Sudan University Of Science & Technology
College of Graduate Studies

Nature and Role of Internal Field in Amplification and conductivity of Electromagnetic Wave

طبيعة ودور المجال الداخلي في التضخيم والموصلية للموجات الكهرومغناطيسية

B.sc(SUST) & M.sc(SUST)

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الاية

قال تعالى:

{قَالُواْ سُبْحَانَكَ لاَ عِلْمَ لَنَآ إِلاَّ مَا عَلَّمْتَنَآ إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ}

سورة البقرة الاية (٣٢)

Dedication;

To my life father

And the soul of my mother

To my wife and children

Acknowledgements

This work was carried out in cooperative research and I would like to express my gratefulness and gratitude is due to my supervisor, Prof. Mubarak Dirar Abd-Alla for his guidance, encouragement ,and kind helpful which made this work possible, special thanks to Sudan University of Science & Technology and College of Graduate Studies and also to the college of science. We are very much great full to my friends and every one for their help. And of course thanks again to all of my family for everything do to use .

ABSTRACT

Interaction of radiation plays an important role in determining the optical properties of matter. In this work amplification of electromagnetic waves depends on the relation between the external with internal field .It is shown that amplification exists when the external and internal field are in phase. When the two field are normal to each other no amplification exists . While absorption happens when the two fields apposes each other. The internally generated electric field due to the interaction of matter with electromagnetic field is usually described by polarization. An alternative way based on the notion of internal field and current density is introduced here. The mathematical model is based on RCL circuits and effective values and complex representations. It shows that the external field induces internal field perpendicular to it ,beside two current components ,One parallel and the other is perpendicular to it .The material act as a resister and an inductor connected in series.

المستخلص

يلعب تفاعل الموج الكهرومغنطيسي مع المادة دوراً مهماً في تحديد الخواص الضوئية للمادة هذا العمل وضح إعتماد تضخيم الموجات الكهرومغنطيسية على علاقة المجال الداخلي بالخارجي. وهذا يدل علي ان التضخيم يكون حاضرا متي ما كان المجالان في طور واحد . وعندما يكون المجالان متعامدان لبعضهما البعض ينعدم التضخيم . بينما يحدث الامتصاص عندما يعاكس المجالان بعضهما البعض.والمجال الكهربي المتولد يعزى لتفاعل المادة مع المجال الكهرومغنطيسي وعادة يوصف بالاستقطاب. ويمكن استبداله بمفهوم المجال الداخلي وكثافة التيار. النموذج الرياضي يعتمد على دائرة تحتوي (مقاومة ومكثف وملف) والقيم الفعالة والتمثيل المركب. وهذا يوضح أن المجال الخارجي يولد مجال داخلي حتي معامداً له بجانب مركبتين للتيار أحداهما موازية والاخرى عمودية عليه. حيث تعمل المادة كمقاومة ومحاثة موصلتين على التوالي.

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Chapter one

Introduction

1.1 Electromagnetic

Electromagnetic field results from as collation of electric and magnetic fields perpendicular to each other. The electromagnetic field E_F is used widely in many applications. One of these very interesting one is Laser. Laser is the light amplification by the stimulated emission of radiation, which serves to explain most but not all the critical physical interactions that occur within a laser generation cavity [1]. The first actual continuously generating la ser was attributed to Javan and colleagues in 1961 that used a mixture of helium and neon. One of the most practical lasers used in oral and maxilla facial surgeries was developed by Patel in 1964[2]. Laser is a highly intensive light All light consist of waves traveling through space, the color of the light is determined by the frequency of these waves .The beam of a laser is a very pure red color – it consists of an extremely narrow range of wave lengths within the red portion of the spectrum, it is said to be nearly “monochromatic “ or nearly “single –colored”. Near monochromatic is a unique property of laser light meaning that consists of light of almost a single wavelength [3]. Four functional elements are necessary in lasers to produce coherent light by stimulated emission of radiation Active medium, Excitation mechanism, Feedback mechanism, and output coupler. The type of lasers it is solid crystalline and glass lasers, gas lasers, liquid bye lasers, and semiconductor lasers (diode laser). The importance of emission and absorption comes from wide variety of applications light source and laser technologies depend on The emission phenomena [4] Laser is now widely used in computer to store information in CDs. it is also used in telecommunication to transmit information and calls through optical fibers. Laser is also useful in medicine especially in surgery.[5,6]. As well as it is importance in industry. Absorption process plays an important role in the efficiency of solar cells . solar cells are considered as an alternative to petroleum energy which is expensive and causes pollution. Despite the wide variety of application of laser in modern technology, its theoretical background suffers from noticeable setbacks. One of the most problem is related to the theory of amplification which needs to be promoted to relate amplification to more physical parameters,[7,8,9,10] so as to produce new laser types. The subject matter of electronics may be divided into two broad categories: the application of physical properties of materials in the development of electronic control devices and the utilization of electronic control devices in circuit applications. [11,12] the interaction of light with matter has aroused interest – at least among poets, painters, and physicists.[13] This interest stems not so much from our curiosity about materials themselves, but rather to applications, should it be the exploration of distant stars, the burning of ships of ill intent, or the discovery of new paint pigments.[14,15] Optics, as defined is concerned with the interaction of electromagnetic radiation with matter. The theoretical description of the phenomena and the analysis of the experimental results are based on Maxwell's equations and on their solution for time-varying electric and magnetic fields. The optical properties of solids have been the subject of extensive treatise. [16] This interaction induces internal field and polarization. The interaction of conductors, dielectrics, ionic crystals and field shows different characteristics.

1.2 Research problem

The research problem is related to the fact that there is no direct relationship between the internal and external field and processes of interaction of electromagnetic field with the bulk matter.

1.3 Aim of The work

The aim of this work is to study the nature of internal field , complex conductivity on the basis of polarization and current density relation with electric field. And also study new amplifications conditions.

1.4 Literature review

The relation between optical properties and the electrical properties of matter are discussed by many authors,[17,18,19,20,21,22] some of them like the paper of R.Abd Elhai, Using the tight binding approximation in deriving the quantum critical temperature superconductivity equation.[23] In some papers and works, like the work H.G.I.Hamza,in which he uses the resistance depending on the magnetic and electric susceptibility to derive the expression of the critical temperature.[24] And Lutfi Mohammed Abdalgadir, with others researcher derive Schrodinger quantum equation from classical and quantum Harmonic Oscillator.[25] K.G.Elgaylani,derive of Klein-Gordon equation from Maxwell's electric wave equation.[26], and Almahdi.A.Alhaj, studies the relation between the amplification of the electromagnetic radiation and the phase between the external electric field intensity(E)and the velocity of the electron and its relation to amplification.[27] Mohammed Ismail derive new Maxwell's equation which accounts for the effect of diffusion current an them.[28]

1.5 Research methodology

The methodology is as follows

1. find the electric conductivity of Direct and alternating current :
2. find the amplification conditions on the basis of phase relation to the electric susceptibility
3. The Relation between current and electric field in terms of a conductivity
4. find the Electric conductivity by using RLC circuits Relations.
5. find the Electric conductivity by using Effective values.
6. find the Electric conductivity using complex Representation

1.6 presentation of the thesis

The thesis consists of five chapters. Chapter1 is the introduction. Chapter2 is concerned with the theoretical background and chapter3 is devoted for laser and light amplification. Literature review is in chapter4,while the contribution is in chapter5.

Chapter two

Theoretical Background

2.1 Introduction

This chapter concerned Background with Maxwell's equation and optical properties of solids and Polarization and susceptibility and The RLC Circuit.

2.2 Maxwell's Equations

Until Maxwell's work , the known basic laws of electricity and magnetism were. Gauss law applied to electrostatics [29,30]

$$\nabla . D = \rho \tag{2.2.1}$$

corresponding result for magnetic field yields

$$\nabla . B = 0 \tag{2.2.2}$$

faraday's law of induction

$$\nabla X E = -\frac{\partial B}{\partial t} \tag{2.2.3}$$

Amperes law for magneto motive force

$$\nabla X H = j \tag{2.2.4}$$

The first there of these are general equations and are valid for static as well as dynamic fields The fourth equation was derived From steady state observations and we have to examine its validity for time varying fields taking the divergence of both sides of (4) we have

$$\nabla .(\nabla X H) = \nabla . j = 0$$

$$\nabla . \mathrm{j} + \frac{\partial \rho}{\partial \mathrm{t}} = 0$$

$$\nabla . \mathrm{j} = -\frac{\partial \rho}{\partial \mathrm{t}} = -\frac{\partial (\nabla . D)}{\partial \mathrm{t}} = \nabla . (\frac{-\partial D}{\partial \mathrm{t}})$$

Maxwell replaced j in Amperes' law by $\mathrm{j} + \frac{\partial D}{\partial \mathrm{t}}$ with this modification Amperes law take the from

$$\nabla X H = \mathrm{j} + \frac{\partial D}{\partial \mathrm{t}} \quad (2.2.5)$$

The four equations whish .the field E,D,B,H satisfy everywhere are

$$\nabla . D = \rho$$

$$\nabla . B = 0$$

$$\nabla X E + \frac{\partial \mathrm{B}}{\partial \mathrm{t}} = 0$$

$$\nabla X H = \mathrm{j} + \frac{\partial D}{\partial \mathrm{t}} \quad (2.2.6)$$

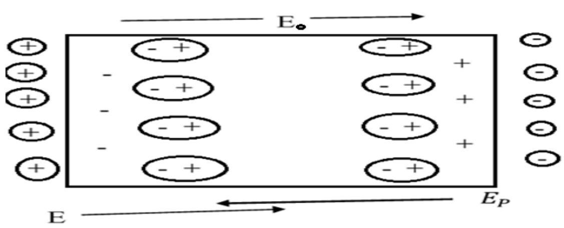
These equations are the fundamental equations of electromagnetic field and are known as Maxell's equations.

2.3 optical properties of solids

we mean those properties that relate to the interaction of solids with electromagnetic radiation whose wavelength is in the infrared to the ultraviolet. There are several aspects to optical properties of solids and looking at the subject in full generality can often lead to complexity, whereas treating each part as a separate case often leads to confusion. We will try to keep to a middle ground between these, by emphasizing only one topic (absorption) but treating it in some detail. Although we will concentrate on absorption, we will mention other optical phenomena including emission, reflection, scattering, and photoemission of electrons. There are several processes involved in absorption, but the main five seem to be:[31,32] (a) Absorption due to electronic transitions between bands that involve wavelengths typically less than ten micrometers. (b) Absorption by excitons at wavelengths with energies just below the absorption edge due to valence–conduction band transitions (in semiconductors). (c) Excitation and ionization of impurities that involve wavelengths ranging fromabout one micrometer to one thousand micrometers. d) Excitation of lattice vibrations (optical phonons) in polar solids for which the usual wavelengths are ten to fifty micrometers. (e) Free-carrier absorption for frequencies up to the plasma edge. Free-carrier absorption is particularly important in metals, of course. By gathering data about any optical process, we can gain information about the inner workings of the solid.

2.4 Polarization and susceptibility

When a dielectric is placed in an external electric field E the positive and negative charge are displaced from their equilibrium positions by very small distance (less than an atomic diameter) throughout the volume of dielectric this results in the formation of a large number of dipoles each having some dipole moment in the direction of the field .The material is said to be polarized with polarization P defined as the dipole moment per unit volume of the material.[33,34]



Fig(2.4.1) A dielectric slap placed in an electric field E_0 produced by fixed charges (encircled) outside the slap. The internal polarization field E_p is assumed to be due to fictitious bound. Charges at the surface of the slap and is directed apposite to E_0

As shown in the effect of polarization is to reduce the magnitude of external E_0 field thus the magnitude of resultant field is less than the applied field $E < E_0$ in vector notation we may write

$$E = E_0 + E_P \qquad (2.4.1)$$

The fieldE_P is called the polarization field as it tends to oppose the applied field E_0 within the material for ordinary electric fields ,the polarization P is proportional to the Macroscopic field E . In (SI) units, it is expressed as

$$P = \varepsilon_0 x_e E \qquad (2.4.2)$$

Where ε_0 is the permittivity of free space and x_e is the electric susceptibility Thus ,except for a constant factor ,the electric susceptibility is a measure of the polarization produced in the material per unit resultant electric field. The total dipole moment is defined as

$$P = \Sigma q_n r_n \ , \qquad (2.4.3)$$

Where r_n is the position vector of the charge q_nThe electric field at a point r from a point of moment P is given by

$$E_{(r)} = \frac{3(P.r)r - r^2 P}{4\pi\epsilon_0 r^5} \qquad (2.4.4)$$

One contribution to the electric field inside a body is that of the applied electric field , defined as

$E_{0\equiv}$ field produced by fixed charges external to the body.

The other contribution to the electric field is the sum of the fields of all charges that constitute the body. If the body is neutral, the contribution to the average field may be expressed in terms of the fields of atomic dipoles of the form of (2.4.4). We define the average electric field $E(_{r_0})$ as the average field over the volume of the crystal cell that contains the lattice point r_0 :

$$E(_{r_0}) = \frac{1}{V_c}\int dVe(r) \ ,$$

Where $e(r)$ is the microscopic electric field at the point r . The field E is a much smoother quantity than the field e.

The electric field due to these charge has a simple from at any point between the plates, but comfortably removed from their edges. By Gauss's law

$$E_1 = -\frac{|\sigma|}{\epsilon_0} = -\frac{P}{\epsilon_0} \qquad (2.4.5)$$

We add E_1 to the applied field to obtain the total macroscopic field inside the slab:

$$E = E_0 + E_1 = E_0 - \frac{P}{\epsilon_0}\hat{z} \qquad (2.4.6)$$

Where $\hat{z}$ is the unit vector normal to the plane of the slab. $E_1 \equiv$ Field of the surface charge density $\hat{n}.P$ on the boundary of a simply-connected body. This field is smoothly –varying in space inside and outside the body and satisfies the Maxwell equation for the macroscopic field E . The reason is E_1a smooth function when viewed on an atomic scale is that we have replaced the discrete lattice of dipoles P_i with the smoothed polarization$P$.[35]

2.4.1 Dielectric constant and polarizability

The dielectric constant ϵ of an isotropic medium relative to vacuum is defined as

$$\epsilon = \frac{\epsilon_0 E + P}{\epsilon_0 E} = 1 + \boldsymbol{\chi} \qquad (2.4.7)$$

The susceptibility is related to the dielectric constant by

$$\chi = \frac{P}{\epsilon_0 E} = \epsilon - 1 \qquad (2.4.8)$$

Here is the macroscopic electric field. In a noncubic crystal the dielectric response is described by the components of the susceptibility tensor or of the dielectric constant tensor:

$$P_\mu = \chi_{\mu v}\epsilon_0 E_v \;\; ; \qquad \epsilon_{\mu v} = 1 + \chi_{\mu v}$$

2.4.2 Sources of polarizability

The net polarizability of dielectric material results mainaly from the following there types of contributions:[36]

Ionic polarizability

Dipolar polarizability

Electronic polarizability

The extent to which a particular polarizability contributes depends on the nature of the dielectric and the frequency of the applied electric field **.**

2.4.3 Ionic polarizability

The ionic polarizability arises due to displacement of charged ion relative to other ions in a solid.[37]

Consider the equation of motion of two particle of masses m_1 and m_2 respectively.

$$m_1 \ddot{u}_{n+1} = -c(2u_{n+1} - u_n - u_{n+2}) + eE \qquad (2.4.9)$$

$$m_2 \dot{u}_n = -c(2u_n - u_{n-1} - u_{n+1}) - eE \qquad (2.4.10)$$

Consider the solution

$$u_n = u_0 e^{-iwt} e^{-ikna}$$

$$u_{n+1} = u_0 e^{-iwt} e^{-ik(n+1)a}$$

$$u_{n+1} = u_+$$

$$u_+ = u_{0+} e^{-iwt}$$

$$u_n = u_{n+2} = u_-$$

$$u_- = u_{0-} e^{-iwt} \qquad (2.4.11)$$

Inserting equation(2.4.11)in(2.4.9) and (2.4.10) yields

$$-m_1 \omega^2 u_+ = -2c[u_+ - u_-] + eE$$

$$-m_2 \omega^2 u_- = -2c[u_- - u_+] - eE$$

$$= 2c[u_+ - u_-] - eE \qquad (2.4.12)$$

Adding the two equations yields

$$-m_1 \omega^2 u_+ - m_2 \omega^2 u_- = 0$$

$$u_- = \frac{-m_1}{m_2} u_+ \qquad (2.4.13)$$

Thus inserting (2.4.13)in(2.4.12) yields

$$-m_1\omega^2 u_+ = -2c[1+\frac{m_1}{m_2}]u_+ + eE \quad (2.4.14)$$

But the electric field intensity is given by

$$E = E_0 e^{-iwt} \quad (2.4.15)$$

substituting(2.4.15) and (2.4.11)in (2.4.14) yields

$$m_1\left[2C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)-\omega^2\right]u_{0+} = eE_0 \quad (2.4.16)$$

$$m_1[\omega_0{}^2-\omega^2]u_{0+} = eE_0$$

$$u_{0+} = \frac{e}{m_1[\omega_0{}^2-\omega^2]}E_0 \quad (2.4.17)$$

There fore

$$u_+ = \frac{e}{m_1[\omega_0{}^2-\omega^2]}E \quad (2.4.18)$$

Similarly inserting equation(2.4.12)in(2.4.13) yields

$$-m_2\omega^2 u_- = -2c[\frac{m_2}{m_1}+1]u_- - eE_-$$

$$m_2\left[2C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)-\omega^2\right]u_- = -eE_-$$

$$m_2[\omega_0{}^2-\omega^2]u_- = -eE_-$$

$$u_- = \frac{-e}{m_2[\omega_0{}^2-\omega^2]}E \quad (2.4.19)$$

Thus the ionic polarization is given by

$$P_i = en\chi = en(u_+ + u_-) \quad (2.4.20)$$

$$P_i = \frac{e^2 n}{(\omega_0{}^2-\omega^2)}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)E = \chi_i E \quad (2.4.21)$$

Thus the ionic susceptibility is given by

$$\chi_i = \frac{e^2 n(m_1+m_2)}{m_1 m_2(\omega_0{}^2-\omega^2)} \quad (2.4.22)$$

2.4.4 Dipolar polarizability

A molecule, such as H_2o, having a permanent dipole moment is called a dipolar or polar molecule and a substance comprising such molecule is called a dipolar substance, the dipolar polarizability is the property of dipolar substance. In the absence of an external electric field , the dipoles have random orientations and there is no net polarization, However, when the field is present the dipoles orient themselves along the field and produce orientational or dipolar polarizability the thermal agitation of molecules tends to counteract the ordering effect of the electric field and equilibrium state is reached where in the different dipolar make all possible angles varying from zero to **π** radians with the field direction, the potential energy of such a molecule of dipole moment P oriented at an angle Θ with the field direction fig(2.4.2) is given by[38]

$$V = -P.E = -pEcos\Theta \qquad (2.4.23)$$

WhereΘ is the angle between the moment and the field direction. Then

$$P = Np\langle cos\Theta\rangle, \qquad (2.4.24)$$

Where N is the concentration of molecules and $\langle cos\Theta\rangle$ is the thermal average. According to the Boltzmann distribution law the relative probability of finding a molecule in an element of solid angle $d\Omega$ is proportional to $\exp(-\frac{V}{k_B T})$, and

$$\langle cos\Theta\rangle = \frac{\int e^{-\beta V} cos\Theta d\Omega}{\int e^{-\beta V} d\Omega} \qquad (2.4.24)$$

Where $\beta \equiv \frac{1}{k_B T}$. The integration is to be carried out over all solid angle, so that

$$\langle cos\Theta\rangle = \frac{\int_0^{\pi} 2\pi sin\Theta cos\Theta e^{\beta pEcos\Theta} d\Theta}{\int_0^{\pi} 2\pi sin\Theta e^{\beta pEcos\Theta} d\Theta} \qquad (2.4.25)$$

We let $s \equiv cos\Theta$ and $x \equiv \frac{pE}{k_B T}$, so that

$$\langle cos\Theta\rangle = \frac{\int_{-1}^{1} e^{sx} s ds}{\int_{-1}^{1} e^{sx} ds} = \frac{d}{dx} log \int_{-1}^{1} e^{sx} ds$$

$$= \frac{d}{dx} log(e^{x} - e^{-z}) - \frac{d}{dx} logx = ctnhx - \frac{1}{x} \equiv L(x)$$

In the limit of $x \ll 1$, we have

$$ctnhx = \frac{1}{x} + \frac{x}{3} + \frac{x^3}{45} + \cdots ; L(x) \cong \frac{x}{3} = \frac{pE}{3k_B T}, \qquad (2.4.26)$$

And the polarization is

$$P = Np\langle cos\Theta\rangle = \frac{\mathrm{pE}}{3k_BT} \qquad (2.4.27)$$

The Dipolar polarizability per molecule is given by

$$P_d = \frac{p^2\mathrm{E}}{3k_BT} \qquad (2.4.28)$$

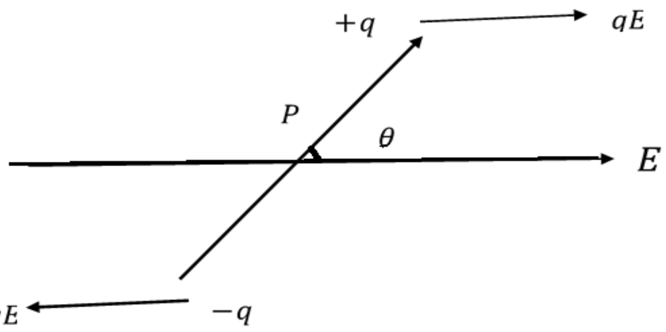


Fig (2.4.2) : A molecule dipole moment P placed in on electric field E

2.4.5 Electronic polarizability

The electronic polarizability arises due to displacment of the electron cloud of an atom relative to its nucleus in the presence of an applied electric field as shown in fig (2) the polarization as well as the dielectric constant of a material at optical frequencies results mainly from the electronic polarizability.[39]

The equation of motion in the local electric field is

$$m\frac{d^2x}{dt^2} + m{\omega_0}^2x = -eE_{0L}e^{-i\omega t} \qquad (2.4.29)$$

When one assumes the solution

$$x = x_0e^{-i\omega t} \qquad (2.4.30)$$

$$-m\omega^2x + m{\omega_0}^2x = -eE_{0L}e^{-i\omega t} = -eE_L$$

$$x = \frac{e}{m({\omega_0}^2 - \omega^2)}E_L$$

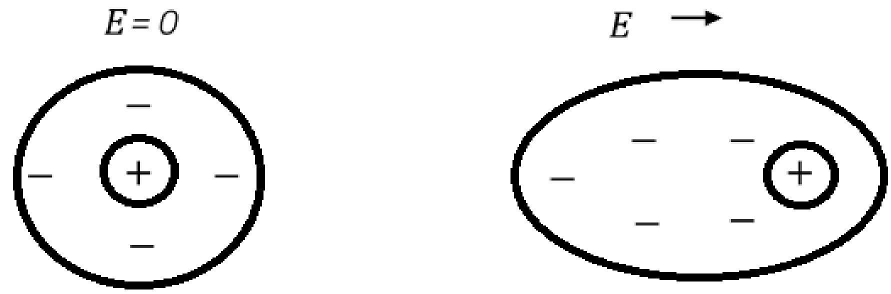
Therefore the polarization is given by

$$P_e = \frac{nze^2/m}{{\omega_0}^2 - \omega^2}E = \chi_e\, E \qquad (2.4.31)$$

P_e=Electronic polarizability

Thus the electric susceptibility is given by

$$\chi_e = \frac{nze^2}{\varepsilon_0 \mathrm{m}({\omega_0}^2 - \omega^2)} \tag{2.4.32}$$



Un polarized atom polarized atom

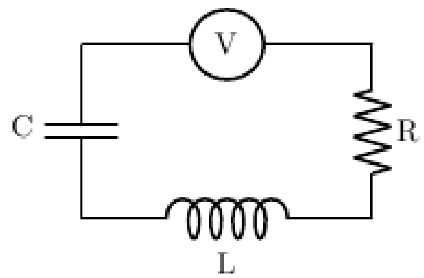
Fig (2.4.3)Electronic polarization. In the presence of external field

The total polarizability of a dielectric is given as a sum of electronic ,ionic, and dipolar terms, i.e.

$$P = P_{\mathrm{e}} + P_{\mathrm{i}} + P_{\mathrm{d}} \tag{2.4.33}$$

2.5 The RLC Circuit

The RLC circuit is the electrical circuit consisting of a resistor of resistance R, a coil of inductance L, a capacitor of capacitance C and a voltage source arranged in series. If the charge



on the capacitor is Q and the current flowing in the circuit is I, the voltage across R, L and C are RI , L $\frac{dI}{dt}$ and $\frac{Q}{c}$ respectively. By the Kirchhoff's law that says that the voltage between any two points has to be independent of the path used to travel between the two points[40]

$$\mathrm{LI'(t)} + \mathrm{RI(t)} + \frac{1}{C}\mathrm{Q(t)} = \mathrm{V(t)} \tag{2.5.1}$$

Assuming that R, L, C and V are known, this is still one differential equation in two unknowns, I and Q. However the two unknowns are related by $I(t) = \frac{dQ}{dt}(t)$ so that

$$LQ''(t) + RQ'(t) + \frac{1}{C} Q(t) = V(t) \tag{2.5.2}$$

or, differentiating with respect to t and then subbing in $\frac{dQ}{dt}(t) = I(t)$,

$$LI''(t) + RI'(t) + \frac{1}{C} I(t) = V'(t) \tag{2.5.3}$$

For an ac voltage source, choosing the origin of time so that V(0) = 0, $V(t) = E_0 \, sin(\omega t)$

and the differential equation becomes

$$LI''(t) + RI'(t) + \frac{1}{C} I(t) = \omega E_0 \, cos(\omega t) \tag{2.5.4}$$

The General Solution

We first guess one solution of (2.5.4) by trying

$$I_P(t) = A \, sin(\omega t - \phi) \tag{2.5.5}$$

with the amplitude A and phase ϕ to be determined. That is, we are guessing that the circuit responds to an oscillating applied
voltage with a current that oscillates with the same rate. For $Ip(t)$ to be a solution, we need

$$LI''p\,(t) + RIp'\,(t) + \frac{1}{C} Ip(t) = \omega E_0 \, cos(\omega t) \tag{2.5.6}$$

$$-L\omega^2 A \, sin(\omega t - \phi) + R\omega A \, cos(\omega t - \phi) + \frac{1}{C} A \, sin(\omega t - \phi)$$

$$= \omega E_0 \, cos(\omega t)$$

$$= \omega E0 \, cos(\omega t - \phi + \phi)$$

and hence

$$(\frac{1}{C} - L\omega^2) \; A \, sin(\omega t - \phi) + R\omega A \, cos(\omega t - \phi)$$

$$= \omega E_0 \, cos(\phi) \, cos(\omega t - \phi) - \omega E_0 \, sin(\phi) \, sin(\omega t - \phi)$$

Matching coefficients of $sin(\omega t - \phi)$ and $cos(\omega t - \phi)$ on the left and right hand sides gives

$$L\omega^2 - \frac{1}{C} A = \omega E_0 \, sin(\phi) \tag{2.5.7}$$

$$R\omega A = \omega E_0 \, cos(\phi) \tag{2.5.8}$$

It is now easy to solve for A and ϕ

$$tan(\phi) = \frac{\omega L - \frac{1}{\omega C}}{R} \Longrightarrow \phi = tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$E_0 = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} \quad A$$

$$\Longrightarrow A = \frac{E_0}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}} \tag{2.5.9}$$

Naturally, different input frequencies ω give different output amplitudes A. Here is a graph of A against ω, with all other parameters held fixed.

Now back to finding the general solution. Note that subtracting (2.5.6) from (2.5.4) gives

$$L(I - Ip)''(t) + R(I - Ip)'(t) + \frac{1}{C}(I - Ip)(t) = 0$$

That is, any solution of (2.5.4) differs from Ip(t) by a solution of

$$LI''(t) + RI'(t) + \frac{1}{C} I(t) = 0 \tag{2.5.10}$$

This is called the complementary homogeneous equation for (2.5.4).

Chapter three

Laser and light Amplification

3.1 Introduction:

Laser plays an important role in ours day life. Thus it is important study The laser properties . The stimulated emission beside spontaneously emission processes are studied here . The laser production is also discussed.

3.2 Emission and Absorption of light:

The atom now a day is considered as a system consisting of a central**,** positively charged nucleus surrounded by a number of negatively charged electrons revolving around the nucleus in certain orbits .Each orbit describes energy level .The energy is characterized by a principal quantum number denoted by n . The nearest level to the nucleus is called the ground state and its principal quantum number is equal to one. Each type of atom contains a certain amount of energy levels. If the atom contains additional energy states over and above its ground state it can emit or absorb photons .The absorption takes place when an electron makes a transition from a lower to a higher energy state , with a photon being absorbed in this process. In the emissions process the electrons move from a higher state to a lower one . A photon with energy equal to the energy difference between the two levels E_1 and E_2 is released or absorbed in the emission or absorption process .The frequency f of the photon is related to the energy difference between the two levels E_1 and E_2 according to the relation [41,42]

$$E_2 - E_1 = hf \qquad (3.2.1)$$

Where h is Planck's constant.

3.2.1 Absorption process

Absorption is the process by which a photon is absorbed by atom, the photon of frequency f passes through an atomic system with energy levels E1 and E2 can absorb this photon if

$$hf = E_1 - E_2 \qquad (3.2.2)$$

As a result an electron leave E_1 to E_2 . The population of the lower level E_1 will be depleted at a rate proportional both , i.e.

$$\frac{dN1_{\circ}}{dt} = - \beta_{12}\, \rho N_1 \qquad (3.2.3)$$

Where β_{12} is a constant of proportionality called Einstein coefficient. The produced $\beta_{12}\, \rho$ can be interpreted as .The probability per unit frequency that transitions are induced by the effect of the field. [43,44]

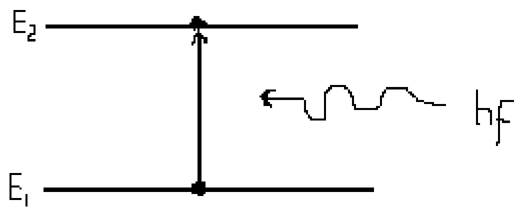


Fig (3.2.1) The atomic absorption

3.2.2 spontaneous emission process

spontaneous emission is the process by which an electron spontaneously " without any outside influence, decays from a higher energy level to a lower one after an electron has been raised to the upper level by absorption. The population of the upper level2 decays spontaneously to the lower level1 at a rate proportional to the upper level population N_2 ,i.e. [45]

$$\frac{dN_{2\circ}}{dt} = - A_{21}\, N_2 \qquad (3.2.4)$$

Where A_{21} is a constant of proportionality

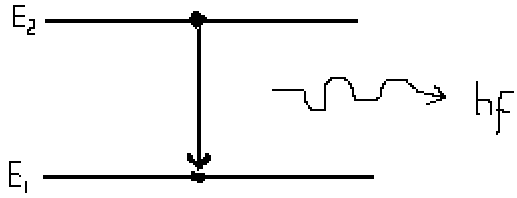


Fig (3.2.2) shown the spontaneous emission of light

3.2.3 Stimulated emission process

The process is described by the Einstein coefficient E_2 which gives the probability per unit energy density of the radiation field that electrons from the excited state E_2 are forced to return to its ground state E_1 is a photon of energy $hf = E_2 - E_1$ is incident on the atom .The rate of transition of electron from E_2 to E_1 is given by[46]

$$\frac{dN_{2\circ}}{dt} = -\beta_{21} N_2 \rho \qquad (3.2.5)$$

The emitted and incident photon have the same frequency, direction and are in phase.

Stimulated emission is one of the fundamental processes that led to the development of laser. This is because the coherence of the incident and emitted photon increases the light intensity .

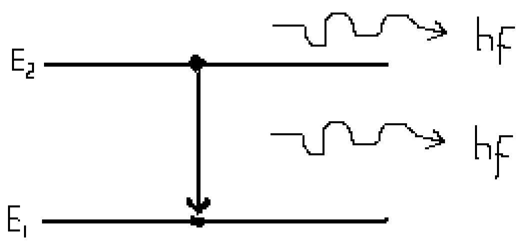


Fig (3.2.3) the stimulated Emission of light

3.3 Amplification of light and population inversion

If a light of intensity I_0 is incident on a medium, it's intensity in active medium increases. [48,49]The intensity, I, of light at a distance Z inside the medium is given by

$$I = {}_{\circ}I_0 \quad e^{\beta z} \qquad (3.3.1)$$

Where β is called amplification factor.

If a light of radiation density ρ is incident on a medium the rate of electrons leaving level E1 is given by $\beta_{12} N_1$, While the rate of electron coming to E1 from E2 by spontaneous and stimulated emission are given by $A_{12}N_2$ and $\beta_{21}N_2$ respectively.[50] Thus the rate of change of electrons in level1 E_1 is given by

$$\frac{dN1_{\circ}}{dt} = (-\beta_{12}N_1 + \beta_{21}N_2)\rho + A_{12} N_2 \qquad (3.3.2)$$

Similarly the rate of change of electrons in level2 E_2 is given by

$$\frac{dN_{2\circ}}{dt} = (\beta_{12} N_1 - \beta_{21}N_2)\rho - A_{21} N_1 \qquad (3.3.3)$$

At equilibrium the number of atom N_2 in level E_2 is constant. Thus the rate of change of N_2 vanishes, 1.e

$$\frac{dN_{2\circ}}{dt} = 0$$

Thus a equation (3.3.3) becomes

$$(\beta_{12} N1 - \beta_{21}N_2)\rho - A_{21} N_1 = 0$$

If $\beta_{12} = \beta_{21} = \beta$, Then

$$\rho\, \beta(N_1 - N_2) = A21\, N_1$$

On the other hand the rate of electron transition

$$-\frac{dN_{2\circ}}{dt} A\Delta z$$

from level2 is equal to the rate of photon emission

$$\frac{\Delta I_{\circ}}{hf} A$$

Through the area A,1.e

$$- \frac{dN2_\circ}{dt} A\Delta Z = \frac{\Delta I_\circ}{hf} A \tag{3.3.4}$$

By reviewing of equation (3.3.4) and neglecting the process of spontaneous emission one gets

$$(\beta_{12} N_1 - \beta_{21} N_2)\rho \, A\Delta Z = \frac{\Delta I_\circ}{hf} A$$

But since I= ρc, then,

$$(\beta_{12} N_1 - \beta_{21} N_2) \frac{IA\Delta Z_\circ}{c} = \frac{\Delta I_\circ}{hf} A \tag{3.3.5}$$

Bearing in mind that

$$\beta = \beta_{12} = \beta_{21}$$

$$\frac{\Delta I_\circ}{\Delta Z} = \frac{dI_\circ}{dz} = \beta(N_2 - N_1) \frac{hfI_\circ}{c} \tag{3.3.6}$$

Hence

$$\int \frac{dI_\circ}{I} = \beta(N_2 - N_1) \frac{hf_\circ}{c} \int dZ; \tag{33.7}$$

$$LnI = \beta(N_2 - N_1) \frac{hf_\circ}{c} Z + Co$$

$$I = I_0 e^{\beta(N_2 - N_1) \frac{hf_\circ}{c} Z} \tag{3.3.8}$$

Comparing (2.3.1) with (2.3.8) one finds that the amplification coefficient β is given by

$$\beta = \beta(N2 - N1) \frac{hf_\circ}{c} \tag{3.3.9}$$

Thus according to equation (2.3.8) I increases when the radiation enters more and more inside the medium when

$$\beta(N_2 - N_1) \frac{hf_\circ}{c} > o$$

this requires that

$$N_2 \ > \ N_1 \tag{3.3.10}$$

Which means that the population $N_2$of the upper levelE_2 should be more than the population N_1 of lower level E_1 . this condition is called population inversion.

3.4 properties of laser

Laser is a highly intensive light All light consist of waves traveling through space, the color of the light is determined by the frequency of these waves .The beam of a laser is a very pure red color – it consists of an extremely narrow range of wave lengths within the red portion of the spectrum, it is said to be nearly “monochromatic “ or nearly “single –colored”. Near monochromatic is a unique property of laser light meaning that consists of light of almost a single wavelength [51]

3.4.1 Directionality

Devices such as automobile head lights and spot lights contain optical systems that collimate the emitted light, such that it leave the device in a directional beam.However, the beam produced always (diverges spreads). Rapidly parallel beams of directional light, which we refer to, as collimated light cannot be produced. All light beams eventually spread (diverge) as they wave through the space. But laser light is more highly collimated, that is it is more directional than the light from any conventional source and thus les divergent. In some applications optical system are employed with laser to improve the directional of the output beam. One system of this type can produce a spot that can reach the moon.[53]

3.4.2 Coherence

Coherence is the most fundamental (property) of laser light and can distinguish it from the light from other sources. Thus a laser may be defined, as a source of coherent light. The importance of coherence cannot be understood until other concepts have been introduced. But evidence of the coherence of laser light can be observed easily.

3.5 Elements of a laser

Four functional elements are necessary in lasers to produce coherent light by stimulated emission of radiation.[54]

3.5.1 Active medium

The active medium is a collection of atoms or molecules that can be excited to a state of inverted population that is, where more atoms or molecules are in an excited state than is some lower energy state. The two state chosen for the lasing transition must possess certain characteristics. First atoms must remain in the upper lasing level for a relatively long time to provide more emitted photons by stimulated emission than by spontaneous emission in other lower energy levels, more photons will be lost by spontaneous emission- giving off randomly directed out of phase light .Photons are coherent in the process of stimulated emission. The active medium may be gas, a liquid , a solid material ,or a junction between two slabs of semiconductor materials.

3.5.2 Excitation mechanism

The excitation mechanism is a source of energy that excites or "pumps" the atoms in the active medium from a lower to a higher energy state in order to produce a population inversion. In gas lasers and semiconductors lasers the excitation mechanism usually consists of an electrical current flow through the active medium. Solid and liquid lasers most often employ optical pumps, for example, in a ruby laser the chromium atoms inside the ruby crystal may be pumped into an excited state by means of a powerful burst of light from a flash lamp containing xenon gas.

3.5.3 Feedback mechanism

The feedback mechanism usually consists of two mirrors, one at each and of the active medium. Aligned in such a manner that they are highly parallel to each other.

3.5.4 Output coupler

The output coupler allows apportion of the laser light contained between the two mirrors to leave the laser in the form of a beam. One of the mirrors of the feedback mechanism, allows some light to be transmitted through it at the laser wavelength. The fraction of the coherent to beam is less than one percent for some helium neon lasers more than 80 percent for many solid state lasers.

Chapter four

Literature review

4.1 Introduction:

The relation between optical properties and the electrical properties of matter are discussed by many author's,[55,56,57,58] some of them use the tight binding approximation in deriving the quantum critical temperature superconductivity equation. In some papers and works, Resistance Depending on the Magnetic and Electric Susceptibility was used derive the Equation of the critical temperature. Major attempts to study the nature of electric field inside matter and its interaction with individual atoms are exhibited in this review.

4.2 Using the tight binding approximation in deriving the quantum critical temperature superconductivity equation

In Rasha paper, she used plasma equation to derive Schrödinger temperature dependent equation according to plasma equation, a fluid of particles of mass m, number density n, velocity v, force F and pressure P can be described by the equation[59]

$$mn\left[\frac{\partial v}{\partial t}+v.\nabla v\right]=F-\nabla P \qquad (4.2.1)$$

The force F can be defined as

$$F=-n\nabla V$$

Where V is the potential of one particle. In one dimension

$$mn\left[\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right]=-\nabla V-\nabla P=-n\frac{dV}{dx}-\frac{dP}{dx}$$

$$dv=\frac{\partial v}{\partial t}dt+\frac{\partial v}{\partial x}dx$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\frac{dx}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}$$

Thus according to Equation (4.2.I), in one dimension

$$mn\frac{dv}{dt} = -n\frac{dv}{dx} - \frac{dp}{dx} \quad (4.2.2)$$

Schrodinger equation can be derived by using Anew expression of energy can be obtained from the plasma equation to do this one can use (4.2.2) to get

$$mn\frac{dv}{dx}\frac{dx}{dt} = -n\frac{dV}{dx} - \frac{dP}{dx}$$

Multiplying both sides by dx and integrating yields

$$mn\int vdv = -n\int dV - \int dP$$

Considering the pressure to be p $P = ynkT$ in general, thus

$$mn\frac{v^2}{2} = -nV - P = -nV - \gamma nkT$$

Hence $$m\frac{v^2}{2} = +V + \gamma kT = const$$

This constant conserved quantity looks like the ordinary energy beside the ordinary thermal energy term ykT

$$E = \frac{P^2}{2m} + V + \gamma kT \quad (4.2.3)$$

To find Schrödinger equation for it, consider the ordinary wave function

$$\psi = Ae^{i/\hbar(Px-Et)}$$

Differentiating both sides by t and x yields

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi \Rightarrow i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{P^2}{\hbar^2}\psi \Rightarrow -\hbar^2\nabla^2\psi = P^2\psi \quad (4.2.4)$$

Multiplying both sides of Equation (4.2.3) by yields ψ yields

$$E\psi = \frac{P^2}{2m}\psi + V\psi + V\psi + \gamma kT\psi$$

Substituting Equation (4.2.4), one gets

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + \gamma kT\psi$$

This equation represents Schrödinger equation when thermal motion is considered. The solution for time free potential can be

$$\psi = e^{-i/\hbar(Et)}u \Longrightarrow \frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi \quad E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + \gamma kT\psi$$

The time independent Schrödinger equation thus takes the form

$$Eu = -\frac{\hbar^2}{2m}\nabla^2 u + Vu + \gamma kTu \qquad (4.2.5)$$

For constant potential, the solution can be

$$u = e^{ikx}, V = V_0$$

Inserting this solution in Equation (4.2.5) yields

$$Eu = \frac{\hbar^2 k^2}{2m}u + V_0 u + \gamma kTu \qquad E = \frac{\hbar^2 k^2}{2m}u + V_0 + \gamma kT$$

If one set the kinetic term to be $E_0 = \frac{\hbar^2 k^2}{2m}$, one can thus write the energy in the form $\quad E = E_0 + V_0 + \gamma kT \quad$ (4.2.6)

This quantum energy expression involves a thermal term beside kinetic and potential term. The resistance, z, per unit length ($L = 1$) per unit area ($A = 1$) can be found from the ordinary definition of, z.

The resistance z is defined to be the ratio of the potential, u, to the current per Lülit area, $J, i.e.$

$$z = \frac{u}{I} = \frac{u}{JA} = \frac{u}{J} = \frac{u}{nev} = \frac{mu}{nep} \qquad (4.2.7)$$

With n and e standing for the free hole or electron density and charge respectively, while p represents the momentum of electron of mass m, where $P = mv$

This resistance (it actually stands for resistivity) can be found by using the laws of quantum mechanics for a free charge which are responsible for generating the electric current, where the wave function takes the form[60]

$$\psi = Ae^{ikt} \tag{4.2.8}$$

This selection of ψ comes from the fact that the resistance property comes from the motion of the free charges. The potential u is related to the Hamiltonian H through the relation $H = eu$

Thus for freely moving charge one gets:

$$\hat{H} = eu = \frac{1}{2}mv^2 = \frac{\hat{P}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

In view of Equation (4.2.8) and according to the correspondence principle Vtakes the form $u = \frac{\langle \hat{H} \rangle}{e} = \frac{\int \bar{\psi}\hat{H}\psi dx}{e} = \frac{\int \bar{\psi}\hat{P}^2\psi dx}{2me}$

$$= \frac{\hbar^2 k^2}{2me}\int \bar{\psi}\psi dx = \frac{\hbar^2 k^2}{2me} \tag{4.2.9}$$

While P becomes

$$P = \langle \hat{P} \rangle = \int \bar{\psi}\hat{P}\psi dx = \hbar K \int \bar{\psi}\psi dr = \hbar K \tag{4.2.10}$$

Thus inserting Equations (4.2.9), (4.2.10) in (4.2.7) one obtains

$$Z = \frac{m\hbar^2 k^2}{2me^2\hbar kn} = \frac{\hbar k}{2e^2 n} = \left(\frac{h}{2\pi}\right)\left(\frac{2\pi}{\lambda}\right)\frac{1}{2e^2 n}$$

$$z = \frac{h}{2\lambda e^2 n} = \frac{hf}{2f\lambda e^2 n} = \frac{hf}{2e^2 nv} = \frac{hf\sqrt{u\varepsilon}}{2e^2 n}\frac{\hbar\omega\sqrt{u\varepsilon}}{2e^2 n} \tag{4.2.11}$$

Where the expression $f\lambda$ for velocity is found by as suming charges to be waves, then following the electromagnetic theory (EMT), the speed of the waves is affected by electric permittivity ε and magnetic permeability through the relation

$$v = \lambda f = \frac{1}{\sqrt{\mu\varepsilon}} \quad (4.2.12)$$

where the effect of medium changes the wave length, λ, while the frequency, f, is unchanged. Thus assuming the charge density, n, to be constant, the only change of, Z, can be caused by μ and ε. It is also important to note that, in superconductors, the current can flow without the aid of deriving potential u. the role of u is confined only in enabling electrons to gain kinetic energy through the relations

$$eu = \frac{1}{2}mv^2 = k \quad (4.2.13)$$

where this potential can be applied between any two arbitrary points m the superconductors then remove it. The role of resistive force is neglected here as done in deriving London equations.

The expression for Z can also be found by inserting Equation (4.2.13) in to get

$$Z = \frac{u}{J} = \frac{u}{nev} = \frac{mv^2}{2ne^2 v} = \frac{mv}{2ne^2}\frac{P}{2ne^2} = \frac{h}{2\lambda ne^2}$$

$$Z = \frac{hf}{2\lambda f e^2 n} = \frac{hf}{2e^2 m} = \frac{hf\sqrt{\mu\varepsilon}}{2e^2 n}\frac{\hbar\omega\sqrt{\mu\varepsilon_o(1+\mathcal{X})}}{2e^2 n} \quad (4.2.14)$$

It is important to note that this quantum resistance expression resembles the ones found by Tsui where one uses De Broglie hypothesis $i.e.\ P = h/\lambda$. Consider holes in a conductor having resistive force Fm, magnetic force Fm and pressure force Fp, beside the electric force Fe, the equation motion then becomes [61]: $F = F_r + F_m + F_e - F_p$

where $F_p = -\nabla P, F_r = -\frac{mv}{\tau}, F_m = Bev, F_e = eE = eE_o e^{iox}$

$P,\ x,\ m, v, \tau\ , B, e$ and E stands for the pressure, displacement, mass, velocity, relaxation time, magnetic flux density, electron charge and electric field intensity respectively. Thus the equation of motion takes the form

$$m\ddot{x} = -\frac{mv}{\tau} + Bev + eE - \nabla\mathrm{P} \tag{4.2.15}$$

The solution of this equation can be suggested to be:

$$x = x_0 e^{iox} \quad v = v_0 e^{iox} \quad E = E_0 e^{iox} \tag{4.2.16}$$

Inserting (4.2.16) in (4.2.15) yields

$$-m\varpi^2 x = \left(-\frac{mv_0}{E_0\tau} + \frac{Bev_0}{E_0} - \frac{kt\nabla n}{E_0} + e\right)E \tag{4.2.17}$$

$$x = \frac{\left(-\frac{mv_0}{E_0\tau} + \frac{Bev_0}{E_0} - \frac{kt\nabla n}{E_0} - e\right)E}{m\varpi^2}$$

This expression of x can be utilized in the formula which relates the electric polarization vector P to the susceptibility χ on one hand and to the number of atoms N via the following relation [62]

$$P = \varepsilon_0\ \chi E = +eNx \tag{4.2.18}$$

Motivated by the important role of holes in HTSC, displacement can be assumed to result from the motion of holes or positive nuclear charges, thus inserting Equation (4.2.17) in (4.2.18) yields

$$\varepsilon\ \mathrm{o}\chi E = eN\frac{\left(-\frac{mv_0}{E_0\tau} + \frac{Bev_0}{E_0} - \frac{kt\nabla n}{E_0} - e\right)E}{m\varpi^2}$$

$$\chi\frac{eN}{m\varpi^2\varepsilon_0 E_0}\left(\frac{mv_0}{\tau} - Bev_0 + kT\nabla n - eE_0\right) \tag{4.2.19}$$

The electric flux density assumes the following relation

$$D = \varepsilon E = \varepsilon_0 E + \chi\varepsilon_0 E(1+\chi)E = P + \varepsilon_0 E$$

The electric permittivity is given by $\quad \varepsilon = \varepsilon_0(1+\chi)$ (4.2.20)

The electric permittivity is thus given according to Equation (4.2.20) to be

$$\varepsilon = \varepsilon_0(1+\chi) = \varepsilon_0\left[1+\frac{eN}{m\varpi^2 E_0}\left(\frac{MV_0}{\tau} - Bev_0 + KT\nabla n - eE_0\right)\right]. \qquad (4.2.21)$$

The resistance Z can be found by inserting (4.2.21) in (4.2.14) to get:

$$m\varpi^2\varepsilon_0 E_0 + eN\left(KT\nabla n + \frac{mv_0}{\tau} - Bev_0 - eE_0\right) < 0$$

$$kT\nabla n < +Bev_0 + eE_0 - \frac{m\varpi^2\varepsilon_0 E_0}{eN} - \frac{mv_0}{\tau}$$

$$T < +\frac{Bev_0}{k\nabla n} + \frac{(e - m\varpi^2\varepsilon_0)E_0}{eNk\nabla n} - \frac{mv_0}{\tau k\nabla n}$$

$$z = \frac{\hbar\varpi}{2ne^2}\sqrt{\mu\varepsilon\sqrt{1+\frac{eN}{m\varpi^2\varepsilon_0 E_0}\left(kT\nabla n + \frac{mv_0}{\tau} - Bev_0 - eE_0\right)}} \qquad (4.2.22)$$

$$z = \frac{\hbar\varpi}{2ne^2}\sqrt{\mu\varepsilon_0\sqrt{\frac{m\varpi^2\varepsilon_0 E_0 - eN\left(kT\nabla n + \frac{mv_0}{\tau} - Bev_0 - eE_0\right)}{m\varpi^2\varepsilon_0 E_0}}}$$

Thus the critical temperature is given by

$$T_e = \frac{(Be\tau - m)v_0}{\tau k\nabla n} + \frac{(e - m\varpi^2\varepsilon_0)E_0}{eNk\nabla n} \qquad (4.2.23)$$

If the internal field B results from N_0 atoms each having a verge flux density μB then: $B = \mu_B N_0$ (4.2.24)

Therefore T_0 can take the form

$$T_e = \frac{(\mu_B N_0 e\tau - m)v_0}{\tau k\nabla n} + \frac{(e - m\varpi^2\varepsilon_0)E_0}{eNk\nabla n} \qquad (4.2.25)$$

In tight binding model [63] the energy of electrons in the crystal is given by

E +2ycoska $\varepsilon = \varepsilon_0 + a_1 + 2y cos ka$ (4.2.26) where ε_0 is the energy in the absence of crystal field, while the other terms describe the effect of the crystal field. The energy ε_0 can split into two terms the kinetic part which can describe the thermal motion in the form $\frac{f_0}{2}kT$ beside the potential term $-V_0$ for atffactive force or bounded particle.

Thus one can write $$\varepsilon_0 = \frac{\hbar^2 k_0^2}{2m} + \frac{f_0}{2}kT - V_0 \qquad (4.2.27)$$

$$E = \frac{\hbar^2 k_0^2}{2m} + ykT + V \quad \varepsilon_0 = \frac{f_0}{2}kT - V_0 - \alpha_0 \quad \alpha_0 = \frac{\hbar^2 k_0^2}{2m}$$

f_0 represents the degrees of freedom.

The terms describing the effect of the crystal force are

$$\alpha_1 = \langle \emptyset_m | \hat{\mathrm{H}}_{cry} | \emptyset_m \rangle, \qquad y = \langle \emptyset_J | \hat{\mathrm{H}}_{cry} | \emptyset_m \rangle, \qquad \alpha = \alpha_0 + \alpha_1 \qquad (4.2.28)$$

In view of Equations (26) and (27)

$$\varepsilon_0 = \frac{f_0}{2}kT - V_0 + \alpha + 2ycoska \qquad (4.2.29)$$

Here H_{cry} stands for the crystal force Hamiltonian part, while $\emptyset_m$ and $\emptyset_J$ the states of particles located at the site m and j respectively.

The superconductor is characterized by the existence of energy gap. This gap can be under stood here in two ways. If the electrons or holes are not free. This requires E to negative. Thus Equations (4.2.27) and (4.2.26) needs

$$\varepsilon = \frac{f_0}{2}kT - V_0 + \alpha + 2ycoska < 0 \qquad (4.2.30)$$

Or the max value of ε where $coska = -1$ is less than zero, $i.e.$

$$\varepsilon_{max} = \frac{f_0}{2}kT - V_0 + \alpha + 2ycoska < 0 \qquad (4.2.31)$$

$$\frac{f_0}{2}kT \leq V_0 - \alpha + 2y$$

For constant attractive crystal force

$H_{cry} = -V_{cry}$

$\alpha_1 = \langle \emptyset_m | H_{cry} | \emptyset_m \rangle = -\langle \emptyset_m | V_{cry} | \emptyset_m \rangle = -V_{cry}\delta_{mm}$

$$y = \langle \emptyset_J | -V_{cry} | \emptyset_m \rangle = -V_{cry}\langle \emptyset_J | \emptyset_m \rangle = -V_{cry}\delta_{jm=0} \qquad (4.2.32)$$

Thus $\frac{f_0}{2}kT \leq V_0 - a$

Thus the critical temperature is given by

$$\frac{f_0}{2}kT_c = V_0 - \alpha \qquad (4.2.33)$$

Substituted Equation (4.2.33) beside Equation (4.2.32) in Equation (4.2.30) one gets

$$\varepsilon = \frac{f_0}{2}kT - \frac{f_0}{2}kT_c \qquad (4.2.34)$$

The energy Δ s equal to the difference between zero energy in conduction band and the negative energy in the valence band. Thus

$$\Delta = 0 - \varepsilon\frac{f_0}{2}kT_c - \frac{f_0}{2}kT$$

Since this relation holds for $T < T_c$ one can neglect T since it is small to get

$$\Delta = \varepsilon\frac{f_0}{2}kT_c$$

Equation (4.2.30) can also be utilized to get the forbidden energy states which characterizes superconductors, where

$$coska = \frac{\varepsilon - \frac{f_0}{2}kT + V_0 - \alpha}{2y} \qquad (4.2.35)$$

The energy is forbidden when $coska \geq 1$

$$\frac{\varepsilon - \frac{f_0}{2}kT + V_0 - \alpha}{2y} \geq 1 \; \varepsilon - \frac{f_0}{2}kT + V_0 - \alpha \geq 2y$$

$$\frac{f_0}{2}kT + \alpha - \varepsilon - V_0 \leq -2 \quad \frac{f_0}{2}kT \leq \varepsilon + V_0 - 2y - \alpha$$

Thus the critical temperature $\frac{f_0}{2}kT \leq \varepsilon + V_0 - 2y - \alpha$

The forbidden energy is thus related to the critical temperature through the relation $\varepsilon - \frac{f_0}{2}kT_c - V_0 + 2y + \alpha$ (4.2.36)

4.3 Using the Resistance Depending on the Magnetic and Electric Susceptibility to Derive the Equation of the critical temperature

Some authors uses temperature dependent Schrödinger equation to study the effect of magnetic field an so. When the temperature of a conductor approach to the absolute zero, the friction resistance can be ignored[64], if an electron e is induced by an electric field E, then the force on it is given by

$$m\frac{dv}{dt} = eE \tag{4.3.1}$$

Including the position variable x in Equation (4.3.1) it can be written as

$$m\frac{dv}{dx}\frac{dx}{dt} = eE \tag{4.3.2}$$

Then $\int mvdv = \int eEdx$ (4.3.3)

According to the definition of the potential V , we get

$$E = -\frac{dV}{dx} \tag{4.3.4}$$

From Equation (4.3.3) $\frac{mv^2}{2} = e\int \frac{dV}{dx}dx =$ (4.3.5)

Then $v = \frac{2eV}{mv}$ (4.3.6)

While m is constant, and when the potential difference is constant, then the velocity v is being also constant. Using Equation (4.3.6) and substituting the value of v in the equation of current, that given due to the electron velocity v, charges density n, and the area $A, I = nevA$, then the current I is found to be

$$I = \frac{2Ane^2V}{mv} \tag{4.3.7}$$

Then the resistance R is given

$$R = \frac{V}{I} = \frac{V}{nevA} = \frac{Vmv}{Ane[2Ve]} = \frac{mv}{2Ane^2} \tag{4.3.8}$$

On other hand R can be written due to the resistivity ρ, the length l, and the crossection area as $\quad R = \frac{\rho L}{A}$ (4.3.9)

Considering the electron as a wave, its velocity becomes [65]

$$V = \frac{1}{\sqrt{\mu\varepsilon}} \tag{4.3.10}$$

Accordingly the resistivity is given by

$$\rho = \frac{m}{2ne^2L\sqrt{\mu\varepsilon}} \tag{4.3.11}$$

If a magnetic field with a flux density B, an electric force F_e, besides a friction resistance γv , and a pressure

force $\Delta P = \nabla\left(\frac{1}{3}mnv^2\right)$ act together, then the centripetal forces which balance this force is given by [66].

$$\frac{mv_0^2}{r} = Bev_0 + F_e - \gamma v_0 - \frac{m}{3}v_0^2\nabla n \tag{4.3.12}$$

Where v_0 is the radial velocity, while the friction force and the pressure are given by $\quad F_r = \gamma v_{0,}\ \nabla P = \frac{1}{3}mv^2\nabla n$ (4.3.13)

where y is the friction coefficient.

$$m\varpi_0^2 r = B_0 e\varpi_0 r + F_e - \gamma\varpi_0 r\frac{m}{3}\varpi_0^2 r^2\nabla n$$

when the outer magnetic field vanishes, then the radial velocity becomes

$$v_{0=\varpi_0}r \tag{4.3.14}$$

And $$F_e = m\varpi_0^2 r + B_0 e\omega_e r + \frac{m}{3}\varpi_0^2 r^2 \nabla n \tag{4.3.15}$$

where B_0 denotes the ineer magnetic field. And when an outer magnetic field B is applied, then

$$m\frac{dv}{dt} = -\nabla P + F_e + F_r + eB_0 v + Bev \tag{4.3.16}$$

where F_r is the radial force, and F_m , F_e are the magnetic and the electric forces respectively, which are given by

$$F_B = Bev \; and \; F_e = eE \tag{4.3.17}$$

The equation of motion in the presence of the outer magnetic field is given in the form[67] .

$$\frac{mv^2}{r} = -\frac{1}{3}mv^2\nabla n + F_e - \gamma v + B_0 ev + Bev \tag{4.3.18}$$

where v is the radial velocity, and while $v = \varpi r$ then

$$m\omega^2 r = -\frac{1}{3}m\omega^2 r^2 \nabla n + F_e - y\omega r + B_0 e\omega r + Be\omega r$$

$$= \left(-\frac{1}{3}m\omega^2 r^2 \nabla n + F_e - y\omega r + B_0 e\omega r + Be\omega r\right) + m\omega_0^2 r - B_0 e\omega r$$
$$+ y\omega_0 r + \frac{1}{3}\omega_0^2 r\nabla n \tag{4.3.19}$$

When ω is so closed to $\omega_0$then

$$\omega \rightarrow \omega_0 and \; \omega + \omega_0 = 2\omega_0$$

$$\omega - \omega_0 = \Delta\omega = \omega_L$$

where $\omega_L$is Larmar frequency, substitute Equation (4.3.15) and Equation $m(\omega^2 - \omega_0^2)r = -\frac{1}{3}m(\omega^2 - \omega_0^2)r\nabla n - y(\omega - \omega_0)r + B_0 e(\omega - \omega_0)r + Be\omega r$

$$m\left[1+\frac{r}{3}\nabla n\right]r(\omega-\omega_0)(\omega+\omega_0)=-ry\omega_L+B_0e\omega_Lr+Bew_0r$$

$$m\left[1+\frac{r}{3}\nabla n\right]r(2\omega_0)\omega_L=-ry\omega_L+B_0e\omega_Lr+Bew_0r$$

Dividing both sides by $\omega_0 r$ we get

$$\left[2m\left[1+\frac{r}{3}\nabla n\right]+\frac{y}{\omega_0}-\frac{B_0e}{\omega_0}\right]\omega_L=Be \quad (4.3.20)$$

$$\omega_L=\frac{e}{\left[\left[2m\left[1+\frac{r}{3}\nabla n\right]+\frac{y}{\omega_0}-\frac{B_0e}{\omega_0}\right]\right]}B$$

The current for one atom with Z electrons, moving around its nucleus with a frequency f is $i=+Zef=+\frac{Ze}{2\pi}\omega_L$ (4.3.21)

where Z is the atomic number, e is the electron charge, and $\omega_L$is Larmar frequency. The magnetic torque for one atom is given by

$$M_a=iA \quad (4.3.22)$$

where A is the area surrounded by the current which is equal

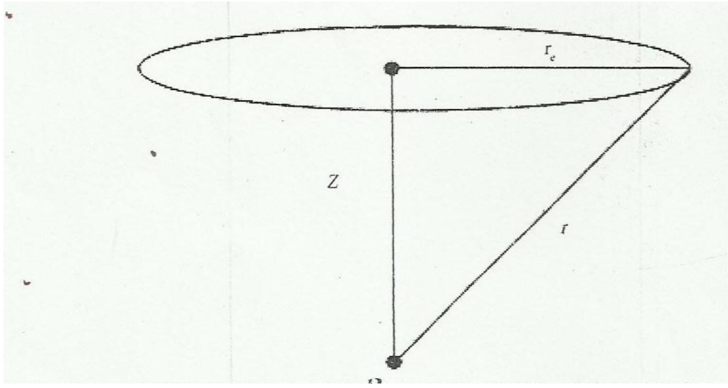
$$A=\pi r_e^2$$

And from Figure l, one get:

$$x=y=z,\ \ r^2=x^2+y^2+z^2,\ \ r^2=3z^2, \therefore z^2=\frac{1}{3}r^2$$

But $r^2=z^2+r_e^2,\ \ \therefore r^2=\frac{1}{3}r^2+r_e^2\ ,\ r_e^2=\frac{2}{3}r^2$ (4.3.23)

So the magnetic torque for one atom M_a becomes



Figure(4.3.1), Magnetic torque in Zdirectiene

$$M_a = \frac{2\pi i r^2}{3}$$

If the number of atoms per unit volume is assumed to be N then, the magnetic torque for the matter is

$$M = NM_a = \frac{2\pi}{3} N r^2 \frac{ze\omega_L}{2\pi} = \frac{Nzer^2}{3}\omega_L \quad (4.3.25)$$

$$M = + \frac{Nzer^2\mu_0}{3\left[2m\left[1+\frac{r}{3}\nabla n\right]+\frac{y}{\omega_0}-\frac{B_0 e}{\omega_0}\right]} H \quad (4.3.26)$$

According to the definition of susceptibility χ_m then[68].

$$M = \chi_m H \quad (4.3.27)$$

Comparing Equations (4.3.26) and (4.3.27) the susceptibility being

$$\chi_m = + \frac{Nzer^2\mu_0\omega_0}{3\left[2m\left[1+\frac{r}{3}\nabla n\right]\omega_0+y-B_0 e\right]} \quad (4.3.28)$$

Then the resistivity in Equation (4.3.11) becomes

$$\rho = \frac{m}{2neL\sqrt{\varepsilon_0\mu}} = \frac{m}{2neL\sqrt{\varepsilon_0\mu\frac{Nzer^2\omega_0}{3\left[\frac{2m}{\hbar}\left[1+\frac{r}{3}\nabla n\right]\frac{1}{2}kT+y-B_0 e\right]}}} \text{ m}$$

where $\hbar\omega_0 = \frac{1}{2}kT$ denotes the photon energy.

The resistivity ρ is imaginary, and the real resistivity vanishes when

$$3\left[\frac{2m}{\hbar}\left[1+\frac{r}{m}\nabla n\right]\frac{1}{2}kT+y-B_0e\right]\leq 0 \quad or\; 3\left[\frac{2m}{\hbar}\left[1+\frac{r}{3}\nabla n\right]\frac{1}{2}kT\right]\leq B_0e-y \quad (4.3.30)$$

Accordingly the critical temperature becomes

$$T_e=\frac{2(B_0e-y)}{3\left[\frac{2m}{\hbar}\left[1+\frac{r}{3}\nabla n\right]\right]k} \tag{4.3.31}$$

Assuming that the charges in the conductor are acted by a resistance force F_r , and a magnetic force F_m , besides the electric force F_e , and then the equation of motion becomes [69].

$$F=F_r+F_m+F_e \tag{4.3.32}$$

The previous forces are given by the formulas

$$F_0=k_0x,\; F_r=\frac{nmv}{\tau} \qquad F_m=Bev,\; F_e=eE$$

where $n, k, x, m, v. e, B, \tau$and E denotes the density, rigidity coefficient, displacement, mass, velocity,
electron charge, magnetic flux density, resolving time, and the electric field respectively.

The equation of motion takes the formula

$$ma=\frac{nmv}{\tau}+Bev+eE \tag{4.3.33}$$

When the electron moves with a uniform constant velocity, the Equation (4.3.33) becomes

$$\left(\frac{nm}{\tau}-Be\right)v=eE,\; v=\frac{e}{\left(\frac{nm}{\tau}-Be\right)}E \tag{4.3.34}$$

And the conductivity is given by $J=n_eev=\frac{n_ee^2}{\left(\frac{nm}{\tau}-Be\right)}\,E$ (4.3.35)

where n_e the electrons density, while n denotes the density of the medium atoms, accordingly the conductivity being

$$\sigma=\frac{n_ee^2}{\left(\frac{nm}{\tau}-Be\right)}\,E=\sigma E \tag{4.3.36}$$

And the conductivity approaches to infinity when

$$\frac{nm}{\tau} - Be = 0 \tag{4.3.37}$$

According to the Maxwell-Boltzmann statistics the density of the atoms in the medium takes the formula [70].

$$n = n_e e^{\frac{E}{kT}} \approx n_0 \left(1 - \frac{E}{kT}\right)$$

Then $$\frac{nm_0}{\tau}\left(1 - \frac{E}{kT}\right) = Be$$

$$\frac{E}{kT} = 1 - \frac{Be\tau}{mn_0} = \frac{mn_0 - Be\tau}{mn_0}$$

$$T_e = \frac{mn_0 E}{k(mn_0 Be\tau)} \tag{4.3.38}$$

Equation (4.3.38) represents the critical temperature in which the conductivity becomes very huge, and when

$$\frac{nm}{\tau} - Be \ll 1 \tag{4.3.39}$$

The conductivity also becomes very high, and then

$$\frac{nm}{\tau}\left[1 - \frac{E}{kT}\right] - Be \ll 1 \;, 1 - \frac{E}{kT} \ll \frac{Be\tau}{mn_0}, \frac{E}{kT} \ll \frac{Be\tau}{mn_0} - 1, \frac{E}{k\left(1 - \frac{Be\tau}{mn_0}\right)} \gg T \tag{4.3.41}$$

And finally the critical temperature is found to be

$$T_C = \frac{mn_0 E}{k(mn_0 - Be\tau)} \tag{4.3.42}$$

4.4 Schrodinger quantum equation from classical and quantum Harmonic Oscillator

Maxwell's equation and plank quantum theory wave used here to relate conductivity to absorption coefficient Maxwell's theory is one many theories that can describe electromagnetic field[71]. The equation of the electric field intensity E inside a polarized medium field

$$-\nabla^2 E + \mu_0 \sigma \frac{\partial E}{\partial \tau} + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial \tau^2} = -\mu_0 \sigma \frac{\partial^2 P}{\partial \tau^2} \qquad (4.4.1)$$

The solution of this equation can be expressed in terms of time attenuation coefficient , wave member K and angular frequency ω, to be in the form [72]

$$E = E_0 e^{-\alpha t} e^{(kx-\omega t)} \qquad (4.4.2)$$

The displacements of charges can be described by:

$$E = E_0 e^{-\alpha t} e^{(kx-\omega t)} = \frac{X_0}{E_0} E \qquad (4.4.3)$$

The electric dipole moment can thus define in terms of charge density:

$$\rho = -en_e X = -en_e X_0 e^{-\alpha t} e^{/(Kx-\omega t)\alpha t} = \frac{n_{ec} X_0}{E_0} E \qquad (4.4.4)$$

Differentiating (4.4.2) with respect to X and t yields:

$$\frac{\partial E}{\partial x} = iKE, \nabla^2 E = \frac{\partial^2 E}{\partial x^2} = -K^2 E$$

$$\frac{\partial E}{\partial x} = (-\alpha - i\omega)E\,, \frac{\partial^2 E}{\partial t^2} = (-\alpha - i\,\omega)^2 \qquad (4.4.5)$$

Differentiating (4.4.2) with respect to t gives:

$$\frac{\partial E}{\partial t} = -e\, n_{elec} = \frac{\partial x}{\partial t} = i - e\, n\omega x$$

$$\frac{\partial^2 E}{\partial x^2} = -e\, n_{elec} \omega^2\, x = e\, n_{elec} \omega^2 x \qquad (4.4.6)$$

Inserting equations (4.4.5)and (4.4.6) in equation (4.4.1) yield:

$$[K^2 E + \mu\varepsilon(-\alpha - \omega)^2 E + \mu\sigma(-\alpha i\omega)E] = \frac{-en\,\omega^2 \mu x_0}{E_0} E$$

$$K^2 + \mu\varepsilon\alpha^2 - \omega(\alpha^2 - \omega^2 + 2i\alpha\omega) - E + \mu\sigma\alpha - \mu\sigma\omega i = \frac{-e\,n\,\omega^2 \mu x_0}{E_0} \qquad (4.4.7)$$

Thus equation (4.4.7) reads:

$$K^2 + \frac{(\alpha^2 - \omega^2 + 2i\alpha\omega)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i \ \frac{-e\ n\ \omega^2\ \mu x_0}{E_0} E = -\frac{\mu\partial^2 P}{t^2}$$

$$K^2 + \frac{(\alpha^2)}{c^2} - \frac{\omega^2}{c^2} + \frac{(2i\alpha\omega)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i = \frac{-e\ n\ \omega^2\ \mu x_0}{E_0}$$

But since the speed of light is given by:

$$\mu\ \varepsilon = \frac{1}{c^2}$$

Thus equation (4.4.7) reads

$$K^2 + \frac{(\alpha^2 - \omega^2 + 2i\alpha\omega)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i \ \frac{-e\ n\ \omega^2\ \mu x_0}{E_0} E = -\frac{\mu\partial^2 P}{t^2} \qquad (4.4.8)$$

$$K^2 + \frac{\alpha^2}{c^2} - \frac{\omega^2}{c^2} + \frac{(2i\alpha\omega)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i = \frac{-e\ n\ \omega^2\ \mu x_0}{E_0}$$

But:

$$K^2 = +\left(\frac{2\pi}{\lambda}\right) = \left(\frac{2\pi f}{\lambda f}\right)^2 = \frac{(\omega^2)}{c^2}$$

Thus:

$$\frac{\alpha^2}{c^2} + \frac{2i\alpha\omega}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i = \frac{-e\ n\ \omega^2\ \mu x_0}{E_0} \qquad (4.4.9)$$

Equation real parts on both sides on (4.4.9)

$$\frac{\alpha^2}{c^2} - \mu\sigma\alpha = \frac{-e\ n_e\ \mu x_0}{E_0}\omega^2 \qquad (4.4.10)$$

Since the speed of light is given large $c \gg \alpha$

Hence:

$$\frac{\alpha^2}{c^2} \to 0 \qquad (4.4.11)$$

Thus the absorption coefficient is given by:

$$\alpha = \frac{-e\, n_{elec}\, x_o}{E_o} \quad \frac{\omega^2}{\sigma} \tag{4.4.12}$$

Expression (4.4.12) can be simplified by using the conductivity expression for direct current:

$$\sigma = \frac{ne^2\tau}{m} \tag{4.4.13}$$

And by using the electron equation under the action of electromagnetic field, where:

$$m\ddot{x} = -\omega_n^2 x_o\, e^{i\omega e} = -e\, x_o e^{i\omega t}$$

Thus:

$$\frac{x_o}{E_O} = \frac{e}{m\,\omega^2} \tag{4.4.14}$$

Thus a direct substitution of (4.4.13) and (4.4.14) in 12) yields:

$$\alpha = \frac{e\, n\, \omega^2 m}{n\, e^2 \tau} \left(\frac{e}{m\,\omega^2}\right) = \frac{1}{\tau} \tag{4.4.15}$$

One can also engorge polarization term in the equation (a) to get:

$$\frac{\alpha^2}{c^2} - \frac{2i\alpha\omega}{c^2} - \mu\sigma\omega i - \mu\sigma\alpha -= 0 \tag{4.4.16}$$

Since for osculating electron the equation of motion is:

$$m\ddot{x} = -\omega_n^2 x_o\, e^{i\omega t} = eE_o e^{i\omega t} - \frac{mv_o e^{i\omega t}}{\tau} \tag{4.4.17}$$

Since $\quad v = dx/dt = i\omega x \quad$ (4.4.18)

$$\left[i\,\omega + \frac{1}{\tau}\right] m\, v = e\, E \tag{4.4.19}$$

Using the relation

$$J = n\, e\, v = \frac{n\, e^{\,2}}{m}\left[i\,\omega + \frac{1}{\tau}\right]^{-1} E = \sigma\, E[\sigma_1 + i\sigma_2]E^2 \tag{4.4.20}$$

Therefore:

$$\sigma_1 = \frac{n\, e^2 (T)^{-1}}{m}[(T)^{-1} + \omega^2]\,, \sigma_2 E = \frac{n\, e^2 \omega}{m}[\tau^{-1} + \omega^2]^{-1} \qquad (4.4.21)$$

Inserting the complex conductivity of (4.4.20) in equation (4.4.16) yields:

$$\frac{\alpha^2}{c^2} - \frac{2i\alpha\omega}{c^2} - \mu(\sigma_1 + i\,\sigma_2)\,\alpha - \mu\,(\sigma_1 + i\,\sigma_2)\omega i = 0 \qquad (4.4.22)$$

Equating imaginary parts one gets:

$$\alpha = \left[\frac{2\,\omega}{c^2} + \mu\,\sigma_2\right] = -\mu\,\sigma_1\,\omega \qquad (4.4.23)$$

Using C^2 is very large, one can neglect the first term in the ???

$$2\,\omega/_{c^2}\ \ .\rightarrow \qquad (4.4.24)$$

To get:

$$\alpha = \frac{-\sigma_1\omega}{\sigma_2} \qquad (4.4.25)$$

In view of in equation (4.4.24). The absorption coefficient (4.4.25) is given by:

$$\alpha = \frac{1}{\tau} \qquad (4.4.26)$$

Consider an electron osculating naturally with frequency $w_0$and affected by the osculating electric field [73,74]

$$E = e\ \ E_0\, e^{i\ \omega t} \qquad (4.4.27)$$

The equation of motion for such electron is given by:

$$m\frac{dv}{dt} = -kx + e\,E - \frac{m\,v}{\tau} \qquad (4.4.28)$$

Where:

$$K = m\,\omega_0^2 \qquad X = x_0\, e^{i\,\omega t} \qquad V = i\omega x \qquad (4.4.29)$$

There for equation (4.4.28) becomes:

$$i\ m\omega v = -m\,\omega_0^2 x + e\,E\ \frac{-m\,v}{\tau} = i\,m\,\omega_0 v\ \frac{-m\,v}{\tau} + e\,E$$

$$\left[i\,(\omega - \omega_0) + \frac{1}{\tau}\right] m\,v = eE \qquad (4.4.30)$$

becomes:

By setting:

$$\omega - \omega_o = \nabla\omega \qquad (4.4.31)$$

Thus:

$$V = \frac{eE}{M\left[i\nabla\omega + \frac{1}{\tau}\right]} = \frac{e\left[\frac{1}{\tau} - i\nabla\omega\right]E}{M\left[i\nabla\omega + \frac{1}{\tau}\right]} \qquad (4.4.32)$$

According to the relation between current density, velocity and conductivity one gets:

$$J = nev = \frac{ne^{2\left[\frac{1}{\tau} - i\nabla\omega\right]}}{M\left[i\nabla\omega + \frac{1}{\tau}\right]} \qquad (4.4.33)$$

Hence the conductivity is given by:

$$\sigma = \frac{ne^{2\left[\frac{1}{\tau} - i\nabla\omega\right]}}{M\left[i\nabla\omega + \frac{1}{\tau}\right]} \qquad (4.4.34)$$

For non-polarized medium, equation (4.4.27) can be solved by assuming:

$$E = E_O e^{i(k\,x - \omega t)}$$

To get:

$$k^2 - \frac{\omega^2}{c^2} + i\mu_0\omega\sigma = 0$$

In view of this equation beside (34) one gets:

$$K^2 - \frac{\omega^2}{c^2} = \mu\sigma\omega i = \frac{\mu_0\omega\left[\nabla\omega + \frac{1}{\tau}\right]}{M\left[i\nabla\omega + \frac{1}{\tau}\right]} ne^2 \qquad (4.4.35)$$

If the wave length in the medium is near to that of free space it follows that:

$$k^2 = \left(\frac{2\pi}{\lambda}\right)^2 = \left(\frac{2\pi f}{\lambda f}\right)^2 = \frac{\omega^2}{c^2} \qquad (4.4.36)$$

As a result:

$$K^2 - \frac{\omega^2}{C^2} = 0 \qquad (4.4.37)$$

Hence using equation (4.4.37) in equation (4.4.35) yields:

$$\omega - \omega_0 = -\Delta\omega = \frac{-i}{\tau}\omega_0 - \omega = \frac{i}{\tau} \qquad (4.4.38)$$

A similar result can be found for polarized material According to equation (4.4.3):

$$X = x_0 e^{i\omega t} \qquad \dot{X} = v = -i\omega x_0 e^{-i\omega t} \qquad V_0 e^{-i\omega t}$$

$$\ddot{X} = - - i\omega v \qquad P = e\, n\, x \qquad (4.4.39)$$

Thus: $\mu \frac{\partial^2 P\, E}{\partial t^2} = \mu\, e\, n\, \ddot{x} = -i\,\omega\,\mu\, n\, e\, v = -i\,\omega\,\mu\, J = -i\,\omega\,\mu\sigma\sigma E$

Thus equation (4.4.8) for α neglected becomes: $K^2 - \frac{\omega^2}{C^2} = \mu\sigma\omega i = 0$

This is typical equation (4.4.35) thus gives again:

$$\omega_0 - \omega = \frac{i}{\tau} \qquad (4.4.40)$$

According to plank hypothesis the original energy and the energy medium are is given by:

$$E_f = E_0 - E = \hbar\,\omega = \left(\frac{\hbar}{\tau}\right) i \qquad (4.4.41)$$

This expression for frictional energy agrees with equation (4.4.26) and (4.4.2)in view or equation (4.4.2) in a resistive medium with the aid of equation(4.4.2) one can write the wave function ψ similar to

E. This is justifiable as far as [75,76]

$E^2\alpha$ Number of photon

$\psi^2\alpha$ Number of particle:

Thus, one can write ψ to be:

$$\psi = A\, e^{\frac{i}{\hbar}}\left(E - \frac{i\hbar}{\tau}\right) t \quad (4.4.42)$$

$$\frac{\partial \psi}{\partial t} = \frac{-i}{\hbar}\left(E - \frac{i\hbar}{\tau}\right)\psi\,, \quad i\hbar\frac{\partial \psi}{\partial t} + \frac{i\hbar}{\tau}\psi = E\psi$$

$$i\hbar\left[\frac{\partial}{\partial t} + \frac{1}{\tau}\right]\psi = E\psi \quad (4.4.43)$$

$$\hat{H}\psi = E\psi \quad (4.4.44)$$

Thus the energy operator takes the form:

$$\hat{H}\psi = E\psi \quad (4.4.45)$$

Using:

$$E = \frac{P^2}{2m} + V \quad (4.4.46)$$

$$E\psi = \frac{P^2}{2m}\psi + V\psi \quad (4.4.47)$$

$$\frac{\partial \Psi}{\partial x} = \frac{i}{\hbar}P\psi$$

In 3 dimensions: $-\hbar^2\nabla^2\Psi = \mathrm{P}^2\Psi$ (4.4.48)

$$i\hbar\left[\frac{\partial}{\partial t} + \frac{1}{\tau}\right]\Psi = \frac{-\hbar^2}{2m}\nabla^2\Psi + V\,\Psi$$

$$i\hbar\frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi - \frac{i\hbar}{\tau}\psi \quad (4.4.49)$$

To find solution for harmonic oscillator, it is important to separate variables Thus one can write the wave function as[77]

$$\psi(r,t) = f\,(t)u(r) \quad (4.4.50)$$

Inserting of equation (4.4.50) in (4.8) yields:

$$\frac{i\hbar\partial f}{f\partial t} = \frac{\hbar}{2mu}\nabla^2 u + V = \frac{i\hbar}{\tau} + E_0 \qquad (4.4.51)$$

Therefore:

$$i\hbar\frac{\partial f}{\partial t} \quad = E_0 f \qquad (4.4.52)$$

The solution or this equation is:

$$f \;=\; A_0\, e^{-i\,B_O t} \qquad (4.4.53)$$

Substituting (4.4.53) in (4.4.52) yields:

$$\hbar\,\beta_O = E_0 \qquad (4.4.54)$$

For Harmonic Oscillator the potential is given by:

$$V = \frac{1}{2}kx^2 \qquad (4.4.55)$$

This equation (4.4.51) reduces to:

$$\frac{-\hbar^2}{2m}\nabla^2 u + \frac{1}{2}kx^2 u = \left(E_0 + \frac{i\hbar}{\tau}\right)u = Eu \qquad (4.4.56)$$

But the energy of harmonic oscilIator is given by:

$$E = E_0 + \frac{i\hbar}{\tau} = \left(N + \frac{1}{2}\right)\hbar\,\omega \qquad (4.4.57)$$

$n = 1,2,3 \ldots ....$

The harmonic Oscillator satisfies periodicity condition, 1 .e:

$$f(t - T) = f(t) \qquad (4.4.58)$$

In view equation (4.4.53) this requires:

$$e^{-i\,B_O t} = \cos\beta_0 T - i\;s\;in\;\;\beta_0 T = 1$$

This means that:

$\cos\beta_0 T - 1$ $\quad\quad\quad$ $\sin \quad \beta_0 T = 0$

Hence: $\beta_0 T = 2\,\pi\, s$ $\quad$ S = 1,2,3…………..

$$\beta_0 = \frac{2\,\pi}{T} S \quad S\,\omega \tag{4.4.59}$$

Inserting (4.4.59) in (4.4.54), the energy is given by:

$$E_O = \hbar\,\omega \tag{4.4.60}$$

This energy IÖSI by friction is thus gives according to equation (4.4.57) and (4.4.60) given by: $E_f = E - E_O \hbar\,\omega\left(N - S + \frac{1}{2}\right)$ (4.4.61)

4.5 Derivation of Klein-Gordon equation from Maxwell's electric wave equation

In the work done by kamil Maxwell's equation were used to derive klein-Gordon equation can be described by Maxwell's equations,where [78,79]

$$\nabla . D = \rho,\ \nabla, B = 0,\ \nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}}, \nabla \times \mathrm{H} = \mathrm{J} + \frac{\partial \mathrm{D}}{\partial \mathrm{T}} \tag{4.5.1}$$

where D, B, E, H and J represent the electric flux density, the magnetic flux density, the electric field and the current density, respectively. Satisfying the following relations, we have

$$B = \mu_0 H, J = \sigma E, D = \varepsilon_0 E + P \tag{4.5.2}$$

where P, μ_0 go and ε_0 are the macroscopic polarization of the medium, the permittivity of free space and the permeability of free space, respectively. Applying the curl operator to both sides of the 3rd equation in (4.5.1), the following equation is obtained.

$$\nabla \times (\nabla \times E) = -\nabla \times \frac{\partial \mathrm{B}}{\partial \mathrm{t}} = -\frac{\partial}{\partial \mathrm{t}}(\nabla \times \mathrm{B}) \tag{4.5.3}$$

Using the identity [80]

$$\nabla\times(\nabla\times E)=\nabla(\nabla.E)-\nabla^2 E \tag{4.5.4}$$

Equation (4.5.3) gives: $\nabla(\nabla.E)-\nabla^2 E=-\frac{\partial}{\partial t}(\nabla\times B)$ (4.5.5)

From (4.5.2) since: $B=\mu_0 H$ (4.5.6)

Then (4.5.5) becomes: $\nabla(\nabla.E)-\nabla^2 E=-\frac{\partial}{\partial t}(\nabla\times\mu_0 H)$ (4.5.7)

From equation (4.5.7), since: $\nabla\times H=J+\frac{\partial D}{\partial t}$ (4.5.8)

From (4.5.1) we have:

$$\nabla(\nabla.E)-\nabla^2 E=-\frac{\partial}{\partial t}\left(\mu_0 J+\mu_0\frac{\partial D}{\partial t}\right) \tag{4.5.9}$$

But: $D=\varepsilon_0 E+P$ (4.5.10)

Therefore: $\nabla(\nabla.E)-\nabla^2 E=-\mu_0\frac{\partial J}{\partial t}-\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}-\mu_0\frac{\partial^2 E}{\partial t^2}$ (4.5.11)

Also: $J=\sigma E$ (4.5.12)

Then $\nabla(\nabla.E)-\nabla^2 E=-\mu_0\frac{\partial J}{\partial t}\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}=-\mu_0\frac{\partial^2 P}{\partial t^2}$ (4.5.13)

The polarization, P , thus acts as a source term in the equation for radiation field [81]

Since: $D=\varepsilon_0 E,\nabla.D=\rho,\rho=0$ (4.5.14)

Therefore: $\varepsilon\nabla.E=\rho=0,\nabla.E=0$ (4.5.15)

Therefore equation (13) becomes:

$$-\nabla^2 E=-\mu_0\frac{\partial J}{\partial t}+\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}=-\mu_0\frac{\partial^2 P}{\partial t^2} \tag{4.5.16}$$

This represents the wave equation for electric field.

Klein-Gordon equation for free particles is usually derived by using Einstein relativistic energy equation:

$$E^2 = P^2c^2 + m_0^2c^4 \tag{4.5.17}$$

where E, p and m_0 are the energy, momentum and rest mass, respectively.

This equation is then multiplied by the wave function ψ to get:

$$E^2\psi = P^2c^2\psi + m_0^2c^4\psi \tag{4.5.18}$$

The energy and momentum terms are replaced by considering the particles as free waves having the wave function:

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \;\; \psi = Ae^{\frac{1}{\hbar}(Px-Et)} \tag{4.5.19}$$

This equation is differentiated with respect to t and x to τ and x to get:

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \; = \hbar^2\frac{\partial^2\psi}{\partial t^2} = E^2\psi$$

$$\frac{\hbar}{i}\frac{\partial\psi}{\partial x} = \frac{\hbar}{i}\nabla\psi = p\psi - \hbar^2\nabla^2\psi = P^2\psi \tag{4.5.20}$$

Inserting (4.5.20) in (4.5.18), the following equation is obtained:

$$- \, i\hbar\frac{\partial\psi}{\partial t} - c^2\hbar^2\nabla^2\psi + m_0^2c^4\psi \tag{4.5.21}$$

This is Klein-Gordon equation.

Maxwell's equation for an electric of field intensity E in a dielectric insulating non-charged medium material of electric dipole moment P is given by equation (4.5.16) to be. $-\nabla^2E + \varepsilon_0\mu_0\frac{\partial^2E}{\partial t^2} = -\mu_0\frac{\partial^2P}{\partial t^2}$ (4.5.22)

Where for non-charged insulating material:

$$\rho = 0, \sigma = 0$$

Where for simplification it is better to consider current density J as a constant, that is: $\frac{\partial J}{\partial t} = 0$ (4.5.23)

The electric dipole moment is given by:

$$P = nq_0x = \frac{Nq_0x}{Ax} = \frac{Q}{A} = \frac{\emptyset}{A} = D = \varepsilon E \quad (4.5.24)$$

Where n is the number density of charge, N is the total number, A is the area and x is the distance.

V= Volume = , Q = Total charge = Nq_0

q_0 = Charge of a single pole according to Gauss law.

The charge Q and total flux $\emptyset$ are related by: $Q = \emptyset$ (4.5.25)

To solve equation (4.5.22), one can assume the electric field intensity in free space E to be. $E = E_0e^{i(kx-wt)}$ (4.5.26)

Thus: $\frac{\partial^2 E}{\partial t^2} = -\omega^2 E \quad \nabla^2 E = -k^2 E$ (4.5.27)

From equations (26) and (24):

$$\frac{\partial^2 E}{\partial t^2} = -\mu_0 \varepsilon \omega^2 E \quad (4.5.28)$$

The speeds in vacuum c and in the medium v are given:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad v = \frac{1}{\mu_0 \varepsilon} \quad (4.5.29)$$

Thus (28) reads:

$$-\mu_0 \frac{\partial^2 P}{\partial t^2} = -\mu_0 \varepsilon \omega^2 E = -\frac{1}{v^2}\omega^2 E = -\left(\frac{2\pi f}{f\lambda_m}\right) E = -{k_m}^2 E \quad (4.5.30)$$

Inserting (4.5.27) and (30) in (4.5.22) yields:

$$k^2 - \frac{\omega^2}{c^2} = -{k_m}^2 \quad (4.5.31)$$

Multiplying both sides byc^2 and $\hbar^2$, one gets:

$$c^2\hbar^2k^2 - \hbar^2\omega^2 = c^2\hbar^2{k_m}^2 \quad (4.5.32)$$

Using De Broglie and Plank hypotheses:

$$P = \frac{h}{\lambda} = \hbar k \quad E = hf = \hbar \tag{4.5.33}$$

Equation (4.5.32) can thus be given by:

$$c^2 P^2 + c^2 {P_m}^2 = E^2 \tag{4.5.34}$$

Since the electromagnetic waves can be assumed as a photon moving with the speed of light c, the photon momentum rest mass mo is given by:

$$\hbar k_m = P_m = m_0 c \tag{4.5.35}$$

Here the rest mass is assigned to a medium since the medium lower photon speed and it can even stop it when it is absorbed. Thus inserting (4.5.35) in (4.5.34) yields: $c^2 P^2 = {m_0}^2 c^4 = E^2$ (4.5.36)

This is the Einstein expression that relates momentum to energy. The derivation of this relation can be done by using the classical equation of energy and Plank hypothesis only. The classical energy for an electromagnetic wave photon oscillating particle with maximum velocity is given by:

$$E = \frac{1}{2} m {v_m}^2 \tag{4.5.37}$$

Since for waves or any harmonic system, the root mean square (r. m. s) velocity v_{rmx} is given by: $v_{rmx} = \frac{1}{\sqrt{2}} v_m$ (4.5.38)

By assuming the photon speed c equal to the r. m. s speed, that is:

$$c = \frac{1}{\sqrt{2}} v_m \tag{4.5.39}$$

It follows that: $E = m \left(\frac{v_m}{\sqrt{2}} \right) = mc^2$ (4.5.40)

According to Plank theory: $E = hf = \frac{hc}{\lambda} = mc^2$ (4.5.41)

Therefore, the momentum p is given by:

$$P = mc = \frac{mc^2}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda} \tag{4.5.42}$$

The Klein-Gordon equation can be obtained by replacing the electric dipole moment term in equation (4.5.17) by the term standing for photon rest mass in equation (4.5.30) to get:

$$-\nabla^2 E + \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = -{k_m}^2 E \tag{4.5.43}$$

Multiplying both sides by $c^2\hbar^2$ and using equation (4.5.29), the following equation is obtained:

$$-c^2\hbar^2\nabla^2 E + \hbar^2 \frac{\partial^2 E}{\partial t^2} = c^2\hbar^2{k_m}^2 E \tag{4.5.44}$$

According to relation (4.5.35): $\quad {P_m}^2 = \hbar^2{k_m}^2 = {m_0}^2c^2$

Thus (4.5.44) reads: $\quad -c^2\hbar^2\nabla^2 E + {m_0}^2c^4 E = -\hbar^2 \frac{\partial^2 E}{\partial t^2}$ (4.5.45)

The incorporation of mass for photon in Maxwell's equations corresponds to adding the term $m_0 A^\mu A_\mu$ to the electromagnetic field lagrangian.

Since in the electromagnetic (e. m) theory the oscillating electric wave E is related to its e. m, the energy or intensity is obtained according to the relation:

$$I \propto c\varepsilon_0 E^2 \tag{4.5.46}$$

And since the e. m intensity, when treated as a stream of photons of density n is given by: $\quad I \propto nhf \ \propto |\psi|^2 hf$ (4.5.47)

Where the photon density is related to the wave function ψ according to the relation: $\quad n = |\psi|^2$ (4.5.48)

Comparing (46) and (47) it follows that:

$$E^2 \Leftrightarrow |\psi|^2 \quad E \Leftrightarrow \psi \tag{4.5.49}$$

Thus the correspondence between E and ψ secure the replacement of E by ψ in equation (45) to get:

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -c^2 \hbar^2 \nabla^2 \psi + {m_0}^2 c^4 \psi \qquad (4.5.50)$$

This represents Klein-Gordon equation for free electron.

4.6 Phase effect between the Electric Internal Current Field and the External Current Field on Amplification of the total Field and Intensity of the Electromagnetic Radiation

The behavior of electromagnetic field inside any medium was tackled also by some authors [82]The equation of motion of the electron of mass m and velocity v under the action of electric force eE and resistive force γv can be described by the equation of motion[83]

$$m \frac{dv}{dt} = eE - \gamma v \qquad (4.6.1)$$

Where e is the electron charge ,E is the electric field intensity , and ɣ is the resistance coefficients. To solve equation (4-2-1) consider the velocity v and the electric field intensity E to be in the form:

$$\mathrm{V} = \mathrm{V}_0 \sin(\omega t + \emptyset) \qquad (4.6.2)$$

$$\mathrm{E}=\mathrm{E}_0\mathrm{E} = \sin \omega t \qquad (4.6.3)$$

Where $\emptyset$ is the phase , V_0 and E_0 are velocity and electric intensity amplification respective while ω is the angular frequency . but the current density is given by

$$J = env \qquad (4.6.4)$$

Where n is electron density Substituting (2)in (4) yields

$$J = en_{\circ} v_0 \sin(\omega t + \phi) \qquad (4.6.5)$$

$$J = env_{0\circ}(\cos\phi \sin\omega t + \sin\phi \cos \omega t) \qquad (4.6.6)$$

Looking at (4.6.6), it is clear that the phase ϕ between E and $v$produces an additional oscillating cosine term.Therefore the definition of conductivity is terms of J and E should include two terms one corresponds to the external field as in (4.6.3) , beside .an additional term which may be defined to be representing an internal field E_m in the form

$$E_m = E_{m_0} cos\omega t \qquad (4.6.7)$$

WhereE_{m_0} is the medium field intensity amplitude

Thus J can be written in terms of

$$J = \sigma_1 E + \sigma_2 E_m \qquad (4.6.8)$$

By substituting the value of $E_m$from equation(4.6.7)and equation (4.6.8) we find;

$$\sigma_1 E_0 \sin\omega t + \sigma_2 E_0 \cos\omega t \qquad (4.6.9)$$

Comparing the coefficients of sinωɯt and cos ωt in (4.6.7)and (4.6.9) yields.

$$nev_0 \cos\phi = \sigma_1 E_0$$

$$nev_0 \sin\phi = \sigma_2 E_m \qquad (4.6.10)$$

where $E_m$is the medium internal filed thus the external and internal field conductivities$\sigma_1$andσ_2 are given by

$$\sigma_1 = \frac{nev_0}{E_0} \cos\phi$$

$$\sigma_2 = \frac{nev_0}{E_m} \sin\phi \qquad (4.6.11)$$

This last one is an imaginary conductivity.

to see how the phase angle ϕ between E and v affect the amplification of electromagnetic radiation , it is important to utilize the expression for the light intensity I which penetrates a distance z inside a medium , which is given by[84]

$$I = I_{\circ}\, e^{\beta z} \qquad (4.6.12)$$

Where$I_0$is the initial light intensity, β is the amplification factor it is given by [85]

$$\beta = \frac{\mu\, c\, \sigma_1}{n_1} \qquad (4.6.13)$$

with μ , c ,n_1 , standing for the magnetic permeability , speed of light and the refractive index ., inserting (4.6.11) in (3)gives

$$\beta = \frac{\mu\, c}{n_1}\frac{nev_0}{E_0}\cos\phi \qquad (4.6.14)$$

it is clear from this relation that amplification takes place when ϕ vanishes $i.e$

$$\beta = \frac{\mu\, c}{n_1}\frac{nev_0}{E_0} \qquad (4.6.15)$$

when $\phi = 0$

but if the velocity v and E are normal to each other . $i - e$when

$$\phi = 90\beta = 0 \qquad (4.6.16)$$

no amplification or absorption takes place ,however ,when v and E are out of phase by

$$\phi = 180 = \pi$$

in this case equation (16) reads

$$\beta = -\frac{\mu\, c}{n_1}\frac{nev_0}{E_0} \qquad (4.6.17)$$

The current density can be explained by associating with v and internal filed $.E_i$ the conductivity defined by

$$J = nev = \sigma_0 E_i \tag{4.6.18}$$

So $E_i = \frac{ne}{\sigma_0} v = \frac{ne}{\sigma_0}{}_{\circ} v_0 sin(\omega t + \phi)$

$$\mathrm{E_i} = \mathrm{E}_{i_0} \sin(\omega t + \phi) \tag{4.6.19}$$

Where$\mathrm{E}_{i_0}$is the initial field intensity amplitude

E_i and E are in phase when $\phi = o$ .thus using the lows of vectors the total filed E_T in the direction of E is given by

$$\mathrm{E_T} = (\mathrm{E_0} + E_{i0})\ s\,in(\omega t) + (E_{\mathrm{T0}} \sin(\omega t) \tag{4.6.20}$$

WhereE_{T_0}total field intensity amplitude

And the total intensity of the electromagnetic radiation is given by

$$I = (E_0 + E_{i0})^2 \tag{4.6.21}$$

Thus the total field increases and amplification takes place ,,which is consistent with (4.6.18)and(4.6.21) .But if V and E are out of phase by 90 .1.e $\Phi = 90$ in this case (4.6.19) becomes

$$E_i = E_{i_0} \sin(\omega t + 90) = E_{i_0}[\sin \omega t \cos 90 + \cos \omega t \sin 90]$$

$$E_i = E_{i_0} \cos \omega t \tag{4.6.22}$$

Thus no component of $E_i$in the E direction According to the law of vectors the total field in direction of E is given by

$$E_{T_0} = (E_0 + E_{i0}) = E_0 + 0 = E_0 \tag{4.6.23}$$

Substituting (4.6.20)in (4.6.23)one be found that

$$E_T = E_{T_0} \sin \omega t = E_0 \sin \omega t \tag{4.6.24}$$

And I = $E_{T_0}{}^2 = E_0{}^2 = I_0$ (4.6.25)

Thus the light passes without attenuation on amplification This result is again in agreement with equation (4.6.18) and (4.6.22) , which indicates that

Φ =90, $\beta = 0$, $I = I_0$ (4.6.26)

But when E_i and E are out of phase by π , in this case the resultant filed in the x direction is given by

$E_T = E + E_{i_0} = E_0 \ \sin\omega t + \ E_{i_\circ} \sin(\omega t + \pi)$ (27)

$= E_0 \ \sin\omega t + \ E_{i_0}[\sin\omega t \cos\pi + \cos\omega t \sin\pi]$

$= E_0 \ \sin\omega t - \ E_{i_0} \sin\omega t$ (4.6.28)

By using equation (4.6.24),equation(4.6.28) become;

$E_{T_0} \sin\omega t = \left(E_0 \ - E_{i_0}\right) \sin\omega t$ (4.6.29)

Divided two side of equation (4.6.29)$\sin\omega t$

$E_{T_0} = E_0 \ - E_{i_0}$ (4.6.30)

Thus the total field decreases $i.e$

$E_{T_0} < E_0$, $E^2{}_{T_0} < E^2{}_0$, $I < I_0$ (4.6.31)

This is absorption takes place , this is again is consistent with (4.6.18) and (4.6.23) where

$I = I_\circ e^{-\alpha z}$ (4.6.32)

4.7 Derivation of Maxwell's Equation for Diffusion Current and Klein-Gordon Equation beside New Quantum Equation Form Maxwell's Equation for Massive Photon

Mohamed. I used also Maxwell's equation to derive Klein-Gordon equation.

From Maxwell's equation [86] $\nabla \times H = J + G$ (4.7.1)

Taking into account diffusion effect the equation of continuity takes the form

$$\nabla . J + \frac{\partial \rho}{\partial t} - \frac{\partial \rho_h}{\partial t} + c_d \nabla^2 \rho = 0 \quad (4.7.2)$$

The current density J is assumed to result from external ohmic fieldJ_O, beside bounded charge j b and diffusion process J_d

$$J = J_0 + J_b + J_d \quad (4.7.3)$$

Where $J_0 = \frac{-\partial D}{\partial t}$

$$\Longrightarrow \nabla . J_0 = -\frac{-\partial}{\partial t}(\nabla . P) \quad = \frac{-\partial}{\partial t} \quad (4.7.4)$$

$$J_b = \frac{-\partial D}{\partial t}$$

$$\Longrightarrow \nabla . J_b = -\frac{-\partial}{\partial t}(\nabla . P) \quad = \frac{-\partial \rho_b}{\partial t} \quad (4.7.5)$$

$$J_d = c_d \nabla \rho$$

$$\Longrightarrow \nabla . J_d = -c_d \nabla^2 \rho \quad (4.7.6)$$

Thus the divergence of both sides of equation (4.7.3) gives

$$\nabla . J = \nabla . J_0 + \nabla . J_b + \nabla . J_d \quad (4.7.7)$$

In view of equations (4.7.4) , (4.7.5) and (4.7.6)

$$\nabla . J = -\frac{\partial \rho}{\partial t} + \frac{\partial \rho_b}{\partial t} - c_d \nabla^2 \rho \quad (4.7.8)$$

By rearranging the above equation

$$\nabla . J = +\frac{\partial \rho}{\partial t} - \frac{\partial \rho_b}{\partial t} - c_d \nabla^2 \rho = 0 \tag{4.7.9}$$

To find the unknown G, one uses

$$\rho = \nabla . D = \varepsilon . \nabla . E \tag{4.7.10}$$

$$\rho_b = -\nabla . P \tag{4.7.11}$$

Taking the divergence of equation (4.7.1), one have

$$\nabla . \nabla \times H = 0$$

$$\nabla . \nabla \times H = \nabla . J + \nabla . G = 0 \tag{4.7.12}$$

Insert equation (4.7.12) in (4.7.8) yields

$$-\frac{\partial \rho}{\partial t} - \frac{\partial \rho_b}{\partial t} - c_d \nabla^2 \rho = -\nabla . G \tag{4.7.13}$$

Using equation (4.7.10) and (4.7.11) yields

$$-\frac{\partial \rho}{\partial t}(\nabla . D) + \frac{\partial \rho}{\partial t}(-\nabla . P) - c_d \nabla . (\nabla \rho) = -\nabla . G \tag{4.7.14}$$

But $\quad \nabla . D = \rho$

Thus $\quad \nabla \rho = \nabla . (\nabla . D) \qquad (4.7.15)$

Using relations (4.7.10) and (4.7.15) yields

$$-\frac{\partial}{\partial t}(\nabla . \varepsilon E) + \frac{\partial}{\partial t}(-\nabla . P) - c_d \nabla . (\nabla(\nabla . D)) = -\nabla . G$$

$$-\frac{\partial}{\partial t}(\nabla . \varepsilon E) + \frac{\partial}{\partial t}(-\nabla . P) - c_d \nabla . (\nabla(\nabla . \varepsilon E)) = -\nabla . G$$

$$-\varepsilon \nabla . \frac{\partial E}{\partial t} - \nabla . \frac{\partial \rho}{\partial t} - \varepsilon c_d \nabla . (\nabla(\nabla . E)) = -\nabla . G$$

Comparing both sides of above equations yields

$$\varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla (\nabla . E) = G$$

$$G = \varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla (\nabla . E) \qquad (4.7.16)$$

Thus from equation (I) and the fact that $J = \sigma_0 E$, $\nabla \times H = J + G$

$$\nabla \times H = \sigma_0 E + \varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla (\nabla . E) \qquad (4.7.17)$$

Also from Maxwell's equations we have

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times \nabla \times E = -\mu \frac{\partial (\nabla \times H)}{\partial t} \qquad (4.7.18)$$

From equation (4.7.16) and (4.7.1) one found that

$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d \nabla (\nabla . E) \qquad (4.7.19)$$

Multiplying both sides of equation (4.7.19) by μ and differentiate over time t yields

$$\mu \frac{\partial}{\partial t} (\nabla \times H) = \mu \frac{\partial J}{\partial t^2} + \mu \frac{\partial^2 E}{\partial t} + \varepsilon \mu c_d \nabla \left(\nabla . \frac{\partial E}{\partial t}\right) \qquad (4.7.20)$$

But $J = \sigma E$ (4.7.21)

$$\mu \frac{\partial}{\partial t} (\nabla \times H) = \mu \sigma \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} + \mu \frac{\partial^2 P}{\partial t^2} + \varepsilon \mu c_d \nabla \left(\nabla . \frac{\partial E}{\partial t}\right) \qquad (4.7.22)$$

Also we have $\nabla \times \nabla \times E = -\nabla^2 E + \nabla (\nabla . E)$ (4.7.23)

From equations (4.7.23), (4.7.22) and (18) yields

$$-\nabla^2 E + \nabla(\nabla . E) = \mu \varepsilon \frac{\partial^2 E}{\partial t^2} + \mu \sigma \frac{\partial E}{\partial t} + \mu \frac{\partial^2 E}{\partial t^2} + \varepsilon \mu c_d \nabla(\nabla . \frac{\partial E}{\partial t}) \qquad (4.7.24)$$

From Maxwell's equation

$$-\nabla^2 E + \mu \sigma \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} + \mu \frac{\partial^2 P}{\partial t^2} + \frac{m^2 c^2}{\hbar^2} E = 0 \qquad (4.7.25)$$

Neglecting polarization effect and considering the propagation in free space where $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$ (4.7.26)

$$\mu_0\varepsilon_0 = \frac{1}{c^2} \qquad (4.7.27)$$

Where c is speed of light Equation(4.7. 25) reduce to

$$-\nabla^2 E + zero + \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} + zero + \frac{\mathrm{m}^2\mathrm{c}^2}{\hbar^2} E = 0 \qquad (4.7.28)$$

$$-\nabla^2 E + \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} + \frac{\mathrm{m}^2\mathrm{c}^2}{\hbar^2} E = 0$$

$$\hbar^2 \left(-\nabla^2 E + \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}\right) + m^2c^2E = 0 \qquad (4.7.29)$$

inserting equation (4.7.27) in (4.7.29) , one gets

$$-\hbar^2\nabla^2 E + \hbar^2 \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} + m^2c^2E = 0$$

Multiplying both sides of above equation byc^2

$$-\hbar^2c^2\nabla^2 E + \hbar^2 \frac{\partial^2 E}{\partial t^2} + m^2c^4E = 0 \qquad (4.7.30)$$

If the rest mass equals the relativistic mass,[87] when no potential exist then,

$$m = m_0\left(1 - \frac{v^2}{c^2} + \frac{2\phi}{c^2}\right)$$

$$= m_0(1 - \frac{v^2}{c^2})$$

When $v \ll<$

Thus equation (4.7.30) reduces to

$$m = m_0 \qquad (4.7.31)$$

$$-\hbar^2 \frac{\partial^2 E}{\partial t^2} = -\hbar^2c^2\nabla^2 E + m^2c^4E \qquad (4.7.32)$$

Replacing E by Ψ in equation (4.7.32), one gets

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \Psi + m^2 c^4 \Psi \qquad (4.7.33)$$

This is the ordinary Klein-Gordon Equation

Schrodinger equation deals only with non-relativistic particles, thus it does not take into account the rest mass energy. On contrary Klein-Gordon equation can account for rest mass energy but does not have potential energy term for fields other than electromagnetic fields. Thus there is a need to find a new quantum equation that accounts for rest mass energy, beside potential energy. This can be done with the aid of equation (4.7.25), where one uses the mass expression of the generalized special relativity which is given by:

$$m = m_0 \left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right) \frac{1}{2} \qquad (4.7.34)$$

$$m^2 = {m_0}^2 \left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)$$

$$m^2 = {m_0}^2 + 2m_0 \left(\frac{m_0 \varphi}{c^2}\right) - \frac{{m_0}^2 v^2}{c^2} \qquad (4.7.35)$$

But we have

$$m_0 \varphi = V$$

$$m_0 v = V$$

Substituting equation (4.7.37) and (4.7.36) in (4.7.35), one gets

$$m^2 = {m_0}^2 + 2m_0 \frac{V}{c^2} - \frac{P^2}{c^2} \qquad (4.7.38)$$

Multiplying both sides of equation (38) by Ec^4

$$m^2 c^4 E = {m_0}^2 E + 2m_0 c^2 V E - P^2 c^2 E \qquad (4.7.39)$$

But for oscillating electric field

$$E = E_0 e^{i(kx - \omega t)}$$

$$\frac{\partial E}{\partial x} = ikE_0 e^{i(kx-\omega t)}$$

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} = i^2 k^2 E_0 e^{i(kx-\omega t)}$$

$$\nabla^2 E = -k^2 E$$

$$\hbar^2 \nabla^2 E = -\hbar^2 k^2 E$$

$$\hbar^2 \nabla^2 E = -p^2 E \qquad (4.7.40)$$

Thus equation (4.7.39) becomes

$$m^2 c^4 E = {m_0}^2 c^4 E + 2m_0 c^2 VE - c^2 \hbar^2 \nabla^2 E \qquad (4.7.41)$$

By using the identity $\mu\varepsilon = \frac{1}{c^2}$ and inserting equation (4.7.41) in equation (4.7.25)

$$-c^2\hbar^2\nabla^2 E + -c^2\hbar^2\mu\sigma\frac{\partial E}{\partial t} + \hbar^2\frac{\partial^2 E}{\partial t^2} + {m_0}^2 c^4 E + 2m_0 c^2 VE - c^2\hbar^2\nabla^2 E = 0$$

Replacing E by ψ and collecting similar terms leads to the new quantum equation of the form

$$-c^2\hbar^2\nabla^2\psi + -c^2\hbar^2\mu\sigma\frac{\partial \psi}{\partial t} + \hbar^2\frac{\partial^2 \psi}{\partial t^2} + {m_0}^2 c^4 \psi + 2m_0 c^2 V\psi - c^2\hbar^2\nabla^2\psi = 0 \qquad (4.7.42)$$

4.8 summary and critique

In all attempts mentioned in this paper the properties of bulk matter in relation to the electric field is studied . some of them find is so resistance vanishing conditions, while others tries to link electric field to the quantum behavior of the system .other attempts tries to relate lasing condition to the relation between external and magnetic field. Unfortunately none of them tries to study the real nature of internal medium electric field and its role in the resistance and conductivity of the medium.

Chapter five

The nature of Internal Field In relation to conductivity and Light Amplification

5.1 introduction:

This chapter is concerned with the relation between the velocity and current generated by an electric field which exerts force an electron It also discusses the relation between the electric field generated by an alternating current or an oscillating electron with periodic velocityalso Electric conductivity by using RLC circuits Relations and using Effective values and using complex Representation.[88]

5.2 the electric field generated by electric current

The generation of the electric field is described by Maxwell's equations which takes the form[89]

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t} = \mu \frac{\partial H}{\partial t}$$

$$\overrightarrow{\nabla} \times \underline{H} = J + \frac{\partial D}{\partial t} = \sigma E + \epsilon \frac{\partial E}{\partial t} \qquad (5.2.1)$$

Where $\underline{B}$, $\underline{H}$, $\underline{J}$,and $\underline{D}$ stands for magnetic flux density magnetic field intensity , current density and electric field intensity respectively ,while μ and ϵ, σ represents magnetic permeability ,electric permittivity and electric conductivity where

$$\underline{D} = \epsilon\, \underline{E} ,\quad B = \mu\, \underline{H}$$

The electric charge density generated the electric field according the relation

$$\overrightarrow{\nabla} . D = \rho = ne, \qquad \overrightarrow{\nabla} . B = 0$$

$$\underline{J} = \sigma \underline{E} = nev = \rho v \qquad (5.2.2)$$

Hence n stand for the number of electrons per unit volume .

The magnetic flux density can be defined in terms of a vector potential to be

$$\underline{B}=\vec{\nabla}\times A \qquad (5.2.3)$$

In view of (5.2.1) , the curl of $\underline{E}$ become

$$\vec{\nabla}\times E=-\vec{\nabla}\times\frac{\partial A}{\partial t}$$

Thus

$$\vec{\nabla}\times(\underline{E}+\frac{\partial A}{\partial t})=0$$

Hence one can define another scalar potential V to be

$$\underline{E}+\frac{\partial A}{\partial t}=-\vec{\nabla}V$$

i.e.

$$\underline{E}=-\frac{\partial A}{\partial t}-\vec{\nabla}V \qquad (5.2.4)$$

A is called magnetic vector potential , while V is multiplying (5.2.1) by μ and using (5.2.2) yields

$$\vec{\nabla}\times\mu H=\mu J+\mu\epsilon\frac{\partial E}{\partial t}$$

$$\vec{\nabla}\times\underline{B}=\mu J+\mu\epsilon\frac{\partial \mathrm{E}}{\partial t}$$

Using (5.2.3) the above equation become :

$$\vec{\nabla}\times\vec{\nabla}\times A=\mu\epsilon\frac{\partial E}{\partial t}+\mu J \qquad (5.2.5)$$

This expression can be simplified by using the vector identity[90]

$$\vec{\nabla}\times\vec{\nabla}\times A=\vec{\nabla}(\vec{\nabla}.\underline{A})-\nabla^2\underline{A} \qquad (5.2.6)$$

To get

$\overrightarrow{\nabla}(\overrightarrow{\nabla}.\underline{A})$ $-\nabla^2\underline{A}$ $= -\mu\epsilon\left[\frac{\partial^2 A}{\partial t^2} + \overrightarrow{\nabla}\left(\frac{\partial V}{\partial t}\right)\right] + \mu J$

$\nabla^2\underline{A}$ $= \mu\epsilon\frac{\partial^2 A}{\partial t^2} + \overrightarrow{\nabla}\left[(\overrightarrow{\nabla}.A) + \mu\epsilon\frac{\partial V}{\partial t}\right] - \mu J$ (5.2.7)

This expression can be simplified by selecting the following gauge

$$\overrightarrow{\nabla}.A + \mu\epsilon\frac{\partial V}{\partial t} = 0 \quad (5.2.8)$$

As a result equation (5.2.7) reduce to

$$\nabla^2\underline{A} - \mu\epsilon\frac{\partial^2 A}{\partial t^2} = -\mu J \quad (5.2.9)$$

This equation can solved by assuming A to be in the form

$$A = A_0 sin\,(kx - \omega t) \quad (5.2.10)$$

Differentiating worth x ant t, one gets

$$\nabla^2\underline{A} = -k^2 A \qquad \frac{\partial^2 A}{\partial t^2} = -\omega^2 A \quad (5.2.11)$$

Inserting (5.2.10) and (5.2.11)in (5.2.9) yields

$$(-k^2 + \mu\epsilon\omega^2)A = -\mu J$$

$$\mu J = (k^2 - \mu\epsilon\omega^2)A \quad (5.2.12)$$

The current density is becomes

$$\mu J = A_0(k^2 - \mu\epsilon\omega^2)sin\,(kx - \omega t) \quad (5.2.13)$$

One can re write J in the form

$$\mu J = J_0 sin\,(kx - \omega t) \quad (5.2.14)$$

Where

$$J_0 = \frac{A_0}{\mu}(k^2 - \mu\epsilon\omega^2) \quad (5.2.15)$$

On the other hand the current density can be defined in terms of the velocity v in the form

$$J = nev$$

Where n stands for the number density of electrons. hence

$$\underline{V} = \frac{J}{ne} = \frac{J_0}{ne} sin\,(kx - \omega t) \qquad (5.2.16)$$

The velocity $\underline{v}$ can thus be rewritten in the form

$$\underline{v} = v_0 \sin(kx - \omega t) \qquad (5.2.17)$$

where

$$v_0 = \frac{J_0}{ne}$$

Substituting (5.2.10) in (5.2.4) yields

$$\underline{E} = -\frac{\partial A}{\partial t} - \overrightarrow{\nabla} V \qquad (5.2.18)$$

$$E = +\omega A_0 \cos(kx - \omega t) - \overrightarrow{\nabla} V \qquad (5.2.19)$$

Utilizing (5.2.8) and (5.2.10) yields:

$$\mu\epsilon \frac{\partial V}{\partial t} = -\overrightarrow{\nabla} . \underline{A}$$

$$\mu\epsilon \frac{\partial V}{\partial t} = -kA_0 \cos(kx - \omega t) \qquad (5.2.20)$$

Equation (5.2.20) can be solved by suggesting v to be in the form

$$V = V_0 \sin(kx - \omega t) \qquad (5.2.21)$$

thus (5.2.20) reads

$$-\mu\epsilon\omega V_0 \cos(kx - \omega t) = -kA_0 \cos(kx - \omega t)$$

This requires V_0 to be in the form

$$V_0 = \frac{kA_0}{\mu\epsilon\omega} \tag{5.2.22}$$

Inserting (5.2.21) in (5.2.19) yields

$$E = \omega A_0 \cos(kx - \omega t) - kV_0 \cos(kx - \omega t) \tag{5.2.23}$$

Hence the electric field generated by the current density J becomes

$$E = E_0 \cos(kx - \omega t) \tag{5.2.24}$$

Thus the electric field E in equation (5.2.24) generated by the current J in equation (5.2.16) due to the motion of charges with velocity v in equation (5.2.17) Indicates that the electric field E 90^0 out of phase w.r.t to J and v.

This is confirmed by the fact that when :

$$v = v_0 \sin(kx - \omega t) \quad see \quad (5.2.23)$$

$$m\frac{\partial v}{\partial t} = eE$$

$$-mwv_0 \cos(kx - \omega t) = eE$$

Thus $\qquad E = \frac{-mwv_0}{e} \cos(kx - \omega t)$

This can be explained also by circular motion= $E_0 \cos(kx - \omega t)$

Where $v = v_0 \sin(\omega t)$

$$a = \frac{\partial v}{\partial t} = wv_0 \cos(\omega t)$$

$$F = eE = ma = mwv_0 \cos(\omega t)$$

The acceleration and force are perpendicular to v

5.3 The electric conductivity of Direct and alternating current

To find the conductivity of any material consider a electron v in an alternative medium of coefficient[91]

$$m\frac{dv}{dt} = -eE - \gamma v \tag{5.3.1}$$

Far steady state flow the velocity v is constant and the acceleration vanishes i.e.

$$\frac{dv}{dt} = 0 \tag{5.3.2}$$

Thus (5.3.1) reads

$$\gamma v = eE \qquad v = \frac{eE}{\gamma}$$

Hence the current density is given by

$$J = nev = \frac{ne^2E}{\gamma} = \sigma E \tag{5.3.3}$$

Thus the conductivity is given by

$$\sigma = \frac{ne^2}{\gamma} \tag{5.3.4}$$

In the case when the acceleration does not vanish ,when E is oscillation the equation of motion is given by

$$m\frac{dv}{dt} = -eE_0 \sin\omega t - \gamma v \tag{5.3.5}$$

The solution of this equation ,thus is given by assuming v to be

$$v = v_0 \sin(\omega t + \emptyset)$$

$$= v_{0_1} \sin\omega t + v_{0_2} \cos\omega t \tag{5.3.6}$$

$$v_{0_1} = v_0 \cos\emptyset \; , v_{0_2} = v_0 \sin\emptyset$$

The current density J is given by

$$J = nev$$

$$= ne(v_{0_1} \sin \omega t + v_{0_2} \cos \omega t) \quad (5.3.7)$$

The conductivity is defined by in terms of J and E to be

$$J = \sigma E = \sigma E_0 \sin \omega t \quad (5.3.8)$$

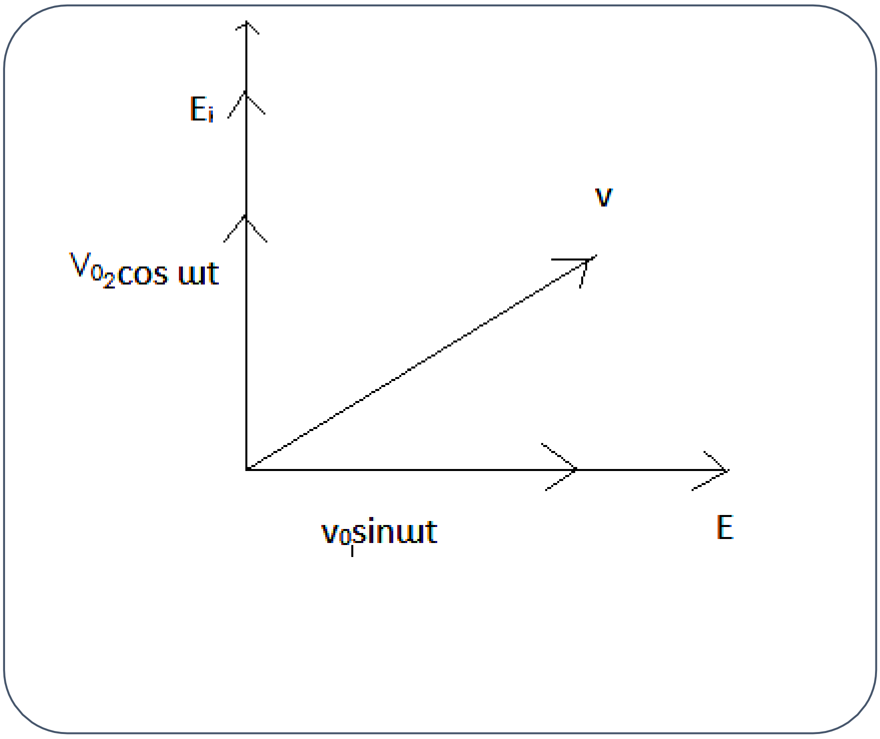
Which is not consistent with relation (5.3.7) thus one needs to redefine the conductivity to be in the form

$$J = ne(v_{0_1} \sin \omega t + v_{0_2} \cos \omega t)$$

$$J = \sigma_1 \mathrm{E}_0 \sin \omega t + \sigma_2 E_{0_i} \cos \omega t$$

$$J = \sigma_1 E + \sigma_2 E_i \quad (5.3.9)$$

This indicates the existence of internal electric field , which results in a velocity of the form $v = v_0 \sin(\omega t + \emptyset)$ (5.3.10)



Fig(5.1) external field E and the internal field E_i

But since the external field E and the internal field E_i are given by equation (5.3.9) to be

$$E = E_0 \sin \omega t$$

$$E_i = E_{0_i} \cos \omega t$$

Thus we have two conductivity types , σ_1 which reflect the response of charges to external field E and σ_2 which reflects the response to the internal filed E_i

5.4 amplification conditions on the basis of phase relation to the electric susceptibility

The electric dipole moment P is related to the displacement x Between the nucleus and electron cloud according to the equation:[92]

$$P = Zex \tag{5.4.1}$$

Where Z is the atomic number ,thus Ze is the charge of each dipole the displacement x is given with :

$$x = \int v\, dt \;=\; v_\circ \int \cos(\omega t + \emptyset)\, dt$$

$$x = \frac{v_\circ}{\omega} \sin(\omega t + \emptyset)$$

$$x = x_\circ \sin(\omega t + \emptyset) \;,\; x_\circ = \frac{v_\circ}{\omega} \tag{5.4.2}$$

Inserting (5.4.2) in (5.4.1) yields

$$P = Zex_\circ \sin(\omega t + \emptyset)$$

$$P = Zex_\circ cos\emptyset sin\omega t - Zex_\circ sin\emptyset \cos \omega t \qquad (5.4..3)$$

The electric dipole moment can also be written in terms of the applied external electric field E and the medium field perpendicular to it E_m in the form

$$p = x_1 E + x_2 E_m$$

$$p = \mathcal{X}_1 E_{m_\circ} \cos \omega t + \mathcal{X}_2 E_\circ \sin \omega t \qquad (5.4.4)$$

Comparing equation (5.4.3) and (5.4.4) yields :

$$x_1 E_{0m} = -\text{Z}ex_0 \sin \emptyset$$

$$x_2 E_0 = \text{Z}ex_0 \cos \emptyset \qquad (5.4.5)$$

The electric dipole moment can be write in a complex form in terms of x_1 and x_2 to b e

$$\text{P}=(x_1+jx_2)\text{E} =(x_1+jx_2)E_0 \ e^{j\omega t} \qquad (5.4..6)$$

But the current generated by p is given by :

$$\text{j} = \frac{dp}{dt} = x\frac{dE}{dt} = x\frac{dE_0 \ e^{-j\omega t}}{dt}$$

$$-j\omega \, x\underline{\text{E}} \ = -\text{j}\omega(x_1+jx_2 \ )\underline{\text{E}} \ =(\omega x_2 - j\omega x_1)\text{E}$$

$$= (\omega x_2 - j\omega x_1 \ ) \ \underline{\text{E}} \qquad (5.4.7)$$

Since $x = x_1+jx_2$ one can write

$$x_2 = x cos\varphi \, , x_1 = x sin\varphi \qquad (5.4.8)$$

The current density J can also be written in terms of σ_1 and σ_2 to be

$$J = (\sigma_1 + j\sigma_2)\underset{\sim}{\text{E}} \qquad (5.4.9)$$

Thus comparing (5.4.7) and (5.4.9) yields

$$\sigma_1 = \omega x_2 \quad , \sigma_2 = -\omega x_1 \qquad (5.4.10)$$

Thus according to equation for (4.6.13) amplification factor is given by

$$\beta = \frac{\mu\, c\, \omega}{n_1}\, x_2 \tag{5.4.11}$$

to express β in terms of the phase ∅ , one uses equation (5.4.2) and(5.4.5) and (5.4.8)to get from(5.4.11)

$$\beta = \frac{\mu\, c\omega z e x_\circ}{n_1 E_\circ} \cos \emptyset$$

where $q = ze$ and $x_\circ = \frac{v_\circ}{\omega}$

$$\beta = \frac{\mu\, c\, q v_\circ}{n_1 E_\circ} \cos \emptyset \tag{5.4.12}$$

again ,when v and E are in phase ∅ =0 , and

$$\beta = \frac{\mu\, c\, q v_\circ}{n_1 E_{0_m}} \tag{5.4.13}$$

and amplification takes place , as far as β does not vanish . if $\emptyset = 90$, $\beta = 0$ and no amplification takes place for $\emptyset = \pi$

$$\beta = -\frac{\mu\, c\, q v_\circ}{n_1 E_{0_m}}$$

the incident radiation is absorbed by the medium .

5.5 Relation between current and electric field in terms of a Circuit

Equation (5.3.1)which describes electron motion , under the action of E only ,indicates that on electric field E generates a current J with electrons moving with velocity V out of phase w.r.t E by 90 . to see how can this understood on the basis of AC electric circuits consider [93]

$$E = E_0 e^{i(wt-kx)} \qquad v = v_0 e^{i(wt-kx)} \tag{5.5.1}$$

Thus the equation of motion(5.3.4) in the absence of resistive force becomes

$$im\omega v = -eE$$

Thus :-

J= $nev = \frac{-ne^2}{im\omega}E = \sigma E$ (5.5.2)

The electric current is defined to result from the motion of positive charge .therefore we have to replace – e by e in the equation of motion .thus the impendence becomes

$$z = \frac{\rho d}{A} = \frac{d}{\sigma A} = \frac{md}{Ane^2}\omega = \omega l \quad (5.5.3)$$

This solution can describe electron current conductivity for electron generated by thermionic emission

Thus the material here behaves as an inductor of inductive reactance

$$x_l = \omega l = \omega\left(\frac{md}{Ane^2}\right) \quad (5.5.4)$$

When the medium becomes resistive ,the equation of motion becomes

$$[im\omega + \gamma]v = eE$$

Thus

$$J = nev = \frac{ne^2}{(\gamma + im\omega)} = \sigma E$$

Thus the impendence becomes

$$z = \frac{\rho d}{A} = \frac{d}{\sigma A} = \frac{d\ (\gamma + im\omega)}{Ane^2} = \frac{d\gamma}{Ane^2} + i\omega\left(\frac{md}{Ane^2}\right) = R + i\omega l$$
$$= R + ixl \quad (5.5.5)$$

This indicates that in the absence of resistive force the electric field generates a current out of phase by 90. The medium in this case acts as an inductor. In the presence of resistive frictional force in the medium the electric field and the velocity becomes

$$E = E_0 \sin \omega t \qquad v = v_0 \sin(\omega t + \emptyset) \quad (5.5.6)$$

Thus E and V are out of phase by $\emptyset$ degree .this situation corresponds to the existence of inductor and resistor in series.

The relation between v and E when E is generated by current is discussed in section (5.2). While E is given by (5.2.4). To find relation between E and V beside J in a complex form, one can insert (5.5.1) 1n (5.2.2) and then in (5.2.9) by suggesting

$$A = A_0 e^{i[\omega t - kx]} \qquad (5.5.7)$$

To get

$$A_0(-k^2 + \mu\epsilon\omega^2) = \mu J_0 \qquad (5.5.8)$$

5.6 Electric conductivity by using RLCcircuits Relations.

In resistance, Capacitor and inductor circuit, There is a phase difference between currents and voltages . For resistor and coil connected in series, the total voltage is given by [94]

$$\mathrm{V} = L\frac{di}{dt} + Ri \qquad (5.6.1)$$

Where the current is given by $i = i_{0\ sin\omega t}$

Thus

$$V = wli_0\, coswt + Ri_0\, sinwt$$

$$= V_0 \sin(\omega t + \emptyset) = x_L icoswt + Ri_0\, sinwt$$

$$V_0 \sin \omega tcos\emptyset + v_0 cos\emptyset sinwt = x_L i_0\, coswt = Ri_0\, sinwt$$

$$V_0 sinw = x_L i_0 \qquad v_0 cos\emptyset = Ri_0$$

$${V_0}^2 sin^2\emptyset + {v_0}^2 cos^2\emptyset = [{x_L}^2 + R^2]{i_0}^2$$

$$V_0 = \sqrt{{x_L}^2 + R^2}\; i_0 \qquad (5.6.2)$$

This situation resembles that of the electron vibrating in an oscillating electric field The equation of motion of electron is given by

$$m\frac{dv}{dt} = -eE - \gamma mv \qquad \gamma = \frac{1}{\tau} \tag{5.6.3}$$

Consider the electron velocity be in the form

$$V = v_0 \sin wt \tag{5.6.4}$$

Thus inserting this expression (5.6.4) in (5.6.3) yields

$$mwv_0 \cos wt = eE - \gamma mv_0 \sin wt$$

$$m[wv_0 \cos wt + \gamma v_0 \sin wt] = eE$$

Multiplying both sides by the conductor length α the potential is given by

$$\frac{md}{ne^2}[newv_0 \cos wt + \gamma nev_0 \sin wt] = Ed = V \tag{5.6.5}$$

But the maximum current i_0 and current density j_0 are given by

$$nev_0 = j_0 \qquad i_0 = j_0 A$$

Hence equation(5.6.5)can be re written as

$$V = \frac{md}{ne^2 A}[\frac{x_L}{L} i_0 \cos wt + \gamma i_0 \sin wt] \tag{5.6.6}$$

Where $x_L = wL$,thus

$$V = [\frac{mdx_L}{ne^2 AL} i_0 \cos wt + \frac{md\gamma}{ne^2 A} i_0 \sin wt] \tag{5.6.7}$$

Comparing (5.6.6)and(5.6.7) yields

$$\sigma = \frac{ne^2\tau}{m} \qquad \boldsymbol{\rho} = \frac{1}{\sigma} = \frac{m}{ne^2\tau} = \frac{m\gamma}{ne^2} \qquad L = \frac{m\mathrm{d}}{ne^2 A}$$

But the resistance R and inductive reactance x_L are given by

$$R = \frac{\rho d}{A} = \frac{m\gamma d}{ne^2 A} \qquad x_L = \frac{mdw}{ne^2 A} = wL \tag{5,6.8}$$

Thus the potential can be written as

$$V = x_L i_0 coswt + R i_0 sinwt \qquad (5.6.9)$$

But one can write

$$V = V_0 \sin(\omega t + \emptyset) = v_0 sin\emptyset coswt + v_0 cos\emptyset sinwt \qquad (5.6.10)$$

Comparing equation (5.6.9)and (5.6.10)

$$V_0 sin\emptyset = x_L i_0 \qquad V_0 cos\emptyset = \ R i_0$$

There for

$$V_0{}^2 sin^2\emptyset + V_0{}^2 cos^2\emptyset = V_0{}^2 = [x_L{}^2 + R^2] i_0{}^2$$

$$V_0 = \sqrt{x_L{}^2 + R^2}\ i_0 \qquad (5.6.11)$$

Hence the impedance is given by

$$Z = \frac{V_e}{i_e} = \frac{\frac{V_0}{\sqrt{2}}}{\frac{i_0}{\sqrt{2}}} = \frac{V_0}{i_0} = \sqrt{x_L{}^2 + R^2} = \frac{1}{y}$$

The admittance can thus be given to be

$$y = \frac{i_0}{V_0} = \frac{\sigma A}{d} = \frac{1}{\sqrt{x_L{}^2 + R^2}} \qquad (5.6.12)$$

Using equation (5.6.8) the total conductivity is given by

$$\sigma = \frac{1}{\rho} = \frac{d}{AR} = \frac{d}{AZ} = \frac{d}{A} y = \frac{d}{A} \frac{1}{\sqrt{x_L{}^2 + R^2}} \qquad (5.6.13)$$

where the potential takes the form [see(5.6.9)and(5.6.10)]

$$V = Ed = E_0 \mathrm{d}\sin(wt + \emptyset)$$

$$= E_0 \mathrm{d}\sin(wt + \emptyset) = v_0 \sin(wt + \emptyset) \qquad (5.6.14)$$

Thus according to equation (5.6.14) the electric field inside the medium is given by

$$\mathrm{E} = E_0 \sin(wt + \emptyset)$$

$$\mathrm{E} = E_0 \sin\emptyset\, coswt + E_0 cos\emptyset sinwt \quad (5.6.15)$$

In view of equation (5.6.5) the part including w is related to inductance which is proportional w thus the internal field is generated electromagnetic induction, since the resistive term in equation (5.6.5) most stand for resistance to the external field E through friction coefficient

$$V_i = L\frac{di}{dt}$$

But the current i takes the form : $i = neAv$ thus the inductive voltage is related to the internal generated potential according to the relation

$$V_i = E_i d = \frac{enALdv}{dt} = \frac{enALd(v_0 sinwt)}{dt}$$

$$= enALv_0 coswt = dE_{0i} coswt = dE_i$$

Thus the internal field is given by

$$E_i = E_{0i} coswt \quad (5.6.16)$$

Hence according to equation (5.6.15) $E_i = E_0 \sin\emptyset\, coswt$

which means that

$$E_{0i} = E_0 \sin\emptyset \quad (5.6.17)$$

the external field is thus given by

$$E_e = E_{0i} \mathrm{sinwt} = E_i = E_0 \cos\emptyset\, sinwt \quad (5.6.18)$$

Thus σ_1 can be found to be from equation (8) to get

$$R = \frac{d}{\sigma_1 A} = \frac{\rho_1 d}{A} \quad (5.6.19)$$

But $x_L = wl = \frac{d}{\sigma_2 A}$

$$x_L = \frac{wmd}{ne^2A} = c_0\frac{d}{A} = \frac{1}{σ_2}\frac{d}{A} \tag{5.6.20}$$

Thus the conductivity for inductance takes the form

$$c_0 = \frac{wm}{ne^2} = \frac{1}{σ_2} = \boldsymbol{ρ}_2 \tag{5.6.21}$$

$$σ = (\frac{d}{A}) = \frac{1}{(\frac{d}{A})\sqrt{(\frac{1}{σ_1})^2 + (\frac{1}{σ_2})^2}}$$

thus

$$\frac{1}{σ} = \sqrt{(\frac{1}{σ_1})^2 + (\frac{1}{σ_2})^2} \tag{5.6.22}$$

Therefore the net resistivity is given by

$$\boldsymbol{ρ} = \sqrt{\boldsymbol{ρ}_1{}^2 + \boldsymbol{ρ}_2{}^2} \tag{5.6.23}$$

5.7 Electric conductivity by using Effective values.

The electric conductivity can also be found by using directly the concept of current density and electric field. The electric field E and current density J are related by

$$j_e = σE_e \tag{5.7.1}$$

Where J_e and E_e are the effective values which are related to the maximum values J_0 and E_0 , thus

$$J_e = \frac{J_0}{\sqrt{2}}$$

$$E_e = \frac{E_0}{\sqrt{2}}$$

$$j_0 = σE_0 \tag{5.7.2}$$

According to equation The electron equation of motion becomes.

$$mwv_0 \cos wt + \gamma m v_0 \sin wt \ = eE_0 \sin(wt + \emptyset) \tag{5.7.3}$$

$$= eE_0 \sin\emptyset \cos wt + eE_0 cos\emptyset sinwt)$$

$$= eE_{e0} coswt + eE_{i0} sinwt \tag{5.7.4}$$

Where the total electric field is given by

$$E \ = E_0 \sin(wt + \emptyset) = E_0 \sin\emptyset \cos wt + E_0 cos\emptyset sinwt \tag{5.7.5}$$

Comparing equation (5.7.3)and (5.7.5) yields

$$mwv_0 = eE_{i0} = eE_0 \sin\emptyset$$

$$m\gamma v_0 = eE_{e0} = eE_0 \cos\emptyset \tag{5.7.6}$$

But the current density is related to the velocity by

$$j_0 = nev_0 \tag{5.7.7}$$

Using equation (5.7.6) by squaring both sides one gets

$$[m^2w^2 + m^2\gamma^2]{v_0}^2 = e^2{E_0}^2[sin^2\emptyset + \cos^2\emptyset] = e^2{E_0}^2$$

Hence

$$eE_0 = m\sqrt{w^2 + \gamma^2}\, v_0 \tag{5.7.8}$$

$$v_0 = \frac{eE_0}{m\sqrt{w^2+\gamma^2}} \tag{5.7.9}$$

Inserting equation (5.7.9) in (5.7.7) yields

$$j_0 = \frac{ne^2E_0}{m\sqrt{w^2+\gamma^2}} \tag{5.7.10}$$

Comparing equations (5.7.10) and (5.7.2)yields

$$\sigma = \frac{1}{\sqrt{\frac{m^2w^2}{n^2e^4}+\frac{m^2\gamma^2}{n^2e^4}}} \tag{5.7.11}$$

in view of equations (5.6.8) and (5.6.20)

$$\sigma = \frac{1}{\sqrt{(\frac{1}{\sigma_1})^2 + (\frac{1}{\sigma_2})^2}}$$

$$\frac{1}{\sigma} = \sqrt{(\frac{1}{\sigma_1})^2 + (\frac{1}{\sigma_2})^2} \qquad (5.7.12)$$

Hence

$$\boldsymbol{\rho} = \sqrt{\boldsymbol{\rho}_1{}^2 + \boldsymbol{\rho}_2{}^2} \qquad (5.7.13)$$

5.8 Electric conductivity using complex Representation

Another alternative can be also used to find the total conductivity and resistivity by suggesting V to be in a complex form as

$$v = v_0 e^{i\omega t} \qquad (5.8.1)$$

But the electron equation in a resistive medium is given by

$$m\frac{dv}{dt} = -eE - \gamma m v \qquad (5.8.2)$$

Substituting equation (5.8.1) in (5.8.2)yields

$$imwv = -eE - \gamma m v$$

$$(imw + \gamma m)v = eE \qquad (5.8.3)$$

Since external field is related to conductor resistance through V and γ Which recognizes resistance thus $E_e = E_0 \cos\emptyset\, sinwt$ Which is the real part of E stands for external field .Hence

Let $E = (E_e + iE_i)e^{i\omega t}$

$$= (E_0 cos\emptyset + iE_0 sin\emptyset)e^{i\omega t} = E_0 \; e^{i(\omega t+\emptyset)} \qquad (5.8.4)$$

Inserting equation(5.8.4) in equation (5.8.3)yields

$$m(iw+\gamma)v_0 e^{i\omega t} = eE_0\ e^{i(\omega t+\emptyset)}$$

Therefor

$$m(iw+\gamma)v_0 \ = eE_0\ e^{i\emptyset}$$

$$m(iw+\gamma)v_0 \ = eE_0(cos\emptyset + isin\emptyset)$$

Hence

$$eE_0 cos\emptyset = m\gamma v_0 \qquad eE_0 sin\emptyset = mwv_0 \tag{5.8.5}$$

Squaring both sides ,gives

$$e^2{E_0}^2[sin^2\emptyset + \cos^2\emptyset] = m^2[w^2+\gamma^2]{v_0}^2$$

$$eE_0 = \ m\sqrt{w^2+\gamma^2}\, v_0$$

$$v_0 = \frac{eE_0}{m\sqrt{w^2+\gamma^2}} \tag{5.8.6}$$

But the current density is given by

$$j_0 = \ nev_0 = \sigma E_0 \tag{5.8.7}$$

Substituting (5.8.6)gives

$$j_0 = \frac{ne^2 E_0}{m\sqrt{w^2+\gamma^2}} = \frac{E_0}{\sqrt{\frac{mw^2}{n^2e^4} + \frac{m^2}{n^2e^4\tau^2}}}$$

As a result

$$\sigma E_0 = \frac{E_0}{\sqrt{\frac{mw^2}{n^2e^4} + \frac{m^2}{n^2e^4\tau^2}}} \tag{5.8.8}$$

thus the view of equation(5.6.8) and (5.6.21)yields

$$\sigma = \frac{1}{\sqrt{(\frac{1}{\sigma_1})^2 + (\frac{1}{\sigma_2})^2}}$$

$$\frac{1}{\sigma} = \sqrt{(\frac{1}{\sigma_1})^2 + (\frac{1}{\sigma_2})^2} \quad (5.8.9)$$

$$\boldsymbol{\rho} = \sqrt{\boldsymbol{\rho}_1{}^2 + \boldsymbol{\rho}_2{}^2} \quad (5.8.10)$$

5.9 Travelling wave solution and internal current.

Conceder travelling wave electric field of the form

$$E = E_0 \sin(kx - wt) = E_0 sin\Theta(x,t) \quad (5.9.1)$$

The electron equation of motion is

$$m\frac{dv}{dt} = -eE - \gamma mv \quad (5.9.2)$$

The velocity which satisfy this equation must be

$$v = v_0 \sin(kx - wt + \varphi) = v_0 \sin(\Theta + \varphi) \quad (5.9.3)$$

Thus

$$m\frac{dv}{dt} = -wv_0 \cos(\Theta + \varphi) \quad (5.9.4)$$

Inserting equation(5.9.1),(5.9.3),(5.9.4)in(5.9.2)yields

$$-mwv_0 cos\varphi cos\Theta + mwv_0 sin\varphi sin\Theta =$$

$$= eE_0 sin\Theta - m\gamma v_0 cos\varphi sin\Theta - m\gamma v_0 sin\varphi cos\Theta$$

Equating coefficients of $cos\Theta$ and $sin\Theta$ an both sides yields

$$-mwv_0 cos\varphi = m\gamma v_0 sin\varphi$$

$$mwv_0 sin\varphi = eE_0 - m\gamma v_0 cos\varphi \quad (5.9.5)$$

$$tan\varphi = \frac{w}{\gamma} \qquad sin\varphi = \frac{w}{\sqrt{w^2+\gamma^2}} \qquad cos\varphi = \frac{\gamma}{\sqrt{w^2+\gamma^2}} \quad (5.9.6)$$

$$m[wsin\varphi + \gamma cos\varphi]v_0 = eE_0 \quad (5.9.7)$$

$$m[\frac{w^2}{\sqrt{w^2+\gamma^2}} + \frac{\gamma^2}{\sqrt{w^2+\gamma^2}}]v_0 = eE_0$$

$$m\left[\frac{w^2+\gamma^2}{\sqrt{w^2+\gamma^2}}\right]v_0 = eE_0 \quad (5.9.8)$$

$$v_0 = \frac{eE_0}{m\sqrt{w^2+\gamma^2}} \quad (5.9.9)$$

Using the same procedures as in equation (5.7.8)and (5.8.12),one gets

$$\boldsymbol{\rho} = \sqrt{\boldsymbol{\rho}_1{}^2 + \boldsymbol{\rho}_2{}^2} \quad (5.9.10)$$

This means that the incidence of travelling electromagnetic wave causes electron to travel also with speed shown by equation (5.9.3).The materials behaves as inductor and resistor in series . In view of equations (5.9.1)(5.9.3) and(5.9.9) one have two currents , the one resulting from the external field (5.9.1) and it takes the form

$$J_e = \sigma_e E_e = nev_0 cos\varphi \sin(wt - kx) \quad (5.9.11)$$

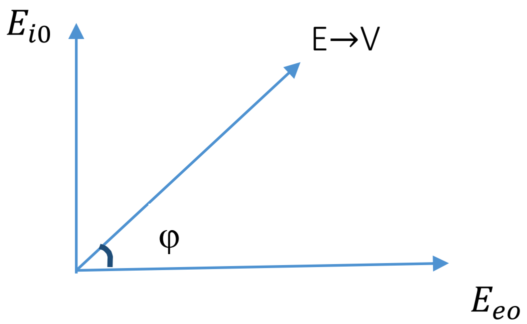
The other is due to an internally induced electromagnetic field by oscillating charge ,and is given by

$$J_i = \sigma_i E_i = nev_0 sin\varphi \sin(wt - kx) \quad (5.9.12)$$

Such that

$E_{eo} = nev_0 cos\varphi$

$E_{i0} = nev_0 sin\varphi$



Fig(5.2) external field E and the internal field $E_i$andφ

The resultant field is thus due to the effect of external and internal field.

5.10 Discussion

The amplification factor in equation (5.4.12) depends on the angle Ø between The total field or polarization field see equation (5.4.3) on external field E and internal field E_m as shown by equations (5.4.4) and (5.4.5). It is very interesting to note that when the external field is parallel to the polarization field, the atom absorb the incident photon and eremite it , and amplification take place (see equation(5.4.13)) This agrees with the fact that the polarization and external field are in phase thus they are coherent .But when the polarization field and external one are out of phase by 90^0 no amplification takes place , This is due to the fact that the two waves are out of phase by 90^0.This does means that there is no field component the polarization field in the direction of external field .Thus the amplitude does not increase and remains constant thus no amplification is observed . However when the external and polarization field apposes each other β is negative which means that the external photon is absorbed by the polarized atom. The interaction of electromagnetic field with matter resuts in generating internal electromagnetic field as equation (5.6.15).The material respond to the external field by dissipating energy by friction ,while the response to the internal field manifests itself through inducing inductive current and inductive reactance as shown by equation(5.7.6) .The net electric field inside matter subtends an angle φ w.r.t to E_e as equation(5.7.3) reads. The net current flowing inside matter faces a total resistance resulting from friction and induction current. This resistance is that of a resistor and inductor connected in series as shown by equations(5.6.23),(5.7.1) ,(5.7.13) and (5.9.9). When friction is neglected equation (5.7.6) indicates that φ is 90, and only induction current and electromotive internal field exists . If electromagnetic field is a travelling wave [see equation (5.9.1), this wave induces frictional current generated by external field see equation (5.9.6),(5.6.10)beside induction current generated by internal field [see equation (5.9.6),(5.9.11).

5.11 Conclusions

If one consider the polarization atom to emit two components one is parallel to the external applied field and the other to the internal field which is perpendicular to the external field, in this case one can predict amplification and absorption process easily in terms of the angle between polarization vector and the external field. The transmition of electromagnetic field through amedium induces electromotive internal field ,acting as an inductor. The net effect of the interaction corresponds to resistor and inductor connected in series.

5.12 Recommendation

1. The nature of internal field need to be generalized to be applicable for quantum systems and a Nano scale.
2. The behavior of the internal field for Nano materials need to beinvestigated this can help in fabricating Nano inductors and capacitor .
3.The role of internal field for plasma and super fluids requires deep studies.

References

[1] Hitz B,Ewingj,and Heehtj,In introduction to lasers,3redition ,iEEEpress,pisc atmay,Newjersey ,(2001)

[2] Samyeh,kamlesh Jain ,Sebastiano Andreana, Using adiode laser to uncover dental implants in second-stage surgery-general Dentistry November-Decem ber ;414-417(2005)

[3] Siham.A.Kandela,Laser physics in medicine ,Saddam college of medicine,199

[4] Davis, C. C., Lasers and Electro-Optics: Fundamentals and Engineering, Cambridge University Press, New York (1996)

[5] William,t.silfvast,laserfundamentals,U.S.A(1999).

[6] P.othow,Ph.D thesis,sust, khartum(2010)

[7] A. Haggar ,ph.Dt hesis ,sust,Khartoum(2007)

[8] Marvin J. Weber, Handbook of Lasers,(University of California,(2001)

[9] Sihtay ehutesfa,quan.ph.v.i15868(2010)

[10] Amd Ra: and G.s Agarawal,quan.ph.v2.1462(2009)

[11] Jimmie J. Cathey ,Electronic dives and circuits ,university of Kentucky, New York: 2002

[12] Martin Dressel ,Elctrodynumics of solids ,Cambrdge ,New york ,USA:2002

[13] Benjamin crowell ,Electricity and magnetism ,fullerton,california,copyright1999:rev2010

[14] Jimmie J. Cathey ,Electronic dives and circuits ,university of Kentucky, New York: 2002

[15] Martin Dressel ,Elctrodynumics of solids ,Cambrdge ,New york

[16] Jasprit singh ,Electronic and Optoelectronic properties of semiconductor structures ,university of michigan ,Ann Arbor ,Cambridge:2003

[17] R.Abd Elhai,Using the tight binding approximation in deriving the quantum critical temperature superconductivity equation, Natural Science:2013

[18] H.G.I.Hamza, Using the Resistance Depending on the Magnetic and Electric Susceptibility to Derive the Equation of the critical temperature, Natural Science:2014

[19] Ludfi Mohmmed AbdAlgadir, Schrodinger quantum equation from classical and quantum Harmonic Oscillator,ISSN:2277-9955,IJESRT,2016

[20] K.G.Egaylani,Derivation of Klein-Gordon equation from Maxwell's electric wave equation,IJPS,ISSN2331-1827,Academe Research Journals:2014

[21] A.A.Alhaj,Phase effect between the Electric Internal Current Field and the External Current Field on Amplification of the total Field and Intensity of the Electromagnetic Radiation, JBASR, J. Basic. Appl. Sci.Res..3(9)1-6,:2013

[22] Mohammed Asmail Adam,Derivation of Maxwell's Equation for Diffusion Current and Klein-Gordon Equation beside New Quantum Equation Form Maxwell's Equation for Massive Photon, (JOSR-JAP) e-ISSN:2278-4861.Volume7,Issue 2 Ver II,:2015

[23] Nguyen, D.N., et alTemperature Dependence of Total AC Loss in High-Temperature Superconducting Tapes. IEEE Transactions on Applied Superconductivity, 19, 3637-3644. . 2009

[24] Slooten, E., et al. Enhancement of Superconductivity near the Ferromagnetic Quantum Critical Point in UCoGe.Physical Review Letters, 103, Article ID: 097003. (2009)

[25] Mc Graw Hill, Quantum Mechanics , Tokyo, 2015

[26] The M.Es used to describe the behavior of electromagnetic waves are (Wolf, 1976; Griffths,; 1999

[27] A.A.Alhaj,Phase effect between the Electric Internal Current Field and the External Current Field on Amplification of the total Field and Intensity of the Electromagnetic Radiation, JBASR, J. Basic. Appl. Sci.Res..3(9)1-6,:2013

[28] Mohammed Asmail Adam,Derivation of Maxwell's Equation for Diffusion Current and Klein-Gordon Equation beside New Quantum Equation Form Maxwell's Equation for Massive Photon, (JOSR-JAP) e-ISSN:2278-4861.Volume7,Issue 2 Ver II,:2015

[29] B B laud ,Electromagnetic , second Edition Newdelhi,india , Reprint(2006)

[30] Fleisch D ,A student's Guide to Maxwell's Equations, Cambridge University Press, Cambridge, :2008.

[31] M.Brunger,photon and Electron interactions with Atoms molecules and ions,Edited by –Itikawa,Springer-verlag-Berlin,Heidelierg,:2003

[32] Dale M.Grimes, The Electromagnetic origin of quantum Theory and light ,second edition ,British library,:2005

[33] P.Kpuri, solid state physics, indian Institute of Technology, Newdelhi,india:2004

[34] Kittle,Charles ,introduction of solid state physics,fourth Edition,U.S.A,(2005)

[35] Neil W,Ashcroft,solid state physics,cornell university,1976

[36] P.Kpuri, solid state physics, indian Institute of Technology, Newdelhi,india:2004

[37] Kittle,Charles ,introduction of solid state physics,fourth Edition,U.S.A,(2005)

[38] James D.patterson, solid state physics ,Springer,Rapid city,SD 57701,USA,:2007

[39] Neil W,Ashcroft,solid state physics,cornell university,1976

[40] B.L.Theraja,Atext book of Electrical Technology,ins,1.units,volume

[41] P.M.Mathews,K.Venkatesan,Atext book of quantum Mechanics (Tata mcgraw-hill,new Delhi,2007)

[42] Yariv, A., *Quantum Electronics*, John Wiley & Sons, New York (1989).

[43] Glen.F.Knol,Radiation Detection and measurement(2000)

[44] Wa.lter.koechner,Michacl Bass,solid-statelaser(Agraduatext springer,New York,2003)

[45] C.L.Tang,Fundamental of quantum mechanics for solid state Electronics and optics,Cambridge university press New York,:2005

[46] Wa.lter.koechner,Michacl Bass,solid-statelaser(Agraduatext springer,New York,2003

[47] A. Hajar ,ph.Dthesis ,sust,Khartoum(2007)

[48] jan.Tunar and lars.Hode,laser theropy ,prima book's AB,2002

[49] Williams.c.chang,Principles of lasers and optics , Cambridge press, New York.(2005)

[50] Davis,C.C.,*Lasersand Electro-Optics: Fundamentals and Engineering*, Cambridge University Press, New York (1996).

[51] Ajohnwiley&sons,fundamentals of light sources and lasers(United state of America,2004)

[52] Hecht, J., *Understanding Lasers* (second edition), IEEE Press, New York (1994).

[53] Marvin J. Weber, Handbook of Lasers,(University of California,2001)

[54] Siham.A.Kandela,Laser physics in medicine ,Saddam college of medicine,1991

[55] R.Abd Elhai,Using the tight binding approximation in deriving the quantum critical temperature superconductivity equation, Natural Science:2013

[56] H.G.I.Hamza, Using the Resistance Depending on the Magnetic and Electric Susceptibility to Derive the Equation of the critical temperature, Natural Science:2014

[57] Ludfi Mohmmed AbdAlgadir, Schrodinger quantum equation from classical and quantum Harmonic Oscillator,ISSN:2277-9955,IJESRT,2016

[58] K.G.Egaylani,Derivation of Klein-Gordon equation from Maxwell's electric wave equation,IJPS,ISSN2331-1827,Academe Research Journals:2014

[59] Nguyen, D.N., et alTemperature Dependence of Total AC Loss in High-Temperature Superconducting Tapes.IEEE Transactions on Applied Superconductivity, 19, 3637-3644. (2009)

[60] Millican, J.N., Phelan, D., Thomas, E.L., Leao. J.B. and Carpenter, E. Solid State Communications, 149, 707. (2009)

[61] Sales, B.C., et al. Transport, Thermal, and Magnetic Properties of the Narrow Gap Semiconductor CrSb2. Physical Review B, 86, Article ID: 235136.Http://dx.doi.org/10.1103/PhysRevB.86.235136. (2012)

[62] Kantorovich, L. Quantum Theory of the Solid State: An Introduction. Kluwer Academic Publishers, London. http://dx.doi.org/l0.1007/978-1-4020-2154-1. (2004)

[63] Weyeneth, S., Puzniak, R., Zhigadlo, N.D., Katrych, S., Bukowski, Z., Karpinski, J. and Keller, H.J. Evidence for Two Distinct Anisotropies in the Oxypnictide Superconductors SmFeAsOo.8Fo.2 and NdFeAsOo.8Fo.2. Journal of Superconductivity and Novel Magnetism, 22, 347-351. http://dx.doi.org/l0.1007/s10948-009-0445-I. (2009)

[64] Nguyen, D.N., et al. Temperature Dependence of Total AC Loss in High-Temperature Superconducting Tapes. (2009)

[65] Slooten, E., et al. Enhancement of Superconductivity near the Ferromagnetic Quantum Critical Point in UCoGe.Physical Review Letters, 103, Article ID: 097003. (2009)

[66] Millican, J.N., Phelan, D., Thomas, E.L., Leao. J.B. and Carpenter, E. Solid State Communications, 149, 707. (2009)

[67] Kantorovich, L. (2004) Quantum Theory of the Solid State: An Introduction. Kluwer Academic Publishers, London. http://dx.doi.org/l0.1007/978-1-4020-2154-1

[68] Weyeneth, S., Puzniak, R., Mosele, U., Zhigadlo, N.D., Katrych, S., Bukowski, Z., Karpinski, J., Kohout, S., Roos, J. and Keller, H. Anisotropy of Superconducting Single Crystal $SmFeAsOo_{.8}F_{o.2}$ Studied by Torque Magnetometry.Journal of Superconductivity and Novel Magnetism, 22, 325-329. http://dx.doi.org/10.1007/s10948-008-0413-1 . (2009)

[69] Weyeneth, S., Puzniak, R., Zhigadlo, N.D., Katrych, S., Bukowski, Z., Karpinski, J. and Keller, H.J) Evidence for Two Distinct Anisotropies in the Oxypnictide Superconductors $SmFeAsO_{o.8}F_{o.2}$ and $NdFeAsO_{o.8}F_{o.2}$. Journal of Superconductivity and Novel Magnetism, 22, 347-351. http://dx.doi.org/l0.1007/s10948-009-0445-I . (2009)

[70] De Visser, A., et al.) Muon Spin Rotation and Relaxation in the Superconducting Ferromagnet UCoGe. Physical Review Letters, 102, Article ID: 167003.http://dx.doi.org/10.1103/PhysRevLett.102.167003. (2009)

[71] Paden M., Pocrzyvnck, S., Stalinisk:i B., Phys. state sol., 24, K73 (2011).

[72] S.widle,A. Sparenber, G.L.J.A.Rekk'&n, D.Lacoste, and B.A.Van Tiggelen,photonic Hall effect in absorbing media ,E62,8636-8639,:2014

[73] T.M.karek, B.C.Crooker, Miotkowski and A.K.Ramdas, Magnetic measurement on nvae semicouctor Gal -KMnxSe Volume 83, Issua 1 1, pp.6557-6559 (2013

[74] Heisenberg Werner .In Rechenberg, Helmut. Deutsche und Jüdische Physik. Piper. ISB. (2011).

[75] Journal –phys.RevB, Volume 38 Issue I-Functional integral theories of low dimensional quantim ??Iber-2013.

[76] L.I.Schiff, Quantum Mechanics , Mc Graw Hill Tokyo, 2015

[77] Ludonco Anna Effetto Heisenbev. La rivoluzione scientifica che ha cambiato la storia. Rio. p. 224. ISBN 88-8358-182-2 .liesrt.com . (2011).

[78] Abbot AF .Ordinary Level Physics, Third Edition, Heinemann Educational Book-London. (1977).

[79] Bruce S, Minning P .The Klein-Gordon Oscillator, IL Nuovo Cimento, 106 A:5. (1993).

[80] Salih BEA, Teich MC Fundamentals of Photonics, Second Edition, John Wiley and sons, New York. (2007).

[81] Hand LN. Finch JD .Analytical Mechanics, Cambridge University press, Cambridge. (1998).

[82] C.L.Tang,Fundamental of quantum mechanics for solid state Electronics and optics,Cambridge university press New York,:2005

[83] A.Messiah .Quantum Mechanics.Dover Puplications. (1999)

[84] A.Neil.Solid State physics.NewYork: Holt,Rinehart and Winston. (1976).

[85] A.H.Abdelrahman ,M.D,Abdella,Mahgoub Salih , The effect of External Electric Field on the Lasing Mechanism in the Field,Scientic Research and Impact,1(4):80-85 December2012

[86] Fleischer D,A students Guide to Maxwells Equation,Camberidge University press ,Camberidge,:2008

[87] Sakurai,J,J,Fu Tuan and son ,Modern quantum mechanics revised edition,Addison Wesley ,California,1994.

[88] Dale M.Grimes, The Electromagnetic origin of quantum Theory and light ,second edition ,British library,:2005

[89] Fleischer D,A students Guide to Maxwells Equation,Camberidge University press ,Camberidge,:2008

[90] Griffths DJ .Introduction to Electrodynamics, Third Edition, Prentice Hall New Jersey. (1999).

[91] C.L.Tang,Fundamental of quantum mechanics for solid state Electronics and optics,Cambridge university press New York,:2005

[92] Davis, C. C., Lasers and Electro-Optics: Fundamentals and Engineering, Cambridge University Press, New York (1996)

[93] Lal B .Mathematical Theory of Electromagnetism, Asia Publishing House, Delhi. (1965).

[94] Jimmie J. Cathey ,Electronic dives and circuits ,university of Kentucky, New York: 2002