Dedications

To my father, mother, brothers, sisters, children and dear friend.

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Abstract

We study the bounded variation , tensor products of Banach Lattices, tensor norms , operators in the category , the interpolation of injective or projective tensor products of Banach spaces . We give the decomposition of spaces of distributions induced by tensor product bases, homogeneous orthogonally additive polynomials on Banach Lattices , on spaces of continuous functions and positive tensor products . We determine the localized polynomial frames on the interval with Jacobi weights and on the ball. We also determine the sub-exponentially localized kernels and frames induced by orthogonal expansions. We show the operator space UMD property and constants for non-commutative Lp- spaces and for a class of iterate $Lp(lq)$ spaces .

الخلاصة

درسنا التغير المحدود وضرب تنسر لشبكات باناخ وأنظمة تنسر والمؤثرات في الطبقة واستكمال ضرب تنسر الواحد لواحد والاسقاطي لفضاءات باناخ . أعطينا تفكيك للفضاءات للتوزيعات المحدثة بواسطة أساس ضرب تنسر وكثيرات الحدود المضافة المتعامدة المتجانسة علي شبكات باناخ وعلي فضاءات الدوال المستمرة وضرب تنسر الموجب . حددنا إطارات كثيرة الحدود الموضعية علي الفترة مع مرجحات الجاكوبي وعلي الكرة . أيضا حددنا النويات الموضعية جزئية – الأسية والإطارات المحدثة بواسطة التفكيكات المتعامدة . أوضحنا خاصية UMD فضاء المؤثر والثوابت لأجل فضاءات L_P غير التبديلية ولأجل عائلة فضاءات $L_P(L_q)$ المتكررة.

Introduction

The theory of dual functors in the category β of Banach spaces is applied to the study of tensor norms in the sense of Grothendieck. The dual functors of the tensor norms arising from the projective and inductive tensor product as well as from more general tensor norms, such as the norms d_p of Saphar, are identified as various spaces of operators, which include p-integral and absolutely p-summing operators. We show result on tensor products of general vector lattices to give a construction for the projective tensor product of the Banach space .

We show a general result on the factorization of matrix-valued analytic functions. We deduce from it that if (E_0, E_1) and $(F_0, F_1,)$are interpolation pairs with dense intersections, then under some conditions on the spaces $E_i,, F_i$, we have $[E_0 \hat{\otimes} F_0, E_1 \hat{\otimes} F_1]_\theta = [E_0, E_1]_\theta \hat{\otimes} [F_0, F_1]_\theta, 0 < \theta < 1.$

As is well known the kernel of the orthogonal projector onto the polynomials of degree $n$in $L^2(\omega_{\alpha,\beta},[-1,1]), \omega_{\alpha,\beta}(t) = (1-t)^\alpha (1+t)^\beta$, can be written in terms of Jacobi polynomials.

Almost exponentially localized polynomial kernels are constructed on the unitball B^d in R^d with weights $\omega_\mu(x) = (1-|x|^2)^{\mu-1/2}, \mu \geq 0$, by smoothing out the coefficients of the corresponding orthogonal projectors.

Rapidly decaying kernels and frames (needlets) in the context of tensor product Jacobi polynomials are developed based on several constructions of multivariate C^∞ cutoff functions. These tools are further employed to the development of the theory of weighted Triebel–Lizorkin and Besovspaceson $[-1, 1]^d$.

We study the operator space UMD property, introduced by Pisier in the context of noncommutative vector-valued L_p-spaces. It is unknown whether the property is independent of p in this setting. We show that for

$1 < p,q < \infty$, the Schatten q-classes S_q are $OUMD_p$. The proof relies on properties of the Haagerup tensor product and complex interpolation.

We mention that the first super-reflexive non-UMD Banach lattices were constructed by Bourgain. Our results yield another elementary construction of super-reflexive non-UMD Banach lattices, i.e. the inductive limit of X_n, which can be viewed as iterating infinitely many times $L_p(L_q)$.

We introduce bilinear maps of order bounded variation, semivariation and norm bounded variation.

The main result is a representation theorem for homogeneous orthogonally additive polynomials on Banach lattices. The representation theorem is used to study the linear span of the set of zeros of homogeneous real-valued orthogonally additive polynomials.

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