Dedication

To my Parents

To my Brothers,

To my Sisters

To my Teachers

To my colleagues

and friends

Acknowledgments

First All praise is to ALLAH, the Almighty, with whose gracious help was possible to accomplish this work.

I would like to thanks supervised **Dr:Emad Eldeen Abdallah Abdelrahim** Who did not spare us for his information at all and thanks go to all my teachers in the Department of mathematics, I am deeply indebted to all staff of typists those help me to finish this study.

Abstract

In this research we discuss some models and develop the stochastic calculus and some application to finance. By using the stochastic processes and as applications, we present Poisson process and Wiener process which used for modeling Brownian motion. Also we discuss Ito's integrals and formulas, we used stochastic integration techniques. Also we discuss some examples to find mean and variance from the stochastic differential equations.

الخلاصة

في هذا البحث ناقشنا بعض نماذج لتطوير حساب التفاضل والتكامل العشوائي وبعض التطبيقات المالية. باستخدام العمليات العشوائية وكتطبيق قدمنا عملية بواسون وعملية وينر المستخدمة لنمذجة الحركة البراونية . ايضا ناقشنا تكاملات وصيغ ايتو، استخدامنا تقنيات التكامل العشوائي . ايضا ناقشنا بعض الامثلة لإيجاد المتوسط والتباين من المعادلات التفاضلية العشوائية.

The contents

Subject **Page**

Chapter 4

Stochastic Calculus Techniques and Stochastic Differential Equations

Introduction

In this work we consider the stochastic calculus as the mathematics used for modeling financial options. Stochastic calculus is used to model system that have a random behavior.

Also develop professional skill in stochastic calculus and its application to problem in finance.

Our research will be organized as follows:

Firstly we discuss the Sample space, the events and probability, and the random variables. we study the distribution functions, the basic distribution, and the independent random variables. Also we discuss the Radom-Noikody's theorem, the conditional expectation, the inequalities of the random variables. Also we study the limit of sequences of random variable and the stochastic processes.

In chapter 2 we study the geometric Brownian motion, the integrated Brownian motion, and the exponential integrated Brownian motion, the Brownian bridge, the Brownian motion with drift. Also we discuss the Bessel process, the Poisson process, and the definition and properties. Also the interarrival times, the integrated Poisson process, and the fundamental relation $dM_t^2 = dN_t$, the relation $dtdM_t = 0, dW_t dM_t = 0$, the limits of stochastic processes. Also we investigate the convergence theorem, the Martingale convergence theorem, and the squeeze theorem.

In chapter 3 we present the nonanticipating processes, the increment of Brownian motion, and the Ito integral. Also we study the existence of Ito integral, the examples of Ito integrals, and the case $F_t = c$,the case $F_t = W_t$. And also we discuss the fundamental relation $dW_t^2 = dt$, the properties of the Ito integral, also an workout example, the differentiation rules, the basic rules. Also we study the Ito's formula, the Ito's formula for diffusions and the Ito's multidimensional formula.

Finally in the following we discuss fundamental theorem of stochastic calculus, the stochastic calculus by parts. Also we study the heat equation

methods, the finding mean and variance, the integration technique, the exact stochastic equation.

Also we discuss the integration by inspection, the linear stochastic equations, and the methods of variance of parameters, also the integration factors and the existence and uniqueness. With some examples and applications.