

## **Dedication**

To my Family , Friends and colleagues.

## **Acknowledgments**

At first I would like to thank Allah who gives me the ability to complete this work.

I would like to express my deep thanks to my supervisor Prof.Dr.Shawgy HusseinAbd Alla for his great effort and help. My thanks to Sudan University of Science and Technology, for permission to use the facilities required.

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## Abstract

We study the nonparametric function estimation involving time series and change-point estimation. We show the cusum test for parameter changes in GARCH (1, 1), and time series models with cusum of squares test for variance change in nonstationary and nonparametric time series models. We also study the weak convergence of the sequential empirical processes of residuals in ARMA models. Endpoint and moving estimates for the circular maximal function, test with time varying bandwidth and for an oscillatory integral operator related to restriction to space curves are considered. We establish the application of bilinear approach to cone multipliers and restriction estimates for surface with curvatures of different signs. We discuss the Fourier and weighted restriction theorems for curves in  $\mathbf{R}^3$  and space curves. Similarly we investigate the restriction theorems and estimates for a surface with negative curvature and some surfaces with vanishing curvatures.

## الخلاصة

تم دراسة تقدير الدالة غير الوسيطة محتوية متسلسلة الزمن وتقدير تغير - النقطة. أوضنا اختبار كيسيم لتغيرات الوسيط في (1, 1) GARCH ونماذج متسلسلة الزمن مع كيسيم لاختبار المربعات لاجل تغير التباين في نماذج متسلسلة الزمن غير الثابتة وغير الوسيطة. أيضاً درسنا التقارب الضعيف للعمليات التجريبية التسلسلية للمتبقيات في نماذج ARMA. تم إعتبار تقديرات نقطة النهاية والمتحركة للدالة الأعظمية الدائرية والإختبار مع سعة التوحد متغير الزمن ولجل مؤثر التكامل التذبذي ذو العلاقة إلى القصر إلى منحيات الفضاء. تم تأسيس التطبيق للمقاربة ثنائية الخطية إلى مضاعفات المخروط وتقديرات القصر للسطح مع الانحناءات للعلامات المختلفة. درسنا مبرهنات فورير والقصر المرجحة للمنحنيات في  $R^3$  ومنحنيات الفضاء. وبالمثل ناقشنا مبرهنات القصر والتقديرات لاجل السطح مع الانحناء السالب وبعض السطوح مع الانحناءات المتلاشية.

## Introduction:

Consider a stationary time series  $(\mathbf{X}_t, Y_t), t = 0, \pm 1, \dots$ , with  $\mathbf{X}_t$  being  $\mathbb{R}^d$ -valued and  $Y_t$  real-valued. The conditional mean function is given by  $\theta(\mathbf{X}_0) = E(Y_0|\mathbf{X}_0)$ . The conditional mean function is given by  $\theta(\mathbf{X}_0) = E(Y_0|\mathbf{X}_0)$ . Under appropriate regularity conditions, a local average estimator of this function based on a finite realization  $(\mathbf{X}_1, Y_1), \dots, (\mathbf{X}_n, Y_n)$  can be chosen to achieve the optimal rate of convergence  $n^{-1/(2+d)}$  both pointwise and in  $L_2$  norms restricted to a compact; and it can also be chosen to achieve the optimal rate of convergence  $(n^{-1} \log(n))^{1/(2+d)}$  in  $L_\infty$  norm restricted to a compact.

We consider the problem of testing parameter constancy in GARCH (1, 1) models. A cusum of squares test is proposed in analogy of Inclán and Tiao (1994) 's statistic. Consider a sequence of independent random variables  $\{X_i: 1 \leq i \leq n\}$  having cdf  $F$  for  $i \leq \theta n$  and cdf  $G$  otherwise. A class of strongly consistent estimators for the change-point  $\theta \in (0, 1)$  is proposed. The estimators require no knowledge of the functional forms or parametric families of  $F$  and  $G$ . Furthermore,  $F$  and  $G$  need not differ in their means (or other measure of location). The only requirement is that  $F$  and  $G$  differ on a set of positive probability.

We study the weak convergence of the sequential process  $\hat{K}_n$  of the estimated residuals in ARMA(p, q) models when the errors are independent and identically distributed. We consider the problem of endpoint estimates for the circular maximal function defined by

$$Mf(x) = \sup_{1 < t < 2} \left| \int_S f(x - ty) d\sigma(y) \right|$$

where  $d\sigma$  is the normalized surface area measure on  $S$ .

We continue the study of three-dimensional bilinear restriction and Keakeya estimates which was initiated. In particular, we give new linear and bilinear restriction estimates for the cone, sphere, and paraboloid in  $R^3$ , building upon and unifying previous work in this direction by Bourgain, Wolff, and others. This is a continuation of [99], in which new bilinear estimates for surfaces in  $R^3$  were proven. We give a concrete improvement to the square function estimate of Mockenhaupt. We consider the problem of testing for parameter changes in time series models based on a cusum test. Although the test procedure is well established for the mean and variance in time series models, a general parameter case has not been discussed in the literature.

We consider the problem of testing for a variance change in nonstationary and nonparametric time series models. The models under consideration are the unstable AR(q) model and the fixe design nonparametric regression model with a strong mixing error process. We consider the problem of testing for parameter changes in time series models on a moving estimates (ME) test. It is widely accepted that detecting some changes, for instance, those caused by temporary parameter Shifts by the exiting cusum test is difficult.

We obtain estimates for restriction of the Fourier transform to certain curves in  $R^3$ . We consider the oscillatory integral operator defined by

$$T_\lambda f(x) = \int_{\mathbb{R}} e^{i\lambda\phi(x,t)} a(x,t) f(t) dt$$

where  $\lambda > 1$ ,  $a \in C_c^\infty(\mathbb{R}^n \times \mathbb{R})$  and  $\phi$  is a real -valued function in  $C^\infty(\mathbb{R}^n \times \mathbb{R})$ . This operator may be thought of as a variables-curve version of the adjoint of the Fourier restriction operator for space curves. Consider a nondegenerate  $C^n$  curve  $\gamma(t)$  in  $\mathbb{R}^n$ ,  $n \geq 2$ , such as the curve  $\gamma_0(t) = (t, t^2, \dots, t^n)$ ,  $t \in I$ , where  $I$  is an interval in  $\mathbb{R}$ . We first show a weighted Fourier restriction theorem for such curves, with a weight in a Wiener amalgam space, for the full range of exponents  $p, q$ , where  $I$  is a finite interval. Next, we obtain a generalization of this result to some related oscillatory integral operators. In particular, our results suggest that this is a quite general phenomenon which occurs, for instance, when the associated oscillatory integral operator acts on functions  $f$  with a fixed compact support. We show a bilinear restriction theorem for a surface of negative curvature. This is the analogue of the results of T. Wolff and T. Tao for cones and paraboloids. We generalize the bilinear restriction estimates to surfaces with curvatures of different signs. We obtain bilinear restriction estimates for surfaces with vanishing curvatures.

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