CHAPTR TWO

Literature review

This chapter focuses on the conceptual framework, theoretical background, estimation procedures and the empirical models of the stochastic production frontier (SPF) and linear programming (LP).

2.1 Definition and Measures of Efficiency

2.1.1 Efficiency Concept

Efficiency is a very loose term indeed; to an engineer efficiency may mean the ratio of output / input or output/ theoretical capacity, percent. While the cost account use the ratio standard cost / actual cost, percent, or its inverse to measure the productive efficiency of a firm. The economist, when he refers to the efficiency of a firm generally means one of two ratios, the first concerns the firm's success in producing as large as possible an output from a given set of inputs; or what amounts to the same thing, producing a given output with the least inputs; this called productivity, or technical efficiency (Amey; 1969).

2.1.2 Production Efficiency

Production efficiency refers to a firm´s costs of production and can be applied both to the short and long run. It is achieved when the output is produced at minimum average total cost (ATC). For example we might consider whether a business is producing close to the low point of its long run average total cost curve. When this happens the firm is exploiting most of the available economies of scale. Productive efficiency exists when producers minimize the wastage of resources in their production processes (Tutor2u, 2006).

Rahman, 2002 cited that productive efficiency has two components. The purely technical, or physical, component refers to the ability to avoid waste by producing as much output as input usage allows, or by using as little input as output production allows.

Rahman, 2002 stated that production efficiency is one of the three conditions necessary for an economy to be economically efficient is that it be on its production-possibilities frontier. If it is not on the production-possibilities frontier, more could be produced with the given resources and technology. Because greater production would increase value, any position below the production-possibilities frontier is inefficient. Notice that a great many points satisfy this condition of production efficiency every point on the production-possibilities frontier is production efficient.

To be on production possibilities frontier, all resources must be used. Unemployed resources indicate that more goods and services could be produced, which means that the economy was not on the frontier initially. In addition, resources must be used properly.

2.1.3 Production Possibility Frontier

The Production Possibilities Frontier (PPF) shows the maximal combinations of two goods that can be produced during a specific time period given fixed resources and technology and making full and efficient use of available factor resources. A PPF is normally drawn as concave to the origin because the extra output resulting from allocating more resources to one particular good may fall. This is known as the law of diminishing returns and can occur because factor resources are not perfectly mobile between different uses, for example, re-allocating capital and labour resources from one industry to another may require re-training, added to a cost in terms of time and also the financial cost of moving resources to their new use.

To be on the production-possibilities frontier, all resources must be used. Unemployed resources indicate that more goods and services could be produced, which means that the economy was not on the frontier initially. In addition, resources must be used properly. If society randomly assigns people to jobs or if it assigns jobs on the basis of political reliability, it will not produce as much as it could. It will require some people with little intellectual ability to perform jobs that require great intellectual ability, and it will require some people with little strength and endurance to perform jobs that demand much strength and endurance. If switching people among jobs can increase output, the original situation was not on the production-possibilities frontier and thus not economically efficient (Rahman, 2002).

2.1.4 Economic Efficiency

Economic efficiency is a general term in economics describing how well a system is performing, in generating the maximum desired output for given inputs with available technology. Efficiency is improved if more output is generated without changing inputs, or in other words, the amount of "friction" or "waste" is reduced. Ahmed, (2004) cited that the measure of firm efficiency consists of two components: technical efficiency, which reflects the ability of a firm to obtain the maximal output from a given set inputs, and the allocative efficiency, which reflects the ability of a firm to use the inputs in optimal proportion, given their respective price. These two measures combined to provide a measure of the total economic efficiency.

Economic Efficiency = *Technical Efficiency x Allcocative Efficiency*

Economic efficiency is used to refer to a number of related concepts. A system can be called economically efficient if:

* No one can be made better off without making someone else worse off.

* More output cannot be obtained without increasing the amount of inputs.

* Production proceeds at the lowest possible per unit cost.

These definitions of efficiency are not exactly equivalent. However, they are all encompassed by the idea that nothing more can be achieved given the resources available.

An economic system is more efficient if it can provide more goods and services for society without using more resources. Market economies are generally believed to be more efficient than other known alternatives. The first fundamental welfare theorem provides some basis for this belief, as it states that any perfectly competitive market equilibrium is efficient (but only if no market imperfections exist).

Microeconomic reforms is policies that aim to reduce economic distortions, and increase economic efficiency. However, there is no clear theoretical basis for the belief that removing a market distortion will increase economic efficiency. The Theory of the Second Best states that if there is some unavoidable market distortion in one sector, a move toward greater market perfection in another sector may actually decrease efficiency.

There are several alternate criteria for economic efficiency, these include:

- Pareto efficiency
- Kaldor-Hicks efficiency
- X-efficiency

- Allocative efficiency
- Distributive efficiency
- Productive efficiency
- Optimization of a social welfare function
- Utility maximization

(http://www.economicsentwork.ac.uk/copy right.html).

2.1.5 Allocative and Technical Efficiency

Technical efficiency is just one component of overall economic efficiency. However, in order to be economically efficient, a firm must first be technically efficient. Profit maximization requires a firm to produce the maximum output given the level of inputs employed (i.e. be technically efficient), use the right mix of inputs in light of the relative price of each input (i.e. be input allocative efficient) and produce the right mix of outputs given the set of prices (i.e. be output allocative efficient) (Kumbhaker and Lovell 2000). These concepts can be illustrated graphically using a simple example of a two input (x_1, x_2)-two output (y_1, y_2) production process (Figure 2.1). Efficiency can be considered in terms of the optimal combination of inputs to achieve a given level of output (an input-orientation), or the optimal output that could be produced given a set of inputs (an output-orientation).

In Figure 2.1(a), the firm is producing a given level of output (y_1^*, y_2^*) using an input combination defined by point A. The same level of output could have been produced by radially contracting the use of both inputs back to point B, which lies on the isoquant associated with the minimum level of inputs required to produce (y_1^*, y_2^*) (i.e. Iso (y_1^*, y_2^*)). The input-oriented level of technical efficiency ($TE_I(y, x)$) is defined by 0B/0A. However, the least-cost combination of inputs that produces (y_1^*, y_2^*) is given by point C (i.e. the point where the marginal rate of technical substitution is equal to the input price ratio w_2/w_1). To achieve the same level of cost (i.e. expenditure on inputs), the inputs would need to be further contracted to point D. The cost efficiency (CE(y, x, w)) is therefore defined by 0D/0A. The input allocative efficiency (AE_I(y, w, w)) is subsequently given by CE(y, x, w) / TE_I(y,x), or 0D/0B in Figure 2.1(a) (Kumbhaker and Lovell 2000).

The production possibility frontier for a given set of inputs is illustrated in Figure 1(b) (i.e. an output-orientation). If the inputs employed by the firm were used efficiently, the output of the firm, producing at point A, can be expanded radially to point B. Hence, the output oriented measure of technical efficiency (TE_O(y, x)), can be given by 0A/0B. This is only equivalent to the input-oriented measure of technical efficiency under conditions of constant returns to scale. While point B is technically efficient, in the sense that it lies on the production possibility frontier, higher revenue could be achieved by producing at point C (the point where the marginal rate of transformation is equal to the price ratio p_2/p_1). In this case, more of y_1 should be produced and less of y_2 in order to maximize revenue. To achieve the same level of revenue as at point C while maintaining the same input and output combination, output of the firm would need to be expanded to point D. Hence, the revenue efficiency (RE(y, x, p)) is given by 0A/0D. Output allocative efficiency (AE_O(y, w, w)) is given by RE(y, x, w)/TE_I(y, x), or 0B/0D in Figure2. 1(b) (Kumbhaker and Lovell 2000).

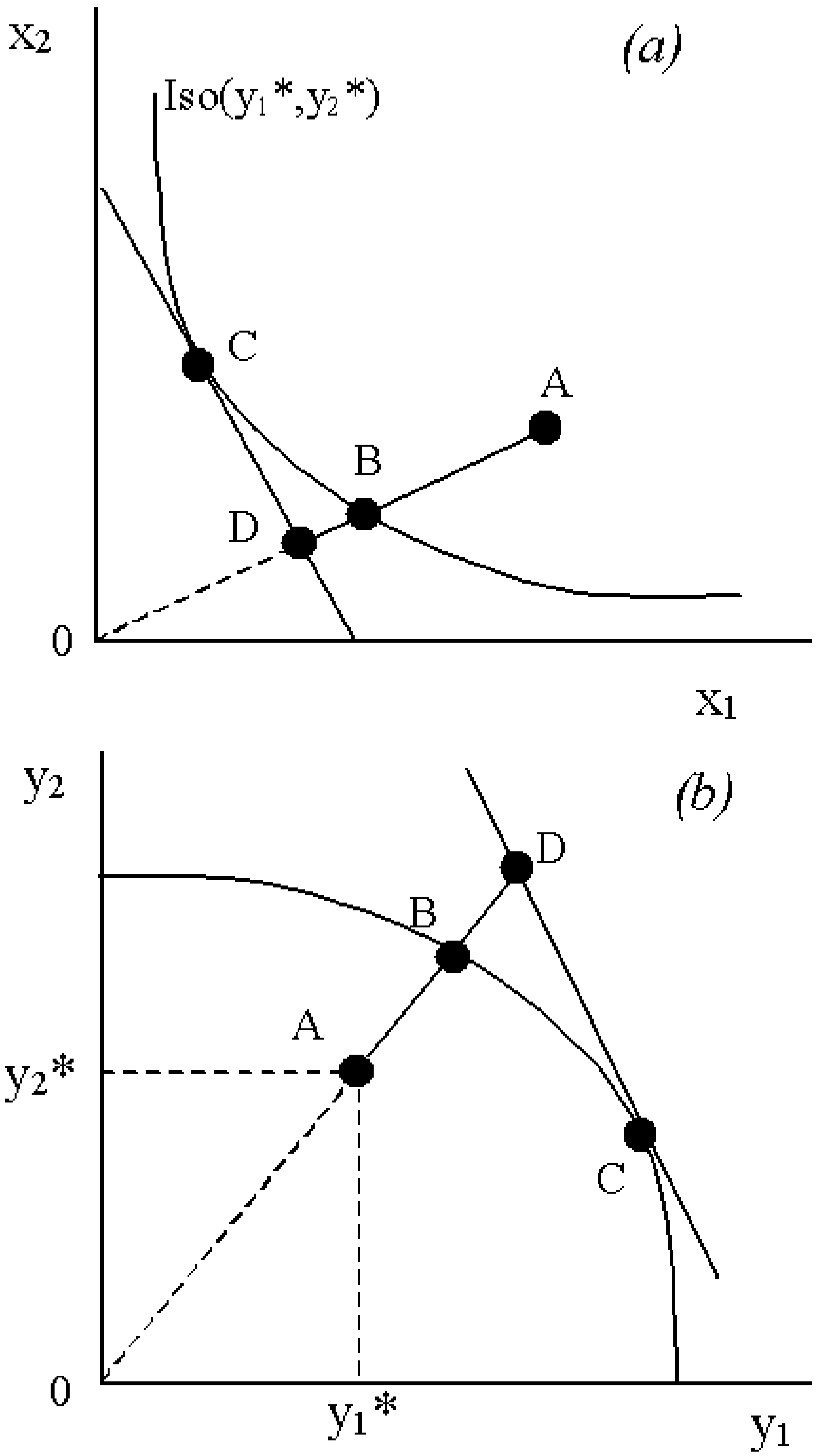

Figure2. 1: Input (a) and output (b) oriented efficiency measure

2.1.6 Stochastic production frontier (SPF)

Farrell´s, (1957) seminal article has led to development of several techniques for the measurement of efficiency of production. These techniques can be broadly categorized into two approaches: parametric and non-parametric. The parametric stochastic frontier production function approach non-parametric mathematical programming approach, commonly referred to as data envelopment analysis (DEA) are the two most popular techniques used in efficiency analysis. The main strengths of the stochastic frontier approach are that it deals with stochastic noise and permits stochastic tests of hypotheses pertaining to production structure and the degree of inefficiency (Sharma el al, 1999). www.en.wikipedia.org/wiki/stochasticfrontier analysis. Stochastic frontier production functions (SFPF) have been the subject of considerable econometric research during the past two decades, originating with a general discussion of the nature of inefficiency in Farrel, (1957). In traditional economic theory, efficiency is generally assumed as an outcome of price-taking, competitive behavior. In this context (and assuming no uncertainty), a production function shows the maximum level of output that can be obtained from given inputs under the prevailing technology. However, variation in maximum output can also occur either as a result of stochastic effects (e.g., good and bad weather states), or from the fact that firms in the industry may be operating at various levels of inefficiency due to mismanagement, poor incentive structures, less than perfectly competitive behavior or inappropriate input levels or combination. The econometric technique developed by Battese and Coelic (1998), www.unedu.an/staff/g Battese, allows for a decomposition of these effects and precise measure of technical inefficiency defined by the ratio of observed output to the corresponding (estimated) maximum output defined by the frontier production function, given inputs and stochastic variation (kompas,2001). The stochastic production frontier (Aginer, Lovell, and Schmidt (1977), Battese and Corra (1977) and Meesusen and Van den Broeck (1977)) is motivated by the idea that deviations from the production frontier may not be entirely under the control of the production unit under study. These models allow for technical inefficiency, but they also acknowledge the fact that random shock outside the control of producers can affect output. They account for measurement error and other factors, such as effects of weather, luck, etc, on value of the output variable, together with combined effects of unspecified input variables in the production function. The main virtue of stochastic frontier models is that at least in principle these effects can be separated from the contribution of variation in technical inefficiency (Kebede, 2001). Rahman, (2002) stated that several methods have been developed for the empirical estimation of the frontier models. These different methods to estimate the frontier efficiency models can be categorized according to:

(a) The way the frontier is specified: the frontier may be specified as parametric function of inputs or as deterministic nonparametric function. The main distinguishing characteristics of the parametric frontier is the assumption of an explicit function from the given technology and thus the frontier is expressed in a mathematical from. Nonparametric is not based on any explicit model of the frontier or of the relationship of the observations to the frontier (Forsund, et al., 1980).

(b) The frontier may be estimated either through programming techniques or through the explicit use of statistical procedures;

(c) The deviation from the frontier is interpreted; deviations may be interpreted simply as inefficiencies or they could be treated as mixtures of inefficiency and statistical noise; that is, frontier may be deterministic or stochastic;

(d) The frontier is optimized (dual approach); the frontier may be production frontier or cost frontier.

Stochastic frontier production function was thereafter developed to overcome the deficiency (Ogundari and Ojo, 2006). The frontier production function model is estimated using maximum likelihood procedures. This is because it is considered to be asymptotically more efficient than the corrected ordinary least square estimators (Coelli, 1995), (Battese and Coelli, 1995), www.springerlink.com/index/h5x6j80852428mp1. The maximum likelihood estimates for all the parameters of the stochastic frontier and inefficiency model, defined by equation simultaneously obtains by using the programme, FRONTIER VARTION4.1, which estimates variance parameters in terms of the parameterization.

2.1.7 The stochastic production frontier with the Cob-Douglas production function

The Cob-Douglas production function is probably the most widely used form for fitting agricultural production data, because of its mathematical properties, ease of interpretation and computational simplicity (Heady and Dillon, 1969, Fuss et al, 1978). The Cob-Douglas production function has convex isoquants, but it has unitary elasticity of substitution, it does not allow for technically independent or competitive factors, nor does it allow for stage I and III along with stage II. That is MPP and APP are monotonically decreasing function for all X- the entire factor – factor space is stage II given $0<b<1$, which is the usual case. However, the Cob-Douglas may be a good approximation for the production processes for which factors are imperfect substitutes over the entire range of inputs values. Also, the Cob-Douglas is easy to estimate because, in logarithmic form, its linear in parameters, its parsimonious in parameters (Beattie and Taylor, 1985).

A stochastic Cob-Douglas production frontier model may be written as:

$$Y_i = f(X_iB)\ \exp.(V_i - U_i) \qquad I = 1,2,\ldots\ldots..N$$

Where the stochastic production frontier is (X_iB) exp. (V_i), V_i having some symmetric distribution to capture the random effects of measurement error and exogenous stocks which cause the placement of the deterministic kernel (X_iB) to vary across firm. The technical inefficiency relative to the stochastic production frontier is then captured by the one side error component $U_i \geq 0$. The explicit form of the stochastic Cob-Douglas production frontier is given by:

$$\mathbf{yi = \beta 0 + \sum_{j=1}^{n} \beta j \ln Xij + Vi - Ui}$$

Where y_i is the frontier output, β_0 is intercept, β_j the elasticity of y_i with respect to X_{ij}, X_{ij} is the physical input, V_i-U_i a composed error.

2.1.8 FRONTIER 4.1

FRONTIER 4.1 has been created specifically for the estimation of production frontiers. As such, it is a relatively easy tool to use in estimating stochastic frontier models. It is flexible in the way that it can be used to estimate both production and cost functions, can estimate both time-varying and invariant efficiencies, or when panel data is available, and it can be used when the functional forms have the dependent variable both in logged or in original units.

FRONTIER offers a wide variety of tests on the different functional forms of the models that can be conducted easily by placing restrictions on the models and testing the significance of the restrictions using the likelihood ratio test. The

FRONTIER program is easy to use. A brief instruction file and a data file have to be created. The executable file and the start-up file can be downloaded from the Internet free of charge at the CEPA http://www.uq.edu.au/economics/cepa/frontier.htm.

2.2 Linear Programming (LP) in brief:

2.2.1 Introduction:

From an application perspective, mathematical (and therefore, linear) programming is an optimization tool, which allows the rationalization of many managerial and/or technological decisions required by contemporary techno-socio-economic applications .

2.2.2 Definitions of LP:

Gass (1964) stated that, programming is concerned with the efficient use or allocation of limited resources to meet desired objectives . Heady and Candler (1973) defined Linear programming as an efficient way of determining optimum plans only if there are numerous enterprises or processes and numerous restrictions in attaining a specific objective such as maximizing farm profits or minimizing production costs. Bazaraa and Jarvis (1977) see a linear programming problem as a problem of minimizing or maximizing a linear function in the presence of linear constraints of the inequality and / or the equality type. Another definition reported by Dent, Harrison and Woodford (1986) is that linear programming is one of a class of operations research methods referred to as mathematical programming; the linear programming technique is a general methodology that can be applied to a wide range of problems. While Hazell and Norton (1986) see linear programming as a method of determining a profit maximization combination of farm enterprises that are feasible with respect to a set of fixed farm constraints. Mohamed (1986) reported that, LP provides a means to find the level of decision variable(s) that would maximize the objective function subject to a set of constraints. A linear programming problem is a special case of a mathematical programming problem. From an analytical perspective, a mathematical program tries to identify an extreme (minimum or maximum) point of a function which furthermore satisfies a set of constraints i.e., linear programming is the specialization of mathematical programming to the case where both, function and the problem constraints are linear (Kourouma, 1982).

2.2.3 Assumptions of LP:

Several assumptions are used in linear programming. If these assumptions do not apply to the problem under consideration, linear programming may not provide a sufficient precise solution. These assumptions are explained below:

- **Additivity and linearity:**

The activities must be additive in the sense that when two or more are used, their total product must be the sum of their individual products. An equivalent statement is that, the total amount of resources used by several enterprises must be equal to the sum of the resources used by each individual enterprise. Thus no interaction is possible in the amount of resources required per unit of output regardless of whether activities are produced alone or in various proportions.

- **Divisibility:**

  It is assumed that factors can be used to produce commodities that can be produced in quantities which are fractional units. That is, resources and products are considered to be continuous to be infinitely divisible.

- **Finiteness:**

  It is assumed that there is a limit to the number of alternative activities and to the resource restrictions which need to be considered.

- **Single-value expectations:**

  In general, the linear programming method used widely to date employs the standard linear programming assumption that resources supplies, input-output coefficients, and prices are known with certainty (Heady and Candler, 1973).

Other assumptions summarized by Hazel and Norton (1986) are:

- **Optimization:**

  It is assumed that an appropriate objective function is either maximized or minimized.

- **Fixedness:**

  At least one constraint has a non-zero right hand side coefficient.

- **Homogeneity:**

  It is assumed that all units of the same resource or activity are identical.

- **Proportionality:**

  The gross margin and resource requirements per unit of activity are assumed to be constant regardless of the level of the activity used.

2.2.4 Why use LP:

The great advantage of programming is that it allows one to test a wide range of alternative adjustments and to analyze their consequences thoroughly with a small input of managerial time (Beneke and Winterboer, 1973). Linear programming is a powerful tool of analysis which can be used to look at several budgets of a farm at a time and depict the optimal enterprises in a profit-maximization or cost minimization context (Kourouma, 1982) . Bazaraa and Javis (1977) emphasize that the simplex method of linear programming enjoys wide acceptance because of 1) its ability to model important and complex management decision problems and 2) its capability for producing solutions in a reasonable amount of time. Malik (1994) sees the most important advantages of linear programming is the flexibility in stating objectives that will satisfy the consumption requirements of the household. Furthermore, the by-product of the solution provides rich information on economic issues like shadow prices and average productivities. One should be careful in utilizing linear programming results in explaining farmers behavior, because of the normative nature of LP analysis and due to its dependence on the degree of accuracy of the coefficients and assumptions which were used in the model formulation. Nevertheless, LP still provides an essential indicator of the degree to which farmers are market-oriented and gives an adequate analysis of input-output relationships (Malik, 1994) .

2.2.5 Limitations of the LP model:

The LP technique suffers from several limitations which can be stated as follows:

1) Programming cannot help the manager in the difficult task of formulating price expectations.

2) Activities that involve decreasing costs cannot be treated adequately with programming methods.

3) Restraints are sometimes difficult to specify.

4) LP is of little help in estimating input-output relationship. It can only specify data needed but the planning must supply estimates of the amount and distribution of labour, land and capital needed to produce the crop. Estimates of such types are difficult to make.

5) LP proceeds as if the price and input-output expectations we have formulated were equally reliable for all farm products and the result is that farms treated as they were equally without risk i.e. risk preference of the operator does not taken into consideration.

6) One of the assumptions of the LP is that each additional unit of the output requires the same quantity of the input. But if you recall the law of diminishing return to scale, the amount of crop output declines as more fertilizer is used per feddan.

2.2.6 Application of linear programming technique in the agricultural sector.

There are several individuals who contributed to the development of linear programming among them Von Neumann, Leontief, Laplace, Weyl, Wood Stigeler, Cornfield, Koopmans and Dantzing Abdel aziz, 1999. Abdala (2005) mentioned that many researchers in the world applied linear programming in the last years among them Majmder (1998), Darwish et al (1999), Neto et al (1997), Salinas et al (1999), Pennel (1999), Frizzone et al (1997), Kassie et al (1998), Goswami (1997) and Zahoor(1997). On the other hand, Heyer (1996), Delgado(1979), Schultz(1964) and Metson(1978) contributed to the application of linear programming in African agriculture (Abdelaziz, 1999).

Linear programming was applied also in the agricultural sector of Sudan by some researchers among them Abdelaziz (1999), Ahmed (1988), Brima (2004), Elbadawi (1990) and Ahmed (2005). Faki and Ahmed (1992 and 1994) applied linear programming for investigating the prospects of technology in small pump scheme in Wad Hamid and Rubatab areas in the River Nile State (Abdelaziz, 1999).

Elbadawi (1990) and Abdelraouf (2010) applied linear programming method in New Halfa Agricultural Production Corporation for the following reasons and justifications which are similar to reasons that justify application of LP technique in this study:

1- LP is suitable to examine constraints of production and the behavior of the farmers .

2- Homogeneity of the farming in the area of study.

3- Studying farm-income and crop combination of varying resource.