Y_Q}{2})^2 \left(\frac{i}{\not{p} - \not{k}}\right) \left(\frac{i}{p^2}\right) \qquad (3.100)$$

$\frac{\mathrm{Y_Q}}{2}$ is the hyper charge

$$(\frac{Y_Q}{2})^2 = (\frac{1}{6})^2 \qquad (3.101)$$

Then we get

$$\frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_{q_L} = (\frac{1}{6})^2 g_1^2 \frac{1}{(16\pi^2)}\frac{1}{2} \qquad (3.102)$$

Likewise calculation of fig (b), using the following relation

$$(\frac{Y_u}{2})^2 = (\frac{2}{3})^2 \qquad (3.103)$$

We obtain

$$Z_{u_R} = 1 - g_1^2(\frac{2}{3})^2 \frac{1}{(16\pi^2)}\frac{1}{\epsilon}(\mu^2)^{-\epsilon} i \frac{1}{2} \qquad (3.104)$$

Thus

$$\frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_{u_R} = (\frac{2}{3})^2 g_1^2 \frac{1}{(16\pi^2)}\frac{1}{2} \qquad (3.105)$$

Similarly calculation of fig (a) gives

$$\frac{Y_Q}{2}\frac{Y_u}{2} = \frac{1}{6}\cdot\frac{2}{3} \qquad (3.106)$$

$$Z_{coupling} = 1 - g_1^2 \frac{1}{6}\cdot\frac{2}{3}\frac{1}{(16\pi^2)}\frac{1}{\epsilon}(\mu^2)^{-\epsilon} i \qquad (3.107)$$

Then

$$-\mu\frac{\partial}{\partial\mu}\ln Z_{coupling} = -\frac{1}{6}\cdot\frac{2}{3} g_1^2 \frac{1}{(16\pi^2)}\cdot 2 = -\frac{2}{9}\ g_1^2 \frac{1}{(16\pi^2)} \qquad (3.108)$$

And calculation of fig (d) + fig (e) gives

$$= -\frac{17}{12}\ g_1^2 \frac{1}{(16\pi^2)} \tag{3.109}$$

Consider adding now Eqs ((3.102), (3.105), (3.108) and (3.109)) to give

$$\begin{aligned}&(\frac{1}{6})^2 g_1^2 \frac{1}{(16\pi^2)}\frac{1}{2} + (\frac{2}{3})^2 g_1^2 \frac{1}{(16\pi^2)}\frac{1}{2} - \frac{2}{9}\ g_1^2 \frac{1}{(16\pi^2)} - \frac{17}{12}\ g_1^2 \frac{1}{(16\pi^2)} \\ &= g_1^2 \frac{1}{(16\pi^2)}\left[-\frac{17}{12} + \frac{1}{72} + \frac{2}{9} - \frac{2}{9}\right] = -\frac{101}{72}\ g_1^2 \frac{1}{(16\pi^2)}\end{aligned} \tag{3.110}$$

Rescale it with SU (5) normalization.

$$-\frac{101}{72}\ g_1^2 \frac{1}{(16\pi^2)} \to -\frac{101}{72}\ g_1^2 \frac{1}{(16\pi^2)} \times \frac{3}{5} = -\frac{101}{120}\ g_1^2 \frac{1}{(16\pi^2)} \tag{3.111}$$

Finally this result agrees with the coefficient of g_1^2 in equation (3.70).

Chapter IV

Numerical Result and discussion

4.1 Introduction:

This chapter present the numerical results and discussion for the gauge coupling constants and Yukawa couplings behavior in 4D and 5D Standard model. We set the compactification energy scale to be$\mathrm{R}^{-1} = 1$ TeV, 5 TeV and 13 TeV. Only some selected plots will be shown and we will comment on the other similar cases not explicitly presented here. We quantitatively analyses and explore these quantities in UED model, though we observed similar behaviors for all values of R^{-1}. Initial values we shall adopt at the M_z scale are given in table 4.1.

4.2 Numerical results for gauge couplings evolution in 4D and 5D Standard Model:

The generic structure of the one-loop RGEs for the gauge couplings is given by:

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 + \pi(s(t) - 1) b_i^{5D} g_i^3 \tag{4.1}$$

Where

$$t = \ln\left(\frac{E}{M_z}\right) \quad and \quad s(t) = e^t M_z R \quad for \quad M_z < E < \Lambda \tag{4.2}$$

Where Λ is the cut-off energy scale, s(t) is the sum of KK states.

The numerical coefficients appearing in equation (4.1) are given by:

$$b_i^{SM} = \left[\frac{41}{10}, -\frac{19}{6}, -7\right] \tag{4.3}$$

$$b_i^{5D} = \left[\frac{81}{10}, \frac{7}{6}, -\frac{5}{2}\right] \tag{4.4}$$

Table 4.1 shows initial values at M_z scale (where $M_z = 91.1876 GeV$) used in our numerical calculations. Data are taken from Ref (Z.~-z.~Xing H. a., 2008).

Parameter	**Value (90% CL)**
$g_1(M_z)$	0.1184
$g_2(M_z)$	0.0338
$g_3(M_z)$	0.0169
$V_{11}(M_z)$	0.9492
$V_{12}(M_z)$	0.0508
$V_{21}(M_z)$	0.0507
$V_{22}(M_z)$	0.9477
$Y_u(M_z)$	0.0073×10^{-3}
$Y_c(M_z)$	0.0036
$Y_t(M_z)$	0.9871
$Y_d(M_z)$	0.0167×10^{-3}
$Y_s(M_z)$	0.3162×10^{-3}
$Y_b(M_z)$	0.0166
$Y_e(M_z)$	0.0028×10^{-3}
$Y_\mu(M_z)$	0.5905×10^{-3}
$Y_\tau(M_z)$	10.0388×10^{-3}

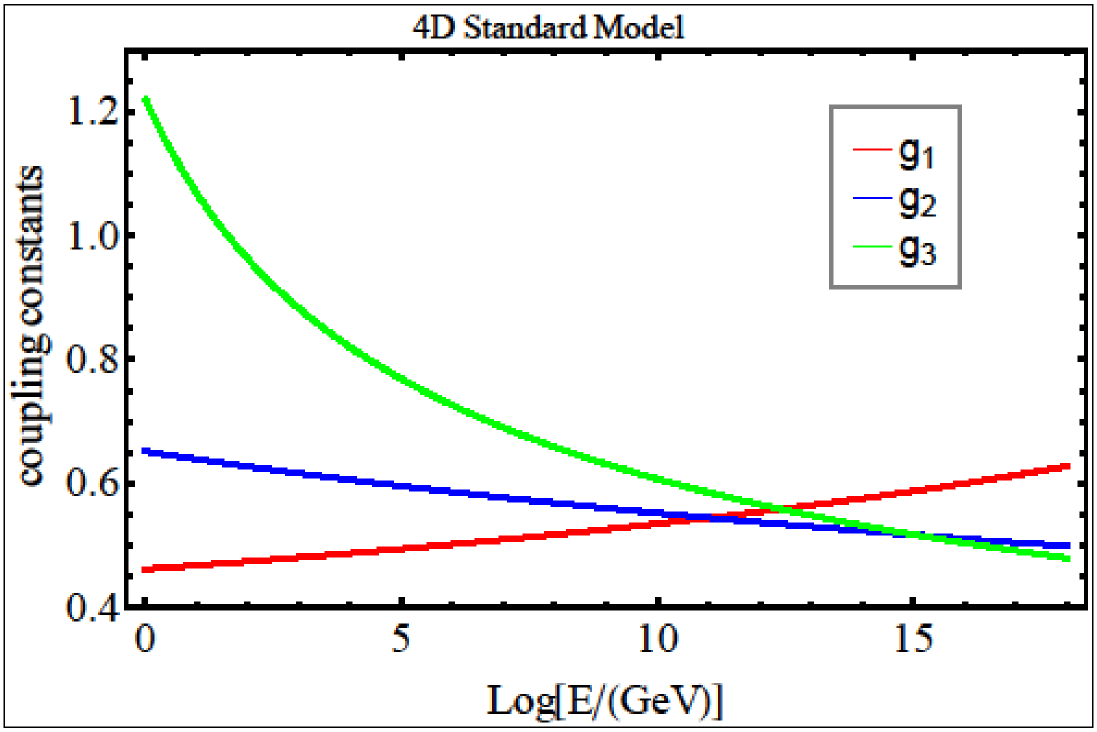

Figure 4.1: shows relation between the gauge coupling constants behavior and energy scale in 4D Standard Model.

Figure 4.1 present the evolution of gauge coupling constants in the standard model as function of energy scale, as can be seen from this figure the three gauge couplings tend to unify at some high energy scale approximately at $E = 10^{14}\ GeV$. But these couplings constant do not unify at one point. The gauge couplings run in the usual logarithmic fashion. Therefore the gauge coupling constants in the SM do not unify.

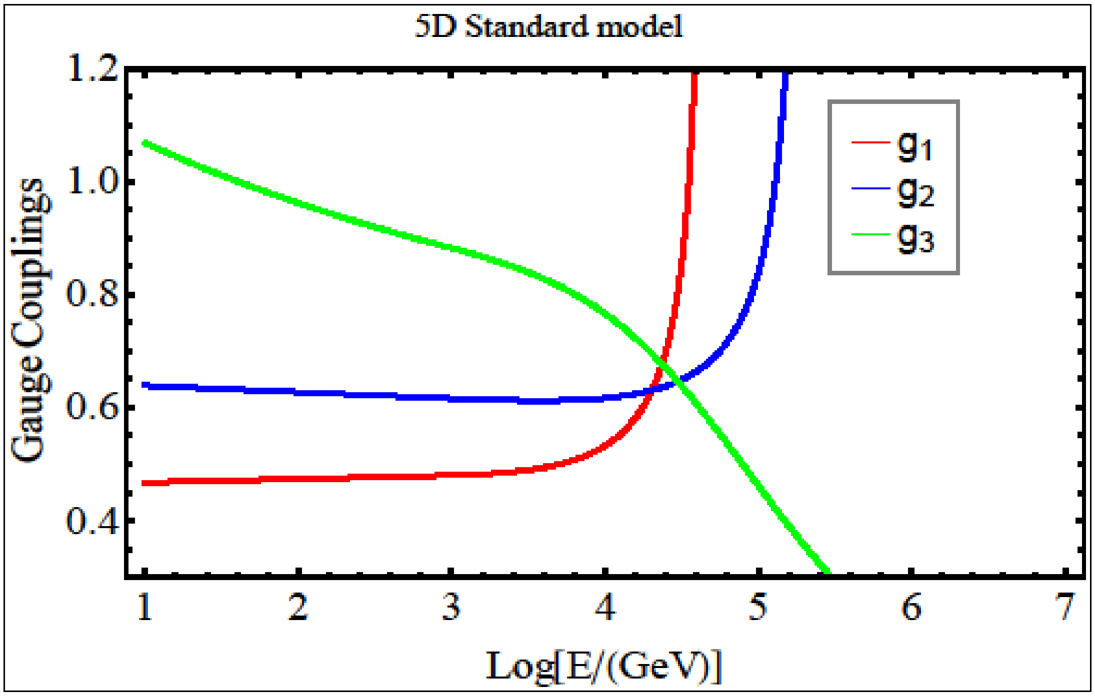

Figure 4.2: shows the evolution of the gauge coupling constants as function of energy scale in 5D Standard Model.

Furthermore, as depicted in figure 4.2 the running of the three gauge couplings g_1, g_2 and g_3 in 5D SM changes from logarithmic running to power law running this is due to the contribution of KK states at energy greater than $E = \frac{1}{R}$. As expected extra dimension lowered the unification energy scale. We found that the evolution of g_3 decrease faster, we also found that g_1 and g_2 approximately remain constant for energies below $10^4\ GeV$ and suddenly increase faster for energies exceeds $10^4\ GeV$ and the unification scale occur approximately at $E = 10^4\ GeV$ which is lower compared to the unification scale in the standard model at $E = 10^{14}\ GeV$ in other words, by comparing the gauge coupling constants running between the 4D SM and 5D SM we can see that the behavior of the gauge couplings in both cases is difference and the unification in 5D SM occur at lower energy scale, this can be explained due to the sum of KK state $S(t)$ that present in equation(4.1).

4.3 Numerical results for Yukawa coupling evolution in 4D and 5D Standard Model:

This is a theory project in which analytic computations are carried out. However it is expected that some calculations need to be performed numerically by using dedicated numerical packages (Mathematica).

The figure (4.3), figure (4.4) and figure (4.5) represent the evolution of down quarks Yukawa coupling h_d in the bulk case where the (solid line) represents the SM case with three different radii of compactification scale, 1TeV (red-dot line), 5 TeV (blue-dashed line) and 13 TeV (green-dashed line).

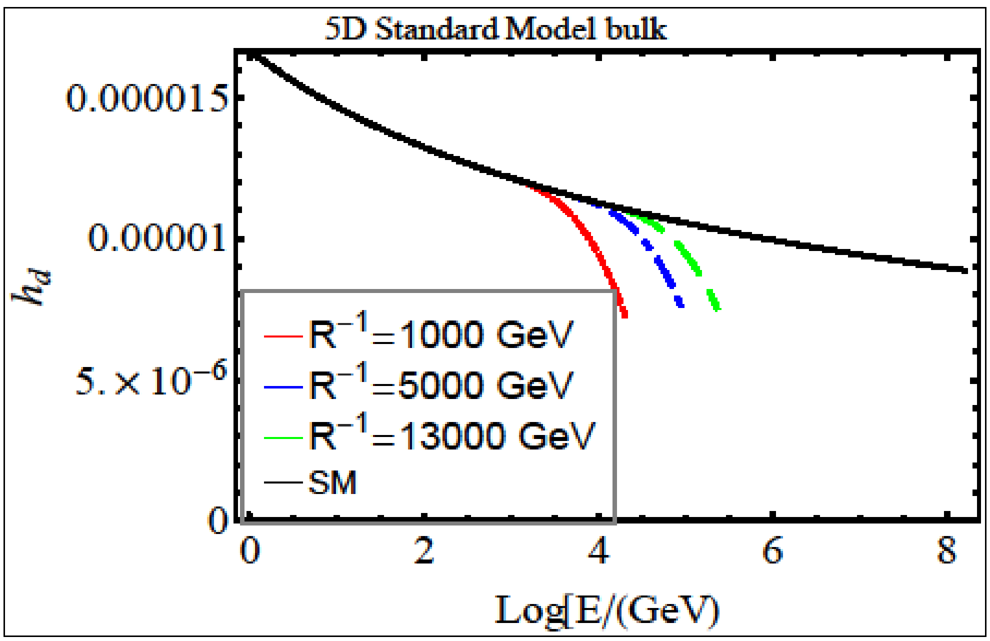

Figure 4.3: shows the evolution of the down quark Yukawa coupling as function in energy scale in 5D SM for three different values of the compactification scales R.

Once the first KK threshold is reached, the contributions from the KK states become more and more important due to the power law running where the second term on the right hand side of Eq. (4.1) depends explicitly on the cutoff, which has finite quantum corrections to the beta functions at each massive KK excitation level. Therefore, the running of the Yukawa couplings deviates from their normal trajectories and starts to run faster. Similarly, for the Yukawa couplings, the up Yukawa couplings decrease by increasing the energy scale as shown in figure (4.9), figure (4.10) and figure (4.11).

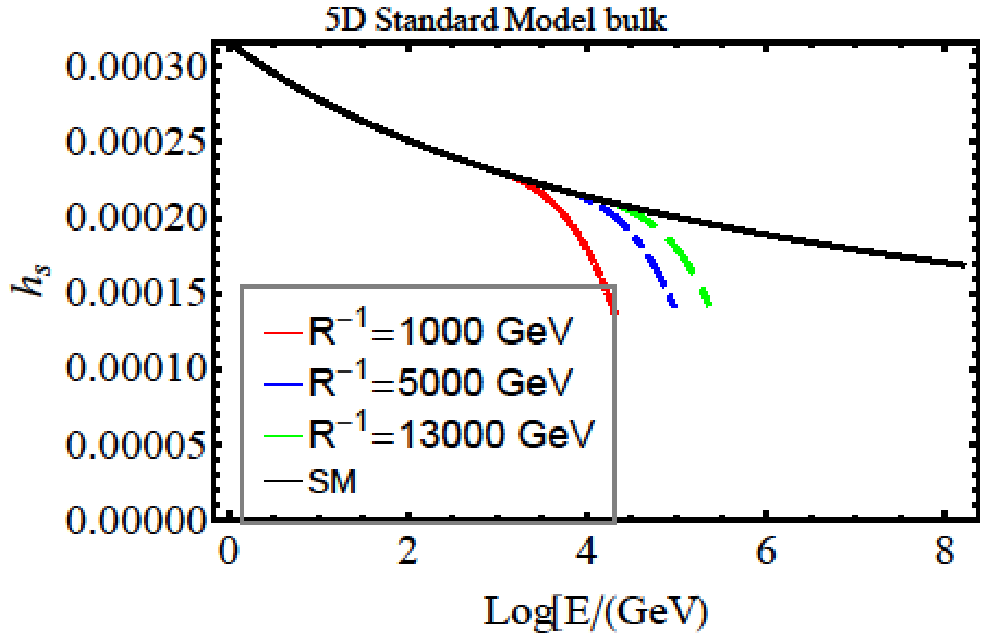

Figure 4.4: show the evolution of the strange quark Yukawa coupling as function of the energy scale in 5D SM for three different values of the compactification scales R.

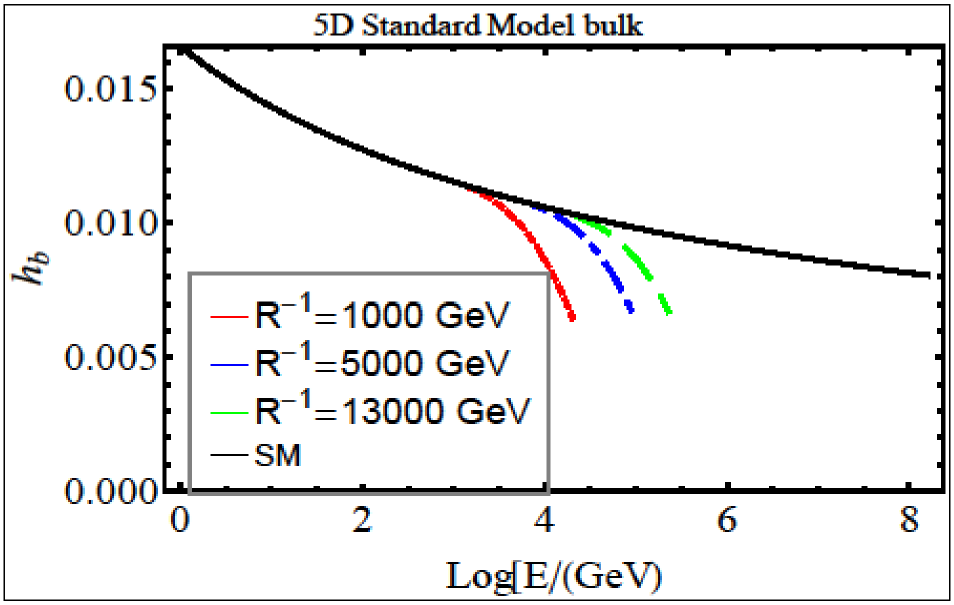

Figure 4.5: show the evolution of bottom quark Yukawa coupling as function of energy scale in 5D SM for three different values of the compactification scales R.

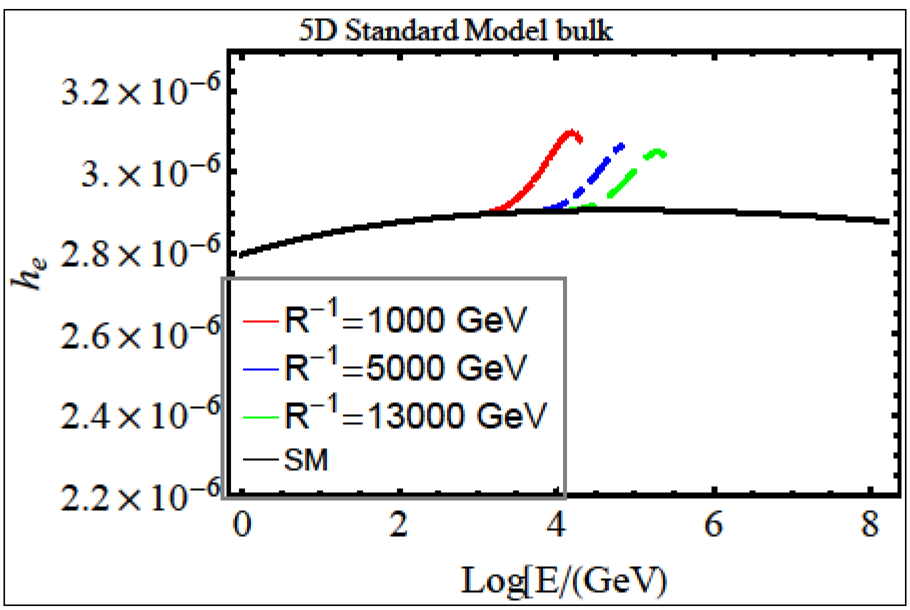

Figure 4.6: show the evolution of electron Yukawa coupling as function of energy scale in 5D SM for three different values of the compactification scales R.

In fact, the Yukawa couplings also receive finite one-loop corrections at each KK level and whose magnitudes depend upon the cut off energy scale we take it where the gauge couplings do unify. We present the numerical analysis of the one-loop calculation of the lepton Yukawa couplings (electron, muon and tau Yukawa couplings) in figure (4.6), figure (4.7) and figure (4.8). As can be seen from these figures the evolutions of leptons sector have small variation in five dimensional models. This implies that the Yukawa couplings of the leptons have a slowed evolution well before the unification scale. Beyond that point new physics would come into play.

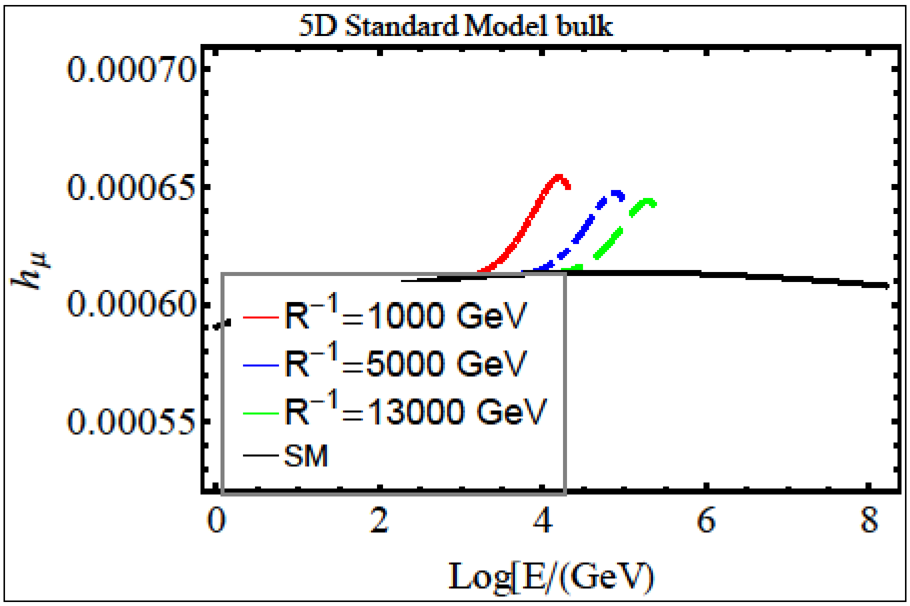

Figure 4.7: show the evolution of muon Yukawa coupling as function of energy scale in 5D SM for three different values of the compactification scales R.

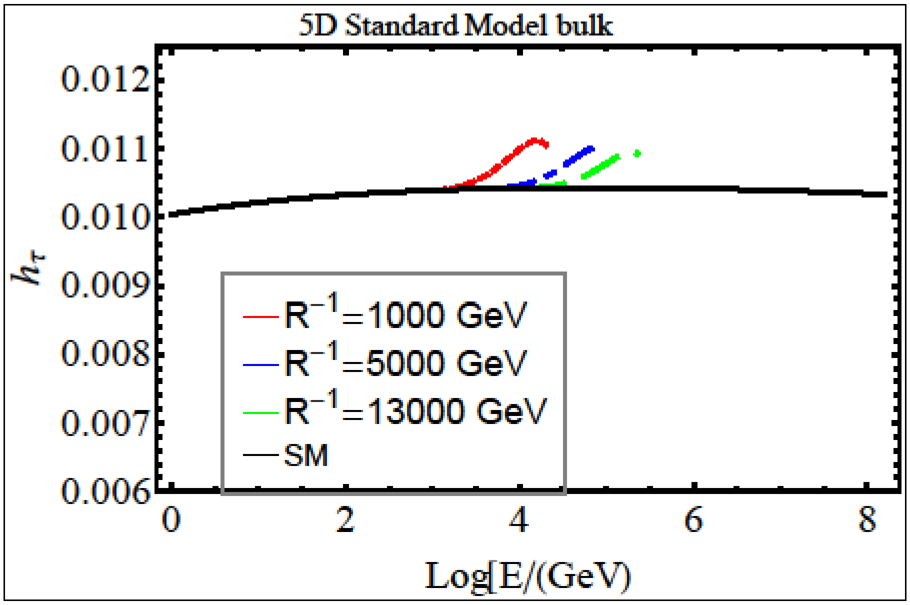

Figure 4.8: show the evolution of tau Yukawa coupling as function of energy scale in 5D SM for three different values of the compactification scales R.

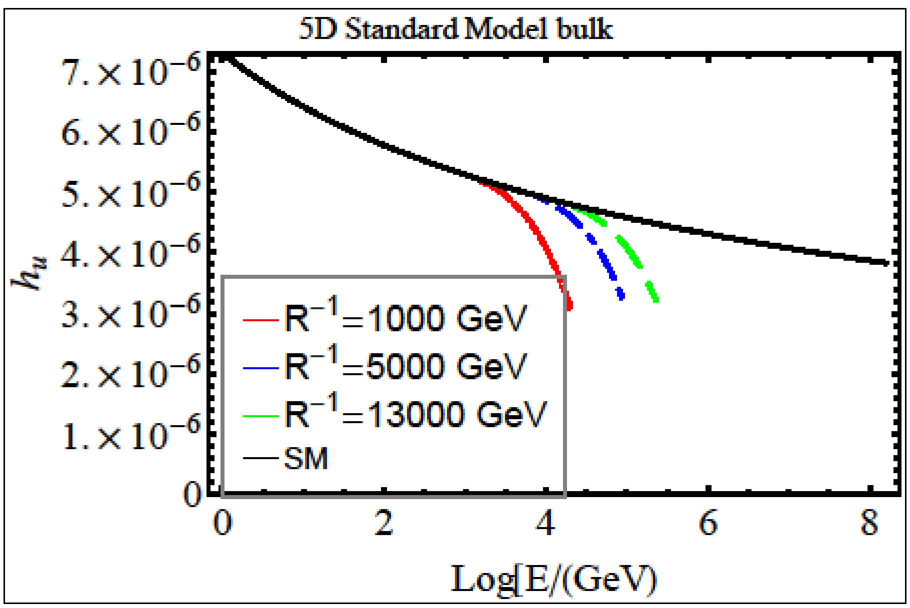

Figure 4.9: show the evolution of up quark Yukawa coupling as function of energy scale in 5D SM for three different values of the compactification scales R.

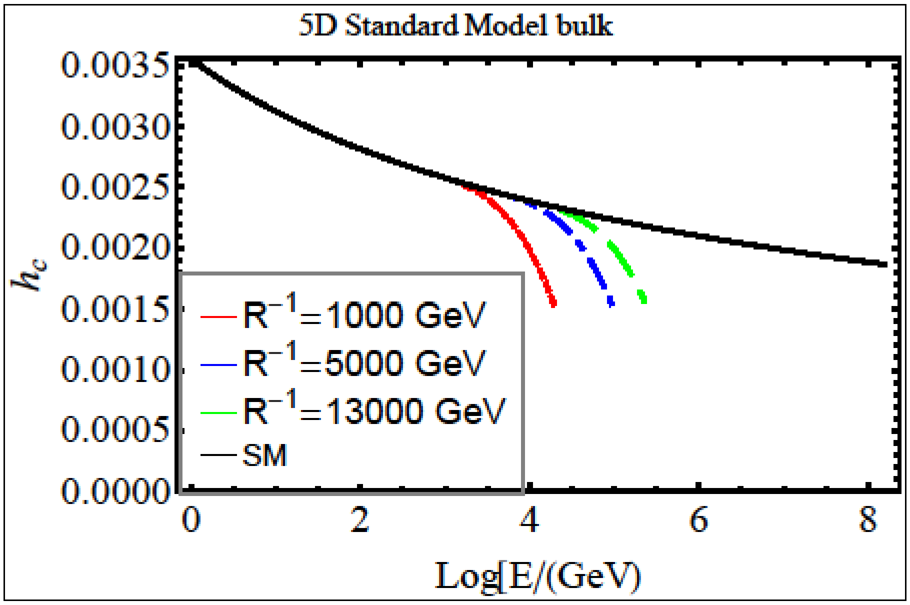

Figure 4.10: show the evolution of charm quark Yukawa coupling as function of energy scale in 5D SM for three different values of the compactification scales R.

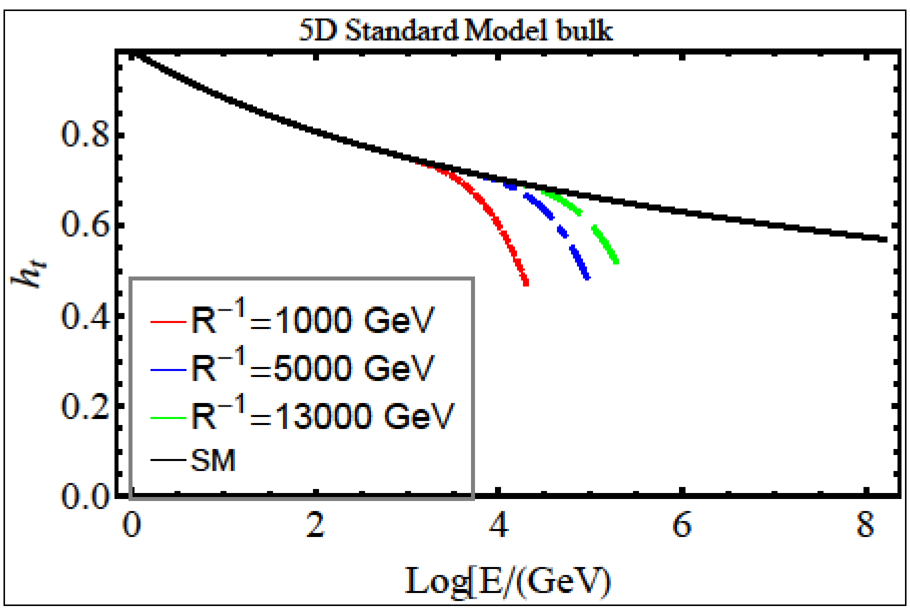

Figure 4.11: show the evolution of top quark Yukawa coupling as function of energy scale in 5D SM for three different values of the compactification scales R.

4.4 Conclusions:

In conclusion, the five dimensional standard models with compactification radius near the TeV scale imply exciting phenomenology for collider physics. It is found that the evolution of the gauge couplings has a rapid variation in the presence of the KK modes and this leads to a much lower unification scale than the SM. The running of Yukawa couplings for the three families has a sizable variation in five dimensional models. We quantitatively discussed these quantities for $R^{-1} = 1\ \mathrm{TeV}, 5\ \mathrm{TeV}$ and $13\ \mathrm{TeV}$ observing similar behaviors for all values of the compactification radius below these scale their trajectory run in the usual SM logarithmic fashion. We have shown that the scale dependence is not logarithmic; it shows a power law behavior. Therefore, the five dimensional models has substantial effects on the Yukawa couplings and promises exciting phenomenology for upcoming collider physics results, especially with the Large Hadron Collider now being operational and already starting explorations of the TeV scale where the possibility of KK excitations to SM particles exists. After all, only experiments will tell us if any of these ideas are relevant.

4.5 Recommendation:

This work can be extended in a number of ways and we discuss just a few. In this work we considered only the bulk scenarios in which all SM field have access to full space. We leave other possibilities for future work in which the 1st and 2nd generation are in the bulk, with the 3rd generation either in the bulk or on a brane.

It is Also important to confirm these results and conclusions made at one loop that are sensitive to this scale are still consistent and under control at two (and higher) loops. For instance one might be concerned that one loop linear sensitivity to the cutoff behaving as ΛR do not result in terms of the form $(\Lambda R)^2$ at two-loop, which would then indicate a breakdown of perturbation theory at renormalization scales of the order of the compactification radius.

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