Dedication

Dedicated to:

- To my Beloved Family with love.

Acknowledgments

Firstly my Great thanks to Allah for helping me to complete this research and asking Allah to help me in any work and any destination.

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Abstract

In this research we study Brownian motion and we discuss continuity and Holder continuity of Brownian paths, and local martingales. We present some simple integrands and processes and L^2 properties, and we applied Brownian motion on Ito's integrals and formulas, and we also discuss some applications to Brownian motion and martingales. Then we discuss relation between Brownian motion and differential equations.

الخلاصة

في هذا البحث درسنا الحركة البراونية وناقشنا الاستمرارية واستمرارية هولدر للمسارات البراونية، ومارتنجل المحلي. وقدمنا بعض التكاملات والعمليات البسيطة وخصائص L^2، ثم طبقنا الحركة البراونية على تكاملات وصيغ إيتو.أيضاً ناقشنا بعض تطبيقات الحركة البراونية ومارتنجل. كذلك ناقشنا العلاقة بين الحركة البراونية والمعادلات التفاضلية.

The contents

Stochastic Differential Equations and Diffusion Approximation

Introduction

In this research we will develop the tools needed to Handle continuous-time Markov processes in$\mathbb{R}^d$.We will restrict our attention to continuous processes, although the theory we develop also a well-suited for dealing with processes that exhibit jumps. The most basic example of such processes is Brownian motion $(W_t \geq 0)$ with constant diffusivity. That is, for all $t \geq 0, W_t \sim \mathcal{N}(0, at)$.

Our research will be organized as follows:

Firstly we beginning with continuity and H$\ddot{o}$lder continuity of Brownian paths, basic properties, the bigger picture, in a nutshell, finite variation integrals, previsible processes and local Martingales, with some examples and applications.

In chapter 2 we discusssimple integrands and L^2properties, quadratic variation, It$\hat{o}$'s integrals, covariation and It$\hat{o}$'s formula, with some examples and applications.

In chapter 3 we study Brownian Martingales, Dubins-Schwarz theorem, planar Brownian motion. We discuss the Donsker's invariance principle , Brownian motion and the dirichlet problemand Girsanov's theorem, with some examples and applications.

Finally we illustrate some general definition, Lipschitz coefficients, strong Markov property, diffusion processes, some Links with PDEs, Martingale problems, notions of weak convergence of processes and Markov chains and diffusions, with some examples and applications.