Chapter One

Introduction

1.1 Quantum History

Classical physics divide physical laws into two main parts . Newton's laws which describe matter particles , and Maxwell's equations which take care of electromagnetic energy waves . This classification survives till the trial of scientists to does not be black body radiation phenomena . Maxplank found that the explanation of black body radiation is impossible unless light wakes behaves as describe energy quanta known as photons [1,2] . This means that light has dual wave and particle nature. The particle behavior of light waves motivates DeBrogglie to suggest that particles like electrons can be have some times like waves . This suggestion was confirmed experimentally , by Davison and Krimer which observe electron diffraction by a solid crystal [3,4,5] .

The dual nature of atomic world entitles encourages Schrodinger and independently Heisenberg to construct the so called quantum mechanics to describe the behavior of the atomic world [6,7,8] .

The quantum theory succeeded in describing a wide variety of atomic world phenomena, like atomic spectra Zeeman effect and hyper fine interaction [9,10,11] .

1.2 Quantum Mechanics Problems

Dispite the fat that quantum mechanics succeeded in explaining a wide variety of physical phenomena, it suffers from noticeable setbacks . For example it cannot explain high temperature superconductivity phenomena [12,13] .

More one there is no well established quantum gravitational theory [14,15,16] .

1.3 Attempts to Modify Quantum Mechanical Laws

To cure the afore noted set backs different were made to modify quantum mechanics [17,18,19,20] . In Haroun model Maxwell's equations were used to account for the effect of frictional force by deriving Schrodinger equation from the electromagnetic wave equation in a conductive medium [21,22] .

In Lutfimodel , the equation for the harmonic oscillator is used to account for the effect of friction [23,24] .

1.4 Aim of the Work

The aim of this work is to construct a new quantum relativistic equation, based on generalized special relativity (GSR) . This model should at least share the ordinary quantum mechanics its successes , beside solving some of the problems facing the physicals now .

1.5 Presentation of the Thesis

Chapter one is the introduction . Chapter two is devoted for Schrodinger equation , which chapter three is concerned with the theory of generalized special relativity . Chapters four and five are devoted for literature review and contribution .

Chapter Two

Quantum Equation

2.1 Introduction

To describe the behavior of particles in the atomic world , one have to take into account the dual nature of particles .

This description was made by Schrodinger equation which describes how the quantum state of a physical system change with respect to time and space .

Schrodinger equation can defer mine the energy level of atomic electrons moving around the nucleus . The chapter is concerned with deriving Schrodinger equation beside relativistic quantum equation .

2.2 Derivation of Schrodinger Equation

To derive Schrodinger equation are uses the classical expression of energy and the wave nature of atomic particles according to DeBrogglie Hypotheses . The atomic particles can be treated as waves having wave function .

$\Psi = A\ e^{i\,(kx-\omega t)}$(2.2.1)

This equation describes the behavior of a wave having wave number k , angular frequency w and amplitude A

Where :

K $= \frac{2\pi}{\lambda}$ ω = 2 π f (2.2.2)

With λ and f standing for the wave length and frequency , respectively .

The particles cannot be described by λ and f , due to the dual particle-wave nature of the atomic world .

Thus it is more preferable to use common language for them , using Plank theory , one can relate the energy E to f and ω , in the form :

$E = h\ f = \frac{h}{2\pi}\,(2\pi f) = \hbar\ \omega$ (2.2.3)

Utilizing also DeBrogglie Hypothesis , one can relate the momentum p to λ and k to be :

$P = \frac{h}{\lambda} = \frac{h}{2\pi}\left(\frac{2\pi}{\lambda}\right) = \hbar\ k$ (2.2.4)

Thus one can rewrite Ψ to be expressed interm of E and p , which is a common physical quantity for both waves and particle.

Thus is review of equation (2.2.3), (2.2.4) one can rewrite equation (2.2.1) in the form :

$\Psi = A\ e^{i/\hbar\,(px-Et)}$(2.2.5)

The energy E of the classical system is given by

$$E = \frac{p^2}{2m} + V \qquad (2.2.6)$$

P is the momentum

V is the potential

Multiply both sides by Ψ yields

$$E\Psi = \frac{p^2}{2m}\Psi + V\Psi \qquad (2.2.7)$$

To write equation (2.2.4) in differential form , one can use equation (2.2.5) by differentiation (2.2.5) with respect to t to get :

$$\frac{\partial\psi}{\partial t} = -\frac{E}{i\hbar}\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \qquad (2.2.8)$$

Differentiation again with respect to x twice one gets :

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar}P\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = i^2\frac{P^2}{\hbar^2}\psi = \frac{-P^2}{\hbar^2}\psi$$

$$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = P^2\psi$$

In three dimension

$$-\hbar^2\nabla^2\psi = P^2\psi \qquad (2.2.9)$$

Substituting (2.2.8) and (2.2.9) in (2.2.7) one gets :

$$\imath\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2\text{m}}\nabla^2\psi+V\psi \qquad (2.2.10)$$

Which is the Schrodinger equation .

2.3 Klein - Gordon Relativistic Equations

Schrodinger equation is based on the Newton's energy formula which cannot describe the behavior of relativistic fast particles .

This means that there is a need for a quantum equation which is based on relativistic energy equation

$$E^2 = c^2p^2+m_o^2c^4 \qquad (2.3.1)$$

To do this multiply both side of (2.3.1) by Ψ to get :

$$E^2\Psi= c^2\,p^2\,\Psi+m_oc^4\Psi \qquad (2.3.2)$$

Using equation (2.2.8) yields

$$\frac{\partial^2\psi}{\partial t^2}=\frac{-\text{E}}{i\hbar}\frac{\partial\psi}{\partial t}=\left(\frac{-\text{E}}{i\hbar}\right)^2\psi=\frac{-\text{E}^2}{\hbar^2}\psi$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2}=\text{E}^2\psi \qquad (2.3.3)$$

Inserting equation (2.2.9) and equation (2.3.3) in equation (2.3.2) , one gets :

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2}=-\text{c}^2\hbar^2p^2\psi+m_0^2\text{c}^4\psi \qquad (2.3.4)$$

This equation is known as Klein – Gordon .

2.4 Dirac Relativistic Equation

Klein - Gordon equation fails in describing the instructive which results from the electron – spin .

This motivates Dirac to derive a new quantum mechanical expression based on a linear expression of the relativistic energy , which given by

$$E = c\,\alpha\,.\,p + \beta\,m_\circ c^2 \qquad (2.4.1)$$

Where the parameters α and β were determined by the boundary conditions imposed by the quantum relativistic . It was found that these parameters related to the electron - spin and are known as Pauli matrice to derive Dirac equations multiply both sides of equation (2.4.1) by ψ to gets:

$$E\,\Psi = c\,\alpha\,.\,p\,\Psi + \beta\,m_\circ\,c^2\,\Psi \qquad (2.4.2)$$

To write equation (2.2.4) in a differential form , one can use equation (2.2.5) .

With the aid of equations (2.2.8) , (2.2.9) one gets :

$$i\hbar\cdot\frac{\partial\psi}{\partial t} = E\psi , \frac{\hbar}{i}\nabla\psi = P\psi \qquad (2.4.3)$$

Inserting (2.4.3) in (2.4.2) , yields

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar}{i}c\alpha\cdot\nabla\psi + \beta m_o c^2\psi \qquad (2.4.4)$$

Which is the relativistic Dirac equation .

2.5 Harmonic Oscillator

The time independent Schrodinger equation take the form :

$$\frac{-\hbar^2}{2m}\nabla^2 U+Vu=Eu \qquad (2.5.1)$$

The potential of a harmonic oscillator in one dimension is given by

$$V=\frac{1}{2}Kx^2 \qquad (2.5.2)$$

Inserting equation (2.5.2) in equation (2.5.1) for one dimension yields

$$\frac{-\hbar^2}{2m}U^{\backslash\backslash}+\frac{1}{2}Kx^2 = Eu \qquad (2.5.3)$$

To simplify this equation lets $Y=\alpha X$

$$\alpha=\left(\frac{mK}{\hbar^2}\right)^{\frac{1}{4}}$$

$$\lambda=\frac{2mE}{\hbar^2\alpha^2}=\frac{2E}{h}\left(\frac{m}{K}\right)^{\frac{1}{2}}=\frac{2E}{\hbar\omega_\circ} \qquad (2.5.4)$$

To get :

$$U^{\backslash\backslash}+(\lambda-Y^2)\,u=o \qquad (2.5.5)$$

To simplify further let

$$U=He^{-\frac{Y^2}{2}} \qquad u^1=\left[H^1-YH\right]e^{-Y^2/2}$$

$$U^{\backslash\backslash} = \left(H^{\backslash\backslash} - 2YH^{1} + Y^{2}H\right)e^{\frac{-Y^2}{2}} \qquad (2.5.6)$$

Thus

$$H^{\backslash\backslash} - 2YH^{\backslash\backslash} + (\lambda - 1)H = O \qquad (2.5.7)$$

Consider the solution

$$H = \sum_n a_n Y^n \qquad H^{\backslash} = \sum_n n a_n Y^{n-1} \qquad (2.5.8)$$

$$H^{\backslash\backslash} = \sum_n n(n-1)a_n Y^{n-2}$$

To get :

$$\sum_n = (n-1)a_n Y^{n-2} + \sum_n [\lambda - 1 - 2n] a_n Y^n = 0 \qquad (2.5.9)$$

To make all terms a function (2.5.9) of y^n replace n by (n+2) in the first term to get :

$$\sum_n (n+2)(n+1) a_{n+2}\, y^n + \sum_n [\lambda - 1 - 2n] a_n y^n = 0 \qquad (2.5.10)$$

One of the possible solution is to equate coefficients of y^n to zero to get :

$$(n+1)(n+2)\, a_{n+2} = [-\lambda + 1 + 2n]\, a_n$$

$$a_{n+2} = \frac{[-\lambda + 1 + 2n]}{(n+1)(n+2)} a_n \qquad (2.5.11)$$

$$\text{For } n \to \infty \frac{\left[-\frac{(\lambda-1)}{n^2}+\frac{2}{n}\right]a_n}{\left[1+\frac{3}{n}+\frac{2}{n^2}\right]^{-1}} = \frac{2}{n} \qquad (2.5.12)$$

Which means : $H \to \infty$ as $y \to \infty$ (2.5.13)

This is in conflict with the fact that the wave function should be finite .

Thus the finite probability requires that equation (2.5.8) to be have finite terms

Thus

$$H = \sum_{n=1}^{s} a_n y^n \qquad (2.5.14)$$

This means that : $a_s \neq 0$ $a_{s+1} = 0$ $a_{s+2} = 0$ (2.5.15)

Hence from (2.5.11) for n= s

$$0 = a_{s+2} = \frac{[1-\lambda+2s]}{(s+1)(s+2)} a_s$$

The only possible solution s : $1-\lambda + 2S = 0$

$\lambda = 1+2s$ (2.5.16)

From (2.5.4)

$$E = (\lambda \hbar\omega_\circ)/2 = (s+½)\hbar\omega_\circ \qquad (2.5.17)$$

Replacing s by n , the energy of a harmonic oscillator is quantized and is given by

$$E = (n + \frac{1}{2}) \hbar\omega \qquad (2.5.18)$$

Chapter Three

Generalized Special Relativity

3.1 Introduction

Einstein Generalized special relativity (EGSR) is one of the most promising physical theories that cure the defects of SR .

In this section the expressions of time , length and mass beside energy is derived .

3.2 Space and Time Relations in Generalized Special Relatively

The generalization of SR stems from the expression of proper time T where :

$$c^2 dt^2 = g_{\mu\upsilon} dx^{\mu} dx^{v} \qquad (3.2.1)$$

With C representing the speed of light , $g_{\mu\upsilon}$ is the coordinate metric of coordinates x^{μ}. The subscripts and superscripts . The terms μ and υ denotes indicies of contravariant and covariant tensors . The proper time is the common language for both Special Relativity (SR) Equation (3.2.1) reduces to

$$c^2 d\tau^2 = c^2 dt^2 - dx^i dx^j \quad (3.2.2)$$

Where : $\kappa^o = ct$, where i denotes the particles position covariant tensor

Rearranging equation (3.2.2) one gets :

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{1}{c^2} \cdot \frac{dx^i}{dt} \frac{dx^j}{dt}} = \sqrt{1 - \frac{v^2}{c}} = \gamma^{-1} \qquad (3.2.3)$$

It is easy to generalize γ to include the effect of field potential by using equation (3.2.1) and the fact that

$$g_{11} = g_{22} = g_{33} = -1 \quad ,$$

$$g_{00} = 1 + \frac{2\emptyset}{c^2} \qquad (3.2.4)$$

Where Ørepresents the potential per unit mass . As a result equation (3.2.1) together with the definition of γ in equation (3.2.3) yields

$$\gamma^{-1} = \frac{d\tau}{dt} = \sqrt{g_{00} - \frac{1}{c^2} . \frac{dx^i}{dt} . \frac{dx^j}{dt}} = \sqrt{g_{00} - \frac{v^2}{c^2}}$$

$$\gamma^{-1} = \sqrt{1 + 2\emptyset/c^2 - v^2/c^2} \qquad (3.2.5)$$

When the effect of motion only is considered , i.e. by ignoring the field effect through the potential , the expression for time is given according to SR and (3.2.5) to be

$$dt = \gamma dt_o = \frac{dt_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.2.6)$$

Where the subscript o stands for the quantity measured in the rest frame .

This expression is the ordinary SR time dilation . However , when the clock is at rest in a certain field the expression of time becomes according to equation (3.2.6) and (3.2.5)in the form

$$dt = \gamma dt_\circ = \frac{dt_\circ}{\sqrt{g_{00}}} \qquad (3.2.7)$$

This expression conforms with the time dilation in gravity field .

In view of equations (3.2.6) and (3.2.7) , the expression

$$dt = \frac{dt_o}{\gamma} \qquad (3.2.8)$$

can be generalized to recognize the effect of motion as well as the field on time to get :

$$dt = \frac{dt_o}{\sqrt{g_{00} - \frac{v^2}{c^2}}} = \frac{dt_o}{\sqrt{1 + \frac{2\text{Ø}}{c^2} - \frac{v^2}{c^2}}} \qquad (3.2.9)$$

The same result can be obtained for length contraction by substituting the expression for γ in equation (3.2.5) instead of that of SR to get :

$$L = \gamma^{-1} L_\circ = \sqrt{g_{00} - \frac{v^2}{c^2}}\, L_\circ \qquad (3.2.10)$$

Where this expression reduces to that of SR when no field exists , i.e. when : Ø =0

To get :

$$L = \sqrt{\frac{1-v^2}{c^2}} L_\circ \qquad (3.2.11)$$

3.3 The Mass and Energy in EGSR

The first attempt to find a useful expression for the energy and mass in EGSR as made by M. Dirac where the Hamiltonian is given by

$$H = p\,c^2 = g_{00}\,T^{\circ\circ} = g_{00}\,\rho_\circ\,[\,\frac{dx^\circ}{dt}\,]^2 = g_{00}\frac{pc^2}{\gamma} = g_{00}\,\frac{m_o c^2}{v_o \gamma} \qquad (3.3.1)$$

ρ stands for matter density in the presence of any field , m_o and v_o are the mass and volume in free space .

Where His Hamiltonian and T^{oo} is energy tensor [11.16.17] using the definition of mass density and that of volume in EGSR , i.e. equation (3.2.10) one get :

$$pc^2 = \frac{mc^2}{v} = g_{oo}\frac{m_o c^2}{v\,\gamma} \qquad (3.3.2)$$

Therefore the mass moving with speed v in a field of potential Ø is given by

$$m = \frac{g_{oo} m_o}{\sqrt{g_{oo} - \frac{v^2}{c^2}}} \qquad (3.3.3)$$

Inserting the expression of g_{oo} in equation (3.3.2) yields

$$m = \frac{m_o\left[1 + \frac{2Ø}{c^2}\right]}{\sqrt{1 + \frac{2Ø}{c^2} - \frac{v^2}{c^2}}} \qquad (3.3.4)$$

It is interesting to note that , when no field exists ; i.e. when Ø = 0 :

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.3.5)$$

Which is the ordinary expression of mass in SR .

The energy in EGSR can be found with the aid of (3.3.1) , to be in the form

$$E = mc^2 = \frac{g_{oo} m_o \ c^2}{\sqrt{g_{oo} - v^2 / c^2}}$$

$$= \frac{\left(1 + 2Ø / c^2\right) m_o c^2}{\sqrt{1 + \frac{2Ø}{c^2} - \frac{v^2}{c^2}}} \qquad (3.3.6)$$

3.4 The Mass and Energy in EGSR Using Momentum conservation

Another expression for mass was obtained by Ahmed Zakarea and M. Dirar by using energy conservation law. The momentum conservation law for two identical particles mass m_o , one is at rest is given by.

$$m_1 v_1 = m_2 v_2 \qquad (3.4.1)$$

The speed of them is given by

$$v_1 = L / t_o \ , v_2 = \frac{L}{t} \qquad (3.4.2)$$

Where : $t = \gamma \ t_o$ (3.4.3)

Inserting (3.4.2 , 3)in (3.4.1) yields

$$m_1 = \gamma^{-1} m_2 \qquad (3.4.4)$$

But , since

$$m_1 = m_0 \qquad m_2 = m \qquad (3.4.5)$$

It follows that

$$m = \gamma\, m_0 = \frac{m_o}{\sqrt{g_{oo} - \frac{v^2}{c^2}}} \qquad (3.4.6)$$

$$= \frac{m_o}{\sqrt{1 + \frac{2\phi}{c_2} - \frac{v^2}{c^2}}}$$

The energy is thus given by

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \qquad (3.4.7)$$

When no field exist

$$\text{Ø} = 0$$

$$E = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.4.8)$$

Which is the ordinary expression of energy in SR .

3.5 Advantage of EGSR

Special relativity suffers from noticeable set backs . For example the expression of energy does not satisfy Newtonian limit since

$$E = mc^2 = m_o \left[1 - \left(\frac{v^2}{c^2}\right)\right]^{\frac{-1}{2}} \quad (3.5.1)$$

For small velocity , i.e. when :

$v/c << 1$ (3.5.2)

$$E = m_o\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = m_oc^2 + \frac{1}{2}m_ov^2$$

$$E = m_oc^2 + T \qquad (3.5.3)$$

The energy does not reduce to Newtonian formula

$$E = T + V \qquad (3.5.4)$$

Which include potential energy, beside kinetic energy .

This expression for E does not also explain red shift phenomena in which the frequency f_o of light changes to f when it enter the gravitational field of potential V , according to the relation

$$hf = hf_o + V \qquad (3.5.5)$$

This is since according to (3.5.3)

$$mc^2 = E = m_oc^2 + T$$

$$hf = hf_o + T \qquad (3.5.6)$$

Where :

$$mc^2 = hf \qquad m_oc^2 = hf_o \qquad (3.5.7)$$

However the situation is different for E in EGSR , where according to equation (3.3.6)

$$E = m_oc^2\left(1 + \frac{2Ø}{c^2}\right)\left(1 + \frac{2Ø}{c^2} - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

For weak field and small speed :

$$\frac{Ø}{c^2}, \frac{v^2}{c^2} << 1 \qquad (3.5.8)$$

Thus

Neglecting higher order terms

$$E = m_o c^2 \left(1 + \frac{2Ø}{c^2}\right)\left(1 - \frac{Ø}{c^2} + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$= m_o c^2 \left(1 - \frac{Ø}{c^2} + \frac{1}{2}\frac{v^2}{c^2} + \frac{2Ø}{c^2}\right)$$

$$= m_o c^2 \left(\frac{1}{2}\frac{v^2}{c^2} + \frac{Ø}{c^2} + 1\right) = \frac{1}{2} m_o v^2 + m_o Ø + m_o c^2$$

$$= T + V + m_o c^2 \qquad (3.5.9)$$

This expression is typical to that of Newton , when one neglects rest mass term .

Similarly equation (3.4.7) gives

$$E = m_o c^2 \left(1 + \frac{2Ø}{c^2} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$= m_o c^2 \left(1 - \frac{Ø}{c^2} + \frac{v^2}{2c^2}\right)$$

$$E = m_o c^2 - m_o Ø + \frac{1}{2} m_o v^2$$

$$E = m_o c^2 + V + T \qquad (3.5.10)$$

However there is a difference in the two expression (3.5.8) and (3.5.9) due to the presence of a minus sign in the latter equation .

Equation (5.5.8) and (5.5.9) predicts the gravitational red shift phenomena, where :

$$E= mc^2 = hf \qquad m_o c^2 = hf \qquad (3.5.11)$$

Since for a photon

$$m_o \rightarrow 0 \qquad T=\frac{1}{2} m_o v^2 \rightarrow 0 \qquad (3.5.12)$$

Thus

According to the two equations

$$hf = hf_o + V \qquad (3.5.13)$$

Thus

EGSR can explain the gravitational red shift phenomenon .

Chapter 4

Literature Review

4.1 Introduction

The setbacks of quantum laws motivates scientist to search for new quantum laws . Different attempts were made to derive new quantum equations . some of them take care of medium fiction , while others accounts for thermal effects . In this chapter some of them are presented .

4.2 Derivation of Schrodinger and Einstein Energy Equations from Maxwell's wave Equation

In the work done by Mohammed Ismail Adam and Mubarak Dirar AbdAlla Maxwell's equations are one of the biggest achievements that describes generation, reflection, transmittance and interaction of electromagnetic waves the matter [1,2 ,3]. The light was accepted as having a wave nature for long time. But, unfortunately, this nature was unable to describe black body radiation phenomenon. This forces Max Plank to propose that light and electromagnetic waves behave as discrete particles known later as photons. This particle nature succeeded in describing a number of physical phenomena, like atomic radiation photoelectric, Compton and pair production effect. The pair production effect needs particle nature of light as well as special relativity (SR) to be explained [4-5-6]. This dual nature of light encourages De Broglie to propose that particles like electrons can behave sometimes as waves. The experimental confirmation of this hypothesis leads to formation of new physical laws known as quantum mechanics.

Quantum Mechanics (QM) is formulated by Heisenberg first and independently by Schrodinger, to describe the dual nature of the atomic world [7.8].

Despite the fact that the DeBrogglie hypothesis is based on Max Plank energy expression beside spatial relativity. There is no link made with

Maxwell equation, some attempts were made by K. Algeilani [a] to derive Klein- Gordon and special relativity energy relation. This chapter devoted to make further links, by deriving Schrodinger equation as well as (SR) energy momention relation from Maxwell's equation. This done in section two and three respectively. Section four and five are devoted for discussion and conclusion.

Derivation of Schrodinger Equation from Maxwell's Equations. Maxwell's electric wave Equation for massive photon can be written as :
$-h^2 c^2 \nabla^2 E + h^2 c^2 \mu\sigma \frac{\partial E}{\partial t} + h^2 \frac{\partial^2}{\partial t^2} + h^2 \mu c^2 \frac{\partial^2 \rho}{\partial t^2} + m^2 c^4 E = 0$ (4.2.1)

Where :

$$\mu_\circ \varepsilon_\circ = \frac{1}{c^2}$$

Neglecting the dipole moment contribution and taking into account that fact that

$$C \gg 1$$

Thus the terms that do not consist of c can be neglected to get :

$-h^2 c^2 \nabla^2 \mathrm{E} + h^2 c^2 \mu\sigma \frac{\partial E}{\partial t} + zero + zero + m^2 c^4 E = 0$ (4.2.2)

Dividing both sides of equation (4.2.2) by $m^2 c^2$ yields

$$-\frac{h^2 c^2 \nabla^2 E}{2\,mc^2} + \frac{h^2 c^2 \mu\sigma}{2\,mc^2}\frac{\partial E}{\partial t} + \frac{m^2 c^4}{2\,mc^2} E = 0$$

$-\frac{h^2}{2m}\nabla^2 E + \frac{h^2}{2m}\mu\sigma \frac{\partial E}{\partial t} + \frac{1}{2} mc^2 E = 0$(4.2.3)

To find conductivity consider the electron equation for oscillatory system. Where the electron velocity is given by

$$v = v_{\circ} e^{iwt} \ (4.2.4)$$

And its equation of motion takes the form

$$m\frac{dv}{dt} = eE \ (4.2.5)$$

Differentiation equation (4.2.4) one gets :

$$\frac{dv}{dt} = iwv_{\circ}e^{iwt} (4.2.6)$$

$$\frac{dv}{dt} = iwv$$

Inserting equation (4.2.6) in (4.2.5)

$$iwmv = eE$$

$v = \frac{e}{iwm}E$ (4.2.7)

But for electron, the current J is given by

$$J = nev$$

$J = \frac{ne^2E}{imw} = -\frac{ine^2E}{wm}$ (4.2.8)

Also we known that

$J = \sigma E$ (4.2.9)

Comparing equation (4.2.8) and (4.2.9) one gets :

$\sigma = -\frac{ine^2}{mw}$(4.2.10)

The coefficient of the first order differentiation of E with respect to time is given with the aid of equation (4.2.3) and (4.2.10)

$\frac{h^2\mu\sigma}{2m} = \frac{-ih^2\mu ne^2}{2m^2w}$ (4.2.11)

Using Gauss Law

$\varepsilon EA = Q = neAx$(4.2.12)

Where x is the average distance of oscillator and is related to the maximum displacement according to the relation

$x = \frac{1}{\sqrt{2}}x_\circ$ (4.2.13)

$\frac{h^2\mu\sigma}{2m} = -\frac{ih^2\mu ne^2}{2m^2w} = -\frac{ih(hw)\mu\left(ne^2Ax_\circ^2\right)}{4\left(\frac{1}{2}mw^2x_\circ^2\right)mA}$ (4.2.14)

By using equation (4.2.13)

$$x_\circ^2 = 4x^2$$

Thus equation (4.2.14) becomes

$$\frac{h^2\mu\sigma}{2m} = -\frac{ih(hw)\mu(ne^2A\,4x^{2)}}{4\left(\frac{1}{2}mw^2x_\circ^2\right)mA}$$

$= -\frac{ih(hw)(neAx)e(x)\mu}{\frac{1}{2}(mw^2x_\circ^2)mA}$ (4.2.15)

By using equation (4.2.12), one gets :

$$= -\frac{ih(hw)(\varepsilon EA)e(x)\mu}{\left(\frac{1}{2}m\omega^2x_\circ^2\right)mA} = -\frac{ih(hw)c^2(\mu\varepsilon)eEAx}{(mc^2)A(\frac{1}{2}mw^2x_\circ^2)}$$

$$= -\frac{ih(hw)c^2\left(\frac{1}{c^2}\right)(Fx)}{(mc^2)\left(\frac{1}{2}mwx_\circ^2\right)} \quad (4.2.16)$$

But according to quantum mechanical and classical energy formula

$$hw = mc^2 = E(quantum)$$

$$Fx = \frac{1}{2}mw^2x_\circ^2 = E(classical)$$

There for equation (4.2.16) reduce to

$$\frac{h^2\mu\sigma}{2m} = -ih \quad (4.2.17)$$

As a result equation (4.2.3) becomes

$$-\frac{h^2}{2m}\nabla^2E - ih\frac{\partial E}{\partial t} + \frac{1}{2}mc^2E = 0 \quad (4.2.18)$$

We have

$$m = m_\circ\left(1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

Since Schrodinger deals with low speed therefore

$$\frac{v}{2} \ll 1$$

Thus one can neglect the speed term to get :

$$m = m_\circ\left(1 + \frac{2\emptyset}{c^2}\right)^{\frac{1}{2}} \quad (4.2.19)$$

Taking c as a maximum value of light speed. Such that the average light speedc_e is given by

$$c_{e=\frac{1}{\sqrt{2}}c}$$

$c_e^2{}_{=\frac{c^2}{2}}$ (4.2.20)

For small $\emptyset$ compared to c (4.2.20)

$m = m_\circ \left(1 + \frac{2\emptyset}{2\, c_e^2}\right)$ (4.2.21)

Thus

$\frac{1}{2} mc^2 = mc_e^2 = m_\circ c_e^2 (1 + \frac{\emptyset}{c_e^2})$

$$= m_\circ c^2 + m_\circ \emptyset \quad (4.2.22)$$

$$= m_\circ c^2 + V$$

Since atomic particle which are describes by quantum laws are very small , thus one can neglects $m_\circ$ compared to the potential V to get :

$m_\circ c^2 + V = V$ (4.2.23)

Hence from equations (4.2.22) and (4.2.23)

$\frac{1}{2} mc^2 = m_\circ c^2 + V = V$ (4.2.24)

Thus

Equation (4.2.18) reduce to

$$-\frac{h^2}{2m}\nabla^2 E - ih\frac{\partial E}{\partial t} + VE = 0$$

Taking in to account that electromagnetic energy density is proportional to E^2 and sinceΨ is also reflects photon density. Thus one can easily replace E byΨ, in the above equation, to get :

$$-\frac{h^2}{2m}\nabla^2\Psi - ih\frac{\partial\Psi}{\partial t} + V\Psi = 0 \quad (4.2.25)$$

This is Schrodinger equation.

- The electric polarization and special Relativity:

We have

$$P = -nex \quad (4.2.26)$$

$$\mu\frac{\partial^2 P}{\partial t^2} = -n\mu ex \quad (4.2.27)$$

Where :

$$x = x_\circ e^{i\omega t}$$

$$\dot{x} = \mathrm{i}\omega_0 x_\circ e^{\mathrm{i}\omega \mathrm{t}}$$

$$\ddot{x} = i^2\omega_0{}^2 x_o^2 e^{i\omega \mathrm{t}} = -\omega_0^2 x \quad (4.2.28)$$

$$\mu\frac{\partial^2 P}{\partial t^2} = \mu\omega_0 nex \quad (4.2.29)$$

From equation (4.2.12) one have

$$\varepsilon EA = Q = neAx$$

$$\varepsilon E = nex \quad (4.2.30)$$

Inserting equation (4.2.30) in equation (4.2.29), one gets :

$\mu \frac{\partial^2 P}{\partial t^2} = \mu\omega_\circ^2 \varepsilon E = \mu\varepsilon\omega_\circ^2 E = \frac{\omega_\circ^2}{c^2} E$ (4.2.31)

Equation (4.2.1) can be written as

$-\nabla^2 E + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} + \mu \frac{\partial^2 P}{\partial t^2} = 0$ (4.2.32)

From equation (4.2.31) and (4.2.32), one gets :

$-\nabla^2 E + {}^{1}/_{c^2} \frac{\partial^2 E}{\partial t^2} + \frac{\omega_\circ}{c^2} E = 0$ (4.2.33)

Consider

$$E = E_\circ e^{i(kx - \omega t)} \quad (4.2.34)$$

$\nabla^2 E = -k^2 E$(4.2.35)

$$\frac{\partial E}{\partial t} = -i\omega E_\circ e^{i(Kx - \omega t)}$$

$$\frac{\partial^2 E}{\partial t^2} = i^2 \omega^2 E_\circ e^{i(Kx - \omega t)}$$

$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E$ (4.2.36)

Substituting equation (4.2.35) and (4.2.36) in (4.2.33)

$$+k^2 E - \frac{1}{c^2}\omega^2 E + \frac{\omega^2}{c^2} E = 0$$

Multiplying both sides of above equation by $\frac{c^2}{E}$

$$\frac{K^2 E c^2}{E} - \frac{1}{c^2}\omega^2 \frac{E c^2}{E} + \frac{\omega_\circ^2}{c^2}\frac{E c^2}{E} = 0$$

$K^2c^2 - \omega^2 + \omega_\circ^2 = 0$ (4.2.37)

Multiplying both sides of equation (4.2.37) by $\hbar^2$

$\hbar^2K^2c^2 + \hbar^2\omega_\circ^2 = \hbar^2\omega^2$ (4.2.38)

For a photon the energy and a momentum are given by Plank and DeBrogglie hypothesis to be :

$$P = \frac{h}{\lambda} E = hf$$

$E = h\frac{c}{\lambda} = Pc$ (4.2.39)

But the photon momentum is given by

$$P = mc$$

Thus

$E = Pc = mc^2$(4.2.40)

Therefore :

$$\hbar w = hf = E = mc^2$$

$\hbar w_\circ = hf_\circ = E_\circ = m_\circ c^2$ (4.2.41)

Substituting equations (4.2.39) , (4.2.40) and (4.2.41) in equation (4.2.38) , one gets :

$P^2c^2 + m_\circ^2c^4 = m^2c^4$(4.2.42)

Since Schrodinger equation is first order in time, thus the second order time term should disappear in equation (4.2.1) .

This achievement by taking into account that all terms that consist of C are larger compared to terms free of C. this since the speed of light is very larger ($c \approx 10^8$). The dipole term in (4.2.1) is neglected, which is also natural as well as Schrodinger equation deals only with particles moving in a field potential through the term V which is embedded in the mass term according to (GSR) [see equation (4.2.1)]. In deriving the conductivity term the effect on the particles is only the electric field, while the effect of friction is neglected. This also compatible with Schrodinger hypothesis which consider the effect of themedium is only through the potential according to the energy wave equation.

$$E\Psi = \frac{P^2}{2m}\Psi + V\Psi \quad (4.2.43)$$

$$\Psi = Ae^{\frac{i}{\hbar}(px-Et)}$$

$$ih\frac{\partial\Psi}{\partial t} = E\Psi$$

$$-h^2\nabla^2\Psi = P^2\Psi \quad (4.2.44)$$

The fact that the velocity in equation (4.2.4) represents oscillating particle reflects the wave nature of particles, on which one of the main quantum hypothesis is based.

By using this hypothesis together with Blank expression of energy. Beside classical energy of an oscillating system, the coefficient of the first time derivative ofels found to be equal to (-I h), in view of equation (4.2.18) and the GSR expression of mass (19) the potential term in Schrodinger equation is clearly stems from the mass term again the wave nature of particles relates the maximum light speed to its average speed according to equation (4.2.20).

Neglecting the rest mass, in the third term in equation (4.2.18) the coefficient of E is equal to the potential.

The final Schrodinger equation was found by the replacing E by ψ.

This is not surprising since number of photon d $[4]^2$d E^2. The relation of energy and momentum is SR by assuming oscillating atoms in the media with frequency W_0 as representing the background rest energy as shown by equation (4.2.28 , 4.2.31) the energy gained by the system is the electromagnetic energy of frequency W {see equation (4.2.34, 4.2.36). Using Plank hypothesis for a photon, beside momentum mass relation in equation (4.2.39, 4.2.40, 4.2.41) the special relativity momentum energy relation was found.

The derivation of Schrodinger quantum equation and special relativity energy-momentum relation from Maxwell electric equation shows the possibility of unifying the wave and particles nature of electromagnetic waves. It shows also of unifying Maxwell's equations, SR and quantum equations.

The behavior of nano system are now far from being described fully by quantum mechanics.

The situation for elementary particles, field is even worse. There is no theoretical model that can put gravity under the umbrella of quantum mechanics.

The dream of unification of forces is too difficult to be achieved within the present physical theories including quantum mechanics.

These failures maybe related to mathematical and physical laws are based on the dual nature of wave pickets beside the energy expression in classical mechanics and relativity [10] . Unfortunately the energy expression take care of the effect of the field potential only, without accounting other effects that can change the behavior of the particles under study.

These effects include friction, collision and Scaling effects that are closely related to the density of particles and relaxation time.

Thus there is a need for a quantum model that can accounts for the effect of the surrounding medium one of the approach is based on deriving quantum equations from Maxwell's equations as done by K.Algeilani and others [11].

This approach is reasonable. Since Maxwell's equations have terms like conductivity and electric dipole moment (polarization) that account for medium density, relaxation time and internal electric charge [12]. Unfortunately Algeilani model don't accounts for the field effect through the potential term [13]. Maxwell's equations for diffusion current and polarized current is derived in section two. A new approach based on Maxwell equations is used to derive. Klein-Gordon equation in section three section four is devoted for deriving new generalized quantum equation based also on Maxwell's equation section five and six are concerned with discussion and conclusion.

4.3 Derivation of Quantum Equation from Maxwell's equations

According to Maxwell's equations , one has

$\nabla X H = J + G$ (4.3.1)

Where H , J and G stands for magnetic intensity , current density and displacement current respectively

$\nabla . J + \frac{\partial \rho}{\partial t} + \frac{\partial \rho_b}{\partial t} + c_d \nabla^2 \rho = 0$ (4.3.2)

The current density J is assumed to result from external ohm ($j_\circ$) $b$eside bounded charge$j_b$and diffusion process J_α

$J = j_\circ + J_b + J_d$(4.3.3)

Where :

$$j_\circ = -\frac{\partial \rho}{\partial t}$$

$\Rightarrow \nabla . j_\circ = -\frac{\partial}{\partial t}(\nabla . D) = -\frac{\partial \rho}{\partial t}$(4.3.4)

$$J_b = -\frac{\partial \rho}{\partial t}$$

$\Rightarrow \nabla . J_b = -\frac{\partial}{\partial t}(\nabla . P) = \frac{\partial \rho_b}{\partial t}$(4.3.5)

Where p , ρ_b stands for polarization and bound charge respectively

$$J_d = -c_d \nabla \rho$$

$\Rightarrow \nabla . J_d = -c_d \nabla^2 \rho$ (4.3.6)

Thus the divergence of both sides of equation (4.3.3) gives

$\nabla.J = \nabla.J_\circ + \nabla.J_b + \nabla.J_d$ (4.3.7)

$\overleftrightarrow{\nabla.J} = -\frac{\partial\rho}{\partial t} + \frac{\partial\rho_b}{\partial t} - c_d\nabla^2\rho$ (4.3.8)

By rearranging the above equation

$\nabla.J + \frac{\partial\rho}{\partial t} - \frac{\partial\rho_b}{\partial t} - c_d\nabla^2\rho = 0$ (4.3.9)

To find the unknown G, one uses

$\rho = \nabla.D = \varepsilon.\nabla.E$ (4.3.10)

$\rho_b = -\nabla.P$(4.3.11)

Taking the divergence of equation (4.3.1) ,one have

$$\nabla.\nabla \times H = 0$$

$\nabla.\nabla \times H = \nabla.J + \nabla.G = 0$ (4.3.12)

Inserting equation (4.3.12) in (4.3.8) yields

$-\frac{\partial\rho}{\partial t} + \frac{\partial\rho_b}{\partial t} - c_d\nabla^2\rho = -\nabla.G$ (4.3.13)

Using equation (4.3.10) and (4.3.11) yields

$-\frac{\partial}{\partial t}(\nabla.D) + \frac{\partial}{\partial t}(-\nabla.P) - c_d\nabla.(\nabla\rho) = -\nabla.G$ (4.3.14)

But $\nabla.D = \rho$

Thus

$\nabla\rho = \nabla.(\nabla.D)$(4.3.15)

Using relating (4.3.10) and (4.3.15) yields

$$-\frac{\partial}{\partial t}(\nabla.\varepsilon E) + \frac{\partial}{\partial t}(-\nabla.P) - c_d\nabla.(\nabla(\nabla.D)) = -\nabla.G$$

$$-\frac{\partial}{\partial E}(\nabla.\varepsilon E) + \frac{\partial}{\partial t}(-\nabla.P) - c_d\nabla.(\nabla(\nabla.\varepsilon E)) = -\nabla.G$$

$$-\varepsilon\nabla.\frac{\partial E}{\partial t} - \nabla.\frac{\partial P}{\partial t} - \varepsilon c_d\nabla.(\nabla(\nabla.E)) = -\nabla.G$$

Comparing both sides of above equation yields

$$\varepsilon\frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d\nabla(\nabla.E) = G$$

$G = \varepsilon\frac{\partial E}{\partial t} + \frac{\partial \rho}{\partial t} + \varepsilon c_d\nabla(\nabla.E)$ (4.3.16)

Thus from equation (4.3.1) and the fact that $J = \sigma_\circ E$

$$\nabla \times H = J + G$$

$\nabla \times H = \sigma_\circ E + \varepsilon\frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d\nabla(\nabla.E)$ (4.3.17)

Also from Maxwell's equations we have

$$\nabla \times E = -\mu\frac{\partial H}{\partial t}$$

$\nabla \times \nabla \times E = -\mu\frac{\partial(\nabla\times H)}{\partial t}$ (4.3.18)

From equation (4.3.16) and (4.3.1) one found that

$\nabla \times H = J + \varepsilon\frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} + \varepsilon c_d\nabla(\nabla.E)$(4.3.19)

Multiplying both sides of equation (4.3.19) by $\mu$and differentiation over time t yields

$$\mu \frac{\partial}{\partial t}(\nabla \times H)\mu \frac{\partial j}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} + \mu \frac{\partial^2 P}{\partial t^2} + \varepsilon\mu c_d \nabla \left(\nabla . \frac{\partial E}{\partial t}\right) \quad (4.3.20)$$

But

$$J = \sigma E \quad (4.3.21)$$

$$\mu \frac{\partial}{\partial t}(\nabla \times H) = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} + \mu \frac{\partial^2 P}{\partial t^2} + \varepsilon\mu c_d \nabla \left(\nabla . \frac{\partial E}{\partial t}\right) \quad (4.3.22)$$

Also we have

$$\nabla \times \nabla \times E = -\nabla^2 E + \nabla(\nabla . E) \quad (4.3.23)$$

From equations (4.3.23), (4.3.22) and (4.3.18) yields

$$-\nabla^2 E + \nabla(\nabla . E) = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} + \mu\sigma \frac{\partial^2 E}{\partial t^2} + \mu \frac{\partial^2 P}{\partial t^2} + \varepsilon\mu c_d \nabla \left(\nabla . \frac{\partial E}{\partial t}\right) \quad (4.3.24)$$

This is the Maxwell equation when diffusion is considered .

Derivation of Klein-Gordon equation from Maxwell's Equation for Massive photon is possible by using Maxwell's equation for massive photon to be

$$-\nabla^2 E + \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \mu \frac{\partial^2 P}{\partial t^2} + \frac{m^2 c^2}{\hbar^2} E = 0 \quad (4.3.25)$$

Neglecting polarization effect and considering the propagation in free space where:

$$\begin{bmatrix} \sigma = 0 \\ \mu = \mu_0 \\ \varepsilon = \varepsilon_0 \end{bmatrix} \quad (4.3.26)$$

$$\mu_0 \varepsilon_0 = \frac{1}{c^2} \quad (4.3.27)$$

Where c is speed of light .

Equation (4.3.25) reduce to

$$-\nabla^2 E + zero + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} + zero + \frac{m^2 c^2}{\hbar^2} E = 0 \quad (4.3.28)$$

$$-\nabla^2 E + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} + \frac{m^2 c^2}{\hbar^2} E = 0$$

$$\hbar^2 \left(-\nabla^2 E + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}\right) + m^2 c^2 = 0 \quad (4.3.29)$$

Inserting equation (4.3.27) in (4.3.29), one gets :

$$-\hbar^2 \nabla^2 E + \hbar^2 \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + m^2 c^2 = 0$$

Multiplying both sides of above equation by c^2

$$-\hbar^2 C^2 \nabla^2 E + \hbar^2 \frac{\partial^2 E}{\partial t^2} + m^2 c^4 E = 0 \quad (4.3.30)$$

If the rest mass equals the relativistic mass . When no potential exist then

$$m = m_0 \left(1 - \frac{v^2}{c^2} + \frac{2\phi}{c^2}\right)$$

$$= m_0 \left(1 - \frac{v^2}{c^2}\right)$$

When :

$$v << c$$

Thus equation (4.3.30) reduces to

$$m = m_0 (4.3.31)$$

$$-\hbar^2 \frac{\partial^2 E}{\partial t^2} = -c^2\hbar^2\nabla^2 E + m_\circ^2 c^4 E \quad (4.3.32)$$

Replacing E by Ψ in equation (4.3.32), one gets :

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = -c^2\hbar^2\nabla^2 \Psi + m_\circ^2 c^4 \Psi (4.3.33)$$

This the ordinary Klein-Gordon Equation. Schrodinger equation deals only with non-relativistic particles, thus it does not take into account the rest mass energy. On contrary Gordon equation can account for rest mass energy but does not have potential energy term for fields other than electromagnetic fields.

Thus there is a need to find a new quantum equation that accounts for rest mass energy, beside potential energy. This can be done with the aid of equation (4.3.25). Where one uses the mass expression of the generalized special relativity which is given by

$$m = m_o \, (1 + \; + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2})^{\frac{1}{2}} \; (4.3.34)$$

$$m^2 = m_0^2 \left(1 - \frac{2\emptyset}{c^2} + \frac{v^2}{c^2}\right) (4.3.35)$$

But we have

$$m_o \emptyset = V \quad (4.3.36)$$

$$m_o v = P \quad (4.3.37)$$

Substituting equation (4.3.37) and (4.3.36) in (4.3.35) one gets :

$$m^2 = m_\circ^2 + 2m_o \frac{V}{c^2} - \frac{P^2}{c^2} (4.3.38)$$

Multiplying both sides of equation (4.3.38) by E C^4

$m^2c^4E = m_0^2c^4E + 2m_\circ c^2VE - p^2c^2E$ (4.3.39)

But for oscillating electric field

$$E = E_\circ e^{i(kx-wt)}$$

$$\frac{\partial E}{\partial x} = ikE_\circ e^{i(kx-wt)}$$

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} = i^2k^2E_\circ e^{i(kx-wt)}$$

$$\nabla^2 E = -k^2E$$

$\hbar^2\nabla^2 E = -\hbar^2k^2\mathrm{E}$ (4.3.40)

$$\hbar^2\nabla^2 E = -P^2E$$

Thus

equation (4.3.39) becomes

$m^2c^4E = m_\circ^2c^4E + 2m_\circ c^2VE - c^2\hbar^2\nabla^2 E$ (4.3.41)

By using identify$\mu\varepsilon = \frac{1}{c^2}$and inserting equation (4.3.41) in equation (4.3.25)

$$-c^2\hbar^2\nabla^2 E + c^2\hbar^2\mu\sigma\frac{\partial E}{\partial t} + \frac{\hbar^2\partial^2 E}{\partial t^2} + m_\circ^2c^2E + 2m_\circ c^2VE - c^2\hbar^2\nabla^2 E = 0$$

Replacing E by ψ and collecting similar terms leads to the new quantum equation of the form

$-2c^2\hbar^2\nabla^2\Psi + c^2\hbar^2\mu\sigma\frac{\partial\Psi}{\partial t} + \frac{\hbar^2\partial^2\Psi}{\partial t^2} + m_\circ^2c^4\Psi + 2m_\circ c^2V\Psi = 0$ (4.3.42)

The fact that Maxwell's equation is used to derive Klein-Gordon equation is related to the fact that quantum mechanical laws are based on Plank quantum light equation.

The replacement of the electric field Intensity Vector E by the wave function ψ is reasonable as far as the electromagnetic energy density which is related to the number of photons is proportional to E^2, i.e.nα E^2

While it is also related to $|\psi|^2$

i.e

$n \alpha |\psi|^2$

Thus

$E \rightarrow \psi$

The new quantum mechanical law shown in equation (4.3.42) is more general than Schrodinger and Klein-Gordon equation it consist of conductivity of the medium which is related to the friction of the system . The conductivity term can also feel the existence of the bulk matter through the particle density term n where

$$\sigma = \frac{ne^2\tau}{m}$$

Unlike Schrodinger equation the new quantum equation consist of a term representing rest mass energy. This equation is also more general than Klein-Gordon equation by having terms accounting for the effect of friction, collision through conductivity, besides having potential term accounting for all fields other than electromagnetic field.

4.4Schrodinger Quantum Thermal Equation

Dr. R. AbdElhai, M.H.M.Hilo, R.AbdElgani, and M.D. Abd Allah derive also a new quantum equation accounting for temperature effect.

They use plasma equation according to plasma equation, fluid of particles of mass m, number density. Velocity V force F and pressure P is given by

$$mn\left[\frac{\partial v}{\partial t} + v.\nabla v\right] = F - \nabla p \qquad (4.4.1)$$

If F is a field force then

F = - n∇V

Where V is the potential of one particle in one dimension

$$mn\ \left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = -n\nabla V - \nabla p = -n\frac{dV}{dx} - \frac{dp}{dx}$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial x}dx$$

$$\frac{dv}{dt} = \frac{dv}{\partial t} + \frac{\partial v}{\partial x}\frac{dx}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} \quad (4.4.2)$$

Thus

According to equation (4.4.2), in one dimension

$$mn\frac{dv}{dt} = -n\frac{dV}{dx} - \frac{dp}{dx} \quad (4.4.3)$$

Schrodinger Temperature dependent Equation. Schrodinger equation can be devived by using new expression of energy obtained from the plasma equation to do this one can use (4.4.3) to get :

$$mn\frac{dv}{dx}\frac{dx}{dt} = -n\frac{dV}{dx} - \frac{dp}{dx}$$

Multiplying both sides by dx and integrating yields

$$mn\int vdv = -n\int dV - \int dp$$

Considering the pressure to be $p = \gamma nkT$ in general

Thus

$$mn\frac{v^2}{2} = -nV - p = -nV - \gamma nkT$$

Hence

$$m\frac{v^2}{2} + V + \gamma nkT = const$$

This constant conserved quantity looks like the ordinary energy beside the ordinary thermal energy term γKT

$E = \frac{p^2}{2m} + V + \gamma kT$ (4.4.4)

To find Schrodinger equation for it, considering the ordinary wave function

$$\Psi = Ae^{\frac{i}{\hbar}(px-Et)}$$

Differentiating both sides by t and x yields

$$\frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}E\Psi$$

$$i\hbar\frac{\partial\Psi}{\partial t} = E\Psi$$

$$\frac{\partial^2\Psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\Psi \Longrightarrow -\hbar^2\nabla^2\Psi = p^2\Psi$$

Multiplying both sides of equation (4.4.4) by ψ yields

$$E\Psi = \frac{P^2}{2m}\Psi + \mathrm{V}\Psi + \gamma kT\Psi$$

Substituting Equation (4.4.3), one gets :

$\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + \mathrm{V}\Psi + \gamma kT\Psi$ (4.4.5)

This equation represents Schrodinger equation when thermal motion is considered. The solution for time free potential can be obtained by suggesting

$$\Psi = e^{-\frac{i}{\hbar}(E_t)}u \Longrightarrow \frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}E\Psi$$

Thus

From equation (4.4.5) one gets :

$$\mathrm{E}\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + \mathrm{V}\Psi + \gamma kT\Psi$$

The time independent Schrodinger equation thus takes the form

$Eu = -\frac{\hbar^2}{2m}\nabla^2 u + Vu + \gamma kTu$ (4.4.6)

For constant potential, the solution can be

$$u = e^{ikx} \quad , \quad V = V_\circ$$

Inserting this solution in equation (4.4.6) yields

$$Eu = \frac{\hbar^2 k^2}{2m} u + V_\circ u + \gamma k T u$$

$$E = \frac{\hbar^2 k^2}{2m} + V_\circ + \gamma k T$$

If one set the Kinetic term to be $E_\circ = \frac{\hbar^2 k^2}{2m}$, one can thus write the energy in the form

$E = E_\circ + V_\circ + \gamma k T$ (4.4.7)

This quantum energy expression involves a thermal term beside Kinetic and potential term.

The resistance per unit length (L=1) per unit area (A=1) can be found from the ordinary definition of Z, the resistance Z is defined to be the ratio of the potential, to the current per unit area, j,i.e.

$$Z = \frac{u}{\mathrm{I}} = \frac{u}{\mathrm{J}A} = \frac{u}{\mathrm{J}} = \frac{u}{nev} = \frac{mu}{neP} \quad (4.4.8)$$

With n and e standing for the free hole or electron density and charge respectively, while prepresents the momentum of electron of mass m where p= mn.

This resistance (it actually stands for resistivity) can be found by using the laws of quantum mechanic for a free charge which responsible for generating the electric current, where the wave function take the form

$\Psi = Ae^{ikx}$ (4.4.9)

This selection of ψ comes from the fact that the resistance property comes from the motion of the free charge. The potential U is related to the Hamiltonians H through the relation H= eu.

Thus for freely moving charge one gets :

$$\hat{H} = eu = \frac{1}{2}mv^2 = \frac{\hat{P}^2}{2m} = \frac{\hbar^2}{2m}\nabla^2$$

In view of equation (4.4.9) and according to the correspondence principle V takes the form

$$u = \frac{<\hat{H}>}{e} = \int \frac{\overline{\Psi}\hat{H}\Psi dx}{e} = \int \frac{\overline{\Psi}\hat{P}^2\Psi dx}{2me} = \frac{\hbar^2 K^2}{2me}\int \overline{\Psi}\Psi \mathrm{dx} = \frac{\hbar^2 K^2}{2me} \qquad (4.4.10)$$

While P becomes

$$P =< \hat{P} >= \int \overline{\Psi}\hat{P}^2\Psi dx = \hbar K \int \overline{\Psi}\Psi dr = \hbar K \quad (4.4.11)$$

Thus inserting Equations (4.4.10), (4.4.11) in (4.4.8) one obtains

$$Z = \frac{m\hbar^2 K^2}{2m\hbar kne^2} = \frac{\hbar K}{2e^2 n} = \left[\frac{h}{2\pi}\right]\left[\frac{2\pi}{\lambda}\right]\frac{1}{2e^2 n}$$

$$Z = \frac{h}{2\lambda e^2 n} = \frac{hf}{2f\lambda e^2 n} = \frac{hf}{2ve^2 n} = \frac{hf\sqrt{\mu\varepsilon}}{2e^2 n} = \frac{\hbar\omega\sqrt{\mu\varepsilon}}{2e^2 n} \quad (4.4.12)$$

Where the expression f for velocity is found by assuming charges to be waves, then following the electromagnetic theory (EMT), the speed of the waves is affected by electric permittivity E and magnetic permability through the relation.

$$v = \lambda f = \frac{1}{\sqrt{\mu\varepsilon}} \quad (4.4.13)$$

Where the effect of medium chargers the wave length,λ, while the frequency f, is unchanged thus assuming the charge density n, to be constant, the only charge of Z can be caused by M,E.

It is also important to note that, in superconductor, the current can flow without the aid of deriving potential U, the role of U is confined only in enabling electrons to gain Kinetic energy throughout the relations

$eu = \frac{1}{2}mv^2 = K$ (4.4.14)

Where this potential can be applied between any two arbitrary points in the superconductors then remove it. The role of resistive force is neglected here as done in deriving London equations.

The expression for Z can also be found by Inserting Equation (4.4.14) in to get :

$$Z = \frac{u}{J} = \frac{u}{nev} = \frac{mv^2}{2ne^2v} = \frac{mv}{2ne^2} = \frac{P}{2ne^2} = \frac{h}{2\lambda ne^2}$$

$$Z = \frac{hf}{2f\lambda ne^2} = \frac{hf}{2e^2\, nv} = \frac{hf\sqrt{\mu\varepsilon}}{2e^2n} = \frac{\hbar\omega\sqrt{\mu\varepsilon(1+x)}}{2e^2n} \text{(4.4.15)}$$

It is important to note that this quantum resistance expression resembles the ones found by Tsui three where one uses De Broglie hypothesis four i.e.$P = \frac{h}{\lambda}$

4.5 Calculation HTSC by Electric Susceptibility

Consider holes in a conductor having resistive force Fr magnetic force Fm and pressure force Fp beside the electric force Fe, the equation motion then becomes [3]

$$F = F_r + F_m + F_e - F_p$$

Where :

$$F_p = -\nabla p \ , \qquad F_r = \frac{mv}{t} \ , \quad F_m = Bev \ , \ \ F_e = eE = eE_\circ e^{ikx}$$

P, x, m, v, I, b, e and E stands for the pressure, displacement, mass, velocity, relaxation time, magnetic fluid density, electron charge and electric fluid density.

Thus

The equation of motion takes the form

m$\ddot{x} = -\frac{mv}{\tau} + Bev + eE - \nabla p$ (4.5.1)

The solution of this equation can be suggested to be

$$\left.\begin{aligned} x &= x_\circ e^{ikx} \\ v &= v_\circ e^{ikx} \\ E &= E_\circ e^{ikx} \end{aligned}\right] \qquad (4.5.2)$$

Inserting (4.5.2) in (4.5.1) yields

$-mw^2 x = \left[\frac{-mv_\circ}{E_\circ \tau} + \frac{Bev}{E_\circ} - \frac{KT\nabla n}{E_\circ} + e\right] E$(4.5.3)

$$x = \left[\frac{\frac{mv_\circ}{E_\circ \tau} - \frac{Bev}{E_\circ} + \frac{KT\nabla n}{E_\circ} - e}{mw^2}\right] E$$

This expression of X can be utilized in the formula which relates the electric polarization vector P to susceptibility$x$ on one hand and to the number of atoms N V12 the following relation

$P = \varepsilon_{\circ} x E = eNx$ (4.5.4)

Motivated by the important role of holes in HTSC, displacement can be assumed to result from the motion of holes or positive nuclear charges, thus inserting equation (4.5.3) in (4.5.4) yield

$$\varepsilon_{\circ} x E = \left[\frac{\frac{mv_{\circ}}{E_{o}\tau} - \frac{Bev_{\circ}}{E} + \frac{kT\nabla n}{E_{o}} - e}{mw^{2}}\right] E$$

$x = \frac{eN}{mw^{2}\varepsilon_{o}E_{o}}\left[\frac{mv_{\circ}}{\tau} - Bev + kT\nabla n - eE_{\circ}\right]$(4.5.5)

The electric flux density assumes the following relation

$$D = \varepsilon E = \varepsilon_{\circ}(1 + x)E = P + \varepsilon_{\circ}E$$

The electric permittivity is given by

$\varepsilon = \varepsilon_{\circ}(1 + x)$(4.5.6)

The electric permittivity is thus given according to equation (4.5.6) to be

$$\varepsilon = \varepsilon_{\circ}(1 + x)$$

$= \varepsilon_{\circ}\left[\frac{eN}{mw^{2}E_{o}}\left[\frac{mv_{\circ}}{\tau} - Bev_{o} + kT\nabla n - eE_{\circ}\right]\right]$(4.5.7)

The resistance Z can be found by inserting (4.5.7) in (4.5.1) to get :

$Z = \frac{\hbar\omega}{2ne^2}\sqrt{m\varepsilon_o\sqrt{1 + \frac{eN}{mw^2\varepsilon_\circ E_\circ}(KT\nabla n + \frac{mv_o}{\tau} - Bev_o - eE_\circ)}}$ (4.5.8)

$$Z = \frac{\hbar\omega}{2ne^2}\sqrt{\mu\varepsilon_o\sqrt{\frac{mw^2\varepsilon_\circ E_\circ + eE(KT\nabla n + \frac{mv_o}{\tau} - Bev_o - eE_o}{mw^2\varepsilon_\circ E_\circ}}}$$

The resistance Z is zero when it is imaginary . This happens when

$$mw^2\varepsilon_o + eN\left[kT\nabla n + \frac{mv_\circ}{t} - Bev_o - eE_\circ\right] < 0$$

$$kT\nabla n < +Bev_o + eE_o - \frac{mw^2\varepsilon_\circ E}{eN} - \frac{mv_\circ}{t}$$

$$T < +\frac{mv_\circ}{k\nabla n} + \frac{(e - mw^2\varepsilon_\circ)E_\circ}{eNk\nabla n} - \frac{mv_\circ}{t}$$

Thus

The critical temperature is given by

$T_e = \frac{(Be\tau - m)v_o}{\tau k\nabla n} + \frac{(e - mw^2\varepsilon_\circ)E_\circ}{eNk\nabla n}$ (4.5.9)

If the internal field B results from no atoms each having average flux density μB: [5]

$B = \mu_o N_o$ (4.5.10)

Therefore,T_c can take the form

$T_c = \frac{(\mu_o N_o e\tau - m)v_o}{\tau k\nabla n} + \frac{(e - mw^2\varepsilon_\circ)E_\circ}{eNk\nabla n}$(4.5.11)

In tight binding model [5] the energy of electrons in the crystal is given by

$\varepsilon = \varepsilon_o + \alpha_1 + 2\gamma cosKa$ (4.5.12)

Where$\varepsilon_o$is the energy in the absence of crystal field while the other term describe the effect of the crystal field. The energy$\varepsilon_o$can split into term the Kinetic part which, can describe the thermal motion in the form$\frac{f_o}{2}kT$beside the potential term $-V_o$ For attractive force or bounded particle.

Thus one can write

$\varepsilon_o = \frac{\hbar^2 k_o^2}{2m} + \frac{f_o}{2}kT - V_o$ (4.5.13)

$$\alpha_o = \frac{\hbar^2 k_o^2}{2m}$$

$f_o$represent the degrees of freedom.

The terms describing the effect of the crystal force are

$\alpha_1 = \langle \emptyset_m | \widehat{H}_{cry} | \emptyset_m \rangle$ (4.5.14)

$$\gamma = \langle \emptyset_j | \widehat{H}_{cry} | \emptyset_m \rangle$$

In view of equations (4.5.12) and (4.5.13)

$\varepsilon = \frac{f_o}{2}kT - V_o + \alpha + 2\gamma cosKa$(4.5.15)

Where :

$$\alpha = \alpha_o + \alpha_1$$

Here $\widehat{H}_{cry}$ stands for the crystal force Hamiltonian part while $\emptyset_m$and$\emptyset_j$ are the states of particles located at the site m and j respectively.

Chapter Five

Quantum Generalized Special Relativistic Equation

5.1 Introduction

Einstein special relativity suffers from the lack of a term representing the potential energy in the expression of relativistic energy .

Klein - Gordon quantum equation (KGQE) thus suffers from disappearance of potential term . Although , in an electromagnetic fields the electric and magnetic potentials have a room in (KGQE) . But the potential of other fields cannot be represented . Thus (KGQE) cannot differentiate between particles in a potential field and particle in free space , since for both , the wave function is the same which is physically wrong .

Thus one needs a relativistic quantum equation having a potential term in it . This is done in this chapter with some application .

5.2 Generalized Special Relativistic Quantum Equation

According to GSRE the energy given by

$$g_{oo}E^2 = c^2p^2 + g_{00}^2 m_o{}^2c^4 \qquad (5.2.1)$$

The feeling of this equation by the wave function ψ can be made by multiplying both sides (5.2.1) by ψ to get :

$$g_{oo}E^2 = c^2P^2\psi + g_{00}^2 m_o c^4\psi \qquad (5.2.2)$$

The GSRE for quantum system can be obtained by taking into account the dual nature of atomic particles which are assumed to be in the form of a wave bracket [*]

$$\psi = A \; e^{\frac{i}{\hbar}(Px - Et)} \qquad (5.2.3)$$

Differentiating both sides with respect to time and space twice yields .

$$\frac{\partial\psi}{\partial t} = \frac{i}{h} E\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = i^2 \frac{E^2}{\hbar^2}\psi = -\frac{E^2}{\hbar^2}\psi$$

$$-\hbar^2 \frac{\partial^2\psi}{\partial t^2} = E^2\psi \qquad (5.2.4)$$

$$\frac{\partial\psi}{\partial x} = \frac{i}{h} p\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{i^2}{\hbar^2} p^2\psi = -\frac{p^2}{\hbar^2}\psi$$

$$-\hbar^2 \frac{\partial \; \psi}{\partial x^2} = p^2\psi$$

In 3 dimensions

$$-\hbar^2\nabla^2 = \rho^2\psi \qquad (5.2.5)$$

Substituting (5.2.4) and (5.2.5) in (5.2.2) yield

$$-\; \hbar^2 g_{oo} \frac{\partial^2\psi}{\partial^2 t} = C^2 \; \hbar^2\nabla^2\psi + g_{oo}{}^2 \; m_o{}^2 \; C^4 \gamma \qquad (5.2.6)$$

Where

$$g_{oo} = 1+\frac{2\phi}{C^2} = 1+\frac{2m_o\phi}{m_oC^2} = 1+\frac{2V}{m_oC^2} \qquad (5.2.7)$$

Where the relativistic mass assumed to be equal to the rest mass . This approximation is true for slow particles in a week field , i.e., where

$$\frac{\phi}{C^2} < 1 \qquad \frac{\upsilon^2}{C^2} < 1 \qquad m \approx m_o \qquad (5.2.8)$$

Thus the quantum generalized special relativistic equation becomes

$$-\hbar^2\left(1+\frac{2V}{m_oc^2}\right)\frac{\partial^2\psi}{\partial t^2} = c^2\hbar^2\nabla^2\psi + \left(1+\frac{2V}{m_oc^2}\right) m_0^2 c^4\psi \qquad (5.2.9)$$

Clearly this equation reduced to Klein - Gordon equation when

$V = 0$

Where :

$$-\hbar\frac{\partial^2\psi}{\partial t^2} = -c^2\hbar^2\nabla^2\psi + m_o{}^2 c^4\psi \qquad m_0^2 \quad 5.2.10$$

)**5.3 Time Independent Quantum Equation for Time Independent Potential**

Most of electron atoms are affected by nuclear time independent potential resulting from the electrostatic potential in the form :

$$V = v(r, q, \phi) = v(x, y, z) \qquad (5.3.1)$$

According to equation (5.2.7)

$$g_{oo} 1+\frac{2\nu}{m_oC^2} = g_{oo}(x, y, z) \qquad (5.3.2)$$

Dividing both sides of (5.2.6) yields

$$-\hbar^2 \frac{\partial^2 \psi}{\partial \tau^2} = -\frac{C^2 \hbar^2}{g_{oo}} \nabla^2 \psi + m_o{}^2 C^4 g_{oo} \psi \qquad (5.3.3)$$

If one write ψ in the form

$$\psi = e^{iwt} u = e^{iwt} u(x, y, z) \qquad (5.3.4)$$

Where :

$$E = \hbar\omega \qquad (5.3.5)$$

$$\frac{\partial \psi}{\partial \tau} = \iota \omega \psi$$

$$\frac{\partial^2 \psi}{\partial \tau^2} = \iota^2 \omega^2 \psi = -\omega^2 \psi$$

Thus

Equation (5.3.5) yields

$$\hbar^2 \omega^2 u e^{iwt} = \left[-\frac{c^2 \hbar^2}{g_{oo}} \nabla^2 u + g_{oo} m_o{}^2 c^4 u \right] e^{iwt}$$

Using equation (5.3.5) and eleminating exponential term yields

$$E^2 u^2 = - c^2 \hbar^2 \nabla^2 = g_{oo} E^2 u + m_o{}^2 c^4 g_{oo} u \qquad (5.3.6)$$

Equation is the time independent quantum GER equation . It is very interesting to note this equation reduced to the time independent Klein - Gordon equation when :

$$V = 0 \qquad g_{oo} = 1 \qquad (5.3.7)$$

Where :

$$-c^2 \hbar^2 \nabla^2 u = E^2 u - m_o^2 c^4 u \qquad (5.3.8)$$

Consider particles with energy E

$$E = mc^2$$

$$g_{oo} = 1 + \frac{2\phi}{c^2} = 1 + \frac{2m\phi}{mc^2} = 1 + \frac{2V}{E} \qquad (5.3.9)$$

Divide both sides of (5.3.6) by $2mc^2$ to get :

$$\frac{-\hbar^2}{2m}\nabla^2 u = \left(\frac{1}{2mc^2} + \frac{2V}{2mc^2 E}\right)E^2 - \frac{m_o^2 c^4 g_{oo}^2 u}{2mc^2}$$

ε

Neglecting m_o , yields

$$\frac{-\hbar^2}{2m}\nabla^2 u = \left(\frac{1}{2E} + \frac{2V}{E^2}\right)Eu^2$$

$$\frac{-\hbar^2}{2m}\nabla^2 u = \nu u \quad \frac{1}{2}E \qquad (5.3.10)$$

The Newtonian energy is given by

$$E_N = \frac{1}{2}mc^2 = \frac{1}{2}E \qquad (5.3.11)$$

Replacing V by -V , i.e.

$$V \rightarrow -V \qquad (5.3.12)$$

And substituting (5.3.11) and (5.3.12) in (5.3.10) yields

$$-\frac{\hbar^2}{2m}\nabla^2 u + \nu u = E_N\, u \qquad (5.3.13)$$

Which is the ordinary time independent Schrodinger equation .

5.4 Time Independent Quantum Equation for Time Dependent Potential

Consider now time dependent and spatial in dependent potential of the form

$$v = v(t) \qquad (5.4.1)$$

In this ease , equation (5.2.7) becomes

$$g_{oo} = 1 + \frac{2V(t)}{m_o c^2} = g_{oo}(t) \qquad (5.4.2)$$

Thus equation (5.2.6) becomes

$$c^2 \hbar^2 \nabla^2 \psi = g_{oo} \hbar^2 \frac{\partial^2 \psi}{\partial \tau^2} + m^2 c^4 \psi \qquad (5.4.3)$$

Consider the solution of the form

$$\psi = e^{ikr} . f(t) \qquad (5.4.4)$$

Thus

$$\nabla^2 \psi = -\kappa^2 \psi = -\kappa^2 e^{i k.r} f \qquad (5.4.5)$$

Inserting (5.4.5) and (5.4.4) in (5.4.3) yields

$$-c^2 \hbar^2 k^2 f\, e^{ik.r} = g_{oo} \hbar^2 e^{ik.r} \partial f^2 + g_{oo}^2 m_o^2 c^4 e^{ik.v} f$$

$$-c^2 \hbar^2 k^2 f\, e^{ik.r} = g_{oo} \hbar^2 \frac{\partial f^2}{\partial t^2} + g_{oo}^2 m_o^2 c^4 f \qquad (5.4.6)$$

But

$$\hbar\, k = p \qquad (5.4.7)$$

Thus

$$g_{oo}\hbar^2 \frac{\partial^2 f}{\partial^2 t} + g_{oo}^2\, m_o^2\, c^4 f = - c^2 p^2 f \qquad (5.4.8)$$

With the aid of equation (5.4.2) , (5.4.8) becomes

$$(1 + \frac{2V(t)}{m_o c^2})\left[\hbar^2 \frac{\partial^2 f}{\partial t^2} + m_0^2\, c^4 f\right] = -c^2 p^2 f \qquad (5.4.9)$$

5.5 General Form of Time Independent Quantum Equation for Time Independent Potential

Let the potential be in the form

$$V = (v, r) = v(x, y, z) \qquad (5.5.1)$$

Which represents the potential for stable atoms. In this ease equation(5.2.6) becomes

$$-\hbar^2 g_{oo}(\underline{r})\, \frac{\partial^2\psi}{\partial t^2} = -c^2 \hbar^2 \nabla^2 \psi + g_{oo}{}^2 m_o^2\, c^4 \psi$$

$$g_{oo}(\underline{r})\, m_o^2\, c^4 \psi \qquad (5.5.2)$$

Try the solution

$$\psi = f(t)\, u(v) \qquad (5.5.3)$$

$$-\hbar^2 u \frac{\partial^2 f}{\partial t^2} = -\frac{c^2 \hbar^2}{g_{oo}} f\, \nabla^2 u + g_{oo} m_o^2\, c^4 f u$$

Divide both sides by u f to get

$$-\frac{\hbar^2}{f}\frac{\partial^2 f}{\partial t^2} = -\frac{c^2\hbar^2}{g_{oo}} + \frac{\nabla^2 u}{u} + g_{oo} m_o^2 c^4 = c_o = E^2 \quad (5.5.4)$$

Thus

One have two separate equations

$$-\hbar^2 \frac{\partial^2 f}{\partial t^2} = E^2 f \quad (5.5.5)$$

And

$$-c^2 \hbar^2 \nabla^2 u^2 = g_{oo} E^2 u - g_{oo}^2 m_o^2 c^4 u \quad (5.5.6)$$

Which is a time independent equation .

5.6 General Time Independent Quantum Equation for Time Dependent Potential

For Time Dependent Potential

$$v = v(t) \qquad g_{oo} = g_{oo}(t) \quad (5.6.1)$$

$$-\hbar^2 g_{oo} \frac{\partial^2 \psi}{\partial t^2} - g_{oo}{}^2 m_o^2 c^4 \psi = c' g_{00}^2 {}'^2 \psi \quad (5.6.2)$$

Consider the solution

$$\psi(\underline{r}, t) = f(t)\, u(\underline{r}) = fu \quad (5.6.3)$$

Inserting equation (5.6.2) in equation (5.5.4) yields

$$-\frac{\hbar^2 g_{oo}}{f}\frac{\partial^2 f}{\partial t^2} g_{00}^2 m_0^2 c^4 = \frac{c^2\hbar^2}{u}\nabla^2 u = c_o = p^2 c^2 = E^2 \quad (5.6.4)$$

Thus separating time dependent and the spatial dependent part result in the equations :

$$c^2 \hbar^2 \nabla^2 u = c_o \qquad (5.6.5)$$

$$\hbar^2 g_{oo} \frac{\partial^2 f}{\partial t^2} - g_{oo}^2 m_o^2 c^4 f = g_{00}^2 \qquad (5.5.6)$$

5.7General Quantum Equation for Time Independent Potential

$$v = v(\underline{r}) \qquad g_{oo} = g_{oo}(r) \qquad (5.7.1)$$

Consider again the solution

$$\psi(r, t) = f(t)\, u(\underline{v}) \qquad (5.7.2)$$

Inserting equations (5.7.1) and (5.7.2) in equation (5.6.2) yields

$$-\frac{\hbar^2}{f}\frac{\partial^2 f}{\partial t^2} = \frac{g g_{00}^{2-}}{g_{oo}} m_o^2 c^4 - \frac{c^2 \hbar^2}{g_{oo} U} \nabla^2 u = c_o = E^2$$

Thus the time dependent part is given by

$$-\hbar^2 \frac{\partial^2 f}{\partial t^2} = E^2 f \qquad (5.7.3)$$

The spatial part takes the form

$$-c^2 \hbar^2 \nabla^2 u = g_{oo} E^2 u - g_{oo}^2 m_o^2 c^4 u \qquad (5.7.4)$$

5.8 Harmonic Oscillator Wave Function and Energy

For particle like pendulum the displacement should be small as to execute simple harmonic motion i.e.

$\varkappa << 1$ (5.8.1)

Thus one expects the potential v to be a small , i.e.

$$V = \tfrac{1}{2} \quad k\,\varkappa^2 << 1 \qquad (5.8.2)$$

Since the displacement is in the form

$$\varkappa = \varkappa_o \sin \omega t \qquad (5.8.3)$$

From equation (5.6.6) by dividing both sides by g_{oo}

$$\hbar^2 \frac{\partial^2 f}{\partial t^2} + g_{oo} m_o^2 c^4 f = - c_o\, g_{oo}^{-1} f \quad (5.8.4)$$

Since v is small .

Thus

$$g_{oo} = \left(1 + \frac{2\phi}{c^2}\right) = \left(1 + \frac{2m_o\phi}{m_o c^2}\right) = \left(1 + \frac{2V}{m_o c^2}\right) = (1 + c_1 V)$$

$$g_{oo}^{-1} = (1 + c_1 v)^{-1} = 1 - c_1 V \quad (5.8.5)$$

where :

$$c_1 = \frac{2}{m_o c^2}$$

A direct substitution of

$$\hbar^2 f^{11} + (1 + c_1 V) m_o^2 c^4 \; f = - c(1 - c_1 v)\, f \; (5.8.6)$$

Let the solution be

$$f = A \sin \alpha t \qquad f^{11} = -\alpha^2 f \quad (5.8.7)$$

$$-\alpha^2\hbar^2 f + m_o{}^2 c^4 f + c_1 m_o{}^2 c^4 V f = -c_o f + c_o c_1 V f$$

Equating the coefficients of *f* and *v f* on both sides gives

$$-\alpha^2\hbar^2 + m_o{}^2 c^4 = -c_o \quad (5.8.8)$$

$$c_1 m_o{}^2 c^4 = c_o c_1$$

$$c_o = m_o{}^2 c^4$$

$$\alpha^2\hbar^2 = m_o{}^2 c^4 + c_o = 2m_o{}^2 c^4$$

$$\alpha^2 = \frac{2m_o{}^2 c^4}{\hbar^2} \quad (5.8.9)$$

$$f = A \sin \alpha t$$

The periodicity condition requires

$$f(t) = f(t+T) \quad (5.8.10)$$

$$A \sin \alpha t = A \sin(\alpha t + \alpha T) = A[\sin \alpha t \cos \alpha T + \cos \alpha t \sin \alpha T]$$

This requires

$$\cos \alpha T = 1 \qquad \text{sing } \alpha T = 0$$

Hence

$$\alpha T = 2\pi n \quad (5.8.11)$$

$$\alpha = \frac{2n\pi}{T} = n(2\pi f) = n\omega \qquad (5.8.12)$$

$$m_o c^2 = \frac{1}{\sqrt{2}}\hbar\alpha = \frac{1}{\sqrt{2}} n\hbar\omega$$

In view of equations (5.6.4) , (5.6.5)

$$c_o = E^2 = m_o^2\, c^4 \quad (5.8.13)$$

$$E = \pm m_o c^2 = \pm\frac{1}{\sqrt{2}} n\hbar\omega$$

$$E = m_\circ c^2 = \frac{n\hbar\omega}{\sqrt{2}} \quad (5.8.14)$$

But the approximation used in equation (5.2.7)

$$V = m\phi \approx m_o\phi$$

Requires

$$E = mc^2 \approx m_o c^2 = \frac{1}{\sqrt{2}} n\hbar\omega \quad (5.8.15)$$

5.9 Harmonic Oscillator for Time independent Potential

From equation (5.7.4)

$$E^2 u - g_{oo} m_o^2\, c^4 u = -\, c^2\, \hbar^2\, g_{oo}^{-1} \nabla^2 u \quad (5.9.1)$$

Consider the solution

$$u = A \sin \alpha x \quad u^{\backslash\backslash} = -\alpha^2 u \quad (5.9.2)$$

Using equation (5.7.5)

$$E^2 u - (1+c_1 v) m_o^2\, c^4\, u = c^2 \hbar^2\, (1-c_1 v)\, \alpha^2 u$$

$$[E^2 - m_o^2\, c^4 - \alpha^2 c^2 \hbar^2] u = [c_1 m_o^2\, c^4 - c_1\, c^2 \hbar^2 \alpha^2]\, vu \quad (5.9.3)$$

Equating the coefficients of u and vu requires

$$c^2 \hbar^2 \alpha^2 - m_o^2 c^4 = 0$$

$$\alpha^2 = \frac{m_o^2 c^4}{c^2 \hbar^2} \Rightarrow \alpha = \frac{m_o c^2}{\hbar} \quad (5.9.4)$$

$$E^2 = \alpha^2 C^2 \hbar^2 + m_o^2 C^4 = 2 m_o^2 C^4 = 2 C^2 \hbar^2 \alpha^2 \quad (5.9.5)$$

The periodicity condition requires

$$u(x + \lambda) = u(x) \qquad (5.9.6)$$

$A \sin \alpha (x + \lambda) = A \sin \alpha x$

$$\sin \alpha x \cos \alpha\lambda + \cos \alpha x \sin \alpha\lambda = \sin \alpha x \qquad (5.9.7)$$

This requires

$$\cos \alpha\lambda = 1 \qquad \sin \alpha\lambda = 0 \quad (5.9.8)$$

$\alpha \lambda = 2n\pi \qquad n = 1,2,3,4,\ldots\ldots$

$$\alpha = n(2\pi/\lambda) = n(2\pi f / \lambda f) = \frac{n\omega}{c} \quad (5.9.9)$$

Substitute (5.9.9) in (5.9.5) yields

$$E = 2c^2\hbar^2\alpha^2 = 2n^2\hbar^2\omega^2 \qquad (5.9.10)$$

Again the energy (5.9.10)is quantized .

5.10 Travelling Wave Solution

$$- \hbar^2 g_{oo} \frac{\partial^2 \psi}{\partial t^2} - g_{oo}^2 \, m_o c^4 \psi = - c^2 \hbar^2 \nabla^2 \psi$$

$$- \hbar \frac{\partial^2 \psi}{\partial t^2} - g_{oo} \, m_o^2 \, {}_o c^4 \psi = - c^2 \hbar^2 \; g_{oo}^{-1} \nabla^2 \psi \qquad (5.10.1)$$

Consider the solution

$$\psi = A \sin \left(\beta x - \alpha t \right) \qquad (5.10.2)$$

Where :

$g_{oo} = (1 + c_1 v)$ $g_{oo}^{-1} = (1 - c_1 v)$ (5.10.3) Inserting (5.10.2 , 3)in (5.10.1) yields

$$\hbar^2 \alpha^2 \psi - \left(1 + c_1 v \right) m_o^2 \, c^4 \psi = c^2 \hbar^2 \beta^2 \left(1 - c_1 v \right) \psi$$

Rearranging both sides one gets :

$$\left(\hbar^2 \alpha^2 - m_o^2 \, c^4 \right) \psi - c_1 m_o^2 c^4 V \psi = c^2 \hbar^2 \beta^2 \psi - c^2 \hbar^2 \beta^2 c_1 v \psi \qquad (5.10.4)$$

Equating the coefficients of ψ and $v\psi$ yields

$$\hbar^2 \alpha^2 - m_o^2 \, c^4 = c^2 \hbar^2 \beta^2$$

$$\hbar^2\alpha^2 = c^2\hbar^2\beta^2 + m_o^2 c^4 \qquad (5.10.5)$$

The coefficients of $\nu\psi$ yields

$$m_o^2 c^4 = c^2\hbar^2\beta^2$$

$$c^2\hbar^2\beta^2 = m_o^2 c^4 \qquad (5.10.6)$$

Applying periodicity condition i.e.

$$\psi(x+\lambda, t+T) = \psi(x,t)$$

The resulting equation takes the form

$$\text{Sin}\,[\beta(x+\lambda) - \alpha(t+T)] = \sin[\beta x - \alpha t]$$

$$\text{Sin}\,[(\beta x - \alpha t) + (\beta\lambda - \alpha T)] = \sin\,[(\beta\,x - \alpha t) + 2\pi n_1 - 2\pi n_2]$$

$$\beta\lambda = 2\pi n_1 \qquad \alpha T = 2n_2$$

$$\beta = \frac{2\pi n_1}{\lambda} = n_1\kappa \qquad \alpha = \frac{2\pi n}{T} = n_2\,[2\pi f] \quad (5.10.7)$$

$$\alpha = n_2\,\omega \qquad (5.10.8)$$

In view of equations (5.10.5) , (5.10.6) , (5.10.7) , and (5.10.8)

$$\hbar^2\alpha^2 = c^2\,\hbar^2\beta^2 + m_o^2 c^4 = 2m_o^2 c^4 \qquad (5.10.9)$$

$$m_o c^2 = \frac{1}{\sqrt{2}}\hbar\alpha = \frac{1}{\sqrt{2}} n_2 \hbar\omega \qquad (5.10.10)$$

$$c^2\hbar^2\beta^2 = c^2\hbar^2 n_1^2 k^2 \qquad (5.10.11)$$

But from (5.10.6)

$$m_o^2 c^4 = c^2\hbar^2\beta^2 \qquad (5.10.12)$$

Thus from (5.10.11)

$$m_o c = \hbar\beta = n_1\hbar k$$

But since one makes an approximation

$$\nu = m\phi \approx m_o\phi \qquad (5.10.13)$$

Thus

$$E = mc^2 \approx m_o c^2 \qquad (5.10.14)$$

$$E = \frac{1}{\sqrt{2}} n_2 \hbar\omega \quad , \quad n_2 = 0,1,2,\ldots.. \qquad (5.10.15)$$

But

$$E^2 = c^2p^2 + m_o^2 c^4$$

For small m_o , this relation becomes

$$c^2 p^2 = E^2$$

$$cp = E$$

$$P = \frac{E}{c} = \frac{mc^2}{c} = mc \approx m_o c$$

But from equation (5.10.12)

$$m_o c = n_1 \hbar \kappa$$

Thus

$$p = n_1 \hbar \kappa$$

$$n_1 = 0, 1, 2, \ldots \qquad (5.10.16)$$

The momentum is quantized .

5.11 New Version of GSR Energy Formula and GSR Quantum Theory

Recently a paper of A-Zakarea and M. Dirac shows that the mass expression is given according to the energy momentum conservation is given by

$$m = \gamma\, m_o = \frac{m_o}{\sqrt{g_{oo} - \frac{v^2}{c^2}}} \qquad (5.11.1)$$

This appears to be in direct conflict with the expression.

$$m = g_{oo}\,\gamma\, m_o \qquad (5.11.2)$$

But this conflict can be removed by rederiving the expression of energy.

Where :

$$T^{oo} = \gamma\, m_o\, c^2 \qquad (5.11.3)$$

$$E = T_o^{\ o} = g_{oo}T^{oo} = g_{oo}\,\gamma\, m_o c^2 \qquad (5.11.4)$$

This conflict can be removed by lowering the indicies in inflate space by taking

$$E = T_o^o = L_{oo}T^{oo} = 1X\gamma\, m_o c^2 = \frac{m_o}{\sqrt{g_{oo} - \frac{v^2}{c^2}}} \qquad (5.11.5)$$

There for expression (5.11.1) and (5.11.5) are the same .

Recalling equation (5.11.4) one get :

$$E = c^2 m_o \left(g_{oo} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \left(1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} m_o c^2$$

$$E = \left[\frac{m^2 c^4 + 2(m\Phi)\left(m c^2\right) - m^2 v^2}{m^2 c^4}\right] m_o c^2$$

$$m_o c^4 E^{-2} = \frac{\left(E^2 + 2\nu E - P^2 c^2\right)}{E^2}$$

$$E^2 + 2VE = P^2c^2 + m_o^{\,2}\, c^4 \qquad (5.11.6)$$

It is very interesting to note that

When : v= o

Equation (5.11.6) reduces to

$$E^2 = p^2c^2 + m_o c^4 \qquad (5.11.7)$$

Which is the ordinary SR energy momentum relation . The quantum new GSR equation can be obtained by using equations (5.2.4) and (5.2.5) , where

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = E\psi \qquad \imath\hbar \frac{\partial \psi}{\partial t} = E\psi$$

$$-\hbar^2 \nabla^2 \psi = P^2 \psi \qquad (5.11.8)$$

There by multiplying equation (5.11.6) by ψ and substituting (5.11.8) , one gets :

$$E^2\psi + 2VE\psi = c^2P^2\psi + m_o^{\,2}\, c^4\psi \qquad (5.11.9)$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial \tau^2} + 2i\hbar V \frac{\partial \psi}{\partial \tau} = -c^2\hbar^2\nabla^2\psi + m_o^2 c^4 \psi \qquad (5.11.10)$$

5.12 Josephson Effect Current Expression According to New GSR

Consider solution of (5.11.10) in the form

$$\psi(\underline{r},t) = f_o(t)\, e^{\iota\kappa r} \quad (5.12.1)$$

A direct substitution of (5.12.1) in (5.11.10) yields

$$\left[-\hbar^2 \frac{\partial^2 f_o}{\partial t^2} + 2i\hbar r \frac{\partial f_o}{\partial t}\right] e^{ik.r} = C^2\hbar^2 k^2 f_o e^{ik.r} + m_o^2 c^4 f_o e^{ik.r}$$

$$\left[-\hbar^2 \frac{\partial^2 f_o}{\partial t^2} + 2i\hbar r \frac{\partial f_o}{\partial t}\right] = [\, c^2\hbar^2 k^2 + m_o^2 c \;\; m_o^2 c^4 \quad 5.12.2)$$

Consider very small mass and momentum such that

$$C^2\hbar^2 k^2 = P^2 c^2 = 0 \qquad m_o^2 c^4 = 0$$

In this case equation (5.12.2) reads

$$-\hbar^2 \frac{\partial^2 f_o}{\partial t^2} + 2i\hbar V_e \frac{\partial f_o}{\partial t} = 0 \quad (5.12.3)$$

Where V_e is the potential affecting one electron consider the solution

$$f_o = e^{\pm(-\alpha t + \Phi)i}$$

$$f_o = 0 \qquad \frac{\partial f}{\partial t} = \pm i\alpha t \qquad \frac{\partial^2 f}{\partial t^2} = \alpha^2 f \qquad (5.12.4)$$

Substituting

$$\left[\hbar^2\alpha^2 \pm 2\alpha V_e\right] f_o = 0 \qquad \hbar^2\alpha^2 = \pm 2\alpha V_e$$

$$\alpha = \pm\frac{2V_e}{\hbar} \qquad (5.12.5)$$

The general solution is a super position of solution in equation (5.12.4) , i.e.

$$f = D_1 e^{i(\phi - \alpha t)} + D_2\, e^{-i(\phi - \alpha t)} \qquad (5.12.6)$$

The current is given by

$$I = \frac{dQ}{dt} = e\frac{dn}{\partial t} = e\frac{d|\psi|^2}{dt} = e\frac{d\psi}{dt}\bar{\psi}$$

$$I = e\,\frac{df}{dt}\bar{f} \qquad (5.12.7)$$

When no potential is applied and when no separation is made by an in sulator between the super conductor sides

$$f = 0 \qquad V_e = p \qquad \alpha = 0 \quad (5.12.8)$$

Since no current flows .

Thus

According to(5.12. 7) one of the possible solutions is to set

$F= 0$ (5.12.9)

In view of (5.12.8 ,1 ,11) one gets :

$$0=(D_1+D_2)\ e^{i\phi}$$

Thus

$D = D_1= - D_2$ (5.12.10) Hence the solution will be according to equation (5.12.6) in the form

$$f=D\ [\ e^{i\,(\Phi-\alpha t\,)}\ -\ e^{-i\,(\Phi-\alpha t\,)}\ ] = D\ [e^{i\,\theta}\ -\ e^{-i\,\theta}\ ]$$

$$f =2\ i\ D \sin (\ \Phi-\alpha t\ ) \qquad (\ 5.12.11\ )$$

where $\theta\ = \phi - \alpha t$ (5.12.12)

$$V = 2e(½V_o) = eV_o \qquad (\ 5.12.13\ )$$

The total potential of the cooper electron pair is also double that of a single electron .

Thus

$$V =2V_e \qquad (\ 5.12.14\ )$$

Thus α in equation (5.12.5)

$$\alpha = \frac{V}{\hbar} = \frac{2V_e}{\hbar} \qquad (5.12.15)$$

The electric current can be obtained by inserting (5.12.11) in (5.12.7) to get :

$$I = e\frac{d}{dt}[4(i)(-i)D^2 \sin^2(\phi - \alpha t)]$$

$$= 8eD^2 \sin(\phi - \alpha t)\,[-\alpha \cos(\phi - \alpha t)]$$

$$I = 8\,e^2 \alpha\, D^2 \sin(\phi - \alpha t)\cos(\phi - \alpha t)$$

$$= 4e^2 \alpha\, D^2 \sin 2(\phi - \alpha t)$$

$$I = D_o \sin(\phi_o - 2\alpha t) \qquad (5.12.16)$$

$$\phi_o = 2\phi \qquad (5.12.17)$$

where one uses the relation

$$\text{Sin } 2\theta = 2 \sin\theta \cos\theta$$

To simplify the expression of I .

To find the relation between k and the frequency , one use the periodicity condition , i.e.

$$f(t+T) = f(t) \qquad (5.12.18)$$

Thus from (5.12.11)

$$\sin\left[\phi - \alpha(t + \mathrm{T})\right] = \sin\left[\phi - \alpha t\right]$$

$$\sin\left[(\phi - \alpha t) - \alpha \mathrm{T}\right] = \sin(\phi - \alpha t)\cos\alpha\mathrm{T} + \cos(\phi - \alpha t)\sin\alpha\mathrm{T} = \sin(\phi - \alpha t)$$

This can be satisfied if

$$\cos\alpha\mathrm{T} = 1 \;\Rightarrow\; \sin\alpha\mathrm{T} = 0 \qquad (5.12.19)$$

Which is satisfied by setting

$$\alpha\mathrm{T} = 2\mathrm{n}\pi$$

$$\alpha = \frac{2\mathrm{n}\pi}{\mathrm{T}} = 2\mathrm{n}\pi \mathrm{f} = \mathrm{n}\,\omega \qquad (5.12.20)$$

According to equations (5.12.14) and (5.12.15)

The super current is given by

$$\mathrm{I} = \mathrm{D_o}\sin\left(\phi_o - \frac{\mathrm{e}v_o t}{\hbar}\right) \qquad (15.12.21)$$

How ever periodicity

$$\mathrm{I}\,(t+\mathrm{T}) = \mathrm{I}\,(t) \qquad (5.12.22)$$

According to equation (5.12.16)

$$\mathrm{Sin}\left[\phi_o - 2\alpha(t + \mathrm{T})\right] = \sin\left[\phi - 2\alpha t\right]$$

$$\mathrm{Sin}\left[\phi - 2\alpha t\right]\cos\alpha\mathrm{T} - \cos\left[\phi_o - 2\alpha t\right]\sin 2\alpha\mathrm{T} = 0$$

Thus requires

$$\cos 2\alpha\mathrm{T} = 1 \quad \sin 2\alpha\mathrm{T} = 0 \qquad (5.12.23)$$

$$2\alpha T = 2n\pi$$

$$\alpha = \frac{n\pi}{T} = n\pi \text{ f F} \qquad (5.12.24)$$

If one choose n to by unity then

$$n=1 \qquad \alpha = \pi \text{ f} \qquad (5.12.25)$$

Thus equation (5.12.16) becomes

$$I=D_o \sin\left(\phi_o - 2\pi ft\right) \qquad (5.12.26)$$

Comparing equations (5.12.21) and (5.12.26)

$$2\pi \text{ f} = \frac{2e\nu_o}{\hbar}$$

Hence

$$f=\frac{2e\nu_o}{\hbar}$$

It is interesting to note that expression (5.12.21 , 26 , 27) for super current frequency are completely consistent with Josephson super current formula or expression .

5.13 discussion

Anew quantum GSR equation was obtained as shown by equation (5.2.9) unlike Schrodinger it is second order in t and consists of terms representing rest mass energy . This is equation is also unlike relativistic equations it consists of terms standing for potential of any field . This equation (5.2.9) also reduces to Klein – Gordon equation (5.2.10) when the potential vanishes .

It is also very striking to note that time independent new quantum equation (5.3.6) reduces to ordinary time independent Schrodinger equation (5.3.13) when one neglects rest mass energy and when relativistic energy is replaced by Newtonian energy in equation (5.3.11) .

Different expression for this new quantum GSR equations were obtained as shown by equations (5.4.9) , (5.5.6) , (5.6.6) and (5.7.4) .

These equations are different due to the spatial and time dependence of the potential . Using (5.6.6) for time potential , the equation was solved for harmonic Oscillator assuming very small displacements , one obtained quantized energy expression (5.6.7) . This expression shows that the rest mass m_o is quantized .It is very interesting to note that according to equation (5.6.7) , by adjusting the quantum number n and frequency ω , one can predict the mass of any elementary particle which can be considered as a vibrating string .

Another approach based on the full quantum equation (5.6.2) and time independent potential predicts a travelling wave solution for harmonic

Oscillator in equation (5.9.2) . Thus this equation can describe a photon . According to equations (5.9.14) and (5.9.15)

$m_{\circ\circ} \mathbf{c}^2 = n_2 \hbar \omega \sqrt{2}$

Again the mass is quantized and can predict any elementary particle mass .

A second new quantum GSR equation based on an alternative GSR energy expression (5.10.5) was obtained . This new equation is shown in equation (5.10.10) .

When solving it for super conductors its predicts Josephson effect by using simple mathematics as in equation (5.11.26) .

5.14 Conclusion

The new quantum GSR equation is more general than Schrodinger and relativistic equations .

Since it reduces to time independent Schrodinger equation and to Klein – Gordon equation .

It can also describe photon behavior as for as it predict travelling wave solution . it also predict Josephson equation by using simple mathematics .

5.15 Recommendation

1 . It is essential to extend this equation to be reducable to time dependent Schrodinger equation and Dirac equation .

2 . This equation should be applied to describe super conducting materials at high temperature .

3 . The photon equation need to be used to describe interaction of electromagnetic radiation with matter .

4 . This equation can be applied to describe elementary particles interactions .

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