Sudan University of Science and Technology
College of graduate

Applications of the Spectral Local Linearisation Method on Different Types of Fluid Flow

تطبيقات الطريقة الخطية المحلية الطيفية علي أنواع مختلفة لتدفق المائع

A thesis Submitted in Partial Fulfillment for the Degree of M.Sc in Mathematics

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November 2016

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Dedication

To my dear father, his mercy and forgiveness

…

To my dear mother

…

To my brothers and sisters

…

To all those who helped me in my educational career

…

Acknowledgment

Firstly, thank Allah Almighty, which helped me of completed this work. Thanks my efficient supervisor Dr. Mohammed Hassan Mohammed Khabir, which has the great virtue of tips. Thanks my efficient supervisor Dr. Faiz Gad Elmula Awad, who is widely represented in the great merit support and assistance. Thanks ustaza. Khalda Elkhair and Lamia Elmansour. Thanks my friend ustaz. Mohammed Ibrahim Abdrahim, who has been serviced me throughout of two years ago with activity. I would like to express my profound gratitude to my all family members to persevere through the previous period. Finally thank everyone who supported me throughout the study period.

Abstract

In this thesis we paid a tension on two kind of fluid flow problems, the the non-Newtonian fluids flow include the Couette flow, Poiseuille flow, Couette Poiseuille flow, and the convection heat and mass transfer in non-Darcy porous medium. The highly nonlinear differential equations governing the fluid flow transferred into ordinary differential equations. Later the resulting equations solved numerically by using the spectral local linearization method (SLLM). The effects of the governing parameters such as Brinkman number, Nusselt number, Sherwood number, Lewis number, the buoyancy ratio, the pore diameter dependent Rayleigh number, and other parameters have been shown graphically.

Abstract (Arabic)

في هذا البحث ركّزنا في دراسة نوعين من مسائل تدفق المائع .النوع الأول تدفق الموائع غير النيوتونية ويحتوي علي تدفق كويت و تدفق بواسويل و تدفق كيوت - بواسويل، والنوع الثاني ظاهرة الحمل الحراري ونقل الكتلة في وسط ذو مسامي غير دارسى. المعادلات التفاضلية عالية اللاخطية تُحّكم بتدفق المائع المحّول إلي معادلات تفاضلية عادية. المعادلات الناتجة حلت عددياً بإستخدام طريقة الخطية المحلية الطيفية. تأثيرات المعاملات الحاكمة مثل رقم برينكمان، رقم نيوسلت، رقم شورود، نسبة الطفو، رقم ريااليغ و رقم لويس توضُع عددياً في صورة رسومات بيانية.

Chapter 1

Introduction

The study of the phenomenon of thermal and solutal transport by fluid flow through porous media is of great interest. The flow phenomenon relatively complex rather than that of the pure thermal convection process. Heat and mass transfer processes in porous media are often encountered in the study of dynamics of hot and salty springs of a sea, and in the chemical industry, in reservoir engineering in connection with thermal recovery process. Underground spreading of chemical wastes and other pollutants, grain storage, evaporation cooling, and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed. Coupled heat and mass transfer in porous media has been analyzed in [5-8]. Cheng [8] presented a comprehensive review about heat transfer in geothermal systems. Plumb and Huenefeld [10] and Nakayman et al. [11] used the Forchheimer extension to study the non-Darcy natural convection from the vertical wall.

Study of the thermal dispersion effects become prevalent in the porous media flow region. The thermal and solutal dispersion effects become more important when the inertial effects are prevalent. Fried and Combarnous [12] proposed a linear function to expressed the thermal dispersion. Also, a linear dispersion model takes the porosity of the porous medium into account is used for free convection in a horizontal layer heated from below was introduced by Georgiadis and Catton [13]. Cheng [14] and Plumb[15] gave another model for flow and heat transfer in porous media by taking thermal dispersion effects into consideration. An analysis of thermal dispersion effect on vertical plate natural convection in porous media is presented by Hong and Tien [16]. Lai and Kulacki [17] investigated thermal dispersion effect on non-Darcy convection from horizontal surface in saturated porous media. Effects of thermal dispersion and lateral mass flux on non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium were studied by Murthy and Singh [18].

The complexity of the flow increases when higher order effects like thermal and solutal dispersion are considered in the medium. Karimi-Fard et .[19] has been presented a numerical study of double-diffusive free convection heat and mass transfer in a square cavity filled with a porous medium. Began [20] has been analyzed the effect of solutal and thermal dispersion in Darcian porous medium. The double dispersion phenomenon in a free convection boundary layer adjacent to a vertical wall in Darcian porous medium, using scale analysis arguments, has been investigated byTelles and Trevisan [21]. Effects of double dispersin on mixed convection heat and mass transfer in non-Darcy porous medium has been investigated by Murthy [22].

The present investigation is devoted to study the combined effect of solutal and thermal dispersion on Forehheimer natural convection heat and mass transfer over a vertical flat plate in a fluid saturated porous medium. The Forehheimer flow model is considered and the porosity of the porous medium is assumed to be low, so that the boundary effects in the medium may be neglected. The heat and mass transfer in the boundary region has been analyzed for aiding and opposing buoyancies. The flow, temperature and concentration field in Darcy and non-Darcy porous media are observed to governed by complex interactions among the diffusion rate Le, buoyancy ratio N, Ra_{d}, the dispersion thermal and solutal diffusivity parameter. The wall temperature and the wall concentration distribution are assumed to be uniform.

Chapter 2

Spectral Local Linearization Method (SLLM)

To describe the spectral local linearization method .We are illustrating the behaviors and approach for this method through the following steps. Considering a system of m non-linear ordinary differential equations in m unknown functions $Z_i(\eta), i = 1,2, \ldots, m,$ where $\eta \epsilon [a, b]$ is the dependent variable. The system can be written in terms of Z_i as a sum of its linear $(L_i)$and nonlinear components N_i as

$$L_i[Z_1, Z_2, \ldots, Z_m] + N_i[Z_1, Z_2, \ldots, Z_m] = 0, \qquad i = 1, \ldots, m. \quad (2.1)$$

To develop the iteration scheme, we apply local linearization of N_i about $Z_{i,r}$ (the previous iteration) to the i^{th} nonlinear equation assuming that all other $Z_{k,r}(k \neq i)$ are known. Thus, at the i^{th} equation, N_i is linearized as follows.

$$N_i[Z_1, Z_2, \ldots, Z_m] =$$

$$N_i\left[Z_{1,r}, Z_{2,r}, \ldots, Z_{m,r}\right] + \frac{\partial N_i}{\partial Z_i}\left[Z_{1,r}, Z_{2,r}, \ldots, Z_{m,r}\right]\left(Z_i - Z_{i,r}\right). \quad (2.2)$$

Thus, at the current iteration with $Z_i = Z_{i,r+1}$,equation (2.1) becomes

$$L_i\left[Z_{1,r+1}, \ldots, Z_{m,r+1}\right] + \frac{\partial N_i}{\partial z_i}[\ldots]Z_{i,r+1}$$

$$= \frac{\partial N_i}{\partial z_i}[\ldots]Z_{i,r} - N_i\left[Z_{1,r}, Z_{2,r}, \ldots, Z_{m,r}\right], (2.3)$$

where$[\ldots]$ denotes $[Z_{1,r}, Z_{2,r}, \ldots, Z_{m,r}]$ and $Z_{i,r+1}$ and $Z_{i,r}$ are the approximations of Z_i at the current and the previous iteration, respectively. To obtain a decoupled iteration scheme, we appeal to the Gauss-Seidel approach of decoupling linear algebraic systems in linear algebra applications. We therefore arrange the equations in a particular order and solve them in a chronological order. In seeking the solution of Z_i in the current iteration level, $Z_{i,r+1}$,we use updated solutions of $Z_s(s < i)$ obtained as solutions of the previous $(i = 1,2, \ldots s)$equations. Thus, for a system of m equations, the local linearization iteration scheme becomes

$$L_1\left[Z_{1,r+1}, Z_{2,r} \ldots, Z_{m,r}\right] + \frac{\partial N_1}{\partial z_1}[\ldots]Z_{1,r+1} = \frac{\partial N_1}{\partial z_1}[\ldots]Z_{1,r} - N_1\left[Z_{1,r},, \ldots, Z_{m,r}\right],$$

$$L_2\left[Z_{1,r+1}, Z_{2,r+1}, Z_{3,r} \ldots, Z_{m,r}\right] + \frac{\partial N_2}{\partial z_2}[\ldots]Z_{2,r+1}$$

$$= \frac{\partial N_2}{\partial z_2}[\ldots]Z_{2,r} - N_2\left[Z_{1,r+1}, Z_{2,r}, \ldots, Z_{m,r}\right],$$

$$\vdots$$

$$L_m\left[Z_{1,r+1}, Z_{2,r+1} \ldots, Z_{m,r+1}\right] + \frac{\partial N_m}{\partial z_m}[\ldots]Z_{m,r+1}$$

$$= \frac{\partial N_m}{\partial z_m}[\ldots]Z_{m,r} - N_m\left[Z_{1,r+1}, \ldots, Z_{m-1,r+1}, Z_{m,r}\right], \quad (2.4)$$

where$[\ldots] \equiv \left[Z_{1,r+1}, Z_{2,r+1} \ldots, Z_{i-1,r+1}, Z_{i,r}, \ldots, Z_{m,r}\right]$ at the ith equation.Thus, starting from an initial approximation $Z_{1,0}, Z_{2,0}, \ldots, Z_{m,0}$, the proposed iterative scheme (2.4) is then solved as a loop until the system converges at a consistent solution for all the variables. To solve the iteration scheme (2.4), it is convenient to use the Chebychev-Pseudo spectral method. For this reason the proposed method is referred to as the spectral local linearization iteration method (SLLM. Spectral methods are now becoming the preferred tools for solving ordinary and partial differential equations because of their elegance and high accuracy in resolving problems with smooth functions.

Before applying the spectral method, it is convenient to transform the domain on which the governing equation is defined to the interval $[-1,1]$ on which the spectral method can be implemented. We use the transformation $\eta = (b - a)(\tau + 1)/2$ to map the interval $[a, b]$ on $[-1,1]$. The basic idea behind the spectral collocation method is the introduction of a differentiation matrix

D which is used to approximate the derivatives of the unknown variables $Z_i(\eta)$ at the collocation points as the matrix vector product

$$\frac{dZ_i}{d\eta} = \sum_{k=0}^{\bar{N}} D_{lk} z_i(\tau_k) = \mathbf{D} Z_i, \;\; l = 0,1,\dots,\bar{N}, \tag{2.5}$$

where$\bar{N}+1$ is the number of the collocation points (grid points),

$\mathbf{D} = 2D/(b-a)$, and $Z = [z(\tau_0), z(\tau_1), \dots, z(\tau_N)]^T$ is the vector function at the collocation points. Higher order derivatives are obtained as of D, that is

$$Z_j{}^{(p)} = \mathbf{D}^p Z_j, \tag{2.6}$$

where p is the order of the derivative.

We can express equation (2.5) in a matrix form in following

$$\begin{bmatrix} D_{0,0} & \cdots & D_{0,\bar{N}} \\ \vdots & \ddots & \vdots \\ D_{\bar{N},0} & \cdots & D_{\bar{N},\bar{N}} \end{bmatrix} \begin{bmatrix} Z_1(\tau_0) \\ Z_2(\tau_1) \\ \vdots \\ Z_m(\tau_{\bar{N}}) \end{bmatrix} = \begin{bmatrix} \mathrm{R}_{1,r} \\ \mathrm{R}_{2,r} \\ \vdots \\ \mathrm{R}_{m,r} \end{bmatrix}, \tag{2.7}$$

And after apply Chebyshev pseudospectralmethod, notice that every interval $[a,b]$, to be transform to interval $[-1,1]$ then we can write that in the form

$$\begin{bmatrix} D_{1,1} & \cdots & D_{1,,\bar{N}+1} \\ \vdots & \ddots & \vdots \\ D_{\bar{N}+1,1} & \cdots & D_{\bar{N}+1,\bar{N}+1} \end{bmatrix} \begin{bmatrix} Z_m(\tau_{\bar{N}+1}) \\ Z_2(\tau_{\bar{N}}) \\ \vdots \\ Z_1(\tau_0) \end{bmatrix} = \begin{bmatrix} \mathrm{R}_{1,r} \\ \mathrm{R}_{2,r} \\ \vdots \\ \mathrm{R}_{m,r} \end{bmatrix}, \tag{2.8}$$

where $z_i(\tau_k), \mathrm{R}_{i,r}$ are vectors of size $(\bar{N}+1) \times 1$ and $D_{i.j}$ are $(\bar{N}+1) \times (\bar{N}+1)$ matrixes.

Chapter 3

The Couette Flow

3.1 – Third Grade Fluid

Third grade fluid is considered one of the subclasses of the non-Newtonian fluids and depends on the shear stress and shear rate. Refs [1-4].

In general the governing equations for conservation of mass, momentum and energy for an incompressible fluid in tensor notation which are given by Refs. [3-4] as follows:

$$u_{j,j} = 0, \tag{3.1}$$

$$\rho\left(\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right) = \rho f + \tau_{ij,j}, \tag{3.2}$$

$$\rho c_p\left(\frac{\partial \theta}{\partial t} + u_j\frac{\partial \theta}{\partial x_j}\right) = \kappa\theta_{jj} + \tau_{ij}u_{i,j}, \tag{3.3}$$

$$i,j = 1,2,3.$$

where u is the velocity, f is the body force, τ is stress tensor, θ is the temperature, ρ is the constant fluid density, κ is the thermal conductivity, c_p is the specific heat. The constitutive equation for a third grade fluid given by Refs. [3-4] as follows:

$$\tau_{ij} = -p\delta_{i,j} + \mu S_{1ij} + \alpha_1 S_{2ij} + \alpha_2 {S^2}_{1ij} + \beta_1 S_{3ij}$$
$$+\beta_2\left(S_{1ij}S_{2ij} + S_{2ij}S_{1ij}\right) + \beta_3\left(\mathrm{tr}S_{2ij}\right)S_{1ij}, \tag{3.4}$$

where p is the fluid pressure, μ is the coefficient of viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are material constants and $S_{1ij},\ S_{2ij},\ S_{3ij}$ are line kinematics tensors defined by

$$S_{1ij} = \left(u_{j,i} + u_{i,j}\right), \tag{3.5}$$

$$S_{nij} = \frac{DS_{(n-1)ij}}{Dt} + S_{(n-1)ij}u_{i,j} + u_{j,i}S_{(n-1)ij}, \qquad n = 2,3. \tag{3.6}$$

3.2 – Couette flow

Consider the steady state flow of a third grade fluid all a long tow parallel plates the distance between them 2h. The lower plate is stationary and the upper plate is moving with a constant speed U. The temperature of the lower plate is θ_0 and that of the upper plate is θ_1.The lower and upper plates are located in the planes $y = -h$ and $y = h$. The pressure gradient is zero, and the velocity and temperature fields are assumed to be of the form

$$u = u(y), \qquad v(y) = 0, \qquad w(y) = 0, \qquad \theta = \theta(y) \tag{3.7}$$

The equation of continuity is satisfied and the momentum and energy equations become ($\beta_1 = \beta_3$)

$$\mu \frac{d^2u}{dy^2} + 6(\beta_1 + \beta_2)\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} = 0, \tag{3.8}$$

$$\kappa \frac{d^2\theta}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + 2(\beta_1 + \beta_2)\left(\frac{du}{dy}\right)^4 = 0. \tag{3.9}$$

As a result, the problem reduces for solving the equations (3.8) - (3.9) subject to the conditions of no slip and no temperature jump at both of the plates

$$u(-h) = 0, \quad u(h) = U \text{ and } \theta(-h) = \theta_0, \qquad \theta(h) = \theta_1. \tag{3.10}$$

We consider h as the characteristic length, U as the characteristic velocity, and θ_0 and θ_1 as characteristic temperature and rewrite the above equation in dimensionless form by using transformations

$$y^* = \frac{y}{h}, \quad u^* = \frac{u}{U}, \theta^* = \frac{\theta - \theta_0}{\theta_1 - \theta_0}. \tag{3.11}$$

Therefore $y = hy^*$, since

$$\frac{\partial}{\partial y^*} = \frac{\partial y}{\partial y^*}.\frac{\partial}{\partial y} \quad \Rightarrow \frac{\partial u^*}{\partial y^*} = \frac{\partial y}{\partial y^*}.\frac{\partial u^*}{\partial y} \text{ and } \frac{\partial u}{\partial y} = \frac{U}{h}.\frac{\partial u^*}{\partial y}.$$

Then

$$\frac{\partial^2 u}{\partial \mathrm{y}^2} = \frac{U}{h}\frac{\partial^2 u^*}{\partial y^{*2}}. \tag{3.12}$$

By substituting equation (3.12) in equation (3.8), we have

$$\frac{\partial^2 u^*}{\partial y^{*2}} + 6\left(\frac{\beta_1 + \beta_2}{\mu}\right)\left(\frac{U}{h}\right)^2 \left(\frac{\partial u^*}{\partial y^*}\right)^2 \frac{\partial^2 u^*}{\partial y^{*2}} = 0. \tag{3.13}$$

And on the other side, $y = hy \Rightarrow \theta^*(y) = \theta^*(hy^*) = h.\theta^*(y^*)$

$$\frac{\partial \theta^*}{\partial y^*} = \frac{\partial y}{\partial y^*}.\frac{\partial \theta^*}{\partial y} = \left(\frac{h^2}{\theta_1 - \theta_0}\right)\frac{\partial \theta}{\partial y}. \tag{3.14}$$

Then

$$\frac{\partial^2 \theta}{\partial \mathrm{y}^2} = \left(\frac{\theta_1 - \theta_0}{h^2}\right)\frac{\partial^2 \theta^*}{\partial y^{*2}} \ . \tag{3.15}$$

By substituting equation (3.15) in equation (3.9), we have

$$k\left(\frac{\theta_1 - \theta_0}{h^2}\right)\frac{\partial^2 \theta^*}{\partial y^{*2}} + \left(\frac{U}{h}\frac{\partial u^*}{\partial y^*}\right)^2 + 2(\beta_1 + \beta_2)\left(\frac{U}{h}\frac{\partial u^*}{\partial y^*}\right)^4 = 0, \tag{3.16}$$

$$\frac{\partial^2 \theta^*}{\partial y^{*2}} + \frac{\mu U^2}{k(\theta_1 - \theta_0)}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + 2\left(\frac{\beta_1 + \beta_2}{\mu}\right)\left(\frac{U}{h^2}\right)^2 \frac{\mu U^2}{k(\theta_1 - \theta_0)}\left(\frac{\partial u^*}{\partial y^*}\right)^4 = 0, \tag{3.17}$$

In a non-dimensional form, after dropping the asterisks and using ODEs style, equations (3.13) and (3.17) become

$$\frac{d^2 u}{dy^2} + 6\beta\left(\frac{du}{dy}\right)^2 \frac{d^2 u}{dy^2} = 0, \tag{3.18}$$

$$\frac{d^2 \theta}{dy^2} + \lambda\left(\frac{du}{dy}\right)^2 + 2\beta\lambda\left(\frac{du}{dy}\right)^4 = 0. \tag{3.19}$$

where $\beta = \left(\frac{\beta_1 + \beta_2}{\mu}\right)\left(\frac{U}{h}\right)^2$, $\quad \lambda = \frac{\mu U^2}{\kappa(\theta_1 - \theta_0)} = \frac{\mu c_p}{\kappa}\frac{U^2}{c_p(\theta_1 - \theta_0)} = PrEc,$

$$Pr = \frac{\mu c_p}{\kappa}, \qquad Ec = \frac{U^2}{c_p(\theta_1 - \theta_0)},$$

where $PrEc$ is the Brinkman number that is the product of the Prandtl number Pr and Eckert number Ec.The corresponding boundary conditions are

$$u(-1) = 0\,, \qquad u(1) = 1, \qquad \theta(-1) = 0, \qquad \theta(1) = 1. \tag{3.20}$$

3.3 – Numerical Solution

The momentum and energy in which governing Couette flow along with the boundary can be written as

$$u'' + 6\beta u'^2 u'' = 0, \tag{3.21}$$

$$\theta'' + \lambda u'^2 + 2\beta\lambda u'^4 = 0, \tag{3.22}$$

$$u(-1) = 0\,, \qquad u(1) = 1, \qquad \theta(-1) = 0, \qquad \theta(1) = 1, \tag{3.23}$$

where β, λ are constants. To linearize the above system of nonlinear ordinary differential equations, one can use the so called local linearization method. We appeal to the Gauss-Sedeil approach of decoupling linear algebraic, then the solutions can be obtained in the following steps:

In equation (3.21) the term u'' is the linear denoted by L and the term $u'^2 u''$ is non-linear say N_1, now rewrite (3.21)

$L[u] + N[u] = 0.$

From Taylor series,

$$N_1(u', u'') = N_1(u', u'') + \frac{\partial N_1}{\partial u'}(u'_{r+1} - u'_r) + \frac{\partial N_1}{\partial u''}(u''_{r+1} - u''_r). \tag{3.24}$$

Applying the local linearization on the nonlinear term gives

$$u'^2 u'' = 2u'_r u''_r u'_{r+1} - 2u'^2_r u''_r + u'^2_r u''_{r+1}. \tag{3.25}$$

Substituting equation(3.25) into equation (3.21) gives decoupled equations, from this point an iteration scheme is developed by evaluating linear terms at the current iteration level $r+1$ and the nonlinear terms at the previous iteration level r, now one can write the following

$$\left(u''_{r+1} + 6\beta u'^2_r\right) u''_{r+1} + 12\beta u'_r u''_r u'_{r+1} = 12\beta u'^2_r u''_r, \tag{3.26}$$

$$\theta''_{r+1} = -\lambda (u'_r)^2 - 2\beta\lambda (u'_r)^4, \tag{3.27}$$

subject to

$$u_{r+1}(-1) = 0, \qquad u_{r+1}(1) = 1, \qquad \theta_{r+1}(-1) = 0, \qquad \theta_{r+1}(1) = 1.$$

Applying the Chebyshev Pseudo-spectral method on equations $(3.26) - (3.27)$ we obtained the following decoupled system of equations

$$(\text{diag}[1 + 6\beta u'^2_r]\mathbf{D^2} + 12\beta\text{diag}[u'_r u''_r]\mathbf{D})u_{r+1} = 12\beta u'^2_r u''_r, \tag{3.28}$$

$$\mathbf{D^2}\theta_{r+1} = -\lambda(u'_r)^2 - 2\beta\lambda(u'_r)^4, \tag{3.29}$$

subject to the boundary conditions

$$u_{r+1}(-1) = 0, \quad u_{r+1}(1) = 1, \qquad \theta_{r+1}(-1) = 0, \qquad \theta_{r+1}(1) = 1.$$

Starting from given initial approximations u_0 and θ_0, equations (3.28)–(3.29) can be solved iteratively the spectral collocation methods for its accuracy. We find the unknown function at collocation points by requiring that equations (3.28)–(3.29) be satisfied exactly at these points. A convenient set of collocation points is the Gauss-Lobat to points defined by

$$\omega_j = cos\frac{\pi j}{N}\ ,\ j = 0,1\ ,2,3,\cdots,N,$$

where $N+1$ the number of collocation points is, $\mathbf{D}$ is defined by equation (2.15) and $u = [u(\omega_0),\ u(\omega_1),\ u(\omega_2), \dots, u(\omega_N),]^T$ is the vector of unknown functions at the collocation points. In the matrix form, one can rewrite equations (3.28) - (3.29) in the form

$$\mathbf{A_1}u_{r+1} = R_1\ ,\ \ u_{r+1}(\omega_N) = 0,\ \ \ u_{r+1}(\omega_0) = \ 1, \tag{3.30}$$

$$\mathbf{A_2}\theta_{r+1} = R_2\ ,\ \ \theta_{r+1}(\omega_N) = 0,\ \ \ \theta_{r+1}(\omega_0) = \ 1, \tag{3.31}$$

where,

$$\mathbf{A_1} = \text{diag}[1 + 6\beta(u'_r)^2]\mathbf{D^2} + 12\text{diag}[\beta u'_r u''_r]\mathbf{D}, \quad R_1 = 12\beta(u'_r)^2 u''_r,$$

$$\mathbf{A_2} = \mathbf{D^2}, \quad R_2 = -\lambda(u'_r)^2 - 2\beta\lambda(u'_r)^4,$$

where, diag[.] denotes a diagonal matrix. We choose suitable initial guesses u_0 and θ_0 which satisfy the boundary conditions of governing equations.

$3.4-$ Results and Discusses

The governing equations of the momentum and energy of the Couette flow, which are represented in equations (3.21) – (3.22) along with the boundary conditions (3.23), are solved numerically by using the spectral local linearization method. The results showing the effects of Brinkman number λ, and the parameters β on the velocity and temperature are given graphically.

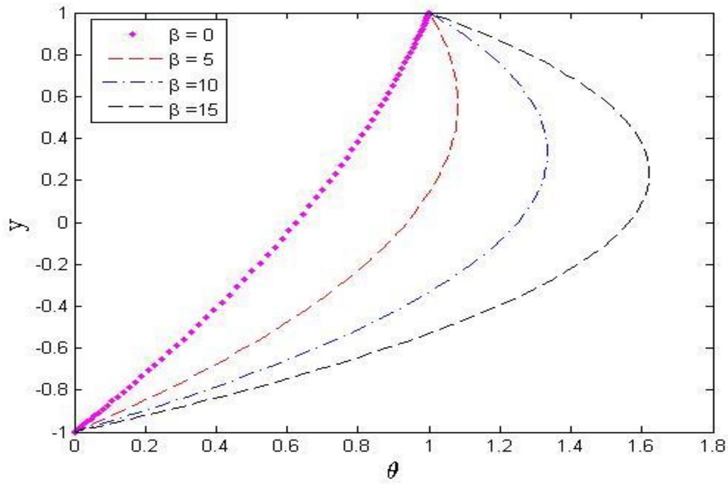

Fig3.1 Effects ofβon temperature $\theta$with$\lambda = 1$.

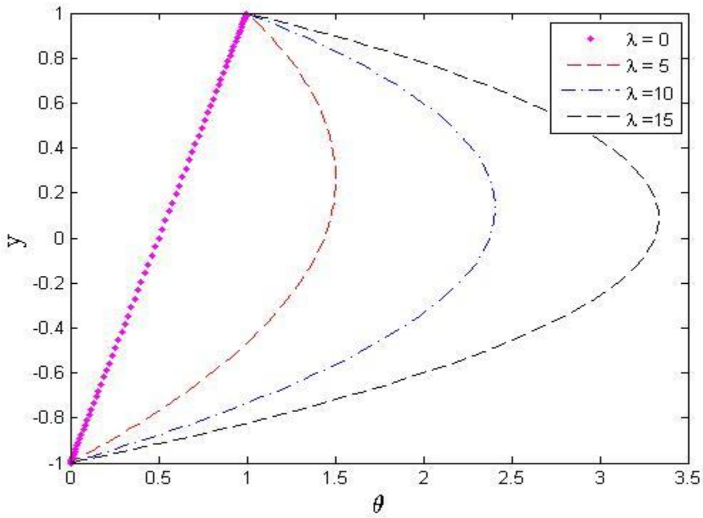

Fig 3.2 Effects of the Brinkman number λ on temperature θ for $\beta = 1$.

Fig 3.1 illustrates the variation of temperature profile θ with β as Brinkman number λ is taken to be one. It clear that, the temperature profile increases with increasing β leading to a thinness in the thermal boundary layer, hence increases the fluid temperature. Fig 3.2 shows the effects of the Brinkman parameter λ on the temperature profile when $\beta = 1$. We notice that thermal boundary layer thickness decreases when the Brinkman parameter increases, and hence enhancing the fluid temperature.

Chapter 4

The Poiseuille flow

This a type of the fluid flow ,the mathematical formulations for it are reduced from third grade fluid through Chapter 3.

$\mathbf{4.1} -$ The governing Equations

Let us consider the problem in Chapter 3 when both plates are stationary and the fluid motion is induced by constant pressure gradient, other conditions on the velocity and the temperature fields remain unchanged. In this case, the momentum and energy equations with $(\beta_1 = \beta_3)$ yield

$$\mu \frac{d^2u}{dy^2} + 6(\beta_1 + \beta_2)\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} = \frac{\partial \hat{p}}{\partial x}, \qquad (4.1)$$

$$\frac{\partial \hat{p}}{\partial y} = \frac{\partial \hat{p}}{\partial z} = 0, \qquad (4.2)$$

$$\kappa \frac{d^2\theta}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + 2(\beta_1 + \beta_2)\left(\frac{du}{dy}\right)^4 = 0, \qquad (4.3)$$

where $\hat{p}$ denotes the generalized pressure which given by

$$\hat{p} = p - (2 \propto_1 + \propto_2)\left(\frac{du}{dy}\right)^2. \qquad (4.4)$$

We find from (4.1) that

$$\frac{\partial \hat{p}}{\partial x} = \text{constant}. \qquad (4.5)$$

Thus, the problem reduces to the following differential equations

$$\mu \frac{d^2u}{dy^2} + 6(\beta_1 + \beta_2)\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} = \frac{\partial \hat{p}}{\partial x},$$

$$\kappa\frac{d^2\theta}{dy^2}+\mu\left(\frac{du}{dy}\right)^2+2(\beta_1+\beta_2)\left(\frac{du}{dy}\right)^4=0, \tag{4.6}$$

$$u(-h)=0, \qquad u(h)=0, \qquad \theta(-h)=\theta_0, \qquad \theta(h)=\theta_1. \tag{4.7}$$

By using transformations in equation (3.11), we will obtain

$$\frac{d^2u}{dy^2}+6\beta\left(\frac{du}{dy}\right)^2\frac{d^2u}{dy^2}=-B, \tag{4.8}$$

$$\frac{d^2\theta}{dy^2}+\lambda\left(\frac{du}{dy}\right)^2+2\beta\lambda\left(\frac{du}{dy}\right)^4=0, \tag{4.9}$$

Subject to the boundary conditions

$$u(-1)=0, \qquad u(1)=0, \qquad \theta(-1)=0, \qquad \theta(1)=1, \tag{4.10}$$

where $B=-\frac{h^2}{U\mu}\frac{d\hat{p}}{dx}$.

4.2 – Numerical Solution

Equations(4.8) $-$ (4.9) can be written in form

$$u''+6\beta(u')^2u''=-B, \tag{4.10}$$

$$\theta''+\lambda(u')^2+2\beta\lambda(u')^4=0, \tag{4.11}$$

with the boundary conditions,

$$u(-1)=0, \qquad u(1)=0, \qquad \theta(-1)=0, \qquad \theta(1)=1, \tag{4.12}$$

where β, B, λ are constants. Also we use the same style to get the solutions of these equations. Applying the local linearization method we end up by

$$u''_{r+1}+6\beta u'^2_r u''_{r+1}+12\beta u'_r u''_r u'_{r+1}=12\beta u'^2_r u''_r-B, \tag{4.13}$$

$$\theta''_{r+1}=-\lambda(u'_r)^2-2\beta\lambda(u'_r)^4, \tag{4.14}$$

Subject to the boundary conditions,

$$u_{r+1}(-1)=0, \qquad u_{r+1}(1)=0, \qquad \theta_{r+1}(-1)=0, \qquad \theta_{r+1}(1)=1.$$

Applying the Chebyshev Pseudo-spectral method on equations $(4.13) - (4.14)$, we obtain the following decoupled system

$$\left(diag[1+6\beta {u'_r}^2]\boldsymbol{D^2} + diag[12\beta u'_r u''_r]\boldsymbol{D}\right)u_{r+1}$$
$$= 12\beta {u'_r}^2 u''_r - B, \tag{4.15}$$

$$\mathbf{D^2}\theta_{r+1} = -\lambda(u'_r)^2 - 2\beta\lambda(u'_r)^4, \tag{4.16}$$

subject to the boundary conditions

$$u_{r+1}(-1) = 0,\ \ u_{r+1}(1) = 0,\ \theta_{r+1}(-1) = 0,\ \theta_{r+1}(1) = 1. \tag{4.17}$$

Starting from given initial approximations u_0 and θ_0, equations (4.15)–(4.16) can be solved iteratively by using the spectral collocation method to that end, it is convenient set of collocation points in terms of Gauss- Lobat to points

$$\omega_j = cos\frac{\pi j}{N},\ j = 0,1\ ,2,3,\cdots,N,$$

where $N+1$ denotes the number of collocation points, $\mathbf{D}$ is differentiation matrix and $u = [u(\omega_0),\ u(\omega_1),\ u(\omega_2),\dots,u(\omega_N),]^T$ is the vector of unknown functions at the collocation points. In the matrix form, one can rewrite equations (4.15) - (4.16) in the form

$$\mathbf{A_1}u_{r+1} = R_1,\ \ u_{r+1}(\omega_N) = 0,\ \ \ u_{r+1}(\omega_0) = \ 0, \tag{4.20}$$

$$\mathbf{A_2}\theta_{r+1} = R_2\ ,\ \ \theta_{r+1}(\omega_N) = 0,\ \ \ \theta_{r+1}(\omega_0) = \ 1, \tag{4.21}$$

where,

$$\boldsymbol{A_1} = \mathrm{diag}\left[1+6\beta {u'_r}^2\right]\boldsymbol{D^2} + \mathrm{diag}[12\beta u'_r u''_r],\ \ R_1 = 12\beta {u'_r}^2 u''_r - B$$
$$\mathbf{A_2} = \mathbf{D^2}, \qquad R_2 = -\lambda(u'_r)^2 - \ 2\beta\lambda(u'_r)^4,$$

where, diag[.] denotes a diagonal matrix. We choose suitable initial guesses u_0 and θ_0 which satisfy the boundary conditions of governing equations.

4.3 − Results and Discusses

The governing equations (4.10) – (4.11) along with the boundary conditions (4.12) solved numerically using spectral local linearization method (SLLM). The effects of the governing physical parameters on the properties of the fluid have been presented graphically.

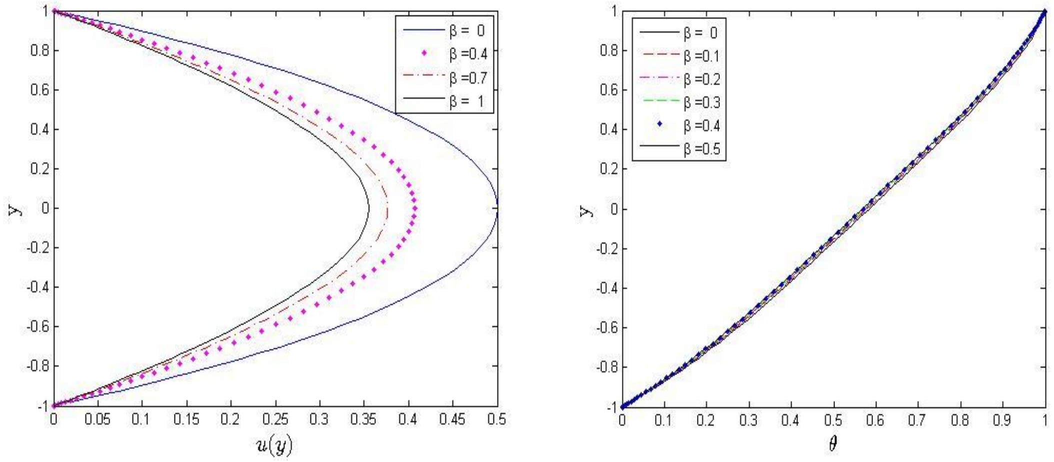

Fig. 4.1 Effects of β on the velocity and temperature respectively.

Fig 4.1 shows the velocity and temperature profiles for difference values of β. It clear that the velocity profile decreases with increase in β. On the other hand increasing in β leads to slight increases in the temperature field.

Chapter 5

The Couette- Poiseuille Flow

The Couette-Poiseuille flow is the flow which is component between the Couette flow and Poiseuille flow , the mathematical formulations for it are reduced from third grade fluid through Chapter 3.

5.1 – The governing Equations

For the Couette-Poiseuille flow, we assume that the fluid motion is produced by both the motion of the upper plate with constant velocity U and by a constant pressure gradient. All other conditions on the temperature and the velocity remain unchanged. Thus, the momentum and energy equations with $(\beta_1 = \beta_3)$ take the form

$$\mu \frac{d^2u}{dy^2} + 6(\beta_1 + \beta_2)\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} = \frac{\partial \hat{p}}{\partial x},$$

$$\frac{\partial \hat{p}}{\partial y} = \frac{\partial \hat{p}}{\partial z} = 0, \tag{5.1}$$

$$\kappa \frac{d^2\theta}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + 2(\beta_1 + \beta_2)\left(\frac{du}{dy}\right)^4 = 0,$$

with the boundary conditions

$$u(-h) = 0, \qquad u(h) = U, \qquad \theta(-h) = \theta_0, \qquad \theta(h) = \theta_1. \tag{5.2}$$

From equation (5.1), we obtain the non-dimensional form as follows:

$$\frac{d^2u}{dy^2} + 6\beta\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} = -B, \tag{5.3}$$

$$\frac{d^2\theta}{dy^2} + \lambda\left(\frac{du}{dy}\right)^2 + 2\beta\lambda\left(\frac{du}{dy}\right)^4 = 0,$$

with the boundary conditions

$$u(-1) = 0, \qquad u(1) = 1, \qquad \theta(-1) = 0, \qquad \theta(1) = 1. \quad (5.4)$$

5.2 – Numerical Solution

To solve the system (5.3) along with the boundary conditions (5.4), first of all we have to linearize the non-linear term using Taylor's series as follows

$$N_1(u', u'') = N_1(u', u'') + \frac{\partial N_1}{\partial u'}(u'_{r+1} - u'_r) + \frac{\partial N_1}{\partial u''}(u''_{r+1} - u''_r). \qquad (5.6)$$

Applying the idea of (5.6) in the system (5.4), yields

$$u''_{r+1} + 6\beta u'^2_r u''_{r+1} + 12\beta u'_r u''_r u'_{r+1} = 12\beta u'^2_r u''_r - B, \qquad (5.10)$$

$$\theta''_{r+1} = -\lambda(u'_r)^2 - 2\beta\lambda(u'_r)^4, \qquad (5.11)$$

subject to the boundary conditions,

$$u_{r+1}(-1) = 0, \qquad u_{r+1}(1) = 1, \; \theta_{r+1}(-1) = 0, \qquad \theta_{r+1}(1) = 1.$$

Again by applying the Chebyshev Pseudo-spectral method on equations (5.10)-(5.11) we obtain the following decoupled system of equations:

$$\left(\mathrm{diag}[1 + 6\beta {u'_r}^2]\mathbf{D^2} + \mathrm{diag}[12\beta u'_r u''_r]\mathbf{D}\right)u_{r+1} = 12\beta {u'_r}^2 u''_r - B, \quad (5.12)$$

$$\mathbf{D^2}\theta_{r+1} = -\lambda(u'_r)^2 - 2\beta\lambda(u'_r)^4, \qquad (5.13)$$

subject to the boundary conditions

$$u_{r+1}(-1) = 0, \quad u_{r+1}(1) = 1, \qquad \theta_{r+1}(-1) = 0, \qquad \theta_{r+1}(1) = 1. \quad (5.14)$$

Then the matrix form for the above system is

$$\mathbf{A_1}u_{r+1} = R_1, \;\; u_{r+1}(\omega_N) = 0, \;\; u_{r+1}(\omega_0) = 1, \qquad (5.15)$$

$$\mathbf{A_2}\theta_{r+1} = R_2, \;\; \theta_{r+1}(\omega_N) = 0, \;\; \theta_{r+1}(\omega_0) = 1, \qquad (5.16)$$

where,

$$\mathbf{A_1} = \mathrm{diag}\left[1 + 6\beta {u'_r}^2\right]\mathbf{D^2} + \mathrm{diag}[12\beta u'_r u''_r], R_1 = 12\beta {u'_r}^2 u''_r - B$$
$$\mathbf{A_2} = \mathbf{D^2}, \qquad R_2 = -\lambda(u'_r)^2 - 2\beta\lambda(u'_r)^4.$$

We choose suitable initial guesses u_0 and $\theta_0$which satisfy the boundary conditions.

$5.3-$ Results and Discusses

The highly nonlinear ordinary differential equations which governed the Couette- Poiseuille flow have been solved numerically using the spectral local linearization method (SLLM). The effects of the physical parameters such as Brinkman number λ, β and the parameter B on the properties of the fluid as well as the velocity and temperature have been presented graphically.

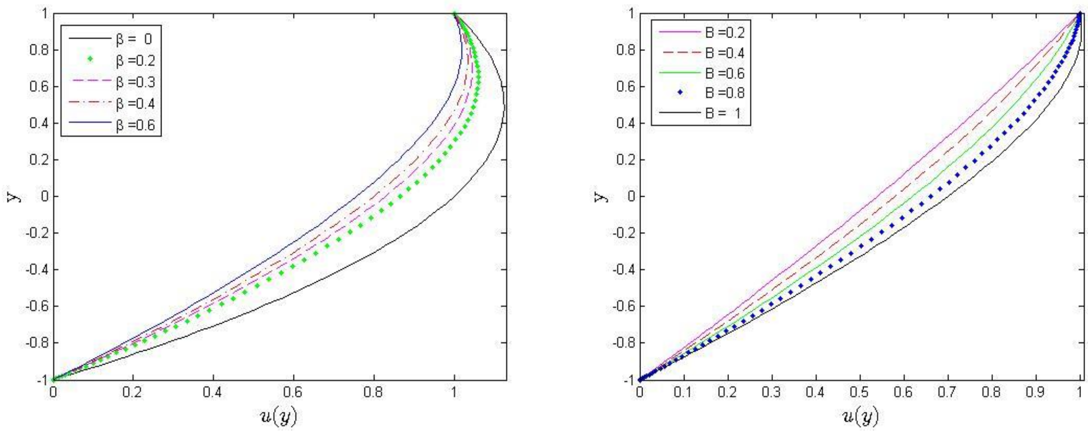

Fig. 5.1 Effects of β and Bon the velocity profile respectively.

Fig 5.1demonstrates the effects of β and B the velocity profile respectively. As it is shown, increasing inβ leads to an increasing in momentum boundary layer thickness and hence reduced the velocity field. On the other hand side which the velocity profile increases with increasing B.

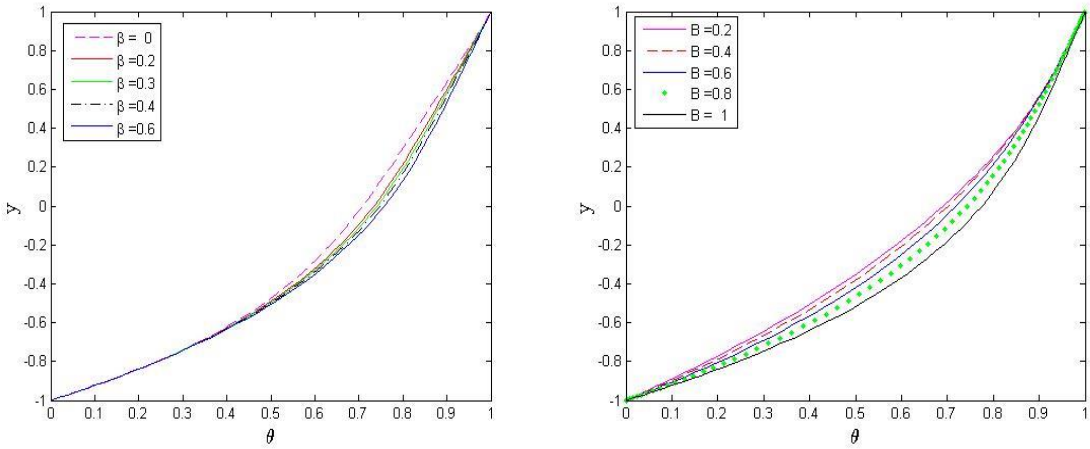

Fig 5.2 Effects of $\beta$and Bon temperature profile, respectively.

Fig. 5.2 illustrates effects of both β and B on the temperature profile respectively. It is clear that the thermal boundary layer thickness decreases with increasing values of both β and B this is as expected, thermal conductivity will be reduced, hence the temperature decreases.

Chapter 6

Double dispersion effects on convection heat and mass transfer in non-Darcy porous medium

Consider the non-Darcy natural convection heat and mass transfer over a semi-infinite vertical surface in a fluid saturated porous medium. The x-axis is taken along the plate and y- axis is normal to it. The wall is maintained at constant temperature and concentration, $T_w$and C_w respectively, and these values are assumed to be greater than the ambient temperature and concentration, T_∞ and C_∞, respectively. The governing equations for this problem are given through next section.

$\mathbf{6.1-}$ The governing equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6.1}$$

$$u+\frac{c\sqrt{K}}{v}uq=-\frac{K}{\mu}\left(\frac{\partial p}{\partial x}+\rho\, g\right), \tag{6.2}$$

$$v+\frac{c\sqrt{K}}{v}vq=-\frac{K}{\mu}\left(\frac{\partial p}{\partial y}\right), \tag{6.3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial x}\left(\alpha_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\alpha_y\frac{\partial T}{\partial y}\right), \tag{6.4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial x}\left(D_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_y\frac{\partial C}{\partial y}\right), \tag{6.5}$$

$$\rho=\rho_\infty[1-\beta(T-T_\infty)-\beta^*(C-C_\infty)] \tag{6.6}$$

Along with the boundary conditions

$$y=0,:v=0,T_w=\text{constant},C_w=\text{constant}, \tag{6.7}$$

$$y\to\infty: u\to 0, T\to T_\infty, C\to C_\infty,$$

where u and v are the velocity component in the x and y directions, respectively, p is the pressure, T is the temperature, C is the concentration, K is the permeability constant, c is an empirical constant, β is the thermal expansion coefficient, β^* is the solutal expansion coefficient, μ is the viscosity of the fluid, ρ is the density, and g is the acceleration due to gravity, α_x and α_y are the components of the thermal diffusivity in x and y directions, respectively, D_x and D_y are the components of the mass diffusivity in x and y directions, respectively. The normal component of the velocity near the boundary is small compared with the other component of the velocity and the derivatives of any quantity in the normal direction are large compared with derivatives of the quantity in direction of the wall. Under these assumptions, equations$(6.1)-(6.6)$ become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6.8}$$

$$u+\frac{c\sqrt{K}}{v}u|u|=-\frac{K}{\mu}\left(\frac{\partial p}{\partial x}+\rho\, g\right), \tag{6.9}$$

$$\frac{\partial p}{\partial y}=0, \tag{6.10}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_y\frac{\partial T}{\partial y}\right), \tag{6.11}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_y\frac{\partial C}{\partial y}\right), \tag{6.12}$$

Following Telles and Trevisan [21], the variables α_y and D_y are defined as $\alpha_y=\alpha+\gamma d|u|$ and $D_y=D+\zeta\, d|u|$, where, α and D are the molecular thermal and solutal diffusivities, respectively, whereas $\gamma d|u|$, $\zeta\, d|u|$ represent dispersion thermal and solutal diffusivities, respectively, where γ and ζ are the coefficients of dispersion thermal and solutal diffusivities, respectively. This model for thermal dispersion has been used extensively by researchers like Cheng [14], Pumb [15], Hong and Tien [16], Lai and Kulack [17] and, Murthy and Singh [18] in studies of convective heat transfer in non-Darcy porous media.

Having invoked the $Boussinesq$ approximations, with substituting $Eqs.$ (6.9)and (6.10), eliminating the pressure between equations (6.9) and (6.10) and using the stream function ψ to define the velocity component u and v as: $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, we obtain:

$$\frac{\partial^2\psi}{\partial y^2} + \frac{c\sqrt{K}}{v}\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)^2 = -\frac{K\,g\,\beta}{\mu}\frac{\partial T}{\partial y} + \frac{K\,g\,\beta^*}{\mu}\frac{\partial C}{\partial y} \quad , \tag{6.13}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\left(\alpha + \gamma d\frac{\partial\psi}{\partial y}\right)\frac{\partial T}{\partial y}\right], \tag{6.14}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left[\left(D + \zeta\, d\frac{\partial\psi}{\partial y}\right)\frac{\partial C}{\partial y}\right]. \tag{6.15}$$

Introducing the similarity variable and similarity profiles

$$\eta = Ra_x^{\frac{1}{2}}\frac{y}{x}\ , F(\eta) = \frac{\psi}{\alpha Ra_x^{\frac{1}{2}}}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{6.16}$$

Where Ra_x is the Rayleigh number $Ra_x = K\,g\,\beta(T_w - T_\infty)x/\alpha v$.

The problem statement then becomes

$$F'' + 2F_0 Ra_d F'F'' = \theta' + N\phi' \tag{6.17}$$

$$\theta'' + \frac{1}{2}LeRa_d F\theta' + \gamma.Ra_d(F'\theta'' + F''\theta') = 0, \tag{6.18}$$

$$\phi'' + \frac{1}{2}LeRa_d F\phi' + \zeta\, Le.Ra_d(F'\phi'' + F''\phi') = 0, \tag{6.19}$$

with the boundary conditions becomes

$$F(0) = 0, \qquad \theta(0) = \phi(0) = 1,\ \ F'(\infty) = \theta(\infty) = \phi(\infty) = 0, \tag{6.20}$$

where the parameter $F_0 = c\sqrt{K\alpha}/vd$ represents the structural and thermo-physical properties of the porous medium, $Ra_d = K\,g\,\beta(T_w - T_\infty)d/\alpha v$ is the pore diameter dependent Rayleigh number which describes the relative intensity of the buoyancy force, such that d is the pore diameter, $N = \beta^*(C_w - C_\infty)/\beta(T_w - T_\infty)$ is the buoyancy ratio, and $Le = \alpha/D$ is the diffusivity ratio (Lewis number) which is the ratio of Schmidt number v/D and Prandtl number v/α.

It is noteworthy that $F_0 = 0$, corresponds to the Darcian free convection, $\gamma = 0$, representsthe case where the thermal dispersion effect is neglected and $\zeta = 0$, represents the case where the thermal solutal dispersion effect neglected. In equation (6.17), values of $N > 0$ indicates the aiding buoyancy and $N < 0$ indicates the opposing buoyancy.

From the definition of the stream function, the velocity components become

$$u = \frac{\alpha}{x} Ra_x F' , \quad v = -\frac{\alpha}{2x} Ra_x^{\frac{1}{2}} [F - \eta F'].$$

The local heat transfer rate which is the primary interest of the study is given by

$$q_w = -k_e \frac{\partial T}{\partial y} \Big|_{y=0} = -(k + k_d) \frac{\partial T}{\partial y} \Big|_{y=0} , \tag{6.21}$$

where k_e is the effective thermal conductivity of the porous medium which is the sum of the molecular thermal conductivity k and the dispersion thermal conductivity k_d.

The local Nusselt number $\boldsymbol{Nu_x}$ can be obtained from

$$Nu_x = \frac{q_w}{T_w - T_\infty} \frac{x}{k_e}, \tag{6.22}$$

one can write

$$Nu_x Ra_x^{-1/2} = -[1 + \gamma Ra_d F'(0)] \theta'(0). \tag{6.23}$$

Also, the local mass flux is given by

$$j_w = -D_y \frac{\partial C}{\partial y} \Big|_{y=0}. \tag{6.24}$$

Therefore, Sherwood number is defined by

$$Sh_x Ra_x^{-1/2} = -\left[1 + \zeta\, Ra_d F'(0)\right] \phi'(0). \tag{6.25}$$

6.2 – Numerical solution

We have given three governing equations for momentum fluid and temperature and concentration respectively

$$F'' + 2F_0Ra_dF'F'' = \theta' + N\phi' , \quad (6.26)$$

$$\theta'' + \frac{1}{2}LeRa_dF\theta' + \gamma.Ra_d(F'\theta'' + F''\theta') = 0, \quad (6.27)$$

$$\phi'' + \frac{1}{2}Le.Ra_dF\phi' + \zeta\,Le.Ra_d(F'\phi'' + F''\phi') = 0, \quad (6.28)$$

with the boundary conditions

$$F(0) = 0, \qquad \theta(0) = \phi(0) = 1, \quad F'(\infty) = \theta(\infty) = \phi(\infty) = 0, \quad (6.29)$$

Where $F_0, Ra_d, Le, \gamma, \zeta$ are constant parameters.

Again, we appeal to the spectral local linearization method, equations (6.27) – (2.29) give,

$$F''_{r+1} + 2F_0Ra_dF''_{\mathrm{r}}\mathrm{F}'_{\mathrm{r+1}} + 2F_0Ra_dF'_{\mathrm{r}}\mathrm{F}''_{\mathrm{r+1}} \\ = 2F_0Ra_dF''_{\mathrm{r}}\mathrm{F}'_{\mathrm{r}} + \theta'_r + N\phi'_r \quad (6.31)$$

$$\theta''_{r+1} + \frac{1}{2}F_{r+1}\theta'_{\mathrm{r+1}} + \gamma.Ra_d(F'_{r+1}\theta''_{r+1} + F''_r\theta'_{r+1}) = 0 , \quad (6.32)$$

$$\phi''_{\mathrm{r+1}} + \frac{1}{2}\mathrm{Le.Ra_dF_r}\phi'_{\mathrm{r+1}} + \zeta\,Le.Ra_d\left(\mathrm{F}'_{\mathrm{r+1}}\phi''_{\mathrm{r+1}} + \mathrm{F}''_{\mathrm{r+1}}\phi'_{\mathrm{r+1}}\right) \\ = 0 , \quad (6.33)$$

with boundary conditions

$$F_{\mathrm{r+1}}(0) = 0, \qquad \mathrm{F}'_{\mathrm{r+1}}(\infty) = 0 , \theta_{r+1}(0) = 1, \;\; \theta_{r+1}(\infty) = 0,$$

$$\phi_{r+1}(0) = 1, \qquad \phi_{r+1}(\infty) = 0. \quad (6.34)$$

Applying the Chebyshev spectral collocation method to the system Eqs(6.31)-(6.33) along with boundary conditions, we obtain the following matrix equations

$$\mathbf{A_1}F_{r+1} = R_1 , \;\; F_{r+1}(\omega_N) = 0, \;\; F'_{r+1}(\omega_0) = 0, \quad (6.39)$$

$$\mathbf{A_2}\theta_{r+1} = R_2 , \;\; \theta_{r+1}(\omega_N) = 1, \;\; \theta_{r+1}(\omega_0) = 0, \quad (6.40)$$

$$\mathbf{A_3}\phi_{r+1} = R_2 , \;\; \phi_{r+1}(\omega_N) = 1, \;\; \phi_{r+1}(\omega_0) = 0, \quad (6.41)$$

where,

$$\mathbf{A_1} = (1 + 2F_0Ra_dF'_r)\mathbf{D^2} + (2F_0Ra_dF''_r)\mathbf{D},$$

$R_1 = 2F_0 Ra_d F'_r F''_r + \theta'_r + N\phi'_r,$

$\mathbf{A_2} = (1 + \gamma . Ra_d \text{diag}[F'_{r+1}])\mathbf{D^2} + (\frac{1}{2}\text{diag}[F_{r+1}] + \gamma . Ra_d \text{diag}[F''_{r+1}])\mathbf{D},$

$R_2 = 0,$

$\mathbf{A_3} = 1 + \zeta\, Le\, . Ra_d \text{diag}[F'_{r+1}])\mathbf{D^2} + (\frac{1}{2}Le. \text{diag}[F_{r+1}] + \zeta\, Le. Ra_d F''_{r+1})\mathbf{D},$

$R_3 = 0.$

6.3 – Results and discussion

We have started from suitable initial guesses F_0 , $\theta_0$and $\phi_0$which satisfy the boundary conditions (6.29), the governing equations (6.26) – (2.28) have been solved numerically using the local linearization method. Effects of the physical governing parameters on the fluid properties have been obtained graphically.

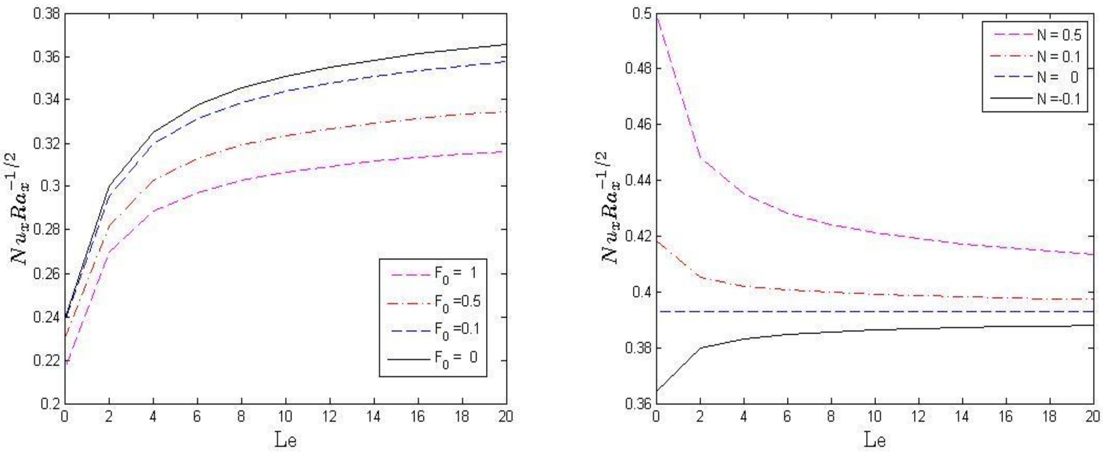

Fig 6.1 The heat transfer rate in terms of Nusselt number $Nu_x Ra_x^{-1/2}$ with Lewis number Le, for varying F_0, N when $N = -0.5,\ \gamma = \zeta = 0$, and $Ra_d = 1$.

The combined effects of Le, F_0 and N on the heat transfer rate in terms of the Nusselt number $Nu_x Ra_x^{-1/2}$ is depicted in Fig 6.1. Clearly increase in F_0 leads to decrease in the heat transfer rate. While increase in N with leads enhanced the heat transfer rate through**.**

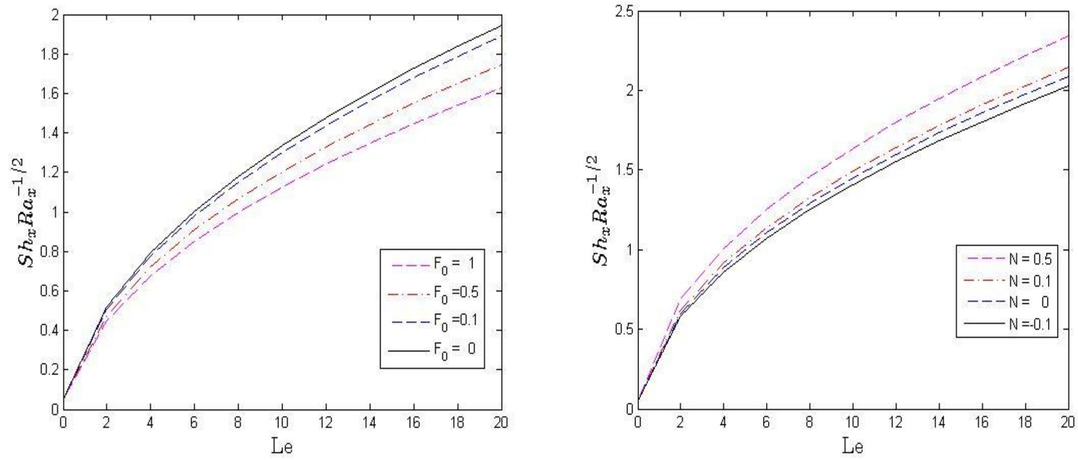

Fig 6.2 The mass transfer rate in terms of Sherwood number $Sh_x Ra_x{}^{-1/2}$ with Lewis number Le, for varying F_0, when $N = -0.5, \gamma = \zeta = 0$, and $Ra_d = 1$.

Fig 6.2 Illustrates the Sherwood number $Sh_x Ra_x{}^{-1/2}$as a function of Lewis number Le at difference values of heat F_0 and N, respectively. The results show that the rate of heat transfer reduced with F_0, the clear fact is that, increasing N enhances the mass transfer rate.

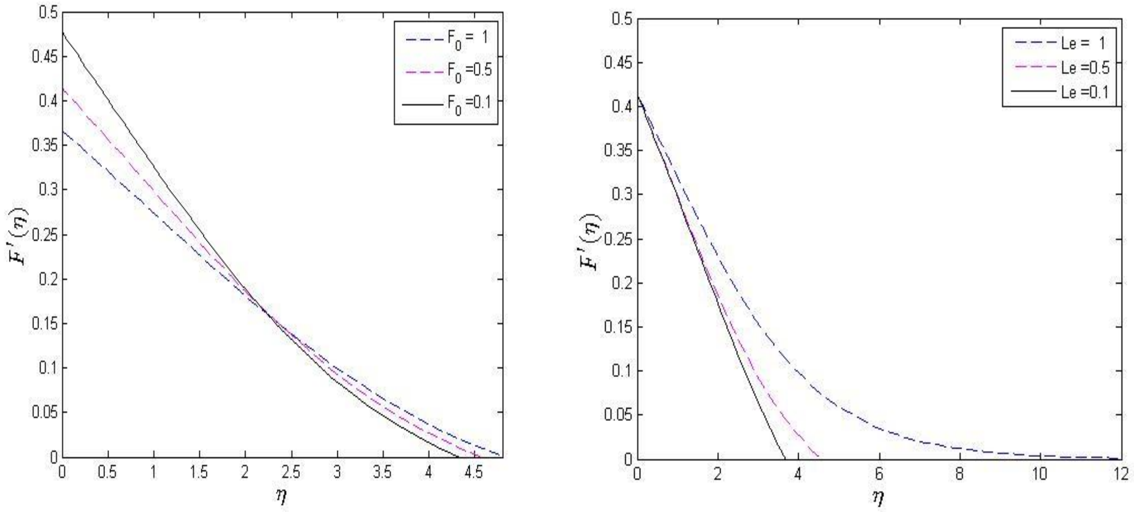

Fig 6.3 Effects of $F_0$and Le on the velocity profiles for $N = -0.5,\ Ra_d = 1.0,\ \gamma = \zeta = 0.5$.

Fig 6.3 represents the variation of velocity component $F^{'}(\eta)$ with F_0 and Le, respectively. The velocity of the fluid is found to be decrease with $\mathrm{F_0}$and increase with Le.

From Fig 6.4 we notice that slight effects of Ra_d on the velocity of the fluid. On the other hand as N increase the velocity increases.

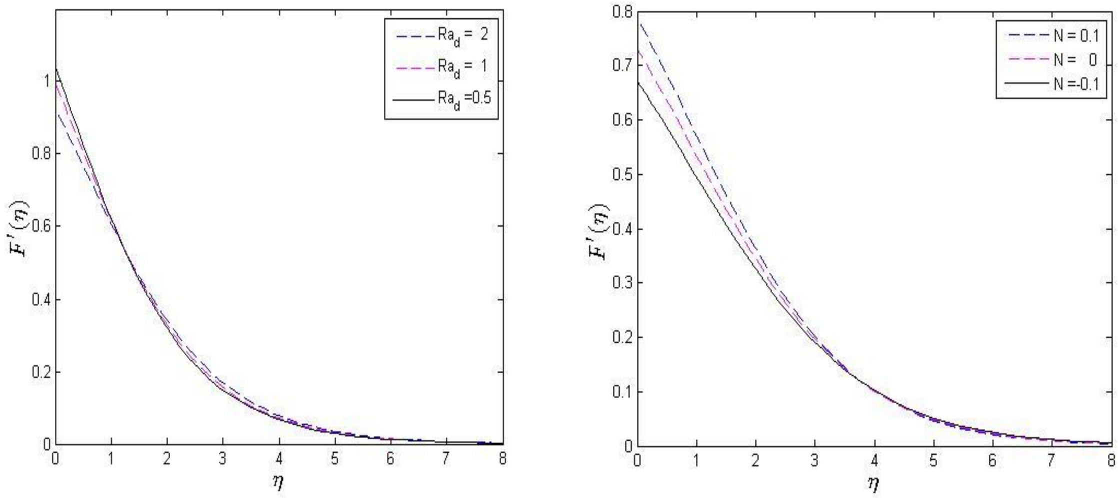

Fig 6.4 The velocity profile varying the buoyancy Ra_d and N, when $F_0 = 0.5,\ Le = 1.0$ and $\gamma = \zeta = 0.5$.

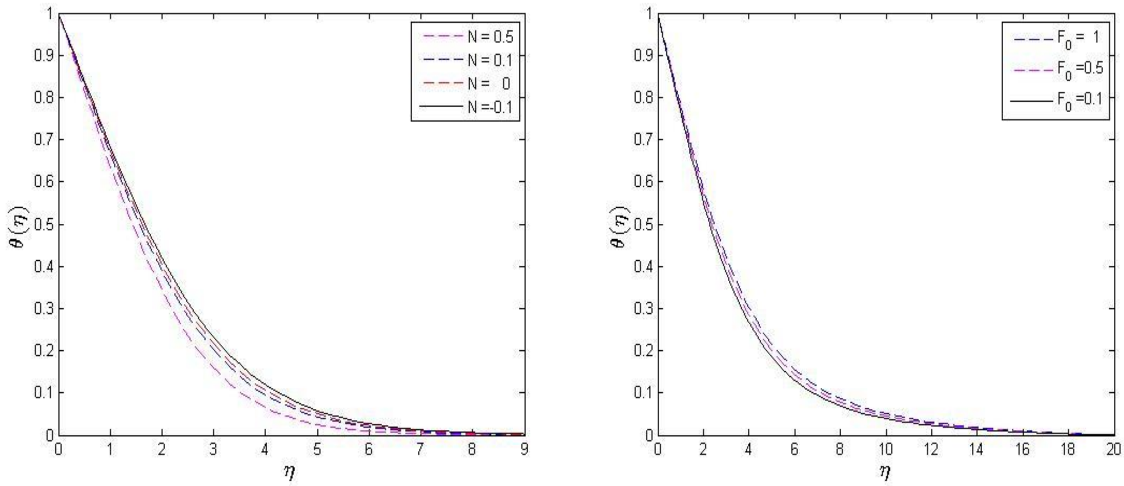

Fig 6.5. Effects of N and F_0 on the temperature profile, respectively for $Ra_d = 1.0, Le = 0.5$ and $\gamma = \zeta = 0.5$.

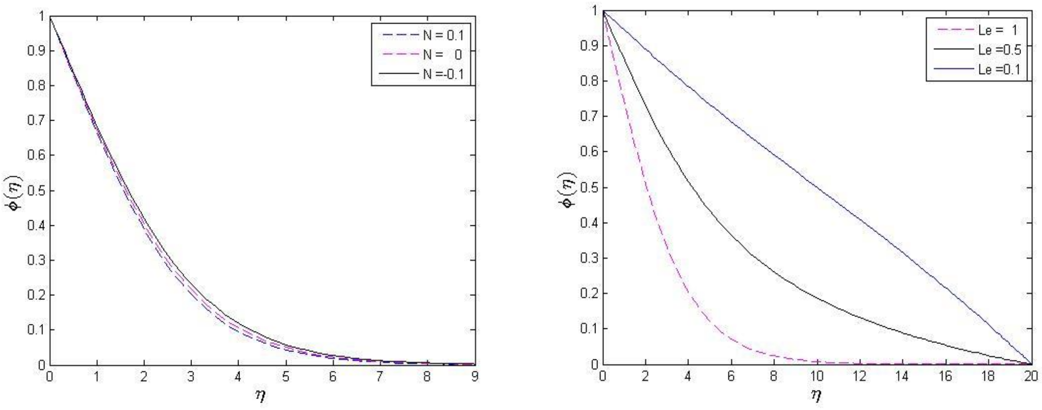

Fig. 6.6 Concentration profile for varying N andLe, when $Ra_d = 1, F_0 = 0.5$, and $\gamma = \zeta = 0.1$.

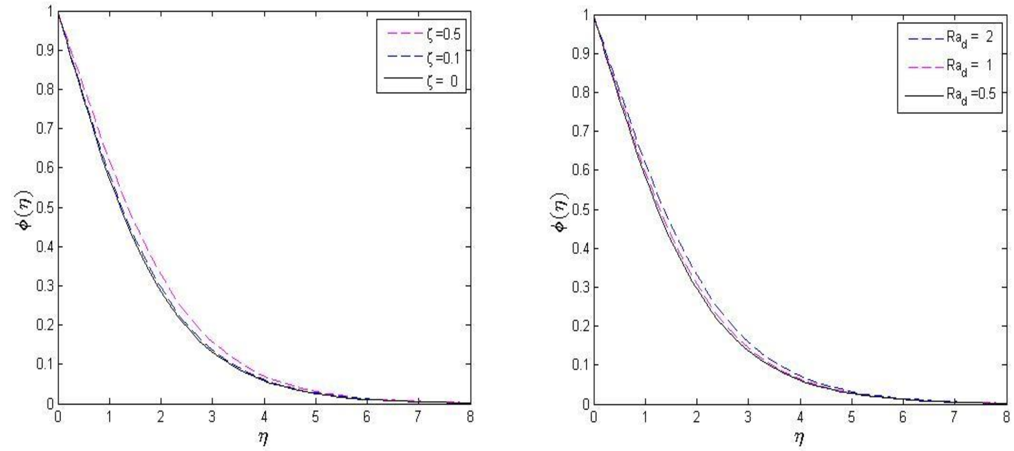

Fig 6.7 Effects of ζ and Ra_d,on the concentration for$N = 0, F_0 = 0.1, Le = 1.0$, and $\gamma = 0.1$.

Fig. 6.5 shows the effects of N and F_0 on the dimensionless temperature respectively. It can be seen from this figure that the thermal boundary layer thickness decreases with an increasing in N, thus leading to temperature profile decreasing in the boundary. On the other hand, we observe that dimensionless temperature increases with an increase in Ra_d that is due to increase in the thermal boundary layer thinness.

The effect of N and Le on the solute concentration is shown in Fig. 6.6. An increase in both N and Le reduces concentration within the solute boundary.

Fig. 6.7 shows the effects of ζ and Ra_d, on the concentration profile respectively. It is clear that increase in both ζ and Ra_d, leads to increase in the concentration profile.

conclusion

In this work, we have studied the non- Newtonian fluids, convection heat and mass transfer in non-Darcy porous medium. The classical boundary conditions of the velocity and temperature and concentration has been substituted by the more realistic condition where the velocity is not controlled at the boundary. the effects of the governing parameters such as the Brinkman number, parameter β, Nusselt number, Sherwood number, heat transfer coefficient, pore diameter dependent Rayleigh number, buoyancy ratio, Lewis number and fluids flow characteristic have been studied. Here β represents the viscoelastic properties of the fluid and resists the motion of the fluid, and B represents the viscoelastic properties with the variation of pressure rate, and The effects of the heat transfer coefficient , buoyancy ratio, Lewis number and pore diameter dependent Rayleigh number enhances and some cases reduces the velocity and temperature and concentration with according of variation the parameters values.

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