

# CHAPTER FOUR

## SCALAR CONTROL METHOD

### 4.1 Two-Inductance Equivalent Circuits Of The Induction Motor:

As a background for scalar control methods, it is convenient to use a pair of inductance per-phase equivalent circuits of the induction motor. They differ from the three-inductance circuit introduced in Section 2.8, which can be called a *T-model* because of the configuration of inductances. Introducing the transformation coefficient  $\gamma$  given in [13][7]:

$$\gamma = \frac{X_s}{X_r} \quad (4.1)$$

The T-model of the induction motor can be transformed into the so-called  $\Gamma$ -model, shown in Figure 4.1. Components and variables of this equivalent circuit are related to those of the T-model as follows [13]:

1. Rotor resistance (referred to stator),

$$R_R = \gamma^2 R_r \quad (4.2)$$

2. Magnetizing reactance,

$$X_M = \gamma X_m = X_s \quad (4.3)$$

3. Total leakage reactance.

$$X_L = \gamma X_{ls} + \gamma^2 X_{lr} \quad (4.4)$$

4. Rotor current referred to stator.

$$I_R = \frac{I_r}{\gamma} \quad (4.5)$$

5. Rotor flux.

$$\Lambda_R = \gamma \Lambda_r \quad (4.6)$$

The actual angular frequency,  $\omega_r$ , of currents in the rotor the induction motor is given by:

$$\omega_r = s\omega \quad (4.7)$$

This frequency, subsequently called *rotor frequency*, is proportional to the slip velocity,  $\omega_{sl}$ , as

$$\omega_r = p_p \omega_{sl} \quad (4.8)$$

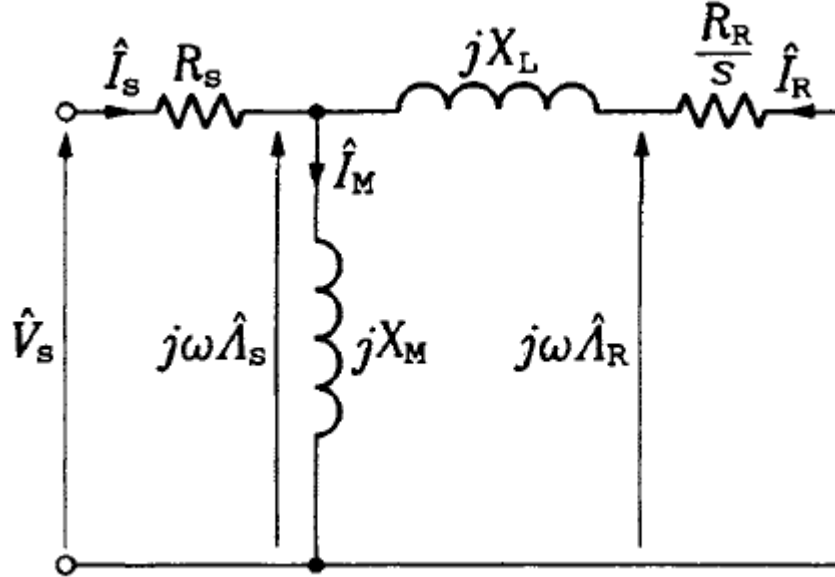


Fig 4.1: The  $\Gamma$  equivalent circuit of the induction motor

Taking into account that  $R_R/s = R_R \omega/\omega_r$ , current  $I_R$  in the  $\Gamma$ -model can be expressed as:

$$I_R = |\hat{I}_R| = \frac{A_s}{R_R} \frac{\omega_r}{\sqrt{(\tau_\Gamma \omega_r)^2 + 1}} \quad (4.9)$$

where  $\tau_\Gamma = L_L/R_R$ . Symbol  $L_L$  denotes the total leakage inductance ( $L_L = X_L/\omega$ ). The electrical power,  $P_{elec}$  consumed by the motor is:

$$P_{elec} = 3R_R \frac{\omega}{\omega_r} I_R^2 \quad (4.10)$$

and the mechanical power,  $P_{mech}$ , can be obtained from  $P_{elec}$  by subtracting the resistive losses,  $3R_R I_R^2$ . Finally, the torque,  $T_M$ , developed in the motor can be calculated as the ratio of  $P_{mech}$  to the rotor angular velocity,  $\omega_M$  which is given by:

$$\omega_M = \frac{\omega - \omega_r}{p_p} \quad (4.11)$$

The resultant formula for the developed torque is:

$$T_M = 3p_p \frac{\Lambda_s^2}{R_R} \frac{\omega_r}{(\tau_r \omega_r)^2 + 1} \quad (4.12)$$

Another two-inductance per-phase equivalent circuit of the induction motor, called an *inverse- $\Gamma$*  and  $\Gamma'$ -model, is shown in Figure 4.2. The coefficient,  $\gamma'$  for transformation of the T-model into the  $\Gamma'$ -model is given by:

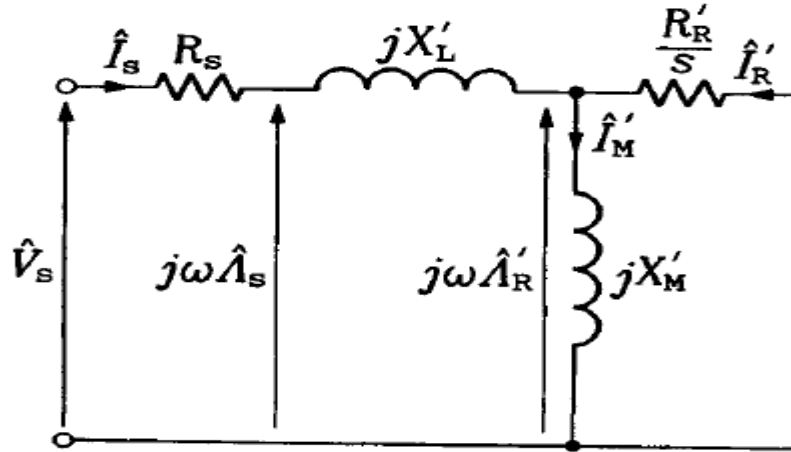


FIGURE 4.2: The  $\Gamma'$  equivalent circuit of the induction motor.

$$\gamma' = \frac{X_m}{X_r}, \quad (4.13)$$

and the rotor resistance, magnetizing reactance, total leakage reactance, rotor current, and rotor flux in the latter model are:

$$R'_R = \gamma'^2 R_r \quad (4.14)$$

$$X'_M = \gamma' X_m \quad (4.15)$$

$$X'_L = X_{ls} + \gamma' X_{lr} \quad (4.16)$$

and

$$I'_R = \frac{I_r}{\gamma'} \quad (4.17)$$

$$\Lambda'_R = \gamma' \Lambda_r \quad (4.18)$$

respectively.

The electrical power is given by an equation similar to Eq. (4.10), that is :

$$P_{\text{elec}} = 3R'_R \frac{\omega}{\omega_r} I_R'^2 \quad (4.19)$$

and the developed torque can again be calculated by subtracting the resistive losses and dividing the resultant mechanical power by the rotor velocity. This yields:

$$T_M = 3p_p R'_R \frac{I_R'^2}{\omega_r} \quad (4.20)$$

which, based on the  $T'$  equivalent circuit, can be rearranged to:

$$T_M = 3p_p L'_M I'_R I'_M \quad (4.20)$$

where  $L'_M = X'_M/\omega$  denotes the magnetizing inductance in the  $\Gamma'$ -model.

## 4.2 Open-Loop Scalar Speed Control (Constant Volts/Hertz)

Analysis of Eq. (4.12) leads to the following conclusions[13]:

1. If ( $\omega_r = 1/\tau_\Gamma$ ) then the maximum (pull-out) torque,  $T_{M,\text{max}}$ , is developed in the motor. It is given by [13]:

$$T_{M,\text{max}} = 1.5p_p \frac{\Lambda_s^2}{L_L} \quad (4.22)$$

and the corresponding critical slip,  $s_{cr}$  is

$$s_{cr} = \frac{1}{\tau_\Gamma \omega} \quad (4.23)$$

2. Typically, induction motors operate well below the critical slip, so that  $\omega \ll 1/\tau_\Gamma$ . then,  $(\tau_\Gamma \omega_r)^2 + 1 \approx 1$  and the torque is practically proportional to  $\omega_r$ . For a stiff mechanical characteristic of the motor, possibly high flux and low rotor resistance are required.

3. When the stator flux is kept constant, the developed torque is independent of the supply frequency,  $f$ . On the other hand, the speed of the motor strongly depends on  $f$ . It must be stressed that Eq. (4.12) is only valid when the stator flux is kept constant, independently of the slip. In practice, it is usually the stator voltage that is constant, at least when the supply frequency does not change. Then, the stator flux does depend on slip, and the critical slip is different from that given by Eq. (4.23). Generally, for a given supply frequency, the mechanical characteristic of an induction motor strongly depends on which motor variable is kept constant.

Assuming that the voltage drop across the stator resistance is small in comparison with the stator voltage, the stator flux can be expressed as:

$$\Lambda_s = \frac{V_s}{\omega} = \frac{1}{2\pi} \frac{V_s}{f} \quad (4.24)$$

Thus, to maintain the flux at a constant, typically rated level, the stator voltage should be adjusted in proportion to the supply frequency. This is the simplest approach to the speed control of induction motors, referred to as *Constant Volts/Hertz (CVH)* method. It can be seen that no feedback is inherently required, although in most practical systems the stator current is measured, and provisions are made to avoid overloads.

For the low-speed operation, the voltage drop across the stator resistance must be taken into account in maintaining constant flux, and the stator voltage must be appropriately boosted. Conversely, at speeds exceeding that corresponding to the rated frequency,  $f_{rat}$ , the CVH condition cannot be satisfied because it would mean an overvoltage. Therefore, the stator voltage is adjusted in accordance to the following rule:

$$V_s = \begin{cases} (V_{s, rat} - V_{s, 0}) \frac{f}{f_{rat}} + V_{s, 0} & \text{for } f < f_{rat} \\ V_{s, rat} & \text{for } f \geq f_{rat} \end{cases}, \quad (4.25)$$

where  $V_{s,0}$  denotes the raised value of the stator voltage at zero frequency.

Relation (4.25) is illustrated in Figure 4.3. For the example motor,  $V_{s,0} = 40$  V. With the stator voltage so controlled, its mechanical characteristics for various values of the supply frequency are depicted in Figure 4.4.

Frequencies higher than the rated (base) frequency result in reduction of the developed torque. This is caused by the reduced magnetizing current, that is, a weakened magnetic field in the motor. Accordingly, the motor is said to operate in the *field weakening* mode. The region to the right from the rated frequency is often called the *constant power area*, as distinguished from the *constant torque area* to the left from the said frequency. Indeed, with the torque decreasing when the motor speed increases, the product of these two variables remains constant.

Note that the described characteristics of the motor can easily be explained by the

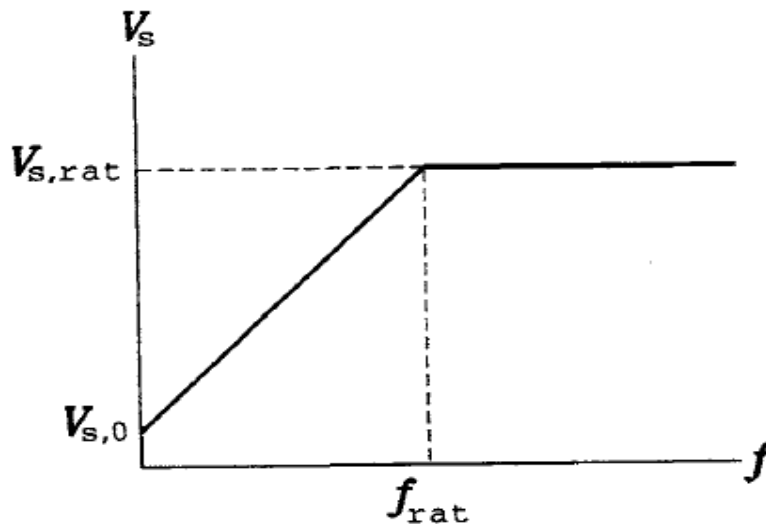


FIGURE 4.3 Voltage versus frequency relation in the CVH drives.

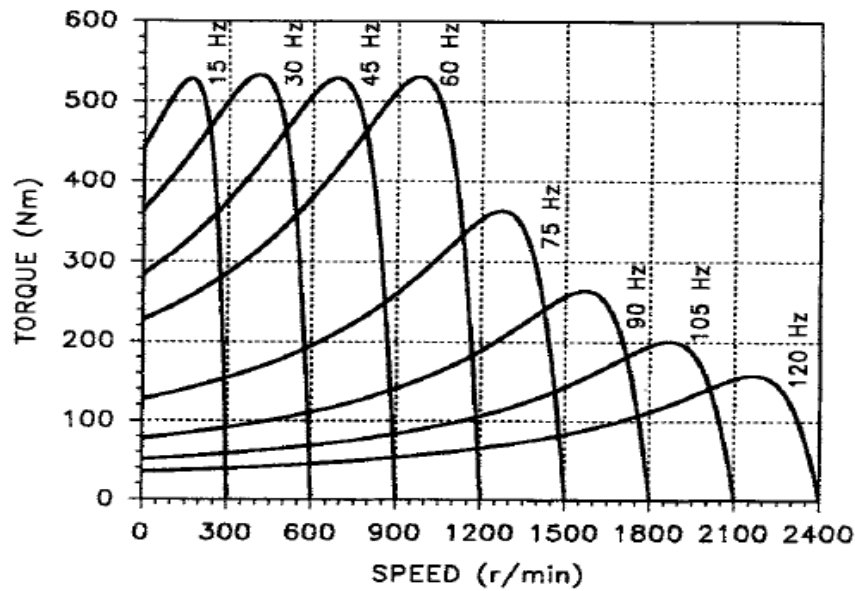


FIGURE 4.4 Mechanical characteristics of the example motor with the CVH control.

impossibility of sustained operation of an electric machine with the output power higher than rated.

A simple version of the CVH drive is shown in Figure 4.5. A fixed value of slip velocity,  $\omega_{sl}$ , corresponding to, for instance, 50% of the rated torque, is added to the reference velocity,  $\omega_M^*$ , of the motor to result in the reference synchronous frequency,  $\omega_{syn}^*$ . This frequency is next multiplied by the number of pole pairs,  $p_p$ , to obtain the reference output frequency,  $\omega^*$ , of the inverter, and it is also used as the input signal to a voltage controller. The controller generates the reference signal,  $V^*$ , of the inverter's fundamental output voltage. Optionally, a current limiter can be employed to reduce the output voltage of the inverter when too high a motor current is detected. The current,  $i_{dc}$ , measured in the dc link is a dc current, more convenient as a feedback signal than the actual ac motor current.

Clearly, highly accurate speed control is not possible, because the actual slip varies with the load of the motor. Yet, in many practical applications, such as pumps, fans, mixers, or grinders, high control accuracy is unnecessary. The basic CVH scheme in Figure 4.5 can be improved by adding slip compensation based on the measured dc-link current. The  $\omega_{sl}$  signal is generated in the slip compensator as a variable proportional to  $i_{dc}$ . A so modified drive system is shown in Figure 4.6.

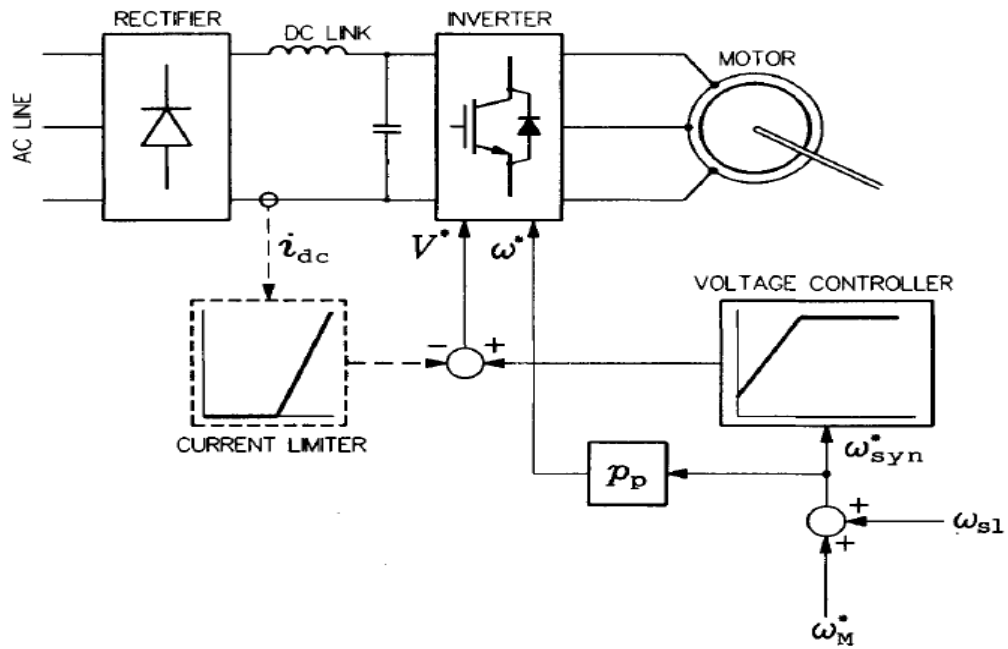


FIGURE 4.5 Basic CVH drive system.

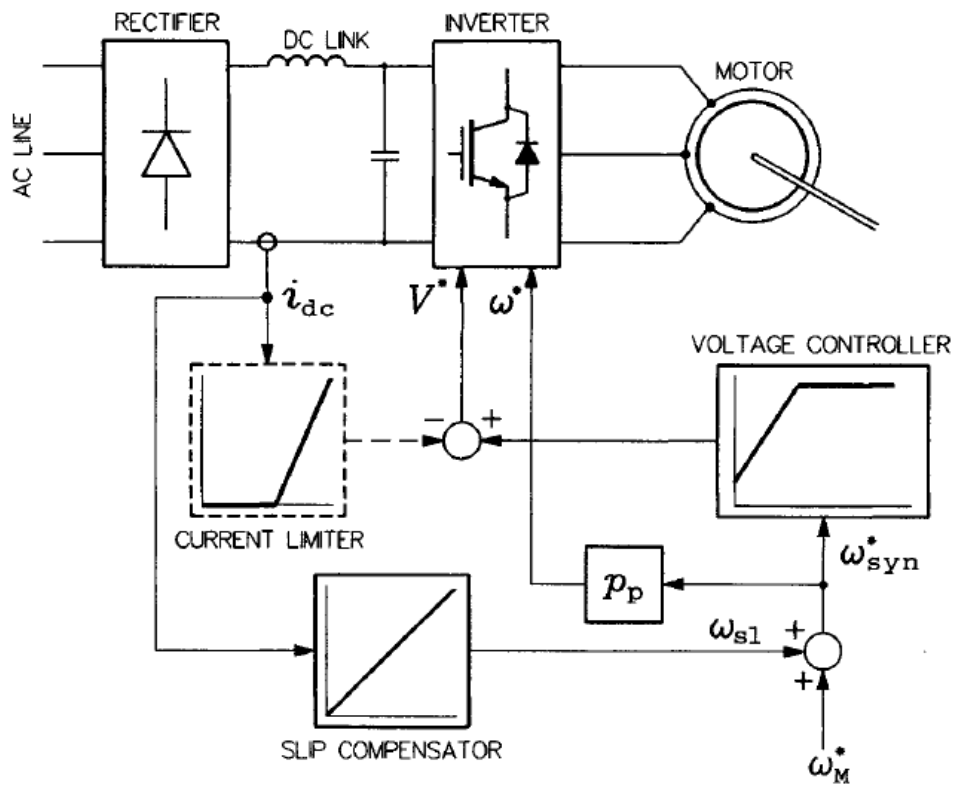


FIGURE 4.6 CVH drive system with slip compensation.



## 4.3 Summary of The Chapter:

The  $\Gamma$  and  $\Gamma'$  two-inductance steady-state equivalent circuits of the induction motor facilitate explanation of scalar speed and torque control methods. The scalar control, consisting in adjusting the magnitude and frequency of stator voltages or currents, does not guarantee good dynamic performance of the drive, because transient states of the motor are not considered in control algorithms.

In many practical applications, such performance is unimportant, and the CVH drives, with open-loop speed control, are quite sufficient. In these drives, the stator voltage is adjusted in proportion to the supply frequency, except for low and above-base speeds. The voltage drop across stator resistance must be taken into account for low-frequency operation, while with frequencies higher than rated, a constant voltage to frequency ratio would result in overvoltage. Therefore, above the base speed, the voltage is maintained at the rated level, and the magnetic field and maximum available torque decrease with the increasing frequency.