Introduction

Lie algebras are vector spaces endowed with a special non-associative multiplication called a Lie bracket. They arise naturally in the study of mathematical objects called Lie groups, which serve as groups of transformations on spaces with certain symmetries. An example of a Lie group is the group O(3) of rotations of the unit sphere $x^2 + y^2 + z^2 = 1$ in R^3. While the study of Lie algebras without Lie groups deprives the subject of much of its motivation, Lie algebra theory is nonetheless a rich and beautiful subject which will reward the physics and mathematics student wishing to study the structure of such objects, and who expects to pursue further studies in geometry, algebra, or analysis. Lie algebras, and Lie groups, are named after Sophus Lie (pronounced "lee"), a Norwegian mathematician who lived in the latter half of the 19th century. He studied continuous symmetries (i.e., the Lie groups above) of geometric objects called manifolds, and their derivatives (i.e., the elements of their Lie algebras). The study of the general structure theory of Lie algebras, and especially the important class of simple Lie algebras, was essentially completed by Elie Cartan and Wilhelm Killing in the early part of the 20th century. The concepts introduced by Cartan and Killing are extremely important and are still very much in use by mathematicians in their research today.

In physics, Emmy Noether showed that if the action of a physical system is invariant under change of coordinates, then the physical system has a conserved quantity: a quantity that remains constant for all time. Knowledge of conserved quantities can reveal deep properties of physical systems. For example, the conservation of energy is by Noether's theorem a consequence of a system's invariance under time-shifting.

Generally, Group theory become a power full method that can be used to solve PDE's, in this a thesis we take a glance of how we can use symmetry to clarify that.