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New Computer Program to Calculate Pressure Drop in Pipes

برنامج جديد لحساب هبوط الضغط في الأنابيب

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New Computer Program to Calculate Pressure Drop in Pipes

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مشروع تخرج مقدم إلي كلية هندسة وتكنولوجيا النفط – جامعة السودان للعلوم والتكنولوجيا
إنجاز جزئي لأحد المتطلبات للحصول على درجة بكالريوس العلوم في هندسة النفط

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Abstract

The pressure drop calculations in any producing system is depends mainly on the multi-phase correlations which is a function of fluid properties and pipe conditions. Many correlations were available for the calculations of the fluid phase behavior properties at various temperature and pressure (PVT correlations); the selection of the best correlation is the key for good production analysis and optimization; any bad selection for the PVT correlations or multiphase flow correlations will lead to no production optimization. The current study address the effect of compressibility factor (Z); oil viscosity (μ_o), gas viscosity (μ_g) and gas solubility (R_s) on the calculations of pressure drop in pipes using three different multiphase flow correlations through Faula North 4-3.

Computer program has been developed to predict pressure drop. Orkiszewski, Hagedorn and Brown and Beggs and Brill methods were programmed with the all available PVT correlations to analyze the effect of PVT correlations. The calculated data have been compared with reference data from vertical pipes in Faula North well 4-3.

The results for the well under study presented that, the best correlation for multiphase flow is Orkisviski which is semi-close to real data. With this method, no effect was found for the gas viscosity (μ_g) correlations and same results were obtained by the different correlations; while little variation on the pressure drop was observed with compressibility factor (Z); oil viscosity (μ_o) and gas solubility (R_s) with the different correlations

Key Words

Production Optimization; Multi-phase Flow; Pressure Drop; Phase Behavior; Pipes

التجريد

إن حسابات هبوط الضغط في أي نظام إنتاج يعتمد بشكل رئيسي على العلاقات الرياضية لحساب الهبوط في الضغط للجريان متعدد الأطوار التي بدورها دالة في خصائص سلوك الموائع والأنابيب. وتتوفر العديد من العلاقات الرياضية لحسابات خصائص سلوك الموائع في مختلف درجات الحرارة والضغط، ويعتبر الإختيار الأفضل لعلاقات الجريان متعدد الأطوار او لعلاقات خصائص سلوك الموائع هو المفتاح للتحليل الأمثل لمنظومة الإنتاج؛ وإن أي اختيار خاطئ يؤدي الى أخطاء فادحة في عملية التحليل. وتهدف الدراسة الحالية لإبراز تأثير معامل الانضغاط (Z)؛ لزوجة النفط (μ_o)، اللزوجة الغاز (μ_g) وذوبانية الغاز (R_S) على حسابات هبوط الضغط في الأنابيب باستخدام ثلاث علاقات مختلفة للجريان متعدد الأطوار وتطبيقها في البئر 4-3 .بحقل الفولة الشمالي.

وقد تم تطوير برنامج كمبيوتر للتنبؤ بهبوط الضغط مستخدمين طرق حسابات الجريان متعدد الأطوار لكل من اوركيزويسكي، هجدورن وبراون وبيجز وبريل مع علاقات خصائص سلوك الموائع المتاحة. وقد تم مقارنة البيانات المحسوبة مع البيانات المرجعية من الأنابيب الرأسية في البئر 4-3 بحقل الفولة الشمالي.

واكدت نتائج الحسابات للبئر قيد الدراسة أن أفضل علاقة لحسابات الجريان متعدد الأطوار هي طريقة اوركيزويسكي والتي تعطي نتائج اقرب للبيانات الحقيقية من البئر. وباستخدام هذه الطريقة أثبتت الدراسة أنه لا يوجد تأثير لعلاقات لزوجة الغاز على حسابات الهبوط في الضغط حيث أن جميع العلاقات كانت نتائجها متشابهة. في حين لوحظ اختلاف بسيط في انخفاض الضغط مع تغير علاقات حساب معامل الانضغاط (Z) لزوجة النفط (μ_o) وذوبانية الغاز (R_S)

الكلمات المفتاحية

الإنتاج الأمثل؛ الجريان متعدد الأطوار؛ هبوط الضغط؛ خصائص سلوك الموائع؛ الأنابيب

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K= permeability of the porous medium,

v= apparent fluid velocity

q= volumetric flow rate

A= area open to flow

μ= fluid viscosity

p= pressure gradient in the direction of flow

P_1= upstream pressure, psia

P_2= downstream pressure, psia

μ_o= oil viscosity, cp

B_o= oil formation volume factor, bbl/STB

L= length of flow path, ft

Z= gas deviation factor evaluated at T, $\bar{P}$

T= flowing temperature, $°R$

γ_g= gas gravity

q_{sc}= gas flow rate at 14.7 psia. $°60F$, scf /day.

μ_g= gas viscosity cp

k_g= permeability to gas, md

h= reservoir thickness, ft

$\bar{P}_R$= average reservoir pressure, psia

P_{wf}= wellbore flowing reservoir, psia

r_e= drainage radius, ft

r_w= wellbore radius, ft

j = productivity index

S = skin factor

k_o= effective oil permeability, md

q_o= inflow rate, STB/day

D = turbulence coefficient

q_o= inflow rate corresponding to wellbore flowing pressure

$q_{(\max)}$= inflow rate corresponding to zero wellbore flowing pressure, (AOF)

FE = flow efficiency

C = flow coefficient

n = exponent depending on the well characteristic

t_s = stabilization time, hrs

ϕ = porosity

A = drainage area, ft^2

Ct= total fluid compressibility, $psia^{-1}$

β = velocity coefficient , ft^{-1}

k_v= effective permeability in the vertical direction, md

k_H= effective permeability to oil in the horizontal direction, md

L = length of the horizontal section , ft

h = vertical thickness of the formation, ft

j_{F^*} = value of $j^*$when$\bar{P}_{RP}$has decline to$\bar{P}_{RF}$

j_{P^*} = value of j^*at the present reservoir pressure

B_2 = turbulence coefficient after recompletion

B_1 = turbulence coefficient before recompletion

h_{P2} = new completion length, ft

h_{P1} = old completion length, ft

A_R = laminar reservoir component

A_P = laminar perforation component

A_g = laminar gravel-pack component

B_R = turbulent reservoir component

B_P = turbulent perforation component

B_G = turbulent gravel-pack component

K_{OR} = unaltered reservoir permeability to oil

K_{gR} = unaltered reservoir permeability to gas

S_d = skin factor due to permeability alteration around the wellbore.

K_d = reservoir permeability

K_R = altered zone reservoir

r_d = altered zone radius

h = total formation thickness

U' = internal energy

pV = energy of expansion or compression

$mv^2/2g_c$ = kinetic energy

mgz/g_c = potential energy

q' = heat energy add to fluid

w' = work done on the fluid by the surroundings

S = entropy

h = enthalpy

H_l = liquid holdup

N_{Re} = Reynolds number

v_g = gas velocity

λ_L = no-slip holdup

ρ_L = liquid density

ρ_g = gas density

f = fraction

H_g = gas holdup

v_{sg} = superficial gas velocity

q_g = gas flow rate

μ_L = viscosity of oil and water

v_m = mixture velocity

q_L = liquid flow rate

v_{sl} = superficial liquid velocity

f_o = fraction of oil

f_w = fraction of water

μ_o = oil viscosity

μ_w = water viscosity

σ_o = oil surface tension

σ_w = water surface tension

A =area, ft2

Chapter 1

General Introduction

1.1 Introduction

Any production well is drilled and completed to move the fluids from it is original location in the reservoir to stock tank or sales line which is called production system. Movement or transport of these fluids requires energy to left the product to the surface. A very important consideration in system analysis is the use of approach correlations or equations while analyzing the inflow and outflow performance. The amount of fluid flowing into the well from the reservoir depend on the pressure drop in the piping system and the pressure drop depend on amount of fluid flowing through it. The pressure gradient must be calculated at several points in the piping system at the pressure and temperature existing at this point

The fluid properties required to calculate pressure drop change with pressure and temperature at any point in the pipe.

The theoretical basis for most fluid flow equations is the general energy equation, an expression for the balance or conservation of energy between two points in a system. The energy equation is developed first and, using thermodynamic principles, is modified to a pressure gradient equation form.

The steady state energy balance simply States that the energy of a fluid entering a control volume, plus any shaft work done on or by the fluid, plus any heat energy added to or taken from the fluid, must equal the energy leaving the control volume.

For single phase flow, the flow equations and procedures presented for evaluating the friction factor as a function of Reynolds number and pipe roughness. while complicated procedures are required when a second phase Introduced into flow stream. The pressure gradient is increased for the same mass flow rate, and the flow may develop a pulsating nature. The fluids may separate because of differences in densities and flow at velocities in the pipe. A rough interface may exist between the liquid and gas phases. Properties such as densities, velocity, and viscosity, which are relatively simple for individual fluids become very difficult to determine.

When designing the Production system; calculation of Pressure involves dividing the flow string into number of length increment and calculating the pressure gradient at average condition and calculating; this requires knowledge of oil and gas properties at various pressure and temperature.

The gas and liquid phases may travel at different velocities in the pipe; the difference between the actual gas and liquid velocities known as slip velocity. The fluid properties used to determine Reynolds number as well as other dimensionless numbers need to be calculated for two-phase based on the slip between the phases using the fractions of oil and water flowing in the mixture as weighting factors; which is rather nebulous and 'is defined differently by various investigators. These fluid properties are a function of temperature and pressure.

Unfortunately, the fluids properties, which are obtained from PVT analysis, are conducted at the reservoir condition and are not applicable to piping system calculations. Therefore, empirical fluids properties correlations will be required. In fact, the only information available about the fluids properties are the separator gas gravity; and the stock tank oil gravity, so all the fluid properties could be correlated as a function of pressure, temperature and oil and gas gravity. Therefore, an accurate correlations are required to estimate the other fluid properties at other conditions. Many correlations are available in the literature for different conditions; the optimization of the best correlation require a real information for the dynamic pressure profile in the well; combination of the different Multi- phase flow correlations with the different PVT correlations lead to several conditions; when compared with the real information the best correlations can be selected and the system will be ready for optimization at any time.

1.2 General Information about the Field

Fula North oilfield is situated in the Fula central structure trend of the Fula sub-basin in the northeast of Block VI in south of Sudan. According to OOIP estimation result, total OOIP is 599.66MMB, in which heavy oil is 89.5%, 536.92MMB; light oil is 10.5%, 62.74MMB. The crude oils have high density, high viscosity, high acid value, low pour point, and low wax and sulfur content. The main Blocks in Fula oilfield can be divided into 3 blocks, i.e. Fula-1 Block, Fula North Block, and Fula Central Block; the main pay zones of heavy oil are Bentiu and Aradeiba reservoir ; while the light oil was found in Abu Gabra formation (Pan et al 206) :

1) Bentiu Formation is a major oil bearing sandstone reservoir in the Muglad rift basin of interior Sudan, with thick massive loose sand. The reservoir has an average reservoir thickness of 83m; it has high porosity ranging from 24.2% to 31.6%, averaging 29.1% and high permeability from 561.5 to 2926×10-3μm2, at an average of 2041.2×10^{-3}μm2. The reservoir is composed of thick beds of sands inter-bedded with thinner beds of clays with a thickness of 1 to 2 meters or less. The average oil viscosity is about 1536.39 cp at 50 ^{O}C.

2) Aradeiba is the second reservoir with stratified unconsolidated pay sand; the reservoir has an average reservoir thickness of 15.5m. It has higher porosity and permeability than Bentiu reservoir, at averages of 32.3% and 3261×10^{-3}μm2 respectively. The average oil viscosity is 400cp; viscosity is up to around 450 cp at 50 ^{O}C

3) Abu Gabra formation is the light oil reservoir with consolidated pay sand; the reservoir has an average reservoir pressure of 2538.3 Psi . It has lowe porosity and permeability than Bentiu reservoir, at averages of 19.3% and 132.9×10^{-3}μm2 respectively. The average oil viscosity is 40cp; viscosity is up to at 50 ^{O}C The oil API is 36.5.

Fula North 4 is located in Fulla North with target formation Abu Gabra; with Average Formation Top of 1755 m; Fig 1.1 presented the Grate Fula fields and Fula North location.

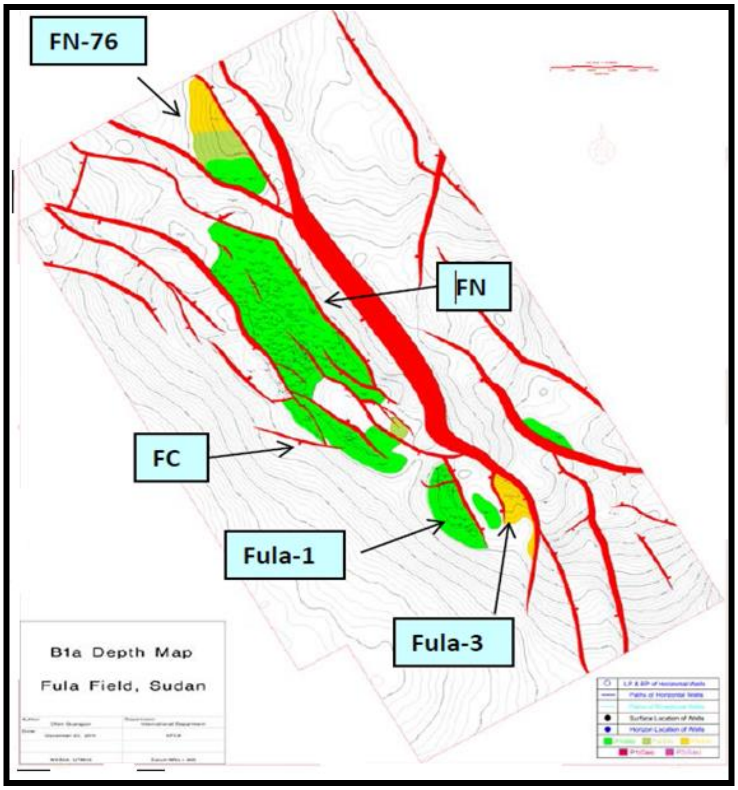


Fig 1.1 Fula North location (Pan et al 206)

Fula North 4-3 (FN-4-N3 in) is one of the infill development wells in Fula North Block; FN 4-3 was completed on Jul.27, 2009, and put into production on Sep.02.2009, totally produced oil as per the below table. The well is vertical well reached true vertical depth (TVD) of 2400 m in Abu Gabra formation, the well was completed with Production Casing grade N80 ; the inside diameter is 124.3 mm for the depth of 1591.51-2398.0 meter while a tubing with inside diameter of 2 7/8 " was settled to depth of 1952.57 m.

1.3 Problem Statement

The pressure drop calculations in any producing system is a function of the mixture fluid properties which can be calculated using different PVT correlations. The selected correlations affect the production optimization. Currently, no work was presented to indentify suitable correlation for the Sudanese field; on the other hand, no computer program is available for the calculations of the Pressure drop in piping system considering all the PVT correlations; and the international programs has a limited considerations. The current study presented the analysis of all the available PVT correlations in Well 4-3 in Fula north oilfield using a new computer program for the calculations of the multiphase pressure drop in piping system.

1.4 Research Objective:

The main objective of this work is to address the effect of PVT correlations on the total pressure drop in the pipe using different Multi- phase flow correlations and developing a computer program for piping system; the specific objectives of the work can be summarized as follows:

1. To calculate the pressure drop in the reservoir using different IPR correlations to make start point for the calculations
2. To calculate the PVT properties using different PVT correlations
3. To calculate the pressure drop in horizontal and vertical pipes using different correlations and the calculated PVT properties
4. To compare the calculated pressure profile with the reference data collected from the well
5. To analyze the results to present the effect of the PVT correlations on the different Pipe correlations.

Chapter 2

Theoretical Background and Literature Review

2.1. Pressure Drop in Pipes

The theoretical basis for most fluid flow equations is the general energy equation, an expression for the balance or conservation of energy between two points in a system. The energy equation is developed first and, using thermodynamic principles, is modified to a pressure gradient equation form. The steady state energy balance simply States that the energy of a fluid entering a control volume, plus any shaft work done on or by the fluid, plus any heat energy added to or taken from the fluid, must equal the energy leaving the control volume; Fig.2.1 illustrate this principle.

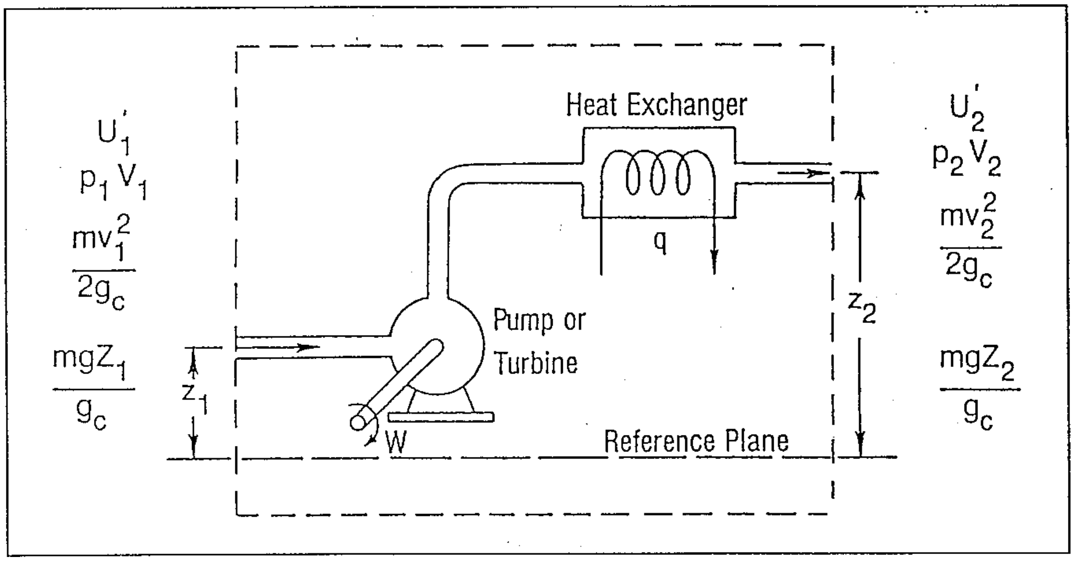


Fig 2.1 Energy balance equation (James P. Brill 1999)

Considering a steady state system, the energy balance may be written as

$$U_1' + p_1V_1 + \frac{mgZ_1}{g_c} + q' + W_z' = U_2' + p_2V_2 + \frac{mV_2^2}{2g_c} + \frac{mgZ_2}{g_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

By rearrange Eq 1

$$\frac{dp}{\rho} + \frac{vdv}{g_c} + \frac{g}{g_c}dZ + dL_w = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

$$\frac{dp}{\rho}+\frac{vdv}{g_c}+\frac{g}{g_c}dL\sin\theta+dL_w=0 \quad \text{(2.3)}$$

Multiplying the equation by $\rho/_{dL}$ gives :

$$\frac{dp}{dL}+\frac{\rho vdv}{g_c dL}+\frac{g}{g_c}\rho\sin\theta+\rho\frac{dL_w}{dL}=0 \quad \text{(2.4)}$$

$$\frac{dp}{dL}=\frac{\rho vdv}{g_c dL}+\frac{g}{g_c}\rho\sin\theta+\left(\frac{dp}{dL}\right)_f \quad \text{(2.5)}$$

In horizontal pipe flow, the energy losses or pressure drop are caused by change in kinetic energy and friction losses only. Since most of the viscous shear occurs at the pipe wall, the ratio of wall shear stress (i to kinetic energy per unit volume (pv2/2gt.) reflects the relative importance of wall shear stress to the total losses. This ratio forms a dimensionless group and defines a friction factor.

2.1.1. Single-phase Flow

Now that equations and procedures have been presented for evaluating the friction factor in single-phase flow, the pressure gradient equation derived previously can be further developed. Combining Equations(2.4) and (2.5), the pressure gradient equation, which is applicable to any fluid at any pipe inclination angle becomes:

$$\frac{dp}{dL}=\frac{\rho vdv}{g_c dL}+\frac{g}{g_c}\rho\sin\theta+\frac{f\rho v^2}{2g_c d} \quad \text{(2.6)}$$

where the friction factor, f, is a function of Reynolds number and pipe roughness. This relationship is shown in form of

$$\frac{dp}{dL}=\left(\frac{dp}{dL}\right)_{el}+\left(\frac{dp}{dL}\right)_f+\left(\frac{dp}{dL}\right)_{acc} \quad \text{(2.7)}$$

Where:

$\left(\frac{dP}{dL}\right)_f$ =pressure drop due to friction

$\left(\frac{dP}{dL}\right)_{acc}$ =pressure drop due to acceleration

$\left(\frac{dP}{dL}\right)_{el}$ =pressure drop due to elevation

Equation 1.7 applies for any fluid in steady state, one-dimensional flow for which f, p, and v can be defined. Definition of these variables is what causes most of the difficulty in describing two-phase flow. In two-phase flow, f may be a function of other variables besides the Reynolds number and relative roughness.

Some aspects of the pressure gradient equation as it applies to single-phase flow are discussed to develop a thorough understanding of each component before modifying it for two-phase flow. The elevation change or hydrostatic component is zero for horizontal flow only. It applies for compressible or incompressible, steady state or transient flow in both vertical and inclined pipes. For downward flow, the sin of the angle is negative, and the hydrostatic pressure increases in the direction of flow. The friction loss component applies for any type of flow at any pipe angle. It always causes a drop of pressure in the direction of flow. In laminar flow, the friction losses are linearly proportional to the fluid velocity. In turbulent flow, the friction losses are proportional to v", where 1.7 Kn.

The kinetic energy change or acceleration component is zero for constant area, incompressible flow. For any flow condition in which a velocity change occurs, such as compressible flow, a pressure drop will occur in the direction of the velocity increase. Although single-phase flow has been studied extensively, it still involves an empirically determined friction factor for turbulent flow calculations. The dependence of this friction Factor on pipe roughness, which must usually be estimated, makes the calculated pressure gradients subject to considerable error. Single-phase, incompressible or slightly compressible liquid flow is a uivial solution of the pressure gradient equation. Single-phase compressible flow of gases is a more complex problem to solve and is covered in detail later. Single-phase, compressible transient flow is an extremely complex problem and is beyond the scope of this book. The preceding descriptions are not meant to be an exhaustive coverage of single-phase flow of Newtonian fluids in pipes. As stated previously, the principal reason for including the material is to form a film foundation for the more complicated analysis of two-phase flow.

2.1.2. Two -phase Flow

Most producing wells (Oil or gas wells) operate under multiphase conditions. Usually there will be some free gas produced along with the oil in oil wells, at the same way the gas wells will produced either water or condensate along with the gas phase. Introduction of a second phase into a flow stream complicates the analysis of the pressure gradient equation- The pressure gradient is increased for the same mass flow rate, and the flow may develop a pulsating nature. The fluids may separate because of differences in densities and flow at velocities in the pipe. Two-phase flow pressure drop calculation involves two types of energy loss; friction loss and slippage loss; slippage is due to the great difference in specific weight between the gas and liquid. Design and analysis of a system in which two-phase flow is occurring requires a thorough understanding of the physical phenomena as well as the basic theory and equations. The spatial arrangement must relative to each liquid, gas, and filling out of space by the individual phase may take a variety of forms of typical arrangements are follow patterns. A rough interface may exist between the liquid and gas phases. Properties such as densities, velocity, and viscosity, which are relatively simple for individual fluids become very difficult to determine.

Before modifying the pressure gradient equation for two-phase flow conditions, certain variables unique to a two-phase, gas-liquid mixture must be defined and evaluated. such as liquid holdup and No slip liquid holdup concept.

A) Liquid Holdup. Liquid Holdup HL

is defined as the fraction of an element of pipe that is occupied by liquid at some instant. That is

$$H_L = \frac{volume\ of\ liquid\ in\ a\ pipe\ element}{volume\ of\ the\ pipe\ element} \quad (2.8)$$

B) No-Slip Liquid Holdup

No-slip holdup, XL, sometimes called input liquid content, is defined as the ratio of the volume of liquid in a pipe clement that would exist if the gas and liquid traveled at the same velocity (noslip page) divided by the volume of the pipe element. It can be calculated directly from the known gas and liquid in-situ flow rates from:

$$\lambda_L = \frac{q_L}{q_L + q_g} \quad (2.9)$$

$$(\lambda_g = 1 - \lambda_L) \quad (2.10)$$

2.2.Mixture Properties Calculation

Calculation of pressure gradients requires values of flow conditions such as velocity, and fluid properties such as density, viscosity, and, in some cases, surface tension. When these variables are calculated for two-phase flow, certain mixing rules and definitions unique 10 this application are encountered. This section will define and analyze some of the more important properties that must be understood before adapting the previously derived pressure gradient equation for two-phase conditions- In this text, two-phase flow implies gas-liquid flow; however, the liquid phase may include two immiscible fluids such as water and oil. Methods for analysis of a liquid phase that consists of any two components are discussed.

I. Fluid Density

All fluid flow equations require that a value of the density of the fluid available. The density is involved in evaluating the total energy changes due to potential energy and kinetic energy changes. Calculation of density changes as pressure and temperature change requires an equation of state for the fluid under consideration. Equations of state are readily available for single-phase fluids and are presented later. When two immiscible liquids such as oil and water flow simultaneously, the definition of density becomes more complicated.

The density of a flowing gas/liquid mixture is very difficult to evaluate because of the gravitational separation of the phases and the slippage between the phases. The density of an oil/water mixture may be calculated from the oil and water densities and flow rates if no slip-page between the oil and water phases is assumed.

$$\rho_L = \rho_o f_o + \rho_w f_w \quad \text{..(2.11)}$$

$$f_o = \frac{q_o}{q_o + q_w} = \frac{q_{o_{SC}} B_o}{q_{o_{SC}} B_o + q_{w_{SC}} B_w} \quad \text{(2.12)}$$

Calculation of the density of a gas/liquid mixture requires knowledge of the liquid holdup. Three equations for two-phase density have been used by various investigators of two-phase flow.

$$\rho_s = \rho_L H_L + \rho_g H_g \quad \text{(2.13)}$$

$$\rho_n = \rho_L \lambda_L + \rho_g \lambda_g \quad \text{(2.14)}$$

$$\rho_k = \frac{\rho_L \lambda_L^2}{H_L} + \frac{\rho_g \lambda_g^2}{H_g} \quad \ldots\ldots(2.15)$$

II. Fluid Viscosity

The viscosity of the flowing fluid is used in determining a Reynolds number as well as other dimensionless numbers used as correlating parameters. The concept of a two-phase viscosity is rather nebulous and 'is defined differently by various investigators. The following equations have been used by various investigators to calculate two-phase, gas/liquid viscosity:

$$\mu_n = \mu_L \lambda_L + \mu_g \lambda_g \;,\; \mu_s = \mu_L^{H_L} \times \mu_g^{H_g} \quad \ldots\ldots(2.16)$$

The viscosity of an oil/water mixture is usually calculated by using the fractions of oil and water flowing in the mixture as weighting factors. The most commonly used equation is

$$\mu_L = \mu_o f_o + \mu_w f_w \quad \ldots\ldots(2.17)$$

This equation is not valid if an oil/water emulsion is formed. The viscosities of natural gas, crude oil and water may be estimated from empirical correlations, described in the next section, if measured viscosities are not available.

III. Fluid Velocity

Many two-phase flow correlations are based on a variable called superficial velocity. The superficial velocity of a fluid phase is defined as the velocity that phase would exhibit if it flowed through the total cross sectional area of the pipe alone. The superficial gas velocity is calculated from:

$$v_{sg} = \frac{q_g}{A} \quad \ldots\ldots(2.18)$$

The actual area through which the gas flows is reduced by the presence of the liquid therefore, the actual gas velocity is calculated from:

$$v_g = \frac{v_{sg}}{H_g} \quad \ldots\ldots(2.19)$$

The superficial and actual liquid velocities are similarly calculated from:

$$v_{sL} = \frac{q_L}{A} \quad \ldots\ldots(2.20)$$

$$v_L = \frac{v_{sL}}{H_L} \qquad (2.21)$$

Since Hg and HL are less than one, the actual velocities are greater than the superficial velocities.

The two-phase or mixture velocity is calculated based on the total in-situ flow rate from the equation.

$$v_m = v_{sg} + v_{sL} \qquad (2.22)$$

As has been stated previously, the gas and liquid phases may travel at different velocities in the pipe. Some investigators prefer to evaluate the degree of slippage and for the liquid holdup by determining a slip velocity Vs. The slip velocity is defined as the difference between the actual gas and liquid velocities by:

$$v_s = v_g - v_L \qquad (2.23)$$

Using the previous definitions for the various velocities, alternate forms of the equations for no-slip and actual liquid holdup are:

$$\lambda_L = \frac{v_{sL}}{v_m} \qquad (2.24)$$

IV. Surface Tension

Correlations for the interfacial tension between water and natural gas and crude oil and natural gas as functions of temperature and pressure are given in the next section. The interfacial tension depends on other fluid properties such as oil gravity, gas gravity and dissolved gas. When the liquid phase contains both water and oil, the same weighting factors as used for calculating density and viscosity are used. That is:

$$\sigma_L = \sigma_o f_o + \sigma_w f_w \qquad (2.25)$$

2.3.Two-Phase Flow Regimes

When two fluids move in a pipe, the continuous liquid dispersal gas bubbles, when this liquid moves up the pressure drops and gas bubbles begin to formation this is known as bubble flow. These bubbles grow and become more numerous and then slug flow forms. Still higher in the tubing, that is, at lower pressure the gas pockets may have grown and the annular flow will form. Continue decrease in pressure the result is increase in gas volume and the oil film become thinner and thinner until finally disappears and then flow regime will be mist flow. According to the above description of the vertical flow, the pressure drops will be calculated under different assumptions, as it will be discussed. The flow pattern are identified according to flow direction; in horizontal pipes, it will be different than that of vertical pipes.

2.3.1. Horizontal Flow pattern

I. Stratified (Smooth and Wavy) Flow:

Stratified flow consists of two superposed layers of gas and liquid, formed by segregation under the influence of gravity.

II. Intermittent (Slug and Elongated Bubble) Flow:

The intermittent flow regime is usually divided into two sub regimes: plug or elongated bubble flow and slug flow. The elongated bubble flow regime can be considered as a limiting case of slug flow, where the liquid slug is free of entrained gas bubbels.

Gas–liquid intermittent flow exists in the whole range of pipe inclinations and over a wide range of gas and liquid flow rates.

III. Annular-Mist Flow:

During annular flow, the liquid phase flows largely as an annular film on the wall with gas flowing as a central core. Some of the liquid is entrained as droplets in this gas core (mist flow).

IV. Dispersed Bubble Flow:

At high liquid rates and low gas rates, the gas is dispersed as bubbles in a continuous liquid phase. The bubble density is higher toward the top of the pipeline, but there are bubbles throughout the cross section. Dispersed flow occurs only at high flow rates and high pressures. This type of flow, which entails high-pressure loss, is rarely encountered in flow lines.

2.3.2. Vertical Flow pattern

I. Bubble flow:

The gas phase is distributed in the liquid phase as variable-size, deformable bubbles moving upward with zigzag motion. The wall of the pipe is always contacted by the liquid phase.

II. Slug Flow

Most of the gas is in the form of large bullet-shaped bubbles that have a diameter almost reaching the pipe diameter. These bubbles are referred to as “Taylor bubbles,” move uniformly upward, and are separated by slugs of continuous liquid that bridge the pipe and contain small gas bubbles. The gas bubble velocity is greater than that of the liquid.

III. Churn Flow:

If a change from a continuous liquid phase to a continuous gas phase occurs, the continuity of the liquid in the slug between successive Taylor bubbles is destroyed repeatedly by a high local gas concentration in the slug. This oscillatory flow of the liquid is typical of churn flow. It may not occur in small-diameter pipes. The gas bubbles may join and liquid may be entrained in the bubbles.

IV. Annular-Mist Flow:

Annular flow is characterized by the continuity of the gas phase in the pipe core. The liquid phase moves upward partly as a wavy film and partly in the form of drops entrained in the gas core.

2.4. Two-Phase Flow Pressure Drop Correlations in Vertical Pipes :

Many correlations have been developed for predicting two-phase flow pressure gradients which differ in the manner used to calculate the three terms of pressure gradients equation (elevation change, friction and acceleration terms. The methods used to predict pressure gradient can be classified as empirical correlations and mechanistic models. The empirical correlations can be placed in one of three categories:

a. No slip, no flow regime considerations:

the mixture density is calculated based on the no slip holdup. No distinction is made for different flow regimes. That is, the gas and liquid are assumed to travel at the same velocity. The only correlation required is for the two-phase friction factor.

b) Slip considered, no flow regime consideration*:*

A correlation is required for both liquid holdup and friction factor. Because the liquid and gas can travel at different velocities, a method must be provided to predict the portion of the pipe occupied by liquid at any location. The same correlations for liquid holdup and friction factor are used for all flow regimes.

c) Slip considered, flow regime considered:

Usually a different liquid holdup and friction factor prediction methods are required in each flow regimes. Not only are correlations required to predict liquid holdup and friction factor, but methods to predict which flow pattern exists are necessary. Once the flow pattern is established, the appropriate holdup and friction... factor correlations are determined. The method used to calculate the acceleration pressure gradient also depends on flow pattern. The following list gives the published empirical correlations for vertical upward flow and the categories in which they belong.

2.4.1. Duns and Ros Method

The Duns and Ros method (1963) is a result of an extensive laboratory study in which liquid holdup and pressure gradients were measured. About 4,000 two-phase-flow tests were conducted in a 185-ft-high vertical-flow loop. Pipe diameters ranged from 1.26 to *5.60* in. and included two annulus configure- ions. Most of the tests were at near atmospheric conditions with air for the gas phase and liquid hydrocarbons or water as the liquid phase. Liquid holdup was measured by use of a radioactive-tracer technique. A transparent section permitted the observation of flow pattern. For each of three flow patterns observed, correlations were developed for friction factor and slip velocity, from which liquid holdup can be calculated.

Duns and Ros performed the first dimensional analysis of two- phase flow in pipes. They identified 12 variables that were potentials y important in the prediction of pressure gradient. Performing a dimensional analysis of these variables resulted in nine dimensionless groups, or it terms. Through a process of elimination, four of the groups were identified as being important and were used to select the range of variables in the experimental program. Fig.(2.2) shows the flow-pattern map developed by Duns and Ros. They identified four separate regions for computation purposes, Regions I through III and a transition region. Fig. 2.2 illustrates the actual flow patterns existing in each region. Duns and Ros also identified the heading region as a fifth region, but this is now considered past of Region II. In this monograph, we will refer to Regions I through Ill as bubble, slug, and mist flow, respectively. The flow pattern transition boundaries are defined as functions of the dimensionless groups Gas Velocity Number (N_{gv} ,) and Liquid Velocity Number *(NL)* For these transition bound arise ; where the dimensionless number can be defined as follows:

$$N_{Lv} = v_{sL}\left(\frac{\rho_L}{g\sigma_L}\right)^{0.25} = 1.938\, v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25} \qquad \text{……………………………………………………}(2.26)$$

$$N_{gv} = v_{sg}\left(\frac{\rho_L}{g\sigma_L}\right)^{0.25} = 1.938\, v_{sg}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25}$$

$$\text{……………………………………………………}(2.27)$$

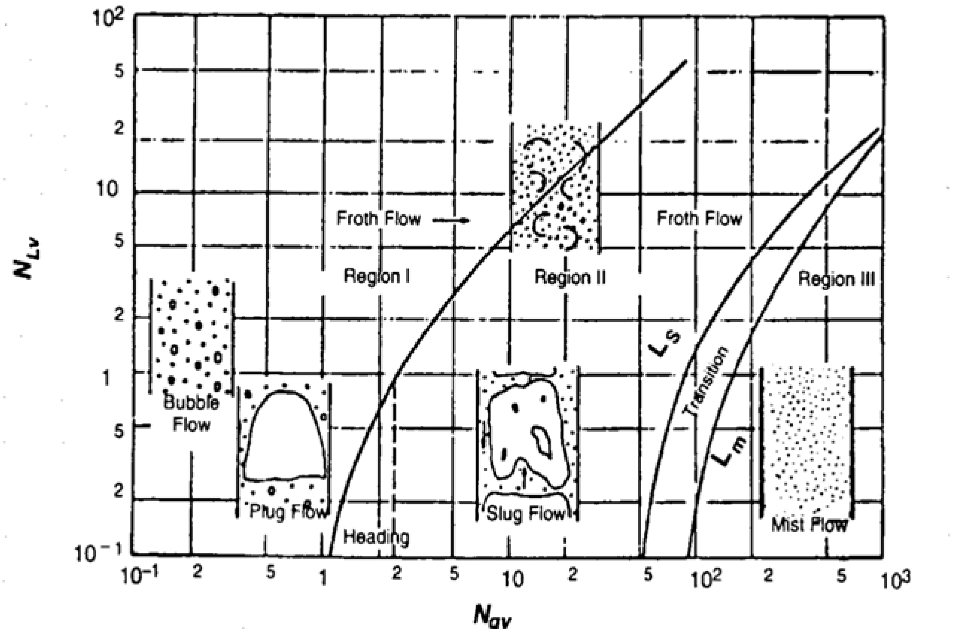


Fig. 2.2 Duns & Ros Flow pattern

2.4.2. Orkiszewski Method

Orkiszewski (1967) tested several published correlations with field data and concluded that none was sufficiently accurate for all flow patterns. He then selected what he considered to be the most accurate correlations for bubble and mist flow and proposed a new correlation for slug flow. The slug-flow correlation was developed with the Hagedorn and Brown data. Orkiszewski selected the Griffith and Wallisl method for bubble flow and the Duns and Ros method for mist flow. Orkiszewski performed a comparison study on some 148 measured well conditions and found that none of the correlations existing at that time (1967) adequately predicted the measured results. He then used the data of Hagedorn and Brown and the field data from the 148 oil well conditions to develop a new correlation to be used in the Bubble- and Slug-flow patterns. He recommended using the Duns and Ros method for Mist-flow. Flow patterns were identified as follows:

a) Bubble Flow

The pipe is almost completely filled with liquid, and the free gas phase is present in small bubbles. The bubbles move at different velocities and, except for their density, have little effect on the pressure gradient. The wall of the pipe is always contacted by the liquid phase.

b) Slug Flow

The gas phase is more pronounced. Although the liquid phase is still continuous, the gas bubbles coalesce and form plugs or slugs that almost fill the pipe cross section. The gas bubble velocity is greater than that of the liquid. The liquid in the film around the bubble may move downward at low velocities. Both the gas and liquid have significant effects on the pressure gradient.

c) Transition Flow

The change from a continuous liquid phase to a continuous gas phase occurs. The gas bubbles may join and liquid may be entrained in the bubbles. Although the liquid effects arc significant. the gas phase effects are predominanl.

d) Mist Flow

The gas phase is continuous. and the bulk of the liquid is entrained as droplets in the gas phase. The pipe wall is coated with a liquid film. But the gas phase predominantIy controls the pressure gradient. Equations were presented for determining the flow pattern existing under various conditions. and methods for calculating friction factor and two-phase density were presented for the Bubble- and Slug-flow patterns. In the Slug-flow pattern the liquid density was calculated using a so-called Liquid Distribution Coefficient, rather than the liquid holdup. A distinction was made as to which equations are used to calculate the liquid distribution coefficient depending on whether oil or water was the continuous liquid phase and if the mixture velocity was greater than 10ft/shsec.

2.4.3. Beggs Method

The Beggs and brill method (1978) was the one to predict flow behavior at all inclination angles, including directional wells. Their first facility was 1 and 1.5 in. sections of acrylic pipe, 90 ft long. The pipe could be inclined at any angle from the horizontal. The fluids were air and water for each pipe size liquid and gas rates were varied so that , when the pipe was horizontal all flow patterns were observed. After a particular set of flow rates was established . The inclination of the pipe was varied through the range of angles so that the effect of angle on holdup and pressure gradient could be observed. Liquid Hold Up and Pressure Gradients were measured at angles from the horizontal of 0 , +-5 , +-10, +-15,+-20, +-35, +-55, +-75 and +-90.The following correlations describe the Beggs and Brill Equation:

$$\frac{dp}{dl} = \frac{\frac{f\rho_n v_m{}^2}{2d} + \rho_s g \sin(\Theta)}{1 - \mathrm{E}\kappa} \text{……………………………………………………………} (2.28)$$

$$\rho_s = \rho_L H_L(\Theta) + \rho_g(1 - H_L(\Theta)) \text{……………………………………………………} (2.29)$$

2.4.4. Hagedorn and Brown Beggs Method

The Hagedorn and Brown (1964) method is based on data obtained from a 1,500-ft-deep vertical experimental well. Air was the gas phase, and four different liquids were used: water and crude oils with viscosities of about 10, 30, and 110 cp. Tubing with 1.0, 1.25, and 1.5 in. nominal diameters were used.

These data represent some of the most extensive large-scale tests ever reported. However, it is important to recognize that Hagedorn and Brown did not measure liquid holdup. Rather, they developed a pressure-gradient equation that, after assuming a friction-factor correlation, permitted the calculation of pseudo liquid-holdup values for each test to match measured pressure gradients. Thus, the values used to develop a liquid-holdup correlation were not true measures of the portion of pipe occupied by liquid. Hagedorn and Brown developed this pressure-gradient equation for vertical multiphase flow

$$\frac{dp}{dz}=\frac{f\rho_n^2 v_m^2}{2\rho_s d}+\rho_s g+\frac{\rho_s\Delta(v_m^2)}{2dz}\ldots\ldots\ldots\ldots(2.30)$$

2.5.PVT Properties:

Ideally, fluid properties are determined from laboratory studies designed to duplicate the conditions of interest. However, experimental data are quite often unavailable because representative samples cannot be obtained or the producing horizon does not warrant the expense of an in-depth reservoir fluid study. Therefore, Pressure Volume Temperature relationship (PVT properties) must be determined by analogy or through the use of empirically derived correlations; which may cause a source of errors in multiphase-flow calculations, the specific correlations that should be used for a specific crude oil or reservoir may vary; therefore, it is always desirable to select the most appropriate PVT correlations for the concerned fluids. The development of correlations for PVT calculations has been the subject of extensive research, resulting in a large volume of publications.

Mawla et al (1996) presented PVT comparative study for the Gulf of Suez crudes; preliminary validation of the reported laboratory results was conducted by mass balance calculations. The reported differential PVT data were corrected to flash data to simulate gas liberation in the production tubing. These identified best correlations for each field have been adopted in the subsequent multiphase pressure-loss calculations. Mawla et al (1996) presented that, a Duns and Ros correlation is best for mist flow regime (high gas velocity), but high water cut reduces its accuracy dramatically. Both Hagedorn and Brown, and Beggs and Brill correlations can give good results in case of high water cut.

Brusilovsky et al (2006) described an efficient approach to preparation of data used to generate reservoir fluid PVT relationships in "black oil" hydrodynamic models which was based on experimental and theoretical data generalization. The algorithm for sequential identification of parameters for the calculated reservoir fluid model was proposed.

Several graphical and mathematical correlations for determining the bubble point pressure (P_b) and the oil formation volume factor (B_{ob}) have been proposed. These correlations are essentially based on the assumption that P_b and B_{ob} are strong functions of the solution gas-oil ratio (R_s) the reservoir temperature (T), the gas and the oil specific gravity .

In 1947, Standing presented graphical correlations for the determination of bubble point pressure (P_b) and the oil formation volume factor (B_{ob}) In developing these correlations, Standing used 105 experimentally measured data points from 22 different crude-oil and gas mixtures from California oil fields. Average relative errors of 4.8% and of 1.17% were reported for P_b and B_{ob} respectively. Later, in 1958, Lasater[9] developed an empirical equation based on Henry's law for estimating the bubblepoint pressure. He correlated the mole fraction of gas in solution to a bubblepoint pressure factor. A total of 137 crude-oil and gas mixtures from North and South America was used for developing this correlation. An average error of 3.8% was reported.

In 1980, two sets of correlations were reported, one by Vasquez and Beggs and the other by Glasø. Vasquez and Beggs used 600 data points from various locations all over the world to develop correlations for P_b and B_{ob}. Two different types of correlations were presented, one for crudes with °API>30 and the other for crudes with °API 30. An average error of 4.7% was reported for their correlation of B_{ob} Glasø used a total of 45 oil samples from the North Sea to develop his correlations for calculating P_b and B_{ob}. He reported an average error of 1.28% for the bubble point pressure and 0.43% for the formation volume factor.

Al-Marhoun et al (2002) used 160 experimentally determined data points from the PVT analysis of 69 Middle Eastern hydrocarbon mixtures to develop his correlations. Average errors of 0.03% and 0.01% were reported for bubble point pressure (P_b) and Oil Formation Volume Factor (B_{ob}) respectively.

Dokla and Osman used a total of 50 data points from reservoirs in the United Arab Emirates to develop correlations for P_b and B_{ob}. They reported an average error of 0.45% for the bubble point pressure and 0.023% for the formation volume factor.

Ridha et al (1999) presented a PVT models based on a successfully trained artificial neural network ANN for Middle East crude oil to estimate P^b and B^{ob} as functions of R_s, T, γ_g, γ_o.

Recently, Ibrahim et al (2013) presents new Black Oil PVT properties correlations for volatile oil and gas condensate reservoir fluids. The new correlations s have the advantage of taking into consideration the effect of surface separator configuration and conditions. The correlations were developed using fourteen actual reservoir fluid samples spanning a wide range of fluid behavior and characteristics.

Chapter 3

Calculations Methodology and Mathematical Model

3.1.Procedures for Iterating on Pressure Increment

When calculating the pressure drop in any pipe, the calculation involves use of an iterative procedure. In calculating these traverse the flow conduit is divided to a number of pressure increment (ΔP) or length increment ΔL) then the fluid properties and pressure gradient are evaluated at average condition of pressure, temperature and pipe inclination angle in the increment. When dividing the flow string into number of length increment (ΔL) , the estimation of the average conditions require two values for pressure and temperature at the bagging and the end of each length; this values can be estimated easily for temperature; for pressure this step complicated the calculation as the procedure included try an error procedures; thus, a value for the pressure need to be assumed at the end of the first length then the average calculated. Then the fluids properties have to be calculated using empirical fluids properties correlations using the information available about the fluids properties such as separator gas gravity; and the stock tank oil gravity. The next step is that the pressure drop have to be calculated according to the selected method; then a new value for the pressure at the end of the first length estimated and compared with the assumed value. When the value was equal to assumed value, new calculation started for new length with increment ΔL. When the two values are different, the estimated value was taken as a new assumption and all the steps were repeated till the two value became equals. This steps require looping function till the required values were reached for that reason a tolerance are required.

The above procedures were followed to estimate the pressure drop with different multi phase flow correlations and PVT correlations. Fig 3.1 presented the flow chart for the computer program. the length increment (ΔL) was taken as The total length divided by 100; while the tolerance was taken as 0.005. The procedures can be summarized as follows

1. Starting with the Known Pressure P_1 at location 1 select a length increment, ΔL
2. Corresponding to the length increment, ΔL, estimate a pressure increment ΔP
3. Calculate the average pressure, and for non-isothermal case calculate the average temperature in the increment.

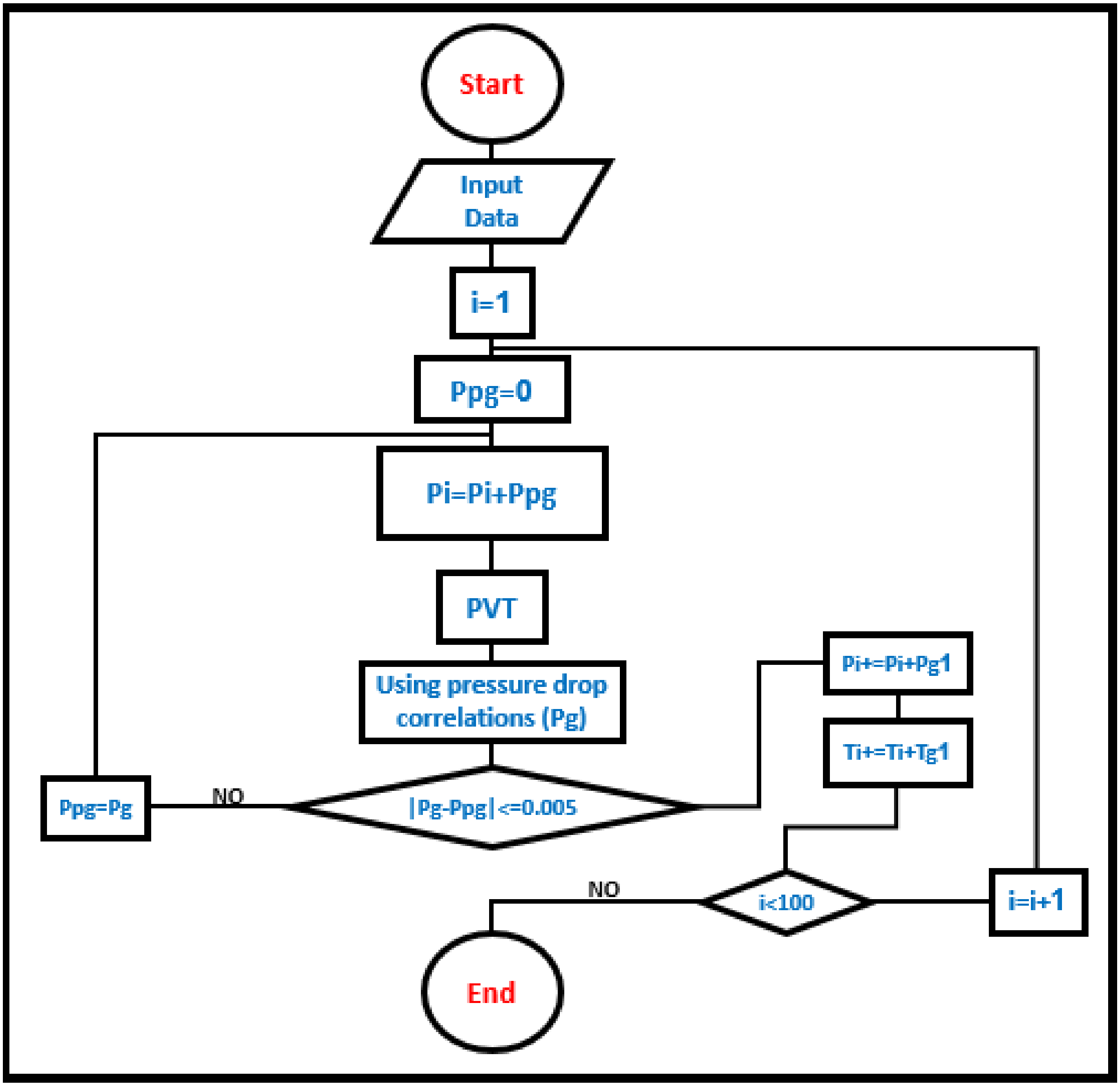


Fig 3.1 The Flow Chart of Pressure Drop Calculations based on Length Increment

4. At this condition of average pressure, and average temperature calculate the fluid and PVT properties from the laboratory data or the empirical correlations.
5. Using the appropriate pressure gradient correlation, calculate the pressure gradient in the increment at average conditions of pressure, temperature, and pipe inclination.
6. Calculate the pressure increment corresponding to the selected length increment,

$\Delta P = \Delta L / (dP / dL)$

7. Compare the estimated and calculated value of ΔP obtained in step2 and 6, if they are not sufficiently close, estimate a new pressure increment and go to step 3. Repeat step 3 through 7 until the estimated and calculated values are sufficiently close.
8. Set $L = L_1 + \sum \Delta L$, $P = P_1 + \sum \Delta P$
9. If L is less than the total conduit length, return to step 2

3.2.Predicting Flowing Temperatures

All the fluid property correlations presented previously require a value of fluid temperature' to calculate the required fluid property. The f1owing temperature profile in a gas well or an oil well is usually assumed to be linear between the surface temperature and the bottom hole temperature. A linear temperature profile is also usually assumed for surface flow-line calculations. The linear assumption for well flow will usually not introduce significant errors if a good value for surface flowing temperature can be obtained. The heat loss from a fluid in a pipe is a function of the mass flow rate in the pipe and will therefore change with a change in producing rate.

An algorithm for coupling pressure and heal loss calculations was presented earlier in this section. The iterative solution was necessary because both the overall heat transfer coefficient and the enthalpy change depend on pressure. If some average heat transfer coefficient can be determined, an approximate temperature profile can be calculated independently of the pressure 1055 calculation. This will of course be less accurate, but in many cases the amount of data available will not be sufficient to perform the more accurate calculation.

1. Flowing Temperature in wells

An equation for temperature in a well as a function of location L, as derived by (Ramey)can be written as:

$$T_L = T_I - g_T\{L - A(1 - EXP(-L/A)\} \quad \ldots\ldots (3.1)$$

When the equation is written in this form it assumes that the fluid and surroundings temperature are equal at the inlet to the pipe. This will be the case foe flowing wells, where TI is the reservoir temperature. Also included is the assumption that the heat loss is independent of time. This assumption limits application of Equation (3.1) to wells that have been producing foe a considerable length of time.

When multiphase flow is occurring in a well. The variables involved in evaluating the relaxation distance, A, are very difficult to determine, especially the overall heat transfer coefficient U. In view of this fact.(Shiu and Begges)developed an empirical method to estimate A based on measured temperature profiles from 270 wells. Using the measured temperatures TL at various locations L, a value of A for each test was calculated from Equation (3.1). An equation lo estimate A was then developed as a function of data which will usually be known. The equation is:

$$A = C_1 W^{C_2} P_L^{C_3} d^{C_4} (API)^{C_5} \gamma_g^{C_6} \quad (3.2)$$

$C_1 = 0.0149$

$C_2 = 0.5253$

$C_3 = 2.9303$

$C_4 = 0.2904$

$C_5 = 0.2608$

$C_6 = 4.4146$

Equation (3.2) is applicable for flowing oil wells only, although a similar approach could be used for gas wells if insufficient data are available to calculate A. Equation (3.2) has been found to give good results foe dry gas wells (no liquid production) by using values for liquid density and oil gravity of 62.4 and 50, respectively.

2. Flowing Temperature in Pipelines

To calculate a temperature profile in a pipeline, it is usually assumed that the temperature of the surroundings is constant. Modification of Equation (3.59) to account for this results in

$$T_L = T_S + (T_1 - T_S)EXP(-L/A) \quad (3.3)$$

where Ts is the surroundings temperature and the other variables are defined in Equation (3.1). For flow of gases, the (Joule-Thomson) effect may be included, but since this effect depends on pressure, an iterative solution is required. The more rigorous equation is:

$$T_L = T_S + \mu A(d_P/d_L) + [T_1 - T_S - \mu A(d_P/d_L)]EXP(-L/A) \quad (3.4)$$

As was discussed earlier the data necessary to calculate the heal transfer· coefficient U is seldom available. A simplified approach to estimating flowing temperatures in either wells or pipelines may be used if at least one measured set of inlet and outlet temperatures is available along with one measured flow rate. This approach can be used for oil and gas wells. A procedure is:

1) Using the measured temperatures and flow rates, solve the flowing temperature equation for A.

Equation (3.1) applies foe wells, while Equation (3.4) applies for pipelines.

2) Considering all the variables in A except flow rate lo be constant, solve for the constant.

$$A = \frac{wC_P}{\pi dU} = Cw \qquad (3.5)$$

Or

$$C = A/W \qquad (3.6)$$

Use this value of C to estimate a value for A for other flow rates.

3.3.Mathematical Models for PVT Calculation

3.3.1. Gas Compressibility Factor

1) Wichert and Aziz Correlation

Wichert and Aziz (1972) calculated the gas compressibility Or Z-factor is a function of the pseudo reduced pressure and temperature of the gas. Corrections to the pseudo critical pressures and temperatures can be made to account for impurities such as N1,CO" and H,S. The pseudo-reduced values are defined as:

$$P_{pr} = \frac{P}{P_{pc}} \qquad (3.7)$$

$$T_{pr} = \frac{T}{T_{pc}} \qquad (3.8)$$

If the gas composition is known, the pseudo critical are calculated from

$$P_{pc} = \sum_{i=1}^{N} y_i \, P_{ci} \qquad (3.9)$$

$$T_{pc} = \sum_{i=1}^{N} y_i \, T_{ci} \qquad (3.10)$$

If the gas composition is unknown, the pseudo critical may be estimated from:

$$T_{pc} = 170.5 + 307.3\gamma_g \qquad (3.11)$$

Brill and Beggs (1980) and modified by Standing which gives have been used as follows:

$$Z = A + (1 - A)EXP(-B) + CP_{pr}^{D} \qquad (3.12)$$

$$A = 1.39(T_{pr} - 0.92)^{0.5} - o.36T_{pr} - 0.101 \tag{3.12-a}$$

$$B = P_{pr}(0.62 - 0.23T_{pr}) + P_{pr}^{2}[0.066/(T_{pr} - 0.86) - 0.037] + 0.32P_{pr}^{6}/EXP[20.723(P_{pr} - 1)] \tag{3.13-b}$$

$$C = 0.132 - 0.32 \log T_{pr} \tag{3.13-c}$$

$$D = EXP(0.715 - 1.128T_{pr} + 0.42T_{pr}^{2}) \tag{3.13-d}$$

If the gas contains impurities, corrections can be made lo Ppc and Tpc according to (Wichcrt and Aziz-19723) as:

$$T_{pc} = T_{pc} - \varepsilon \tag{3.14}$$

$$P_{pc} = \frac{P_{pc}T_{pc}}{T_{pc}+\varepsilon(B-B^{2})} \tag{3.15}$$

$$\varepsilon = 120(A^{0.9} - A^{1.6}) + 15(B^{0.5} - B^{4}) \tag{3.16}$$

2) Hall and Yarborough Correlation

Hall and Yarborough (1973) presented an equation-of-state that accurately represents the Standing and Katz z-factor chart. The proposed expression is based on the Starling-Carnahan equation-of-state. The coefficients of the correlation were determined by fitting them to data taken from the Standing and Katz z-factor chart. Hall and Yarborough proposed the following mathematical form

$$Z = \left[\frac{0.06152P_{pr}t}{Y}\right] EXP[-1.2(1-t)^{2}] \tag{3.17}$$

$$F(Y) = X1 + \frac{Y+Y^{2}+Y^{3}+Y^{4}}{(1-Y)^{3}} - (X2)Y^{2} + (X3)Y^{X4} = 0 \tag{3.18}$$

WHERE

$$X1 = 0.06125P_{pr}texp[-1.2(1-t)^{2}] \tag{3.19-a}$$

$$X2 = (14.761 - 9.76t^{2} + 4.58t^{3}) \tag{3.19-b}$$

$$X3 = (90.7t - 242.2t^{2} + 42.4t^{3}) \tag{3.19-c}$$

$$X4 = (2.18 + 2.82t) \tag{3.19-d}$$

3) The Dranchuk-Abu-Kassem Method

Dranchuk and Abu-Kassem (1975) derived an analytical expression for calculating the reduced gas density that can be used to estimate the gas compressibility factor. The reduced gas density □r is defined as the ratio of the gas density at a specified pressure and temperature to that of the gas at its critical pressure or temperature, or

The critical gas compressibility factor Z is approximately 0.27 which leads to the following simplified expression for the reduced gas density:

$$\rho_r = \frac{0.27P_{pr}}{zT_{pr}} \ldots\ldots \quad (3.20)$$

The authors proposed the following eleven-constant equation-of-state for calculating the reduced gas density:

$$f(\rho_r) = (R_1)\rho_r - \frac{R_2}{\rho_r} + (R_3){\rho_r}^2 - (R_4){\rho_r}^5 + (R_5)(1 + A_{11}{\rho_r}^2)\exp[-A_{11}{\rho_r}^2] + 1 = 0 \ldots\ldots \quad (3.21)$$

With the coefficients, R1 through R5 as defined by the following relations:

$$R_1 = [A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_4}{T_{pr}^4} + \frac{A_5}{T_{pr}^5} \ldots\ldots \quad (3.22)$$

$$R_2 = [\frac{0.27P_{pr}}{T_{pr}}] \ldots\ldots \quad (3.23)$$

$$R_3 = [R_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2} \ldots\ldots \quad (3.24)$$

$$R_4 = A_9[\frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2}] \ldots\ldots \quad (3.25)$$

$$R_5 = [\frac{A_{10}}{T_{pr}^3}] \ldots\ldots \quad (3.26)$$

The constants A1 through A11 were determined by fitting the equation, using nonlinear regression models, to 1,500 data points from the Standing and Katz z-factor chart. The coefficients have the following values:

A1 =0.3265	A2 =1.0700	A3 =0.5339	A4 =0.01569
A5 =□0.05165	A6 =0.5475	A7 =0.7361	A8 =0.1844
A9 =0.1056	A10 =0.6134	A11 =0.7210	

4) The Dranchuk-Purvis-Robinson Method

Dranchuk, Purvis, and Robinson (1974) developed a correlation based on the Benedict-Webb-Rubin type of equation-of-state. Fitting the equation to 1,500 data points from the Standing and

Katz z-factor chart optimized the eight coefficients of the proposed equations. The equation has the following form:

$$1 + T_1 + T_2{\rho_r}^2 + T_3{\rho_r}^5 + [T_4{\rho_r}^2(1 + A_8{\rho_r}^2)\exp(-A_8{\rho_r}^2)] - \frac{T_5}{\rho_r} = 0. \ldots\ldots\ldots(3.27)$$

Where:

$$T_1 = [A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3}] \ldots\ldots\ldots(3.28)$$

$$T_2 = [A_4 + \frac{A_5}{T_{pr}}] \ldots\ldots\ldots(3.29)$$

$$T_3 = [\frac{A_5A_6}{T_{pr}}] \ldots\ldots\ldots(3.30)$$

$$T_4 = [\frac{A_7}{T_{pr}^3}] \ldots\ldots\ldots(3.31)$$

$$T_5 = [\frac{0.27P_{pr}}{T_{pr}}] \ldots\ldots\ldots(3.32)$$

The coefficients A1 through A8 have the following values:

A1 =0.31506237 A2 =1.0467099
A3 =0.57832720 A4 =0.53530771
A5 =0.61232032 A6 =0.10488813
A7 =0.68157001 A8 0.68446549

3.3.2. Solution or Dissolved Gas

1) Standing Correlation

Standing (1947) proposed a graphical correlation for determining the gas solubility as a function of pressure, gas specific gravity, API gravity, and system temperature. The correlation was developed from a total of 105 experimentally determined data points on 22 hydrocarbon mixtures from California crude oils and natural gases. The proposed correlation has an average error of 4.8%. Standing (1981) expressed his proposed graphical correlation in the following more convenient mathematical form

$$R_s = \gamma_g \left[\left(\frac{P}{18.2} + 1.4\right) \times 10^x\right]^{1.2048} \ldots\ldots\ldots(3.33)$$

Where:

$$x = 0.0125\ API - 0.00091(T - 460) \ldots\ldots\ldots(3.34)$$

It should be noted that Standing's equation is valid for applications at and below the bubble-point pressure of the crude oil.

2) Vasquez and Beggs Correlation

Vasquez and Beggs (1980) presented an improved empirical correlation for estimating Rs. The correlation was obtained by regression analysis using 5,008 measured gas solubility data points. Based on oil gravity, the measured data were divided into two groups. This division was made at a value of oil gravity of 30°API. The proposed equation has the following form:

$$R_s = C_1 \gamma_{gs} p^{C_2} exp\left[C_3\left(\frac{API}{T}\right)\right] \dots\dots\dots (3.35)$$

Where the constant C1, C2 and C3 was presented through table 3.1

Table 3.1 The Constant C1, C2 and C3 for Vasquez and Beggs Method (1980)

Coefficient	API≤30	API>30
C_1	0.0362	0.0178
C_2	1.0937	1.1870
C_3	25,7240	23.931

Realizing that the value of the specific gravity of the gas depends on the conditions under which it is separated from the oil, (Vasquez and Beggs) proposed that the value of the gas specific gravity as obtained from a separator pressure of 100 psig be used in the above equation. This reference pressure was chosen because it represents the average field separator conditions. The authors proposed the following relationship for adjustment of the gas gravity γg to the reference separator pressure:

$$\gamma_{gs} = \gamma_g \left[1 + 5.912 \times 10^{-5}\, API\, log\left(\frac{P_{sep}}{114.7}\right)\right] \dots\dots\dots (3.\ 36)$$

The gas gravity used to develop all the correlations reported by the authors was that which would result from a two-stage separation. The first-stage pressure was chosen as 100 psig and the second stage was the stock tank. If the separator conditions are unknown, the unadjusted gas gravity may be used in Equation above. An independent evaluation of the above correlation by(Sutton and Farashad (1984)) shows that the correlation is capable of predicting gas solubility with an average absolute error of 12.7%.

3) Glaso Correlation

Glaso (1980) proposed a correlation for estimating the gas solubility as a function of the API gravity, pressure, temperature, and gas specific gravity. The correlation was developed from studying 45 North Sea crude oil samples. Glaso reported an average error of 1.28% with a standard deviation

of 6.98%. The proposed relationship has the following form:

$$R_s = \gamma_g \left[(P_b^*) \left(\frac{API^{0.989}}{(T-460)^{0.172}} \right) \right]^{1.2255} \quad (3.37)$$

Where

P_b^* is a correlating number and is defined by the following expression:

$$P_b^* = 10^x \quad (3.38)$$

$$X = 2.8869 - [14.1811 - 3.3093\log(p)]^{0.5} \quad (3.39)$$

4) Marhoun Correlation

Marhoun (1988) developed an expression for estimating the saturation pressure of the Middle Eastern crude oil systems. The correlation originates from 160 experimental saturation pressure data. The proposed correlation can be rearranged and solved for the gas solubility to give:

$$R_s = \left[aT^d \gamma_g^c \gamma_o^d P \right]^e \quad (3.40)$$

Where:

a =185.843208

b =1.877840

c =-3.1437

d =-1.32657

e =1.398441

5) Petrosky and Farshad Correlation

Petrosky and Farshad (1993) used a nonlinear multiple regression software to develop a gas solubility correlation. The authors constructed a PVT database from 81 laboratory analyses from the Gulf of Mexico crude oil system. Petrosky and Farshad proposed the following expression:

$$R_S = \left[\left(\frac{P}{112.727} + 12.340\right)\gamma_g^{0,8439}10^x\right]^{1.73184} \tag{3.41}$$

$$X = 7.916 \times 10^{-4}API^{1.5410} - 4.561 \times 10^{-5}(T - 460)^{1.3911} \tag{3.42}$$

6) McCain Correlation

McCain (1991) pointed out that the weight average of separator and stock-tank gas specific gravities should be used for □g. The error in calculating Rs by using the equation will depend only on the accuracy of the available PVT data.The gas solubility can also be calculated rigorously from the experimental measured PVT data at the specified pressure and temperature .The following expression relates the gas solubility Rs to oil density, specific gravity of the oil, gas gravity, and the oil formation volume factor:

$$R_s = \frac{\beta_o\rho_o - 62.4\gamma_o}{0.0136\gamma_g} \tag{3.43}$$

3.3.3. Gas Solubility in Water

The following correlation can be used to determine the gas solubility in water:

$$R_{sw} = A + BP + CP^2 \tag{3.44}$$

Where

$$A = 2.12 + 3.45 \times 10^{-3}T - 3.59 \times 10^{-3}T^2 \tag{3.45}$$

$$B = 0.0107 - 5.26 \times 10^{-5}T + 1.48 \times 10^{-7}T^2 \tag{3.46}$$

$$C = 8.75 \times 10^{-7} + 3.9 \times 10^{-9}T - 1.02 \times 10^{-11}T^2 \tag{3.47}$$

The temperature T in above equations is expressed in °F.

3.3.4. Bubble-Point Pressure

The bubble-point pressure pb of a hydrocarbon system is defined as the highest pressure at which a bubble of gas is first liberated from the oil. This important property can be measured experimentally for a crude oil system by conducting a constant-composition expansion test. In the absence of the experimentally measured bubble-point pressure, it is necessary for the engineer to make an estimate of this crude oil property from the readily available measured producing parameters. Several graphical and mathematical correlations for determining pb have been proposed during the last four decades. These correlations are essentially based on the assumption that the bubble-point pressure is a strong function of gas solubility Rs, gas gravity γg, oil gravity API, and temperature T, or P_b=f (R_S, γg, API, T)

1) Standing Correlation

Standing (1947), based on 105 experimentally measured bubble-point pressures on 22 hydrocarbon systems from California oil fields, Standing (1947) proposed a graphical correlation for determining the bubble-point pressure of crude oil systems. The correlating parameters in the proposed correlation are the gas solubility Rs, gas gravity γ_g, oil API gravity, and the system temperature. The reported average error is 4.8%. In a mathematical form, Standing (1981) expressed the graphical correlation by the following expression

$$P_b = 18.2[\left(R_S/\gamma_g\right)^{0.83}10^a - 1.4] \qquad (3.48)$$

With

$$a = 0.00091(T - 460) - 0.0125(API) \qquad (3.49)$$

2) The Vasquez-Beggs Correlation

Vasquez and Beggs(1980) gas solubility correlation can be solved for the bubble-point pressure pb to give:

$$P_b = [\left(\frac{C_1 R_S}{\gamma_g}\right)10^a]^{C_2} \qquad (3.50)$$

With

$$a = C_3 API/T \qquad (3.51)$$

The gas specific gravity γgs at the reference separator pressure is defined by Equation 3.72. The coefficients C1, C2, and C3 have the following values:

Table 3.2 The Constant C1, C2 and C3 for Vasquez and Beggs Method (1980)

COEFFCIENT	API≤30	API>30
C1	27.624	56.18
C2	0.914328	0.84246
C3	11.172	10.393

3) Glaso's Correlation

Glaso (1980) used 45 oil samples, mostly from the North Sea hydrocarbon system, to develop an accurate correlation for bubble-point pressure prediction. Glaso proposed the following expression:

$$\log(P_b) = 1.7447\log(P_b^*) - 0.30218[\log(P_b^*)]^2 \ldots\ldots\ldots\ldots (3.52)$$

where p*b is a correlating number and defined by the following equation:

$$P_b^* = (\frac{R_S}{\gamma_g})^a (T)^b (API)^c \ldots\ldots\ldots\ldots (3.53)$$

a =0.816

b =0.172

c =-0.989

For volatile oils, Glaso recommends that the temperature exponent b of Equation 3.81 be slightly changed, to the value of 0.130.

4) Marhoun's Correlation

Marhoun (1988) used 160 experimentally determined bubble-point pressures from the PVT analysis of 69 Middle Eastern hydrocarbon mixtures to develop a correlation for estimating pb. The author correlated the bubble-point pressure with the gas solubility Rs, temperature T, and specific gravity of the oil and the gas. Marhoun proposed the following expression:

$$P_b = aR_S^b \gamma_g^c \gamma_o^d T^e \ldots\ldots\ldots\ldots (3.54)$$

a =5.38088× 10^{-3}	b =0.715082
c =-1.87784	d =3.1437
e =1.32657	

The reported average absolute relative error for the correlation is 3.66% when compared with the experimental data used to develop the correlation

5) The Petrosky-Farshad Correlation

Petrosky and Farshad(1991**)** gas solubility equation, i.e., Equation for R_S, can be solved for the bubble-point pressure to give:

$$P_b = \left[\frac{112.727R_S^{0.577421}}{\gamma_g^{0.8439}(10)^x}\right] - 1391.051 \ldots\ldots\ldots\ldots (3.55)$$

$$x = 7.916(10^{-4})(API)^{1.5410} - 4.561(10^{-5})(T-460)^{1.3911} \text{……………………………} (3.56)$$

3.3.5. Formation Volume Factor

The formation volume factor of a fluid is a convenient parameter to use for converting from standard volumes to actual or in-situ volumes existing at any pressure and temperature in the system. Equations are given for gas, oil and water.

A. Formation Volume Factor for Gas.

The gas formation volume factor is defined as the actual volume occupied by a given quantity of gas at some pressure and temperature, divided by the volume, which the gas would occupy at standard conditions. It is ca1culated from

$$B_{g=}\frac{P_{pc}ZT}{T_{pc}P} \text{……………………………………………………} (3.57)$$

For pressure in psia and temperature in °R, using Psc =14.7 psia and Tsc = 520 °R, Equation 3-78 becomes:

$$B_g = \frac{0.0283ZT}{P} \text{……………………………………………………} (3.58)$$

The reciprocal of the gas formation volume factor is called the gas expansion factor and is designated by the symbol Eg, or:

$$E_g = \frac{1}{\beta_g} \text{……………………………………………………} (3.59)$$

B. Formation Volume Factor for Oil

1) **Standing Method**

Standing (1947) presented a graphical correlation for estimating the oil formation volume factor with the gas solubility, gas gravity, oil gravity, and reservoir temperature as the correlating parameters. This graphical correlation originated from examining a total of 105 experimental data points on 22 different California hydrocarbon systems. An average error of 1.2% was reported for the correlation.

Standing (1981) showed that the oil formation volume factor can be expressed more conveniently in a mathematical form by the following equation:

$$\beta_O = 0.9759 + .000120\left[R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25(T-460)\right]^{1.2} \text{……………………………} (3.60)$$

2) Vasquez and Beggs Method

Vasquez and Beggs (1980) developed a relationship for determining Bo as a function of Rs, γ o, γ g, and T. The proposed correlation was based on 6,000 measurements of Bo at various pressures. Using the regression analysis technique, Vasquez and Beggs found the following equation to be the best form to reproduce the measured data:

$$\beta_O = 1.0 + C_1 R_s + (T - 520)\left(\frac{API}{\gamma_{gs}}\right)[C_2 + C_3 R_3] \ldots\ldots\ldots\ldots (3.61)$$

Table 3.3 The Constants for Formation Volume Factor of Oil by Vasquez and Beggs Method (1980)

Constant	API≤30	API>30
C1	4.677×10^{-4}	4.670×10^{-4}
C2	1.751×10^{-5}	1.100×10^{-5}
C3	-1.811×10^{-8}	1.337×10^{-9}

The oil formation volume factor decreases at pressures above the bubble point pressure and is calculated from:

$$\mathrm{B_o = B_{ob} RXP[C_0(P_b - P)]} \ldots\ldots\ldots\ldots (3.62)$$

3) Glaso Method

Glaso (1980) proposed the following expressions for calculating the oil formation volume factor

$$\beta_o = 1.0 + 10^{A} \ldots\ldots\ldots\ldots (3.63)$$

$$A = -6.58511 + 2.91329 log\beta^*_{ob} - 0.27683(log\beta^*_{ob})^2 \ldots\ldots\ldots\ldots (3.64)$$

B*ob is a *correlating number* and is defined by the following equation:

$$\beta^*_{ob} = R_S\left(\frac{\gamma_g}{\gamma_o}\right)^{0{,}526} + 0.968(\mathrm{T} - 460) \ldots\ldots\ldots\ldots (3.65)$$

The above correlations were originated from studying PVT data on 45 oil samples. The average error of the correlation was reported at 0.43% with a standard deviation of 2.18%.

4) Sutton and Farshad Method

Sutton and Farshad (1984) concluded that Glaso's correlation offers the best accuracy when compared with the Standing and Vasquez-Beggs correlations. In general, Glaso's correlation under predicts formation volume factor. Standing's expression tends to over predict oil formation volume factors greater than 1.2 bbl/STB. The Vasquez-Beggs correlation typically over predicts the oil formation volume factor.

5) Marhoun Method

Marhoun (1988**)** developed a correlation for determining the oil formation volume factor as a function of the gas solubility, stock-tank oil gravity, gas gravity, and temperature. The empirical equation was developed by use of the nonlinear multiple regression analysis on 160 experimental data points. The experimental data were obtained from 69 Middle Eastern oil reserves. The author proposed the following expression:

$$\beta_O = 0.497069 + 0.862963 \times 10^{-3}T + 0.182594 \times 10^{-2}F + 0.318099 \times 10^{-5}F^2 \text{…..... (3.66)}$$

with the correlating parameter F as defined by the following equation:

$$F = R_s^a \gamma_g^b \gamma_o^c \text{………………………………………………………...................….. (3.67)}$$

The coefficients a, b and c have the following values:

a =0.742390

b =0.323294

c =-1.202040

6) Petrosky and Farshad Method

Petrosky and Farshad (1993) proposed a new expression for estimating Bo. The proposed relationship is similar to the equation developed by Standing; however, the equation introduces three additional fitting parameters in order to increase the accuracy of the correlation The authors used a nonlinear regression model to match experimental crude oil from the Gulf of Mexico hydrocarbon system. Their correlation has the following form:

$$\beta_O = 1.0113 + 7.2046 \times 10^{-5}\left[R_s^{0.3738}\left(\frac{\gamma_g^{0.2914}}{\gamma_o^{0.6265}}\right) + 0.24626(T - 460)^{0.5371}\right] \text{…...…. (3.68)}$$

C. Formation Volume Factor for Water

the equation given in the HP Petroleum Fluids Pac is

$$\mathrm{B_W} = \mathrm{B_{wp}}(1 + \mathrm{XY} \times 10^4) \text{………………………………………….......................….. (3.69)}$$

$$\mathrm{X} = 5.1 \times 10^{8}\mathrm{P} + (\mathrm{T} - 60)(5.47 \times 10^{-6} - 1.95 \times 10^{-10}\mathrm{P}) + (\mathrm{T} - 60)^{2}(-3.23 \times 10^{8} + 8.5 \times 10^{-13}\mathrm{P}) \quad (3.70)$$

$$\mathrm{B_{wp}} = \mathrm{C_1} + \mathrm{C_2P} + \mathrm{C_3P^2} \dots\dots \quad (3.71)$$

$$\mathrm{C_1} = 0.9911 + 6.35 \times 10^{-5}\mathrm{T} + 8.5 \times 10^{-7\mathrm{T}^2} \dots\dots \quad (3.72)$$

$$\mathrm{C_2} = 1.093 \times 10^{-6} - 3.497 \times 10^{-9}\mathrm{T} + 4.57 \times 10^{-12\mathrm{T}^2} \dots\dots \quad (3.73)$$

$$\mathrm{C_3} = -5 \times 10^{-11} + 6.429 \times 10^{-13}\mathrm{T} - 1.43 \times 10^{-15\mathrm{T}^2} \dots\dots \quad (3.74)$$

The water formation volume factor can be calculated by the following mathematical expression:

$$\beta_w = A_1 + A_2P + A_3P^2 \dots\dots \quad (3.75)$$

where the coefficients A1 to A3 are given by the following expression:

$$A_i = a_1 + a_2(T - 460) + a_3(T - 460)^2 \dots\dots \quad (3.76)$$

with a1–a3 given for gas-free and gas-saturated water thr64gh table 3.4

Table 3.4 The Constants for Formation Volume Factor of Water

A_i	**Gas-Free Water**			**Gas-saturated Water**		
	a_1	a_2	a_3	a_1	a_2	a_3
A_1	0.9947	5.8×10^{-6}	1.02×10^{-6}	0.9911	6.35×10^{-5}	8.5×10^{-7}
A_2	-4.228×10^{-6}	1.8376×10^{-6}	-6.77×10^{-6}	-1.093×10^{-6}	-3.497×10^{-8}	4.57×10^{-12}
A_3	1.3×10^{-10}	-1.3855×10^{-6}	4.285×10^{-6}	-5.0×10^{-11}	6.429×10^{-13}	-1.43×10^{-15}

3.3.6. Isothermal Compressibility

A) Isothermal Compressibility of Crude Oil

1) Vasquez and Beggs Method

Vasquez and Beggs (1980) From a total of 4,036 experimental data points used in a linear regression model, Vasquez and Beggs (1980) correlated the isothermal oil compressibility coefficients with Rs, T, °API, γg, and p. They proposed the
following expression:

$$C_o = \frac{-1{,}433+5R_S+17.2(T-460)-1{,}180\gamma_{gs}+12.62API}{10^5P} \dots\dots \quad (3.77)$$

2) Petrosky and Farshad Method

Petrosky and Farshad (1993) proposed a relationship for determining the oil compressibility for under saturated hydrocarbon systems. The equation has the following form:

$$C_o = 1.705 \times 10^{-7} R_{sb}^{0.69357} \gamma_g^{0.1885} API^{0.3272} (T-460)^{0.6729} P^{-0.5906} \quad (3.78)$$

B) Water Isothermal Compressibility

Brill and Beggs Method

Brill and Beggs (1978) Proposed the following equation for estimating water isothermal compressibility, ignoring the corrections for dissolved gas and solids:

$$C_w = (C_1 + C_2 T + C_3 T^2) \times 10^{-6} \quad (3.79)$$

Where

$C1 = 3.8546 - 0.000134P$

$C2 = -0.01052 + 4.77 \times 10^{-7} P$

$C3 = 3.9267 \times 10^{-5} - 8.8 \times 10^{-10} P$

C) Compressibility of the Natural Gas

1) Standing and Katz (1942)

Standing and Katz (1942) Knowledge of the variability of fluid compressibility with pressure and temperature is essential in performing many reservoir engineering calculations. For a liquid phase, the compressibility is small and usually assumed to be constant. For a gas phase, the compressibility is neither small nor constant. By definition, the isothermal gas compressibility is the change in volume per unit volume for a unit change in pressure or, in equal:

$$C_g = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \quad (3.80)$$

From the real gas equation-of-state:

$$V = \frac{nRTZ}{P} \quad (3.81)$$

Differentiating the above equation with respect to pressure at constant temperature T gives:

$$\left(\frac{\partial V}{\partial P}\right)_T = nRT\left[\frac{1}{P}\left(\frac{\partial V}{\partial P}\right) - \frac{Z}{P^2}\right] \quad (3.82)$$

Substituting into Equation 3.44 produces the following generalized
relationship:

$$C_g = \frac{1}{P} - \frac{1}{Z}\left(\frac{\partial Z}{\partial P}\right)_T \quad (3.83)$$

For an ideal gas, z =1 and$(\frac{\partial Z}{\partial P})_T$=0, therefore:

$$C_g = \frac{1}{P} \qquad (3.84)$$

It should be pointed out that Equation (3.84) is useful in determining the expected order of magnitude of the isothermal gas compressibility. Equation (3.83) can be conveniently expressed in terms of the pseudo reduced pressure and temperature by simply replacing p with (P_{pc}, P_{pr}), or:

$$C_g = \frac{1}{P_{pr}P_{PC}} - \frac{1}{Z}[\frac{\partial Z}{\partial P_{Pr}P_{PC}}]_{T_{Pr}} \qquad (3.85)$$

Multiplying the above equation by P_{pc}yields:

$$C_g P_{PC} = C_{Pr} = \frac{1}{P_{pr}} - \frac{1}{Z}[\frac{\partial Z}{\partial P_{Pr}}]_{T_{Pr}} \qquad (3.86)$$

The term cpr is called the isothermal pseudo-reduced compressibility

and is defined by the relationship

$$C_{Pr} = C_g P_{PC} \qquad (3.87)$$

Values of $[\frac{\partial Z}{\partial P_{Pr}}]_{T_{Pr}}$can be calculated from the slope of the Tpr isotherm on the Standing and Katz z-factor chart.

2) Matter, Brar, and Aziz Method

Matter, Brar, and Aziz (1975) presented an analytical technique for calculating the isothermal gas compressibility. The authors expressed cpr as a function of $\partial P / \partial \rho_r$ rather than $\partial P / \partial P_r$. Equation (3.77) is differentiated with respect to ppr to give:

$$\left[\frac{\partial Z}{\partial P_{pr}}\right] = \frac{0.27}{Z\, T_{pr}}\left[\frac{\left(\partial Z / \partial \rho_r\right)_{T_{pr}}}{1+\frac{\rho r}{Z}\left(\partial Z / \partial \rho_r\right)_{T_{pr}}}\right] \qquad (3.88)$$

pseudo-reduced compressibility as:

$$C_{pr} = \frac{1}{P_{pr}} - \frac{0.27}{Z^{\,2} T_{pr}}\left[\frac{\left(\partial Z / \partial \rho_r\right)_{T_{pr}}}{1+\frac{\rho r}{Z}\left(\partial Z / \partial \rho_r\right)_{T_{pr}}}\right] \qquad (3.89)$$

The partial derivative appearing in Equation (3.79) is obtained and give:

$$\left(\partial Z/_{\partial \rho_r}\right)_{T_{pr}} = T_1 + 2T_2\rho_r + 5T_3\rho_r^4 + 2T_4\rho_r(1 + A_g\rho_r^2 - A_g^2\rho_r^4) \times \exp(A_g\rho_r^2 \text{………..} \quad (3.90)$$

Where the coefficients T1 through T4 and A1 through A8 are defined previously by previous equation.

3.3.7. Viscosity

To calculate the losses due to viscous shear friction a value foe the viscosity of the fluids is required. Calculation of a Reynolds number always requires viscosity. Equations are presented for the viscosity of oil, both above and below bubble point, for water and for natural gas

A. oil Viscosity

1) Begges and Robinson Method

Equations for oil viscosity were presented by Begges and Robioson **(**1975) For $P \leq P_b$

$$\mu_o = A\mu_{oD}^B \text{……} \quad (3.91)$$

$$\mu_{oD} = 10^x - 1.0 \text{……} \quad (3.92)$$

$$X = YT^{-1.163} \text{……} \quad (3.93)$$

$$Y = 10^Z \text{……} \quad (3.94)$$

$$Z = 3.0324 - 0.0203(API) \text{……} \quad (3.95)$$

$$A = 10.715(R_s + 150)^{-0.515} \text{……} \quad (3.96)$$

$$B = 5.44(R_s + 150)^{-0.338} \text{……} \quad (3.97)$$

For Pressers greater than Pb.

$$\mu_o = \mu_{ob}(P/_{P_b})^m \text{……} \quad (3.98)$$

$$m = C_2P^{C2}EXP(C_3 + C_AP) \text{……} \quad (3.99)$$

$$C_1 = 2.6$$

$$C_2 = 1.187$$

$$C_3 = -11.513$$

$$C_A = -8.98 \times 10^{-5}$$

2) Beal's Method

From a total of 753 values for dead-oil viscosity at and above 100°F, Beal (1946) developed a graphical correlation for determining the viscosity of the dead oil as a function of temperature and the API gravity of the crude. Standing (1981) expressed the proposed graphical correlation in a mathematical relationship as follows:

$$\mu_{od} = \left(0.32 + \frac{1.8(10^7)}{API^{4.53}}\right)\left(\frac{360}{T-260}\right)^{a} \quad (3.100)$$

With

$$a = 10^{\left(0.43+\frac{8.33}{API}\right)} \quad (3.101)$$

3) The Beggs-Robinson Correlation

Beggs and Robinson (1975) developed an empirical correlation for determining the viscosity of the dead oil. The correlation originated from analyzing 460 dead-oil viscosity measurements. The proposed relationship is expressed mathematically as follows:

$$\mu_{od} = 10^{X} - 1 \quad (3.102)$$

Where

$$X = Y(T-460)^{-1.163} \quad (3.103)$$

$$Y = 10^{Z} \quad (3.104)$$

$$Z = 3.0324 - 0.02023API \quad (3.105)$$

An average error of 0.64% with a standard deviation of 13.53% was reported for the correlation when tested against the data used for its development. Sutton and Farshad (1980) reported an error of 114.3% when the correlation was tested against 93 cases from the literature.

4) Glaso's Correlation

Glaso (1980) proposed a generalized mathematical relationship for computing the dead-oil viscosity. The relationship was developed from experimental measurements on 26 crude oil samples. The correlation has the following form:

$$\mu_{od} = [3.141(10^{10})](T-460)^{-3.444}[\log(API)]^{a} \quad (3.106)$$

Where

$$a = 10.313[\log(T - 460)] - 36.447 \quad (3.107)$$

The above expression can be used within the range of 50–300°F for the system temperature and 20–48° for the API gravity of the crude. Sutton and Farshad (1986) concluded that Glaso's correlation showed the best accuracy of the three previous correlations.

I. Saturated Oil Viscosity

1) The Chew-Connally Correlation

Chew and Connally (1959) presented a graphical correlation to adjust the dead-oil viscosity according to the gas solubility at saturation pressure. The correlation was developed from 457 crude oil samples. Standing (1977) expressed the correlation in a mathematical form as follows:

$$\mu_{ob} = 10^{a}(\mu_{od})^{b} \quad (3.108)$$

With

$$a = R_S[2.2(10^{-7})R_S - 7.4(10^{-4})] \quad (3.109)$$

$$b = \frac{0.68}{10^{c}} + \frac{0.25}{10^{d}} + \frac{0.062}{10^{e}} \quad (3.110)$$

$$c = 8.62(10^{-5})R_s$$

$$d = 1.1(10^{-3})R_s$$

$$e = 3.74(10^{-3})R_s$$

The experimental data used by Chew and Connally to develop their correlation encompassed the following ranges of values for the independent variables:

Pressure, psia: 132–5,645

Temperature, °F: 72–292

Gas solubility, scf/STB: 51–3,544

Dead oil viscosity, cp: 0.377–50

2) Beggs-Robinson

Beggs-Robinson(1975) From 2,073 saturated oil viscosity measurements, Beggs and Robinson (1975) proposed an empirical correlation for estimating the saturated-oil viscosity. The proposed mathematical expression has the following form:

$$\mu_{ob} = a(\mu_{od})^{b} \quad (3.111)$$

Where

$$a = 10.715(R_S + 100)^{-0.515} \tag{3.112}$$

$$b = 5.44(R_S + 150)^{-0.338} \tag{3.113}$$

The reported accuracy of the correlation is 1.83% with a standard deviation of 27.25%. The ranges of the data used to develop Beggs and Robinson's equation are:

Pressure, psia: 132–5,265
Temperature, °F: 70–295
API gravity: 16–58
Gas solubility, scf/STB: 20–2,070

II. Viscosity of Under Saturated Oil

Vasquez-Beggs Method

From a total of 3,593 data points, Vasquez and Beggs (1980) proposed the following expression for estimating the viscosity of under saturated crude oil:

$$\mu_o = \mu_{ob}(\frac{P}{P_b})^m \tag{3.114}$$

$$m = 2.6P^{1.187}10^a \tag{3.115}$$

$$a = 3.9(10^{-5})P - 5 \tag{3.116}$$

The data used in developing the above correlation have the following ranges:

Pressure, psia: 141–9,151
Gas solubility, scf/STB: 9.3–2,199
Viscosity, cp: 0.117–148
Gas gravity: 0.511–1.351
API gravity: 15.3–59.5

The average error of the viscosity correlation is reported as 7.54%.

B. Water Viscosity (Meehan 1980)

A graphical correlation for water viscosity which was published by(Matthews and Russe) has been converted to equation form by Meehan (1980). Is The correlation accounts for both the effects of pressure and salinity.

$$\mu = \mu_{wD}[1 + 3.5 \times 10^{-2}P^2(T - 40)] \tag{3.117}$$

$$\mu_{wD=}A + {}^{B}/_{T} \tag{3.118}$$

$$\mathrm{A} = -4.518 \times 10^{-2} + 9.313 \times 10^{-7}\mathrm{Y} - 3.93 \times 10^{-12}\mathrm{Y}^2 \quad (3.119)$$

$$\mathrm{B} = 70.634 + 9.576 \times 10^{-10}\mathrm{Y}^2 \quad (3.120)$$

Brill and Beggs (1978) presented a simpler equation, which considers only temperature effects:

$$\mu_w = \exp(1.003 - 1.479 \times 10^{-2}T + 1.982 \times 10^{-5}T^2 \quad (3.121)$$

C. Gas viscosity

1) (Begges and Robinson 1975)

the most widely used method to estimate gas viscosity was presented by(Lee el) The equation is applicable to natural gases containing impurities if the corrected Z-factor is used to calculate the value of gas density required in the equation

$$\mu_{g=}\mathrm{A} \times 10^{-4}\mathrm{EXP}\left(\mathrm{B}\rho_g^c\right) \quad (3.122)$$

Where

$$\mathrm{A} = (9.4 + 0.02\mathrm{M})\mathrm{T}^{1.5}/(209 + 19\mathrm{M} + \mathrm{T}) \quad (3.123)$$

$$\mathrm{B} = 3.5 + 0.01\mathrm{M} + \frac{986}{\mathrm{T}} \quad (3.124)$$

$$\mathrm{C} = 2.4 - 0.2\mathrm{B} \quad (3.125)$$

$$\rho_g = \frac{0.433\gamma_g \mathrm{P}}{\mathrm{ZT}} \quad 3.126)$$

2) The Carr-Kobayashi-Burrows Correlation Method

Carr, Kobayashi, and Burrows (1954) developed graphical correlations for estimating the viscosity of natural gas as a function of temperature, pressure, and gas gravity. The computational procedure of applying the proposed correlations is summarized in the following steps:

Step 1. Calculate the pseudo-critical pressure, pseudo-critical temperature, and apparent molecular weight from the specific gravity or the composition of the natural gas. Corrections to these pseudocritical properties for the presence of the nonhydrocarbon gases (CO2, N2, and H2S) should be made if they are present in concentrations greater than 5 mole percent.

Step 2. Obtain the viscosity of the natural gas at one atmosphere and the temperature of interest from Figure 3.2. This viscosity, as denoted by 1, must be corrected for the presence of non hydrocarbon components by using the inserts of Figure 3.2. The non hydrocarbon fractions tend to increase the viscosity of the gas phase. The effect of non hydrocarbon components on the viscosity of the natural gas can be expressed mathematically by the following relationships:

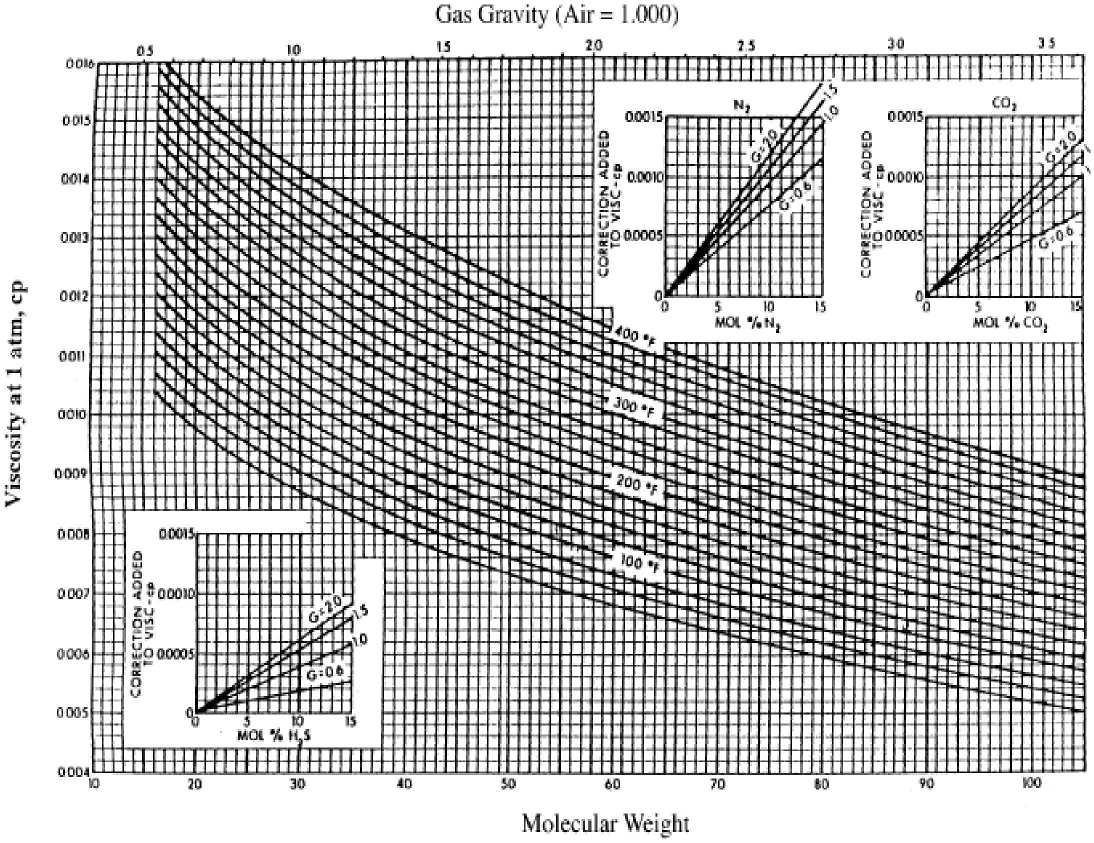


Fig 3,2 viscosity of the Natural Gas at one Atmosphere and the temperature

Step 3. Calculate the pseudo-reduced pressure and temperature.

Step 4. From the pseudo-reduced temperature and pressure, obtain the viscosity ratio from Figure 3.6. The term γg represents the viscosity of the gas at the required conditions.

Step 5. The gas viscosity, γg, at the pressure and temperature of interest is calculated by multiplying the viscosity at one atmosphere and system temperature, 1, by the viscosity ratio.

3) Lee, Gonzalez, and Eakin (1966)

presented a semi-empirical relationship for calculating the viscosity of natural gases. The authors expressed the gas viscosity in terms of the reservoir temperature, gas density, and the molecular weight of the gas. Their proposed equation is given by

$$\mu_g = 10^{-4} K exp[X \left(\frac{\rho_g}{62.4}\right)^{Y}] \quad \text{(3.127)}$$

$$K = \frac{(9.4+0.02M_a)T^{1.5}}{209+19M_a+T} \quad \text{(3.128)}$$

$$X = 3.5 + \frac{986}{T} + 0.01M_a \quad \text{(3.129)}$$

$$Y = 2.4 - 0.2M_a \quad \text{(3.130)}$$

3.3.8. Interfacial Tension

The interfacial tension existing between the gas and liquid phases has very little effect on two-phase pressure gradient calculations. However, some of the pressure gradient prediction methods require a value for interfacial tension to use in calculating certain dimensionless numbers~ Empirical graphs for estimating the gas/oil interfacial tension were presented by (Baker and Swcrdloffl- 1951) and graphs for gas/water interfacial tension were published by (Hough and Regression)analysis was used to fit equations to these graphs for specific temperatures. The effect of temperature can be estimated by linear interpolation.

A. Gas/0il Interfacial Tension. Graphs were presented for dead oil interfacial tension measured at temperatures at 68°f and 100°F. Equations which fit these graphs are:

$$\sigma_{68} = 39 - 0.2571(API) \quad \text{(3.131)}$$

$$\sigma_{100} = 37.5 - 0.2571(API) \quad \text{(3.132)}$$

It has been suggested that if the temperature is greater than 100°f, the value al 100°F should be used. Also, if T< 68, use the value calculated at T = 68. For intermediate temperatures, use linear interpolation between the values obtained at 68 and 100°f. That is:

$$\sigma_T = 68 - \frac{(T-68)(\sigma_{68}-\sigma_{100})}{32} \quad \text{(3.133)}$$

The effect of gas going into solution as pressure is increased 00 the gas/oil mixture is to reduce the interfacial tension. The dead oil interfacial tension can be corrected by multiplying it by the following correction factor.

$$C = 1.0 - 0.024P^{0.45} \quad \text{(3.134)}$$

The interfacial tension at an)' pressure is then obtained from:

$$\sigma_o = C\sigma_T \quad (3.135)$$

The interfacial tension becomes zero at miscibility pressure, and for most systems this will be at any pressure greater than about 5000 psia. Equation (3.135) will give a value of zero al a pressure of 3977 psia. If this occurs, a limiting value at I dyne/cm should be used to calculate the dimensionless numbers in the following section.

B. Gas Water Interfacial Tension.

Equations were fitted to graphs of interfacial tension versus pressure at two temperatures. These equations are:

$$\sigma_{w(74)} = 75 - 1.081P^{4.349} \quad (3.136)$$

$$\sigma_{w(280)} = 53 - 0.1048P^{0.637} \quad (3.137)$$

The same limitations 00 temperature as stated for the gas/oil case apply for gas/water interfacial tension for interpolation purposes. That is, for 74 < T< 280:

$$\sigma_{w(T)} = \sigma_{w(74)} - \frac{(T-74)(\sigma_{w(74)}-\sigma_{w(280)})}{206} \quad (3.138)$$

Sugden (1924) suggested a relationship that correlates the surface tension of a pure liquid in equilibrium with its own vapor. The correlating parameters of the proposed relationship are molecular weight M of the pure component, the densities of both phases, and a newly introduced temperature independent parameter Pch. The relationship is expressed mathematically in the following form:

$$\sigma = [\frac{P_{ch}(\rho_l-\rho_v)^4}{M}] \quad (3.139)$$

where σ is the surface tension and Pch is a temperature independent parameter and is called the *parachor.*

The parachor is a dimensionless constant characteristic of a pure compound and is calculated by imposing experimentally measured surface tension and density data on Equation 3.139 and solving for Pch. The Parachor values for a selected number of pure compounds are given in Table 3.5 as reported by Weinaug and Katz (1943).

Fanchi (1985) correlated the parachor with molecular weight with a simple linear equation. This linear is only valid for components heavier than methane. Fanchi's linear equation has the following form:

$$(P_{ch})_i = 69.9 + 2.3M_i \quad (3.140)$$

Table 3.5 Parachor values for pure compounds as reported by Weinaug and Katz (1943).

Component	Parachor	Component	Parachor
Co2	78.0	n-C4	189,9
N2	41.0	i-C5	225.0
C1	77.0	n-C5	231.5
C2	108.0	n-C6	271.0
C3	150.3	n-C7	312.5
i-C4	181.5	n-C8	351.5

For a complex hydrocarbon mixture, Katz et al. (1943) employed the Sudden correlation for mixtures by introducing the compositions of the two phases into Equation 3.124. The modified expression has the following form:

$$\sigma^{\frac{1}{4}} = \sum_{i=1}^{n}\left[(P_{ch})_i\left(A_{x_i} - B_{y_i}\right)\right] \qquad (3.141)$$

with the parameters A and B as defined by:

$$A = \frac{\rho_o}{62.4M_o} \qquad (3.142)$$

$$B = \frac{\rho_g}{62.4M_g} \qquad (3.143)$$

3.4.Mathematical Models for Pressure Drop Calculations

3.4.1. Orkiszewski Method (1967)

Flow pattern and pressure gradient prediction

$$\lambda_{g_{B/S}} = L_B \qquad (3.144)$$

$$L_B = 1.071 - \left(0.2218 v_m^2 / d\right) \text{and } L_B \geq 0.13 \qquad (3.145)$$

Bubble Flow:

Bubble flow exists if

$$\lambda_g \leq \lambda_{g_{B/S}} \qquad (3.146)$$

Liquid holdup for bubble flow is determined from:

$$H_L = 1 - 0.5\left[1 + \frac{v_m}{v_s} - \sqrt{(1 + v_m / v_s)^2 - 4v_{sg} / v_s}\right] \qquad (3.147)$$

The v_s have a constant value of 0.8 ft/sec.

The friction pressure-gradient component for bubble flow is given by

$$\left(\frac{dp}{dZ}\right)_f = \frac{f\rho_L(v_{SL}/H_L)^2}{2d} \tag{3.148}$$

The friction factor is calculate from moody diagram by Reynold number

$$N_{Re} = \frac{\rho_L(v_{SL}/H_L)d}{\mu_l} \tag{3.149}$$

The acceleration pressure-gradient component for bubble flow was considered negligible.

Slug flow:

Slug flow exists if

$$\lambda_g > \lambda_{g_{B/S}}, and\ N_{gv} < N_{gv_{S/Tr}}$$

$$N_{gv_{S/Tr}} = 50 + 36N_{Lv} \tag{3.150}$$

the slip density is calculated from

$$\rho_s = \frac{\rho_L(v_{sL} + v_b) + \rho_g v_{sg}}{v_m + v_b} + \rho_L\Gamma \tag{3.151}$$

$$v_b = C_1C_2\sqrt{gd} \tag{3.152}$$

$$N_{Re_b} = \frac{\rho_L v_b d}{\mu_L} \tag{3.153}$$

$$N_{Re_L} = \frac{\rho_L v_m d}{\mu_L} \tag{3.154}$$

And the c1 and c2 can be calculated from Griffith and wallis correlation from Fig. (3.3) and Fig. (3.4)

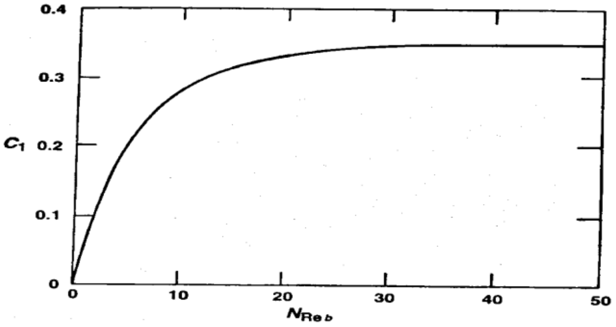


Fig 3.3 The Coefficient C1 of Orkiszewski Correlation (Orkiszewski 1967)

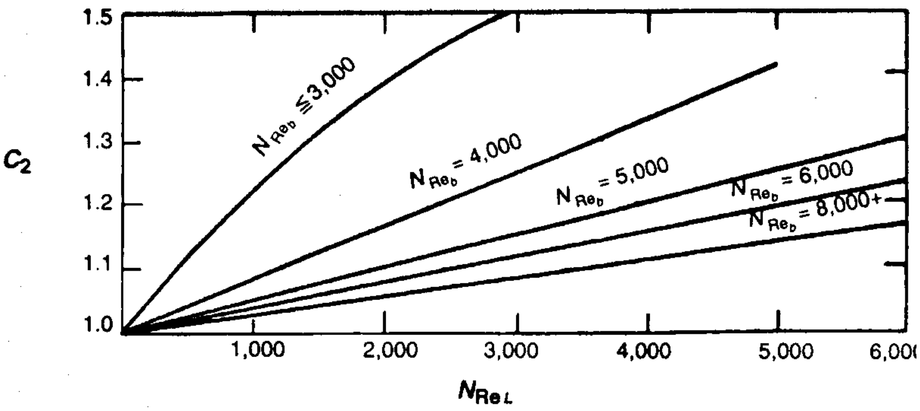


Fig 3.4 The Coefficient C2 of Orkiszewski Correlation (Orkiszewski 1967)

When $N_{Re_b} \leq 3000$

$$v_b = \left(0.546 + 8.74 * 10^{-6} N_{Re_L}\right)\sqrt{gd} \qquad (3.155)$$

When $N_{Re_b} \geq 8000$

$$v_b = \left(0.35 + 8.74 * 10^{-6} N_{Re_L}\right)\sqrt{gd} \qquad (3.156)$$

When $3000 < N_{Re_b} < 8000$

$$v_b = \frac{1}{2}\left(v_{bs} + \sqrt{v_{bs}^2 + \frac{13.59\mu_L}{\rho_L\sqrt{d}}}\right) \qquad (3.157)$$

$$v_{bs} = \left(0.251 + 8.74 * 10^{-6} N_{Re_L}\right)\sqrt{gd} \qquad (3.158)$$

Because the v_b and N_{Re_l} are interrelated

1. Estimate a value of Vb. A good first guess is

   $v_b = 0.5\,(g\,d)^{0.5}$

2 . Calculate NReb using the value of V'b from Step l.

3 . Calculate Vb using previous equations

4. Compare the values of obtained in Steps 1 and 3. If they are not sufficiently close, use the value in Step 3 as the next guess and go to Step l. Continue until convergence is achieved

The friction pressure-gradient component for slug flow is

$$\left(-\frac{\mathrm{d}P}{\mathrm{d}Z}\right)_f = \frac{f_{tp}\,\rho_L\,v_m^2}{2g_c d}\left[\frac{v_{sL} + v_b}{v_m + v_b} + \Gamma\right] \qquad (3.159)$$

Where ⌈ is liquid distribution coefficient

$$\Gamma = \frac{0.013\log\mu_L}{d^{1.38}} - 0.681 + 0.232\log v_m - 0.428\log d\text{-} \qquad (3.160)$$

$$\Gamma = \frac{0.045 \log \mu_L}{d^{0.799}} - 0.709 - 0.162 \log v_m - 0.888 \log d \quad (3.161)$$

$$\Gamma = \frac{0.0127 \log(\mu_L + 1)}{d^{1.415}} - 0.284 + 0.167 \log v_m + 0.113 \log d \quad (3.162)$$

$$\Gamma = \frac{0.0274 \log(\mu_L + 1)}{d^{1.371}} + 0.161 + 0.569 \log d + x \quad (3.163)$$

$$x = -\log v_m \left[\frac{0.01 \log(\mu_L + 1)}{d^{1.571}} + 0.397 + 0.63 \log d\right] \quad (3.164)$$

If $v_m < 10$

$$\Gamma \geq -0.065 v_m$$

And

If $v_m > 10$

$$\Gamma \geq -\frac{v_b}{v_m + v_b}\left(1 - \frac{\rho_S}{\rho_L}\right)$$

Pressure gradient due to acceleration is negligible in slug flow

Transition and mist flow :

Transition (Churn) Flow Limits: $L_s < N_{gv} < L_m$

The same as Duns and Ros method.

Annular-Mist Flow Limits: $N_{gv} > L_m$

The same as Duns and Ros method.

Where:

$$L_s = 50 + 36\, N_{Lv} \quad (3.165)$$

and $$L_m = 75 + 84\, N_{Lv}^{0.75} \quad (3.166)$$

Liquid Velocity Number:

$$N_{Lv} = v_{sL}\left(\frac{\rho_L}{g\sigma_L}\right)^{0.25} = 1.938\, v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25} \quad (3.167)$$

Gas Velocity Number:

$$N_{gv} = v_{sg}\left(\frac{\rho_L}{g\sigma_L}\right)^{0.25} = 1.938\, v_{sg}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25} \quad (3.168)$$

3.4.2. Beggs and Brill (1978)

Beggs and Brill method can be used for vertical, horizontal and inclined two-phase flow pipelines.

Flow Regimes:

The flow regime used in this method is a correlating parameter and gives no information about the actual flow regime unless the pipe is horizontal. The flow regimes boundaries are defined as a functions of the following variables:

$$N_{Fr} = \frac{v_m{}^2}{gd}\ , \tag{3.169}$$

$$L_1 = 316\lambda_L{}^{0.302} \tag{3.170}$$

$$L_2 = 9.252 * 10^{-4}\lambda_L{}^{-2.4684}, \tag{3.171}$$

$$L_3 = 0.10\lambda_L{}^2, \tag{3.172}$$

$$L_4 = \lambda_L{}^{-6.738} \tag{3.173}$$

Segregated Limits:

$$\lambda_L 0.01 and N_{Fr} < L_1$$

Or $\lambda_L \geq 0.01 and N_{Fr} < L_2$

Transition Limits:

$$\lambda_L \geq 0.01 and L_2 \leq N_{Fr} \leq L_3$$

Intermittent Limits:

$$0.1 \leq \lambda_L < o.4 and L_3 < N_{Fr} < L_1$$

or$\lambda_L \geq \mathrm{o.4and}L_3 < N_{Fr} < L_4$

Distributed Limits

$$\lambda_L < o.4 and N_{Fr} \leq L_1$$

or$\lambda_L \geq \mathrm{o.4and}N_{Fr} > L_4$

Liquid Holdup: In all flow regimes, except transition, liquid holdup can be calculated from the following equation:

$$H_{L(\varphi)=}H_{L(0)}\Psi, \quad \mathrm{H}_{\boldsymbol{L}} = \frac{a\lambda_L{}^b}{N_{Fr}{}^c}\ , \tag{3.174}$$

with constrain: $H_L(0) > \lambda$

Where HL(0) is the liquid holdup which would exist at the same conditions in a horizontal pipe. The values of parameters, A, B and C are shown for each flow regimes in this Table:

For transition flow regimes, calculate HL as follows:

$$H_{L(transtion)} = H_{L(segregated)} + BH_L, A = \frac{L_3 - N_{Fr}}{L_3 - L_2} \tag{3.175}$$

The holdup correcting factor (ψ), for the effect of pipe inclination is given by:

$$\Psi = 1 + \mathrm{C}(\sin(1.8\varphi) - 0.333 sin^3 \tag{3.176}$$

Flow Pattern	A	B	C
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

Table (3.6) The Constants A, B and C for each Flow Regimes for Beggs and Brill (1978)

Where φ is the actual angle of the pipe from horizontal. For vertical upward flow, φ = 90o and$\Psi = 1 + 0.3\mathrm{C}$. C is:

$$\mathrm{C} = (1 - \lambda_L)\ln(d' \lambda_L{}^{e} N_{LV}{}^{f} N_{Fr}{}^{g}) \tag{3.177}$$

with restriction that C≥0.

The values of parameters, d’, e, f and g are shown for each flow regimes in this Tabl:

Flow Pattern	d	E	F	g
Segregated uphill	0.011	-3.768	3.539	-1.614
Intermittent uphill	2.96	0.305	-0.4473	0.0978
Distributed uphill	No correction C = 0 , ψ = 1			
All patterns downhill	4.70	-0.3692	0.1244	-0.5056

Table (3.7) The Constants d, E, F and g for each Flow Regimes for Beggs and Brill (1978)

Pressure gradient due to friction factor:

$$(-\frac{dp}{dl}) = \frac{f_{tp}\rho_n v_m}{2 g_c d} \, , \tag{3.178}$$

$$f_{tp} = f_n{}^{s} \tag{3.179}$$

fn is determined from the smooth pipe curve of the Moody diagram, using the following Reynolds number

$$N_{Re} = \frac{\rho_{n} v_m d}{\mu_n} \tag{3.180}$$

for 1<$y = \frac{\lambda_l}{H_L{}^2(\varphi)} < 1.2y, s = \ln(1.2y - 1.2)$and for others:

$$s = \frac{lny}{(-0.0523 + 3.182 lny - 0.8725(lny)^2 + 0.01853(lny)^4} \tag{3.181}$$

Chapter 4

Results and Dissection

4.1 About the Computer Program

The objectives of this work is to analyze the effect of different PVT correlations in the calculation of pressure drop in pipes; for this purpose a new computer program Pipe Cal was developed to simplify the work. The program allow the user to go through different PVT correlations and three multi phase flow correlations as discussed before. Follows the procedures and the flow charts presented through chapter 3, the program divided the pipe length into 100 sections; with tolerance for the loop of try and error equal to ±0.005. The main user interface have five sections in the menu par they are: File, Input Data, S-Conditions, PVT and Run as presented in Fig 4.1.

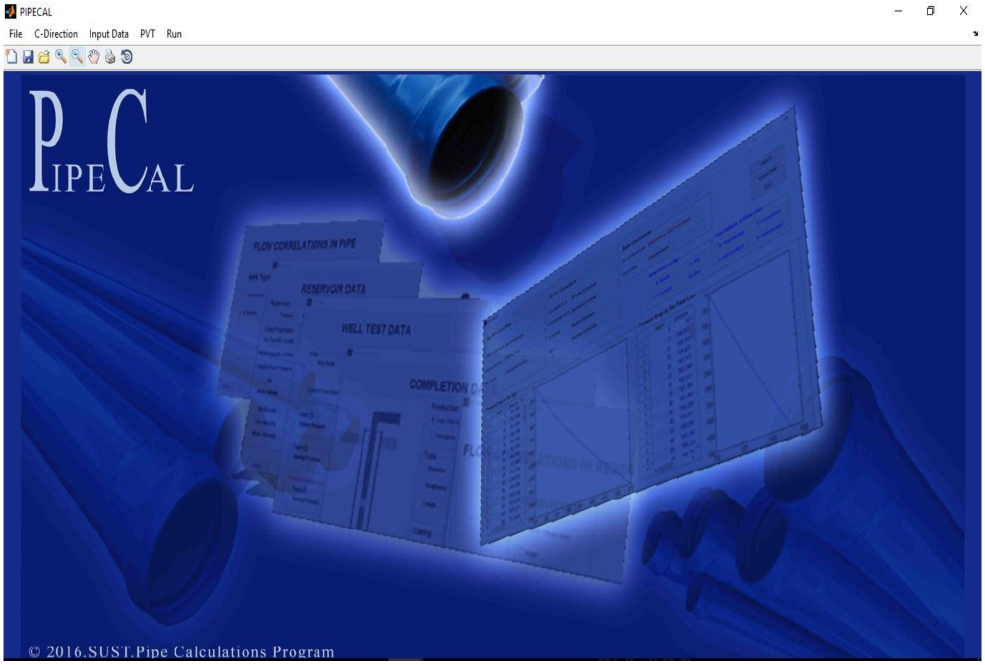


Fig 4.1 PipeCal Main Screen

The user can save or open a new file from the File section; The input data screen allow the used to select the reservoir IPR correlation and the multiphase flow correlation; it is also allow the user to select the temperature calculation method; then the required information have to be inserted according to the selected method. The PVT section allow the user to select the suitable PVT correlation; while S- Condition section allow the user to insure the availability of well test data and select the start point of calculation either it is reservoir or wellhead or separator. Any section contain a number of screens to help the used in put the information; samples of the available screen were presented through Fig. 4.2 to Fig. 4.7

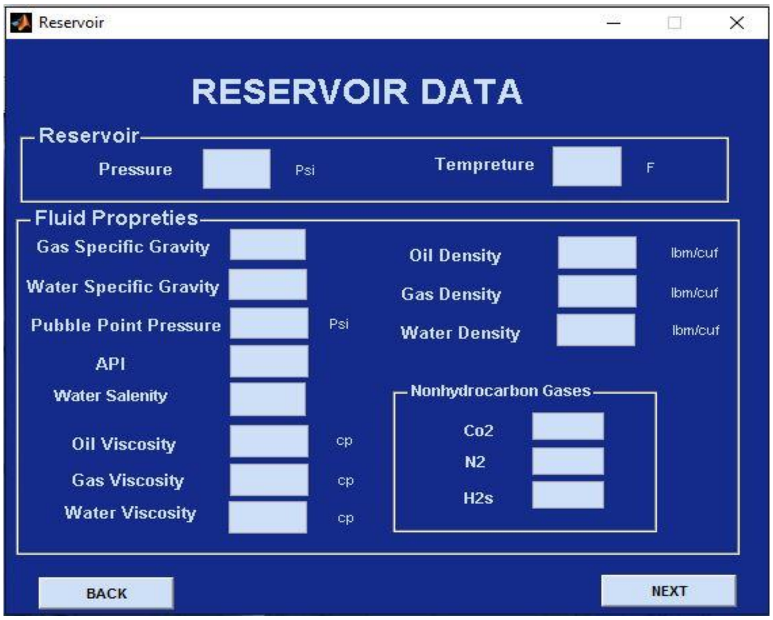


Fig 4.2 PipeCal Reservoir Data Screen

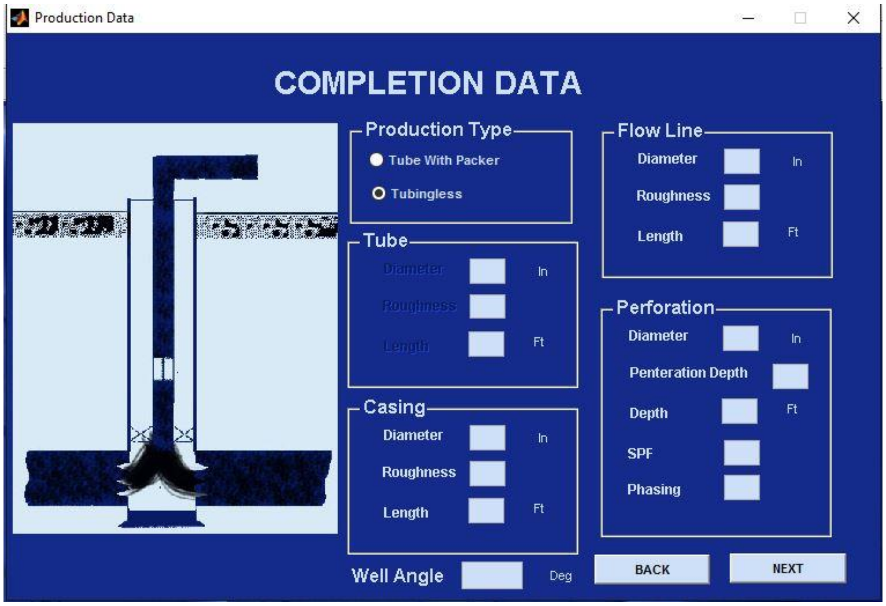


Fig 4.3 PipeCal Completion Data Screen

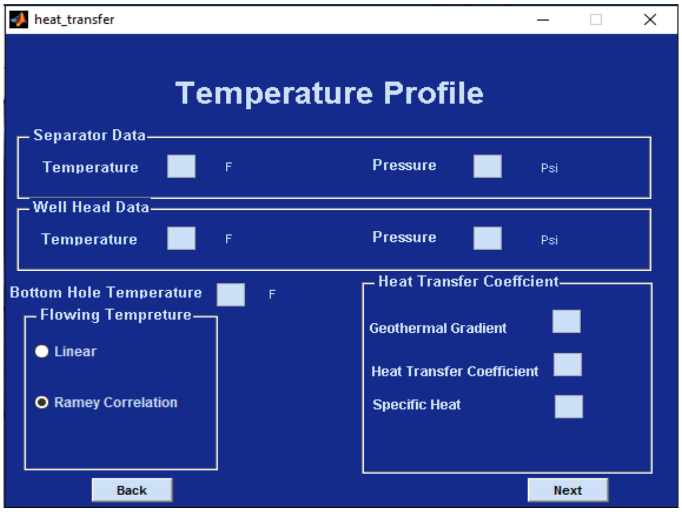


Fig 4.4 PipeCal Heat Transfer Screen

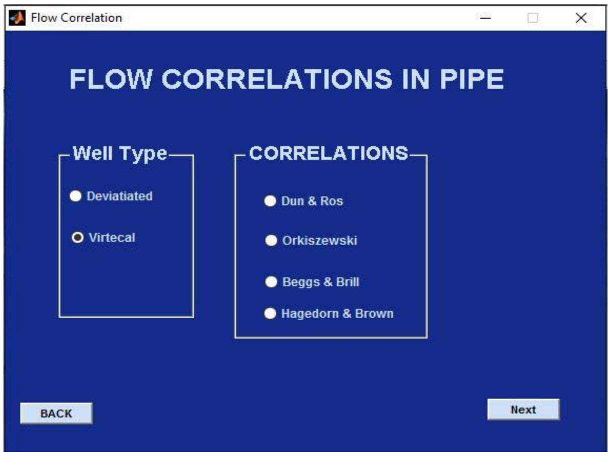


Fig 4.5 PipeCal Multi phase Correlations Screen

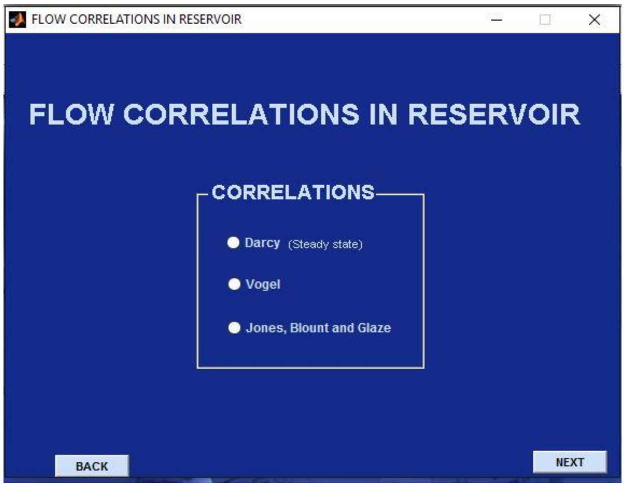


Fig 4.6 PipeCal Reservoir Correlations Screen

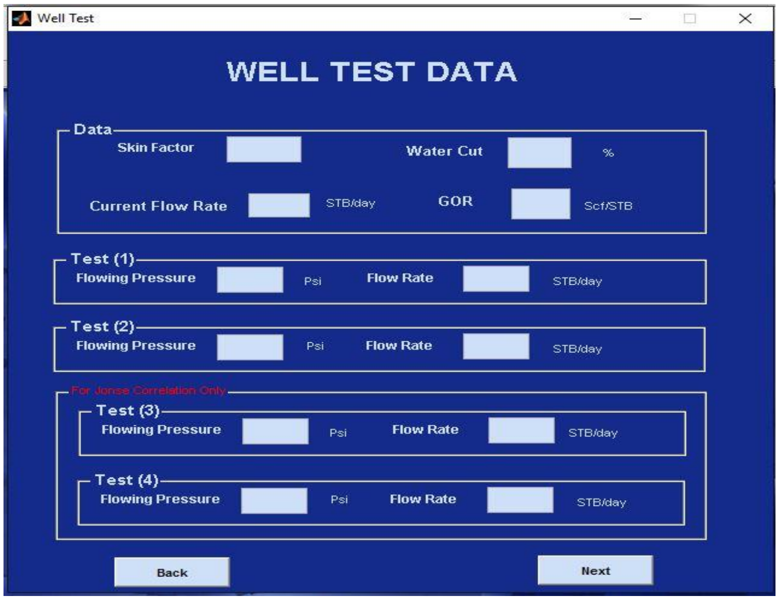
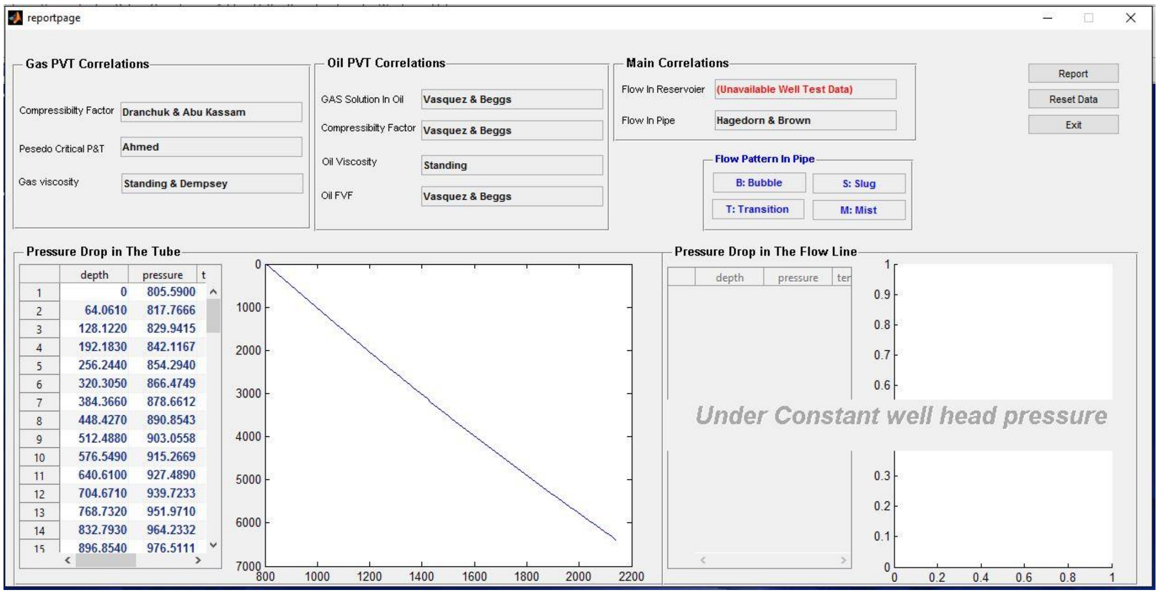


Fig 4.7 PipeCal Test Data Screen

The run screen lead to Result screen (Fig. 4.8) in which the selected correlations, temperature profile, type of flow pattern and pressure profile appear; the result in the screen can be saved as report.



Gas PVT Correlations
Compressibilty Factor: Dranchuk & Abu Kassam
Pesedo Critical P&T: Ahmed
Gas viscosity: Standing & Dempsey

Oil PVT Correlations
GAS Solution In Oil: Vasquez & Beggs
Compressibilty Factor: Vasquez & Beggs
Oil Viscosity: Standing
Oil FVF: Vasquez & Beggs

Main Correlations
Flow In Reservoier: (Unavailable Well Test Data)
Flow In Pipe: Hagedorn & Brown

Pressure Drop in The Tube

	depth	pressure	t
1	0	805.5900	
2	64.0610	817.7666	
3	128.1220	829.9415	
4	192.1830	842.1167	
5	256.2440	854.2940	
6	320.3050	866.4749	
7	384.3660	878.6612	
8	448.4270	890.8543	
9	512.4880	903.0558	
10	576.5490	915.2669	
11	640.6100	927.4890	
12	704.6710	939.7233	
13	768.7320	951.9710	
14	832.7930	964.2332	
15	896.8540	976.5111	

Pressure Drop in The Flow Line

	depth	pressure	ter

Fig. 4.8 PipeCal Final Result Screen

4.2 Pipe Correlations

The program was run several times using the information available from Well 4-3 in Fula North oilfield using different compressibility factor (Z); oil viscosity (μ_o), gas viscosity (μ_g) and gas solubility (R_s) correlations and Orkiszewski, Hagedorn and Brown and Beggs and Brill methods to predict pressure drop. the other PVT properties were stated constant using the correlation presented in table4.1; the inserted data of Well FN 4-3 are as follows::

Flow rate 1800 STB/D
Depth6406.1ft
Well head pressure805.59 psi
Well head temperature 125.6 f
Bottom hole temperature 171.14 f
Tubbing diameter2.875 in
Water cut 20 %
Gas oil ratio 1350 scf/STB
Separator pressure 50 psi
Separator temperature 86 f
Bubble point pressure 347 psi
pipe Roughness 0.00006
API gravity36.5 API
Water specific gravity1
Gas specific gravity 0.65

The has constant well head pressure; hence, the selected options for S- condition is under constant well head pressure; also tubing with packer and linear temperature distribution were selected.

Table 4.1 The Fixed Fluid Properties Correlations Considered During the Analysis

Properties	Correlation
Pseudo pressure & temperature	Wchirt& Aziz
Oil formation volume factor	Petrosky&Farshad
Oil compressibility factor	Petrosky&Farshad
Gas viscosity	Lee Gonzals&Eakin

First, the program has been run with pipe correlations of and Orkiszewski, Hagedorn and Brown and Beggs and Brill methods and fixed PVT correlations. The result was presented as a cross plotted of length vs. Pressure for any length increments to make pressure profile; the dynamic pressure profile of Well FN4-3 was also inserted into the graph to make the comparison and to select the best fitting correlation.

It was observed that Orkisviski is semi close to the real data obtained from the dynamic pressure profile of Well FN4-3. This can be observed in Fig. 4.9.

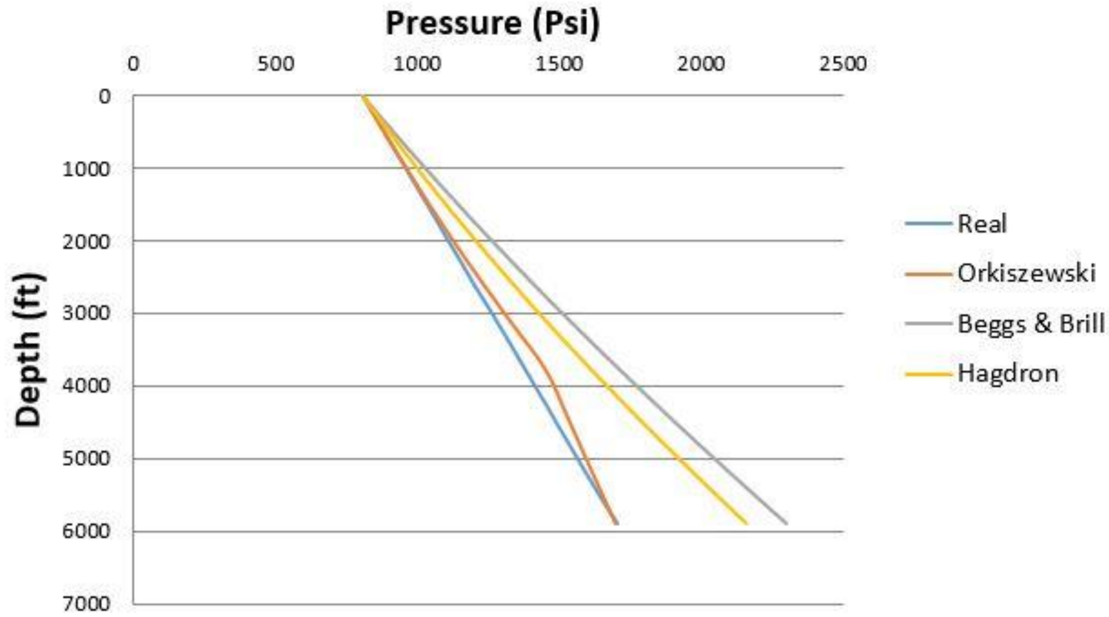


Fig. 4.9. Compression between Orkisviski, Beggs & Brill and Hagdorn

The program then has been run with Orkisviski correlation and different compressibility factor, oil viscosity and gas solubility correlations. Table 4.2 presented the studied correlations for each properties.

The result were also cross plotted against depth with the dynamic pressure profile of Well FN4-3 Here the selection of the best fitting correlation is difficult due to the large amount of information. therefore, the coefficient of determination R^2 was used to select the best correlation for the studied PVT properties. The data point of the dynamic pressure profile of Well FN4 was cross plotted against the results for each correlation.

4.3 PVT Correlations

4.3.1 Gas Solubility

The gas solubility correlations presented through Table 4.2 have been analyzed using the coefficient of determination R^2 was calculated for each plot; samples of coefficient of determination R^2 was presented through Fig. 4.10 to Fig. 4.14. Marhoun correlation was found to be the best fitting correlation for gas solubility with R^2 of 0.993 using Dranchuk and Abu Kassam correlation for compressibility factor and Beggs and Robinson for Oil viscosity.

Table 4.2 The Analyzed Fluid Properties Correlations Considered During the Analysis

Properties	Correlations
Compressibility factor	1. Hall &Yarborugh 2. Dranchuka et al 3. Dranchuk& Abu Kassam 4. Brill &Beggs
Oil viscosity	1. Beggs& Robinson 2. Standing
Gas solubility	1. Petrosky&Farshad 2. Vasquez &Beggs 3. Marhoun 4. Standing 5. Glaso

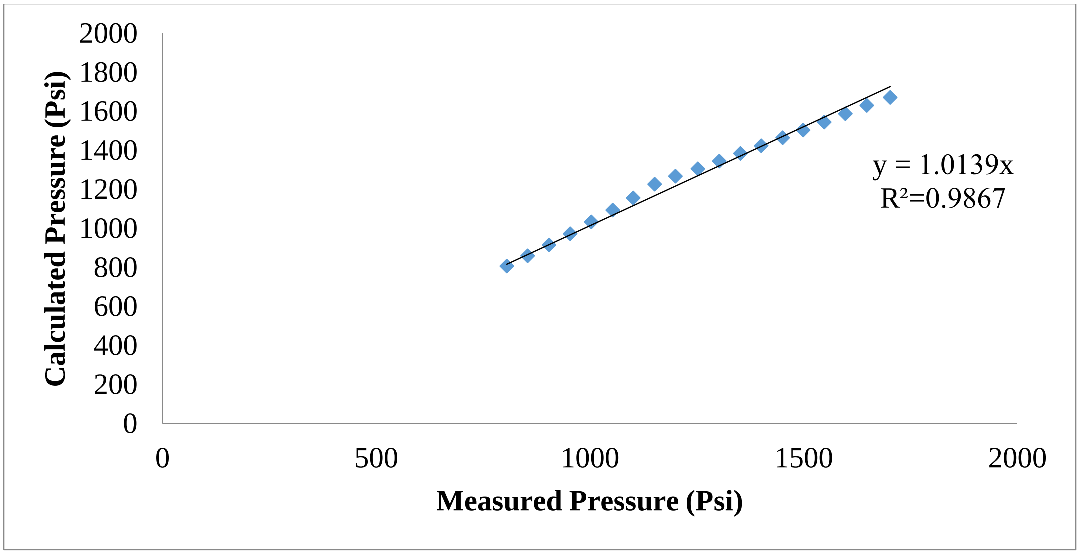


Fig. 4.10 Gas solubility Glaso correlation Vs. Real Data of FN 4 -3 (Compressibility factor by Hall and Yarborugh Method , Oil viscosity by Beggs and Robinson Method)

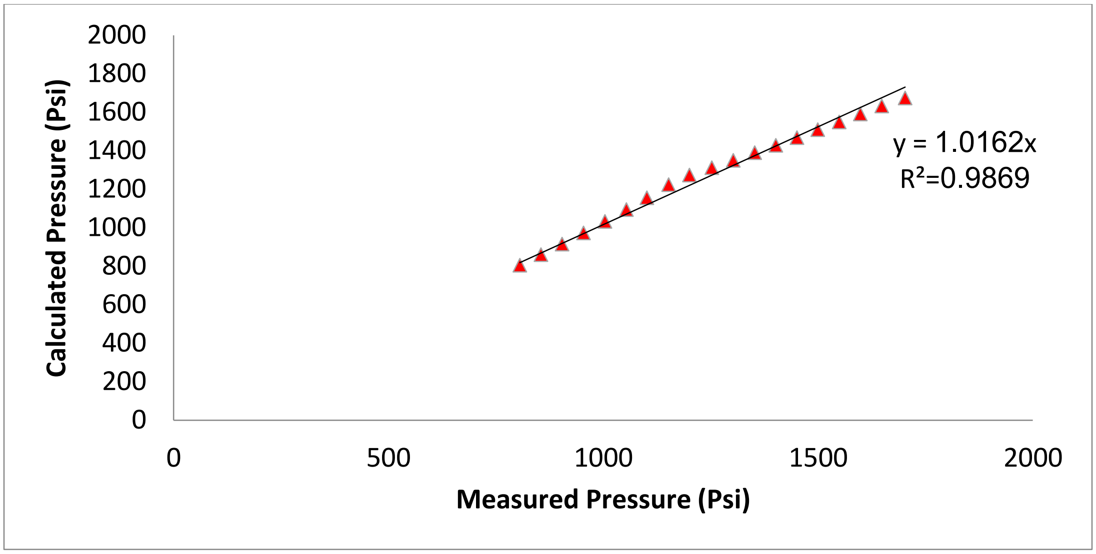


Fig. 4.11 Gas solubility Mahoun correlation Vs. Real Data of FN 4 -3 (Compressibility factor by Hall and Yarborugh Method , Oil viscosity by Beggs and Robinson Method

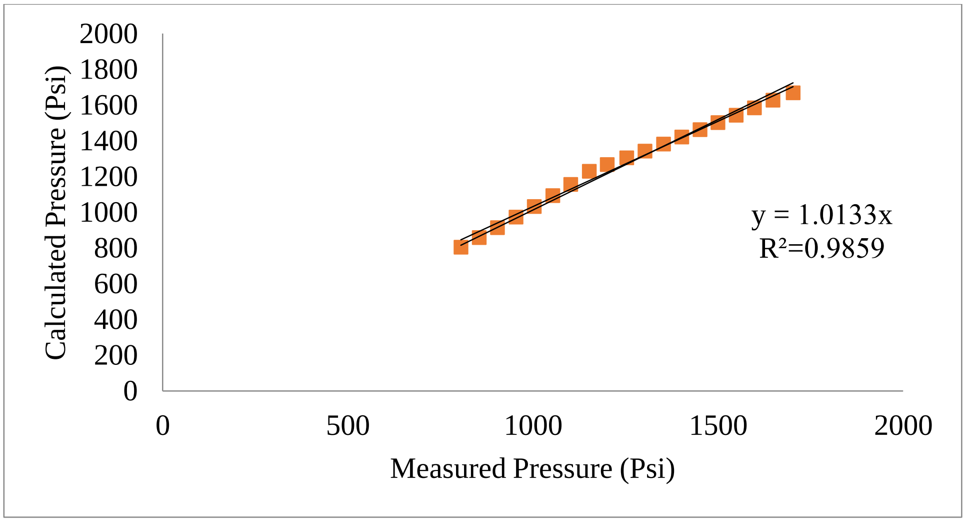


Fig. 4.12 Gas solubility Standing correlation Vs. Real Data of FN 4 -3(Compressibility factor by Hall and Yarborugh Method , Oil viscosity by Beggs and Robinson Method

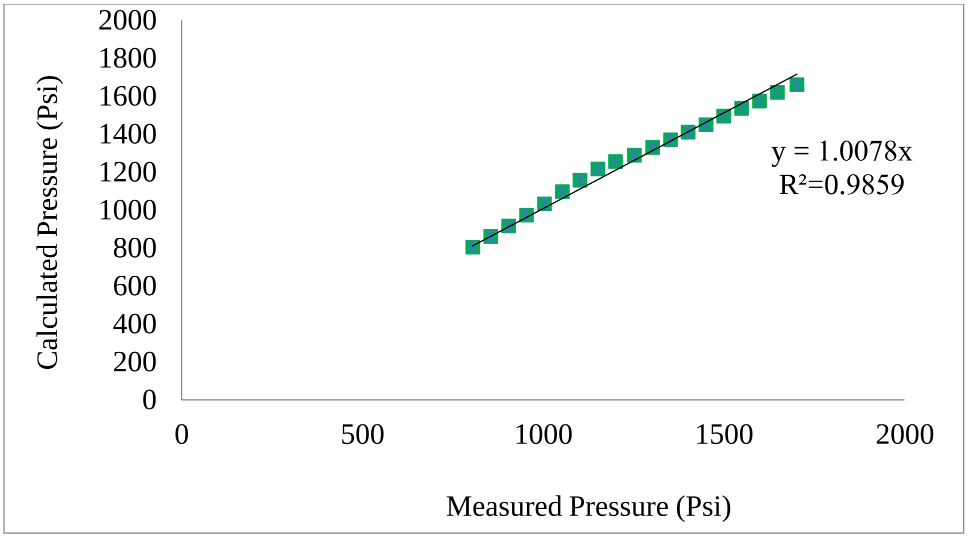


Fig. 4.13 Gas solubility Petrosky & Farshad correlation Vs. Real Data of FN 4 -3 (Compressibility factor by Hall and Yarborugh Method , Oil viscosity by Beggs and Robinson Method

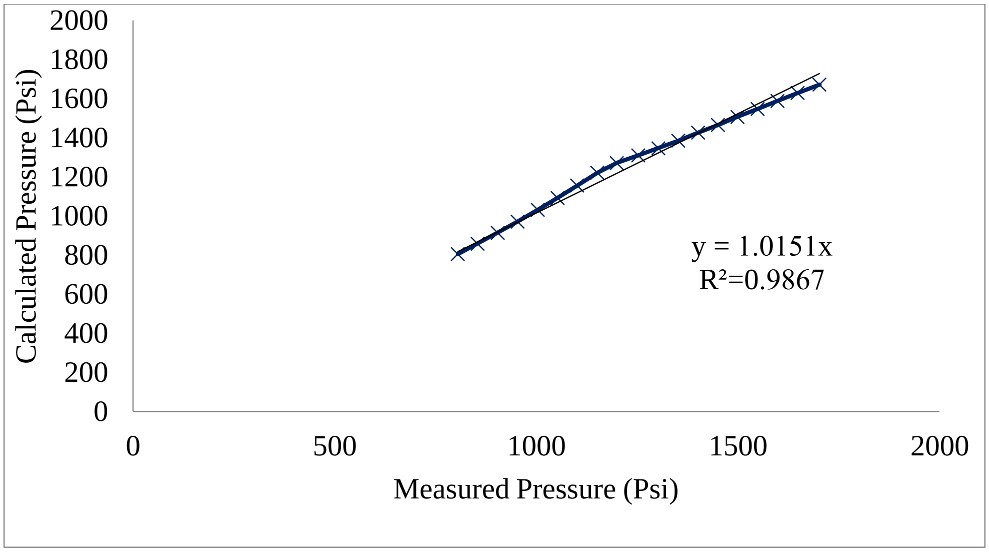


Fig. 4.14 Gas solubility Vasquez and Beggs correlation Vs. Real Data of FN 4 -3 **(**Compressibility factor by Hall and Yarborugh Method , Oil viscosity by Beggs and Robinson Method

4.3.2 Compressibility Factor

The gas Compressibility Factor correlations presented through Table 4.2 have been analyzed using the coefficient of determination R^2 was calculated for each plot as presented through Fig. 4.15 to Fig. 4.18 for some samples. Dranchuk and Abu Kassam correlation with Oil viscosity by Standing correlation and Gas solubility correlation of Glaso has the best fitting correlation with R^2 of 0.993.

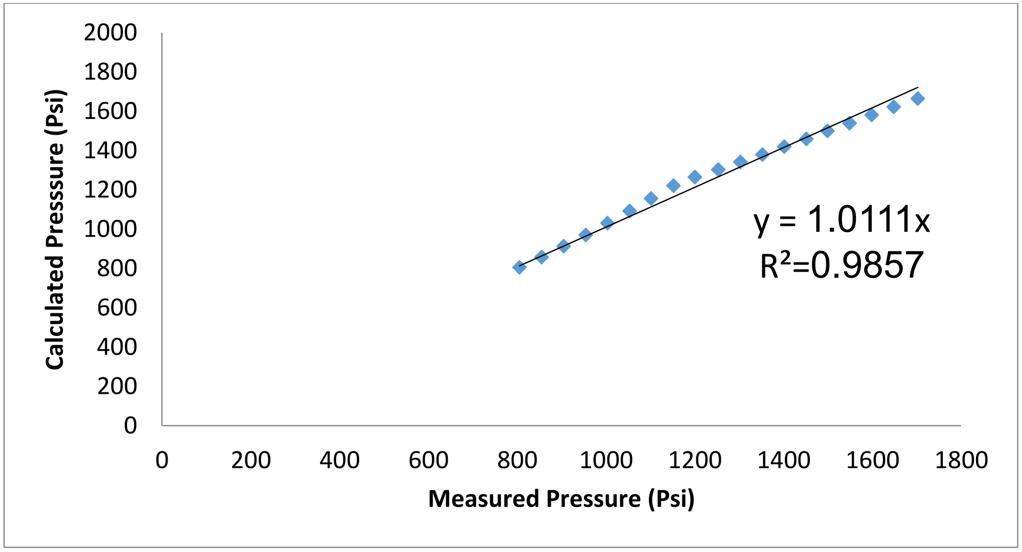


Fig. 4.15 Compressibility Factor Hall &Yarborugh correlation Vs. Real Data of FN 4-3 (Oil viscosity by Standing correlation and Gas solubility correlation Glaso

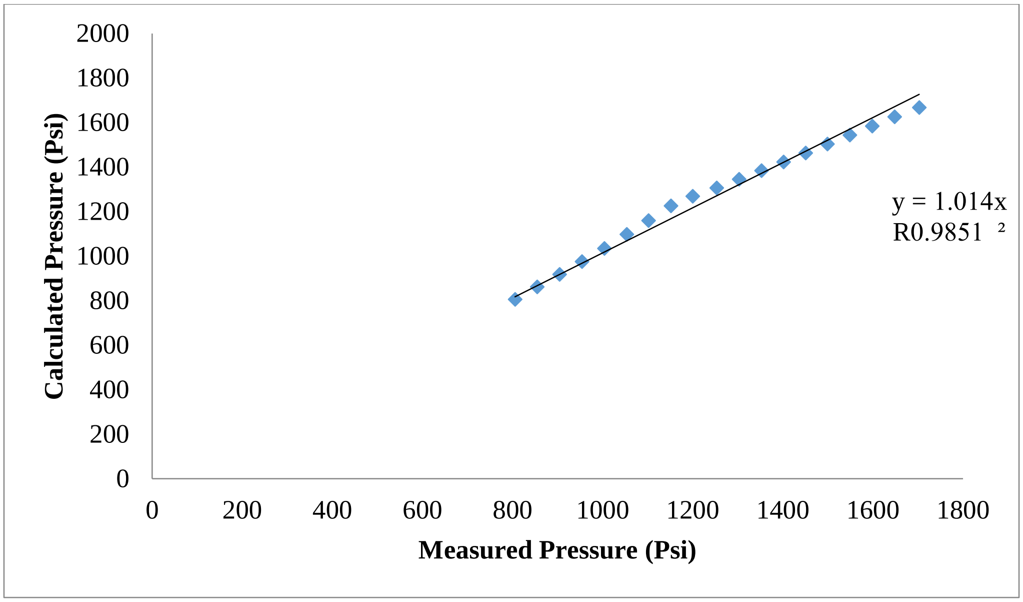


Fig. 4.16 Compressibility Factor Dranchuka et al correlation Vs. Real Data of FN 4-3 (Oil viscosity by Standing correlation and Gas solubility correlation Glaso

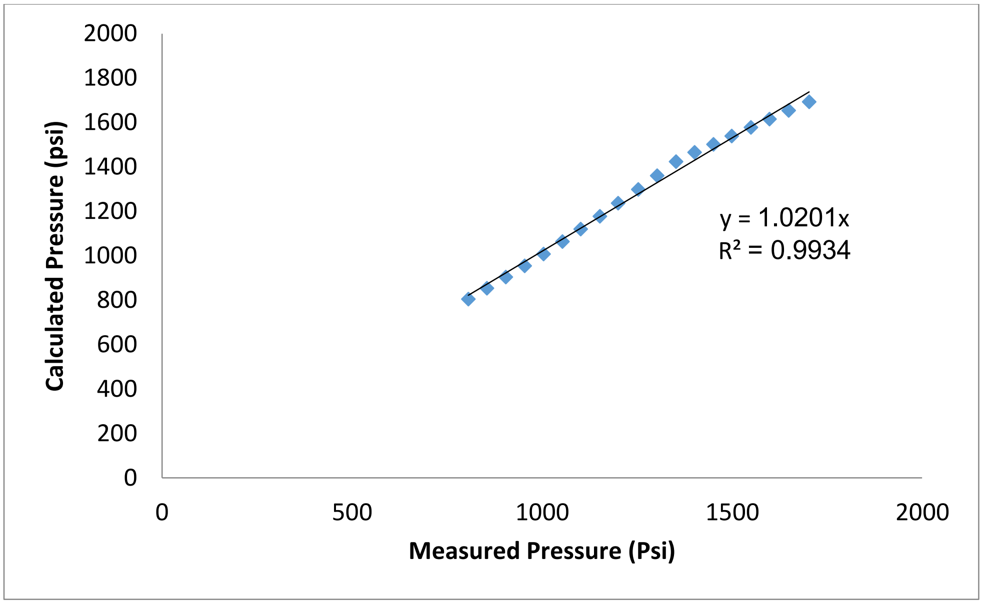


Fig. 4.17 Compressibility Factor Dranchuk and Abu Kassam correlation Vs. Real Data of FN 4-3 (Oil viscosity by Standing correlation and Gas solubility correlation Glaso

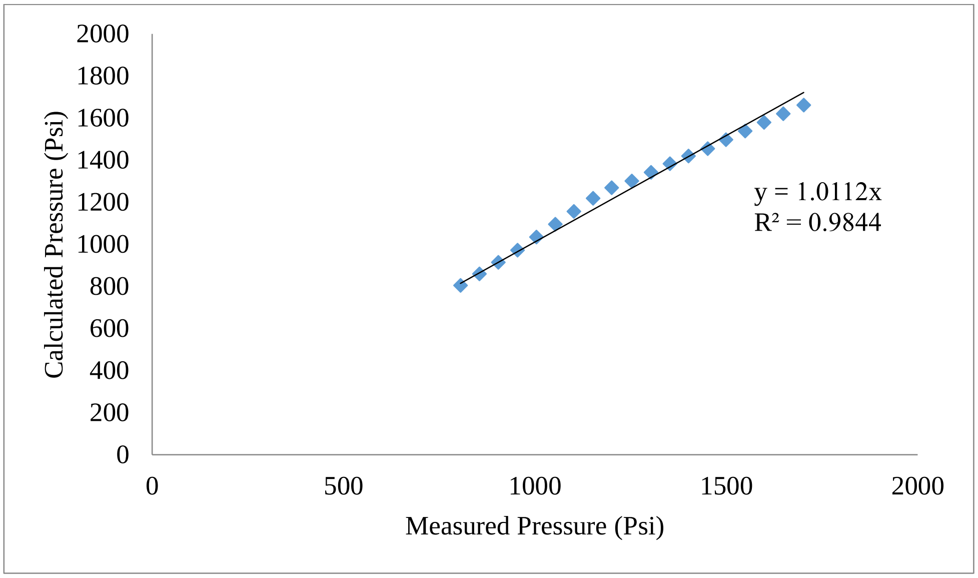


Fig. 4.18 Compressibility Factor Beggs and Brill correlation Vs. Real Data of FN 4-3 (Oil viscosity by Standing correlation and Gas solubility correlation Glaso

4.3.3 Oil viscosity

The Oil viscosity correlations presented through Table 4.2 have been analyzed using the coefficient of determination R^2 was calculated for each plot as presented through Fig. 4.19 to Fig. 4.20 for some samples.

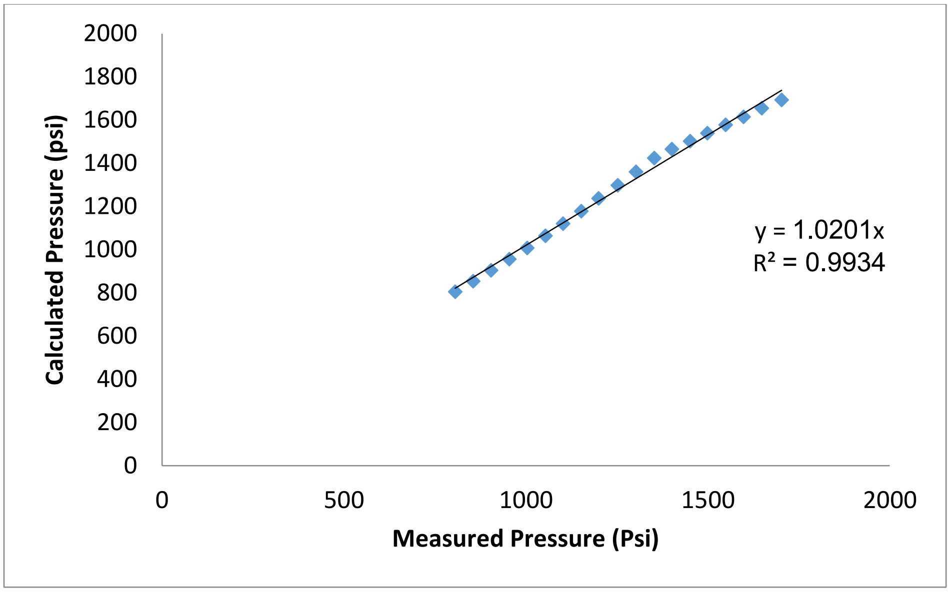


Fig. 4.19 Oil Viscosity Standing Vs. Real Data of FN 4-3 (Compressibility Factor Dranchuk and Abu Kassam correlation and Gas solubility Glaso correlation)

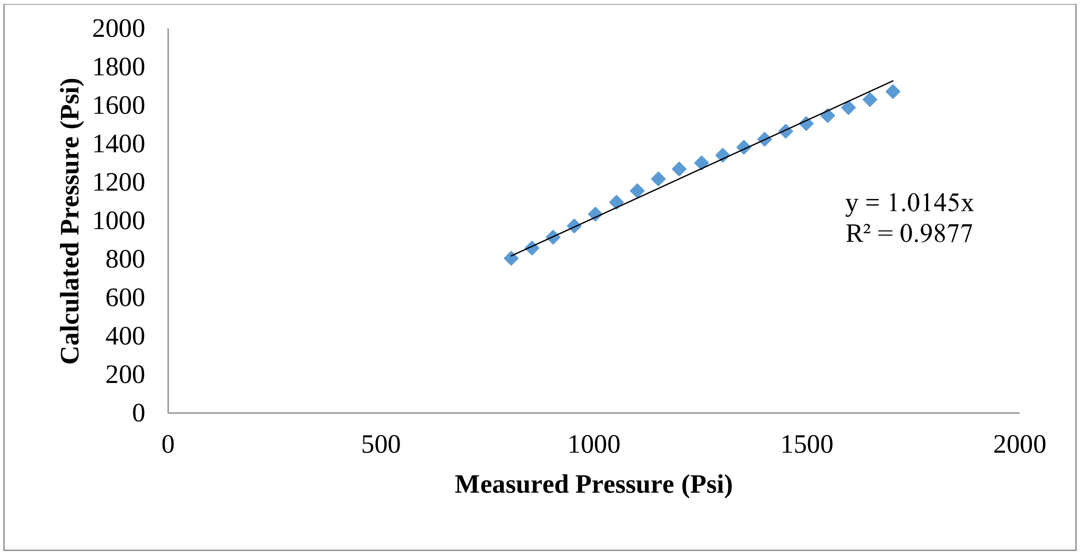


Fig. 4.20 Oil Viscosity Beggs and Robinson Vs. Real Data of FN 4 -3 (Compressibility Factor Dranchuk and Abu Kassam correlation and Gas solubility Glaso correlation)

Chapter 5

Conclusions and Recommendations

5.1 Conclusion

Based on this work the following conclusions are made:

- A new computer program has been developed to calculate two-phase flow pressure drop in pipes using to. Orkiszewski, Hagedorn and Brown and Beggs and Brill methods with a large amount of PVT correlations.
- The program have been used to calculate and analyze the effect of compressibility factor (Z); oil viscosity (μo), gas viscosity (μg) and gas solubility (Rs) on the calculations of pressure drop in pipes using three different multiphase flow correlations through Well F N 4-3 in Fula North.
- The results presented that, the best fitting correlation for multiphase flow for FN 4-3 is Orkisviski; which did not consider the effect of gas viscosity (μ_g) hence same results were obtained by the different gas viscosity correlations
- Small variation on the pressure drop was observed with compressibility factor (Z); oil viscosity (μ_o) and gas solubility (R_s); good correlations fitting was observed with the different correlations; and the coefficient of determination R^2 was used to select the best correlation .

5.2. Recommendations

1. More data is required to address the effect of PVT correlations on the pressure drop calculation.
2. As The current program deal with natural flow only; hence no comparison or fitting can be done for artificial lift or gas lift well
3. Other multi phase flow correlation need to be added to the program and artificial lift or gas lift need to be consider to improve the program to to be used for full production optimization .

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