Chapter 1

Introduction

1.1 Quantum mechanics and matter

Atoms and elementary particles are described by quantum mechanical laws[1,2]. Quantum laws succeeded in describing the atomic spectra and electronic structure of atoms[3], and the behaviour of single isolated atom. However, the description of behavior of the bulk matter, which consists of a large number of interacting atoms, suffers from noticeable setbacks [4].For example the behaviour of high temperature super conductors (SC) cannot be described within the framework of ordinary quantum mechanics [5]. This may be related to the fact that SC are characterized by zero resistance and absence of friction[6].

Quantum Laws are also unable to quantize gravitational field which is described by general relativity [7].This means that there is a need for relativistic quantum theory recognizing the frictional medium, thus it can hopefully quantize gravity and describe high temperature SC.

Einstein special relativity (SR) is one of the biggest achievements that make radical modification to the concepts of space and time. It states that the space-time interval between two events is no longer constant for frames of references moving with constant velocity with respect to each other. As far as the conservation of energy and momentum results from the invariance of space and time, thus it is quite natural to expect SR new concepts to have a direct impact on energy momentum expression [8, 9, and 10]. The relativity of time, space, mass, and energy were verified and confirmed experimentally, these experiments comes as an ultimate reward confirming the key predictions of SR.

Despite these remarkable successes of SR, it suffers from noticeable Setbacks. For instance SR does not satisfy the correspondence principle in the sense that the expression of energy for SR does not reduced to the conventional Newtonian one. This is since the SR energy expression has no expression sensitive to the potential energy [11, 12, and 13].

Lack of SR from a term taking care and feeling the existence of potential energy is in direct conflict with experiments and common sense. For instance, according to SR two particles moving with same velocity, one is in free space, and the other is in gravity field, have the same energy. of course experiments and common sense shows that their energy are different[14,15,16].

This defect encourages some researchers to propose a generalized version of SR, called generalized SR (GSR). This new GSR has an energy term representing a potential energy and satisfies a Newtonian limit [17, 18, and 19]. This success of GSR encourages using the conventional expression of kinetic and potential energy to see how energy conservation looks like in SR and GSR. This task is done in sections. Section 3 and 4 were devoted for discussion and conclusion.

1.2 Research Problem:

Quantum laws cannot easily describe the behavior of particles exiting in bulk matter. The approach used on collision is complex and is incapable of describing many physical phenomena associated with matter interactions. Quantum Lows are also not sensitive to systems having friction, potential and mass at the same time.

1.3 literature review:

A wide variety of materials have mechanical friction. This friction plays an important role in determining the mechanical properties and the electrical properties of the matter .The most popular physical theory that is used to describe the physical properties of matter is quantum mechanics. Recently quantum laws found to be incapable of describing the behavior of some new materials like super conductors and Nano materials[20,21,22]. This may be attributed to the fact that quantum laws have no terms sensitive to friction. Some work was done to derive Schrodinger quantum equation having frictional term [23,24,25]. This equation is used to solve the problem of particle in a box by some researches[26]. The solution shows quantized frictional energy.

In Mineral Exploration Quantum mechanical technique are used. There are many spectral techniques used for identification of elements. Unfortunately these techniques are complex and expensive. There is a need for simple technique for exploration. Some works utilizes simple technique based on electrical conductivity[27,28]. The experimental work shows variation of conductivity with frequency, with line shape similar to absorption line .There is a minimum frequency for each element, which can be used as a finger print characterizing it .Fortunately this conductivity –frequency relation can be explained on the basis of quantum and statistical Physics.

The effective mass Quantum laws are also used to find the effective mass can be related to the wave vector in parabolic or tensor or even in matrix-valued function of wave vector. Some forms are presented in more complex details than the band structure of mass itself. In other aspect one can look at electron-hole, which is attracted to each other by the well-known Coulomb force, as a (exaction), which is very obvious an electrically neutral quasi particle. In this work done by Toum the effective mass role is treated in a form of theoretical method based on classical mechanics, bearing in mind Maxwell equations. The quantum mechanics is utilized also to find the effective mass value. The work also handled the effective mass electromagnetic theory, in both the absence and presence of the binding Energy. Schrödinger equation took an important part in the work where the equation is generalized to present the effective mass in another new form .Lastly very noticeable theoretical relation between the effective mass and the wave vector is obtained, by using generalized special relativity [29]

In Amal work, the Alum material was grinded for different time's to display the spectrum of natural light and laser diode. The dimensions of the bodies were measured by scanner (easy scan microscope). A relationship was found between the change of particle Nano size and the radiation intensity of light. These empirical relations were found to be explained according to Schrodinger equation and Quantum Dots on the basis of tight binding approximation[30].

Kamil and Dirar[31] also proposed also a new derivation for relativity and Klein- Gordon Equation. Maxwell’s equation for electric field was used to derive Einstein energy-momentum relation. This was done by using Plank photon energy relation beside wave solution in insulating no charged matter. Klein-Gordon quantum equation was also derived from the same Maxwell’s equation by utilizing resemblance between electric field vector and wave function in the intensity expression.

A quantum model based on Plasma equation was also suggestion by Rasha and Dirac[32]. They apply it in super conductivity.

Superconductivity is one of the most important phenomena in solid state physics. Its theoretical framework at low critical temperatureT_C is based on Bardeen, Cooper and Schrieffer theory (BCS). But at high T_C above 135, this theory suffers from some setbacks. It cannot explain how the resistivity abruptly drops to zero belowT_C, besides the explanation of the so called pseudo gap, isotope and pressure effect, in addition to the phase transition from insulating to super-conductivity state. The models proposed to cure this drawback are mainly based on Hubbard model which has a mathematical complex framework. In this work a model based on quantum mechanics besides generalized special relativity and plasma physics. It is utilized to get new modified Schrodinger equation sensitive to temperature. An expression for quantum resistance is also obtained which shows existence of critical temperature beyond which the resistance drops to zero. It gives an expression which shows the relation between the energy gap and T_c. These expressions are mathematically simple and are in conformity with experimental results.

A lot of work was done to modify or derive new quantum Equations[33,34,35]. These equation aims to describe bulk matter and elementary particles behavior [36,37,38]

1.4 Aim of the work:

The aim of this work is utilize uncertainty principle to find friction energy. Then one finds also special relativistic energy relation for conserved system to find new a quantum mechanical law.

1.5 Thesis Layout:

This thesis consists of 4 Chapters. Chapter 1 is the introduction. Chapters 2 and 3 are devoted for relativistic quantum equations and literature review. The contribution is done in chapter 4.

Chapter 2

Relativistic Quantum Equation

2.1 Introduction:

This chapter is concerned with basics of Q.M. Beside SR energy relation. The Klein-Gordon and Dirac equations are also presented.

2.2 Relativistic time and length:

From the Lorentz transformation one can derive the correct transformation equation between two inertial frames in special relativity, which modify the Galilean transformation. We consisted two inertial frames S and $\bar{S}$ which hane a relative velocity V between them along x-axis:

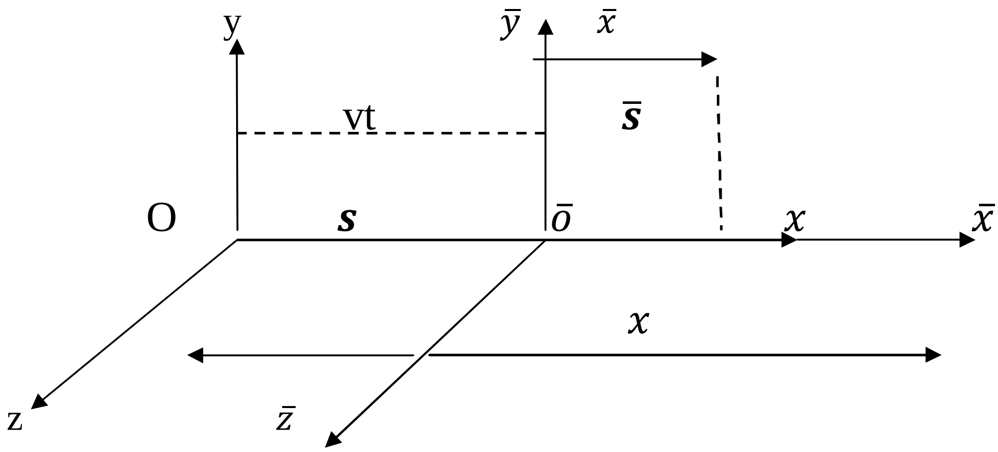


Figure (2.2.1): the reference frames sounds.

Now suppose that there is a single flash at the origin of s and $\bar{s}$ at timeτ, when the two inertial frames happen to coincide the outgoing light wave will be spherical in shape moving outward with a velocity c in both s and $\bar{s}$ by Einstein's second postulate [39,40] from this figure

$$y = \bar{y} \quad , \quad z = \bar{z}$$

Consider the outgoing light wave along the x-axis (y=z=0)

The relation between positions in s and $\bar{s}$ is giving by:

$$x^{-} = \gamma(x - vt) \qquad (2.2.1)$$

$$x = \gamma(x^{-} + vt^{-}) \qquad (2.2.2)$$

Consider a light source emitted photons with speed c. in this case:

$$x^{-} = ct^{-} \qquad x = ct \qquad (2.2.3)$$

A direct substance of (2.2.3) in (2.2.2) yields:

$$ct^{-} = \gamma(ct - vt) = \gamma c\left(1 - \frac{v}{c}\right)t \qquad (2.2.4)$$

$$ct = \gamma(ct^{-} + vt^{-}) = \gamma c\left(1 + \frac{v}{c}\right)t^{-} \qquad (2.2.5)$$

Inserting (2.2.4) in (2.2.5) yields:

$$t = \gamma^2\left[1 + \frac{v}{c}\right]\left[1 - \frac{v}{c}\right]t$$

Thus:

$$\gamma = \frac{1}{\sqrt{1 - {v^2}/{c^2}}} \qquad (2.2.6)$$

In this R is replaced by γ to be consistent with GSR notation the mathematical problem here is to find the relationships of $\bar{x}$ and $\bar{t}$ in terms of x and t the results are the new well known, Lorentz transformation L_e

$$\bar{x} = \frac{x - VT}{\sqrt{1 - \left({v^2}/{c^2}\right)}} = \frac{x - vt}{\sqrt{1 - \beta^2}} = \gamma(x - vt) \qquad (2.2.7)$$

$$\bar{t} = \frac{t-\left(v/_{c^2}\right)x}{\sqrt{1-\left(v^2/_{c^2}\right)}} = \frac{t-\left(v/_{c^2}\right)x}{\sqrt{1-\beta^2}} \tag{2.2.8}$$

$$\bar{t} = \gamma\left(t - \frac{\beta}{c}x\right) \tag{2.2.9}$$

We may also obtain the inverse transformations (from system $\bar{s}$ to s) by replacing v by $-v$ and simply interchanging primed and unprimed coordinates [41,42,43]

$$x = \frac{\bar{x} - v\bar{t}}{\sqrt{1-\left(v^2/_{c^2}\right)}} = \frac{\bar{x} - v\bar{t}}{\sqrt{1-\beta^2}} = \gamma(\bar{x} + v\bar{t}) \tag{2.2.11}$$

$$t = \frac{\bar{t} + \left(v/_{c^2}\right)\bar{x}}{\sqrt{1-\left(v^2/_{c^2}\right)}} = \frac{\bar{t} + \left(v/_{c^2}\right)\bar{x}}{\sqrt{1-\beta^2}} = \gamma\left(\bar{t} + \frac{\beta}{c}\bar{x}\right) \tag{2.2.12}$$

2.3 Relativity of length:

Consider two observers, one on the fixed coordinate system S and the other on the moving system $\bar{S}$ observing the length of a rod. If the two systems are initially at rest and coincided the 60th will measure the same length L, orL_0. The observer on s will express the length as

$L = x_2 - x_1$ .And on the $\bar{S}$ will read it as $L_0 = \bar{x}_2 - \bar{x}_1$

From the Lorentz transformation:

$$\bar{x}_2 = \gamma(x_2 - vt_2) \tag{2.3.1}$$

$$\bar{x}_1 = \gamma(x_1 - vt_1) \tag{2.3.2}$$

$$\bar{x}_2 - \bar{x}_1 = \gamma[(x_2 - x_1) - v(t_2 - t_1)] \tag{2.3.3}$$

Since the observer on S will measure both ends of rod at the same time$t_2 = t_1$, so equation (2.3.3) gives:

$$x_2 = x_1 = \frac{1}{\gamma}(\bar{x}_2 - \bar{x}_1) \quad (2.3.4)$$

Where

$$L_0 = \bar{x}_2 - \bar{x}_1 \quad (2.3.5)$$

$$\frac{1}{\gamma} = \sqrt{1 - \beta^2} \quad (2.3.6)$$

And $x_2 - x_1 = L$ is the Length as measured by the observer on S. thus we have:

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \beta^2} \quad (2.3.7)$$

Which simply says that length appears to shrink as observed forms, as it speeds up along its length[44].

2.4 Relativity of mass:

Assume the two particles A and B moving opposite to each other as shown in Fig (2.4.1). and fig (2.4.2)

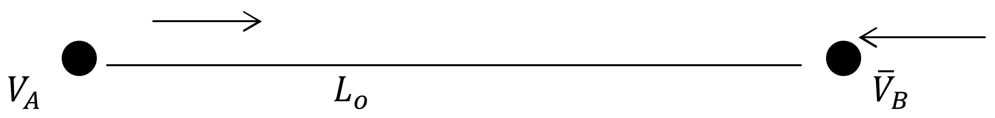


Figure (2.4.1): particles moving opposite to each other

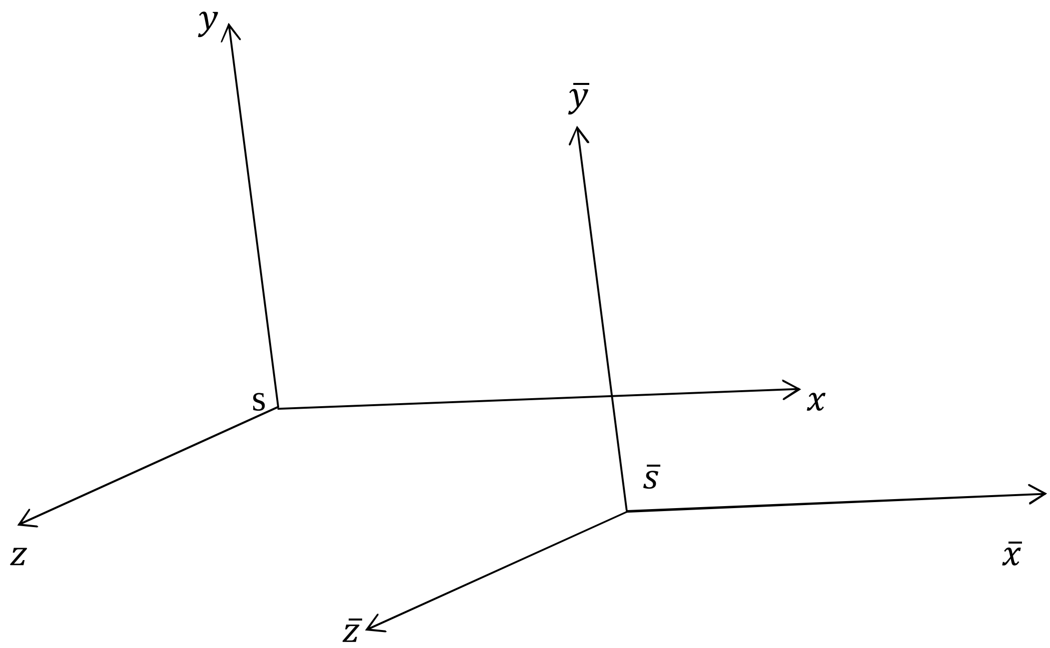


Figure (2.4.2): the frames s and s

Assume that their velocity are equal in magnifies

$$V_A = \bar{V}_B \tag{2.4.1}$$

For particle A in frame S the time is given by

$$t_o = \frac{L_o}{V_A} \tag{2.4.2}$$

whereL_o is the distance between A and B The time for the particle B is given by:

$$t_o = \frac{L_o}{V_B} \tag{2.4.3}$$

From the law of conservation of momentum before collision and after collision:

$$-m_A V_A + m_B V_B = m_A V_A - m_B V_B$$

$$m_A V_A = m_B V_B \tag{2.4.4}$$

The particle velocity B in S frame is given by

$$V_B = \frac{L_o}{t} \tag{2.4.5}$$

But the time in frame s and $\bar{s}$ are related by

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.4.6}$$

From equation (2.4.6) one gets:

$$V_B = \frac{L_o\sqrt{1 - \frac{v^2}{c^2}}}{t_o} \tag{2.4.7}$$

Let:

$$V_A = \frac{\mathrm{L_o}}{\mathrm{t_o}} \tag{2.4.8}$$

From the equations (2.4.4), (2.4.7) and (2.4.8):

We get:

$$m_A = m_B\sqrt{1 - \frac{v^2}{c^2}} \tag{2.4.9}$$

Since the observer is at rest in frame A, thus

$$m_A = m_0$$

$$m_B = m$$

$$m_o = m\sqrt{1 - \frac{v^2}{c^2}}$$

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.4.9)$$

This means that Particle mass increase with the increased speed.

2.5 Relativistic Energy:

According to Newton Second Law the energy gained by a particle of mass:

$$E = \int F.dr = \int \frac{d(mv^2)}{dt} dx = \int_A^B d(mv) \frac{dx}{dt} = \int_A^B vd(mv)$$

$$= \int_A^B vd(mv^2) - \int_1^2 vmd\,v$$

$$= mv^2 \Big]_A^B - \int_1^2 \frac{m_o v dv}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2.5.1)$$

Let:

$$v = c\cos\theta \qquad dv = c\sin\theta d\theta \qquad (2.5.2)$$

Hence:

$$\int \frac{m_o v dv}{\sqrt{1 - \frac{v^2}{c^2}}} = -c^2 \int \frac{m_o \cos\theta \sin\theta \, d\theta}{\sqrt{1 - \cos^2\theta}}$$

$$= -c^2 m_o \int \frac{\cos\theta \sin\theta \, d\theta}{\sin\theta} = -m_o c^2 \int \cos\theta d\theta$$

$$= -m_o c^2 \int d\sin\theta = -m_o c^2 \sin\theta$$

$$= -m_o c^2 \sqrt{1 - \cos^2\theta} = -m_o c^2 \sqrt{1 - \frac{v^2}{c^2}} \qquad (2.5.3)$$

Combining Equations (2.5.1) and (2.5.3) yields:

$$E = mv^2 + \frac{c^2 m_o \left(1 - \frac{v^2}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = mv^2 + mc^2 - mv^2 = mc^2 \qquad (2.5.4)$$

Which is the SR Expression of Energy.

2.6 Energy - momentum Relation:

Any relativistic generalization of Newtonian second Law must satisfied two criterias:

1- Relativistic momentum must be conserved in all frames of reference.

2- Relativistic momentum must reduce to Newtonian momentum at law speed.

The first criterion must be satisfied in order to satisfy Einstein's first postulate while second criteria must be satisfied as its knew that Newtonian's law are:

Correct at sufficiently low speeds. A definition for the relativistic momentum of particle moving with velocity.

And form the equation (2.5.12).

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0 c^2}{\sqrt{\frac{m^2c^4-m^2v^2c^2}{m^2c^4}}} \quad (2.6.1)$$

$$E = \frac{m_0 c^2}{\sqrt{\frac{E^2-P^2c^2}{E^2}}} = \frac{m_0 c^2 E}{\sqrt{E^2-P^2c^2}} \quad (2.6.2)$$

$$\sqrt{E^2-P^2c^2} = m_0c^2, \qquad E^2-P^2c^2 = {m_0}^2c^4$$

$$E^2 = P^2c^2 + {m_0}^2c^4 \quad (2.6.3)$$

2.7 Wave Particle Duality:

Classical physics treat light as a wave for a long time the discovery of the particle of each nature or light was made by Blank. The energy particle is:

$$E = hf = \hbar w \tag{2.7.1}$$

Where $\hbar$ is called Blank constant and f is the frequency this dual nature of light encourages De Broglie to suggest that particles also behaves as waves with wave length λ related to the momentum P as:

De Broglie hypotheses

$$P = \frac{h}{\lambda} = \hbar k \tag{2.7.2}$$

This dual nature encourages scientists to suggest that particles can considered as wave groups with wave function:

$$\Psi = Ae^{i(kx-wt)} \tag{2.7.3}$$

K and w are wave number and angular frequency respectively. Using equations (2.2.1, 2) yields:

$$\Psi = Ae^{\frac{i}{\hbar}(Px-Et)} \tag{2.7.4}$$

2.8 Uncertainty Relation:

The wave packet representation requires that it is impossible to measure position and momentum exactly at the same time the position of the particle is uncertain by Δx and the uncertainty of momentum is Δp.

The uncertainty relation can be derived from the wave backer picture.

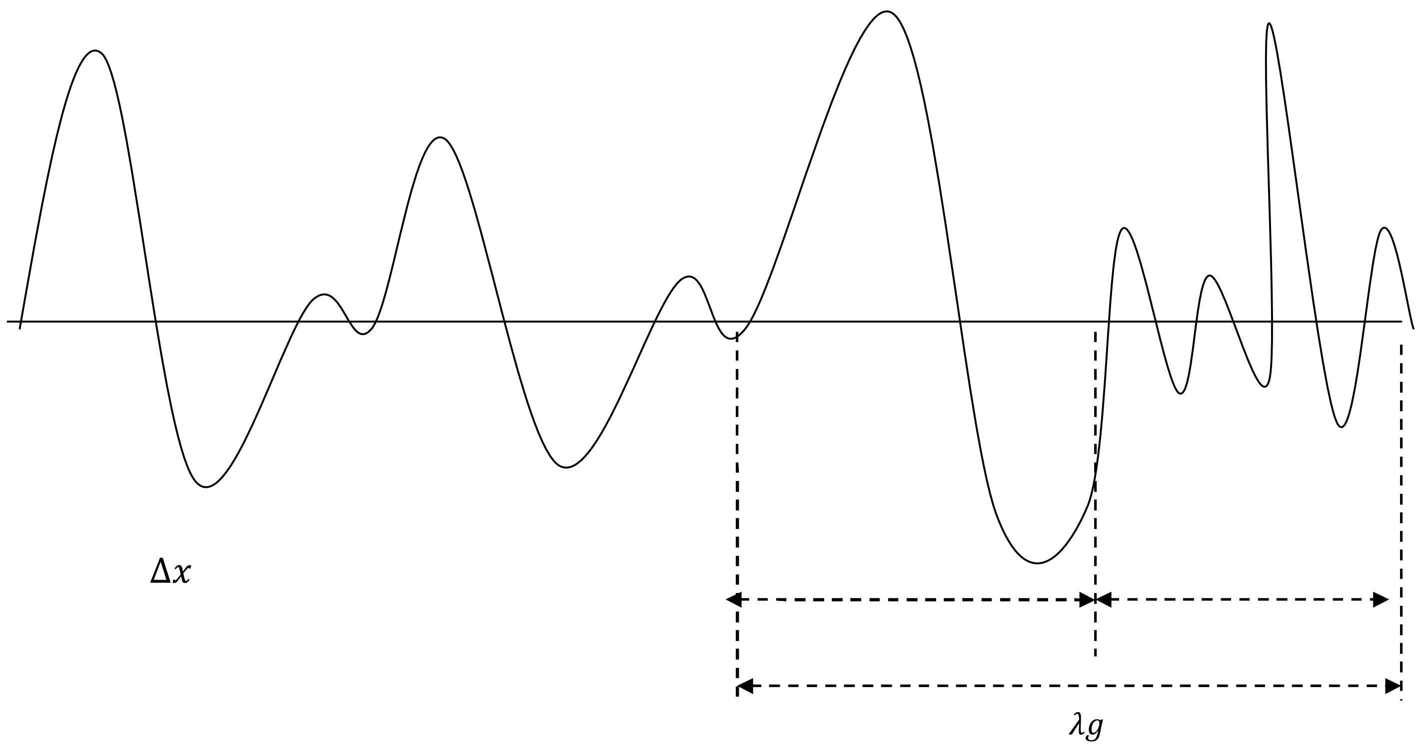


Fig. (2.8.1): wave length and position uncertainty for a wave basket

In view of Fig (2.2.1) the wave length λg of the wave backet is related to the uncertainty in position according to the relation:

$$\lambda g = 2\Delta x \tag{2.8.1}$$

But the wave length is related to the wave number according to the relation:

$$\lambda g = \frac{2\pi}{kg} \tag{2.8.2}$$

Using equations (2.8.1, 2) yields:

$$2\Delta x = \frac{2\pi}{kg} \qquad \Delta x . kg = \pi \tag{2.8.3}$$

But

$$kg = \frac{\Delta k}{2} \qquad k = \frac{2\pi}{\lambda} \tag{2.8.4}$$

From De Broglie equation (2.7.2):

$$P = \frac{h}{\lambda} = \frac{2\pi h}{\lambda 2\pi} = \frac{kh}{2\pi} = \hbar k$$

$$\Delta p = \Delta k \frac{h}{2\pi} = \hbar \Delta k \tag{2.8.5}$$

Hence:

$$\Delta k = \frac{2\pi}{h} \Delta p \tag{2.8.6}$$

Thus from (2.8.3), (2.8.4) and (2.8.5)

$$\Delta x . \frac{2\pi}{h} . \Delta p = 2\pi \tag{2.8.6}$$

Thus the position and momentum uncertainty relation is given by:

$$\Delta x . \Delta p = h \tag{2.8.7}$$

2.9 Relativistic Energy Relation

According to SR the energy equation is given by:

$$E = mc^2 \tag{2.9.1}$$

Where m is the mass and c is the speed of light in vacuum. For a photon, max plank propose that:

$$E = hf \tag{2.9.2}$$

Where h is the Blank constant and f is the frequencies. Comparing equations (2.9.1) and(2.9.2)

$$E = mc^2 = hf \quad \Rightarrow \quad m = \frac{hf}{c^2}$$

Thus the momentum is given by:

$$mc = \frac{hf}{c} = p = mc = \frac{hf}{c} \qquad (2.9.3)$$

But from (2.9.2) and the fact that:

$$c = f\lambda$$

$$E = hf = \frac{hc}{f} \qquad (2.9.4)$$

Hence from (2.9.4, 3, 1)

$$P = mc = \frac{mc^2}{c} = \frac{E}{c} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} \qquad (2.9.5)$$

2.10 Klein-Gordon Equation

Klein and Gordon equation is based on the dual nature of particles as well as SR energy-momentum relation from equation (2.2.4)

$$\Psi = Ae^{\frac{i}{\hbar}(Px-Et)} \qquad (2.10.1)$$

The energy and momentum in SR are related according to the relation:

$$E^2 = c^2P^2 + {m_o}^2c^4 \qquad (2.10.2)$$

Multiplying both sides by Ψ yieds:

$$E^2\Psi = c^2P^2\Psi + {m_o}^2c^4\Psi \qquad (2.10.3)$$

Using equation (2.10.1) yields:

$$i\hbar\frac{\partial\Psi}{\partial t} = E\Psi \qquad , -\hbar^2\frac{\partial^2\Psi}{\partial t^2} = E^2\Psi$$

$$\frac{\hbar}{i}\nabla\Psi = \frac{\hbar}{i}\frac{\partial\Psi}{\partial x} = \mathrm{P}\Psi \qquad , -\hbar^2\nabla^2\Psi = P^2\Psi \qquad (2.10.4)$$

Interesting equation (2.10.4) in (2.10.3) yields:

$$-\hbar^2\frac{\partial^2\Psi}{\partial t^2} = -c^2\hbar^2\nabla^2\Psi + {m_o}^2c^4\Psi \qquad (2.10.5)$$

Which is the Klein-Gordon equation.

2.11 Dirac Equation:

Dirac equation is based mainly on the assumption that E and P in SR are related linearly to obey:

$$E = c\alpha.P + \beta m_o c^2 \qquad (2.11.1)$$

Multiplying both sides by Ψ yields:

$$E\Psi = c\alpha.P\Psi + \beta m_o c^2\Psi \qquad (2.11.2)$$

Using equation (2.10.4) gives Dirac relativistic equation in the form:

This equation is the linear quantum relativistic equation. The terms α and β are matrices.

Chapter 3

Literature Review

3.1 Introduction:

Different attempts were made to conduct quantum model accounting for the effect of friction and collision on particles moving in a bulk matter. This chapter coats some of them.

3.2 Schrodinger Equation in Presence of Thermal and Resistive Energy:

In the work done by Sawsan, Dirar and others, new energy relations was made.

The energy of ordinary Schrödinger equation includes kinetic and potential energy .However, there are other energy types which should be considered, for example the energy lost E by friction for oscillating system which is given by[46]

$$\int F_r dx \tag{3.2.1}$$

The thermal energy is given in terms of temperature and Boltzman constant k as:

$$E_T = kT \tag{3.2.2}$$

Where there is no room in ordinary conventional Schrödinger equation for feeling the effect of friction and heating does not recognize these energy types.

To incorporate energy considers the plasma equation:

$$mn\left(\frac{\partial v}{\partial t} + v.\nabla v\right) = F - \nabla P - F_r \qquad (3.2.3)$$

Where P stands for the force, thermal pressure.

The resistive force is given by

$$F_r = \frac{nmv}{\tau} \qquad (3.2.4)$$

Suggesting the displacement to be

$$x = x_0 e^{-i\omega t}$$

$$\frac{\partial x}{\partial t} = x_0 e^{-i\omega t} = -i\omega x$$

$$v = -\omega x \qquad (3.2.5)$$

If follows that

$$F_r = -i\frac{nm\omega x}{\tau} \qquad (3.2.6)$$

Since v is function of t and x, it follows that

$$dv = \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial x}dx$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial x}dx$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial t}dt + v.\nabla v \qquad (3.2.7)$$

Thus by using equation (3.2.7) and equation (3.2.3) reduces to

$$mn\frac{dv}{dt} = F - \nabla P - F_r \qquad (3.2.8)$$

If the field potential per particle is given by v, then the field force F takes the form

$$F = -\frac{\partial(nv)}{\partial x} \tag{3.2.9}$$

The left hand side of equation (3.2.8) is given by:

$$mn\frac{dv}{dt} = mn\frac{dv}{dx}\frac{dx}{dt} = mnv\frac{dv}{dx}$$

The pressure is given in turn to be:

$$\nabla P = \frac{\partial p}{\partial x} = \frac{dp}{dx} = \frac{d(nkT)}{dx} \tag{3.2.10}$$

Where the thermal pressure takes the form

$$P = nKT \tag{3.2.11}$$

Then equation (3.2.7) can be re expressed with the aid of equations (3.2.1) and (3.2.9) to be

$$\frac{dmnv^2}{dx} = -\frac{d(nv)}{dx} - \frac{d(nKT)}{dx} + i\frac{m\omega xn}{\tau}$$

Thus

$$\frac{d}{dx}\left(\frac{1}{2}mnv^2 + nv + nKT\right) = i\frac{m\omega n}{\tau}\int xdx$$

$$\left(\frac{1}{2}mnv^2 + nV + nKT\right) - i\frac{m\omega nv^2}{2\tau} = C_0$$

$$(nE_{kin} + nv + nKT) - i\frac{m\omega nx^2}{2\tau} = C_0 \tag{3.2.12}$$

The left hand side is a constant of motion and has a dimension of energy. Thus one can define the total energy F_T to be

$$F_T = nE = n\left(\left(\frac{1}{2}mV^2 + nv + nKT\right) - \frac{m\omega A^2}{2\tau}\right) \quad (3.2.13)$$

But

$$\frac{1}{2}mV^2 = \frac{m^2V^2}{2m} = \frac{P^2}{2m} \quad (3.2.14)$$

Thus the total energy of one particle can be rewritten in the form:

$$E = \frac{P^2}{2m} + v + kT - i\frac{m\omega x^2}{2\tau} \quad (3.2.15)$$

This expression stands for the total energy of a single particle, which consists beside kinetic energy, additional terms. The third term represents the thermal energy, while the forth term stands for the frictional energy. To derive Schrödinger equation, for this new energy expression, equation (3.2.14) must be multiplied by the wave function $\psi$to get

$$E\psi = \frac{P^2}{2m}\psi + V\psi + kT\psi - i\frac{m\omega x^2}{2\tau}\psi \quad (3.2.16)$$

The wave function for a free particle is given by

$$\psi = Ae^{-\frac{i}{\hbar}(Px-Et)} \quad (3.2.17)$$

Differentiating $\psi$with respect to t and x yields

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar}p\psi$$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{p^2}{\hbar^2}\psi \tag{3.2.18}$$

Substitute equation (3.2.18) in equation (3.2.16) yields

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2 v^2}{2m}\psi + v\psi + kT\psi - i\frac{m\omega x^2}{2\tau}\psi \tag{3.2.19}$$

If one rewrite the frictional term in the form

$$E_r = i\frac{m\omega A^2}{2\tau} = i\frac{m\omega A^2}{2\tau\omega} = i\frac{m^2V^2}{2\tau m\omega} = i\frac{m^2V^2\hbar}{2\tau m\hbar\omega}$$

$$= i\frac{m\omega v^2}{2\tau m\hbar\omega} = i\frac{\hbar P^2}{2\tau m^2c^2} = i\frac{\hbar}{2\tau m^2c^2}P^2 \tag{3.2.20}$$

Where:

$$\omega\hbar = mc^2$$

And equation (4) gives:

$$v^2 = |v^2| = v.v^* = (-i\omega x)(i\omega x) = \omega x^2$$

Substitute equation (3.2.5) in equation (3.2.14) to get

$$E\psi = \frac{P^2}{2m}\psi + V\psi - i\frac{\hbar}{2\tau m^2c^2}P^2\psi \tag{3.2.21}$$

Substitute equation (3.2.5) in equation (3.2.14) to get

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + kT\psi - i\frac{\hbar^3}{2\tau m^2c^2}\frac{\partial^2\psi}{\partial x^2} + v\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + kT\psi - i\frac{\hbar^3}{2\tau m^2c^2}\nabla^2\psi + v\psi \tag{3.2.22}$$

This is the Schrödinger equation for thermal resistive medium.

Particle in box:

The motion of the particle in box can be described by Schrodinger Equation. The potential per particle is given by

$$V = 0 \tag{3.2.23}$$

Substituting equation (3.2.23) in equation (3.2.22) yields

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + kT\psi - i\frac{\hbar^3}{2\tau m^2c^2}\nabla^2\psi \tag{3.2.24}$$

Consider a solution of the form

$$\psi = e^{-i\omega t}u \tag{3.2.25}$$

Inserting equation (3.2.25) in (3.2.24) yields:

$$\hbar\omega e^{-i\omega t}u$$

$$= -\frac{\hbar^2}{2m}\nabla^2 e^{-i\omega t}u + kTe^{-i\omega t}u$$

$$- i\frac{\hbar^3}{2\tau m^2c^2}\nabla^2\psi e^{-i\omega t}u \tag{3.2.26}$$

By considering

$$E = \hbar\omega$$

And eliminating common terms one gets:

$$Eu = -\frac{\hbar^2}{2m}\nabla^2 u + kTe^{-i\omega t}u - -i\frac{\hbar^3}{2\tau m^2c^2}\nabla^2 e^{-i\omega t}u \tag{3.2.27}$$

To solve this equation consider the solution

$$u = A\sin\alpha x \tag{3.2.28}$$

Differentiating $u$with respect to x yields

$$\frac{\partial u}{\partial x} = \alpha \cos \alpha x$$

Thus

$$\frac{\partial^2 u}{\partial x^2} = -\alpha^2 \sin \alpha x = -\alpha^2 u \qquad (3.2.29)$$

Substituting equation (3.2.29) in equation (3.2.27) to get

$$Eu = \frac{\hbar^2}{2m}\alpha^2 u + kTu + i\frac{\hbar^3}{2\tau m^2 c^2}\nabla^2\alpha^2$$

$$E = \frac{\hbar^2}{2m}\alpha^2 + kT + i\frac{\hbar^3}{2\tau m^2 c^2}\nabla^2\alpha^2\, u \qquad (3.2.30)$$

Rearranging for getting α, yields

$$\left[\frac{\hbar^2}{2m} + i\frac{\hbar^3}{2\tau m^2 c^2}\right]\alpha^2 = E - kT \qquad (3.2.31)$$

$$\alpha^2 = E - kT / \left[\frac{\hbar^2}{2m} + i\frac{\hbar^3}{2\tau m^2 c^2}\right] \qquad (3.2.32)$$

$$\alpha = \sqrt{E - kT / \left[\frac{\hbar^2}{2m} + i\frac{\hbar^3}{2\tau m^2 c^2}\right]} \qquad (3.2.33)$$

$$\alpha = \sqrt{\frac{E - kT}{\frac{\hbar^2}{2m}\left(1 + \frac{i\hbar}{2\tau m^2 c^2}\right)}} \qquad (3.2.34)$$

For particle in a box

$$u(x = L) = 0$$

Thus from (28)

$$u = A \sin \alpha L = 0$$

$$\alpha L = 0, \pm\pi, \pm 2\pi, \ldots$$

$$= 2n$$

$$\alpha = \frac{n\pi}{L} \qquad n = 0, \pm 1, \pm 2, \ldots \tag{3.2.35}$$

Thus from (12)

$$E - kT = \frac{n^2\pi^2}{L^2}\frac{\hbar^2}{2m}\left(1 + \frac{i\hbar}{\tau mc^2}\right)$$

$$E = kT + \frac{n^2\hbar^2}{8L^2m}\left(1 + \frac{i\hbar}{\tau mc^2}\right) \tag{3.2.36}$$

Where

$$\hbar = \frac{h}{2\pi}$$

The energy in equation (3.2.36) can be written in the form:

$$E = E_1 + E_2$$

Where

$$E_1 = kT + \frac{n^2\hbar^2}{8L^2m} \tag{3.2.37}$$

$$E_2 = kT + \frac{n^2h^2\hbar}{8L^2m^2c^2} \tag{3.2.38}$$

$\boldsymbol{E_1}$Stands for the energy gained by the particle, while$\boldsymbol{E_2}$ is the energy lost by the particle.

Harmonic Oscillator:

The Harmonic Oscillator is characterized by the potential:

$$V = +\frac{1}{2}kx^2 \qquad (3.2.39)$$

Inserting equation (3.2.39) in equation (3.2.27) yields:

$$-\frac{\hbar^2}{2m}\nabla^2 u + kTu - \frac{i\hbar^3}{2\tau m^2 c^2}\nabla^2 u - \frac{1}{2}kx^2 u = Eu \qquad (3.2.40)$$

Consider the solution:

$$u = Ae^{-\alpha x^2}\nabla u = -2\alpha x u$$

$$\nabla^2 u = -2\alpha u - 2\alpha x \nabla u = -2\alpha u + 4\alpha^2 x^2 u \qquad (3.2.41)$$

A direct substitution of equation (3.2.41) in equation (3.2.40) yields

$$+\frac{\hbar^2}{2m}[2\alpha - 4\alpha^2 x^2]u + kTu + \frac{\hbar^2}{2m}\left[\frac{2\alpha - 4\alpha^2 x^2}{2\tau mc^2}\right]\hbar u + \frac{1}{2}kx^2 = Eu$$

Equating the coefficients of u and $x^2 u$ on both sides' yields:

$$-\frac{4\hbar^2}{2m}\left[1 + \frac{i\hbar}{2\tau mc^2}\right]\alpha^2 - \frac{1}{2}k = 0 \qquad 2\frac{\hbar^2}{2m}\left[1 + \frac{i\hbar}{2\tau mc^2}\right]\alpha + kT = E$$

By ignoring temperature term, one gets:

$$E = \alpha\left[1 + \frac{i\hbar}{2\tau mc^2}\right]\frac{\hbar^2}{m} \qquad (3.2.42)$$

The energy quantization can be obtained from equation (3.2.22) by separating variables, and assuming ψ to be:

$$\psi = \omega(t)v(x) \qquad (3.2.43)$$

To get:

$$\frac{i\hbar}{\omega}\frac{d\omega}{dt} = \frac{1}{u}\left[-\frac{\hbar^2}{2m}\left[1 + \frac{i\hbar}{\tau mc^2}\right]\right]\nabla^2 u + V = E$$

Thus:

$$\frac{i\hbar}{\omega}\frac{d\omega}{dt} = E$$

$$\ln\omega = c_1 - i\frac{E}{\hbar}t$$

$$\int\frac{d\omega}{\omega} = \frac{E}{i\hbar}\int dt + c_1$$

$$\omega = e^{c_1}e^{\frac{iEt}{\hbar}} = c_2 e^{\frac{iEt}{\hbar}} \tag{3.2.44}$$

The periodicity condition requires:

$$\frac{\omega(t+T) = \omega(t)}{e^{\frac{iEt}{\hbar}} = 1}$$

$$\cos\frac{E}{\hbar}T = 1$$

$$\frac{E}{\hbar}T = 2n\pi$$

$$e^{\frac{iE}{\hbar}(t+T)} = e^{\frac{iE}{\hbar}(t)}$$

$$\csc\frac{E}{\hbar}T - i\sin\frac{E}{\hbar}T = 1$$

$$\sin\frac{E}{\hbar}T = 0$$

$$E = \frac{n\hbar}{T} = n\hbar f \tag{3.2.44}$$

Therefore energy is quantized according to equation (3.2.42) and equation (3.2.43) beside equation (3.2.44), one gets:

$$n\hbar f = \frac{(mk)^{\frac{1}{2}}}{2\hbar\left[1 + \frac{i\hbar}{\tau mc^2}\right]^{\frac{1}{2}}}$$

$$E = n\hbar f = \frac{\hbar}{2}\left(\frac{k}{m}\right)^{1/2}\left[\frac{i\hbar}{\tau mc^2}\right]^{1/2} = \frac{1}{2}\hbar\omega c\left[1 + \frac{i\hbar}{\tau mc^2}\right]^{1/2} \quad (3.2.45)$$

Discussion:

Plasma equation in the presence of friction and thermal energy equations (3.2.1, 3.2. 3, and 3.2.7) is utilized to derive new Schrödinger Equation which is sensitive to temperature and friction as shown by equation (3.2.13). The friction term and manifests itself through (τ), where

$$\gamma = \frac{m}{\tau}$$

For particle in a box equation (3.2.36) shows the energy is quantized, including the energy loss due to friction which appears as an imaginary part. This imaginary energy resembles the optical potential which describes inelastic collision in a sea herring process. It also resembles the role of imaginary wave number in electromagnetic theory which is related to the damping term in the expression of light intensity that describes the energy loss by light when it enters a certain medium. Equations (3.2.36) and (3.2.37) show that the energy for a particle in a box reduces to the ordinary one in the absence of friction and thermal energy.

For a harmonic oscillator the solution equation (3.2.41) and the value of (α) is complex. This solution reduces to the ordinary one in the ordinary one in the absence of friction. Equation (3.2.44), shows that the energy is quantized. Equation (3.2.45) indicates the existence of friction term as an imaginary part.

Thus the new Schrodinger Equation derived in this work is capable of describing a physical system in which temperature and friction plays an important role. It shows that friction energy appear as an imaginary part and is quantized.

3.3 Using the tight binding approximation in deriving the quantum critical temperature superconductivity equation:

According to plasma equation, a fluid of particles of mass m, number density n, velocity *v*, force F and pressure P is given by [47]

$$mn\left[\frac{\partial v}{\partial t} + v.\nabla v\right] = F - \nabla P \tag{3.3.1}$$

If *F* is a field force then

$$F = -n\nabla V$$

Where Vis the potential of one particle. In one dimension

$$mn\left[\frac{\partial v}{\partial t} + v.\nabla v\right] = -n\nabla V - \nabla P = n\frac{d\mathrm{V}}{dx} - \frac{d\mathrm{P}}{dx}$$

$$dv = \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial x}dx$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} \tag{3.3.2}$$

Thus according to Equation (3.3.1), in one dimension

$$mn\frac{\partial v}{\partial t} = -n\frac{d\mathrm{V}}{dx} - \frac{d\mathrm{P}}{dx} \tag{3.3.3}$$

3.3.1 Schrödinger Temperature Dependent Equation:

Schrodinger equation can be derived by using new exp ression of energy obtained from the plasma equation to do this one can use (3.3.2) to get

$$mn\frac{\partial v}{\partial x}\frac{dx}{dt} = -n\frac{d\mathrm{V}}{dx} - \frac{d\mathrm{P}}{dx}$$

Multiplying both sides by dx and integrating yields

$$mn\int vdv = -n\int dV - \int dP$$

Considering the pressure to be $P = \gamma nRT$ in general, thus

$$mn\frac{v^2}{2} = -nV - P = -nV - \gamma nkT$$

Hence

$$m\frac{v^2}{2} + V + \gamma kT = constant$$

This constant conserved quantity looks like the ordinary energy beside the ordinary thermal energy term

$$E = \frac{p^2}{2m} + V + \gamma kT \qquad (3.3.3)$$

To find Schrodinger equation for it, consider the ordinary wave function

$$\Psi = Ae^{i/\hbar(px-Et)}$$

Differentiating both sides by t and x yields

$$\frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}E\Psi \Rightarrow i\hbar\frac{\partial\Psi}{\partial t} = E\Psi$$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{p^2}{\hbar^2}\Psi \Rightarrow -\hbar^2\nabla^2\Psi \qquad (3.3.4)$$

Multiplying both sides of Equation(3.3.3)by Ψ yields

$$E\Psi = \frac{p^2}{2m}\Psi + V\Psi$$

Substituting Equation(3.3.4), one gets

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi + \gamma kT\Psi$$

This equation represents Schrödinger equation when thermal motion is considered. The solution for time free potential can be

$$\Psi = e^{-i/\hbar(Et)}u \Rightarrow \frac{\partial\Psi}{\partial t} = -\frac{1}{\hbar}E\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi + \gamma kT\Psi$$

The time independent Schrödinger equation thus takes the form

$$Eu = -\frac{\hbar^2}{2m}\nabla^2 u + Vu + \gamma kTu \qquad (3.3.5)$$

For constant potential, the solution can be

$$u = e^{ikx} \quad , \quad V = V_0$$

Inserting this solution in Equation (3.3.5) yields

$$Eu = \frac{\hbar^2 k^2}{2m}u + V_0 u + \gamma kTu$$

$$E = \frac{\hbar^2 k^2}{2m} + V_0 + \gamma kT$$

If one set the kinetic term to be $E_0 = \frac{\hbar^2 k^2}{2m}$, one can thus write the energy in the form

$$E = E_0 + V_0 + \gamma kT \qquad (3.3.6)$$

This quantum energy expression involves a thermal term beside kinetic and potential term.

3.3.2 Quantum Resistance:

The resistance, z, per unit length (L = 1) per unit area (A = 1) can be found from the ordinary definition of, z. The resistance z is defined to be the ratio of the potential, u, to the current per unit area, *J*, *i.e*.

$$z = \frac{u}{I} = \frac{u}{JA} = \frac{u}{J} = \frac{u}{mev} = \frac{mu}{nep} \qquad (3.3.7)$$

With n and e standing for the free hole or electron density and charge respectively, while p represents the momentum of electron of mass m, where

$$P = mv$$

This resistance (it actually stands for resistivity) can be found by using the laws of quantum mechanics for a free charge which are responsible for generating the electric current, where the wave function takes the form

$$\Psi = Ae^{ikx} \qquad (3.3.8)$$

This selection of Ψcomes from the fact that the resistance property comes from the motion of the free charges. The potential u is related to the Hamiltonian H through the relation

$$H = eu$$

Thus for freely moving charge one gets:

$$\hat{H} = eu = \frac{1}{2}mv^2 = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

In view of Equation (3.4.8) and according to the correspondence principle V takes the form

$$u = \frac{\langle \hat{H} \rangle}{e} = \frac{\int \bar{\Psi} \hat{H}\, \Psi dx}{e} = \frac{\int \bar{\Psi} \hat{p}^2\, \Psi dx}{2me} \qquad (3.3.9)$$

$$= \frac{\hbar^2 \mathrm{k}^2}{2me} \int \bar{\Psi} \Psi\, dx = \frac{\hbar^2 \mathrm{k}^2}{2me}$$

While *P* becomes

$$p = \langle \hat{p} \rangle = \int \bar{\Psi} \hat{p}\, \Psi dx = \hbar k \int \bar{\Psi}\, \Psi dr = \hbar k \qquad (3.3.10)$$

Thus inserting Equations (3.3.9), (3.3.10) in (3.3.7) one obtains

$$Z = \frac{m\hbar^2 k^2}{2me^2 \hbar k n} = \frac{\hbar k}{2e^2 n} = \left(\frac{h}{2\pi}\right)\left(\frac{2\pi}{\lambda}\right)\frac{1}{2e^2 n}$$

$$z = \frac{h}{2\lambda e^2 n} = \frac{hf}{2f\lambda e^2 n} = \frac{hf}{2e^2 n v} = \frac{\hbar f \sqrt{\mu\varepsilon}}{2e^2 n} = \frac{\hbar \omega \sqrt{\mu\varepsilon}}{2e^2 n} \qquad (3.3.11)$$

Where the expression $f\lambda$ for velocity is found by assuming charges to be waves, then following the electromagnetic theory (EMT), the speed of the waves is affected by electric permittivity ε and magnetic permeability through the relation

$$v = f\lambda = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (3.3.12)$$

Where the effect of medium changes the wave length, λ, while the frequency, *f*, is unchanged. Thus assuming the charge density, *n*, to be constant, the only change of, *Z*, can be caused by μandε.

It is also important to note that, in superconductors, the current can flow without the aid of deriving potential *u*. the role of *u* is confined only in enabling electrons to gain kinetic energy through the relations

$$eu = \frac{1}{2} m v^2 = k \qquad (3.3.13)$$

Where this potential can be applied between any two arbitrary points in the superconductors then remove it. The role of resistive force is neglected here as done in deriving London equations.

The expression for Z can also be found by inserting Equation(3.3.13) in to get

$$z = \frac{u}{J} = \frac{u}{nev} = \frac{mv^2}{2ne^2 v} = \frac{mv}{2ne^2} = \frac{p}{2ne^2} = \frac{h}{2\lambda ne^2}$$

$$z = \frac{hf}{2\lambda f e^2 n} = \frac{hf}{2e^2 nv} = \frac{\hbar f\sqrt{\mu\varepsilon}}{2e^2 n} = \frac{\hbar\omega\sqrt{\mu\varepsilon_0(1+x)}}{2e^2 n} \quad (3.3.14)$$

It is important to note that this quantum resistance expression resembles the ones found by Tsui (3.4.3) where one uses De Broglie hypothesis (3.4.4), *i.e.*

$$p = h/\lambda$$

3.3.4 Calculation HTSC by Electric Susceptibility:

Consider holes in a conductor having resistive force F_r, magnetic force F_m and pressure force F_p, beside the electric force F_e, the equation motion then becomes (3.3.3):

$$F = F_r + F_m + F_e - F_p$$

Where:

$$F_p = -\nabla P, \; F_r = -\frac{mv}{\tau}, Bev + eE = eE_0 e^{i\omega t}$$

P, x, m, v, τ, B, e and *E* stands for the pressure, displacement, mass, velocity, relaxation time, magnetic flux density, electron charge and electric field intensity respectively. Thus the equation of motion takes the form

$$m\ddot{x} = -\frac{mv}{\tau} + Bev - \nabla P \qquad (3.3.15)$$

The solution of this equation can be suggested to be:

$$x = x_0 e^{i\omega t}$$

$$v = v_0 e^{i\omega t}$$

$$E = E_0 e^{i\omega t} \qquad (3.3.16)$$

Inserting (3.3.16) in (3.4.15) yields

$$-m\omega^2 x = \left(\frac{mv_0}{E_0\tau} + \frac{Bev_0}{E_0} - \frac{kT\nabla n}{E_0} + e\right)E$$

$$x = \frac{\left(\frac{mv_0}{E_0\tau} - \frac{Bev_0}{E_0} + \frac{kT\nabla n}{E_0} - e\right)E}{m\omega^2}$$

This expression of x can be utilized in the formula which relates the electric polarization vector P to the susceptibility P on one hand and to the number of atoms N via the following relation

$$P = \varepsilon_0 x E = +eNx \qquad (3.3.18)$$

Motivated by the important role of holes in HTSC, displacement can be assumed to result from the motion of holes or positive nuclear charges, thus inserting Equation (3.3.17) in (3.4.18) yields

$$\varepsilon_0 x E = eN\frac{\left(\frac{mv_0}{E_0\tau} - \frac{Bev_0}{E_0} + \frac{kT\nabla n}{E_0} - e\right)E}{m\omega^2}$$

$$x = \frac{eN}{m\omega^2\varepsilon_0 E_0}\left(\frac{mv_0}{\tau} - Bev_0 + kT\nabla n - eE_0\right) \qquad (3.3.19)$$

The electric flux density assumes the following relation

$$D = \varepsilon E = \varepsilon_0 E + x\varepsilon_0 E = \varepsilon_0(1 + x)E = P + \varepsilon_0 E$$

The electric permittivity is given by

$$\varepsilon = \varepsilon_0(1 + x) \qquad (3.3.20)$$

The electric permittivity is thus given according to Equation (3.3.20) to be

$$\varepsilon = \varepsilon_0(1 + x)$$

$$= \varepsilon_0\left[1 + \frac{eN}{m\omega^2 E_0}\left(\frac{\mathrm{mv_0}}{\tau} - \mathrm{Bev_0} + \mathrm{kT\nabla n} - \mathrm{eE_0}\right)\right] \qquad (3.3.21)$$

$$z = \frac{\hbar\omega}{2ne^2}\sqrt{\mu\varepsilon_0\sqrt{1 + \frac{eN}{m\omega^2\varepsilon_0 E_0}\left(\mathrm{kT\nabla n} + \frac{\mathrm{mv_0}}{\tau} - \mathrm{Bev_0} - \mathrm{eE_0}\right)}} \qquad (3.3.22)$$

$$z = \frac{\hbar\omega}{2ne^2}\sqrt{\mu\varepsilon_0\sqrt{\frac{m\omega^2\varepsilon_0 E_0 + eN\left(\mathrm{kT\nabla n} + \frac{\mathrm{mv_0}}{\tau} - \mathrm{Bev_0} - \mathrm{eE_0}\right)}{m\omega^2\varepsilon_0 E_0}}}$$

The resistance Z can be found by inserting (3.3.21) in (3. 3.14) to get:

$$m\omega^2\varepsilon_0 E_0 + eN\left(\mathrm{kT\nabla n} + \frac{\mathrm{mv_0}}{\tau} - \mathrm{Bev_0} - \mathrm{eE_0}\right) < 0$$

$$\mathrm{kT\nabla n} < Be\mathrm{v_0} + eE_0 - \frac{m\omega^2\varepsilon_0 E_0}{eN} - \frac{\mathrm{mv_0}}{\tau}$$

$$\mathrm{T} < \frac{\mathrm{Bev_0}}{\mathrm{k\nabla n}} + \frac{(e - m\omega^2\varepsilon_0)E_0}{eN\mathrm{k\nabla n}} - \frac{\mathrm{mv_0}}{\tau\mathrm{k\nabla n}}$$

Thus the critical temperature is given by

$$T_c < \frac{(Be\tau - m)v_0}{\tau k\nabla n} + \frac{(e - m\omega^2\varepsilon_0)E_0}{eNk\nabla n} \qquad (3.3.23)$$

If the internal field B results from No atoms each having a verge flux density μB then: (3.4.5).

$$B = \mu_B N_0 \qquad (3.3.24)$$

Therefore T_c can take the form

$$T_c = \frac{(\mu_B N_0 e\tau - m)v_0}{\tau k\nabla n} + \frac{(e - m\omega^2\varepsilon_0)E_0}{eNk\nabla n} \qquad (3.3.25)$$

3.3.5 Tight Binding Critical Temperature and Energy Gap:

In tight binding model (3.4.5) the energy of electrons in the crystal is given by

$$\varepsilon = \varepsilon_0 + \alpha_1 + 2\gamma \cos ka \qquad (3.3.26)$$

Whereε_0 is the energy in the absence of crystal field, while the other terms describe the effect of the crystal field. The energy ε_0 can split into two terms the kinetic part which can describe the thermal motion in the form $\frac{f_0}{2}kT$ beside the potential term$-V_0$ for attractive force or bounded particle.

Thus one can write

$$\varepsilon_0 = \frac{\hbar^2 {k^2}_0}{2m} + \frac{f_0}{2}kT - V_0 \qquad (3.3.27)$$

$$\varepsilon_0 = \frac{\hbar^2 {k^2}_0}{2m} + \gamma RT + V$$

$$\varepsilon_0 = \frac{f_0}{2}kT - V_0 - \alpha_0$$

$$\alpha_0 = \frac{\hbar^2 k^2}{2m}$$

$f_0$represents the degrees of freedom.

The terms describing the effect of the crystal force are

$$\alpha_1 = \langle \emptyset_m | \hat{H}_{cry} | \emptyset_m \rangle$$

$$\gamma = \langle \emptyset_j | \hat{H}_{cry} | \emptyset_m \rangle$$

$$\alpha = \alpha_0 + \alpha_1$$

In view of Equations (3.4.26) and (3.4.27)

$$\varepsilon = \frac{f_0}{2} kT - V_0 + \alpha + 2\gamma \cos ka \qquad (3.3.29)$$

Here H_{cry}stands for the crystal force Hamiltonian part, while$\emptyset_m$ and $\emptyset_j$are the states of particles located at the site m and j respectively.

The superconductor is characterized by the existence of energy gap. This gap can be under stood here in two ways. If the electrons or holes are not free. This requires E to negative. Thus Equations (3.3.27) and (3.3.26) needs

$$\varepsilon = \frac{f_0}{2} kT - V_0 + \alpha + 2\gamma \cos ka < 0 \qquad (3.3.30)$$

Or the max value of ε where $\cos ka = -1$ is less than zero, i.e.

$$\varepsilon_{max} = \frac{f_0}{2} kT - V_0 + \alpha + 2\gamma \cos ka < 0 \qquad (3.3.31)$$

$$\frac{f_0}{2} kT \leq V_0 - \alpha + 2\gamma$$

For constant attractive crystal force

$$H_{cry} = -V_{cry}$$

$$\alpha_1 = \langle \emptyset_m | H_{cry} | \emptyset_m \rangle = -\langle \emptyset_m | V_{cry} | \emptyset_m \rangle = -V_{cry}\delta_{mm}$$

$$\gamma = \langle \emptyset_j | -V_{cry} | \emptyset_m \rangle = -V_{cry}\langle \emptyset_j | V_{cry} | \emptyset_m \rangle = -V_{cry}\delta_{jm=0} \quad (3.3.32)$$

Thus

$$\frac{f_0}{2}kT \leq V_0 - \alpha$$

Thus the critical temperature is given

$$\frac{f_0}{2}kT_c \leq V_0 - \alpha \quad (3.3.33)$$

Substituted Equation (3.3.33) beside Equation (3.3.32) in Equation (3.3.30) one gets

$$\varepsilon = \frac{f_0}{2}kT - \frac{f_0}{2}kT_c \quad (3.3.34)$$

The energy gap Δ is equal to the difference between zero energy in conduction band and the negative energy in the valence band. Thus

$$\Delta = 0 - \varepsilon = \frac{f_0}{2}kT_c - \frac{f_0}{2}kT$$

Since this relation holds $T < T_c$ for one can neglect$T$

Since it is small to get

$$\Delta = \frac{f_0}{2}kT_c$$

Equation (3.3.30) can also be utilized to get the forbidden energy states which characterizes superconductors, where

$$\cos ka = \frac{\varepsilon - \frac{f_0}{2}kT_c + V_0 - \alpha}{2\gamma}$$

The energy is forbidden when $\cos ka \geq 1$

$$\frac{\varepsilon - \frac{f_0}{2}kT + V_0 - \alpha}{2\gamma} \geq 1$$

$$\varepsilon - \frac{f_0}{2}kT + V_0 - \alpha \geq 2\gamma$$

$$\frac{f_0}{2}kT + \alpha - \varepsilon - V_0 - \alpha \leq -2\gamma$$

$$\frac{f_0}{2}kT \leq \varepsilon + V_0 - 2\gamma - \alpha$$

Thus the critical temperature

$$\frac{f_0}{2}kT_c = \varepsilon + V_0 - 2\gamma - \alpha \qquad (3.3.35)$$

The forbidden energy is thus related to the critical temperature through the relation

$$\varepsilon = \frac{f_0}{2}kT_c - V_0 + 2\gamma + \alpha \qquad (3.3.36)$$

3.4 Derivation of Klein – Gordon equation for Maxwell s electric wave equation:

Kamil uas Maxwell's equation to derive Klein – Gordon equation [49] Maxwell's equation for an electric of field intensity E in a dielectric insulating non-charged medium material of electric dipole moment P is given by equation (3.3.16) to be: [48]

$$-\nabla^2 E + \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} = -\mu_0\frac{\partial^2 P}{\partial t^2} \qquad (3.4.1)$$

Where for non-charged insulating material:

$$\rho = 0 \quad , \quad \sigma = 0$$

Where for simplification it is better to consider current density J as a constant, that is:

$$\frac{\partial J}{\partial t} = 0 \tag{3.4.2}$$

The electric dipole moment is given by:

$$P = nq_2x = \frac{Nq_2x}{Ax}$$

$$= \frac{Q}{A} = \frac{\emptyset}{A} \tag{3.4.3}$$

$$= D = \varepsilon E$$

Where n is the number density of charge, N is the total number, A is the area and x is the distance.

V= Volume =Ax

Q = Total charge = Nq_0

q_0= Charge of a single pole according to Gauss law.

The charge Q and total flux$\emptyset$ are related by:

$$Q = \emptyset \tag{3.4.4}$$

To solve equation (3.4.1), one can assume the electric field intensity in free space E to be:

$$E = E_0 e^{i(kx-\omega t)} \quad . \tag{3.4.5}$$

Thus:

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E \qquad \nabla^2 E = -k^2 E \tag{3.4.6}$$

From equations (3.4.5) and (3.4.3):

$$\frac{\partial^2 E}{\partial t^2} = -\mu_0 \varepsilon \omega^2 E \tag{3.4.7}$$

The speeds in vacuum *c* and in the medium *v* are given:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon}} \qquad v = \frac{1}{\sqrt{\mu_0 \varepsilon}} \tag{3.4.8}$$

Thus (3.4.7) reads:

$$-\mu_0 \frac{\partial^2 P}{\partial t^2} = -\mu_0 \varepsilon \omega^2 E = -\frac{1}{v^2} \omega^2 E \tag{3.4.9}$$

$$= -\left(\frac{2\pi f}{f \lambda_m}\right)^2 E = -{k_m}^2 E$$

Inserting (3.4.6) and (3.4.9) in (3.4.1) yields:

$$k^2 - \frac{\omega^2}{c^2} = -{k_m}^2 \tag{3.4.10}$$

Multiplying both sides by $c^2$and$\hbar^2$, one gets:

$$c^2 \hbar^2 k^2 - \hbar^2 \omega^2 = -c^2 \hbar^2 {k_m}^2 \tag{3.4.11}$$

Using De Broglie and Plank hypotheses:

$$P = \frac{h}{\lambda} = \hbar k \qquad E = hf = \hbar \omega \tag{3.4.12}$$

Equation (3.4.11) can thus be given by:

$$c^2 p^2 + c^2 {p^2}_m = E^2 \tag{3.4.13}$$

Since the electromagnetic waves can be assumed as a photon moving with the speed of light c, the photon momentum rest mass m_0 is given by:

$$\hbar k_m = p_m m_0 c \qquad (3.4.14)$$

Here the rest mass is assigned to a medium since the medium lower photon speed and it can even stop it when it is absorbed. Thus inserting (3.4.14) in (3.4.13) yields:

$$c^2p^2 + c^2{p^2}_m = E^2 \qquad (3.4.15)$$

This is the Einstein expression that relates momentum to energy .The derivation of this relation can be done by using the classical equation of energy and Plank hypothesis only .The classical energy for an electromagnetic wave photon oscillating particle with maximum velocity is given by:

$$E = \frac{1}{2} m{v_m}^2 \qquad (3.4.16)$$

Since for waves or any harmonic system, the root mean square (r.m.s) velocityV_{rms} is given by:

$$V_{rms} = \frac{1}{2} V_m \qquad (3.4.17)$$

By assuming the photon speed c equal to the r. m. s speed, that is:

$$c = \frac{1}{2} V_m \qquad (3.4.18)$$

It follows that:

$$E = \left(\frac{V_m}{\sqrt{2}}\right)^2 = mc^2 \qquad (3.4.19)$$

According to Plank theory:

$$E = hf = \frac{hc}{\lambda} = mc^2 \qquad (3.4.20)$$

Therefore, the momentum p is given by:

$$p = mc = \frac{mc^2}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda} \qquad (3.4.21)$$

3.4.1 Derivation of Klein-Gordon Equation:

The Klein-Gordon equation can be obtained by replacing the electric dipole moment term in equation (3.4.17) by the term standing for photon rest mass in equation (3.4.9) to get:

$$-\nabla^2 E + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = -{k_m}^2 E \qquad (3.4.22)$$

Multiplying sides by $c^2\hbar^2$and using equation (8), the following equation is obtained:

$$-\mathrm{c}^2\hbar^2\nabla^2 E + \hbar^2 \frac{\partial^2 E}{\partial t^2} = -\mathrm{c}^2\hbar^2 {k_m}^2 E \qquad (3.4.23)$$

According to relation (3.4.14):

$${P_m}^2 = \hbar^2 {k_m}^2 = {m_0}^2 c^2$$

Thus (3.4.23) reads:

$$-\mathrm{c}^2\hbar^2\nabla^2 E + {m_0}^2 c^4 E = -\hbar^2 \frac{\partial^2 E}{\partial t^2} \qquad (3.4.24)$$

The incorporation of mass for photon in Maxwell s equations corresponds to adding the term $m_0 A^\mu \mathrm{A}_\mu$ to the electromagnetic field largrangian.

Since in the electromagnetic (e. m) theory theos cillating electric wave E is related to its e. m, the energy or intensity is obtained according to the relation:

$$I \alpha c \varepsilon_0 E^2 \qquad (3.4.25)$$

And since the e. m intensity, when treated as a stream of photons of density n is given by:

$$I\alpha nhf\alpha|\psi|^2 hf \qquad (3.4.26)$$

Where the photon density is related to the wave function ψ according to the relation:

$$n = |\psi|^2 \qquad (3.4.27)$$

Comparing (3.4.25) and (3.4.26) it follows that:

$$E^2 \Leftrightarrow |\psi|^2 \qquad E \Leftrightarrow \psi \qquad (3.4.28)$$

Thus the correspondence between E and ψ secure the replacement of E byψ in equation (3.4.24) to get:

$$-\hbar^2 \frac{\partial^2\psi}{\partial t^2} = -c^2\hbar^2\nabla^2\psi + {m_0}^2c^4\psi \qquad (3.4.29)$$

This represents Klein-Gordon equation for free electron.

3.5 Tight-binding and Energy Relation:

In this model is proposed by Amal [49] and is based tight binding approximation in quantum dot theory.

Tight-binding approximation is an approximation method to account for the effect of the crystal field of the bulk matter on the electron bounded to a single isolated atom. In this model the electron wave function is a superposition of that of all atoms in the crystal. The electron energy is found by averaging the Hamiltonian resulting from atom and crystal contribution. The tight-binding method is most practical when only a few types of electronic interactions are dominant.

In tight binding model the energy of electrons in the crystal is given by

$$E = \varepsilon_0 + \alpha_1 + 2\gamma \cos ka \tag{3.5.1}$$

Where ε_0 is the energy in the absence of crystal field, while the other terms,γdescribe the effect of the crystal field, whereas a is the atomic spacing and k is the wave number. The energyε_0 can split into three terms the kinetic part which can describe the wave motion beside the thermal motion in the form $\frac{f_0}{2}kT$, in addition to the potential term$-V_0$ for attractive force or bounded particle.

$$E_0 = \frac{\hbar^2 {k_0}^2}{2m} + \frac{f_0}{2}kT - V_0 \tag{3.5.2}$$

The terms describing the effect of the crystal force are

$$\alpha_1 = \langle \varphi_m | \hat{H}_{cry} | \varphi_m \rangle \tag{3.5.3}$$

$$\gamma = \langle \varphi_j | \hat{H}_{cry} | \varphi_m \rangle \tag{3.5.4}$$

The termsφ_m , φ_j , $\hat{H}_{crystal}$ represent the wave functions of the *m, j* site and the crystal field Hamiltonian respectively.

3.5.1 Theoretical Relations for Intensity and Nanoparticle Size:

The relation between the intensity and diameter of the nanoparticle one can utilize the relation of the intensity,

$$I = nEc \tag{3.5.5}$$

Where: *I* stand for the intensity, *E* for energy of the photon, *n* for the density of photons and *C* for the speed of light.

According to tight binding approximation the energy of the electron in crystal in the valance band is given by:

$$E_v = E_0 + \alpha + 2\gamma \cos ka \approx 2\gamma \cos ka$$

When E_0, αwere ignored, by assuming them as a back ground.

$$E_v = 2\gamma \cos ka \tag{3.5.6}$$

The photon energy results from transfer of an electron from abound state to become free. Thus

$$E = E_v - 0 = E_v$$

by replacing *n* by$|\psi|$ one gets:

$$I = 2c\gamma|\psi|^2 \cos ka \tag{3.5.7}$$

One can treat an electron confined to small nanoparticle as a particle in box. Thus

One can calculate ψ from the equation of particle in box

For particle in box:

$$V = V_0$$

Thus Schrödinger equation takes the form

$$-\frac{\hbar^2}{2m}\frac{d^2y}{dx^2} + V_0\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2y}{dx^2} = (E - V_0)\psi$$

One can assume a solution in the form:

$$\psi = A \sin kx \; ; \; \frac{d^2\psi}{dx^2} = -k^2\psi$$

Thus a direct substitution of this solution in Schrödinger equation yields:

$$\frac{\hbar^2 k^2}{2m}\psi = (E - V_0) \qquad and \;\; k = \frac{\sqrt{2m(E - V_0)}}{\hbar} \tag{3.5.8}$$

$$\psi = A \sin kx \tag{3.5.9}$$

$$|\psi|^2 = A^2 \sin^2 kx = \frac{A^2}{2}[1 - \cos 2kx] \tag{3.5.10}$$

Incorporating (3.5.6) in (3.5.3) yields:

$$I = C\gamma A^2[1 - \cos 2kx] \cos ka \tag{3.5.11}$$

Since I is positive, it follows that:

$$\cos 2kx = - , \qquad \frac{\pi}{2} \leq 2kx \leq \frac{3\pi}{2} \tag{3.5.12}$$

That means:

$$1 - \cos 2kx = +$$

If one assume that the wave length of the electron is equal to the diameter of the nanoparticle d,

$$\lambda = d = 2r \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi}{d} = \frac{\pi}{r} \tag{3.5.13}$$

Where r is the radius. Thus from (3.5.9) and (3.5.8)

$$\frac{r}{4} \leq x \leq \frac{3r}{4} \tag{3.5.14}$$

In the range determined by (3.5.10)

$$-\cos 2kx = |\cos 2kx|$$

Thus one gets (see (3.5.6))

$$I = C\gamma A^2[1 + |\cos 2kx|] \cos ka \tag{3.5.15}$$

The frequency was high for the value in bracket:

$$x > a \tag{3.5.16}$$

Where a: diameter of the atom

The relation of intensity can be simplified by assuming the

Wavelength λ of electron to be equal to the nanoparticle diameter d

$$\lambda = d$$

This is not surprising, since in the Bohr model the electron wave length is related to Bohr radius r, according to the relation $n\lambda = 2\pi r$. If one grinds a bulk matter one expects inverse relation between grinding time and the formed particle size d,

$$\text{i.e.}\, d \approx \frac{c^2}{\tau}$$

Thus k can be rewritten as

$$k = \frac{2\pi}{r} = \frac{2\pi}{d} = \frac{2\pi}{c^2}\tau = \frac{2\pi}{c_0}\tau \tag{3.5.17}$$

For simplicity one can choose the scale to make

$$A = C = C_0 = \gamma = 1 \qquad (3.5.18)$$

That means equation (11) takes the form:

$$I = [1 + |\cos 4\pi\tau x|] \cos 2\pi\tau a \qquad (3.5.19)$$

This theoretical relation (15) is displayed graphically in (Fig (17)).

3.5.2 Materials and Methods:

The aim of this work is to explain how the change of the size of Alum can affect the spectrum of them on the basis of quantum dot model based on tight binding approximation. The spectral change can manifest itself in a wave length shift or intensity change. One has seven materials. The size of each material is changed six times. This is achieved by grinding different parts of each material for different times ranging from 10 sec to 60 sec in steps of 10 sec. The size of each sample is determined by easy scan electron microscope. The spectrums of these samples are displayed by atomic absorption light spectrometer. The spectrum was displayed by using three different sources the first one is laser diode source followed by laser source beside a light source. These spectra are exhibited in figures below. The variation of spectral intensity of the highest

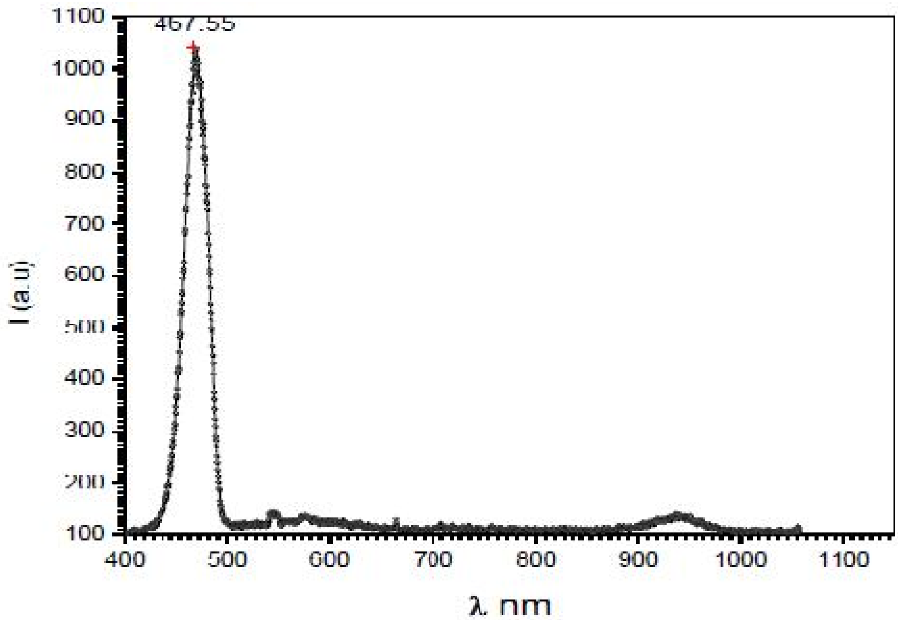


Figure 1. Spectrum of Alum bulk matter before grinding by using laser diode

Beak versus the grinding time is also displayed for each material type.

Material Sample:

The materials used in this work are selected in a special way; because they can be easily ground, beside their abundance locally. This material is:

Alum: Alum is both a specific chemical compound and a class of chemical compounds. The specific compound is the hydrated Potassium aluminum sulfate (potassium alum) with the formula $KAl(SO4)2$·12H2O. More widely, alums are double sulphate salts, with the formula AM (SO4)2·12H2O, where (A) is a monovalent cation such as potassium or ammonium and M is a trivalent metal ion such as aluminum or chromium (III). Alums are useful for a range of industrial processes. They are soluble in water; have a sweetish taste; react acid to litmus; and crystallize in regular octahedral. When heated they liquefy; and if the heating is continued, the water of crystallization is driven off, the salt froths and swells, and at last an amorphous powder remains.

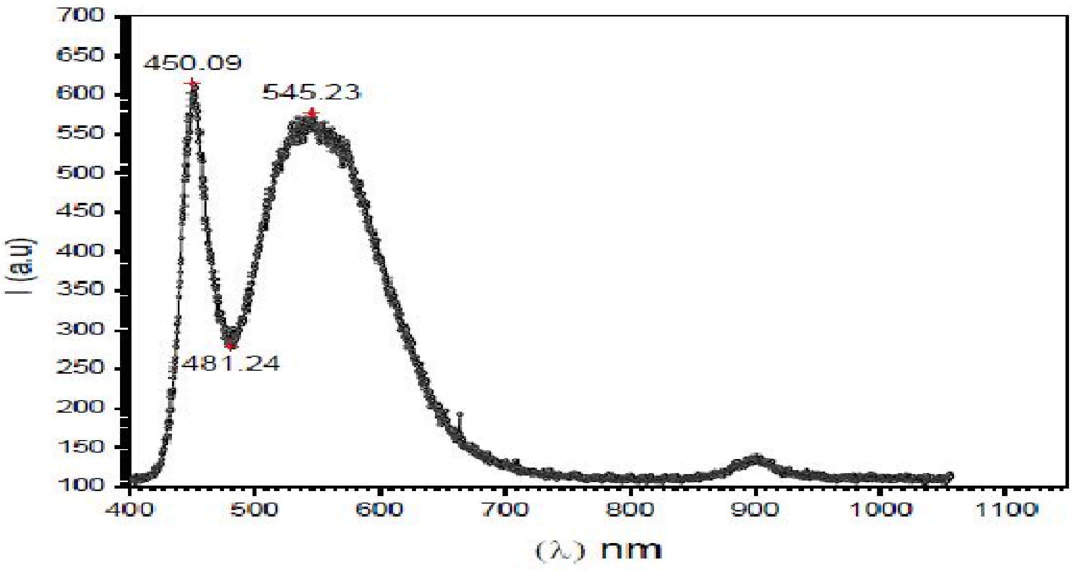


Figure 2 .Spectrum of Alum bulk matter before grinding by using white light

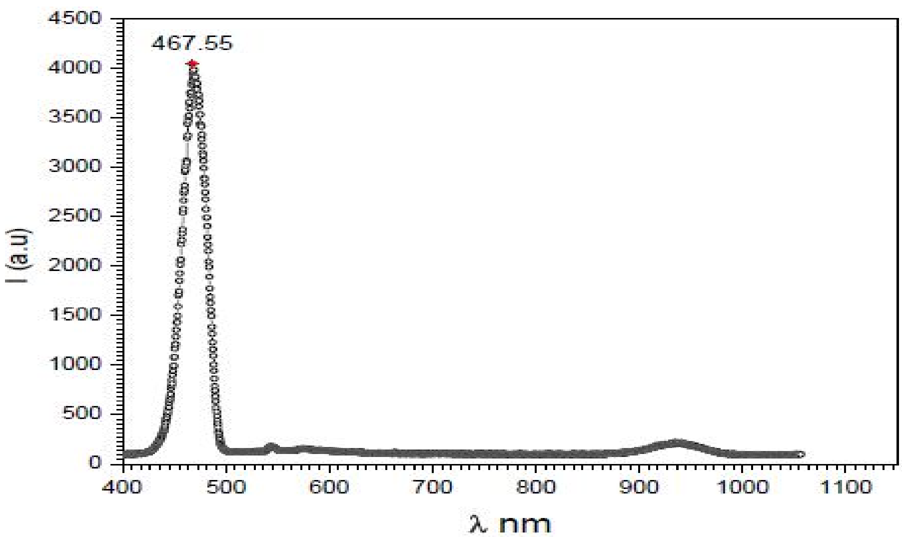


Figure 3. Spectrum of Alum for grinding time 10 sec by using laser diode

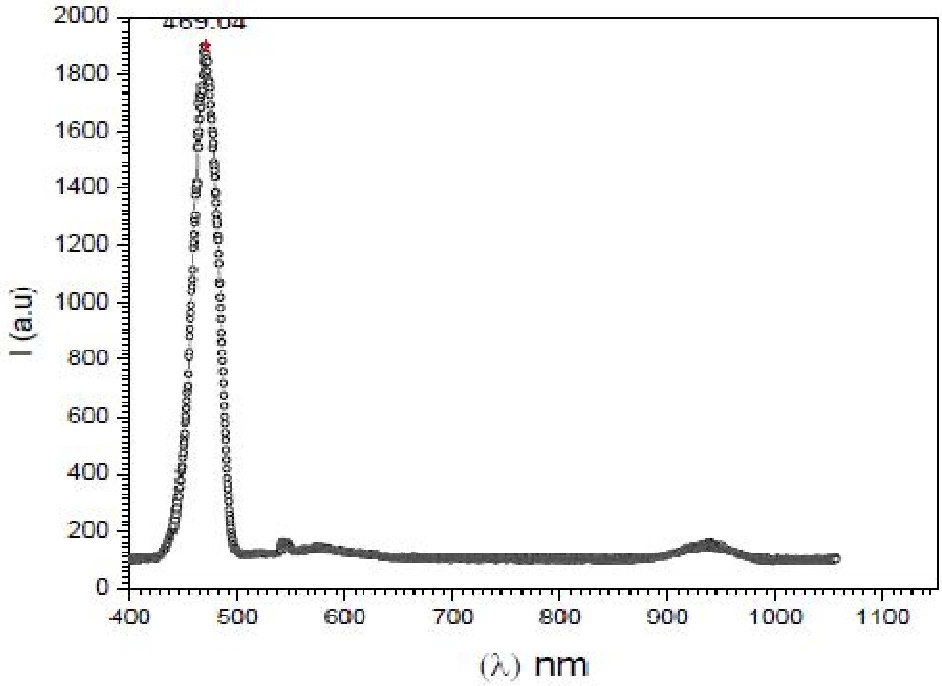


Figure 4.Spectrum of Alum for grinding time 20 sec
by using laser diode

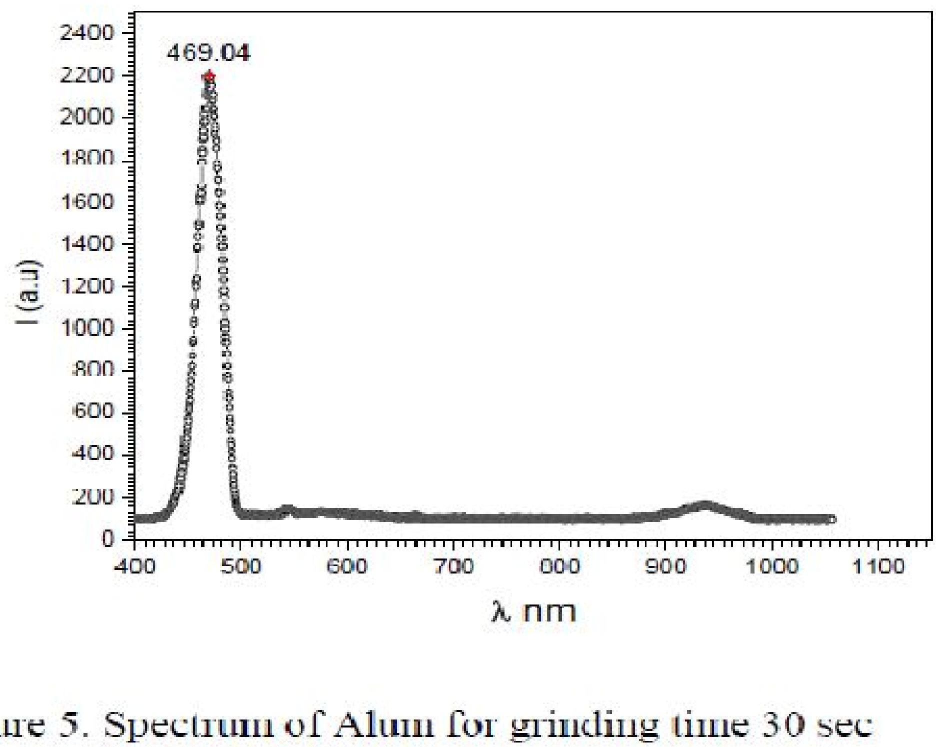


Figure 5. Spectrum of Alum for grinding time 30 sec
by using laser diode

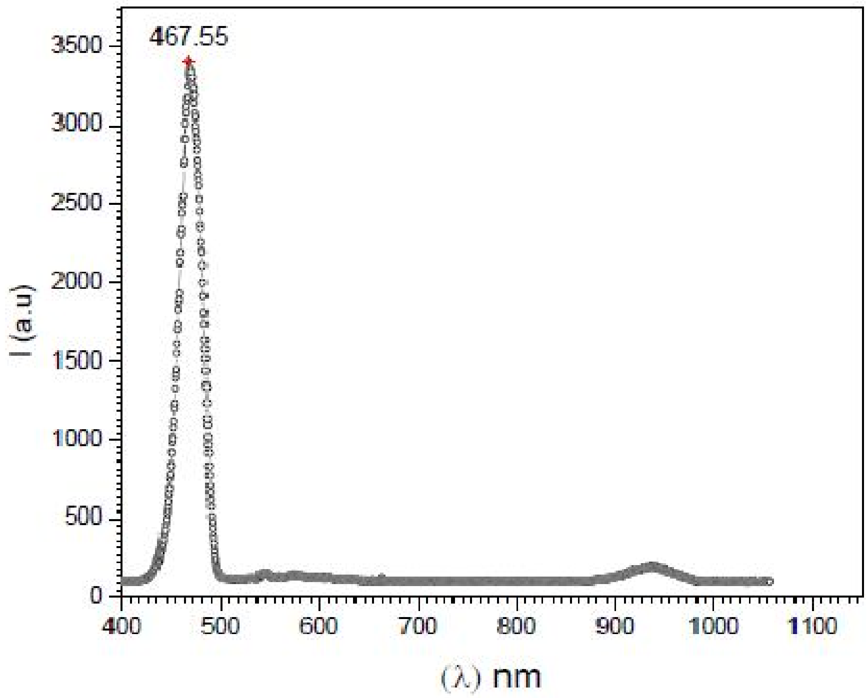


Figure 6. Spectrum of Alum for grinding time 40 sec
by using laser diode

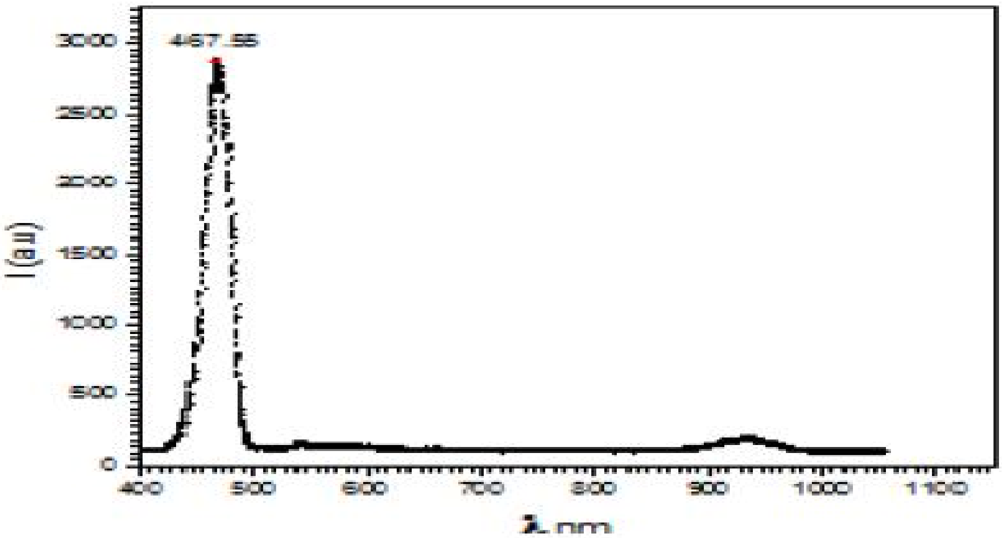


Figure 7. Spectrum of Alum for grinding time 50 sec by using laser diode

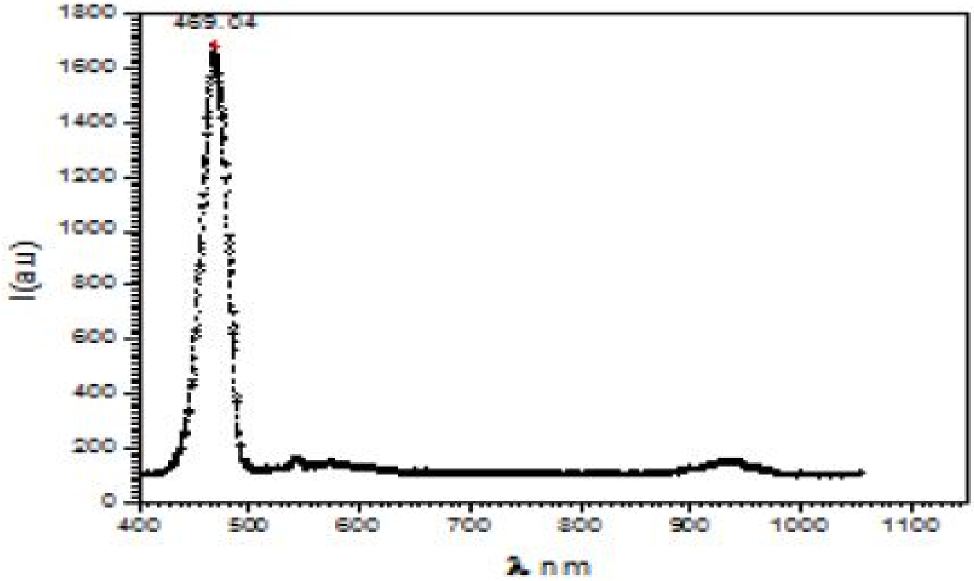


Figure8.Spectrum of Alum for grinding time 60 sec by using laser diode

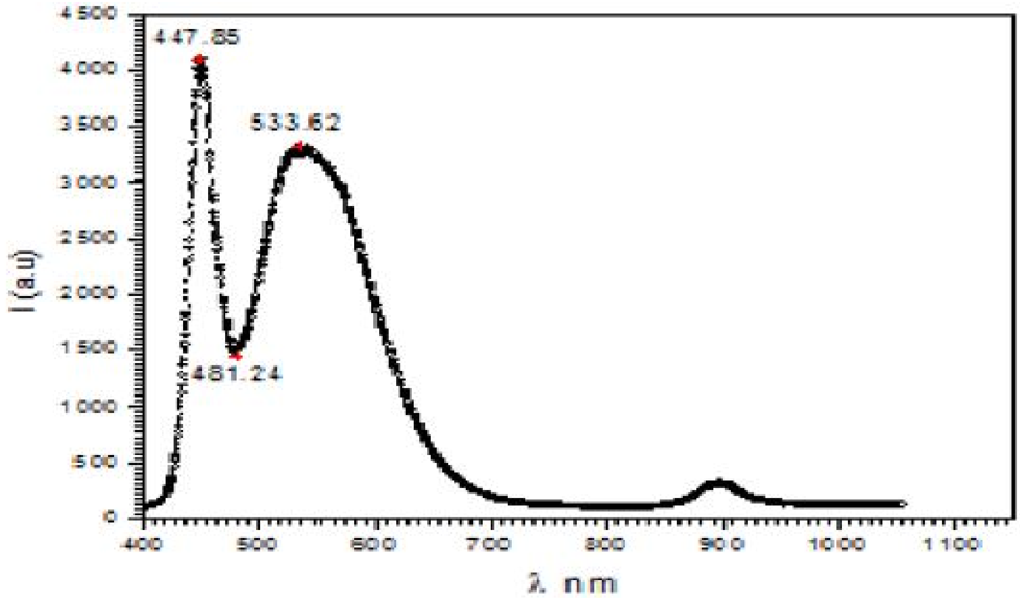


Figure 9.Spectrum of Alum for grinding time 10 sec by using white light

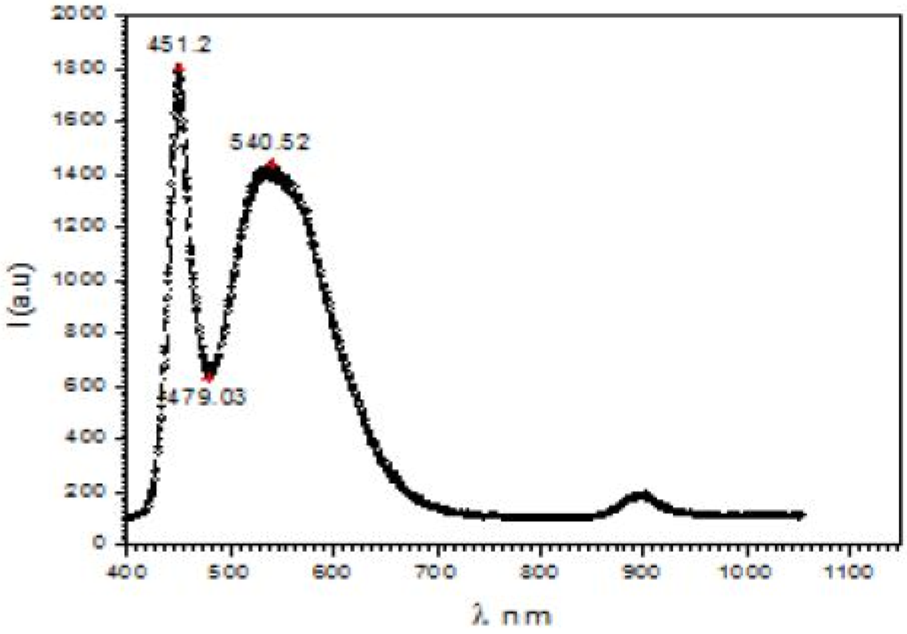


Figure 11. Spectrum of Alum for grinding time 30 sec by using white light

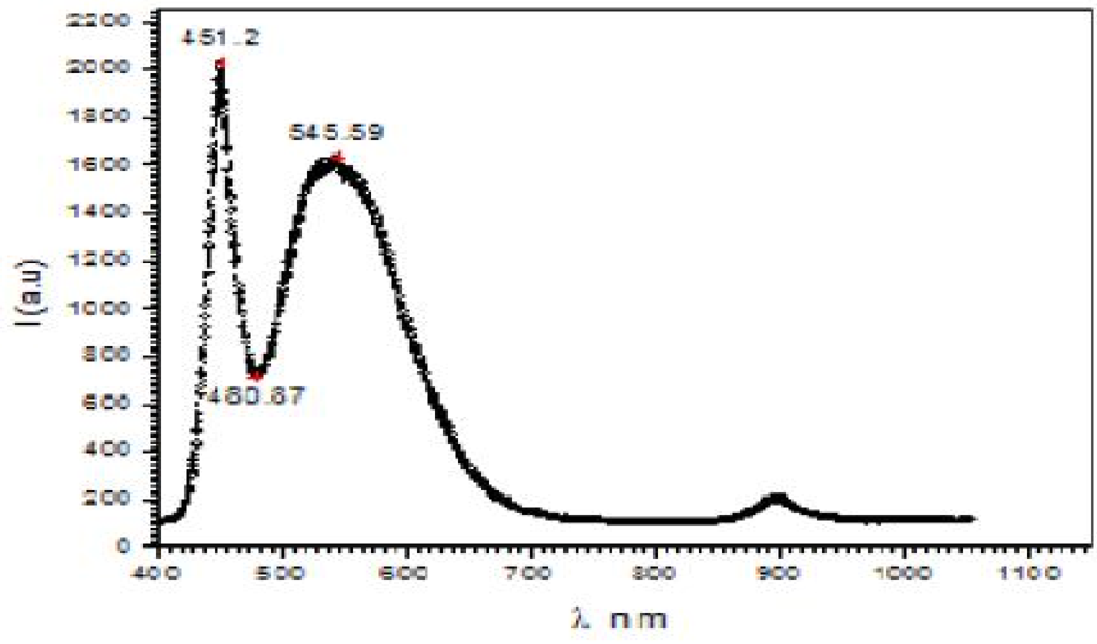


Figure12.Spectrum of Alum for grinding time 40 sec by using white light

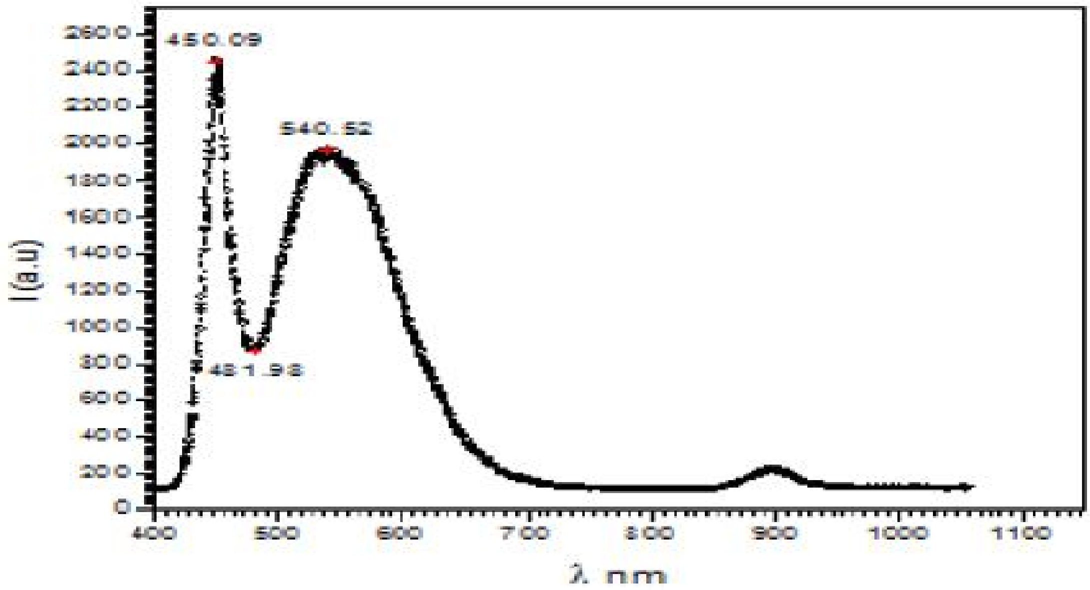


Figure 10. Spectrum of Alum for grinding time 20 sec by using white light

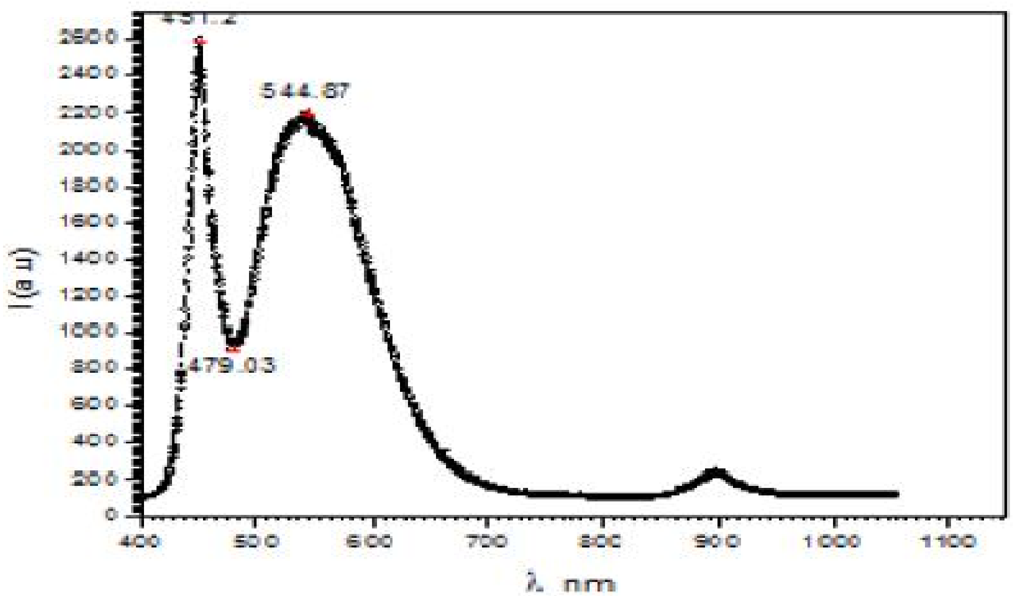


Figure13. Spectrum of Alum for grinding time 50 sec by using white light

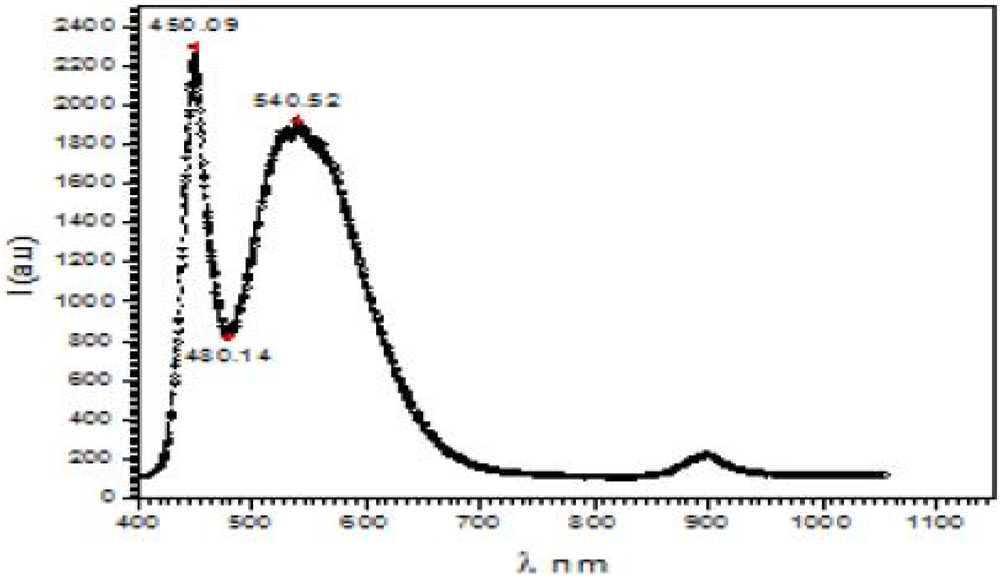


Figure14. Spectrum of Alum for grinding time 60 sec by using white light

Table 1. Time of grind and intensity of light and laser diode of sample Alum

Grinded time	Light Intensity	Laser diode Intensity
0	616.691	1041.884
10	4096.76	4056.828
20	2466.576	1892.356
30	1812.196	2208.703
40	2018.107	3397.867
50	2601.989	2901.088
60	2343.258	1695.391

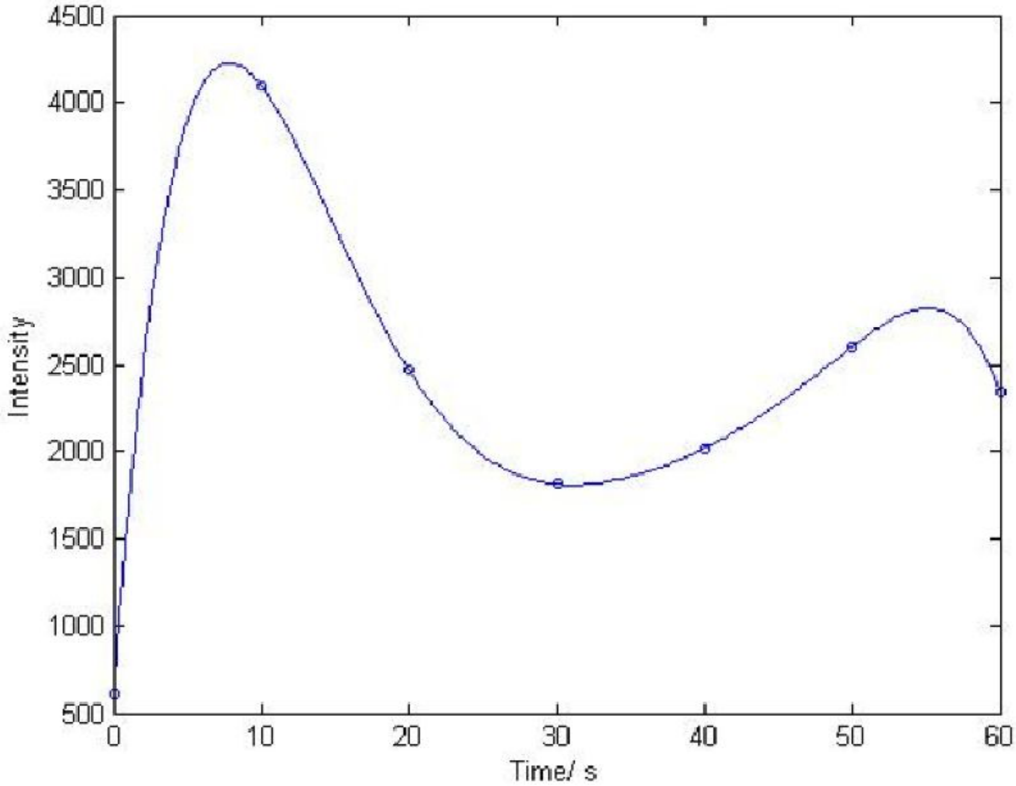


Figure 15. Grinding time versus highest beak intensity of Alum when using white light

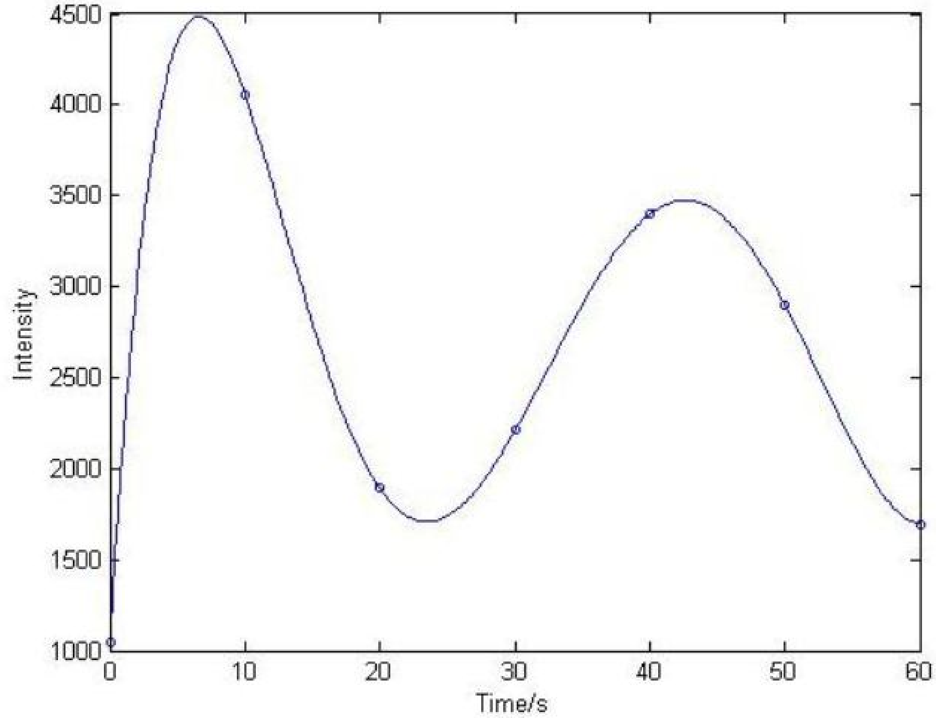


Figure 16. Grinding time versus highest beak intensity of Alum when using laser diode

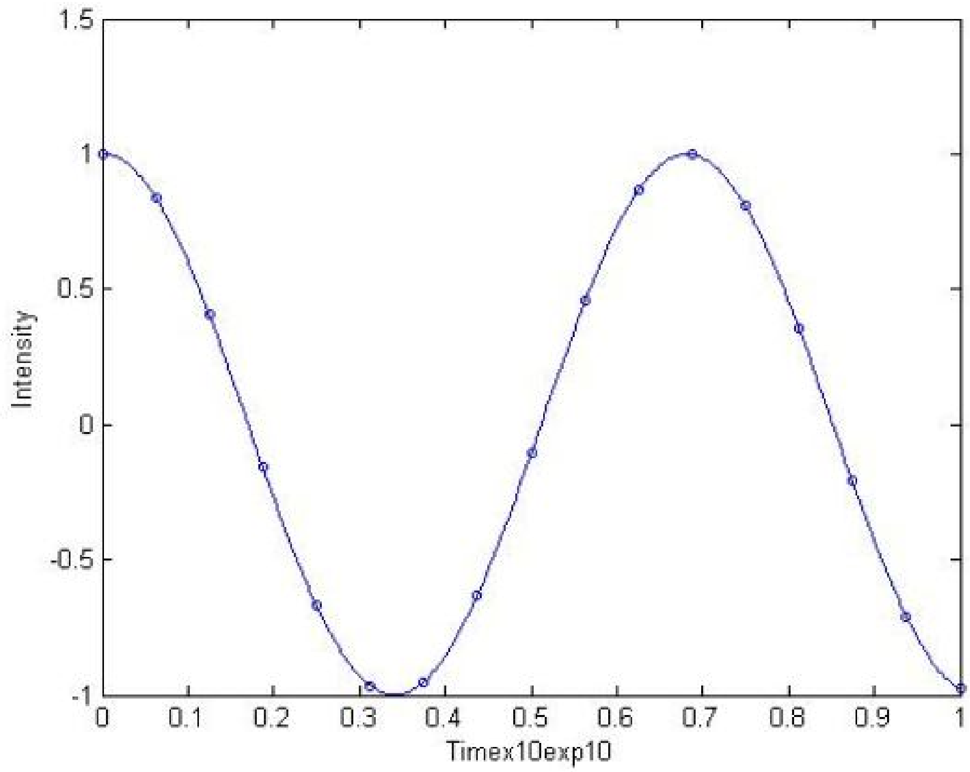


Figure 17 Theoretical relation between Intensity and grind time[see equation (15)]

3.5.3 Results and Discussion:

It is well known that the grinding of crystals convert them to powder consisting of tiny particles with small diameter and size. The increase of grinding time should decrease the particle size as it leads to more fragmentation and disintegration. Thus one expects the particle size (d) to be inversely proportional to the grinding time(τ);

$$d\alpha\frac{1}{\tau}$$

The empirical relations which are displayed graphically in Figs. (15. 16) grinding time versus light intensity shows wavy sine or cosine relation with variable amplitude. This means clearly that the change of Nano size affects and changes optical absorption as well as transmition. These changes are not liner but shows wavy pattern. This means that there critical sizes at which absorption or transmition takes place. This relation between particle size and light intensity can be used in the design and

fabrication of optical sensors and solar cells. These wavy relations resemble the theoretical relations in Fig (17), strictly speaking Fig (17). This means that by fine tuning the parameters in equation (3.5.15) it is possible to find theoretical relations between intensity (I) and time of grinding (resemble to a great extent the empirical relations in Fig (17).
This means that for first time a theoretical relations which relate (I) to the nanosize$d\alpha\frac{1}{\tau}$ is verified empirically and experimentally .The spectrum in figs (15. 16), however, doesnot show appreciable change in the wave length of the incident light.

The theoretical relation which related light radiation intensity (I) to the grinding time (τ) as shown by equation (3.5.15) is based on quantum mechanics. The energy of photons is found by assuming photons resulting from transfer of electrons from the bound state $E_v$toFree State zero. The energy of bound electrons is found by using tight binding model which assumes electron energy as resulting from the effect of the nucleus on its host atom, beside the neighboring atoms. The use of this model is justifiable for nanoparticles since the electrons of each atomis affected by neighboring atoms. The wave function and the electron intensity is found by using Schrodinger equation and treating the electron as particle in a box. This is quite obvious as far as nanoparticles are isolated from each other such that they resemble particle in a box in one dimension.

3.6 The Quantum Expression of the Role of Effective Mass in the Classical Electromagnetic Theory Form & in Absence of Binding Energy

M. El toom proposed a quantum model to the change of electron mass in crystal

The conventional expression for the effective mass was introduced to account for the effect of the crystal field on the mass. This definition is based on the expression of energy (E) for a free particle, which takes the form [50]

$$E = \frac{\hbar^2 k^2}{2m} \tag{3.6.1}$$

With (k) standing for the wave number. The effective mass is thus given by:

$$m^* = \hbar^2 ({\nabla^2}_k E)^{-1} \tag{3.6.2}$$

The external force is satisfied:

$$\frac{dP}{dt} = \hbar \frac{dE}{dt} = F_{ex} \tag{3.6.3}$$

The velocity (v) is related to the energy according to the relation:

$$E = \frac{\hbar^2 k^2}{2m^*}$$

Where

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m^*} = \frac{\hbar P}{m^*} = \hbar v$$

Thus:

$$\hbar \frac{dv}{dk} = \frac{d^2 E}{dk^2} = {\nabla^2}_k E \tag{3.6.4}$$

Therefore the total effect of external force and lattice crystal force (F_L) are responsible for acceleration. Therefore:

$$m_0 \frac{dv}{dt} = m_0 \frac{dv}{dk}\frac{dk}{dt} = F_{ex} + F_L$$

$$m_0 (\hbar^{-2} {\nabla^2}_k E) = \frac{F_{ex} + F_L}{F_{ex}}$$

$$\frac{m_0}{m_*} = \frac{F_{ex} + F_L}{F_{ex}}$$

$$m_* = m_0 \left(\frac{F_{ex} + F_L}{F_{ex}}\right) \tag{3.6.5}$$

In the absence of crystal field the effective mass reduces to the ordinary mass, i.e.:

$$F_L = 0 \quad , \quad m_* = m_0 \tag{3.6.6}$$

However when the external force only disappear the effective mass become:

$$m_* = 0 \tag{3.6.7}$$

The zero mass is physically unacceptable, since the absence of external force leads to acceleration vanishing mass, not vanishing:

$$m_* a = F_{ex} = 0$$

$$a = 0$$

$$m_* \neq 0$$

In the absence of two forces:

$$m_* = m_0 \left(\frac{0}{0}\right) \tag{3.6.8}$$

Where $\left(\frac{0}{0}\right)$is an unknown quantity. The natural and logical result should lead

$$m_* = m_0 \tag{3.6.9}$$

In the absent of the two forces.

It is also illogical to assume a crystal field affecting the mass to change it from (m_0) to (m_*) and prohibiting the external are to affect this mass. Thus argument is in direct conflict with Einstein generalized special relativity (EGSR), which states that the mass is affected by any field, according to the relation [3, 4]:

$$m = g_{00}\, r\, m_0 \tag{3.6.10}$$

With

$$r = \left(g_{00} - {v^2}/{c^2} \right)^{-1} g_{00} = 1 + \frac{2\emptyset}{2c^2} \qquad (3.6.11)$$

Where ($\emptyset$) being representing the potential per unit mass while (c) is the speed of light in vacuum.

3.6.1 Effective Mass Quantum Expectation Value:

According to the laws of quantum mechanics the expectation value of the Hamiltonian coincide with the classical expression of energy. If one assumes the free particle Hamiltonian to be$\hat{H}_0$, its expectation value is equal to the classical kinetic energy [5, 6, and 7].

$$\langle \hat{H}_0 \rangle = \int \bar{\Psi} \hat{H}_0 \Psi = \frac{P^2}{2m} = \frac{\hbar^2 k^2}{2m} \qquad (3.6.12)$$

When the particle enters the crystal its Hamiltonian $\hat{H}$will be the sum of free Hamiltonian$\hat{H}_0$beside the crystal potential V_L. Thus the expectation value can be made equal to the kinetic energy by assuming the particle to be free where the effect of crystal potential manifests itself only through the masses.

$$\langle \hat{H} \rangle = \int \bar{\Psi} \hat{H}_0 \Psi dr = \frac{P^2}{2m^*} = \frac{\hbar^2 k^2}{2m} \qquad (3.6.13)$$

Where

$$\hat{H} = \hat{H}_0 + V_L \qquad (3.6.14)$$

Thus equation (3.6.13) takes the form:

$$\frac{\hbar^2 k^2}{2m^*} = \int \bar{\Psi} \hat{H}_0 \Psi dr = \int \bar{\psi} V_L \psi dv = \langle H_0 + V_L \rangle \quad (3.6.15)$$

In view of equation (3.6.12) and (3.6.15) one gets:

$$\frac{m^*}{m} = \frac{\langle H_0 + V_L \rangle}{\langle H_0 \rangle} = \frac{\int \bar{\Psi} \hat{H} \Psi dr}{\int \Psi \hat{H}_0 \Psi dr} = \frac{\int \bar{\Psi} H_0 \Psi dr + \int \bar{\Psi} V_L \Psi dr}{\int \bar{\Psi} \hat{H}_0 \Psi dr}$$

$$\frac{m^*}{m} = \frac{\langle H_0 + V_L \rangle}{\langle H_0 \rangle} \qquad (3.6.16)$$

From the correspondence principle

$$\langle H_0 \rangle = E_{0c}$$

$$\langle V_L \rangle = V_{Lc} \tag{3.6.17}$$

Where

$$E_{0c} \quad Classical\ value$$

Hence

$$\frac{m^*}{m} = \frac{E_{0c}}{E_{0c} + V_{Lc}} \tag{3.6.18}$$

If one considers the effect of the potential V_e, equation (3.6.16) can be written in the form:

$$\frac{m^*}{m} = \frac{\langle H_0 + V_e \rangle}{\langle H_0 + (V_e + V_L) \rangle}$$

$$= \frac{\langle H_0 \rangle + \langle V_e \rangle}{\langle H_0 \rangle + \langle V_e \rangle + \langle V_L \rangle}$$

$$= \frac{E_{0c} + V_{ec}}{E_{0c} + V_{ec} + V_{Lc}} \tag{3.6.19}$$

On other hand the potentials are related to the forces, according to the identity:

$$V_{ec} = \langle V_e \rangle = \int Fedx$$

Thus for constant forces [3.6.19,3.6.20]:

$$V_{ec} = -\int_0^d F_e dx = F_L d$$

$$V_{Lc} = -\int_0^d F_L dx = F_L d$$

$$V_{ec} = F_{ed}$$

$$V_{Lc} = F_{Ld} \tag{3.6.20}$$

With the aid of (8), (9) one gets:

$$\frac{m^*}{m} = \frac{E_{0c}\, d + V_{ec} d}{E_{0c} d + V_{ec} d + V_{Lc} d} = \frac{E_{0c} + F_e}{E_{0c} + F_e + F_L} \tag{3.6.21}$$

If one neglects E_0 , as far as the zero point energy is equal to zero in classical mechanics' or by considering the external and crystal field to be large then equation (3.6.21) reduces to[3.6.21, 3.6.22]:

$$\frac{m^*}{m} = \frac{F_e}{F_e + F_L} \tag{3.6.22}$$

Thus the quantum effective mass formula reduces to the conventional one.

It is very striking to note that according to equation (3.6.13) the effective mass (m^*) reduces to the ordinary mass or rest mass in the absence of crystal field. The same hold for the equation (3.6.21), where the absent of external and crystal field reduces (m^*) to (m_0) .This result is in conformity with (EGSR).

3.6.2 Effective (EGSR) Mass:

The (EGSR) automatically incorporates the field effect of both external and crystal field. This can be checked with the aid of relations (3.6.21, 3.6.22) by considering the potential (V) to be due to external and crystal field [12, 13]. Thus

$$V = m_0\emptyset = V_e + V_L \tag{3.6.23}$$

Thus equation (3.6.21) reads

$$m = \frac{m_0\left(1 + \frac{2\emptyset}{c^2}\right)}{\sqrt{1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}}}$$

For:

$$\emptyset \ll c^2 \tag{3.6.24}$$

$$m = m_0\left(1 + \frac{2\emptyset}{c^2}\right)\left(1 + \frac{2\emptyset}{c^2} - \frac{v^2}{c^2}\right)^{-1/2}$$

$$= m_0\left(1 + \frac{2\emptyset}{c^2}\right)\left(1 + \frac{2\emptyset}{c^2} + \frac{1}{2}\frac{v^2}{c^2}\right)^{-1/2} \qquad (3.6.25)$$

Neglecting higher power:

$$m = m_0\left(1 - \frac{2\emptyset}{c^2} + \frac{1}{2}\frac{v^2}{c^2} + \frac{2\emptyset}{c^2}\right)$$

$$m = m_0\left(1 + \frac{\emptyset}{c^2} + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$= m_0\left(1 + \frac{m_0\emptyset}{m_0c^2} + \frac{m_0v^2}{2m_0c^2}\right)$$

$$= m_0\left(1 + \frac{m_0\emptyset}{m_0c^2} + \frac{m_0v^2}{2m_0c^2}\right)$$

$$= \frac{m_0}{E_0}(E_0 + V + T) \qquad (3.6.26)$$

For external field only:

$$\frac{\hbar^2k^2}{2m_e} = \frac{P^2}{2m_e} = m_ec^2 = \frac{m_0}{E_0}(E_0 + V_e + T)c^2 \qquad (3.6.27)$$

For external and crystal field:

$$\frac{\hbar^2k^2}{2m^*} = m^*c^2 = \frac{m_0}{E_0}(E_0 + V_e + T)c^2 \qquad (3.6.28)$$

Thus

$$\frac{m_e}{m^*} = \frac{E_0 + V_e + V_L + T}{E_0 + V_e + T} \qquad (3.6.29)$$

With

$$E = E_0 + T \qquad (3.6.30)$$

It is clear that relations (3.6.18) resemble relations (3.6.30).

It is very interesting to note that the two expressions indicate the effect of external field on the mass via the terms $V_e$as well as the effect of the crystal field via the termsV_L. Expression (3.6.18) reduces to [14, 15, and 16]:

$$\frac{m^*}{m} = \frac{E_0 + V_e}{E_0 + V_e + V_L} = \frac{V_e d}{V_e d + V_L d} = \frac{F_e}{F_e + F_L} \quad (3.6.31)$$

Here the rest mass energy is neglected. Also

$$m^* = m \quad (3.6.32)$$

Where there is no field affects the mass, i.e.

$$V_e = 0 \qquad V_L = 0$$

3.6.3 Harmonic Oscillator:

In this approach the particle flux is assumed to be a harmonic oscillator having potential [17, 18]:

$$V_0 = \frac{1}{2} k_0 x^2$$

Thus, its Hamiltonian is given by:

$$H = \frac{\hbar^2}{2m}\nabla^2 + V_0 = \frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}k_0 x^2 = H_0 + \frac{1}{2}k_0 x^2 \quad (3.6.33)$$

When the flux enters the material the lattice field potential, affect the flux. Thus the new Hamiltonian is given by [19]:

$$H = H_0 + \frac{1}{2}k_0 x^2 + V_L = H_0 + V_0 + V_L = H_0 + V_L \quad (3.6.34)$$

If one assumes that the wave function inside the material is not affected so much as the flux enters the bulk matter. In this case the total energy is given by:

$$E = \langle H \rangle = \int \overline{u_n}(H_0 + V)\, u_n dr = \int \overline{u_n} H_0\, u_n dr + u_n V u_n dr$$

$$= \langle H_0 \rangle + \int \overline{u_n} H_0\, u_n dr + \int \overline{u_n} V_L\, u_n dr$$

Thus the average energy is gives as a sum of kinetic part, Harmonic part, and crystal contribution, i.e.:

$$E = \langle H \rangle = \langle H_0 \rangle + \langle V_0 \rangle + \langle V_L \rangle \tag{3.6.35}$$

In the absence of crystal field the average energy for harmonic oscillator is given by [19]:

$$E = \left(n + \frac{1}{2}\right)\hbar\omega \tag{3.6.36}$$

The average contribution of the potential part takes the form:

$$\langle V_0 \rangle = \frac{1}{2}\left(n + \frac{1}{2}\right)\hbar\omega \tag{3.6.37}$$

Thus invies of (3.6.36) and (3.6.37) the kinetic part is given by:

$$\langle H_0 \rangle = E - V = \frac{1}{2}\left(n + \frac{1}{2}\right)\hbar\omega \tag{3.6.38}$$

In the presence of external field V_e . The energy becomes:

$$\frac{\hbar^2 k^2}{2m} = E = \int \overline{u_n}(H_0 + V_0 + V_e)\, u_n dr = \langle H_0 \rangle + \langle V_0 \rangle + \langle V_L \rangle$$

$$\frac{\hbar^2 k^2}{2m} = E = \langle H_0 \rangle + \langle V_0 \rangle + \langle V_L \rangle \tag{3.6.39}$$

But according to quantum mechanics:

$$\langle H_0 \rangle + \langle V_0 \rangle + \langle V_e \rangle = \frac{\hbar^2 k^2}{2m} \tag{3.6.40}$$

In the presence of additional crystal potential (V_c), the energy becomes:

$$\frac{\hbar^2 k^2}{2m^*}\langle H_0 \rangle + \langle V_0 \rangle + \langle V_L \rangle + \langle V_e \rangle \tag{3.6.41}$$

Hence

$$\frac{m}{m^*} = \frac{\langle H_0 \rangle + \langle V_0 \rangle + \langle V_L \rangle + \langle V_e \rangle}{\langle H_0 \rangle + \langle V_0 \rangle + \langle V_e \rangle} \tag{3.6.42}$$

If the contribution of harmonic energy is neglected:

$$\frac{m^*}{m} = \frac{\langle V_e \rangle}{\langle V_e \rangle + \langle V_L \rangle} = \frac{\langle V_e \rangle d}{\langle V_e \rangle d + \langle V_L \rangle d} = \frac{F_e}{F_e + F_L} \tag{3.6.43}$$

Which is the conventional expression for effective mass? In the absence of all fields equation (3.6.43) reads:

$$\frac{m^*}{m} = \frac{\langle H_0 \rangle}{\langle H_0 \rangle} \quad , \quad m^* = m \tag{3.6.44}$$

Where when the force vanishes, i.e.

$$F = 0$$

The acceleration and the mass rate of change vanishes i.e. the acceleration and the mass [22]:

$$a = 0 \qquad \frac{dm}{dt} = 0 \tag{3.6.45}$$

But

$$m \neq 0 \tag{3.6.46}$$

3.6.4 Discussion:

The main drawback of conventional expression for the effective mass, the vanishing of it in the absence of the external force (see equation (3.6.7)) which is in direct conflict with Newton's second law, which states that the acceleration, not the mass, vanishes as shown by equations (3.6.36,3.6.46,and 3.6.46). The second drawback is the equality of the effective mass to the rest mass in the absence of crystal field, according to equation (3.6.5). This result is confusing since it shows that the crystal field affects the mass, while the external field does not. It is very hard to believe that some fields affect the mass, while others cannot. This result also disagrees with (EGSR) which states that the mass is affected by any field.

Models are two free froms. The first model in the absence of kinetic term, as equation (3.6.22) shows. The effective mass in equation (3.6.19) reduces the rest mass, only when all fields vanish (equation (3.6.19)). This result conforms to (EGSR) as shown by equations (3.6.10, 3.6.11).

The second model is based on (EGSR), beside the quantum expression of the energy of the free non relativistic particle. This model also reduces the conventional one when the rest mass energy is neglected which is shown by equation (3.6.31). The mass (m^*) reduces in equation (3.6.32), to (m) when no field exist. Applying the expressions for (m^*) to the harmonic oscillator in section five confirms the fact that the expressions for (m^*) is equal to (m) for free space field.

3.7 quantum explanation of conductivity at resonance

A. M. El hussien proposed a quantum model explain the conductivity at resonance with frequency [51]

In this experiment a transmitter coil emits electromagnetic waves. These electromagnetic waves are allowed to incident on certain materials. The re emitted electromagnetic waves are receipted by a receiver

3.7.1Apparatus:

- 10Resistors (10kΩ, 2.2GΩ, 39kΩ),12 Capacitors (0.1μF, 0.01μF, 220μF), 6 Transistors (NPN), 2 transmitter and receptor Coils (400,500, 600, 700, 1000 turns), Wire connection, Speakers, Cathode Ray Oscillator, Board connection, Battery (9V),Signal generator.

3.7.2 Samples:

A pieces of metal (Cu, Al, Fe, Au,Ag, Sn).

3.7.3 Method:

The transmitter coil current is varied by using signal generator. The emitted photons are allowed to inciden ton the sample. The sample absorbs photons and re emits them. The metal detector receipt photons the signals appearing at oscilloscope were taken before mounting the sample, and after photon emission. The frequency and the corresponding conductivity of sample are recorded and determined from signal generator, current, voltage, and the length and crosssectional area of samples. The current and voltage gives resistance, which allows conductivity determination from the dimensions of the sample.

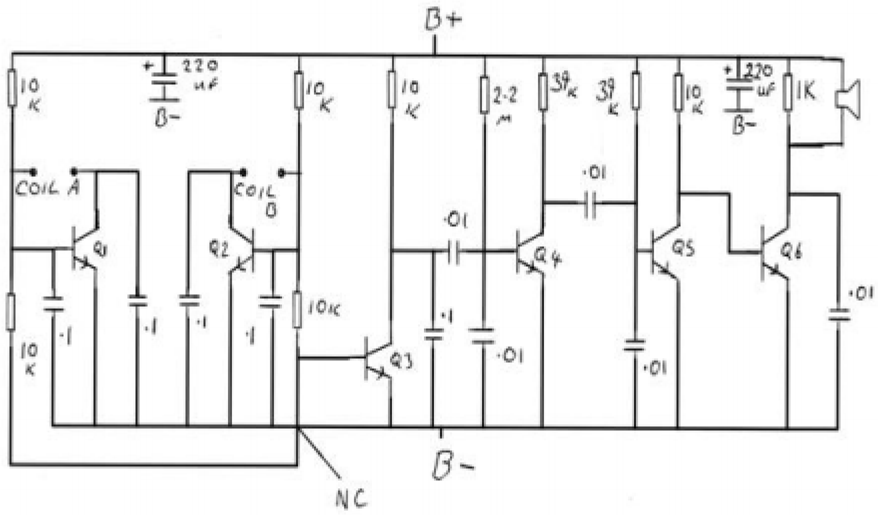


Fig (3.7.1)

3.7.3.1: Tables and Results:

Table (3.7.3.1) Relation between frequency (f) and Conductivity (σ) without applied magnetic field for Cu, Al, Fe, Au, Ag, Sn

Frequency (Hz)	Conductivity ((10^6 cm.Ω)
24	0.452
27	0.596
29	0.0993
34	0.0917
50	0.143
56	0.337

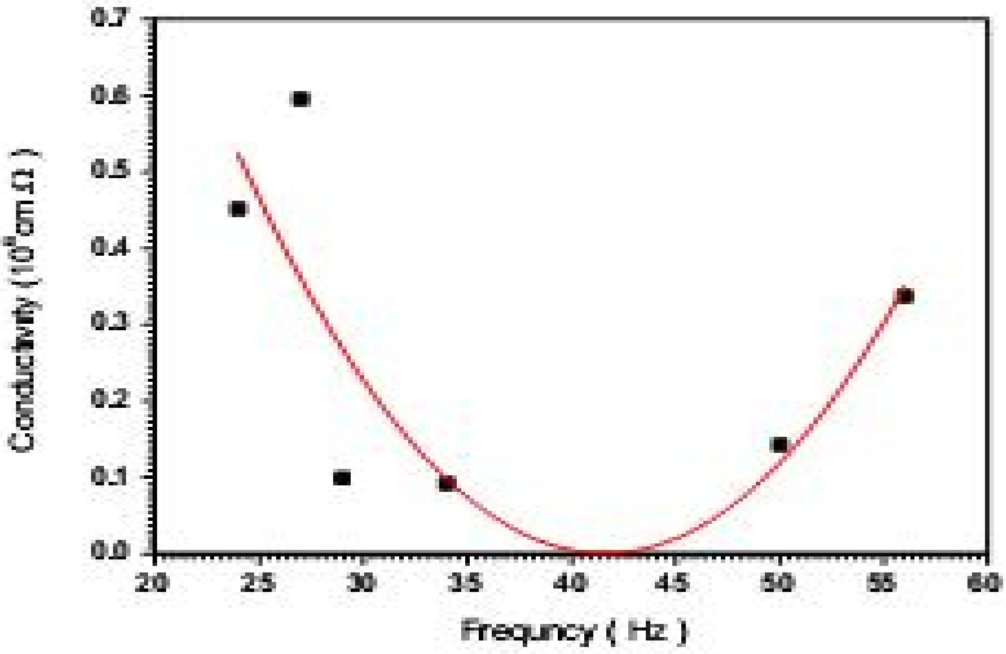


Fig (3.7.1.1) Relation between resonance frequency and Conductivity for Cu, Al, Fe, Au, Ag, Sn

Table (3.7.2) Relation between frequency (f) and Conductivity (σ) for different magnetic flux densities for gold.

Frequency (Hz)	Conductivity (10^6 cm.Ω) In 97.3 μT	Conductivity (10^6 cm.Ω) In 77.μT	Conductivity (10^6 cm.Ω) In 116.7 μT	Conductivity (10^6 cm.Ω) In 136.2μT	Conductivity (10^6 cm.Ω) In 194.53 μT
55.25746	0.50026	0.12637	0.04739	0.15165	0.06824
47.48594	0.26365	0.03724	0.01396	0.04468	0.02011
40.49329	0.16368	0.03439	0.0129	0.04126	0.01857
35.4219	0.24726	0.05362	0.02011	0.06435	0.02896
26.8109	0.39107	0.1695	0.06356	0.2034	0.09153
24.81177	0.58753	0.2235	0.08381	0.2682	0.12069

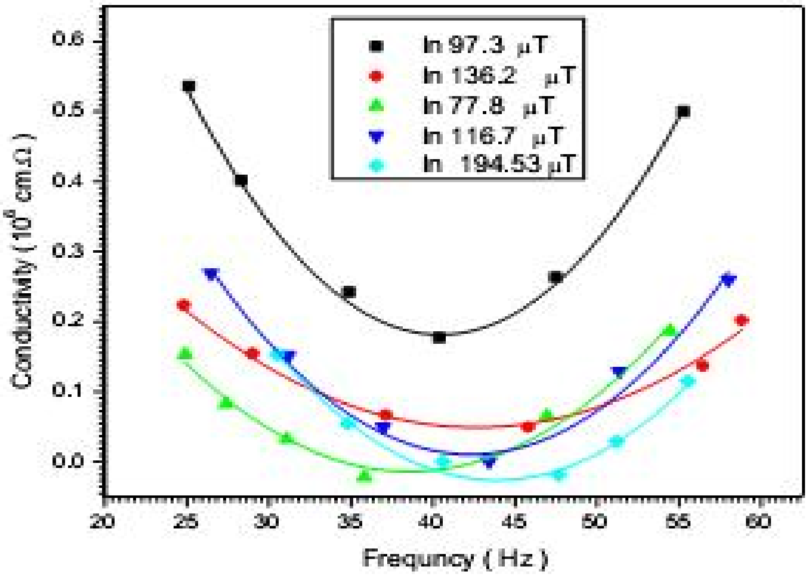


Fig (3.7.1.2) Relation between frequency (f) and Conductivity (σ)for different magnetic flux densities for gold

3.7.4 Theoretical Interpretation

3.7.3.1 Quantum Theoretical Model:

$$\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = -c^2 \hbar^2 \nabla^2 \Psi + {m_0}^2 c^4 \Psi \qquad (3.7.1)$$

Sub

$\Psi = u(r\_) f(t)\ in\ (1) yields$

$$-u\hbar^2 \frac{\partial^2 f}{\partial t^2} = -f c^2 \hbar^2 \nabla^2 u + {m_0}^2 c^4 f u \qquad (3.7.2)$$

$$-\frac{1}{f}\hbar^2 \frac{\partial^2 f}{\partial t^2} = -\frac{1}{u} c^2 \hbar^2 \nabla^2 u + {m_0}^2 c^4 = E^2 \qquad (3.7.3)$$

Where:

$$-\frac{1}{f}\hbar^2 \frac{\partial^2 f}{\partial t^2} = E^2 \qquad (3.7.4)$$

$$-\hbar^2 \frac{\partial^2 f}{\partial t^2} = E^2 f \qquad (3.7.5)$$

Where:

$\hbar\omega_0 = election\ energy\ in\ bounded\ State.$

$\hbar\omega = \ energy\ given\ to\ the\ election.$

$E = \hbar\omega - \hbar\omega_0 = excitation\ energy.$

Consider solution

$$f = \sin \alpha t \tag{3.7.6}$$

$$\hbar^2 \alpha^2 f = E^2 f \tag{3.7.7}$$

$$\therefore \hbar\alpha = E$$

$$\alpha = \frac{E}{\hbar} = \omega - \omega_0 \tag{3.7.8}$$

$$\because \sigma = \frac{ne^2\tau}{m} = \frac{|\Psi^2|e^2\tau}{m} = \frac{e^2\tau}{m}|\sin(\omega - \omega_0)t|^2 \tag{3.7.9}$$

At near resonance $\omega - \omega_0 << 1$

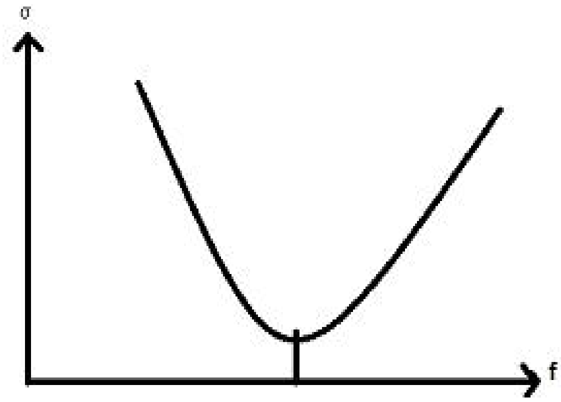


Fig (3.7.3.3) Theoretical relation between frequency (f) and Conductivity (σ).

$$\sin(\omega - \omega_0)t \approx (\omega - \omega_0)t$$

Inserting (3.7.10) in (3.7.9) yields:

$$\therefore \sigma = \frac{e^2\tau}{m}|(\omega - \omega_0)t|^2 \tag{3.7.11}$$

3.7.3.2 Classical Absorption Conductivity Resonance Curve:

Consider an electron of mass m oscillate with natural frequency ω_0.If an electric field of strength

$$E = E_0 e^{i\omega t} \tag{3.7.12}$$

Was applied, then the equation of motion of the electron, in a frictional medium of friction coefficientγ, is given by

$$m\ddot{x} = eE - m{\omega_0}^2 x - \gamma\dot{x} \tag{3.7.13}$$

Consider the solution

$$x = x_0 e^{i\omega t} \tag{3.7.14}$$

Thus

$$v = \dot{x} = i\omega x \qquad \ddot{x} = -\omega^2 x \tag{3.7.15}$$

Interesting (3.7.14) and (3.7.15) and (3.7.12) in (3.7.13) yields

$$-m\omega^2 x = e\frac{E_0}{x_0}x - m{\omega_0}^2 x - \gamma v$$

Thus

$$v = -m\frac{(\omega^2 - {\omega_0}^2)}{\gamma}x + e\frac{E_0}{\gamma x_0}x \tag{3.7.16}$$

For simplicity consider large displacement amplitude compared to the electrical one .Thus the last term in (3.7.14-3.7.15) can be neglected to get

$$v = -m\frac{(\omega^2 - {\omega_0}^2)}{\gamma}x \tag{3.7.17}$$

But the conductivity is given by

$$\sigma = \frac{e\tau}{m}n = \frac{e\tau}{m}n_0 e^{\frac{-\beta m {V_e}^2}{2}}\frac{V_0}{\sqrt{2}} \tag{3.7.18}$$

Where the effective value V_e is related to the maximum value through the relation

$$V_e = \frac{V_0}{\sqrt{2}} \tag{3.7.19}$$

For small value of the power of e, one can expand exponential term to be

$$e^{-x} = 1 - x \tag{3.7.19}$$

Therefore equation (3.7.18) becomes

$$\sigma = \frac{e\tau}{m} n_0 \left[1 + \frac{\beta m {v_0}^2}{4}\right] \tag{3.7.21}$$

Inserting (3.7.15) in (3.7.17) yields

$$\sigma = \frac{e\tau}{m} n_0 \left[1 + \frac{\beta m^2 {x_0}^2 (\omega - \omega_0)^2 (\omega - \omega_0)^2}{4\gamma}\right]$$

$$\sigma = \frac{e\tau}{m} n_0 \left[1 + \frac{\beta m^2 {\omega_0}^2 {x_0}^2 (\omega - \omega_0)^2}{\gamma}\right] \tag{3.7.21}$$

Where near resonance

$$\omega \approx \omega_0 \omega + \omega_0 \approx 2\omega_0 \tag{3.7.22}$$

The relation between conductivity and frequency resembles that of (3.2.21) in its dependence on ω .This relation is displayed graphically in Fig (3.7.3.1)

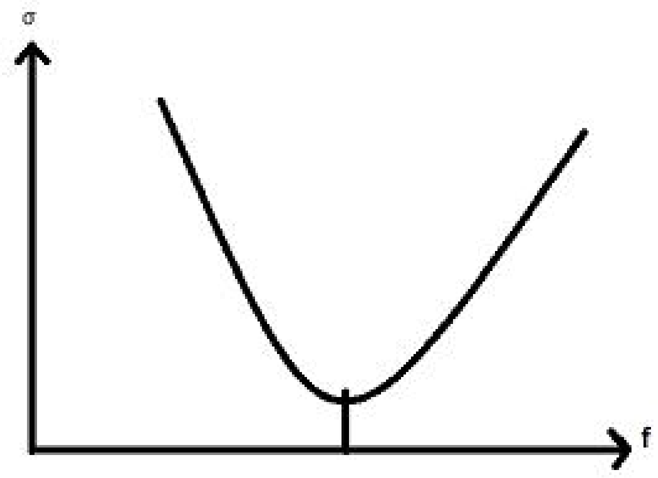


Fig (3.7.3.1) Theoretical relation between frequency (f) and Conductivity (σ).

3.7.5 Discussion

The experimental work which was done shows variation of conductivity for gold according to Figs (3.7.1) and (3.7.2). The conductivity decreases then attains a minimum value in the range of (40 - 50 Hz), then increases again.

The theoretical expression (3.7.11) which is displayed graphically in Fig (3.7.1) is based on the ordinary expression for the conductivity. The electrons density n is found by solving Klein-Gordon equation for free particle. This is obvious as far as conduction electrons are free. The electron density is found from the square of the wave function, which is a sin function. Since at resonance ω is very near to ω_0, thus one can replace$s\, in\, x$ by x. The theoretical relation for f and σobtained by this model resembles the experimental for one in Fig (3.7.11).

Another classical approach based on Maxwell –Boltzmann distribution in section (3) shows a relation between and f in Fig (3.7.3.1) similar to experimental relation. The relations between σ and f resembles that of resonance, with minimum conductivity.

It is very interesting to note that each element has its own resonance conductivity at which conductivity is minimum.

In this model the ordinary expression for σ in equation (3.7.9) is used. But n here is found from Maxwell statistical distribution.

Chapter Four

Introduction:

4.1 Quantum Relativistic Frictional Model and String Theory:

Special relativity is one of the big an achievement that relates mass, space and time. However its energy suffers from the Lack of potential expression in energy relation leads to appearance of new version of SR, called generalized SR (GSR). This new GSR has an energy term representing a potential energy and satisfies a Newtonian limit [61.62.63]. This success of GSR encourages using the conventional expression of kinetic and potential energy to see how energy conservation looks like in SR and GSR. This task is done in this chapter.

4.2 Energy Conservation:

The energy conservation in special Relativity on the Basis of Force relation with kinetic and potential energy .According to the very definition of potential energy V and Kinetic energy T, they can be defined in one dimension as:

Since:

And by defining

c

Equation (4. 2. 4) reads

m

Utilizing equation (4. 2. 5) again

Therefore

Thus in view of equation (4. 2. 3)

Thus the kinetic energy within the frame work of SR is given by:

Clearly the sum of kinetic energy and potential one is a constant of motion, i.e. it represents the total energy E which is conserved according toSR, thus

$E =$

By redefining the energy to be:

Then:

4.3 Energy Conservation on GSR on the Basis of Ordinary Relating which is the Generalized Special Relativistic (GSR).

Then:

By defining:

n

In view of equation (4. 2. 4) and redefining (4. 2. 5) to be:

Equation (4. 2. 4) reads:

$$mv^2 + mc^2 + 2m\emptyset - mv^2 = -V + c_5$$

According to equation (4. 2. 3)

By defining V to be

This equation is inconsistent with equation (4.3.5) even if one define the kinetic energy to be

4.4 Energy Conservation in GSR on the Basis of New Relation between Force and Kinetic Energy.

However one can redefine the relation between the force and kinetic energy to be

For particles having constant mass:

$$F = \frac{1}{2}$$

Hence the definition of kinetic energy in terms of the force is consistent with the formal definition of force for particles having constant mass.

Thus according to this definition (4.4.1) together with definition (4.2.1):

Therefor:

Let:

However for T based on relation (4.2.10) and (4.4.1) requires:

$F = -\frac{d(\ldots}{\ldots}$

Since:

Thus for positive T

One can define:

Where

Thus:

This requires:

$$T$$

Squiring both sides:

$$\frac{1}{2}$$

To make this consistent with the fact that rest mass energy should exit in any relativistic expression, one can suppose that:

To get:

Thus according to relation (4.4.7):

With the aid of equations (4.4.8), (4.4.9) and (4.4.14) requires:

Thus the energy conservation requires:

$$2C_0 = m_0c^2C_0 = \frac{1}{2}m_0c^2 \qquad (4.4.16)$$

4.5 Energy Conservation in GSR Based on new Force and Potential Energy Relation:

Another approach can be tackled by assuming the kinetic energy T and the potential energy to be defined by:

$$T = \int \frac{d}{\ }$$

$$= \int d($$

$$\int$$

Defining:

One gets:

mv^2

Thus the energy conservation requires:

In the classical limit, when:

$E = m_0$

$= m_0$

Which is the conventional ordinary Newton energy relation with additional term standing for rest mass energy

4.6 Relativistic Quantum Frictional Equation:

If a particle move in frictional medium, its velocity and energy are lowered. This is since friction force apposes motion. Thus energy is dissipated to overcome friction effect. Relaxation time τ can be found from uncertainty principle by using the relation:

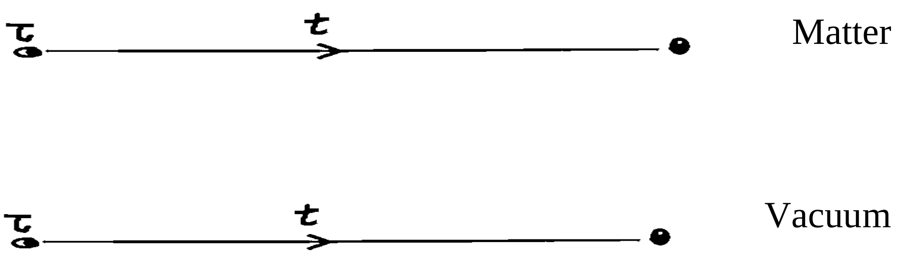
$$\Delta E \Delta \tau = \hbar \quad (4.6.1)$$

It is well known in laser physics that when an election is excited from ground state E_1 to an exited state E_2. The electron takes time τ in an excited state before returning back to the ground state. The relaxation time τ can be found from equation (4.6.1) to be:

Where:

Therefore the energy loss by the electron when it leaves E_2 to E_1 is given by:

This relation can be used to describe the lowering of the photon or light speed inside matter. When a photon is incident on a certain atom, it can be absorbed by it to make an electron leave E_1 to an excited state E_2 by emitting a photon. It return back to E_1. This means that the photon, instead of taking time *t* moving with speed *c*:



in vacuum, it take a time $t + \tau$ inside matter, with delay time τ. This means that as if light moves with apparent speed v inside matter where:

While it moves the same distance in vacuum with speed c, such that:

From (4.6.5) and (4.6.6):

Thus the refractive index n is given by:

Therefor the relaxation time is given by:

Where a here stands for the distance between neigbouring atoms. This delay time thus is responsible for lowering the light speed from $c\ to\ v$.this is equivalent to existence of friction that causes energy loss given by equation (4.6.4) to be:

This means that when a particle of original energy E_0 enters a frictional medium its energy is lowered to become

Where one uses the complex representation proposed Dirar [64.65.65].

According to the special relativistic theory the original energy is given by:

But if one defines the force to be:

Where V stands for potential energy:

Using the relation:

One can easily find that:

This constant of motion is assumed to represent the energy E_0 where:

For very small rest mass energy, one can write:

In view of equation (4.6.5) the energy in the pressure of friction:

E

4.7 Special Relativistic Quantum Frictional Equation:

To find quantum equation for this expression of relativistic energy for frictional medium, one multiply (4.6.18) by the wave:

Using the wave function for quantum system:

One can obtain:

Thus inserting equation (4.7.2) in equation (4.7.1) yields:

Another two new equations can also be obtained from (4.6.16) to get:

The friction effect can be replacing $E_0$by$E$, where:

Thus equation (4.7.4) reads:

$V^2 +$

$V^2 + E_0{}^2$

Multiplying both sides by Ψ yields:

$= $

Using the wave-particle dual nature relation:

Yields:

$-\hbar^2 \frac{\partial^2}{\partial t}$

Inserting (4.7.8) in (4.7.7) results in the following equation:

$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2}$

$= $

Which is the relativistic equation in the presence of friction. In the absence of friction equation (4.7.8) reduces to:

$-\hbar^2 \frac{\partial^2}{\partial}$

4.8 Harmonic Oscillator solution:

To solve (4.7.9) for to simplify the oscillator, it is suitable to simplify the equation by suggesting:

To get:

E

For harmonic oscillator.

Therefore x is small thus one can neglect terms like $V^2$to get:

Since:

Consider now a solution:

Substituting (4.8.6) in (4.8.5) yields:

$[E^2 - m\omega$

Comparing the free terms and coefficients of x^2 on both sides yields:

Dividing (4.8.8) by (4.8.8) after ignoring the rest mass yields:

To find E, by ignoring m_0 for very small mass:

But according to Einstein equation:

Thus:

Hence:

Another solution can be proposed by using periodicity condition of equation (4.8.1), where

Which requires:

Thus:

$$E = \frac{2\pi}{T} n\hbar = n\hbar\omega \qquad (4.8.15)$$

Substituting in (4.8.10) yields:

Thus the mass is quantized.

From (4.8.9), (4.8.15) and (4.8.16):

4.9 Discussion:

Many relativistic expressions for energy that satisfies energy conservation were discussed. In the first one the ordinary SR expression for mass in equation (4.2.4), beside the ordinary definition of force in equation (4.2.1) were used to find expression E- the kinetic energy T be equal to mc^2. T is not like Newtonian one, but for small v

Thus it resembles Newtonian one with additional rest mass energy term. The SR energy reduces to Newtonian one in equation (4.2.12) and is conserved. It is more is advance than SR one since it is conserved and consists of potential term .Another E is based on GSR beside expression for (m) in equation (4.3.1).

The energy is conserved according to equation (4.3.7) but V becomes with a minus sign.

Anew definition of force in equation (4.4.1) with ordinary expression for T is also used to define conservative E. the potential is related to F in a conventional way. Conservation was satisfied in equation (4.4.9) but T is defined in different way, i.e.

The energy lost by friction can be found by using uncertainty principle in section (4.6). Equation (4.6.4) shows that it is inversely proportional to relaxation time τ. This expression conforms to the classical one in which friction energy is inversely proportional to τ. This friction energy is added to the SR energy found by adding potential terms to secure energy conservation as shows in equation (4.6.15). By neglecting rest mass the energy is given by (4.6.17). The final SR term is given in equation (4.6.18) consisting of kinetic, potential and frictional terms. Using the wave equation for particles in equation (4.7.2) a relativistic quantum expressions for neglected rest mass and non-neglected m_0 were found as shown by equations (4.7.3) and (4.7.8).

Neglecting friction, one finds equation (4.7.9) this equation is a modified SR equation. This equation (4.7.9) is used to solve for harmonic oscillator, within the frame work of string theory. The solution shows that the energy and mass are quantized, as shown by equation (4.8.15) and (4.8.16).

4.10 Conclusion:

The concept of force in farms of kinetic and potential energy can lead to energy conservation with in framework of GSR and SR. The new SR quantum based on energy expression including matter energy and potential energy beside friction is promising. It shows that the mass is quantized within the frame work of string theory, the E conservation requires.

4.11 Recommendation:

This work can be extend for future in many respects

1. The conservation of SR energy can be extended to include momentum also.

2. The new SR energy expressions can used for many applications specially that are related to particle physics.

3. The harmonic oscillator solutions can be used to develop strings theory.