Chapter One

Introduction

1.1 History of Quantum Mechanic

In the beginning the starting point of quantum theory will be taken as 1900 and Planck's postulate of the quantization of energy in blackbody radiation. It had been known for some time that the spectrum of thermal radiation contained in a cavity in thermal equilibrium must be a universal function of the temperature, completely independent of the material of the walls of the cavity. Detailed experimental work by Lummer and Pringsheim (1899) and Rubens and Kurlbaum (1900) had determined the shape of the spectrum. Planck was able to present an argument that consisted of essentially phenomenological curve fitting, relying on classical ideas of entropy at long wavelengths and an adhoc conjecture due to Wien (1896) for short wavelengths, but which generated a formula that fitted the data perfectly for all wavelengths. However, he was unable to offer any theoretical justification for the results. Planck himself said of the situation: But even if the absolutely precise validity of the radiation formula is taken for granted, so long as it merely has the standing of a law disclosed by a lucky intuition, it could not be expected to possess more than a formal significance. For this reason, on the very day when I formulated this law, I began to devote myself to the task of investing it with true physical meaning. This question automatically led to study the interrelation of entropy and probability. Although Planck quantized the oscillators in the wall of the cavity he did not quantize the electromagnetic radiation itself. Qualitatively, it can be seen how Planck's energy formula was able to explain the shape of the spectrum for blackbody radiation.

Therefore for any given finite amount of energy in the cavity there is some shortest wavelength which can be excited. Thus for a given temperature, T, each curve peaks at a most probable value.

And then came Albert In 1905, during his Annus Mirabilus, Einstein took the notion of quantization further by suggesting that electromagnetic radiation exists in the form of 'packets' of energy which we now call photons. With this new way of thinking Einstein was able to treat the radiation from the blackbody as a 'gas' of photons, and by application of statistical mechanics, as used in thermodynamics, supply an alternative derivation of Planck's formula. Further application of the photon model supplied Einstein with the means to solve the classically inexplicable

Photoelectric effect. Classical electromagnetic theory predicted that the energy available in light is proportional to the intensity and independent of frequency, but experimental evidence pointed to the opposite result. Einstein argued that if light or any electromagnetic radiation consists of a stream of photons of energy hf, then the maximum energy that an electron can absorb in a collision with a photon must also be hf. This result was verified by Robert Millikan and led to the Nobel Prize for Einstein.

In 1913, Bohr extended the notion of quantization of energy to the hydrogen atom.

Rutherford's work on the scattering of alpha particles by atoms led to the concept of an atom as a small, very hard nucleus around which orbit electrons. In considering the line spectrum of hydrogen Bohr accepted

Rutherford's nuclear atom for further historical details see Heilbronn and Kuhn (1969).In order to obtain a discrete set of stable orbits Bohr postulated, Without any explanation, that electrons are confined to certain stationary states, with circular orbits, and that they emit radiation only when they make transitions from one stationary state to another .The notion of transition from one stationary state to another considers that the atom emits a single photon of energy equal to the difference between orbit energies.

In accordance with Planck's quantization of energy emitted or absorbed by a harmonic oscillator in multiples of hf, Bohr considered an electron infinitely far away from the nucleus falling into an allowed orbit and quantized the energy, hf, of the photon in terms of the energy of the electron in its final orbit .

In developing his theory Bohr considered only circular orbits. Summerfield and Wilson both, independently, extended Bohr's quantization rules for the angular momentum to elliptical orbits with periodic motion. Einstein demonstrated that quantum theory could not predict the timing of such a jump and neither could it predict the direction of the emitted photon of electromagnetic radiation. Quantum theory could, and still can, predict only the probability of such a jump taking place. In Newtonian mechanics understanding the universe was based on stability rather than probability. Einstein's struggle with the probabilistic nature of quantum theory appears to be based on the notion that it violates what is regarded as being 'normal'.

Wave mechanics the early studies in quantum mechanics relied on Newtonian ideas and sought to supplement Newton's laws with quantization conditions. This allowed for the selection of the preferred stationary states in the Bohr model of the atom. Roughly, we can say that the old quantum theory accepted Newtonian kinematics, but sought to modify Newtonian dynamics with supplementary conditions. In the1920s, physicists finally recognized that this attempt to graft quantum structure on the Newtonian roots was unworkable, and they recognized that both Newtonian kinematics and dynamics had to be discarded The first move away from the Newtonian influence was de Broglie's notion that 'particles' have 'wave' properties. By building on the relativistic connection between energy and momentum and frequency and wavelength de Broglie was able to postulate that the wavelength of the wave associated with the particle was related to the momentum of the particle .The widely accepted confirmation of particle diffraction was due to the experimental work of Davisson and Germer (1927), using crystals to scatter electrons, and Thomson, using thin metallic films. The results of these experiments not only established the wave properties of the electron but also that 'particle waves' obeyed the principle of superposition. Whilst this work was in progress Schrödinger formulated what is now commonly called the Schrödinger wave equation.

Application of the three-dimensional version of this equation to the hydrogen atom generated the quantization of both angular momentum and energy. Schrödinger initially interpreted the results as the wave function representing the distribution of electric charge in space. However, this would have allowed for the possibility of an electron being cut in two by an obstacle. The wave function idea was, in some ways, rescued by Born, who proposed that the absolute magnitude of the wave function, $|\psi|^2$, represented the probability distribution for the position of the electron. The wave properties of particles and the description of particles by probability waves implied a profound revision of the foundations of physics. Instead of specifying the state of a particle by position as a function of time, we now have to describe the state by a wave function and we can never predict exactly where the particle will move as a function of time-we can predict only probabilities for motion from one position to another. It is this formulation of the ‘new’ quantum mechanics which was to become the most popular amongst physicists. However, even before the Schrödinger wave equation had been published an alternative formulation was developed by Heisenberg [1, 2].

1.2 Research Problem

The research problem is concerned with the relativistic explanation of particles spins by using simple equations.

1.3 literature review

Many works were done in generalized special relativity and its explanations. In the work done by Mubarak Dirar Abdllah etal [Matter & Antimatter Generation & Repulsive Gravity force] generalized special relativistic energy relations shows that velocities as well as field potential affect the energy. These relations were used to find vacuum energy by minimizing energy. The minimization shows that vacuum energy consists of photons having energy that can produce particle and anti-particle pair. It also shows that the mass of antiparticle is negative, thus it repel ordinary particle. Another expression of vacuum energy shows that vacuum decays and transform may be to ordinary matter as proposed by scientists [3].

A seminal paper was also published by Mubarak Dirar Abdllah etal [Generalized The General Relativity Using Generalized Lorentz Transformation] in this work generalized nonlinear Lorentz transformation is utilized to derive modified special relativistic space-time equations .The equation are found for particles moving in a potential field. The transformation is based on the usual Newtonian relation displacement in terms of initial velocity for constant acceleration. The displacements in all frames are expressed in term of spatial coordinate time and potential per unit mass. The expression for Lorentz transformation parameter, space and tine reduces to that of ordinary special relativity in the absence of field. The energy relation reduces to special relativity for no field and Newtonian one for law velocity [4].

Another work was done by D. Mubarak Dirar Abdllah etal [Utilization of photon equation of motion to obtain electromagnetic momentum, time & length in Einstein generalization of special relativity], Special relativity (SR) in the presence of the gravitational field is obtained from the expression of invariant length and a photon equation of motion. Two expressions explain both mass and energy are obtained; one is generalized special relativity (GSR), the other is of Savakis. The (GSR) is more realistic than Savickas since it is based on the effect of gravity on the time which is not recognized by Savickas expression. Moreover (GSR) model predicts pair production phenomena and its equations of motion which are reduced to Newton's second law. Using the equation of motion of the electron in photon in the electromagnetic (E.M) as a simple expression for the total momentum the sum of photon an electron mechanical momentum was found [5].

Another interesting study were made by D. Mubarak Dirar Abdllah etal [The Effect of Speed and Potential on Time, Mass and Energy on the Basis Newton and Relativity Prediction], the nature of time, mass, and energy, and the effect of speed and potential field on them was experimentally tested. These experiments that show that the time, mass, and energy are affected by both speed and potential.

Newtonian mechanics shows that only energy is affected by speed and potential. Thus it is in direct conflict with experiments that shows the effect of potential on time and mass. However special relativity shows the effect on speed on time, mass, and energy but does not recognized the effect of potential on them.

But generalized special relativity shows that time, mass, and energy are affected by velocity as well as field potentials. Fortunately the theoretical relations agree with the empirical ones [6].

1.4 Aims of the work

The aim of this work is to use generalized special relativity (GSR).To derive spin relativistic quantum equations.

1.5 Thesis Lay Out

The thesis consists of three Chapters. Chapter one is the introduction. Chapter two is devoted for spin quantum equation; finally chapter three is concerned with the quantum generalized special relativity beside conclusion and recommendation.

Chapter Two

Spin Quantum Equations

2.1 Introduction

In quantum mechanics and particle physics, spin is an intrinsic form of angular momentum carried by elementary particles, composite particles (hadrons), and atomic nuclei However, a relativistic formulation of quantum mechanics shows that particles can exhibit an intrinsic angular momentum component known as spin. The discovery of the spin degree of freedom marginally predates the development of relativistic quantum mechanics by Dirac and was achieved in a ground-breaking experiment by Stern and Gerlach (1922),in which particles are observed to possess angular momentum that cannot be accounted for by orbital angular momentum alone. However, the various interpretations which have been proposed and widely accepted all suffer from a serious drawback they are limited to the Schrödinger theory and fail to take into account either spin or relativity. In fact, that spin cannot be ignored. The interpretation of the Pauli theory is determined by its relation to the Dirac theory, so the interpretation of the Schrödinger theory is determined by its relation to the Pauli theory, at least when the theory is applied to an electron rather than a spin less particle like a pi meson. This elementary point of logical consistency has been overlooked or ignored in every discussion of the Uncertainty Relations and the interpretation of non-relativistic quantum mechanics [1, 2].

2.2 Schrödinger Equation without Spin

The classical expression for a particle with mass *m* where the total energy *E* is the sum of the kinetic energy, and the potential energy *V*.

The total energy *E* of a particle is:

$$E = T + V = \frac{p^2}{2m} + V \qquad (2.2.1)$$

The momentum of the particle is **p**, or mass time's velocity. The potential energy is assumed to vary with position.

The wave function of a particle of energy E could most naturally be written as a linear combination of wave functions of the form :

$$\psi = e^{i/\hbar(px-Et)} \qquad (2.2.2)$$

We now generalize this to the situation in which there is both a kinetic energy and a potential energy present in Equation (2.2.1), that yields,

$$E\psi = \frac{p2}{2m}\psi + V\psi \qquad (2.2.3)$$

Where Ψ is now the wave function of a particle moving in the presence of a potential V (x); by derivation the wave function (2.2.2) in time and position, yields:

$$\frac{\partial\psi}{\partial t} = \frac{i}{\hbar}E\psi \qquad (2.2.4)$$

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar}p\psi$$

$$-\hbar^2 \frac{\partial^2\psi}{\partial x^2} = p^2\psi \tag{2.2.5}$$

Substituting Equations (2.2.4) and (2.2.5) in Equation (2.2.3) yields:

$$\frac{\hbar}{i}\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi \tag{2.2.6}$$

This is the famous time dependent Schrödinger wave equation. And it is a fundamental equation in quantum mechanics [7].

2.3 Schrödinger Equation with Spin

Schrödinger Equation in the presence of spin is written as:

$$\left(H_s + V_p\right)\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{2.3.1}$$

Where

H_s ≡ Schrödinger Hamiltonian.

V_p ≡ Spin potential (Pauli potential).

Using the separation of variables, One can split the wave function into Schrödinger and spin (Pauli) wave functions ψ_s and ψ_p respectively,

$$\psi(r,t) = \psi = \psi_s(r,t)\psi_p(t) \tag{2.3.2}$$

By substituting Equation (2.3.2) in Equation (2.3.1) to get:

$$\mathrm{H_s}\,\psi_s\psi_p + V_p\psi_s\psi_p = i\hbar\psi_s\frac{\partial\psi_p}{\partial t} + i\hbar\psi_p\frac{\partial\psi_s}{\partial t} \tag{2.3.3}$$

Dividing both sides by $\psi_s\,\psi_p$ yields

$$\mathrm{H}_s + V_p = i\hbar\frac{1}{\psi_p}\frac{\partial\psi_p}{\partial t} + i\hbar\frac{1}{\psi_s}\frac{\partial\psi_s}{\partial t} \tag{2.2.4}$$

Thus one can write:

$$H_s = i\hbar \frac{1}{\psi_s}\frac{\partial \psi_s}{\partial t} \tag{2.2.5}$$

$$H_s\psi_s = i\hbar \frac{\partial \psi_s}{\partial t}$$

And:

$$i\hbar \frac{1}{\psi_p}\frac{\partial \psi_p}{\partial t} = V_p$$

$$\frac{\partial \psi_p}{\partial \psi_p} = \int \frac{V_p}{i\hbar}\, dt + C_1$$

$$\ln \psi_p = \frac{V_p}{i\hbar}\int dt + C_1$$

$$\psi_p = e^{\frac{V_p}{i\hbar}t + C_1}$$

$$= e^{C_1} e^{\frac{V_p}{i\hbar}t} = C_2 e^{\frac{V_p}{i\hbar}t} \tag{2.2.6}$$

Thus from (2.3.2) and (2.2.6)

$$\psi(r,t) = C_2 e^{\frac{V_p}{i\hbar}t}\psi_s(r,t) \tag{2.2.7}$$

2.4 Dirac Equation

The Dirac Equation is consistent with Quantum Mechanics (QM) and fully consistent with the Special Theory of Relativity (STR). This equation Accounts in a natural way for the nature of particle spin as a relativistic Phenomenon and amongst its prophetic achievements was its successful Prediction of the existence of antiparticles[8], [9].

Dirac derives a new quantum mechanical expression based on a linear expression of the relativistic energy ,which given by: $E = c\alpha.p + \beta m_0 c^2$(2.4.1)

Where the parameters α and β were determined by the boundary conditions imposed by the quantum relativistic. It was found that these parameters related to the electron - spin and is known as Pauli matrices to derive Dirac equations multiply both sides of equation (2.4.1) by ψ to gets:

$$E\psi = c\alpha.p\psi + \beta m_0 c^2 \psi \qquad (2.4.2)$$

To write equation (2.2.2) in a differential form, with the aid of this equations (2.2.4), (2.2.5) and by inserting in equation (2.4.2) yields:

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar}{i}c\alpha.\nabla\psi + \beta m_0 c^2 \psi \qquad (2.4.3)$$

This is the relativistic Dirac equation.

2.5 Dirac Spin Wave Function

The compounds of orbital momentum don't commutate with each other and it satisfies the equation:

$$L_x - L_y = i\hbar L \qquad (2.5.1)$$

Since the spin operator it is appropriate to Dirac matrix, thus:

$$S = \frac{1}{2}\hbar\sigma \qquad (2.5.2)$$

Where σ is Dirac matrix in four compounds from the commutations relation Dirac spin satisfy:

$$S_x S_y - S_y S_x = i\hbar S_z \qquad (2.5.3)$$

Also the total orbital momentums satisfy the same commutation relations:

$$J_xJ_y - J_yJ_x = i\hbar J_z \tag{2.5.4}$$

Where $J = L + S$, and it's a result to the sum of orbital and spin momentum. And to simplify this equation we use the Pauli matrix which takes the form:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{2.5.5}$$

Its two components achieve the total momentum Conservation Law, thus:

$$J^2 \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left(L + \frac{1}{2}\hbar\sigma'\right)^2 \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \hbar^2 j(j+1) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$J_z \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left(L_z + \frac{1}{2}\hbar\sigma_3{}'\right)^2 \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \hbar m_j \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{2.5.6}$$

Where $L = [rp]$ orbital momentum operator σ' in Pauli matrix, and we look for solution to the equation (2.5.6) in the form:

$$\psi_1 = C_1 Y_l{}^{m\prime}$$

$$\psi_2 = C_2 Y_l{}^{m} \tag{2.5.7}$$

$Y_l{}^m$ Spherical

From equation (2.5.7) we found:

$$\frac{1}{\hbar}\left[\left(L_x - iL_y\right)\psi_2 + L_z\psi_1\right] = q\psi_1$$

$$\frac{1}{\hbar}\left[\left(L_x + iL_y\right)\psi_1 - L_z\psi_2\right] = q\psi_2 \tag{2.5.8}$$

And

$$q = j(j+1) - l(l+1) - \frac{3}{4} \tag{2.5.9}$$

In equation (2.5.8), let $m' = m - 1$, so the constant linked together by relation:

$$(q - m + 1)C_1 + \sqrt{(l + 1 - m)(l + m)}C_2 = 0$$

$$\sqrt{(l + 1 - m)(l + m)}C_1 + (q + m)C_2 = 0 \qquad (2.5.10)$$

Let:

$$q = l, \;\; j = l + \frac{1}{2}$$

$$C_2 = -\sqrt{\frac{l-m+1}{l+m}}C_1 \qquad (2.5.11)$$

$$q = -(l + 1), \; j = l - \frac{1}{2}$$

$$C_2 = -\sqrt{\frac{l+m}{l-m+1}}C_1 \qquad (2.5.12)$$

2.6 Spin Pauli equation:

Pauli was added anew limit to the ordinary Hamiltonian in the Schrödinger equation which related to the spin magnetic of the electron [10].

$$i\hbar\frac{\partial\psi}{\partial t} = m_0c^2\psi + v\psi - \frac{\hbar^2}{2m}\nabla^2\psi v = v_s \qquad (2.6.1)$$

$$v_s = \mu\mathcal{H} \qquad (2.6.2)$$

Then the Schrödinger Equation had written as:

$$\{E - H + (\mu\mathcal{H})\}\psi = 0 \qquad (2.6.3)$$

H Is the Hamiltonian operator in Schrödinger equation, which is written in the form:

$$H = \frac{1}{2m_0}\left(p - \frac{e}{c}A\right)^2 + e\varphi \tag{2.6.4}$$

Where

$A \equiv potential.$

$e \equiv electroncharge.$

Pauli chose that ψ form a matrix consist of one column$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$, and the spin magnetic for the electron is written as follows:

$$\mu = -\mu_0\sigma^{'} \tag{2.6.5}$$

Where

$\mu \equiv spinmagneticmoment$.

$\mu_0 \equiv Bohrmagneton$.

$\sigma^{'} \equiv paulimatrix.$

It's convenient to define three useful matrixes 2*2 by the relation:

$$\sigma^{'}{}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^{'}{}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^{'}{}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Together with the unit matrix I,

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.6.6}$$

By substituting Pauli matrix in Equation (2.5.11), thus

$$\left\{(E-H)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}-\mu_0\left[\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\mathcal{H}_{\mathrm{x}}+\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\mathcal{H}_{\mathrm{y}}+\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\mathcal{H}_{\mathrm{z}}\right]\right\}=0$$

(2.6.7)

And its equivalent to the equations:

$$(E-H-\mu_0\mathcal{H}_{\mathrm{z}})\psi_1-\mu_0(\mathcal{H}_{\mathrm{x}}-\mathrm{i}\mathcal{H}_{\mathrm{y}})\psi_2=0$$

$$(E-H-\mu_0\mathcal{H}_{\mathrm{z}})\psi_1-\mu_0(\mathcal{H}_{\mathrm{x}}+\mathrm{i}\mathcal{H}_{\mathrm{y}})\psi_1=0 \qquad (2.6.8)$$

By substituting Hamiltonian Eq (2.5.12), then:

$$\left\{E+e_0\varphi-\mu_0\mathcal{H}\mathrm{m}-\mu_0\mathcal{H}-\frac{p^2}{2m_0}\right\}\psi_1=0$$

$$\left\{E+e_0\varphi-\mu_0\mathcal{H}\mathrm{m}+\mu_0\mathcal{H}-\frac{p^2}{2m_0}\right\}\psi_2=0 \qquad (2.6.9)$$

$\mu_0\mathcal{H}\mathrm{m}$ And $\pm\mu_0\mathcal{H}$ describe the effect of orbital and spin moment respectively, where the magnetic quantum number m is canceled in the case of s, for that the Eqs (2.5.17) yields:

$$\left\{E+e_0\varphi-\mu_0\mathcal{H}-\frac{p^2}{2m_0}\right\}\psi_1=0$$

$$\left\{E+e_0\varphi+\mu_0\mathcal{H}-\frac{p^2}{2m_0}\right\}\psi_2=0 \qquad (2.6.10)$$

Thus:

$$\mu=-\frac{e_0\hbar}{2m_0c}\sigma'$$

$$\mu=-\frac{e_0}{m_0c}S \qquad (2.6.11)$$

The spin operator can be written in terms of the Pauli operators as:

$$S = \frac{1}{2}\hbar\sigma' \qquad (2.6.12)$$

Chapter Three

Quantum Generalized Special Relativistic Equations

3.1 Introduction

Einstein Generalized special relativity (EGSR) is one of the most promising physical theories that cure the defects of SR. Einstein special relativity suffers from the lack of a term representing the potential energy in the expression of relativistic energy .Klein - Gordon quantum equation (KGQE) thus suffers from disappearance of potential term. Although, in an electromagnetic fields the electric and magnetic potentials have a room in (KGQE). But the potential of other fields cannot be represented. Thus (KGQE) cannot differentiate between particles in a potential field and particle in free space, since for both; the wave function is the same which is physically wrong.

3.2 Generalized Special Relativistic Quantum Equation

The Generalized Special Relativity theory is a new form of the special relativity theory that adopts the gravitational potential. Suppose one have a particle of rest mass m_0, momentum p, and energy E, Albert Einstein, from his 1905 reading on the STR, derived the basic energy-momentum equation:

$$E^2 = P^2C^2 + {m_0}^2C^4 \tag{3.2.1}$$

Where E, P stands for energy and momentum, while C represent the speed of light.

In GSR this relation is different and can be found from the energy equation:

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{2\varphi}{c^2}-\frac{v^2}{c^2}}} \tag{3.2.2}$$

where φ stand for the potential per unit mass, rearranging yields:

$$E = \frac{m_0\, c^2}{\sqrt{\frac{m^2c^4-2m^2\varphi^2c^2-m^2v^2c^2}{m^2c^4}}} \tag{3.2.3}$$

Thus

$$E^2\left(\frac{mc^2-2m\varphi c-mvc}{mc^2}\right) = m_0{}^2c^4 \tag{3.2.4}$$

By substituting Equation (3.2.3), and dividing over E^2, Equation (3.2.4), written as:

$$E^2 = 2Ev + c^2p^2 + m_0{}^2c^4 \tag{3.2.5}$$

The feeling of this equation by the wave function ψcan be made by Multiplying both sides (3.2.5) by $\psi$to get:

$$E^2\psi = 2Ev\psi + c^2p^2\psi + m_0{}^2c^4\psi \tag{3.2.6}$$

The GSRE for quantum system can be obtained by taking into account the dual nature of atomic particles which are assumed to be in the form of a wave packet.

$$\psi = e^{i/\hbar(px-Et)} \tag{3.2.7}$$

Differentiating both sides with respect to time and space twice yields:

$$\frac{\partial\psi}{\partial t} = \frac{i}{\hbar}E\psi$$

$$-\hbar^2 \frac{\partial^2\psi}{\partial t^2} = E^2\psi \qquad (3.2.8)$$

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar} p\psi$$

$$-\hbar^2 \frac{\partial^2\psi}{\partial t^2} = E^2\psi \qquad (3.2.9)$$

Substituting (3.2.8) and (3.2.9) in (3.2.6) yield:

$$-\hbar \frac{\partial^2\psi}{\partial t^2} = 2\, i\hbar \frac{\partial\psi}{\partial t} V - \hbar^2 \mathrm{c}^2 \nabla^2 \psi + {m_0}^2 c^4 \psi \qquad (3.2.10)$$

Which is the quantum generalized special relativistic equation. For particles having spin the potential V can be spitted to medium potential V_m and spin potential V_s , i.e.

$$V = V_m + V_s \qquad (3.2.11)$$

Thus the quantum equation (3.2.10) becomes

$$-\hbar \frac{\partial^2\psi}{\partial t^2} = 2i\hbar(V_m + V_s)\frac{\partial\psi}{\partial t} - c^2\hbar + {m_0}^2 c^4$$

(3.2.12)

It's also important to note that equation (3.2.10) reduces to klein-Gordan equation for,$V = 0$ To get:

$$-\hbar \frac{\partial^2\psi}{\partial t^2} = -\hbar^2 \mathrm{c}^2 \nabla^2 \psi + {m_0}^2 c^4 \psi \qquad (3.2.13)$$

3.3 Generalized Special Relativistic Quantum spins Equation

According to GSRE the energy given by:

$$E = \frac{g_{00\,m_0\,c^2}}{\sqrt{g_{00}-\frac{v^2}{c^2}}} \quad (3.3.1)$$

$$g_{00} = 1 + \frac{2\varphi}{c^2} \tag{3.3.2}$$

φ Denotes the gravitational potential, or the field in which the mass is measured.

$$E\left(\frac{m^2c^4-2m^2\varphi c^2-m^2v^2c^2}{m^2c^4}\right)^{\frac{1}{2}} = \left(\frac{mc^2-2\varphi m}{mc^2}\right)m_0{}^2c^4 \tag{3.3.3}$$

By substituting equation (3.2.4), Equation (3.2.5) yields,

$$c - 2Ev - p^2c^2 = \frac{(E-2v)^2}{E^2}m_0{}^2c^4 = \frac{(E^2-2Ev-4v^2)}{E^2}m_0{}^2c^4 \tag{3.3.4}$$

$$E^2 = p^2c^2 + 2Ev + m_0{}^2c^4\left(1-\frac{2v}{E}-\frac{4v^2}{E^2}\right) \tag{3.3.5}$$

$$E\left(1-\frac{2\varphi}{c^2}-\frac{v^2}{c^2}\right)^{\frac{1}{2}} = \left(1-\frac{2\varphi}{c^2}\right)m_0c^2$$

For weak field and small speed for $\varphi, << c$,and Neglecting higher order terms Equation (3.3.5) become:

$$E = m_0c^2\left(1+\frac{\varphi}{c^2}+\frac{v^2}{2c^2}\right) = \left(m_0c^2+m_0\varphi+\frac{m_0v^2}{2}\right)$$

$$E = m_0c^2 + V + T = m_0c^2 + V + \frac{p^2}{2m} \tag{3.3.6}$$

This expression is typical to that of Newton, when one neglects rest

Mass term .where T is the Kinetic energy$T = \frac{p^2}{2m}$, multiplying the both side of Eq (3.3.6) by the wave function ψ, one gets:

$$E\psi = m_0c^2\psi + v\psi + \frac{p^2}{2m}\psi \tag{3.3.7}$$

The potential V is written in the form:

$$V = V_m + V_s \tag{3.3.8}$$

Where

$V_s \equiv spin\ potential$

$V_m \equiv medium\ potential$

$$i\hbar\frac{\partial\psi}{\partial t} = m_0c^2\psi + (V_m + V_s)\psi - \frac{\hbar^2}{2m}\nabla^2\psi \tag{3.3.9}$$

3.4 The Generalized Dirac Relativistic Spin Equation

The energy-momentum relation in GSR takes the form:

$$E = g_{00}(g_{00} - \frac{v^2}{c^2})^{\frac{-1}{2}}m_0c^2 \tag{3.4.1}$$

Thus

$$E = g_{00}(1 - \frac{2\varphi}{c^2} - \frac{v^2}{c^2})^{\frac{-1}{2}}m_0c^2 \tag{3.4.2}$$

Therefore for very weak potential and small speed:

$$E = g_{00}(1 + \frac{1}{2}(\frac{2\varphi}{c^2} + \frac{v^2}{c^2})^{\frac{-1}{2}})m_0c^2 = g_{00}(m_0c^2 + \varphi m_0 + \frac{1}{2}m_0v^2$$

$$E = g_{00}(m_0c^2 + V + T) \tag{3.4.3}$$

Clearly for Minkowskian space:

$$g_{00} = 1 \tag{3.4.4}$$

Thus

$$E = T + V + m_0c^2 \tag{3.4.5}$$

However the more strict linear energy form can be obtained by using equation (3.4.1) to get:

$$E = g_{00}(g_{00} - \frac{v^2}{c^2})^{\frac{-1}{2}}m_0c^2 \tag{3.4.6}$$

Squaring both sides gives

$$E^2(g_{00} - \frac{v^2}{c^2}) = g_{00}{}^2m_0{}^2c^4 \tag{3.4.7}$$

$$E^2(g_{00} - \frac{m^2v^2c^2}{m^2c^4}) = g_{00}{}^2m_0{}^2c^4$$

$$g_{00}E^2 - \frac{\mathrm{p}^2\mathrm{c}^2\mathrm{E}^2}{\mathrm{E}^2} = g_{00}{}^2m_0{}^2c^4$$

Thus finally one gets:

$$g_{00}E^2 = \mathrm{p}^2\mathrm{c}^2 + g_{00}{}^2(m_0{}^2c^4) \tag{3.4.8}$$

The linear form of this equation can be written as:

$$\sqrt{g_{00}}E = \propto \cdot cp + \beta g_{00} m_0 c^2 \quad (3.4.9)$$

Squaring both sides yields :

$$g_{00}E^2 = \propto^2 \cdot c^2 p^2 + \beta^2 g_{00}{}^2 m_0{}^2 c^4 + [(\propto \cdot p) \cdot \beta + \beta \cdot (\propto \cdot p)] m_0 c^3 \quad (3.4.10)$$

$$g_{00}E^2 = \propto^2 \cdot c^2 p^2 + \beta^2 g_{00}{}^2 m_0{}^2 c^4 + [\alpha\beta + \beta\alpha] \cdot p m_0 c^3 \quad (3.4.11)$$

Comparing equations and yields

$$\propto_x{}^2 = \propto_y{}^2 = \propto_z{}^2 = 1$$

$$\propto_x \propto_y = \propto_x \propto_z = \propto_y \propto_z = \propto_z \propto_y = 0$$

$$\alpha_x \beta + \beta \alpha_x = \alpha_y \beta + \beta \alpha_y = \alpha_z \beta + \beta \alpha_z = 0 \quad (3.4.12)$$

Using the same procedures used, in finding $\propto$ *and* β, for Dirac equation, one find that these parameters have the same values as that of Dirac equation.

The new Dirac GSR equation can be obtained by using relation (2.2.4) and (2.2.5), where

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi - \hbar^2 \frac{\partial^2 \psi}{\partial x^2} = p^2 \psi$$

In equation (3.4.9) to get

$$i\hbar \sqrt{g_{00}} \frac{\partial \psi}{\partial t} = -i\hbar c \alpha \cdot \nabla \psi + \beta g_{00} m_0 c^2 \psi \quad (3.4.13)$$

It is important to note that for Minkowskian space:

$$g_{00} = 1$$

And equation (3.4.13) reduces to the ordinary Dirac equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -i\hbar c\alpha\cdot\nabla\psi + \beta m_0 c^2\psi \qquad (3.4.14)$$

3.5 Discussion

Using GSR energy - momentum relation (3.2.5) beside the dual nature of microscopic particles a generalized special relativistic quantum equation found. It is interesting to note that for vanishing potential this equation (3.2.10) reduces to Klein - Gordan equation. The spinning particle can feel the effect of spin through the term V_s for small speed and weak potential the energy is linear in p and can be given by equation (3.3.6). The corresponding quantum equation is given by (3.3.9).

This equation resembles Schrödinger equation for spinning particles with an additional term standing for rest mass energy. This equation reduces to Schrödinger spin equation when the rest mass is neglected. It is also important to note that Dirac GSR quantum equation (3.4.13). For particle having spin was derived in section (3.4).

It is very interesting to note that the parameters $\alpha and\beta$ are related to Pauli spin matrices in the same way as that of Dirac. It is also interesting to note that this equation (3.4.13) reduces to Dirac quantum relation equation for Minkowskian Euclidean space.

3.6 conclusions

Generalized special relativity and energy-momentum relation can be used to derive all quantum spin equations.

3.7 Recommendation

- The generalized special relativistic quantum equation for particles having spin need to be applied for all elementary particles to see how it can explain many physical micro-phenomena.
- This equation should also be tested for fermions and bosons.

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