

List of Symbols

Symbol		page
L^2	Hilbert space	1
L^∞	Essential Lebesgue space	2
ℓ^1	Lebesgue space	2
ℓ^p	Lebesgue space	2
$a. e$	Almost every where	3
$ess\ inf$	essential infimum	4
$ess\ sup$	essential supremum	4
mod	Modular	18
$supp$	Support	22
dim	Dimension	59
\otimes	Tensor product	94
det	Determinant	106
\oplus	Direct sum	116
S_p	Schatten class	161
STFT	Short- time fourier transform	163
OP	Operator	169
$a. a$	almost all	182
L^1	Lebesgue integral on the line	185
$M_m^{p,q}$	Modulation	198
ker	Kernel	216

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