Dedications

To my great mother, father, husband, brothers, sisteres and friends.

Acknowledgements

My gratitude to my supervisor Prof. Shawgy Hussein Abdalla for this ingenious advice and hard efforts. Special thanks are extended to Sudan University of Science & Technology for giving me this chance for higher studies. Last my heart is full thanks to my brothers and sisters whom helped in preparing this project and all dear friends for their help.

ABSTRACT

A Banach space with few operation and an arbitrarily distortable are shown. We also show the sub symmetric sequences ,minimal spaces and subspaces with isomorphically homogenous sequences in a Banach spaces. Renormings, external structures and quantification of weak sequential completeness are obtained .We determine the big slice phenomena in M-embedded and L-embedded , the characterizations of unitaries with

weakly open sets in the unit ball of the projective tensor product of Banach spaces. We study Banach spaces which satisfy linear identities ,polynomials and polynomial norms .We characterize the incomparability , non isomorphic and minimality and α-minimal Banach space.

الخلاصة

تم ايضاح فضاء باناخ مع العملية الاقل وقابلية التشويه الاختيارية .ايضا اوضحنا المتتاليات جزئية التماثل والفضاءات الصغرى والفضاءات الجزئية مع المتتاليات المتجانسة الايزومورفكلي في فضاءات باناخ .تم اعطاء اعادة النظائم والبناءات الخارجية وتكميم التمام المتتابع الضعيف .تم تحديد ظاهرة الشريحة الكبيرة في طمر - M وطمر – L والتشخيصات للاحاديات مع الفئات المفتوحة الضعيفة في كرة الوحدة لضرب الممتد الاسقاطي لفضاءات باناخ.درسنا فضاءات باناخ والتي تحقق المتطابقات الخطية وكثيرات الحدود ونظائم كثيرة الحدود .تم تشيخص التفرد وغير الايزومورفك والاصغرية وفضاء باناخ اصغرية - α .

Introduction

We construct a Banach space with density character N_1 such that every (linear bounded) operator T from B to B has the form a I + T_1, where I is the identity, and T_1 has a separable range. The axiom V = L means that all the sets in the universe are in the class L of sets constructible from ordinals; in a sense this is the minimal universe.
We show that every Banach space saturated with subsymmetric basic sequences contains a minimal subspace.It follows that any Banach space is either ergodic or contains a minimal subspace.

We use renorming techniques to settle several questions on extremal structures of Banach spaces. We construct a unitary vector in a dual space which is not a weak*-unitary. We consider several quantities related to weak sequential completeness of a Banach space and prove some of their properties in general and in L-embedded Banach spaces, improving in particular an inequality of G. Godefroy, N. Kalton and D. Li.

We obtain sufficient conditions on an M-embedded or L-embedded space so that every nonempty relatively weakly open subset of its unit ball has norm diameter 2. We show that, up to renorming, this holds for every Banach space containing c_0 and, as a consequence, for every proper M-ideal. We survey Banach space characterizations of unitary elements of C*-algebras,JB*triples, and JB-algebras. A Banach space is said to have the diameter two property if every non-empty relatively weakly open subset of its unit ball has diameter two. We show that the projective tensor product of two Banach spaces whose centralizer is infinite-dimensional has the diameter two property. The same statement also holds for X $\widehat{\otimes}_\pi$ Y if the centralizer of X is infinite dimensional and the unit sphere of Y* contains an element of numerical index one.

In 1935, Jordan and von Neumann proved that any Banach space which satisfies the parallelogram law $\|x+y\|^2+\|x-y\|^2=2(\|x\|^2+\|y\|^2)$ $(*)$ for all elements x and y must be a Hilbert space Subsequent authors have found norm conditions weaker than (*) which require a Banach space to be a Hubert space. Notable examples include the

results of Day, Lorch, Senechalle and Carlsson . A Banach space X is said to be in the class $\mathscr{p}_{2n}$ if > for all elements x and y, $\|x+ty\|^{2n}$ is a polynomial in real t.These spaces generalize L_{2n} and are precisely those Banach spaces in which linear identities can occur. We shall discuss further properties of $\mathscr{p}_{2n}$ spaces, often in terms of the permissible polynomials $P(t)=\|x+ty\|^{2n}$. For each$n$, the set of such polynomials forms a cone. All spaces in $\mathscr{p}_2$ are Hubert spaces.We study the duality theory for real polynomials and functions on Banach spaces.

A Banach space contains either a minimal subspace or a continuum of incomparable subspaces. A Banach space which contributes to the ongoing project, initiated by W.T. Gowers, of classifying separable Banach spaces by identifying characteristic subspaces.

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