

## **Dedications**

To my mother, father, wife, children, brothers, sisters and friends.

## **Acknowledgments**

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## Abstract

We give a cohomology of the complement of a free divisor and the computation of the logarithmic cohomology of the complement of a plane curve and for plane curves. The pattern-equivariant and comparing different versions of Tiling cohomology with integer coefficients are shown. The projective spectrum and cyclic cohomology are also shown. The  $W_n$ -action on the consecutive commutators, on the lower central series, an upper bound for the lower central series quotients of a free and graded associative algebra are considered. We also consider the lower central series of a free associative algebra over the integers and finite fields. We study the projective spectrum in Banach algebras and the spectrum of operator  $\mathbb{C}$ -valued entire functions.

## الخلاصة

أعطينا الهومولوجيا المصاحبة لمتعم القاسم الحر ولحوسبت الكوهومولوجيا المصاحبة اللوغرتمية لمتعم منحنى المستوى ولأجل منحنيات المستوى. أوضحن النسق متساوى التغير ومقارنة الاصدارات المختلفة للهومولوجيا المصاحبة تلتق مع المعاملات الصحيحة . أيضاً أوضحن طيف الاسقاط والهومولوجيا المصاحبة الدوارة . اعتبرنا الفعل-  $W_n$  على المبدلات المتتابعة وعلى المتسلسلة المركزية الدنيا والحد الأعلى ولأجل قواسم المتسلسلة المركزية الدنيا للجبر المشارك الحر والمدرج . أيضاً اعتبرنا المتسلسلة المركزية الدنيا للجبر المشارك الحر فوق الأعداد الصحيحة والحقول المنتهية . درسنا طيف الاسقاط في جبريات باناخ وطيف مؤثر – الدوال الكاملة القيمة .

## Intoroduction

The class of “strongly quasihomogeneous” free divisors, introduced here, includes free hyperplane arrangements and the discriminants of stable mappings in Mather's nice dimensions (and in particular the discriminants of Coxeter groups). We consider the lower central filtration of the free associative algebra  $A_n$  with  $n$  generators as a Lie algebra we consider the associated graded Lie algebra. It is shown that this Lie algebra has a huge center which belongs to the cyclic words, and on the quotient Lie algebra by the center there acts the Lie algebra  $W_n$  of polynomial vector fields on  $\mathbb{C}^n$ .

We study the structure of  $B_i(A_n)$  for a free algebra  $A_n$  we construt a basis for  $B_2(A_n)$  and determine the structure of  $B_3(A_2)$  and  $B_4(A_2)$  published by Elsevier Inc. feigin and Shoikhet conjectured that successive quotients  $B_m(A_n)$  of the lower central series filtration of a free associative algebra  $A_n$  have polynomial growth. We give a proof of this conjecture, using the structure of a representation of  $W_n$ , the Lie algebra of polynomial vector fields on  $\mathbb{C}^n$ , on  $B_m(A_n)$  which was defined.

We give an example of a free divisor in  $D \in \mathbb{C}^3$  which is not locally weighted homogeneous, but for which this (second) assertion continues to hold. We will give algorithms of computing bases of logarithmic cohomology groups for square-free polynomials in two variables.

We relate Kellendonk and Putnam's pattern-equivariant (PE) cohomology to the inverse-limit structure of a tiling space. This gives an version of PE cohomology with integer coefficients, or with values in any Abelian group. We establish direct isomorphisms between different versions of tiling cohomology. The first version is the direct limit of the cohomologies of the approximants in the Anderson-Putnam-Gähler system, the second is the recently introduced PV-cohomology of Savinien and Bellissard and the third is pattern equivariant cohomology.

We describe  $B_2(A)$  for  $A$  a quotient of the free algebra on two or three generators by the two-sided ideal generated by a generic homogeneous element. We show that it is isomorphic to a certain quotient of Kähler differentials on the non-smooth variety associated to the lianization of  $A$ . we describe the torsion in the reduced quotient  $\overline{B}_1$  and  $B_2$  geometriclly in terms of the De Rham cohomology of  $\mathbb{Z}^n$ . As a corollary we obtain a complete dscription of  $\overline{B}_1(A_n(\mathbb{Z}))$  and  $\overline{B}_1(A_n(\mathbb{F}_p))$ , as well as of  $B_2(A_n(\mathbb{Z}[\frac{1}{2}]))$  and  $B_2(A_n(\mathbb{F}_p))$ ,  $P > 2$ . we also give theoretical and experimental results for  $B_i$  with  $i > 2$ . Formulating a number of conjectures and questicns on their basis. Finally, we discuss the supercase, when some of the generators are odd and some are even, and provide some theoretical results and experimental data in this case.

When  $A$  is commutative,  $P(A)$  is a union of hyperplanes. When  $B$  is reflexive or is a  $C^*$ -algebra, the projective resolvent set  $P^c(A) := \mathbb{C}^{n+1} \setminus P(A)$  is shown to be a disjoint union of domains of holomorphy. Later part studies Maurer-Cartan type  $B$ -valued 1-form  $A^{-1}(z)dA(z)$  on  $P^c(A)$ . As a consequence, we show that if  $B$  is a  $C^*$ -algebra with a trace  $\phi$ , then  $\phi(A^{-1}(z)dA(z))$  is a nontrivial element in the de Rham cohomology space  $H_d^1(P^c(A), \mathbb{C})$ . We study the topology of the resolvent via consideration of the  $B$ -valued Maurer-Crtan type 1-form  $f(z)^{-1}df(z)$ . As an example, we explicitly compute the spectrum of a linear function associated with the tuple of standard unitary generators in a free group factor von Neumann algebra. We show that in non-commutative cases, the cyclic cohomology Of  $B$  does a similar job. Infact, a Chen-Weil type map  $\kappa$  from the cyclic cohomology of  $B$  to the de Rham cohomology  $H_d^*(P^c(A), \mathbb{C})$  is established. As an example, we show a closed high-order form of the classical Jacobi's formula.

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